题一：

四阶导数

边界条件难处理 $\mu=\frac{1}{2}(a+\sqrt{a^2-4b})$

$$\begin{cases}\mu\Delta v-bv=f\\ v=\frac{1}{\mu}g_2-g_1\end{cases}$$

第一类边界条件

五点差分格式

二阶椭圆型方程

可直接调用

$$\begin{cases}\frac{1}{\mu}\Delta u-u=v\\ u=g_1\end{cases}$$

同样，第一类边界条件

五点差分格式有哪结论？

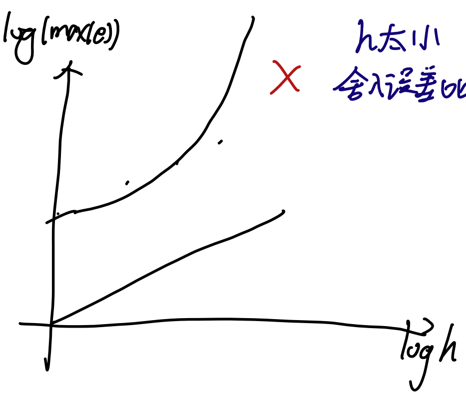
收敛性（二阶） 步长与误差，整体误差（舍入误差，2范数误差）

收敛阶计算 $rate=\dfrac{\log\frac{error_1}{error_2}}{\log(\frac{h_1}{h_2})}$ 分析x方向的误差，固定y方向步长（足够小）

时间精度（相邻）

保证：舍入误差相比截断误差

h小，可能累积



题二：非稳定问题

(1) $c(0,t)=c(l,t)=0$ 第一类边界条件

(2) $\dfrac{\partial c(0,t)}{\partial x}=0,\ c(l,t)=0$

h的大小对数值结果计算有影响

(3) $\dfrac{\partial c(0,t)}{\partial x}=0,\ c(l-vt,t)=0$. 移动边界 (选择) 移动均可选

(4) $\dfrac{\partial c(wt,t)}{\partial x}=0,\ C(l,t)=0$

老师写一下移动边条件的处理

(5) $\frac{\partial c(wt,t)}{\partial x}=0,\ c(l-vt,t)=0$

代码可以借鉴，论文必须原创

毕业论文格式

心得体会

附录可有可无

常见问题：

摘要，关键字，目录，参考文献

latex

图表说明，编号与名称不能分页

精度显示

下节课答疑

(1) $u=\sin x\sin y \Rightarrow g_1=\sin x\sin y$

$g_2=-\sin y\sin x-\sin x\sin y=-2\sin x\sin y$ 边界值

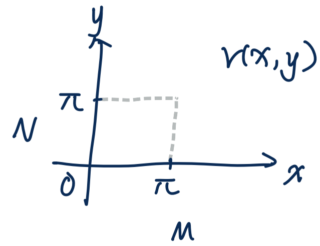
laplace算子 $\Delta u=\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=-\sin y\sin x$

$$\begin{cases}\mu\Delta v-b\,v=f & \text{内点取值}\\ v=\frac{1}{\mu}(-2\sin x\sin y)-\sin x\sin y=(-\frac{2}{\mu}-1)\sin x\sin y & \text{边界点取值}\end{cases}$$

$$\begin{cases}\Delta u-\mu\cdot u=\mu\cdot v & \text{内点取值}\\ u=\sin x\sin y & \text{边界点取值}\end{cases}$$

五点差分格式
(修改版)

$$\begin{cases} \mu\frac{\partial^2 v}{\partial x^2}+\mu\frac{\partial^2 v}{\partial y^2}-b\cdot v=f, & v\in\Omega \\ v=(-\frac{2}{\mu}-1)\sin x\sin y, & v\in\Gamma \end{cases}$$



记 $h_1=\frac{\pi}{M}$, $h_2=\frac{\pi}{N}$, 设 (x_i,y_j) 是正则内点.

$$\mu\left[\frac{v_{i+1,j}-2v_{ij}+v_{i-1,j}}{h_1^2}+\frac{v_{i,j+1}-2v_{ij}+v_{i,j-1}}{h_2^2}\right]-bv_{ij}=f_{ij}$$

若不一定取正方形网络,

$$\mu h_2^2(v_{i+1,j}-2v_{ij}+v_{i-1,j})+\mu h_1^2(v_{i,j+1}-2v_{ij}+v_{i,j-1})-bh_1^2h_2^2v_{ij}=h_1^2h_2^2f_{ij}$$

$$(-2\mu h_2^2-2\mu h_1^2-bh_1^2h_2^2)v_{ij}+\mu h_2^2(v_{i+1,j}+v_{i-1,j})+\mu h_1^2(v_{i,j+1}+v_{i,j-1})=h_1^2h_2^2f_{ij}$$

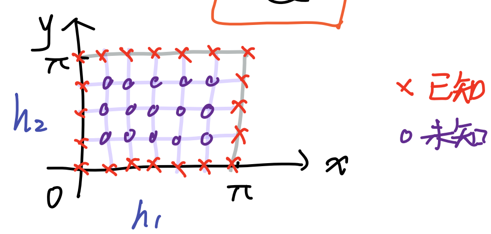
$$(2\mu h_1^2+2\mu h_2^2+bh_1^2h_2^2)v_{ij}=\mu h_2^2(v_{i+1,j}+v_{i-1,j})+\mu h_1^2(v_{i,j+1}+v_{i,j-1})-h_1^2h_2^2f_{ij}$$

其中, $f(x,y)=(4+2a+b)\sin x\sin y$, 记 $2\mu h_1^2+2\mu h_2^2+bh_1^2h_2^2=Q$

$$v_{ij}=\underbrace{\boxed{\frac{\mu h_2^2}{Q}}}_{\alpha}(v_{i+1,j}+v_{i-1,j})+\underbrace{\boxed{\frac{\mu h_1^2}{Q}}}_{\beta}(v_{i,j+1}+v_{i,j-1})-\boxed{\frac{h_1^2h_2^2f_{ij}}{Q}}\ \varepsilon_{ij}$$

边界条件获得 $v_{0,j}, v_{M,j}$, $(j=0,1,\cdots,N)$

$v_{i,0}, v_{i,N}$, $(i=0,1,\cdots,M)$



记 $V_h=[v_{11},v_{21},\cdots,v_{M-1,1},v_{12},\cdots,v_{1,M-1},\cdots,v_{M-1,N-1}]^T$ 从左到右, 从下到上

$$AV_h=F$$

$$F=\begin{pmatrix} -\frac{h_1^2h_2^2}{Q}f_{11}+\cdots \\ -\frac{h_1^2h_2^2}{Q}f_{21} \\ \vdots \\ -\frac{h_1^2h_2^2}{Q}f_{M-1,N-1} \end{pmatrix}$$

非正则内点对应的F项有增加

eg. F_1

y方向 非正则内点 x方向

$$v_{11}=\frac{\mu h_2^2}{Q}v_{21}+\frac{\mu h_1^2}{Q}v_{12}+\underbrace{\left(\frac{\mu h_2^2}{Q}v_{0,1}+\frac{\mu h_1^2}{Q}v_{1,0}-\frac{h_1^2h_2^2f_{11}}{Q}\right)}_{F_1}$$

U_{11} U_{21} $\cdots$ $U_{m-1,1}$

(x,y)

e.g. F_{m-1}

$$V_{m-1,1} = \frac{\mu h_2^2}{Q}\left(\underline{V_{m,1}} + V_{m-2,1}\right) + \frac{\mu h_1^2}{Q}\left(V_{m-1,2} + \underline{V_{m-1,0}}\right) - \frac{h_1^2 h_2^2 f_{ij}}{Q}$$

$$= \frac{\mu h_2^2}{Q} V_{m-2,1} + \frac{\mu h_1^2}{Q} V_{m-1,2} + \underbrace{\left(\frac{\mu h_2^2}{Q} \underset{x-p(1)}{\underline{V_{m,1}}} + \frac{\mu h_1^2}{Q} \underset{y-n(m-1)}{\underline{V_{m-1,0}}} - \frac{\cdots}{Q}\right)}_{F_{m-1}}$$

e.g $F_{(m-1)(n-2)+1}$

$$V_{1,n-1} = \frac{\mu h_2^2}{Q}\left(V_{2,n-1}, \underline{V_{0,n-1}}\right) + \frac{\mu h_1^2}{Q}\left(\underline{V_{1,n}} + V_{1,n-2}\right) - \frac{h_1^2 h_2^2 f_{ij}}{Q}$$

$$= \frac{\mu h_2^2}{Q} V_{2,n-1} + \frac{\mu h_1^2}{Q} V_{1,n-2} + \underbrace{\left(\frac{\mu h_2^2}{Q} \underset{x-n(n-1)}{\underline{V_{0,n-1}}} + \frac{\mu h_1^2}{Q} \underset{y-p(1)}{\underline{V_{1,n}}} - \frac{\cdots}{Q}\right)}_{F_{(m-1)(n-2)+1}}$$

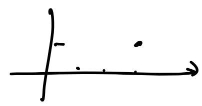
e.g. $F_{(m-1)(n-1)}$

$$V_{m-1,n-1} = \frac{\mu h_2^2}{Q}\left(\underline{V_{m,n-1}} + V_{m-2,n-1}\right) + \frac{\mu h_1^2}{Q}\left(\underline{V_{m-1,n}} + V_{m-1,n-2}\right) - \frac{\cdots}{Q}$$

$$= \frac{\mu h_2^2}{Q} V_{m-2,n-1} + \frac{\mu h_1^2}{Q} V_{m-1,n-2} + \left(\frac{\mu h_2^2}{Q} \underset{y-p(m-1)}{\underline{V_{m,n-1}}} + \frac{\mu h_1^2}{Q} \overset{x-p(n-1)}{\underline{V_{m-1,n}}} - \frac{\cdots}{Q}\right)$$

$$v_{ij} = \underbrace{\boxed{\frac{\mu h_2^2}{Q}}}_{\alpha}\left(V_{i+1,j} + V_{i-1,j}\right) + \underbrace{\boxed{\frac{\mu h_1^2}{Q}}}_{\beta}\left(V_{i,j+1} + V_{i,j-1}\right) - \boxed{\frac{h_1^2 h_2^2 f_{ij}}{Q}} \; \varepsilon_{ij}$$

只需要一个边界点



e.g. $F_{3\times 1}$

$$V_{3,1}=\frac{\mu h_2^2}{\alpha}(V_{4,1}+V_{2,1})+\frac{\mu h_1^2}{\alpha}(V_{3,2}+V_{3,0})-\frac{h_1^2\cdots}{\alpha}$$

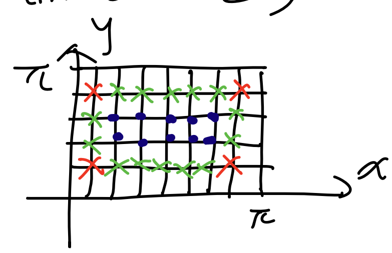
$$=\qquad\qquad +\left(\frac{\mu h_1^2}{\alpha}\underbrace{V_{3,0}}_{y-n(i)}-\frac{\overset{V.}{--}}{\alpha}\right)$$

e.g. $F_{3\times(n-1)}$

$$V_{3,n-1}=\frac{\mu h_2^2}{\alpha}(V_{4,n-1}+V_{2,n-1})+\frac{\mu h_1^2}{\alpha}(\underbrace{V_{3,n}}_{y-p(i)}+V_{3,n-2})-\frac{\cdots}{\alpha}$$

$$=$$

$(m-3)(n-3)$



上式简化为 $V_{ij} = \alpha \cdot V_{i+1,j} + \alpha \cdot V_{i-1,j} + \beta V_{i,j+1} + \beta V_{i,j-1} - \varepsilon_{ij}$

对应改写 $\varepsilon_{ij} = \underset{\alpha_4}{\underset{\Downarrow}{\beta}} V_{i,j-1} + \underset{-\alpha_3}{\underset{\Downarrow}{\alpha}} V_{i-1,j} \underset{\alpha_0}{\underset{\Downarrow}{-}} V_{ij} + \underset{-\alpha_1}{\underset{\Downarrow}{\alpha}} V_{i+1,j} + \underset{-\alpha_2}{\underset{\Downarrow}{\beta}} V_{i,j+1}$

表示为 $f_{ij} = \alpha_4 V_{i,j-1} - \alpha_3 V_{i-1,j} + \alpha_0 V_{ij} - \alpha_1 V_{i+1,j} - \alpha_2 V_{i,j+1}$

$$A = \begin{bmatrix} B & -\alpha_2 I & & \\ -\alpha_4 I & B & -\alpha_2 I & \\ & -\alpha_4 I & \ddots & -\alpha_2 I \\ & & -\alpha_4 I & B \end{bmatrix}_{(M-1)(N-1),\,(M-1)(N-1)}$$

I

$$B = \begin{bmatrix} \alpha_0 & -\alpha_1 & & \\ -\alpha_3 & \alpha_0 & \ddots & \\ & \ddots & \ddots & -\alpha_1 \\ & & -\alpha_3 & \alpha_0 \end{bmatrix}_{(M-1,\,M-1)}$$

$\alpha_0 = -1$

$\alpha_1 = -\alpha$

$\alpha_2 = -\beta$

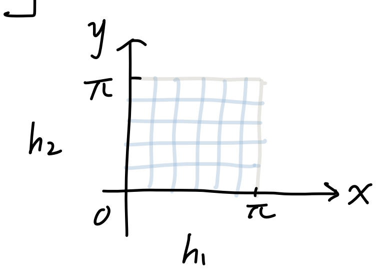
$\alpha_3 = -\alpha$

$\alpha_4 = \beta$

难点：处理非正则内点的 f_{ij}

由上题求出 V_{ij} · 内点

$$\begin{cases} \Delta u - \mu \cdot u = \mu \cdot v, & \Omega \\ u = \sin x \sin y, & \Gamma \end{cases}$$



$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - \mu \cdot u = \mu \cdot v$$

$$\frac{u_{i+1,j} - 2u_{ij} + u_{i-1,j}}{h_1^2} + \frac{u_{i,j+1} - 2u_{ij} + u_{i,j-1}}{h_2^2} - \mu \cdot u_{ij} = \mu \cdot v_{ij}$$

$$(-2h_2^2 - 2h_1^2) u_{ij} + h_2^2 (u_{i+1,j} + u_{i-1,j}) + h_1^2 (u_{i,j+1} + u_{i,j-1}) - \mu h_1^2 h_2^2 u_{ij} = \mu h_1^2 h_2^2 v_{ij}$$

$$\underset{\alpha_4}{\underset{\Downarrow}{h_1^2}} u_{i,j-1} + \underset{-\alpha_3}{\underset{\Downarrow}{h_2^2}} \cdot u_{i-1,j} + \underset{\alpha_0}{\underset{\Downarrow}{(-2h_1^2 - 2h_2^2 - \mu h_1^2 h_2^2)}} u_{ij} + \underset{-\alpha_1}{\underset{\Downarrow}{h_2^2}} u_{i+1,j} + \underset{-\alpha_2}{\underset{\Downarrow}{h_1^2}} u_{i,j+1} = \underset{f_{ij}}{\underset{\Downarrow}{\mu h_1^2 h_2^2 v_{ij}}}$$

同上，代入计算

$$\alpha_4 u_{i,j-1} - \alpha_3 u_{i-1,j} + \alpha_0 u_{ij} - \alpha_1 u_{i+1,j} - \alpha_2 u_{i,j+1} = f_{ij}$$

e.g. $\alpha_4 \underbrace{u_{1,0}}_{y-n(1)} - \alpha_3 \underbrace{u_{0,1}}_{x-n(1)} + \alpha_0 u_{11} - \alpha_1 u_{2,1} - \alpha_2 u_{1,2} = f_{11}$

$$\alpha_4 \underbrace{u_{m-1,0}}_{y-n(m-1)} - \alpha_3 u_{m-2,1} + \alpha_0 u_{m-1,1} - \underbrace{\alpha_1 u_{m,1}}_{x-p(1)} - \alpha_2 u_{m-1,2} = f_{m-1,1}$$

$$\alpha_4 u_{1,n-2} - \alpha_3 \underbrace{u_{0,n-1}}_{x-n(n-1)} + \alpha_0 u_{1,n-1} - \alpha_1 u_{2,n-1} - \underbrace{\alpha_2 u_{1,n}}_{y-p(1)} = f_{1,n-1}$$

$$\alpha_4 u_{m-1,n-2} - \alpha_3 u_{m-2,n-1} + \alpha_0 u_{m-1,n-1} - \underbrace{\alpha_1 u_{m,n-1}}_{x-p(n-1)} - \underbrace{\alpha_2 u_{m-1,n}}_{y-p(m-1)} = f_{m-1,n-1}$$

$$\boxed{\alpha_4 u_{i,j-1} - \alpha_3 u_{i-1,j} + \alpha_0 u_{ij} - \alpha_1 u_{i+1,j} - \alpha_2 u_{i,j+1} = f_{ij}}$$

只缺一个边界点

e.g. $u_{i,1}$ or $u_{i,n-1}$

$$\alpha_4 \underbrace{u_{i,0}}_{y-n(i)} - \alpha_3 u_{i-1,1} + \alpha_0 u_{i,1} - \alpha_1 u_{i+1,1} - \alpha_2 u_{i,2} = f_{i,1}$$

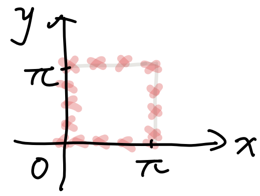
$$\alpha_4 u_{i,n-2} - \alpha_3 u_{i-1,n-1} + \alpha_0 u_{i,n-1} - \alpha_1 u_{i+1,n-1} - \underbrace{\alpha_2 u_{i,n}}_{y-p(i)} = f_{i,n-1}$$

e.g. $u_{1,j}$ or $u_{m-1,j}$

$$\alpha_4 u_{1,j-1} - \underbrace{\alpha_3 u_{0,j}}_{x-n(j)} + \alpha_0 u_{1,j} - \alpha_1 u_{2,j} - \alpha_2 u_{1,j+1} = f_{1,j}$$

$$\alpha_4 u_{m-1,j-1} - \alpha_3 u_{m-2,j} + \alpha_0 u_{m-1,j} - \underbrace{\alpha_1 u_{m,j}}_{x-p(j)} - \alpha_2 u_{m-1,j+1} = f_{m-1,j}$$

边界 1. $v=(-\frac{2}{\mu}-1)\sin x\sin y$, T



$v(0,y)=0$, $v(\pi,y)=0$

$v(x,0)=0$, $v(x,\pi)=0$

边界 2. $u=\sin x\sin y$, T

针对 Q1-1

$u(0,y)=0$, $u(\pi,y)=0$

$u(x,0)=0$, $u(x,\pi)=0$

针对 Q1.2

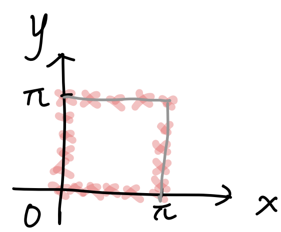
$v=\frac{1}{\mu}g_2-g_1=\frac{1}{\mu}(-y\sin x-x\sin y)-(x\sin y+y\sin x)$

$g_2=\Delta u=\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=(\sin y+y\cos x)'_x+(x\cos y+\sin x)'_y$

$=-y\sin x-x\sin y$

$v=(-\frac{1}{\mu}-1)y\sin x+(-\frac{1}{\mu}-1)x\sin y$

$u=x\sin y+y\sin x$



边界 1. $v(0,y)=0$, $v(\pi,y)=(-\frac{1}{\mu}-1)\pi\sin y$

$v(x,0)=0$, $v(x,\pi)=(-\frac{1}{\mu}-1)\pi\sin x$

边界 2 $u(0,y)=0$, $u(\pi,y)=\pi\sin y$

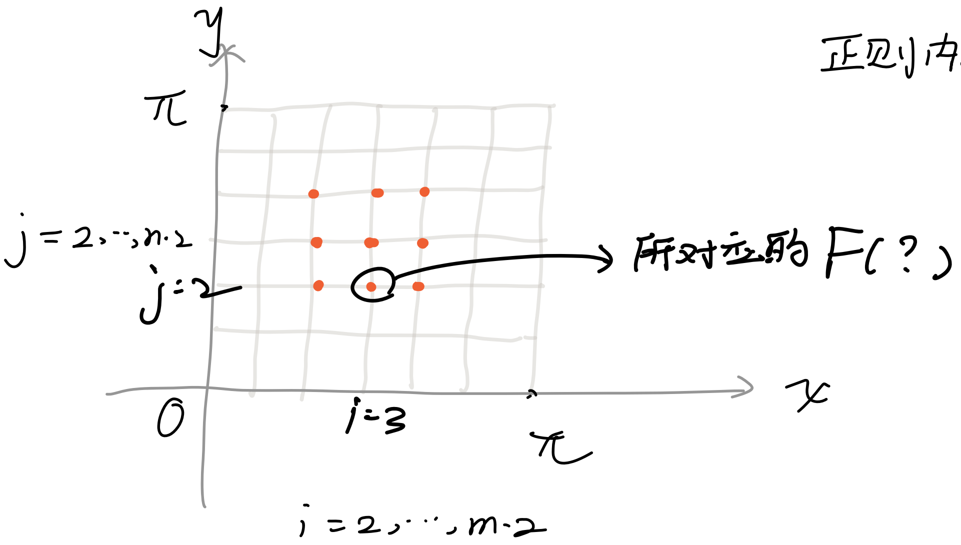
$u(x,0)=0$, $u(x,\pi)=\pi\sin x$

a	b	μ
1	0	1
1	$\frac{1}{8}$	$\frac{1}{2}+\frac{\sqrt{2}}{4}$
2	$\frac{1}{2}$	$1+\frac{\sqrt{2}}{2}$
3	1	$\frac{3}{2}+\frac{\sqrt{5}}{2}$

$\mu=\frac{1}{2}(a+\sqrt{a^2-4b})$

$=\frac{1}{2}(3+\sqrt{9-4})$

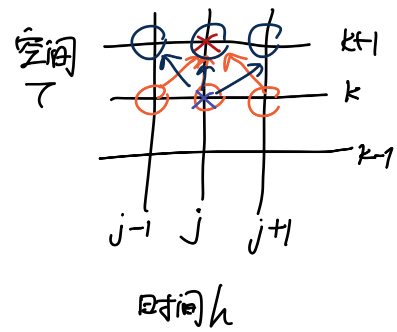
正则内点



$$(m-1)\times(j-1)+i$$

鉴于格式,第二问没有任何可参考的差分格式

$$\frac{\partial C(x,t)}{\partial t}+u\frac{\partial C(x,t)}{\partial x}=D\frac{\partial^2 C(x,t)}{\partial x^2},\quad 0<x<L$$



$$\frac{C_j^{k+1}-C_j^k}{\tau}+u\underbrace{\frac{C_{j+1}^k-C_{j-1}^k}{2h}}=D\underbrace{\frac{C_{j+1}^k-2C_j^k+C_{j-1}^k}{h^2}}$$

向前差分

$$\frac{C_j^{k+1}-C_j^k}{\tau}+u\underbrace{\frac{C_{j+1}^{k+1}-C_{j-1}^{k+1}}{2h}}=D\underbrace{\frac{C_{j+1}^{k+1}-2C_j^{k+1}+C_{j-1}^k}{h^2}}$$

向后差分

区别在于，x方向(时间)

差分表示

向前差分化简

$$C_j^{k+1}-C_j^k+\boxed{\frac{u\cdot\tau}{2h}}_{=a}\left(C_{j+1}^k-C_{j-1}^k\right)=\boxed{\frac{D\cdot\tau}{h^2}}_{=b}\left(C_{j+1}^k-2C_j^k+C_{j-1}^k\right)$$

$$C_j^{k+1}-C_j^k+aC_{j+1}^k-aC_{j-1}^k=bC_{j+1}^k-2bC_j^k+bC_{j-1}^k$$

$$C_j^{k+1}=(1-2b)C_j^k+(b+a)C_{j-1}^k+(b-a)C_{j+1}^k$$

$j=1,2,\cdots,\underline{N-1}$, $k=0,1,\cdots,\underline{M-1}$

与前一问有区别

矩阵格式

$j=1$,

$C_1^{k+1}=(1-2b)C_1^k+(b+a)\underline{C_0^k}+(b-a)C_2^k$ 边界已知

$j=2$

$C_2^{k+1}=(1-2b)C_2^k+(b+a)C_1^k+(b-a)C_3^k$

$j=3$

$C_3^{k+1}=(1-2b)C_3^k+(b+a)C_2^k+(b-a)C_4^k$

$$\begin{pmatrix}C_1^{k+1}\\C_2^{k+1}\\C_3^{k+1}\\\vdots\\C_{N-1}^{k+1}\end{pmatrix}=\begin{pmatrix}1-2b & b-a & & 0\\ b+a & 1-2b & b-a & \\ & \ddots & \ddots & \ddots \\ & & & b-a\\ 0 & & b+a & 1-2b\end{pmatrix}\begin{pmatrix}C_1^k\\C_2^k\\C_3^k\\\vdots\\C_{N-2}^k\\C_{N-1}^k\end{pmatrix}$$

下一页

$j=N-1$ 上一层

$C_{N-1}^{k+1} = (1-2b)C_{N-1}^{k} + (b+a)C_{N-2}^{k} + (b-a)\underbrace{C_{N}^{k}}$ 边界已知

$$\begin{pmatrix} C_1^{k+1} \\ C_2^{k+1} \\ C_3^{k+1} \\ \vdots \\ C_{N-1}^{k+1} \end{pmatrix} = \begin{pmatrix} \boxed{b+a} & 1-2b & b-a & & 0 & \boxed{0} \\ \boxed{0} & b+a & 1-2b & b-a & & \boxed{0} \\ \boxed{0} & & \ddots & \ddots & \ddots & \boxed{\vdots} \\ \boxed{\vdots} & & & & b-a & \boxed{0} \\ \boxed{0} & 0 & & b+a & 1-2b & \boxed{b-a} \end{pmatrix} \begin{pmatrix} \boxed{C_0^k} \\ C_1^k \\ C_2^k \\ C_3^k \\ \vdots \\ C_{N-2}^k \\ C_{N-1}^k \\ \boxed{C_N^k} \end{pmatrix}$$

$R \in (N-1)\times(N+1)$ $(N+1)\times 1$

本题 $C_0^k = C_N^k = 0$，不考虑扩增情况

一步步迭代即可，共M次

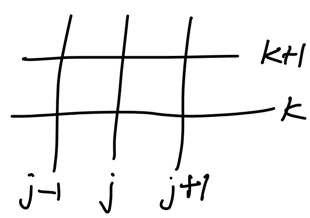
matlab 中 index 写第，有启动步，迭代步 $i=2,\cdots,M$

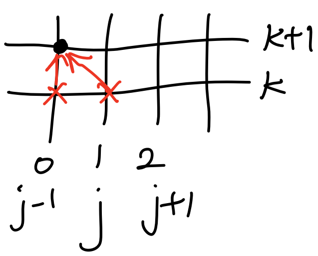
第二类

$\frac{\partial c(0,t)}{\partial x} = 0$，$C_N^k = 0$，$C_j^0 = \sin(\pi j h)$ $j=0,1,2,\cdots,N-1$

$C_0^0 = 0$

$$\boxed{C_0^{k+1} = (1-2b)C_0^k + 2bC_1^k}$$





$k=0,1,2,\cdots,M-1$

$k=0$，$C_j^1 = \underbrace{(a+b)C_{j-1}^0 + (1-2b)C_j^0 + (b-a)C_{j+1}^0}_{\text{down}}$

$k=1$，$C_j^2 = (a+b)C_{j-1}^2 + (1-2b)C_j^1 + (b-a)C_{j+1}^1$

两边各增加一行一列

$$
\begin{pmatrix} C_0 \\ C_1^{k+1} \\ C_2^{k+1} \\ C_3^{k+1} \\ \vdots \\ C_{N-1}^{k+1} \end{pmatrix}_N
=
\begin{pmatrix}
1-2b & 2b & \cdots & & & 0 \\
a+b & 1-2b & b-a & & & 0 \\
\vdots & b+a & 1-2b & b-a & & \\
\vdots & & \ddots & \ddots & \ddots & \\
\vdots & & & & & b-a \\
\vdots & 0 & & & b+a & 1-2b
\end{pmatrix}_{N\times N}
\begin{pmatrix} C_0^k \\ C_1^k \\ C_2^k \\ C_3^k \\ \vdots \\ C_{N-2}^k \\ C_{N-1}^k \end{pmatrix}_N
$$

向后差分基础:

$$-(a+b)C_{j-1}^{k+1}+(1+2b)C_j^{k+1}+(a-b)C_{j+1}^{k+1}=C_j^k \quad , j=0,1,2,\cdots,N-k-2$$

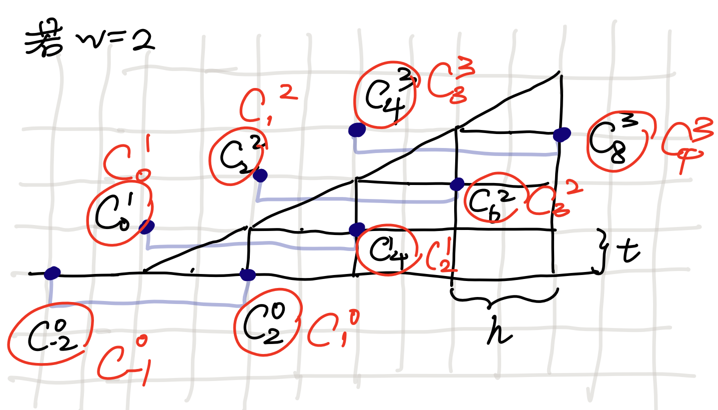
$j=0$时,

$$-(a+b)\underbrace{C_{-1}^{k+1}+(1+2b)C_0^{k+1}+(a-b)C_1^{k+1}}_{\text{相等}}=C_0^k \qquad \because C_{-1}^{k+1}=C_1^{k+1}$$

$$(1+2b)C_0^{k+1}-2b\,C_1^{k+1}=C_0^k \qquad V\text{大小由 } v=\frac{h}{\tau}\text{决定}$$

$$\frac{\partial C(wt,t)}{\partial x}=0 \ , \quad c(l,t)=0 \ , \quad \text{令 } wt=h$$

若 $w=2$



$j=0 \quad 1 \quad 2 \quad 3 \quad 4 \quad \cdots$ 实际网格点?

$$\frac{\partial C(wt,t)}{\partial x}=0$$

$$\frac{C_{wt+h}^t-C_{wt-h}^t}{2h}=0$$

$$C_{(t+1)w}^t=C_{(t-1)w}^t$$

$$C_{(k+1)w}^k=C_{(k-1)w}^k$$

$k=0, \quad C_{-2}^0=C_2^0$ 代替 $\partial c(0,0)$

$k=1, \quad C_0^1=C_4^1 \quad \partial c(2,1)$

$k=2, \quad C_2^2=C_6^2 \quad \partial c(4,2)$

$k=3, \quad C_4^3=C_8^3 \quad \partial c(6,3)$

C_2^0 应该是 C_1^0 ?

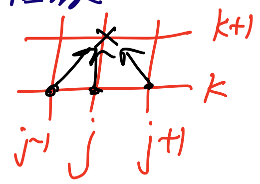
$-(a+b)C_{j-1}^{k+1}+(1+2b)C_j^{k+1}+(a-b)C_{j+1}^{k+1}=C_j^k$ ✗ 向后差分

$k=0$时, $-(a+b)\underbrace{C_{j-1}^1+(1+2b)C_j^1+(a-b)C_{j+1}^1}=C_j^0$

只能用向前差分!!!
如果思路相似

向前差分

$$C_j^{k+1}=(a+b)C_{j-1}^k+(1-2b)C_j^k+(b-a)C_{j+1}^k$$

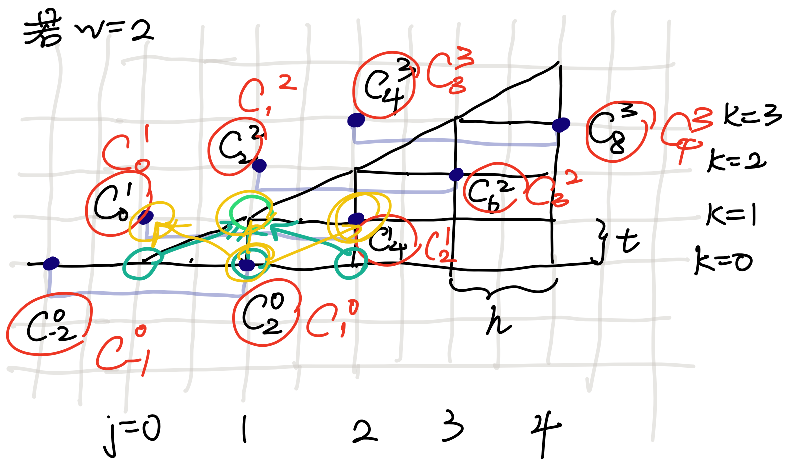


$k=0$时, $C_j^1=(a+b)C_{j-1}^0+(1-2b)C_j^0+(b-a)C_{j+1}^0$

$j=0$, $C_0^1=(a+b)C_{-1}^0+(1-2b)C_0^0+(b-a)C_1^0$ 另外的 $j=1,2,\cdots$ 正常处理

$=(1-2b)C_0^0+2bC_1^0$

若 $w=2$



$C_1^1, C_2^1, \cdots, C_{N-1}^1$

向后差分也可以

P.S. k=0一列所有建立

$\underbrace{1 : N}_{N-i}-i+1$

$N-(N-i)$

$k=1$时, $C_j^2=(a+b)C_{j-1}^1+(1-2b)C_j^1+(b-a)C_{j+1}^1$

从$j=1$开始, $C_1^2=(a+b)\underline{C_0^1}+(1-2b)C_1^1+(b-a)\underline{C_2^1}$

$$C_1^2=(1-2b)C_1^1+2bC_2^1$$

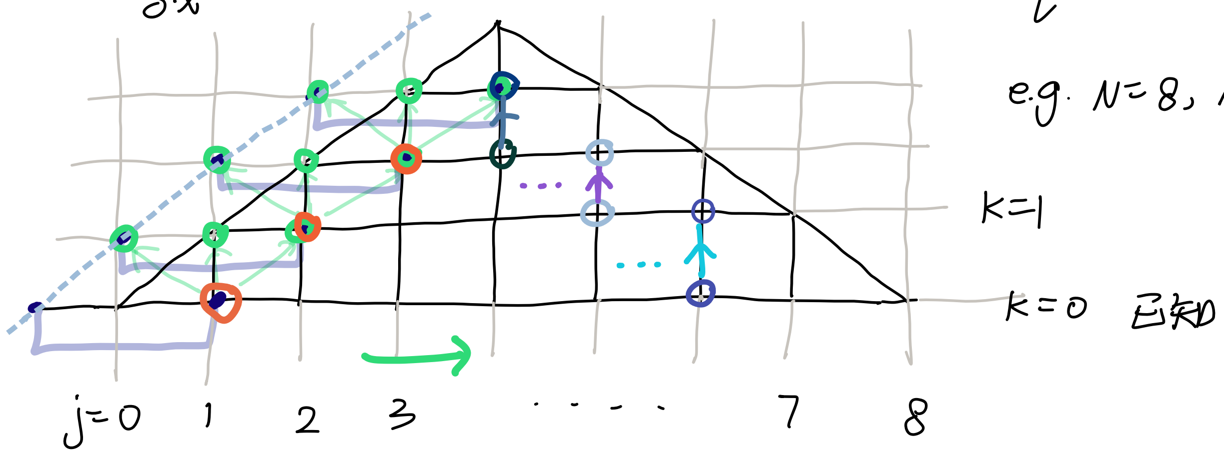
向前差分太不稳定,
用向后差分

$$\begin{pmatrix} C_0^{k+1} \\ C_1^{k+1} \\ C_2^{k+1} \\ C_3^{k+1} \\ \vdots \end{pmatrix} = \begin{pmatrix} 1-2b & 2b & \cdots & & \\ a+b & 1-2b & b-a & & 0 \\ \vdots & b+a & 1-2b & b-a & \\ \vdots & & \ddots & \ddots & \ddots \end{pmatrix} \begin{pmatrix} C_0^k \\ C_1^k \\ C_2^k \\ C_3^k \\ \vdots \end{pmatrix}$$

$$
\begin{pmatrix} \vdots \\ C_{N-1}^{k+1} \end{pmatrix}_{N}
\quad
\begin{pmatrix} \ddots & & b-a \\ & \ddots & \\ O \quad b+a & & 1-2b \end{pmatrix}_{N\times N}
\begin{pmatrix} \vdots \\ C_{N-2}^{k} \\ C_{N-1}^{k} \end{pmatrix}_{N}
$$

一层层写，是最笨的办法

$$\frac{\partial C(wt,t)}{\partial x}=0,\quad C(-vt,t)=0,\quad 令\ w=v=\frac{h}{t}$$



向后差分格式 $-(a+b)C_{j-1}^{k+1}+(1+2b)C_j^{k+1}+(a-b)C_{j+1}^{k+1}=C_j^k$

从k=0开始，$-(a+b)C_{j-1}^1+(1+2b)C_j^1+(a-b)C_{j+1}^1=C_j^0$

当j=1时，$-(a+b)C_0^1+(1+2b)C_1^1+(a-b)C_2^1=C_1^0$

$$(1+2b)C_1^1-2bC_2^1=C_1^0$$

当j=2时，$-(a+b)C_1^1+(1+2b)C_2^1+(a-b)C_3^1=C_2^0$

⋮

当j=6时，$-(a+b)C_5^1+(1+2b)C_6^1+(a-b)C_7^1=C_6^0$，$C_7^1=0$

$$-(a+b)C_5^1+(1+2b)C_6^1=C_6^0$$

$$
\begin{pmatrix}
1+2b & -2b & & & O \\
-a-b & 1+2b & a-b & & \\
 & -a-b & 1+2b & a-b & \\
 & & -a-b & \ddots & \ddots \\
O & & & \ddots & \ddots
\end{pmatrix}
\begin{pmatrix} C_1^1 \\ C_2^1 \\ C_3^1 \\ \vdots \\ C^1 \end{pmatrix}
=
\begin{array}{c} 0\ \ 1\ \ 2 \\ N-1\ \ N-2\ \ N-3 \\ \begin{pmatrix} C_1^0 \\ C_2^0 \\ C_3^0 \\ \vdots \\ C^0 \end{pmatrix} \end{array}
$$

$$\begin{pmatrix} \ddots & & & \\ -a-b & 1+2b & a-b & \\ & -a-b & 1+2b & a-b \\ & & -a-b & 1+2b \end{pmatrix}_{7\times 7} \begin{pmatrix} C_5 \\ C_6^1 \\ C_7^1 \end{pmatrix} = \begin{pmatrix} C_5 \\ C_6^0 \\ C_7^0 \end{pmatrix}$$

$N=8$

$5, 3$

8 2 5

8 3 3

8 4

$i+N-i-1$

$=N-1$

$5=N-i$

$8-2$

$$\begin{pmatrix} 1+2b & -2b & & & \\ -a-b & 1+2b & a-b & & \\ & -a-b & 1+2b & a-b & \\ & & -a-b & 1+2b & a-b \\ & & & -a-b & 1+2b \end{pmatrix} \begin{pmatrix} C_2^2 \\ C_3^2 \\ C_4^2 \\ C_5^2 \\ C_6^2 \end{pmatrix} = \begin{pmatrix} C_2^1 \\ C_3^1 \\ C_4^1 \\ C_5^1 \\ C_6^1 \end{pmatrix}$$

5×5 $\quad 4\times 4$

$N-(i-1)2-1$

$=N-2i+2-1$

$8-3\cdot 1$

$$\begin{pmatrix} 1+2b & -2b \\ -a-b & 1+2b \end{pmatrix} \begin{pmatrix} C_3^3 \\ C_4^3 \end{pmatrix} = \begin{pmatrix} C_3^2 \\ C_4^2 \end{pmatrix}$$

3×3 $\quad 2\times 2$

划分数 → 决定 最终可计算的点数

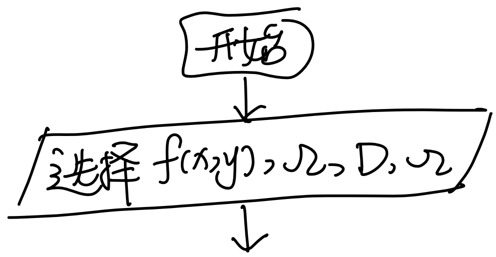
1×1

$(1+2b)_{1\times 1} \cdot C_4^4 = C_3^3$

$i+N-2i+1-1$

$=N-i$

$M-1=2N-1$

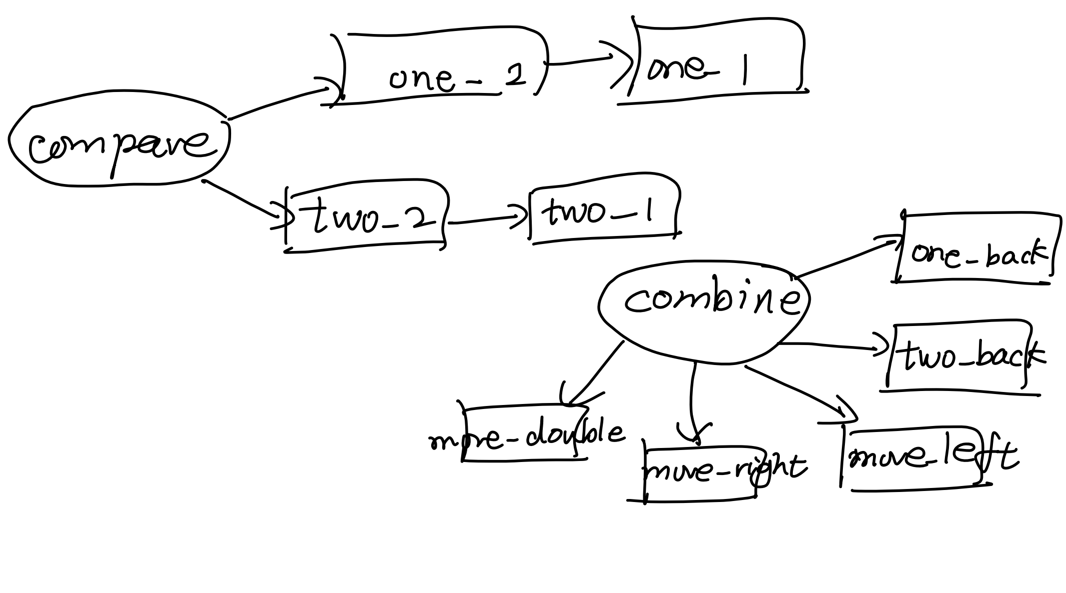


$$wr = h$$

$$x_B^k = wt_k$$

$$\frac{\partial c(wt,t)}{\partial x} = 0 \qquad c^t - c^t$$

$$(1+2b)C_{k-1}^{k+1} - 2bC_k^{k+1} = C_{k-1}^k$$



$$N = 2M$$

$$v = \frac{\frac{L}{N}}{\frac{T}{M}} = \frac{L}{2\cancel{M}} \cdot \frac{\cancel{M}}{T} = \frac{L}{2T}$$

$v = 5, 10, 20$, $L = 1$

$$\begin{array}{r} 0.025 \\ 40\overline{)1.00} \\ \cancel{80} \end{array}$$

$T = 0.1$, 0.05 , 0.025

$=$

$$v = \frac{L}{T}$$

$5, 10, 20$

$L = 1$

$T = 0.2$, 0.1 , 0.05

$5, 10, 20$

$$w = \frac{h}{t} = \frac{\frac{L}{\cancel{M}}}{\frac{T}{\cancel{M}}} = \frac{L}{T} = \frac{1}{T}$$

$T = 0.2, 0.1, 0.05$