Homework 4

作业要求：

提交一份pdf文档，并发送到bianc@lamda.nju.edu.cn，6月25日23:59截止。

- pdf文档命名方式："学号-姓名.pdf"，例如"MG1937000-张三.pdf"；
- 邮件标题命名："随机过程第四次作业-学号-姓名"，
  例如"随机过程第四次作业-MG1937000-张三"。

pdf可以用latex/word/markdown等方式生成，但是不要用手写证明的照片。

作业的评分主要参考以下几点：

1. 证明过程的完整性以及正确性。例如在使用之前的定理时是否充分考虑了其条件，公式推导是否完整、以及是否有错误。
2. 文档的细节。例如是否出现符号错误，文档格式是否混乱。

若发现作业出现雷同的情况，会根据相关规定给予惩罚，详情请参考课程主页中"学术诚信"的相关内容。请同学们务必独立完成作业！

Problem 1

Consider a branching process, and let X_n denote the size of the nth generation.
Prove: if m is the mean number of offspring per individual, then $\{Z_n, n \geq 1\}$ is a martingale when $Z_n = X_n/m^n$.

Problem 2

Consider flipping a coin independently, each time with probability p comes up with H, and with probability $q = 1 - p$ comes up with T. Find the expected time until HHTTHH occurs.

Problem 3

Consider a set of n components that are to be used in performing certain experiments. Let X_i equal 1 if component i is in functioning condition and let it equal 0 otherwise, and suppose that the X_i are independent with $E[X_i] = p_i$. Suppose that in order to perform experiment $j, j = 1, \ldots, m$, all of the components in the set A_j must be functioning. If any component is needed in at most three experiments, show that

$$P\left(X - \sum_{j=1}^{m} \prod_{i \in A_j} p_i \geq 3a\right) \leq \exp\left\{-\frac{a^2}{2n}\right\},$$

and

$$P\left(X - \sum_{j=1}^{m} \prod_{i \in A_j} p_i \leq -3a\right) \leq \exp\left\{-\frac{a^2}{2n}\right\},$$

where X denotes the number of experiments that can be performed, and $a > 0$.

Problem 4

If $X_1, X_2, \ldots$ are independent and identically distributed with mean μ, and $S_n = X_1 + \cdots + X_n$.

Show that for a given $\epsilon > 0$, $P\left(\lim_{n\to\infty} \frac{S_n}{n} \leq \mu - \epsilon\right) = 0$.