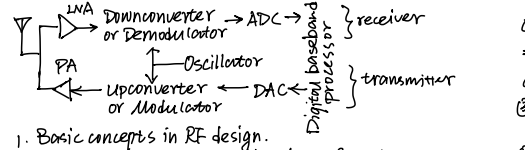


1. Basic concepts in RF design.

(1) Voltage gain: $A_{V,dB} = 20\log \frac{V_{out}}{V_{in}}$. (for LNA)

power gain: $A_{p,dB} = 10\log \frac{P_{out}}{P_{in}}$. (for PA)

only when $R_{in} = R_{out}$, then $A_p = A_V$.

$A_{peak} = 1 \Rightarrow V_{pp} = 2 \cdot V_{rms} = \frac{2}{\sqrt{2}} A_{peak}$.

(2) Effects of nonlinearity: Harmonic distortion

A nonlinear system: $y(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t) + \cdots$

① Input: $x(t) = A\cos\omega_0 t$

$\Rightarrow y(t) = \frac{\alpha_2 A^2}{2} + (\alpha_1 A + \frac{3\alpha_3 A^3}{4})\cos\omega_0 t + \frac{\alpha_2 A^2}{2}\cos 2\omega_0 t + \frac{\alpha_3 A^3}{4}\cos 3\omega_0 t$

gain compression: $A_{in,1dB} = \sqrt{0.145\left|\frac{\alpha_1}{\alpha_3}\right|}$. (peak value)

② Input: $x(t) = A_1\cos\omega_1 t + A_2\cos\omega_2 t$

$y(t) = (\alpha_1 A_1\cos\omega_1 t + \frac{\alpha_2 A_1^2}{4} C_2^1 \cos\omega_1 t + \frac{\alpha_3 A_1 A_2^2}{4} C_3^1 C_3^1 \cos\omega_1 t$

$\Rightarrow (A_2 \gg A_1)\ y(t) = \alpha_1 A_1\cos\omega_1 t + \frac{3\alpha_3 A_1 A_2^2}{2}\cos\omega_1 t = A'\cos\omega_1 t$.

△ Cross modulation (only when the interfer an AM signal).

↳ gain compression: $y(t) = (\alpha_1 + \frac{3}{2}\alpha_3 A_2^2) A_1\cos\omega_1 t$

two signals: $\omega_1 \Rightarrow$ desired signal, $\omega_2 \Rightarrow$ interferer

△ Intermodulation

↳ Third-order IM products: $2\omega_1 - \omega_2$, $2\omega_2 - \omega_1$.

three signals: $\omega_0 \Rightarrow$ desired signal. $\omega_1\omega_2 \Rightarrow$ two interferers

△ Third intercept point.

$\alpha_1 A(\text{fundamental}) = \frac{3}{4}|\alpha_3| A^3$ (IM product) $\Rightarrow A_{IIP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}$.

$A_{IIP3} > A_{in,1dB} \Rightarrow$ 在 A_{IIP3} 处已经被压缩 (gain compression).

△ Cascaded nonlinear stage: $x(t) \to A_{IIP3,1} \to y_1(t) \to A_{IIP3,2} \to y_2(t)$

$\begin{cases} y_1(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 \cdots \\ y_2(t) = \beta_1 y_1(t) + \beta_2 y_1^2(t) + \beta_3 \cdots \end{cases}$ $A_{IIP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1\beta_1}{\alpha_3\beta_1 + 2\alpha_1\alpha_2\beta_2 + \alpha_1^3\beta_3}\right|}$.

$\frac{1}{A_{IIP3}^2} = \left|\frac{1}{A_{IIP3,1}^2} + \frac{2\alpha_2\beta_2}{>\beta_1} + \frac{\alpha_1^2}{A_{IIP3,2}^2}\right| \approx \frac{1}{A_{IIP3,1}^2} + \frac{\alpha_1^2}{A_{IIP3,2}^2}$.

	Input	output	Effect	format
Harmonic distortion	Single signal	Higher order harmonics	Affect other systems.	Any
Gain compression	Single signal	Third order	Desensitize the system	
Cross Modulation	Two signal + interferer	Third order (between signal and interferer)	The BB information carrier by interferer is transferred to signal	AM
Inter-Modulation	Signal + multiple interferers	Third order (between interferers)	The cross modulation of interferers are imposed on the signal	

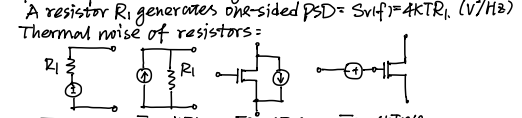
(2) Noise.

Average power of noise: $P_n = \lim_{T\to\infty}\frac{1}{T}\int_{-0}^{T} n^2(t)\,dt$.

Power spectral density: $S(f) = \lim_{T\to\infty} E\left(\frac{|N(f)|^2}{T}\right)$.

A resistor R_1 generates one-sided PSD: $S_V(f) = 4KTR_1$ (V^2/Hz)

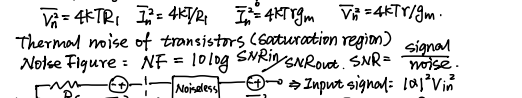
Thermal noise of resistors:



$\overline{V_n^2} = 4KTR_1$ $\overline{I_n^2} = 4KT/R_1$ $\overline{I_n^2} = 4KT\gamma g_m$ $\overline{V_n^2} = 4KT\gamma/g_m$.

Thermal noise of transistors (Saturation region)

Noise Figure: $NF = 10\log \frac{SNR_{in}}{SNR_{out}}$. $SNR = \frac{\text{signal}}{\text{noise}}$.



$\Rightarrow$ Input signal: $|\alpha|^2 V_{in}^2$

$\alpha = Z_{in}/(Z_{in} + R_s)$

Output signal: $|\alpha|^2 V_{in}^2 A_V^2$

Output Noise: $\frac{|\alpha|^2}{V_{Rs}^2} A_V^2 + \overline{V_n^2}$, $SNR_{out} = \frac{|\alpha|^2 V_{in}^2}{|\alpha|^2 \frac{V_{in}^2}{V_{Rs}^2} A_V^2 + \overline{V_n^2}}$

$SNR_{in} = \frac{|\alpha|^2 V_{in}^2}{|\alpha|^2 V_{Rs}^2} \Rightarrow NF = \frac{1}{V_{Rs}^2} \cdot \frac{|\alpha|^2 V_{Rs}^2 A_V^2 + \overline{V_n^2}}{|\alpha|^2 A_V^2}$.

$\because \overline{V_{n,Rs}^2} = 4KTR_s \Rightarrow$ the thermal noise of R_s (in). A_V: LNA's gain.

$\overline{V_n^2} \Rightarrow$ the thermal noise of antenna (out).

$NF = \frac{1}{4KTR_s} \cdot \frac{\overline{V_{n,out}^2}}{A_0^2}$, $A_0 = \frac{V_{out}}{V_{in}} = |\alpha| A_V = \frac{R_{in}}{R_{in} + R_s} A_V$.

For two cascaded stages: $\Rightarrow 4KTR_s A_1 \cdot NF_1$

$\overline{V_{n1,out}^2} = \overline{V_{n1}^2} + \overline{V_{n2}^2} A_1^2 = \overline{V_{n1}^2} + 4KTR_s A_1^2$. $NF_1 = \frac{1}{4KTR_s} \cdot \frac{\overline{V_{n1,out}^2}}{A_1^2}$.

$\overline{V_{n1}^2} = \overline{V_{n1,out}^2} - 4KTR_s A_1^2 = (NF_1 - 1) \cdot 4KTR_s A_1^2$.

$\overline{V_{n2}^2} = (NF_2 - 1) \cdot 4KTR_{out1} A_2^2$.

$\overline{V_{n2,out}^2} = \overline{V_{n1,out}^2} \cdot A_2^2 + (NF_2 - 1) \cdot 4KTR_{out1} A_2^2$

$\Rightarrow NF = \frac{1}{4KTR_s} \cdot \frac{\overline{V_{n2,out}^2}}{A_1^2 A_2^2} = NF_1 + \frac{NF_2 - 1}{A_1^2} \cdot \frac{R_s}{R_{out1}} = NF_1 + \frac{NF_2 - 1}{A_{p1}}$.

Friis' equation: $NF = NF_1 + \frac{NF_2 - 1}{A_1^2} + \cdots + \frac{NF_n - 1}{A_1^2 \cdots A_{n-1}^2}$.

2. Passive RLC Circuits

(1) KVL: $i(t)R + \int\frac{i(t)}{C}dt + L\frac{di(t)}{dt} = V(t)$. $Z(\omega) = R + j(\omega L - \frac{1}{\omega C})$

resonant freq.: $\omega_0 = \frac{1}{\sqrt{LC}}$. $\phi = \arctan\frac{\omega L - 1/\omega C}{R}$ $\begin{cases} \phi > 0. \text{ Inductive} \\ \phi < 0. \text{ Capacitive} \end{cases}$

$\left|\frac{I(\omega)}{I_{max}}\right| = \frac{1}{\sqrt{1 + Q^2(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0})^2}}$. $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$. Q 越大 $|I(\omega)|$ 越小 I_{max} 越大

↳ dB bandwidth (0.707 drop in Amp.): $B = \frac{\omega_0}{Q}$. $\Delta\omega = \frac{\omega_0}{2Q}$. $T = \frac{1}{B} = \frac{Q}{\omega_0}$

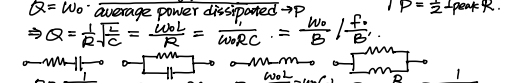
ΔE_g: three TXs: 2.40GHz, 2.41GHz, 2.42GHz. RX → 2.41GHz

↳ $Q = \omega_0/B = \frac{2.41GHz}{0.01GHz} = 241$.

(2) Quality factor

$Q = \omega_0 \cdot \frac{\text{energy stored}}{\text{average power dissipated}}$ → E 与 P: $\begin{cases} E = \frac{1}{2}LI_{peak}^2 \\ P = \frac{1}{2}I_{peak}^2 R \end{cases}$

$\Rightarrow Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC} = \frac{\omega_0}{B} = \frac{f_0}{B}$.



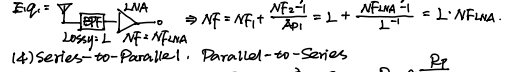
$Q = \frac{1}{\omega_0 RC}$ $Q = \omega_0 RC$ $Q = \frac{\omega_0 L}{R} = \omega_0 GL$ $Q = \frac{R}{\omega_0 L} = \frac{1}{\omega_0 GL}$

$\Rightarrow Q(\text{parallel}) = 1/Q(\text{series})$.

(4) Noise in passive circuits. (Lossy circuits ⇒ RLC circuits

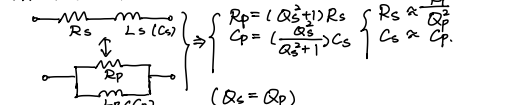
$\text{Loss} = L = \frac{P_{in}}{P_{out}} = \frac{V_{in}^2}{V_{Tnev}^2} \cdot \frac{R_{out}}{R_s}$. $NF = L \cdot \frac{N_{out}}{N_{in}} = L$. $N_{out} = N_{in}$

$\Rightarrow$ The noise not attenuated through the Lossy circuits.

Eg.: 

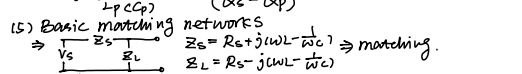
$\Rightarrow NF = NF_1 + \frac{NF_2 - 1}{A_{p1}} = L + \frac{NF_{LNA} - 1}{L^{-1}} = L \cdot NF_{LNA}$.

(4) Series-to-Parallel. Parallel-to-Series



$\Rightarrow \begin{cases} R_p = (Q_s^2 + 1)R_s \\ C_p = (\frac{Q_s^2}{Q_s^2 + 1})C_s \end{cases}$ $\begin{cases} R_s \approx \frac{R_p}{Q_p^2} \\ C_s \approx C_p \end{cases}$ $(Q_s = Q_p)$

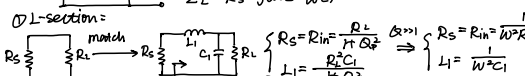
(5) Basic matching networks



$\Rightarrow Z_s = R_s + j(\omega L - \frac{1}{\omega C}) \Rightarrow$ matching.

$Z_L = R_s - j(\omega L - \frac{1}{\omega C})$

① L-section:



$\begin{cases} R_s = R_{in} = \frac{1}{1 + Q^2} \\ L = \frac{R_L C_1}{1 + Q^2} \end{cases}$ $(Q \gg 1)$ $\Rightarrow \begin{cases} R_s = R_{in} = \frac{1}{\omega^2 R_L C_1^2} \\ L = \frac{1}{\omega^2 C_1} \end{cases}$

$R_{in} = \text{Re}[Z_{in}] < R_L$

若特C移动到L前端. 則 $R_{in} = (1 + Q^2) \cdot R_L > R_L$. 互換 L, C 相同.

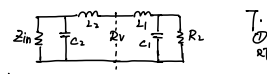
求解元件參數. 先假設 $Q \gg 1$, 計算之后求 Q 是否滿足假設, 否則重新計算.

② π-section:

$Q_{input} = \sqrt{\frac{R_L}{R_V} - 1}$. $Q_{left} = \sqrt{\frac{\text{Re}(Z_{in})}{R_V} - 1}$.

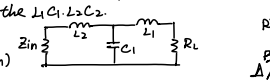
$\Rightarrow Q = \sqrt{\frac{R_L}{R_V} - 1} + \sqrt{\frac{\text{Re}(Z_{in})}{R_V} - 1}$.

Given Q, solve the R_V, then solve the $L_1 C_1$, $L_2 C_2$.

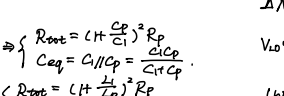


③ T-section:

$R_V = (Q^2 + 1)R_L$. $R_V = (Q^2 + 1)\text{Re}(Z_{in})$



④ Tapped capacitor match:



$\Rightarrow \begin{cases} R_{out} = (1 + \frac{C_p}{C_1})^2 R_p \\ C_{eq} = C_1 // C_p = \frac{C_1 C_p}{C_1 + C_p} \end{cases}$

⑤ Tapped inductor match ⇒ $\begin{cases} R_{out} = (1 + \frac{L_1}{L_p})^2 R_p \\ L_{eq} = L_1 + L_p \end{cases}$

↳ 上圖: L → C. C → L.

(6) Lossy in matching networks

$NF = \text{Loss} = \frac{P_{in}}{P_{out}} = \frac{P_{in}}{P_L} = 1 + \frac{R_s}{R_{in}}$ (串聯電感). $1 + \frac{R_{in}}{R_p}$ (并聯電感)

3. Build a receiver

① Direct filtering: $\left|\frac{V}{V_{max}}\right| = \frac{1}{\sqrt{1 + (2Q \cdot \frac{\Delta\omega}{\omega})^2}}$.

② Add intermediate stages

To use the IRF (Remove the Image), Should have high IF

$\begin{cases} f_{LO} < f_{RF} = RF - LO = IF. \ I_m = LO - IF = LO - (RF - LO) = 2LO - RF \\ f_{LO} > f_{RF} = LO - RF = IF. \ I_m = LO + IF = LO + (LO - RF) = 2LO - RF. \end{cases}$

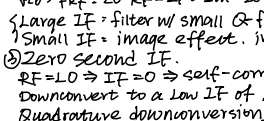
$\begin{cases} \text{Large IF: filter w/ small Q fator, reversible} \\ \text{Small IF: image effect. irreversible.} \end{cases}$

③ Zero second IF.

$RF = LO \Rightarrow IF = 0 \Rightarrow$ self-corruption

Downconvert to a low IF of half the bandwidth ⇒ new Im.

Quadrature downconversion:



④ Sliding IF ⇒ Input f_1. $f_{IF1} = f_1 - f_{LO1}$. $f_{IF2} = f_1 - f_{LO1} - f_{LO2} = 0$

second LO: divide-by-2 circuit: $f_{LO1} = \frac{2f_1}{3}$. $f_{LO2} = \frac{1}{2}f_{LO1} = \frac{f_1}{3}$

$BW_{RF,frac} = \frac{f_2 - f_1}{(f_2 + f_1)/2}$, $BW_{IF,frac} = \frac{f_2/3 - f_1/3}{(f_2/3 + f_1/3)/2}$.

⑤ Direct conversion receiver (DCR)

LO leakage: interference

DC offset: f = 0Hz. caceled by the high pass filter.

Even order distortion and flicker noise

I/Q mismatch: Gain / Phase mismatch

⑥ Image-reject-receiver → RC-CR network

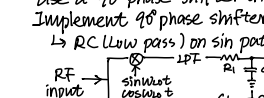
	90° phase shifter	High-side sine	Low-side sine
Positive freq.	$-j$	$-j$	$+j$
Negative freq.	$+j$	$+j$	$-j$

△ Hartley receiver → Adding

Use a 90° phase shifter on Q path to remove the Image.

Implement 90° phase shifter: RC-CR network

↳ RC (low pass) on sin path, CR (high pass) on cos path



gain mis: $\varepsilon = \frac{\Delta R}{R_1} + \frac{\Delta C}{C_1}$

phase mis: $\Delta\theta$

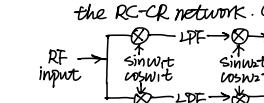
Image reject ratio (IRR) = $\frac{\alpha_{Im}^2(t)}{\alpha_{sig}^2(t)} = \frac{P_{Im}}{P_{sig}}\Big|_{out}$

$IRR = \frac{(1+\varepsilon)^2 + 2(1+\varepsilon)\cos\Delta\theta + 1}{2(1+\varepsilon)\cos\Delta\theta + 1}$, $\begin{matrix} \Delta\theta \ll 1 \\ \varepsilon \ll 1 \end{matrix} \Rightarrow IRR \approx \frac{4}{\varepsilon^2 + \Delta\theta^2}$.

mismatch in LO or in RC-CR network.

△ Weaver architecture

↳ Use second low-side injection mixer to replace the RC-CR network. (Substract I/Q branch)



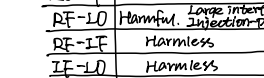
4. Transmitters.

Relative Carrier Leakage = $\frac{\sqrt{V_{DS1}^2 + V_{DS2}^2}}{\sqrt{A_2^2(t)}}$. $\begin{cases} 1\text{dB}: 2.40 \sim 2.48 \\ 1°: 3.15 \sim 3.35 \ 5.75 \sim 5.85 \end{cases}$

Use the feedback loop to cancel it.

	Direct Comersion	Heterodyne
LO-RF	Harmful. DC component	Harmless
LO-IF	Harmless	Harmful. Desentization
RF-LO	Harmful. Large interfer Injection-pull	Harmless
RF-IF	Harmless	Harmless
IF-LO	Harmless	Harmful. Oscillator pulling
IF-RF	Harmless	Harmless

5. MOSFET (Review) Hybrid π-model of NMOSFET



Current gain: $A_I = \left|\frac{I_o}{I_{in}}\right| = \frac{g_m}{\omega(C_{gd} + C_{gs})}$

Unit gain frequency ($A_I = 1$): $\omega_T = \frac{g_m}{C_{gd} + C_{gs}}$.

6. Low noise amplifier (LNA). ⇒ the first stage of receiver

↳ Amplifies weak signal, add low noise (NF ≥ 3dB)

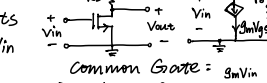
Input matching needed, nonlinearity: IIP3.

High gain ⇒ Smaller both NF and IIP3.

$NF = NF_1 + \frac{NF_2 - 1}{A_1^2} + \cdots + \frac{NF_m - 1}{A_1^2 \cdots A_m^2}$. $\frac{1}{A_{IIP3}^2} \approx \left|\frac{1}{A_{IIP3,1}^2} + \frac{\alpha_1^2}{A_{IIP3,2}^2}\right|$

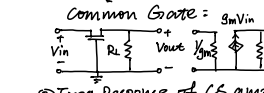
① Single-stage analog amplifier (two types used in LNA)

Common Source:



$\Rightarrow \begin{cases} R_{in} = \infty \\ R_{out} = r_o // R_D \\ A_V = -g_m(r_o // R_D) \end{cases}$ Moderate Large Large

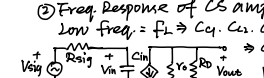
Common Gate:



$\Rightarrow \begin{cases} R_{in} = 1/g_m \\ R_{out} = R_L \\ A_V = g_m R_L \end{cases}$ Low Large Large

② Freq. Response of CS amplifier.

Low freq.: $f_L \Rightarrow C_{g1}, C_{s1}, C_s$. High freq.: $f_H \Rightarrow C_{gs}, C_{gd}, C_{ds}$

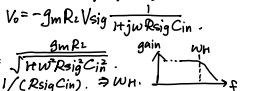


$\Rightarrow C_{in} = C_{gs} + C_{eq} = C_{gs} + C_{gd}(1 + g_m R_L)$

$V_o = -g_m R_L V_{sig} \frac{1}{1 + j\omega R_{sig} C_{in}}$

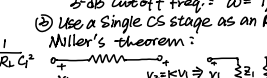
$\Rightarrow$ Voltage gain $= |A_V| = \frac{V_o}{V_{sig}} = \frac{g_m R_L}{\sqrt{1 + \omega^2 R_{sig}^2 C_{in}^2}}$

↳ dB cutoff freq.: $\omega = 1/(R_{sig} C_{in})$. $\Rightarrow \omega_H$.



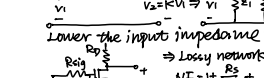
③ Use a single CS stage as an RF amplifier.

Miller's theorem:



$V_2 = KV_1 \Rightarrow V_1 \to Z_1$, $V_2 \to KV_1$ $\begin{cases} Z_1 = \frac{Z}{1 - K} \\ Z_2 = \frac{Z}{1 - \frac{1}{K}} \end{cases}$

Lower the input impedance:



$\Rightarrow$ Lossy network + CS amplifier.

$NF = 1 + \frac{R_s}{R_p} + \frac{\gamma R_s}{g_m(R_s // R_p)^2} + \frac{R_s}{g_m^2(R_s // R_p)^2 R_D}$.

noise

7. Mixer (two means of construct mixer)

① Using an ideal switch as a mixer (switch mixer)

$\begin{cases} y(t) = x(t) \cdot p(t) \\ Y(f) = X(f) * P(f) \end{cases}$ $V_{IF}(f) = \sum_{n=-\infty}^{\infty} \frac{\sin(n\pi/2)}{n\pi} V_{RF}(f - \frac{n}{T})$

switch: $V_{IF}(t) = V_{RF}(t) \cdot g(V_{LO}(t))$. $g(\cdot)$: nonlinear

multiplier: $V_{IF}(t) = V_{RF}(t) \cdot V_{LO}(t)$. $g(\cdot) = \cdot$

Both can be implemented by MOSFET

A MOSFET as a switch

A MOSFET as a multiplier

$\Rightarrow I_D = \frac{\mu_n C_{ox} W}{2L}(V_{LO} - V_{RF} - V_t)^2$

cross term: $\mu_n C_{ox}\frac{W}{L} V_{LO} V_{RF}$.

(work in linear regime) (saturation regime)

Multiplier mixer: conversion gain $= V_{IF}/V_{RF} = \mu_n C_{ox}\frac{W}{2L} V_{LO}$.

↳ conversion gain is higher than switch

switch mixer: better in linearity and feedthrough

Low efficiency (IF output contains DC, harmonic ...).

② Feedthrough in DCR.

LO ⇒ RF = LO leakage, DC offset

LO ⇒ IF: could be filtered out

RF ⇒ LO: injection pulling (large inbanel interferer)

RF ⇒ IF: IP2.

③ Feedthrough in superheterodyne

LO ⇒ RF: LO's freq. is different from RF's by a lot. could be filtered out by LNA, band-select filter, antenna.

LO ⇒ IF: desentize the amplifier. could be severe if f_{LO} is close to f_{IF}.

IF ⇒ LO: Oscillator pulling.

∵ The LO ⇒ RF feedthrough: Cancelled by DCR w/ quadrature.

④ Single-balanced and Double-balanced mixers

Both can cancel the LO ⇒ RF feedthrough.

⑤ Passive downconversion mixers

△ Return-to-zero mixer: switch mixer

switch function: $S(f) = \sum \frac{\sin(n\pi/2)}{n\pi}\delta(f - nf_{LO})$.

$= \frac{1}{2} + \frac{2}{\pi}\cos 2\pi f_{LO} t - \frac{2}{3\pi}\cos 6\pi f_{LO} t + \cdots$

conversion gain: $1/\pi$ (~ 10dB)

△ single-balanced mixer = conversion gain $= \frac{2}{\pi}$ (~ 4dB).

△ double-balanced mixer: $\frac{2}{\pi}$ (~ 4dB)

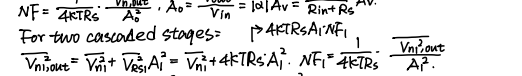
△ Sampling mixer (non return-to-zero). Load resistor → capacitor.

The conversion gain for passive mixers are < 1.

⑥ Active downconversion mixer

Single-balanced active mixer: gain $\frac{2}{\pi} g_m R_D$.

8. Oscillator

(1) 

$A(s) = \frac{Y}{X}(s) = \frac{H(s)}{1 + H(s)}$.

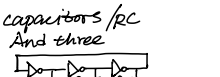
Startup cond: $H(s) = -1$

$\Rightarrow |H(s = j\omega_1)| = 1$. $\angle H(s = j\omega_1) = 180°$.

(2) Tuned oscillator

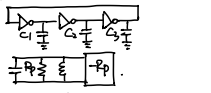
(3) Cross-coupled oscillator

(4) Ring oscillator: use multiple capacitors / RC network to get 180° phase shift. And three invertors provide 180°. Total 360°.

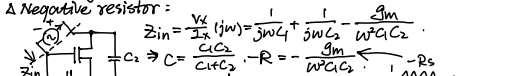


(5) One-port view of oscillators

Use negative resistor to cancle the lossy (resistor) of RLC tank.



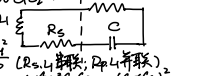
△ Negative resistor:



$Z_{in} = \frac{V_x}{I_x}(j\omega) = \frac{1}{j\omega C_1} + \frac{1}{j\omega C_2} - \frac{g_m}{\omega^2 C_1 C_2}$.

$\Rightarrow C = \frac{C_1 C_2}{C_1 + C_2}$. $-R = -\frac{g_m}{\omega^2 C_1 C_2}$

Resonance: $\omega = \sqrt{\frac{C_1 C_2}{L(C_1 + C_2)}}$



$Q = \frac{\omega L}{R_s} = \frac{R_p}{\omega L} \overset{Q \gg 1}{\Rightarrow} R_s \approx \frac{R_p}{Q^2} = \frac{\omega^2 L^2}{R_p}$ (R_s, L 串聯; R_p, L 并聯)

Startup condition: $R_s = \frac{g_m}{\omega^2 C_1 C_2} \Rightarrow R_p = \frac{\omega^2 L^2 C_1 C_2}{g_m} = \frac{(C_1 + C_2)^2}{g_m C_1 C_2}$

$\Rightarrow g_m R_p = \frac{(C_1 + C_2)^2}{C_1 C_2} \geq 4$.

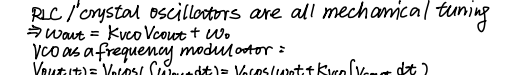
Miller's theorem: $Z_1 = \frac{Z}{1 - K}$. $K = g_m Z_L$. $Z = \frac{1}{j\omega C_1}$.

$\Rightarrow Z_{eq} = \frac{Z}{1 - K} = -\frac{g_m C_1 C_2 + j\omega C_1 C_2}{\omega^2 C_1 C_2 + j\omega C_1 g_m}$.

(6) Three-point oscillators

use negative resistor ⇒ add an inductor

① Colpitts oscillators (gate grounded)



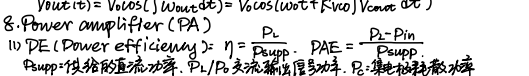
$\Rightarrow KVL: V_2 + (-V_1) + V_3 = 0$. $(V_1 = -\alpha V_2)$. $Z_1 \cdot Z_2$.

Z_1, Z_2 must be both C or L. Z_3 different from

$\Rightarrow Z_1 = Z_2 = C$. $Z_3 = L$. then $f = 2\pi\sqrt{\frac{C_1 C_2}{L(C_1 + C_2)}}$.

Switch the C, L ⇒ Hartley.

(7) Crystal oscillator (XO)



series (nontrual) freq.: $\omega_s = \frac{1}{\sqrt{L_1 C_1}}$.

Parallel freq.: $\omega_p = \sqrt{\frac{C_1 C_2}{L_1(C_1 + C_2)}}$. $\omega_p > \omega_s$.

(8) Voltage controlled oscillators (VCO)

RLC / crystal oscillators are all mechanical tuning

$\Rightarrow \omega_{out} = K_{VCO} V_{control} + \omega_0$

VCO as a frequency modulator:

$V_{out}(t) = V_0\cos(\omega_{out} dt) = V_0\cos(\omega_0 t + K_{VCO}\int V_{control}\,dt)$

8. Power amplifier (PA)

(1) PE (Power efficiency): $\eta = \frac{P_L}{P_{supp}}$. $PAE = \frac{P_L - P_{in}}{P_{supp}}$.

P_{supp}: 供給的直流功率. P_L/P_R 交流輸出信号功率. P_c: 集电极耗散功率

$P_{supp} = P_o + P_c$

(2) LNA: (input) received signal is very small. Input impedance matching is the key. Linearity: compression point, IIP3 ⇒ interferer

Noise must be very low (Friis' equation ⇒ NF).

PA: (output) Output impedance matching not important.

Power efficiency is important. Linearity is extremly important

The input/output signal is huge.

Small signal: $g_m \approx C$. Large signal: g_m: $f \to V_{gs}$.

(3) Generic PA model.

(4) Class A PA.

⇒ Transistors remain on across the entire range.

$i_D = I_{DC} + i_{RF}\sin\omega t$, $V_{out} = -i_{RF} R_L \sin\omega t$. ($I_{DC} > i_{RF}$)

$P_{out} = \frac{V_{out,peak}^2}{2R_L} = \frac{i_{RF}^2 R_L}{2}$. $P_{DC} = I_{DC} V_{DD} \geq i_{RF} V_{DD}$.

$\eta = \frac{P_{out}}{P_{DC}} = \frac{i_{RF}^2 R_L / 2}{I_{DC} V_{DD}} \leq \frac{i_{RF} R_L}{2V_{DD}} \leq \frac{I_{DC} R_{in}}{2V_{DD}} = \frac{I_{DC} R_{in}}{2V_{DD}} = \frac{1}{2} = 50\%$

(If no impedance matching. $R_{in} = R_L$).

(5) Class B PA.

$\eta = \frac{P_{out}}{P_{DC}} = 1 - \frac{P_{DS}}{P_{DC}}$. ($P_{DS}$: dissipated efficiency)

fundamental component:

$i_{fund} = \frac{2}{T}\int_0^{T/2} i_{RF}\sin\omega t \sin\omega t\,dt = \frac{1}{2} i_{RF}$. $V_{out} \approx \frac{1}{2} i_{RF} R \sin\omega t$.

$V_{out}(max) = V_{DD} = \frac{1}{2} i_{RF} R \Rightarrow i_{RF} = \frac{2V_{DD}}{R_L}$ (class A: $i_{RF} = \frac{V_{DD}}{R_L}$)

$P_{out} = \frac{V_{out}^2}{2R_L} \leq \frac{V_{DD}^2}{2R_L}$. $i_{fund} = \frac{1}{2} i_{RF} = \frac{V_{DD}}{R_L}$.

$\overline{i_D} = \frac{1}{T}\int_0^{T/2} \frac{V_{DD}}{R_L}\sin\omega dt = \frac{2V_{DD}}{\pi R_L} \Rightarrow P_{DC} = \overline{i_D} V_{DD} = \frac{2V_{DD}^2}{\pi R_L}$

$\eta = \frac{P_{out}}{P_{DC}} \leq \left(\frac{V_{DD}^2}{2R_L}\right) / \left(\frac{2V_{DD}^2}{\pi R_L}\right) = \frac{\pi}{4} \approx 78.5\%$.