CH02 Signals and random process.
$n_0(t)=n_w(t)*h_R(t)$

1. $\sin\alpha\cos\beta=\frac12[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$ $\sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$
$\cos\alpha\sin\beta=\frac12[\sin(\alpha+\beta)-\sin(\alpha-\beta)]$ $\sin\alpha-\sin\beta=2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$
$\cos\alpha\cos\beta=\frac12[\cos(\alpha+\beta)+\cos(\alpha-\beta)]$ $\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$
$\sin\alpha\sin\beta=-\frac12[\cos(\alpha+\beta)-\cos(\alpha-\beta)]$ $\cos\alpha-\cos\beta=-2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$
2. Energy $E_x=\lim_{T\to\infty}\int_{-T/2}^{T/2}|x(t)|^2dt$. Power $P_x=\lim_{T\to\infty}\frac1T\int_{-T/2}^{T/2}|x(t)|^2dt$.
3. $\Pi(t)=\begin{cases}1, & -1/2<t\le 1/2\\ 0, & \text{o.w.}\end{cases}$ $\Lambda(t)=\begin{cases}t+1, & -1\le t<0\\ -t+1, & 0\le t\le 1\end{cases}$ $\mathrm{sinc}(t)=\begin{cases}\frac{\sin(\pi t)}{\pi t}, & t\ne0\\ 1, & t=0\end{cases}$ $\Lambda(t)\to\mathrm{sinc}^2(f)$
4. correlation $=R_{XY}=E(XY)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}xyf_{XY}(x,y)dxdy$. $\sigma_{XY}=E(XY)-E(X)E(Y)$
Auto-correlation function $=R_X(t_1,t_2)=E(X(t_1)X(t_2))=\int\int x_1x_2f(x_1,x_2;t_1,t_2)dx_1dx_2$
5. WSS: ① $E[X(t)]=\int_{-\infty}^{\infty}xf_X(x)dx=m_X$. ② $R_X(t_1,t_2)$: $\tau=t_2-t_1$.
6. $\overline{x}(t)=E[X(t)]$. $R_X(t_1,t_2)$. $\langle x(t)\rangle=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}x(t)dt$. $\langle x(t)x(t-\tau)\rangle$
7. $E_T=\int_{-\infty}^{\infty}x_T^2(t)dt=\int_{-\infty}^{\infty}|X_T(f)|^2df=\int_{-T}^{T}x^2(t)dt$. $S_X(f)=\lim_{T\to\infty}\frac{|X_T(f)|^2}{2T}$. $P=\int_{-\infty}^{\infty}S_X(f)df$
8. $R_X(\tau)=\int_{-\infty}^{\infty}S_X(f)e^{j2\pi f\tau}df$. $S_X(f)=\int_{-\infty}^{\infty}R_X(\tau)e^{-j2\pi f\tau}d\tau$. $R_X(0)=P=E[X^2(t)]$
9. $\overline{y}(t)=\overline{x}\cdot H(0)$. $R_Y(\tau)=h(\tau)*h(-\tau)*R_X(\tau)$. $S_Y(f)=|H(f)|^2S_X(f)$
10. ① $S_n(f)=N_0/2$. $R_n(\tau)=N_0/2\,\delta(\tau)$. ② $S_n(f)=\frac{N_0}{2}\Pi(\frac{f}{2B})$. $R_n(\tau)=N_0B\,\mathrm{sinc}(2B\tau)$

CH03 Anolog Modulation
1. ① DSBSC $=S(f)=\frac12A_c[M(f-f_c)+M(f+f_c)]$. $B=2W$. $P_s=\frac12A_c^2P_m$.
② conventional AM: $s(t)=A_c[1+am_n(t)]\cos\omega_ct$. $B=2W$. $P_s=\frac12A_c^2+\frac12a^2A_c^2P_{m_n}$
$S(f)=\frac12A_c[\delta(f-f_c)+\delta(f+f_c)]+\frac12aA_c[M_n(f-f_c)+M_n(f+f_c)]$, $E=(a^2P_{m_n})/(1+a^2P_{m_n})$
③ SSB-AM: $B_c=W$. $S=\frac14A_c^2S_x$. ④ VSB: $B=W+f_v$
2. PM: $\theta(t)=k_pm(t)$. FM: $\frac{1}{2\pi}\frac{d\theta(t)}{dt}=k_fm(t)\Rightarrow\theta(t)=2\pi k_f\int_{-\infty}^{t}m(\tau)d\tau$
3. $m(t)=A_m\cos(2\pi f_mt)\Rightarrow\Delta f=k_fA_m$. $\beta=\Delta f/f_m$ (FM) $B=2\Delta f+2f_m=2(1+\beta)f_m$
$s(t)=A_c\sum J_n(\beta)\cos[2\pi(f_c+nf_m)t]$. $S(f)=\frac12A_c\sum J_n(\beta)[\delta(f-f_c-nf_m)+\delta(f+f_c+nf_m)]$
4. $m(t)\to W$. $\beta=\Delta f/W$. $\Delta f=k_f\max|m(t)|$. $B=2(1+\beta)W$.
5. NBFM($\theta(t)\ll1$): $s(t)\approx A_c\cos(2\pi f_ct)-A_c\sin(2\pi f_ct)\theta(t)$. $\theta(t)=2\pi k_f\int_{-\infty}^{t}m(\tau)d\tau$
n次倍频: $s(t)=a_1s(t)+a_2s^2(t)+\dots+a_ns^n(t)\Rightarrow f_c'=nf_c$. $\Delta f'=n\Delta f$. $k_f'=nk_f$

CH04 ADC
$f_s=2W=1/T_s$ (Nyquist).
1. $x_\delta(t)=\sum x(nT_s)\delta(t-nT_s)=x(t)\sum\delta(t-nT_s)$. $X_\delta(f)=X(f)\frac{1}{T_s}\sum\delta(f-\frac{n}{T_s})=\frac{1}{T_s}\sum X(f-\frac{n}{T_s})$
2. LPF $=H(f)=T_s\Pi(\frac{f}{2W'})$. $h(t)=2W'T_s\,\mathrm{sinc}(2W't)$. $W\le W'<\frac{1}{T_s}-W$. $W'=W$.
$x(t)=\sum 2W'T_sx(nT_s)\mathrm{sinc}[2W'(t-nT_s)]=x_\delta(t)*h(t)$.
3. $e=X-Q(X)$. $D=E[(X-Q(X))^2]=\int_{-x_p}^{x_p}(x-Q(x))^2f_X(x)dx$. SQNR $=\frac{E(X^2)}{E[(X-Q(X))^2]}=\frac{P_x}{E[e^2]}$
4. uniform: $N=2^\nu$. $\Delta=\frac{2a}{N}=\frac{a}{2^{\nu-1}}$. $-\Delta/2\le e\le\Delta/2$ $10\lg(SQNR)=10\lg\frac{3\times4^\nu\times P_x}{a^2}$
$P=E(e^2)=\int_{-\Delta/2}^{\Delta/2}\frac1\Delta x^2dx=\Delta^2/12=a^2/3N^2=a^2/(3\cdot4^\nu)$. SQNR $=10\log_{10}\frac{P_x}{a^2}+6\nu+4.8$(dB)
5. ν bits per symbol. f_s samples per second $\Rightarrow R_b=\nu f_s$ (bits/sec).
$f_s=2W\Rightarrow BW=\frac12R_b=\frac12\nu f_s\ge\nu W$ (Hz). ($f_s=2W+f_G$) ($\hat{x}_1=\hat{x}_m=-(\frac{m}{2}-1)\Delta-\frac12\Delta$) ($a_1=a_{m-1}=-(\frac{m}{2}-1)\Delta$)

CH05 Digital Transmission through Bandlimited Channels
1. $y(t)=x_s(t)*p(t)+n_0(t)$. $p(t)=h_T(t)*h_c(t)*h_R(t)$. $n_0(t)=n(t)*h_R(t)$.
$y(t_m)=\sum A_kp(mT-kT)+n_0(t_m)\Rightarrow y(t_m)=A_m+n_0(t_m)$. $P(f)=H_T(f)H_c(f)H_R(f)$ 最小带宽 $\frac{1}{2T}$
$P(f)=1$. $|f|<\frac{1}{2T}\Rightarrow p(t)=\mathrm{sinc}(t/T)=\sin(\pi t/T)/(\pi t/T)$. $B_0=\frac{1}{2T}=\frac12R_s$. $R_s=2B_0$ (矩形)
2. $P(f)=\begin{cases}1 & 0\le|f|<f_1\\ \frac12\{1+\cos[\frac{\pi(|f|-f_1)}{2B_0-2f_1}]\} & f_1\le|f|<2B_0-f_1\\ 0 & |f|\ge2B_0-f_1\end{cases}$ | $\alpha=1-\frac{f_1}{B_0}\Rightarrow B=B_0(1+\alpha)=\frac{1+\alpha}{2T}$; $B_0=\frac{1}{2T}$. $f_1=B_0(1-\alpha)$.
$p(t)=\mathrm{sinc}(2B_0t)\frac{\cos(2\pi\alpha B_0t)}{1-16\alpha^2B_0^2t^2}$. $B_0=\frac{1}{2T}$ | $G(f)=(1+e^{-j2\pi fT})H_d(f)=2Te^{-j\pi fT}\cos\pi fT$. $|f|\le\frac{1}{2T}$
3. Duobinary signal $=g(t)=[\delta(t)+\delta(t-T)]*h_d(f)=\mathrm{sinc}(\frac tT)+\mathrm{sinc}(\frac{t-T}{T})=\frac{T^2}{\pi t}\frac{\sin(\pi t/T)}{(T-t)}$
4. Modified: $G(f)=(1-e^{-j4\pi fT})H_d(f)=2Tje^{-j2\pi fT}\sin2\pi fT$. $|f|\le\frac{1}{2T}$.
$g(t)=\mathrm{sinc}(\frac tT)-\mathrm{sinc}(\frac{t-2T}{T})=-\frac{2T^2\sin(\pi t/T)}{\pi t(t-2T)}\Rightarrow C_k=a_k\oplus C_{k-2}$. $b_k=C_k-C_{k-2}$
5. Precoding for duobinary signal: $C_k=a_k\oplus C_{k-1}$. $b_k=C_k+C_{k-1}$.
6. optimum transmit/receive filter $=|H_T(f)|=|H_R(f)|=\sqrt{P_{rc}(f)}/|H_c(f)|^{1/2}$. $|f|\le W$
$E_{av}=\int_{-\infty}^{\infty}d^2H_T^2(f)df=d^2\int_{-W}^{W}H_T^2(f)df$. $\sigma^2=\frac{N_0}{2}\int_{-W}^{W}|H_R(f)|^2df$. $SNR=\frac{2E_{av}}{N_0}[\int_{-W}^{W}\frac{P_{rc}(f)}{|H_c(f)|}df]^{-2}$
7. Equalizer: $H_0(f)=H_T(f)H_c(f)H_E(f)$.
(ZF) $\to(H_T(f),H_c(f))\to P_c(f)\to(H_E(f))\to P_{eq}(t)$. $h_E(t)=\sum_{n=-N}^{N}C_n\delta(t-nT)$.
$P_{eq}(mT)=\sum_{n=-N}^{N}C_nP_c[(m-n)T]=\begin{cases}1, & m=0\\0, & m=\pm1,\dots,\pm N\end{cases}\Rightarrow P_{eq}(m)=\sum_{n=-N}^{N}C_nP_c(m-n)$ equalized
$P_{eq}=(0,0,0,\dots,0,1,0,\dots,0,0)$
$C_n=(C_{-N},C_{-N+1},\dots,C_{-1},C_0,C_1,\dots,C_N)$. $P_c=\begin{pmatrix}P_c(0) & P_c(-1) & \cdots & P_c(-2N)\\ P_c(1) & P_c(0) & \cdots & P_c(-2N+1)\\ \vdots & & & \\ P_c(2N) & P_c(2N-1) & \cdots & P_c(0)\end{pmatrix}$. $C=P_c^{-1}P_{eq}$
$P_{eq}=P_cC$

CH06 Signal Space Representation.
1. basis $=\{\phi_k(t)\}_{k=1}^{n}$. $\int_{-\infty}^{\infty}\phi_j(t)\phi_k(t)dt=\begin{cases}0, & j\ne k\\1, & j=k\end{cases}$. $x(t)=\sum_kx_k\phi_k(t)$. $x_k=\int_{-\infty}^{\infty}x(t)\phi_k(t)dt$
2. $\vec{x}=(x_1,x_2\dots x_n)\Rightarrow\vec{x}\cdot\vec{y}=\int_{-\infty}^{\infty}x(t)y(t)dt$. $E_x=\vec{x}\cdot\vec{x}=\|x\|^2=\int_{-\infty}^{\infty}x^2(t)dt$.
3. GSO: Given $\{s_1(t),s_2(t)\dots s_M(t)\}$. Find $\{\phi_1(t),\phi_2(t)\dots\phi_N(t)\}$. $N\le M$.
① $\phi_1(t)=\frac{1}{\sqrt{E}}s_1(t)$. $E_1=\int_{-\infty}^{\infty}s_1^2(t)dt\Rightarrow s_{11}=\int_{-\infty}^{\infty}s_1(t)\phi_1(t)dt=\sqrt{E_1}$. $s_1(t)=s_{11}\phi_1(t)=\sqrt{E_1}\phi_1(t)$
② $s_{21}=\int_{-\infty}^{\infty}s_2(t)\phi_1(t)dt$. $g_2(t)=s_2(t)-s_{21}\phi_1(t)\Rightarrow s_{22}=\sqrt{E_{g_2}}=\int_{-\infty}^{\infty}s_2(t)\phi_2(t)dt$. $\phi_2(t)=\frac{1}{\sqrt{E_{g_2}}}g_2(t)$
$g_2(t)=\sqrt{E_{g_2}}\phi_2(t)$. $s_2(t)=s_{21}\phi_1(t)+s_{22}\phi_2(t)=s_{21}\phi_1(t)+\sqrt{E_{g_2}}\phi_2(t)$.
③ For $s_k(t)$: $s_{ki}=\int_{-\infty}^{\infty}s_k(t)\phi_i(t)dt$, $i=1,2,\dots k-1$. $g_k(t)=s_k(t)-\sum_{i=1}^{k-1}s_{ki}(t)\phi_i(t)$.
$E_{g_k}=\int_{-\infty}^{\infty}g_k^2(t)dt\Rightarrow\phi_k(t)=\frac{1}{\sqrt{E_{g_k}}}g_k(t)$. $s_{kk}=\int_{-\infty}^{\infty}s_k(t)\phi_k(t)dt=\sqrt{E_{g_k}}$
$s_k(t)=\sum_{i=1}^{k-1}s_{ki}(t)\phi_i(t)+\sqrt{E_{g_k}}\phi_k(t)$.
$d_{min}\ge 2t+1$; $d_{min}\ge 2t+1$

CH07 Optimal Receivers
1. $\{m(t)\}\to$(Transmitter)$\xrightarrow{s(t)}$ channel $\oplus\to r(t)=s(t)+n_w(t)\to$(Receiver)$\to\{\hat{m}(t)\}$
$\to p(m_1),p(m_2)\dots p(m_M)$. $n_w(t)\sim S_n(f)=N_0/2$ $\to\vec{r}=\vec{s}+\vec{n}$.
$\Rightarrow r(t)=s_m(t)+n_w(t)=\sum_{k=1}^{N}s_{mk}\phi_k(t)+\sum_{k=1}^{N}n_k\phi_k(t)+n_0(t)\Rightarrow r_k=s_{mk}+n_k$.
2. Receiver: $r(t)\to$(demodulator)$\to\vec{r}\to$(detector)$\to\hat{m}$.
3. Matched Filter: output $s_0(t)+n_0(t)$
$d=\frac{s_0^2(t_0)}{E[n_0^2(t)]}=\frac{[\int_{-\infty}^{\infty}A(f)H(f)e^{j\omega t_0}df]^2}{N_0/2\int_{-\infty}^{\infty}|H(f)|^2df}\le\frac{\int_{-\infty}^{\infty}|A(f)|^2df\int_{-\infty}^{\infty}|H(f)|^2df}{N_0/2\int_{-\infty}^{\infty}|H(f)|^2df}=\frac{\int_{-\infty}^{\infty}|A(f)|^2df}{N_0/2}=\frac{2E}{N_0}$
$\Rightarrow H_m(f)=A^*(f)e^{-j\omega t_0}$. $h_m(t)=s_i(t_0-t)$, $0\le t<t_0$, $t_0>T$. $\Rightarrow|H_m(f)|=|A(f)|$.
$y(t)=x(t)*h_m(t)=x(t)*s_i(T-t)=\int_0^Tx(t)s_i(t)dt$. 与相关积分.
$x(t)\to$(matched filter)$\xrightarrow{t=T}y(T)=x(t)\to\otimes(s_i(t))\to\int_0^T(\cdot)dt\to y(T)$.

4. MF output noise $=E[n_0^2(t)]=R_{n_0}(0)=\frac{N_0}{2}\int_{-\infty}^{\infty}s_i^2(t)dt=\frac{N_0}{2}\int_{-\infty}^{\infty}|A(f)|^2df=\frac{N_0}{2}\int|H_m(f)|^2df=\frac{N_0}{2}E$.
5. MF for colored noise: $H(f)=[A^*(f)/S_n(f)]e^{-j2\pi ft_0}$.
6. ① Correlation Type demodulator ② Matched Filter Type ($t=T$)
$r(t)\to\otimes(\phi_i(t))\to\int_0^T(\cdot)dt\to r_i\dots\vec{r}$ $r(t)\to(\phi_i(T-t))\to r_i\dots\vec{r}$
7. $n_k=\int_0^Tn_w(t)\phi_k(t)dt\sim N(0,N_0/2)$ ($p\le\int_{-\infty}^{\infty}r(t)s_m(t)dt=\vec{r}\cdot\vec{s}_m$)
8. $C(\vec{r},\vec{s}_m)=\int_{-\infty}^{\infty}r(t)s_m(t)dt-\frac12\int_{-\infty}^{\infty}|s_m(t)|^2dt+\frac{N_0}{2}\ln p(s_m)$.
$\vec{r}=(r_1,r_2\dots r_N)$: $=\vec{r}\cdot\vec{s}_m-\frac12E_m+\frac12N_0\ln P_m$. ($s_m(t)$ is transmitted $\to M=1,2,\dots M$)
9. MAP $=\|\vec{r}-\vec{s}_k\|^2-N_0\ln P_k$. ML: $\|\vec{r}-\vec{s}_k\|^2$.
10. Binary Data Transmission: $p(m_1)=p(m_2)$. $P_e=Q(\sqrt{\frac{2E}{N_0}})=Q(\frac{d/2}{\sqrt{N_0/2}})=Q(\sqrt{\frac{d_{min}^2}{2N_0}})$
General: $P_e=\frac1M\sum p(e|m_k)\le\frac1M\sum_{k=1}^{M}\sum_{j=1,j\ne k}^{M}Q(\sqrt{d_{kj}^2/2N_0})\le(M-1)Q(\sqrt{\frac{d_{min}^2}{2N_0}})$
union bound $\to$ high SNR $\to$ maximize d_{min}
$P_m=d_m\oplus P_{m-1}$ (mod M). $a_m=2m-(M+1)$. $b_m=a_m+a_{m+1}$. $d_m=\frac{b_m}{2}+(M+1)$ (mod M)

CH08 Digital Modulation Techniques
1. BPSK: "1" $\to s_1(t)=\sqrt{2E_b/T_b}\cos(2\pi f_ct)$. "0" $\to s_2(t)=\sqrt{2E_b/T_b}\cos(2\pi f_ct+\pi)=-\sqrt{2E_b/T_b}\cos(2\pi f_ct)$
$\phi(t)=\sqrt{2/T_b}\cos(2\pi f_ct)\Rightarrow s_1=\sqrt{E_b}$. $s_2=-\sqrt{E_b}$.
$s_1(t)\Rightarrow r_1=s_1+n$. $P(e|s_1)=P(r<0|s_1)=\int_{-\infty}^{0}\frac{1}{\sqrt{\pi N_0}}\exp[-\frac{(r-\sqrt{E_b})^2}{N_0}]dr=Q(\sqrt{\frac{2E_b}{N_0}})$
$r(t)=\pm\sqrt{2E_b/T_b}\cos(2\pi f_ct+\theta)+n(t)\to\otimes\sqrt{2/T_b}\cos(2\pi f_ct+\theta)\to\int_0^{T_b}dt\to r$
2. BFSK: "1" $\to s_1(t)=\sqrt{2E_b/T_b}\cos(2\pi f_1t)$. "0" $\to s_2(t)=\sqrt{2E_b/T_b}\cos(2\pi f_2t)$
$\Delta f=f_1-f_0\Rightarrow\int_0^{T}s_1(t)s_2(t)dt=0$. $\Delta f=1/T_b$.
$\phi_1(t)=\sqrt{2/T_b}\cos(2\pi f_1t)$. $\phi_2(t)=\sqrt{2/T_b}\cos(2\pi f_2t)\Rightarrow s_1=[\sqrt{E_b}\ 0]$. $s_2=[0\ \sqrt{E_b}]$
$s_1(t)\Rightarrow r_1=\sqrt{E_b}+n_1$. $r_2=n_2$. $P(e|s_1)=P(r_2>r_1)=P(n_2-n_1>\sqrt{E_b})=Q(\sqrt{\frac{E_b}{N_0}})$
To the same P_e: $E_b(BFSK)=2E_b(BPSK)$. $E_b(BFSK)=E_b(BPSK)+3$(dB)
3. BASK: "1" $\to s_1(t)=\sqrt{2E/T_b}\cos(2\pi f_ct)$. "0" $\to s_2(t)=0$. $E=2E_b$.
$\phi(t)=\sqrt{2/T_b}\cos(2\pi f_ct)\Rightarrow s_1=\sqrt{E}=\sqrt{2E_b}$. $s_2=0$. $d=\sqrt{2E_b}\Rightarrow P_e=Q(\sqrt{\frac{d^2}{2N_0}})=Q(\sqrt{\frac{E_b}{N_0}})$
4. Non-coherent BFSK: $s_1(t)=\sqrt{2E_b/T_b}\cos(2\pi f_1t+\theta_1)$. $s_2(t)=\sqrt{2E_b/T_b}\cos(2\pi f_2t+\theta_2)$
$\phi_{1c}=\sqrt{\frac2T}\cos f_1$, $\phi_{1s}=\sqrt{\frac2T}\sin f_1$, $\phi_{2c}=\sqrt{\frac2T}\cos f_2$, $\phi_{2s}=-\sqrt{\frac2T}\sin f_2$
$\Rightarrow s_1=[\sqrt{E_b}\cos\theta_1\ \sqrt{E_b}\sin\theta_1\ 0\ 0]$. $s_2=[0\ 0\ \sqrt{E_b}\cos\theta_2\ \sqrt{E_b}\sin\theta_2]$. $P_e=\frac12e^{-\frac{E_b}{2N_0}}$
$\Rightarrow\vec{r}=[r_{1c}\ r_{1s}\ r_{2c}\ r_{2s}]$. $r_{1c}^2+r_{1s}^2\ge r_{2c}^2+r_{2s}^2$ choose s_1.
5. DPSK: $d_i=d_{i-1}\oplus m_i$ (同或). d_0: inital bit.
$m_i\to(\cdot)\oplus(\cdot)\to$(BPSK)$\to$, d_{i-1} delay
$r(t)\to\otimes$ (delay $r(t-T_b)$) $\to\int_0^{T_b}(\cdot)dt\to y$. $y=\int_0^{T_b}r(t)r(t-T_b)=\int_0^{T_b}\cos(\omega_ct+\theta(t))\cos(\omega_ct+\theta(t-T_b))$
if $\theta_k-\theta_{k-1}=0$ ($b(t)=1$) $y>0$. $\theta_k-\theta_{k-1}=\pi$ ($b(t)=0$), $y<0$. $P_e=\frac12\exp(-\frac{E_b}{N_0})$.
6. MPSK: $s_m(t)=\sqrt{2E_s/T}\cos[2\pi f_ct+\theta_m]$. $\theta_m=\frac{2\pi(m-1)}{M}$. $m=1,\dots M$. $M=2^k$
$\phi_1(t)=\sqrt{\frac2T}\cos(2\pi f_ct)$. $\phi_2(t)=-\sqrt{\frac2T}\sin(2\pi f_ct)\Rightarrow s_m=[\sqrt{E_s}\cos\frac{2\pi(m-1)}{M},\sqrt{E_s}\sin\frac{2\pi(m-1)}{M}]$
$\Rightarrow d_{min}=\sqrt{2E_s(1-\cos\frac{2\pi}{M})}=2\sqrt{E_s}\sin\frac{\pi}{M}$. P_e(symbol) $=2Q(\sqrt{\frac{d_{min}^2}{2N_0}})=2Q(\sqrt{\frac{2E_s}{N_0}}\sin\frac{\pi}{M})$
For large M: $E_{b,2M}=4E_{b,M}$. $E_{b,2M}=E_{b,M}+6$dB. to the same P_e.
7. MQAM: $s_i(t)=\sqrt{\frac{2E_0}{T}}a_i\cos(2\pi f_ct)+\sqrt{\frac{2E_0}{T}}b_i\sin(2\pi f_ct)\Rightarrow s_i=[\sqrt{E_0}a_i,\sqrt{E_0}b_i]$
$P_e\le4Q(\sqrt{\frac{3kE_b}{(M-1)N_0}})$. $E_s=kE_b\Rightarrow E_{b,k+1}=2E_{b,k}$. $E_{b,k+1}=E_{b,k}+3$dB, to the same P_e.
8. MFSK: $s_m(t)=\sqrt{2E_s/T}\cos[2\pi(f_c+(m-1)\Delta f)t]$, $m=1\dots M$. $f_m=f_c+m\Delta f$.
$\rho_{mn}=\frac{1}{E_s}\int_0^Ts_m(t)s_n(t)dt=\sin[2\pi(m-n)\Delta fT]/2\pi(m-n)\Delta fT=0\Rightarrow\Delta f_{min}=\frac{1}{2T}$
$\phi_m(t)=\sqrt{\frac2T}\cos[2\pi(f_c+m\Delta f)t]\Rightarrow s_0=(\sqrt{E_s},\dots0)$, $s_{M-1}=(0,\dots\sqrt{E_s})$. $d\equiv\sqrt{2E_s}=\sqrt{2kE_b}$.
9. BER $=P_b=\sum_{i=1}^{M}p(s_i)\sum_{j=1,j\ne i}^{M}\frac{n_{ij}}{\log_2M}P(\hat{s}=\vec{s}_j|\vec{s}_i)$. $\log_2M=K$.
For MPSK with Gray coding: $P_b\approx\frac{P_e}{\log_2M}$. ; For MFSK: $P_b\cong\frac12P_e$.
10. energy efficiency: $\frac{E_b}{N_0}$, bandwidth efficiency $=R_b/B$. B=Hz.
MFSK is more energy efficient than MPSK.
For MPSK/MQAM: $B=\frac{2}{T_s}$ (矩形脉冲). $B=\frac{1+\alpha}{T_s}$ (RC). $\Rightarrow B=\frac{1}{T_s}$. $R_b=\frac{\log_2M}{T_s}$.
$\Rightarrow\rho=\frac{R_b}{B}=\log_2M$ (bits/sec/Hz).
For MFSK: $B=M\Delta f=\frac{M}{2T}$ Hz $\Rightarrow\rho=\frac{R_b}{B}=\frac{2\log_2M}{M}$ (bits/sec/Hz).
11. Channel capacity: $R\le C=B\log_2(1+SNR)=B\log_2(1+\frac{P_s}{N_0B})$. $\begin{cases}P_s=E_b\cdot R\\ \frac12N_0\cdot2B=N_0B\end{cases}$
$\Rightarrow\frac{E_b}{N_0}\ge\frac{B}{R}(2^{\frac{R}{B}}-1)\Rightarrow\frac{E_b}{N_0}=SNR/bit\ge\frac{2^\rho-1}{\rho}$. $\rho=\frac{R_b}{B}$. $\rho\to0$: $\frac{E_b}{N_0}=\ln2=-1.59$dB.

CH09 Information Theory.
1. Information of X: $I=-\log_aP(x)$. $a=2,e$. N symbols
$H(X)=E[I(X)]=\sum_{j=1}^{N}p(x_j)(-\log_2p(x_j))$ (bit/symbol). $0\le H(X)\le\log_2N$.
2. $H(X,Y)=-\sum_{x,y}p(x,y)\log p(x,y)$. $H(X|Y)=-\sum_{x,y}p(x,y)\log p(x|y)=\sum_yp(y)H(X|Y=y)$
$H(X|Y=y)=-\sum_xp(x|y)\log p(x|y)$. $H(X,Y)=H(X|Y)+H(Y)=H(Y|X)+H(X)$
$I(X;Y)=H(X)-H(X|Y)=H(Y)-H(Y|X)$.
2. Channel capacity: $C=\max I(X;Y)$ bits/sec. Transmit source U: $H(U)<C$
3. BSC: $I(X;Y)=H(Y)-H(Y|X)=H(Y)-\sum_xp(x)H(Y|X=x)=H(Y)-\sum p(x)H_b(P_e)$
$C=1-H_b(P_e)\Leftarrow=H(Y)-H_b(P_e)\le1-H_b(P_e)$. $H_b(P_e)=-p\log p-(1-p)\log(1-p)$
4. AWGN: $C=\frac12\log(1+\frac{P}{P_N})$ bits/channel output. $\frac1n\sum x_i^2\le P$.
For bandwidth W, sample at 2W: $C=\frac12\log(1+\frac{P}{N_0W})$ bit/sample $=W\log(1+\frac{P}{N_0W})$ bits/sec
5. ① when SNR≫1. $C\approx W\log\frac{P}{N_0W}$. $P'=2P\Rightarrow C'=C+W$. $P\to\infty$: $C\to\infty$
② when W≫1. $SNR=P/N_0W\approx0$. $C\approx\log_2e\cdot\frac{P}{N_0}=1.44\frac{P}{N_0}$.
6. must have $R\le W\log_2(1+\frac{P}{N_0W})\Rightarrow\rho=\frac{R}{W}\le\log_2(1+\rho\cdot\frac{E_b}{N_0})$. $E_b=\frac{P}{R}\Rightarrow\frac{E_b}{N_0}=\frac{2^\rho-1}{\rho}$.
$\rho=\frac{R}{B}\to0$: $\frac{E_b}{N_0}\to-1.59$dB. (Shannon limit)

CH10 Channel Coding
1. $C=mG$. $C:1\times n$. $m:1\times k$. $G=[I_k|P]_{k\times n}$. $P\Rightarrow k\times(n-k)$.
2. $H=[P^T|I_{n-k}]_{(n-k)\times n}$. $CH^T=0$. $H^T=n\times(n-k)$.
3. (n,k) Hamming code: codeword $n=2^m-1$, $m=n-k$. message $k=2^m-m-1$. parity check $m=n-k$.
$d_{min}=3$. $S=rH^T=eH^T$. total syndromes $=2^{n-k}-1$ = codeword length.
4. k bits $\Rightarrow M=2^k$ $(C_1,C_2\dots C_M)$. error patterns $=2^{n-k}-1$. $(e_1,e_2\dots e_{2^{n-k}-1})$. $2^k\cdot2^{n-k}=2^n$
5. linear block code: detect up to $(d_{min}-1)$ errors. correct up $t=\lfloor\frac{d_{min}-1}{2}\rfloor$ errors.
6. codeword error prob $=P_M\le\sum_{m=t+1}^{n}\binom{n}{m}p^m(1-p)^{n-m}$. $\binom{n}{m}=\frac{n!}{m!(n-m)!}\Rightarrow t\uparrow$. $P_M\downarrow$
7. rate k/n with length L: kL-stage shift register. n mod-2 adders
shift register has $2^{(L-1)k}$ states. k=1 for binary codes.
8. connection representation: $g_1,g_2\dots g_n\Rightarrow g_i(a_1,a_2\dots a_{kL})$. $2^{k(L-1)}$ states.
9. State diagram representation: current state, input, next state, output
10. Trellis diagram: repeat after depth L.
11. soft-decision ML decoder: minimum Euclidean distance: $\log P(z|U^{(m)})=-\sum_j\sum_i(z_{ji}-u_{ji}^{(m)})^2$
hard-decision ML decoder: Minimum Hamming Distance $=\log p(z|U^{(m)})=-d_m$
12. Viterbi decoding: $2^{k(L-1)}$ states at time t_i. 2 paths enter the node.
$\Rightarrow$ correct t errors $t\le\lfloor\frac{d_{free}-1}{2}\rfloor$.
13. Transfer function: start from all-zero return back to all-zero state. D^mN^l
$T(D,N,L)=\sum_{d=d_{free}}a_dD^dN^{f(d)}L^{g(d)}$. Distance, Number of 1 in inputs, Length.
$T(D)=T(D,N,L)|_{N=1,L=1}$.