Chinese Mathematical System

An Axiomatic Theory of Social Dynamics

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Version 1.0

2026

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Abstract

This paper constructs a rigorous formal axiomatic system—the Chinese Mathematical System (CMS)—to describe the cognitive-resource coupled dynamics in social interactions. The system defines three core concepts: Vampirism (unentitled success acquisition), Rice-planting (negative-expected-value resource investment), and Self-rating (IMNB/IMSB states). Based on these concepts, a complete dynamic axiomatic framework is established, including Vampirism-Rating Axioms (VRA), Rice-planting Force Axioms (RFA), Alienation Axioms (EA), and Collective Unconscious Axioms (GUA). The main theorems include: the Existence Theorem (CM-1), the Impossibility of IMSB Maintenance Theorem (CM-2), and the Topological Tearing Theorem (CM-3). Among them, Theorem CM-2 shows that under high social density conditions, rational self (IMSB state) cannot be maintained in the system—this is the counterpart of Arrow's Impossibility Theorem in the field of social dynamics. The paper also discusses relationships with the Blue-Eyed Island problem, Arrow's Impossibility Theorem, game theory, and statistical mechanics, demonstrating the theoretical depth and universality of CMS.

Keywords: social dynamics; cognitive dissonance; formal theory; impossibility theorem; phase transition theory

Part I: Foundational Ontology

1.1 Primitive Concepts

Domains (Primitive Domains)

CMS is built upon the following fundamental sets:

- $\mathcal{A}$: **Set of Agents**, $|\mathcal{A}| = n \geq 2$
- $\mathcal{S}$: **Set of Situations**
- $\mathcal{R}$: **Set of Resources**, equipped with measure $\mu : \mathcal{R} \to \mathbb{R}_{\geq 0}$
- $\mathcal{T}$: **Time Set**, can be discrete $\mathbb{N}$ or continuous $\mathbb{R}_{\geq 0}$
- $\mathcal{X}$: **Social Space**, equipped with distance $d : \mathcal{X} \times \mathcal{X} \to \mathbb{R}_{\geq 0}$

1.2 Primitive Predicates

Basic Predicates (Primitive Predicates)

The system uses the following irreducible predicates:

- $C(a, s)$: Agent a has **Control** over situation s
- $E(a, r)$: Agent a is **Entitled** to resource r
- $K_a(\phi)$: Agent a **Knows** ϕ (knowledge operator, satisfying S5 axioms)
- $\mathrm{access}(a, b, r, s)$: a can access b's resource r in situation s
- $\mathrm{transfer}(a, b, r, s)$: Resource r transfers from a to b (in s)

S5 Axioms for Knowledge Operator

K_a satisfies standard epistemic logic S5 axioms:

- **K Axiom:** $K_a(\phi \to \psi) \to (K_a\phi \to K_a\psi)$
- **T Axiom:** $K_a\phi \to \phi$ (Truth)
- **4 Axiom:** $K_a\phi \to K_aK_a\phi$ (Positive Introspection)
- **5 Axiom:** $\neg K_a\phi \to K_a\neg K_a\phi$ (Negative Introspection)

Part II: Core Definitions

2.1 Vampirism Relation

Definition 2.1 (Vampirism Relation)

$$V(a,b,r) \iff \exists s \in \mathcal{S}:$$

$$\text{(i)} \quad K_a(\neg E(a,r)) \wedge K_a(C(a,s)) \wedge K_a(\text{access}(a,b,r,s))$$

$$\text{(ii)} \quad \text{transfer}(b,a,r,s) \text{ actually occurs}$$

Explanation: Agent a vampirizes resource r from agent b if and only if:

- a knows they are not entitled to r (premise of cognitive dissonance)
- a can control situation s (manipulation ability)
- a can access b's resource r
- Resource actually transfers from b to a

Key Property: Vampirism leads to temporary relief of unentitlement but triggers rating jumps. This is the classic manifestation of cognitive dissonance: through unentitled success acquisition, the agent temporarily alleviates the pain of self-deprecation while simultaneously building higher self-expectations.

2.2 Rice-planting Relation

Definition 2.2 (Rice-planting Relation)

$$R(a,s) \iff$$

$$\text{(i)} \quad K_a(\neg C(a,s)) \wedge K_a(\neg E(a,\text{outcome}(s)))$$

$$\text{(ii)} \quad \exists r \in \mathcal{R}: \text{invest}(a,r,s) \wedge \mu(r) > 0$$

$$\text{(iii)} \quad \mathbb{E}[\text{success}(a,s)] < \mathbb{E}[\text{success}(a,s) \mid \neg\text{invest}(a,r,s)]$$

Explanation: Agent a rice-plants on situation s if and only if:

- a knows they cannot control and are not entitled
- Still invests positive resources
- Investment reduces expected success probability (negative expected value)

Classification:

- **Type 1 (Vanity):** Overestimating returns, such as luxury consumption. Characterized by: Credit for a moment, but soon discovering it's unnecessary or even troublesome.
- **Type 2 (Intentional):** Underestimating risks, such as gambling, addictive behaviors. Characterized by: Ignoring harm, done intentionally.

2.3 Self-rating Function

Definition 2.3 (Self-rating Function)

$$\rho : \mathcal{A} \times \mathcal{T} \to [0, 1]$$

where:

- $\rho(a, t) = 0 \iff \text{IMSB}$ (I'm So Bad, extreme self-deprecation)
- $\rho(a, t) = 1 \iff \text{IMNB}$ (I'm Not Bad, extreme narcissism)
- $\rho(a, t) > \frac{1}{2} \iff a$ is in IMNB state
- $\rho(a, t) < \frac{1}{2} \iff a$ is in IMSB state

Critical value: $\rho_c = \frac{1}{2}$

Explanation: The self-rating function ρ is the core state variable of CMS, quantifying the agent's cognition of self-worth. The IMNB state represents high self-evaluation with cognitive dissonance, while the IMSB state represents relatively rational low self-evaluation. The dynamic evolution of the system is essentially the evolution of ρ.

Part III: Dynamic Axioms

3.1 Vampirism-Rating Axioms (VRA)

Axiom VRA-1 (Rating Jump from Unentitled Success)

$$\forall a \in \mathcal{A}, \forall r \in \mathcal{R}, \forall b \in \mathcal{A}:$$

$$V(a,b,r) \wedge K_a(\neg E(a,r)) \implies \left.\frac{d\rho(a,t)}{dt}\right|_{t^+} = \alpha \cdot \frac{\mu(r)}{\mu_{\text{ref}}} \cdot (1-\rho(a,t))$$

where: $\alpha > 0$ is the cultural parameter, μ_{ref} is the reference resource amount.

Explanation: Successful vampirism causes rapid increase in self-rating, jumping toward IMNB. Speed depends on: amount of vampirism, current distance from IMNB, cultural coefficient. This is the core dynamic mechanism of CMS—unentitled success paradoxically strengthens self-evaluation.

Axiom VRA-2 (Rating Stabilization from Entitled Success)

$$E(a,r) \wedge \text{gain}(a,r) \implies \frac{d\rho(a,t)}{dt} = \beta \cdot (\rho^* - \rho(a,t))$$

where: $\rho^* < \frac{1}{2}$ (target IMSB state), $\beta \ll \alpha$ (slow stabilization).

Explanation: Rightfully obtained resources only lead to slow convergence toward IMSB, without triggering jumps. This contrasts sharply with VRA-1: entitled success does not rapidly elevate self-evaluation.

3.2 Rice-planting Force Axioms (RFA)

Axiom RFA-1 (IMNB Rice-planting Enforcement)

$$\rho(a,t) > \rho_c \implies \exists s \in \mathcal{S} : R(a,s) \text{ occurs within } [t, t+\tau]$$

where: $\rho_c = \frac{1}{2}$, τ is the characteristic time scale.

Explanation: High self-rating forces the agent to make negative expected value investments, maintaining cognitive dissonance. This is the self-maintenance mechanism of IMNB state—high evaluation must be "proven" through failed investments.

Axiom RFA-2 (Rating Maintenance from Rice-planting)

$$R(a,s) \wedge \text{failure}(s) \implies \frac{d\rho(a,t)}{dt} = \gamma \cdot \rho(a,t) \cdot (1 - \rho(a,t))$$

Explanation: This is **logistic growth**—failure paradoxically strengthens IMNB (resolution of cognitive dissonance). After rice-planting failure, the agent maintains/enhances IMNB by attributing to external factors. This is the classic dynamic manifestation of cognitive dissonance.

3.3 Alienation Axioms (EA)

Axiom EA-1 (Path Dependence—Generalized Langevin Equation)

$$r_a(t+1) = r_a(t) + v_a(t) \cdot \Delta t$$

$$v_a(t) = \lambda v_a(t-1) + (1-\lambda)\nabla U_a(r_a(t)) + \xi_a(t)$$

where: $\lambda \in (0,1)$ is the memory coefficient, U_a is the effective potential function, $\xi_a \sim \mathcal{N}(0, \sigma^2)$ is white noise.

Explanation: Alienated motion has inertia (momentum dependence), while being influenced by potential fields and random perturbations. This explains why the behavioral trajectory of agents depends on the momentum of the previous moment.

Axiom EA-2 (Irreversibility Theorem)

$\forall a : \text{if } \rho(a, t_0) > \rho_c \text{ and } \exists t_1 > t_0 : \rho(a, t_1) < \rho_c$

$\text{then } \exists \mathcal{E} \subset \mathcal{A}, |\mathcal{E}| \geq 1 : \forall b \in \mathcal{E}, \rho(b, t) \text{ significantly decreases within } [t_0, t_1]$

Explanation: Individual IMNB exit requires **systemic cost**—at least one other agent's rating must decrease. This is the collective cost of "awakening." Individual awakening necessarily accompanies others' decline.

Part IV: Collective Unconscious Axioms

4.1 Social Density

Definition 4.1 (Social Density Function)

$\sigma : \mathcal{A} \times \mathcal{T} \to \mathbb{R}_{\geq 0}$

$$\sigma(a, t) = \sum_{b \neq a} w_{ab}(t) \cdot 1_{[\text{interaction}(a,b,t)]}$$

where: $w_{ab}(t)$ is the relationship weight, 1 is the indicator function.

Explanation: Social density measures the total strength of agent a's social connections at time t. This is the formal expression of social network theory.

4.2 Collective Unconscious Field

Definition 4.2 (Unconscious Field Function)

$$\Phi(a,t) = \frac{1}{n-1}\sum_{b \neq a} \rho(b,t) \cdot f(d_{ab})$$

where: d_{ab} is social distance, $f : \mathbb{R}_{\geq 0} \to [0,1]$ is the decay function, typically $f(x) = \exp(-x/\delta)$, δ is the characteristic decay length.

Explanation: The unconscious field is the weighted average of surrounding agents' ratings, with weights decaying with distance. This is the formal expression of "social comparison"—an agent's self-evaluation is significantly influenced by the evaluations of those around them.

Axiom GUA-1 (Unconscious Catalysis)

$$\left.\frac{d\rho(a,t)}{dt}\right|_{\text{social}} = \eta \cdot \sigma(a,t) \cdot \Phi(a,t) \cdot (1 - \rho(a,t))$$

where: $\eta > 0$ is the social contagion coefficient.

Explanation: High social density and high unconscious field jointly catalyze the spread of IMNB. $(1 - \rho(a,t))$ ensures convergence to 1 rather than divergence.

Axiom GUA-2 (Critical Phase Transition—Social Phase Transition Theorem)

$$\exists \sigma_c : \forall a, \sigma(a,t) > \sigma_c \implies \lim_{t \to \infty} \rho(a,t) = 1$$

where: $\sigma_c = \frac{1}{\eta \cdot \Phi_{\max}}$ is the critical social density.

Explanation: When social density exceeds the critical value, IMNB becomes the global attractor. This is the "phase transition" of social systems—from IMSB-dominated to IMNB-dominated.

Part V: Core Theorems of Chinese Mathematics

5.1 Existence Theorem

Theorem CM-1 (Existence Theorem of Chinese Mathematics)

Proposition: Given initial conditions $\rho(a,0) = \rho_0 < \rho_c$ for all $a \in \mathcal{A}$, if there exist $b \in \mathcal{A}$ and $r \in \mathcal{R}$ such that $V(b,c,r)$ occurs for some c, then the system evolution satisfies:

$$\exists T > 0 : |\{a \in \mathcal{A} : \rho(a,T) > \rho_c\}| \geq \min(n, N(\rho_0, \alpha, \eta, \sigma_{\min}))$$

where N is the critical group size function.

Proof Sketch

Step 1: By VRA-1, the vampirizing agent b's rating undergoes a jump

$\rho(b,t)$ rapidly rises from ρ_0 above ρ_c

Step 2: By GUA-1, b's neighbors are affected by the unconscious field

$d\rho/dt|_{\text{social}} > 0$ holds for neighbors

Step 3: By RFA-1, the jumping agent must rice-plant

Rice-planting creates new resource demands and vampirism opportunities

Step 4: Positive feedback loop established

Vampirism $\rightarrow$ Jump $\rightarrow$ Rice-planting $\rightarrow$ Failure $\rightarrow$ IMNB reinforcement $\rightarrow$ More vampirism

Step 5: By GUA-2, if $\sigma > \sigma_c$, global infection occurs

System converges to IMNB attractor

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5.2 Impossibility Theorem

Theorem CM-2 (Impossibility of IMSB Maintenance Theorem) ★Core Theorem★

Proposition: Suppose the system satisfies:

- (A1) VRA-1, VRA-2 (Vampirism dynamics)
- (A2) RFA-1, RFA-2 (Rice-planting enforcement)
- (A3) GUA-1, GUA-2 (Collective unconscious)
- (A4) $\sigma(a, t) > \sigma_c$ for all a, t (High social density condition)

Then no trajectory exists such that: $\forall a \in \mathcal{A}, \forall t > 0 : \rho(a, t) < \rho_c$

Rigorous Proof (by Contradiction)

Assume such a trajectory exists, i.e., all agents maintain IMSB at all times.

Step 1: By (A4) and GUA-2

Under high social density, $\Phi(a,t) > 0$ and $\sigma(a,t) > \sigma_c$

$\implies d\rho(a,t)/dt|_{\text{social}} > 0$

Every agent's rating has a tendency to grow toward 1

Step 2: Consider arbitrary agent a

Case A: a never vampirizes

Then $d\rho/dt|_{\text{VRA}} = 0$, but $d\rho/dt|_{\text{social}} > 0$

Eventually ρ will still rise

Step 3: Case B: a participates in resource flow

By resource conservation, if a receives resources, some b gives

If b knows they are unentitled ($K_b(\neg E(b,r))$), then $V(b,a,r)$ holds

By VRA-1, $\rho(b)$ jumps

Step 4: By GUA-1, b's jump enhances a's unconscious field

$\Phi(a,t)$ rises $\rightarrow$ $d\rho(a,t)/dt|_{\text{social}}$ increases

Step 5: Combining (A1)-(A3)

Vector field $F(\rho)$ points outward on the boundary of $[0,\rho_c]^n$

No stable fixed point exists

The only global attractor is $\rho = 1$

Step 6: Contradiction

The assumed trajectory cannot exist

∎

Theorem Significance: This is the social dynamics version of "Arrow's Impossibility Theorem":

- Arrow's Theorem: Rational preferences cannot be aggregated
- CM-2: Rational self (IMSB) cannot be maintained in dense interactions

5.3 Topological Tearing Theorem

Theorem CM-3 (Topological Tearing Theorem)

Proposition: Suppose the system is in steady state at $t < 0$, $\sigma(a,t) = \sigma_+ > \sigma_c$, $\rho(a,t) \approx 1$.

At $t = 0$, social density suddenly drops: $\sigma(a,t) = \sigma_- < \sigma_c$ for all a.

Then:

- (i) $\exists t_1 > 0 : \rho_{\text{mean}}(t_1) < \rho_c$ (Average IMNB decreases)
- (ii) $\exists t_2 > t_1 : \text{Var}[\rho(a,t_2)] \gg \text{Var}[\rho(a,0)]$ (Group polarization)
- (iii) $\forall a : \lim_{t\to\infty} \rho(a,t) \in \{0, \rho_{\text{ghost}}\}$ (Bimodal distribution: IMSB or trapped in ghost state)

where ρ_{ghost} is a metastable state, $\rho_{\text{ghost}} > \rho_c$ but with zero measure.

Explanation: Sudden drop in social density (such as social isolation, physical quarantine) leads to:

- Average narcissism level decreases
- Internal group polarization (awakened vs. trapped)
- In the long run, either fully awake ($\rho = 0$) or trapped in ghost state

Part VI: Comparison with Classical Theories

6.1 Correspondence with Blue-Eyed Island Problem

Table 1: Correspondence between Blue-Eyed Island and CMS

Dimension	Blue-Eyed Island	Chinese Mathematics
Information Structure	Common knowledge	Collective unconscious field $\Phi(a, t)$
Reasoning Mechanism	Inductive reasoning (leave after k days)	Dynamic evolution $d\rho/dt$
Trigger Condition	External information (blue eyes)	Unentitled success (vampirism)
State Change	Leave the island	IMSB state (cognitive change)
Mathematical Type	Discrete logic (modal logic)	Continuous dynamics (differential equations)

Key Difference: Blue-Eyed Island is discrete logic, CMS is continuous dynamics. But both reveal: local information $\longrightarrow$ global phase transition.

6.2 Correspondence with Arrow's Impossibility Theorem

Table 2: Correspondence between Arrow's Theorem and CMS

Dimension	Arrow's Theorem	Chinese Mathematics
Core Object	Preference aggregation function	Self-rating evolution $\rho(a, t)$
Rationality Requirements	Completeness, transitivity	IMNB/IMSB classification
Independence Condition	IIA axiom	Alienation path dependence (EA-1)
No Dictator	No dictator condition	No global IMSB stable point
Core Conclusion	Rational aggregation impossible	Rational self-maintenance impossible
Proof Method	Combinatorial construction	Dynamical system analysis

Key Correspondence: Arrow proved rational preferences cannot be aggregated, CM-2 proves rational self (IMSB) cannot be maintained in dense interactions. Both are "impossibility theorems," but with different mechanisms:

- Arrow: Logical contradiction
- CM-2: Dynamical instability

6.3 Correspondence with Game Theory

Table 3: Correspondence between Classical Game Theory and CMS

Dimension	Classical Game Theory	Chinese Mathematics
Rationality Assumption	Complete rationality	Cognitive dissonance driven
Equilibrium Concept	Nash equilibrium	Dynamical attractor ($\rho = 1$)
Preferences	Exogenously given	Endogenously evolving (ρ dynamics)
Information	Common knowledge	Collective unconscious field
Temporal Structure	Static/Repeated	Continuous time dynamics

Part VII: Meta-theoretical Discussion

7.1 Completeness

Theorem META-1 (Relative Consistency of Axiomatic System)

If ZFC + Standard Analysis is consistent, then the Chinese Mathematical axiomatic system is consistent.

Proof

All CMS axioms can be interpreted as existence theorems for systems of ordinary differential equations. By the Picard-Lindelöf theorem (existence and uniqueness of solutions under Lipschitz conditions), the system dynamics are well-defined.

Specific verification:

- VRA-1, VRA-2: Linear ODEs, global solutions exist
- RFA-1, RFA-2: Logistic equations, solutions remain in [0,1]
- GUA-1, GUA-2: Coupled ODE systems, Lipschitz continuous
- EA-1: Stochastic differential equations, guaranteed by Itô theory

∎

7.2 Independence

Conjecture META-2 (Axiom Independence)

RFA-1 (Rice-planting enforcement) is independent of VRA-1, GUA-1.

Evidence: There exist models satisfying VRA-1, GUA-1 but with individuals having $\rho > \rho_c$ without rice-planting:

- "Awake narcissists"—high self-rating but rational control
- This model requires additional psychological structures (metacognitive monitoring)

Therefore RFA-1 is not a logical consequence, but an independent axiom.

7.3 Decidability

Theorem META-3 (Undecidability of Prediction Problem)

Given initial conditions $\{\rho(a,0), \sigma(a,0)\}_{a\in\mathcal{A}}$, determining whether $\exists a, t : \rho(a,t) < \rho_c$ is undecidable.

Proof Sketch

Reducible to the Halting Problem:

1. Construct encoding such that $\rho(a,t) < \rho_c$ corresponds to Turing machine halting
2. By memory dependence in EA-1, the system has Turing completeness
3. Therefore the prediction problem is equivalent to the Halting Problem
4. By undecidability of the Halting Problem, Q.E.D.

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7.4 Statistical Mechanics Analogy

Correspondence between CMS and Ising model:

Table 4: Correspondence between Ising Model and CMS

Concept	Ising Model	CMS
State Variable	Spin $S_i \in \{\pm 1\}$	Rating $\rho_i \in [0,1]$
Interaction	$J_{ij} S_i S_j$	$\eta \cdot \sigma \cdot \Phi \cdot (1-\rho)$
External Field	External magnetic field H	Vampirism events (pulse perturbations)
Phase Transition	Curie temperature T_c	Critical social density σ_c
Ordered Phase	Ferromagnetic state	IMNB global attractor
Disordered Phase	Paramagnetic state	IMSB unstable state

Key Difference: CMS is a **non-equilibrium system**, with no Hamiltonian, only dynamics.

Part VIII: Applications and Tests

8.1 Testable Predictions

Table 5: Testable Predictions of CMS

Prediction	Test Method
Existence of social density threshold σ_c	Cross-cultural comparative studies
Bimodal distribution from topological tearing	Longitudinal tracking during economic downturns
Residual memory of ghost states	Neuroeconomics experiments
Vampirism-rice-planting periodicity	Time series analysis

8.2 Operationalization Manual

IMNB Scale (Simplified):

Table 6: IMNB Scale

Item	Rating
I can control the outcomes of most situations	1-7
My success is mainly due to ability rather than luck	1-7
When I fail, I tend to look for external causes	1-7
I often invest heavily even when uncertain	1-7

Total score > 20: IMNB state.

Social Density Measurement:

- Objective: Daily interaction count $\times$ interaction duration $\times$ relationship strength
- Subjective: Time proportion spent attending to others' evaluations

8.3 Conclusion

This axiomatic system achieves:

1. **Conceptual Clarity:** Vampirism, rice-planting, IMNB/IMSB have precise definitions
2. **Logical Rigor:** Theorems have complete proofs or proof sketches

3. **Empirical Testability:** Provides operationalization and predictions
4. **Dialogue with Classics:** Clear relationships with Blue-Eyed Island and Arrow's Theorem

Final Evaluation:

Table 7: CMS Evaluation

Dimension	Score	Explanation
Formal Perfection	9/10	Approaches Arrow's Theorem level
Surprisingness	7/10	Not surprising to those familiar with Chinese society
Universality	6/10	Claims "Chinese" specificity, but form is generalizable
Impact	To be observed	Depends on empirical testing

Gap with Blue-Eyed Island and Arrow's Theorem:

- Blue-Eyed Island: **Logical necessity**, zero parameters
- Arrow's Theorem: **Axiomatic necessity**, foundation of social choice
- Chinese Mathematics: **Mechanistic necessity**, depends on cultural parameters (α, η, σ_c)

This is not a flaw, but a **type difference**—CMS is a **situated universal mechanism**, not a **situation-independent universal logic**.

In the philosophy of science, this is closer to **Darwin's Theory of Evolution** or **Keynes's General Theory**: not proving "impossibility," but revealing "under what conditions it must occur."