Homework 2

作业要求：

提交一份pdf文档，并发送到bianc@lamda.nju.edu.cn，5月15日23:59截止。
* pdf文档命名方式："学号-姓名.pdf"，例如"MG1937000-张三.pdf"；
* 邮件标题命名："随机过程第二次作业-学号-姓名"，
  例如"随机过程第二次作业-MG1937000-张三"。

pdf可以用latex/word/markdown等方式生成，但是不要用手写证明的照片。

作业的评分主要参考以下几点：
1. 证明过程的完整性以及正确性。例如在使用之前的定理时是否充分考虑了其条件，公式推导是否完整、以及是否有错误。
2. 文档的细节。例如是否出现符号错误，文档格式是否混乱。

若发现作业出现雷同的情况，会根据相关规定给予惩罚，详情请参考课程主页中"学术诚信"的相关内容。请同学们务必独立完成作业！

Problem 1

Consider a renewal process, define
$A(t) = t - S_{N(t)}$: time from t since the last renewal, called "age" at t,
$Y(t) = S_{N(t)+1} - t$: time from t until the next renewal, called "residual life" at t,

Find $\lim_{t \to \infty} P(Y(t) \leq x)$.

Problem 2

Consider flipping a coin independently, each time with probability p comes up with 1, and with probability $q = 1 - p$ comes up with 0.

Find $E[N_{11\ldots1}]$, i.e., the expected time until pattern $11\ldots1$ (k consecutive 1s) occurs for the first time.

Problem 3

Consider flipping a coin independently, each time with probability p comes up with 1, and with probability $q = 1 - p$ comes up with 0. $A = 1010$, $B = 0100$.

Find $P(A \text{ before } B)$, i.e., the probability that pattern A occurs before pattern B. (Here we set $p = \frac{1}{4}$)

Problem 4

Let $Y_1, Y_2, \ldots$ be iid with

$$P(Y_i = 1) = P(Y_i = -1) = \frac{1}{2}$$

and define

$$Z_0 = 0, Z_n = \sum_{i=1}^{n} Y_i$$

$\{Z_n, n \geq 0\}$ is called the *symmetric random walk* process.
Define

$$u_n = P(Z_{2n} = 0) = \binom{2n}{n} \frac{1}{2^{2n}}$$

Prove $P(Z_1 \neq 0, Z_2 \neq 0, \ldots, Z_{2n} \neq 0) = u_n$.

Problem 5

Express in words what the random variable $X_{N(t)+1}$ represents (*Hint:* It is the length of which renewal interval?) Show that

$$P\left(X_{N(t)+1} \geq x\right) \geq \bar{F}(x)$$