A Bifurcation Analysis using Rpplane

The Rosenzweig-MacArthur model is a simple model for predator-prey population dynamics, somewhat plausible and the starting point for more realistic models. The state variables are n_1 (abundance of prey) and n_2 (abundance of predators):

$$\begin{aligned}\frac{dn_1}{dt} &= r_1 n_1(1 - n_1/K) - \frac{a_1 n_1 n_2}{B + n_1} \\ \frac{dn_2}{dt} &= \frac{a_2 n_1 n_2}{B + n_1} - d_2 n_2\end{aligned} \tag{1}$$

All parameters are positive. The first term in dn_1/dt is how the prey population grows when predators are absent, including births and deaths. $\frac{a_1 n_1 n_2}{B+n_1}$ is the rate of prey-predator encounters that result in a prey being killed and eaten. The numerator is like the βSI infection rate in the SIR model, and the denominator represents the reduction in encounter rate because predators can't hunt and eat at the same time. In dn_2/dt, the first term is conversion of food to babies, and the second is predator mortality.

The exercise is: use Rpplane to construct a bifurcation diagram for this model as a function of K, for other parameters having values $r_1 = d_2 = 1, a_1 = a_2 = 2, B = 200$. Note for any parameter values, there are two equilibria:

- $n_1 = n_2 = 0$ (nobody at all)
- $n_1 = K, n_2 = 0$ (prey only)

and for some parameter values there is also a *coexistence equilibrium* where prey and predators coexist: $n_1 > 0, n_2 > 0$. The value of K affects the existence of the coexistence equilibrium, and the stability of each equilibrium.

The script `Rpplane-RosMac.R` starts you off at $K = 100$. As you will see, $K = 100$ is a low value and nothing will change by making it even smaller. What happens as K increases? Fill in a diagram like the one below, indicating when each equilibrium is stable (solid) vs. unstable (dashed), and anything else of interest. The dashed line represents the prey-only equilibrium $n_1 = K, n_2 = 0$, but don't believe what it says about stability.

Here's a hint: The coexistence equilibrium (when it exists) is a point on the predator nullcline, in the region where $n_2 > 0$. What is the predator nullcline, and what does that say about the value of the prey abundance $\bar{n}_1$ at the coexistence equilibrium (using pencil and paper)?