AAE 575

Introduction to Satellite Navigation and Positioning

Homework 5: PVT Solutions

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Homework 5

Objective:

Using the ephemeris and receiver data, calculate the position of the receiver at the time the data was collected.

Given Data:

Ephemeris Data

Receiver Data

Receiver Position Estimate:

X = -2701206.38 m

Y = -4293642.366 m

Z = 3857878.924 m

Problem 1:

The given ephemeris data provides us with data for 8 different satellites. The initial clock bias is set to 0. Using this data and the code produced from homework 4 the positions of the satellites is calculated. After these are calculated the positions of the satellites are recalculated to account for the rotation of the earth and the time the receiver actually receives the signal. The pseudoranges for all 8 satellites are then calculated between the receiver and each satellite. The linear observation matrix, H, is then calculated and ends up being an 8 by 4 matrix. Once this is computed, MATLAB is used to solve the system of equations and comes up with the receiver position and clock bias estimation. This process is put in a 'while' loop until the desired precision of .5 meters. To complete the problem, it took 3 iterations for the .5 meter precision parameter that was given. The output is shown below.

```
Estimate of Position:

X Position: -2701206.380000

Y Position: -4293642.366000

Z Position: 3857878.924000

t_b:        0.000000


Estimate of Position:
```

```
X Position: -2700363.172502

Y Position: -4292553.165172

Z Position: 3855265.290738

t_b:        0.001733


Estimate of Position:

X Position: -2700363.246131

Y Position: -4292553.664862

Z Position: 3855265.969368

t_b:        0.001733
```

The final position estimate of the receiver in Geodetic Coordinates was calculated to be:

```
Geodetic Coordinates:

Latitude:  37.428187

Longitude: -122.173211

Altitude:  46.380768
```

Problem 2:

The dilution of precision values can be computed from the linear observation matrix as follows:

$$Position\ Dilution\ of\ Precision\ (PDOP) = \sqrt{H_{11} + H_{22} + H_{33}}$$

$$Time\ Dilution\ of\ Precision\ (TDOP) = \sqrt{H_{44}}$$

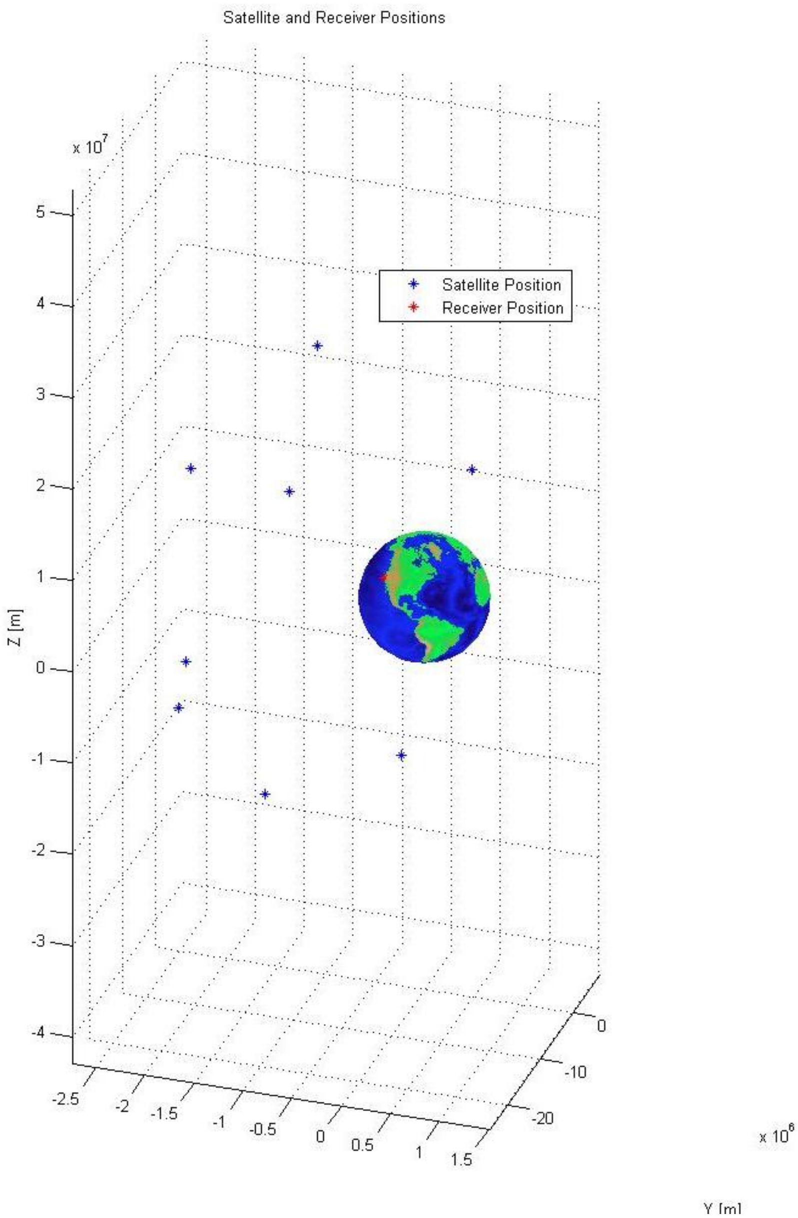
$$Geometric\ Dilution\ of\ Position\ (GDOP) = \sqrt{H_{11} + H_{22} + H_{33} + H_{44}}$$

The following values were calculated in MATLAB:

```
Dilution of Position

GDOP: 1.912511

PDOP: 1.718976

TDOP: 0.838344
```

Conclusion:

In conclusion, the receiver position was calculated in 3 iterations using the ephemeris data given. The final calculated positions of the satellites and the receiver can be seen in the following figure.



Appendix A:

MATLAB Code: hw5.m

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  Author: Josh Wildey
%  Class: AAE57500
%  Homework 5 - 12/2/2011
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
close all
clear all
clc

format long
%%%%%%%%% Problem 1: %%%%%%%%%
load eph.dat
load rcvrc.dat

x_rx = [-2701206.38; -4293642.366; 3857878.924];   %[m]  Initial Position
Guess (given)
tb_R = 0;                        %[s]  Initial Guess in receiver clock bias
x_hat = [.6; .6; .6; 0];            % Initialize x_hat for while loop control

% Display Initial Guess
disp('Estimate of Position:')
fprintf('X Position: %f\n',x_rx(1))
fprintf('Y Position: %f\n',x_rx(2))
fprintf('Z Position: %f\n',x_rx(3))
fprintf('t_b:        %f\n\n',tb_R)

rcvrc_tow = rcvrc(:,1);          %[s]  Receiver Time of Week
svid = rcvrc(:,2);               %[]  Satellite Vehicle ID
pr = rcvrc(:,3);                 %[m]  Pseudorange
c = 299792458;                   %[m/s]  Speed of Light

toc = eph(:,3);                  %[s]  Time of Clock
toe = eph(:,4);                  %[s]  Time of Applicability
I_0 = eph(:,15);                 %[rad]  Orbital Inclination
omega_dot = eph(:,16);           %[rad/s]  Rate of Right Ascen
e = eph(:,9);                    %[]  Eccentricity
sqrt_a = eph(:,10);              %[m^.5]  SQRT(A)
omega_0 = eph(:,14);             %[rad]  Right Ascen at TOA
omega = eph(:,13);               %[rad]  Argument of Perigee
M_0 = eph(:,12);                 %[rad]  Mean Anom
delta_n = eph(:,11);             %[rad/s]  mean motion diff
I_dot = eph(:,17);               %[rad/s]  Rate of inclin
Cuc = eph(:,19);                 %[rad]  lat cosine ampl
Cus = eph(:,18);                 %[rad]  radius sin ampl
Crc = eph(:,23);                 %[m]  inclin cos ampl
Crs = eph(:,22);                 %[m]  radius sin ampl
Cic = eph(:,21);                 %[rad]  inclin cos ampl
Cis = eph(:,20);                 %[rad]  inclin sin ampl
omega_dot_e = 7.2921151467e-5;   %[rad/sec] Number from IS-GPS-200E
mu = 3.986005e14;                %[m^3/s^2] Number from IS-GPS-200E
```

```
t = rcvrc_tow(1);
Y = pr;
I = eye(4);

while (abs(x_hat(1))>.5 && abs(x_hat(2))>.5 && abs(x_hat(3))>.5)

    tau = (pr - c.*tb_R)./c;  %[s]  Time delay of Satellite Signal
    t_tx = t - tau;          %[s]  Time of transmission
    t_k = t_tx - toe;           % Time from ephemeris refence time

    A_k = sqrt_a.^2;  % Semi-Major Axis
    n_0 = sqrt(mu./A_k.^3);  %[rad/s]  Computed Mean Motion
    n = n_0 + delta_n;  % Correct Mean Motion
    M_k = M_0 + n.*t_k;  % Mean Anomaly

    E_k = kepler_E(e,M_k);  % Eccentric Anomaly
    v_k = 2*atan(sqrt((1+e)./(1-e)).*tan(E_k./2));  % True Anomaly

    phi_k = v_k + omega;  % Argument of latitude
    % Second Harmonic Perturbations
    delta_u_k = Cus.*sin(2.*phi_k) + Cuc.*cos(2.*phi_k);  % Argument of
latitude Correction
    delta_r_k = Crs.*sin(2.*phi_k) + Crc.*cos(2.*phi_k);  % Radial Correction
    delta_i_k = Cis.*sin(2.*phi_k) + Cic.*cos(2.*phi_k);  % Inclination
Correction

    u_k = phi_k + delta_u_k;               % Corrected Argument of Latitude
    r_k = A_k.*(1-e.*cos(E_k)) + delta_r_k;  % Corrected Radius
    i_k = I_0 + I_dot.*t_k + delta_i_k;   % Corrected Inclination

    % Positions in Orbital plane
    x_kprime = r_k.*cos(u_k);
    y_kprime = r_k.*sin(u_k);

    % Corrected Longitude of Ascending Node
    omega_k = omega_0 + (omega_dot - omega_dot_e).*t_k - omega_dot_e.*toe;

    % Earth Fixed Coordinates of SV antenna phase center
    x_k = x_kprime.*cos(omega_k) - y_kprime.*cos(i_k).*sin(omega_k);
    y_k = x_kprime.*sin(omega_k) + y_kprime.*cos(i_k).*cos(omega_k);
    z_k = y_kprime.*sin(i_k);

    % Satellite Position at the time of reception
    x_k_rx = cos(omega_dot_e.*tau).*x_k + sin(omega_dot_e.*tau).*y_k;
    y_k_rx = -sin(omega_dot_e.*tau).*x_k + cos(omega_dot_e.*tau).*y_k;
    z_k_rx = z_k;

    R = sqrt((x_k- x_rx(1)).^2 + (y_k-x_rx(2)).^2 + (z_k-x_rx(3)).^2);

    H = [(x_rx(1) - x_k_rx(1))/R(1), (x_rx(2) - y_k_rx(1))/R(1), (x_rx(3) -
z_k_rx(1))/R(1) 1;
         (x_rx(1) - x_k_rx(2))/R(2), (x_rx(2) - y_k_rx(2))/R(2), (x_rx(3) -
z_k_rx(2))/R(2) 1;
```

```
         (x_rx(1) - x_k_rx(3))/R(3), (x_rx(2) - y_k_rx(3))/R(3), (x_rx(3) -
z_k_rx(3))/R(3) 1;
         (x_rx(1) - x_k_rx(4))/R(4), (x_rx(2) - y_k_rx(4))/R(4), (x_rx(3) -
z_k_rx(4))/R(4) 1;
         (x_rx(1) - x_k_rx(5))/R(5), (x_rx(2) - y_k_rx(5))/R(5), (x_rx(3) -
z_k_rx(5))/R(5) 1;
         (x_rx(1) - x_k_rx(6))/R(6), (x_rx(2) - y_k_rx(6))/R(6), (x_rx(3) -
z_k_rx(6))/R(6) 1;
         (x_rx(1) - x_k_rx(7))/R(7), (x_rx(2) - y_k_rx(7))/R(7), (x_rx(3) -
z_k_rx(7))/R(7) 1;
         (x_rx(1) - x_k_rx(8))/R(8), (x_rx(2) - y_k_rx(8))/R(8), (x_rx(3) -
z_k_rx(8))/R(8) 1;];

     M = linsolve(H',I');
     M = M';

     h = [(R(1) + c*tb_R); (R(2) + c*tb_R); (R(3) + c*tb_R); (R(4) + c*tb_R);
          (R(5) + c*tb_R); (R(6) + c*tb_R); (R(7) + c*tb_R); (R(8) +
c*tb_R);];

     y = Y - h;

     x_hat = M*y;

     x_rx = x_rx + x_hat(1:3);
     tb_R = tb_R + x_hat(4)/c;

     disp('Estimate of Position:')
     fprintf('X Position: %f\n',x_rx(1))
     fprintf('Y Position: %f\n',x_rx(2))
     fprintf('Z Position: %f\n',x_rx(3))
     fprintf('t_b:        %f\n\n',tb_R)

end

lla = ecef2lla([x_rx(1) x_rx(2) x_rx(3)], 'WGS84');

disp('Geodetic Coordinates:')
fprintf('Latitude:  %f\n',lla(1))
fprintf('Longitude: %f\n',lla(2))
fprintf('Altitude:  %f\n\n',lla(3))

% Geometry matrix:
G = inv(H'*H);

%GDOP
GDOP = sqrt(trace(G));
% PDOP
PDOP = sqrt(G(1,1) + G(2,2)+ G(3,3));
%TDOP
TDOP = sqrt(G(4,4));

disp('Dilution of Position')
fprintf('GDOP: %f\n',GDOP)
```

```
fprintf('PDOP: %f\n',PDOP)
fprintf('TDOP: %f\n',TDOP)

%  Plot Position of Satellites and Receiver around Earth
figure(1)
hold on
title('Satellite and Receiver Positions')
plot3(x_k,y_k,z_k,'*')
plot3(x_rx(1),x_rx(2),x_rx(3),'r*')
legend('Satellite Position','Receiver Position')
earth_sphere('m')
grid on
hold off
```

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Kepler Equation Solver Function:

```
function E = kepler_E(e, M)
% ------------------------------------------------------------
%

% This function uses Newton’s method to solve Kepler’s
% equation E - e*sin(E) = M for the eccentric anomaly,
% given the eccentricity and the mean anomaly.
%
% E - eccentric anomaly (radians)
% e - eccentricity, passed from the calling program
% M - mean anomaly (radians), passed from the calling program
% pi - 3.1415926...
%
% User M-functions required: none
% ------------------------------------------------------------
%...Set an error tolerance:
error = 1.e-12;
%...Select a starting value for E:
if M < pi
E = M + e/2;
else
E = M - e/2;
end
%...Iterate on Equation 3.14 until E is determined to within
%...the error tolerance:
ratio = 1;
while abs(ratio) > error
ratio = (E - e.*sin(E) - M)./(1 - e.*cos(E));
E = E - ratio;
end
% ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
```

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Earth Sphere Function:

```
function [xx,yy,zz] = earth_sphere(varargin)
%EARTH_SPHERE Generate an earth-sized sphere.
%   [X,Y,Z] = EARTH_SPHERE(N) generates three (N+1)-by-(N+1)
%   matrices so that SURFACE(X,Y,Z) produces a sphere equal to
%   the radius of the earth in kilometers. The continents will be
%   displayed.
%
%   [X,Y,Z] = EARTH_SPHERE uses N = 50.
%
%   EARTH_SPHERE(N) and just EARTH_SPHERE graph the earth as a
%   SURFACE and do not return anything.
%
%   EARTH_SPHERE(N,'mile') graphs the earth with miles as the unit rather
%   than kilometers. Other valid inputs are 'ft' 'm' 'nm' 'miles' and 'AU'
%   for feet, meters, nautical miles, miles, and astronomical units
%   respectively.
%
%   EARTH_SPHERE(AX,...) plots into AX instead of GCA.
%
%  Examples:
%    earth_sphere('nm') produces an earth-sized sphere in nautical miles
%
%    earth_sphere(10,'AU') produces 10 point mesh of the Earth in
%    astronomical units
%
%    h1 = gca;
%    earth_sphere(h1,'mile')
%    hold on
%    plot3(x,y,z)
%      produces the Earth in miles on axis h1 and plots a trajectory from
%      variables x, y, and z

%   Clay M. Thompson 4-24-1991, CBM 8-21-92.
%   Will Campbell, 3-30-2010
%   Copyright 1984-2010 The MathWorks, Inc.

%% Input Handling
[cax,args,nargs] = axescheck(varargin{:}); % Parse possible Axes input
error(nargchk(0,2,nargs)); % Ensure there are a valid number of inputs

% Handle remaining inputs.
% Should have 0 or 1 string input, 0 or 1 numeric input
j = 0;
k = 0;
n = 50; % default value
units = 'km'; % default value
for i = 1:nargs
    if ischar(args{i})
        units = args{i};
        j = j+1;
    elseif isnumeric(args{i})
        n = args{i};
        k = k+1;
    end
```

```
end

if j > 1 || k > 1
    error('Invalid input types')
end

%% Calculations

% Scale factors
Scale = {'km' 'm'  'mile'             'miles'             'nm'                'au'
'ft';
         1    1000 0.621371192237334 0.621371192237334 0.539956803455724
6.6845871226706e-009 3280.839895};

% Identify which scale to use
try
    myscale = 6378.1363*Scale{2,strcmpi(Scale(1,:),units)};
catch %#ok<*CTCH>
    error('Invalid units requested. Please use m, km, ft, mile, miles, nm, or
AU')
end

% -pi <= theta <= pi is a row vector.
% -pi/2 <= phi <= pi/2 is a column vector.
theta = (-n:2:n)/n*pi;
phi = (-n:2:n)'/n*pi/2;
cosphi = cos(phi); cosphi(1) = 0; cosphi(n+1) = 0;
sintheta = sin(theta); sintheta(1) = 0; sintheta(n+1) = 0;

x = myscale*cosphi*cos(theta);
y = myscale*cosphi*sintheta;
z = myscale*sin(phi)*ones(1,n+1);

%% Plotting

if nargout == 0
    cax = newplot(cax);

    % Load and define topographic data
    load('topo.mat','topo','topomap1');

    % Rotate data to be consistent with the Earth-Centered-Earth-Fixed
    % coordinate conventions. X axis goes through the prime meridian.
    %
http://en.wikipedia.org/wiki/Geodetic_system#Earth_Centred_Earth_Fixed_.28ECE
F_or_ECF.29_coordinates
    %
    % Note that if you plot orbit trajectories in the Earth-Centered-
    % Inertial, the orientation of the contintents will be misleading.
    topo2 = [topo(:,181:360) topo(:,1:180)]; %#ok<NODEF>

    % Define surface settings
    props.FaceColor= 'texture';
    props.EdgeColor = 'none';
    props.FaceLighting = 'phong';
```

```
    props.Cdata = topo2;

    % Create the sphere with Earth topography and adjust colormap
    surface(x,y,z,props,'parent',cax)
    colormap(topomap1)

% Replace the calls to surface and colormap with these lines if you do
% not want the Earth's topography displayed.
%     surf(x,y,z,'parent',cax)
%     shading flat
%     colormap gray

    % Refine figure
    axis equal
    xlabel(['X [' units ']'])
    ylabel(['Y [' units ']'])
    zlabel(['Z [' units ']'])
    view(127.5,30)
else
    xx = x; yy = y; zz = z;
end
```