多智能体

HW3

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课后作业 5-1

(1)

$G_1=\begin{pmatrix}a & b\\ d & c\end{pmatrix}=\begin{pmatrix}0 & 3\\ 2 & -1\end{pmatrix}$

$p=\dfrac{c-d}{a-b+c-d}=\dfrac{1}{2}$

$q=\dfrac{c-b}{a-b+c-d}=\dfrac{2}{3}$

Agent I 的最优策略为 $\left(\dfrac{1}{2},\dfrac{1}{2}\right)$

Agent II 的最优策略为 $\left(\dfrac{2}{3},\dfrac{1}{3}\right)$

$V_1=\dfrac{ac-bd}{a-b+c-d}=1$

(2)

$G_2=\begin{pmatrix}a & b\\ d & c\end{pmatrix}=\begin{pmatrix}0 & 1\\ 4 & 3\end{pmatrix}$

$p=\dfrac{c-d}{a-b+c-d}=\dfrac{1}{2}$

$q=\dfrac{c-b}{a-b+c-d}=-1$

因此存在纯策略的纳什均衡解，对应的 $p=0,q=0$

Agent I 的最优策略为 $(0,1)$

Agent II 的最优策略为 $(0,1)$

$V_2=3$

(3)

$G=\begin{pmatrix}a & b\\ d & c\end{pmatrix}=\begin{pmatrix}V_1 & 4\\ 5 & V_2\end{pmatrix}=\begin{pmatrix}1 & 4\\ 5 & 3\end{pmatrix}$

$p = \frac{c-d}{a-b+c-d} = \frac{2}{5}$

$q = \frac{c-b}{a-b+c-d} = \frac{1}{5}$

Agent I 的最优策略为 $\left(\frac{2}{5}, \frac{3}{5}\right)$

Agent II 的最优策略为 $\left(\frac{1}{5}, \frac{4}{5}\right)$

$V = \frac{ac-bd}{a-b+c-d} = \frac{V_1V_2-20}{V_1+V_2-9} = \frac{17}{5}$

课后作业 5-2

(1)

$$\begin{cases} V_1 = \frac{V_2V_3}{V_2+V_3} \\ V_2 = \frac{-1}{V_1-2} \\ V_3 = \frac{-4}{V_1-4} \end{cases} \Rightarrow \begin{cases} V_1 = \frac{2}{5} \\ V_2 = \frac{5}{8} \\ V_3 = \frac{10}{9} \end{cases}$$

所以有

$$G_1 = \begin{pmatrix} \frac{5}{8} & 0 \\ 0 & \frac{10}{9} \end{pmatrix}, G_2 = \begin{pmatrix} \frac{2}{5} & 1 \\ 1 & 0 \end{pmatrix}, G_3 = \begin{pmatrix} \frac{2}{5} & 2 \\ 2 & 0 \end{pmatrix}$$

对于 G_1 博弈有:

$p = \frac{c-d}{a-b+c-d} = \frac{16}{25}$

$q = \frac{c-b}{a-b+c-d} = \frac{16}{25}$

Agent I 的最优策略为 $\left(\frac{16}{25}, \frac{9}{25}\right)$

Agent II 的最优策略为 $\left(\frac{16}{25}, \frac{9}{25}\right)$

对于 G_2 博弈有:

$p=\dfrac{c-d}{a-b+c-d}=\dfrac{5}{8}$

$q=\dfrac{c-b}{a-b+c-d}=\dfrac{5}{8}$

Agent I 的最优策略为 $\left(\dfrac{5}{8},\dfrac{3}{8}\right)$

Agent II 的最优策略为 $\left(\dfrac{5}{8},\dfrac{3}{8}\right)$

对于 G_3 博弈有:

$p=\dfrac{c-d}{a-b+c-d}=\dfrac{5}{9}$

$q=\dfrac{c-b}{a-b+c-d}=\dfrac{5}{9}$

Agent I 的最优策略为 $\left(\dfrac{5}{9},\dfrac{4}{9}\right)$

Agent II 的最优策略为 $\left(\dfrac{5}{9},\dfrac{4}{9}\right)$

课后作业 5-3

$V=\dfrac{ac-bd}{a-b+c-d}=\dfrac{4+\frac{8}{3}V}{4+\frac{1}{3}V}$

$V=2$

$G=\begin{pmatrix}a & b\\ d & c\end{pmatrix}=\begin{pmatrix}4 & \frac{5}{3}\\ 0 & \frac{7}{3}\end{pmatrix}$

$p=\dfrac{c-d}{a-b+c-d}=\dfrac{1}{2}$

$q=\dfrac{c-b}{a-b+c-d}=\dfrac{1}{7}$

Agent I 的最优策略为 $\left(\dfrac{1}{2},\dfrac{1}{2}\right)$

Agent II 的最优策略为 $\left(\frac{1}{7}, \frac{6}{7}\right)$

课后作业 5-4

(1)

$$\begin{cases} V_1 = \frac{ac-bd}{a-b+c-d} = \frac{1}{4}V_2 + 2 \\ V_2 = \frac{ac-bd}{a-b+c-d} = \frac{1}{3}V_1 - \frac{8}{3} \end{cases} \Rightarrow \begin{cases} V_1 = \frac{16}{11} \\ V_2 = -\frac{24}{11} \end{cases}$$

(2)

对于 G_1 博弈有:

$$p = \frac{c-d}{a-b+c-d} = \frac{1}{8}V_2 + 1 = \frac{8}{11}$$

$$q = \frac{c-b}{a-b+c-d} = \frac{1}{2}$$

Agent I 的最优策略为 $\left(\frac{8}{11}, \frac{3}{11}\right)$

Agent II 的最优策略为 $\left(\frac{1}{2}, \frac{1}{2}\right)$

对于 G_2 博弈有:

$$p = \frac{c-d}{a-b+c-d} = \frac{1}{3}$$

$$q = \frac{c-b}{a-b+c-d} = \frac{2}{3} - \frac{1}{12}V_1 = \frac{6}{11}$$

Agent I 的最优策略为 $\left(\frac{1}{3}, \frac{2}{3}\right)$

Agent II 的最优策略为 $\left(\frac{6}{11}, \frac{5}{11}\right)$

(3)

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@func_mat()
def convert_Val(mat):
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    a, b, c, d = mat[0][0], mat[0][1], mat[1][1], mat[1][0]
    return (a * c - b * d) / (a - b + c - d)

@func()
def convert_Val1(v1, v2):
    mat = [[2, 2 + 0.5 * v2], [0, 4 + 0.5 * v2]]
    a, b, c, d = mat[0][0], mat[0][1], mat[1][1], mat[1][0]
    return (a * c - b * d) / (a - b + c - d)

@func()
def convert_Val2(v1, v2):
    mat = [[-4, 0], [-2 + 0.5 * v1, -4 + 0.5 * v1]]
    a, b, c, d = mat[0][0], mat[0][1], mat[1][1], mat[1][0]
    return (a * c - b * d) / (a - b + c - d)
```

$V_0 = (0, 0)$

$$V_1(1) = \mathrm{Val}\begin{pmatrix} 2 & 2 + 0.5 \times 0 \\ 0 & 4 + 0.5 \times 0 \end{pmatrix} = 2$$

$$V_1(2) = \mathrm{Val}\begin{pmatrix} -4 & 0 \\ -2 + 0.5 \times 0 & -4 + 0.5 \times 0 \end{pmatrix} = -2.66666666666667$$

$V_1 = (2, -2.66666666666667)$

$$V_2(1) = \mathrm{Val}\begin{pmatrix} 2 & 2 + 0.5 \times -2.66666666666667 \\ 0 & 4 + 0.5 \times -2.66666666666667 \end{pmatrix} = 1.33333333333333$$

$$V_2(2) = \mathrm{Val}\begin{pmatrix} -4 & 0 \\ -2 + 0.5 \times 2 & -4 + 0.5 \times 2 \end{pmatrix} = -2.00000000000000$$

$V_2 = (1.33333333333333, -2.00000000000000)$

$$V_3(1) = \mathrm{Val}\begin{pmatrix} 2 & 2 + 0.5 \times -2.00000000000000 \\ 0 & 4 + 0.5 \times -2.00000000000000 \end{pmatrix} = 1.50000000000000$$

$$V_3(2) = \mathrm{Val}\begin{pmatrix} -4 & 0 \\ -2 + 0.5 \times 1.33333333333333 & -4 + 0.5 \times 1.33333333333333 \end{pmatrix}$$
$$= -2.22222222222222$$

$V_3 = (1.50000000000000, -2.22222222222222)$

$$V_4(1) = \mathrm{Val}\begin{pmatrix} 2 & 2 + 0.5 \times -2.22222222222222 \\ 0 & 4 + 0.5 \times -2.22222222222222 \end{pmatrix} = 1.44444444444445$$

$$V_4(2) = \mathrm{Val}\begin{pmatrix} -4 & 0 \\ -2 + 0.5 \times 1.50000000000000 & -4 + 0.5 \times 1.50000000000000 \end{pmatrix}$$
$$= -2.16666666666667$$

$$V_4 = (1.44444444444445, -2.16666666666667)$$

$$V_5(1) = \mathrm{Val}\begin{pmatrix} 2 & 2 + 0.5 \times -2.16666666666667 \\ 0 & 4 + 0.5 \times -2.16666666666667 \end{pmatrix} = 1.45833333333333$$

$$V_5(2) = \mathrm{Val}\begin{pmatrix} -4 & 0 \\ -2 + 0.5 \times 1.44444444444445 & -4 + 0.5 \times 1.44444444444445 \end{pmatrix}$$
$$= -2.18518518518518$$

$$V_5 = (1.45833333333333, -2.18518518518518)$$

$$V_6(1) = \mathrm{Val}\begin{pmatrix} 2 & 2 + 0.5 \times -2.18518518518518 \\ 0 & 4 + 0.5 \times -2.18518518518518 \end{pmatrix} = 1.45370370370371$$

$$V_6(2) = \mathrm{Val}\begin{pmatrix} -4 & 0 \\ -2 + 0.5 \times 1.45833333333333 & -4 + 0.5 \times 1.45833333333333 \end{pmatrix}$$
$$= -2.18055555555556$$

$$V_6 = (1.45370370370371, -2.18055555555556)$$

$$V_7(1) = \mathrm{Val}\begin{pmatrix} 2 & 2 + 0.5 \times -2.18055555555556 \\ 0 & 4 + 0.5 \times -2.18055555555556 \end{pmatrix} = 1.45486111111111$$

$$V_7(2) = \mathrm{Val}\begin{pmatrix} -4 & 0 \\ -2 + 0.5 \times 1.45370370370371 & -4 + 0.5 \times 1.45370370370371 \end{pmatrix}$$
$$= -2.18209876543210$$

$$V_7 = (1.45486111111111, -2.18209876543210)$$