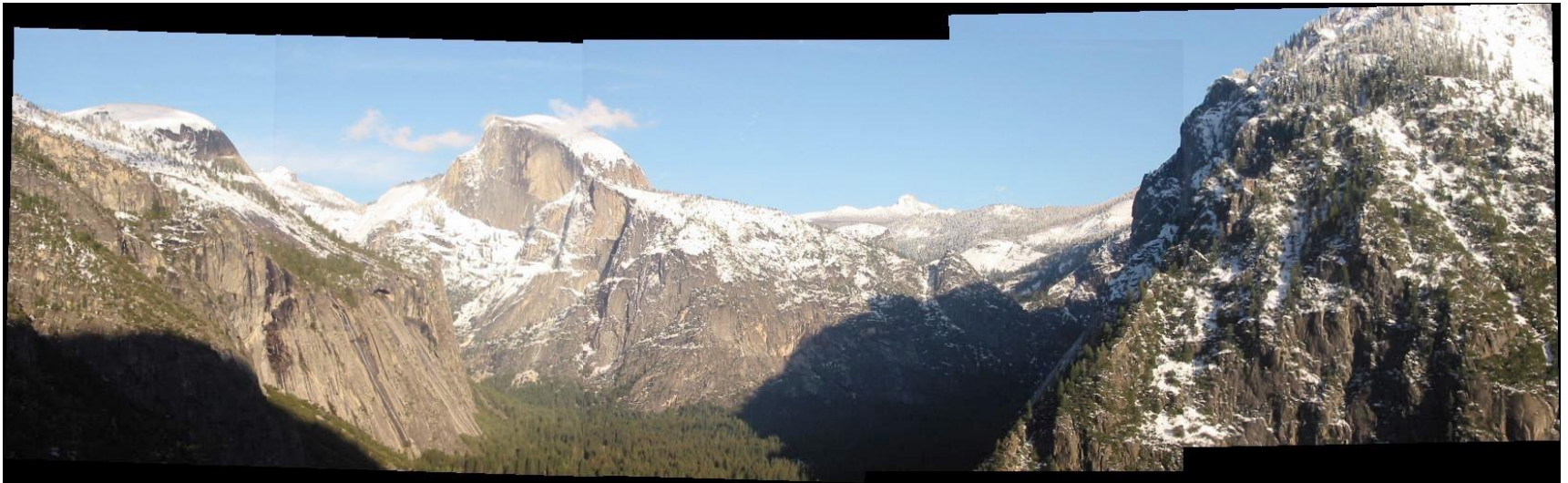


# Image Stitching



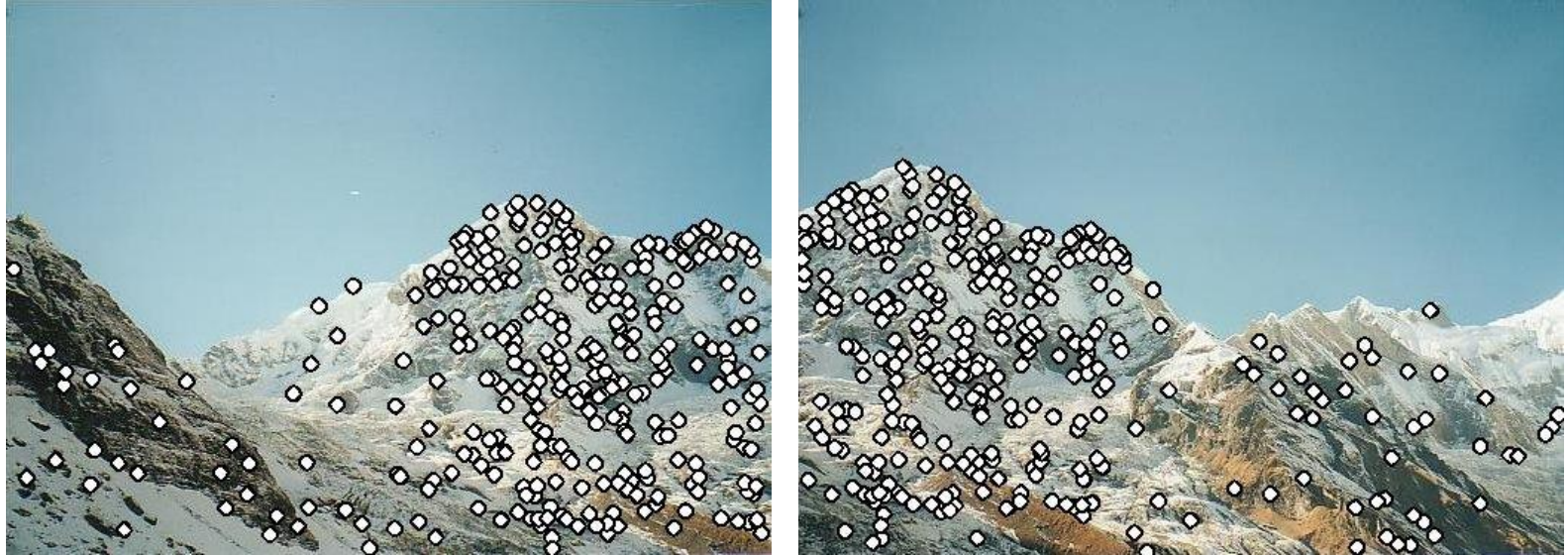
**Gang Pan**  
**[gpan@zju.edu.cn](mailto:gpan@zju.edu.cn)**

# Image Stitching



Slide credit: Darya Frolova, Denis Simakov

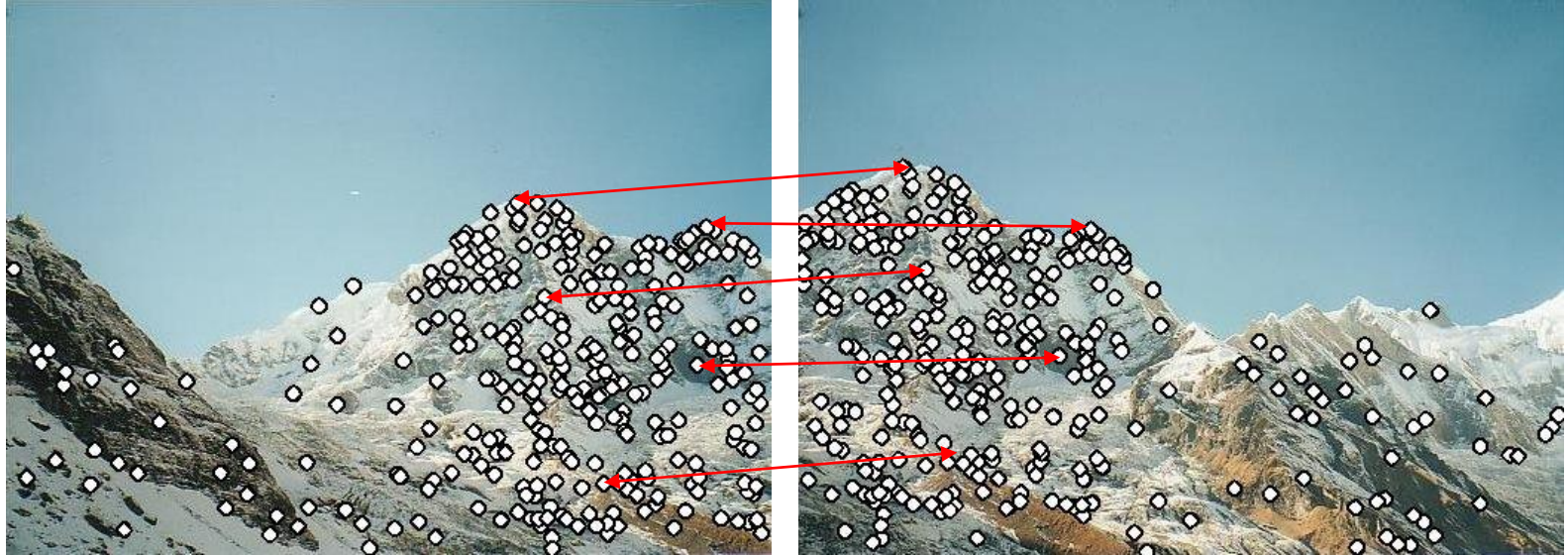
# Application: Image Stitching



- Procedure:
  - Detect feature points in both images



# Image Stitching



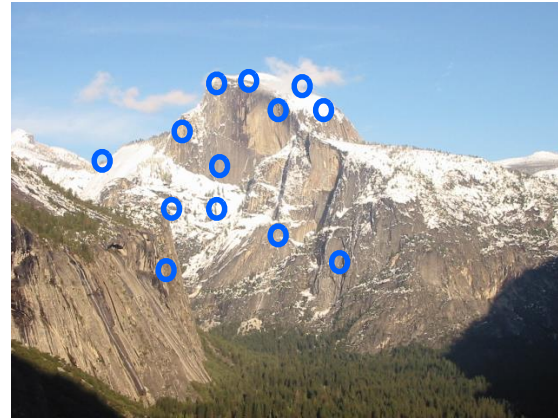
- Procedure:
  - Detect feature points in both images
  - Find corresponding pairs

# Application: Image Stitching



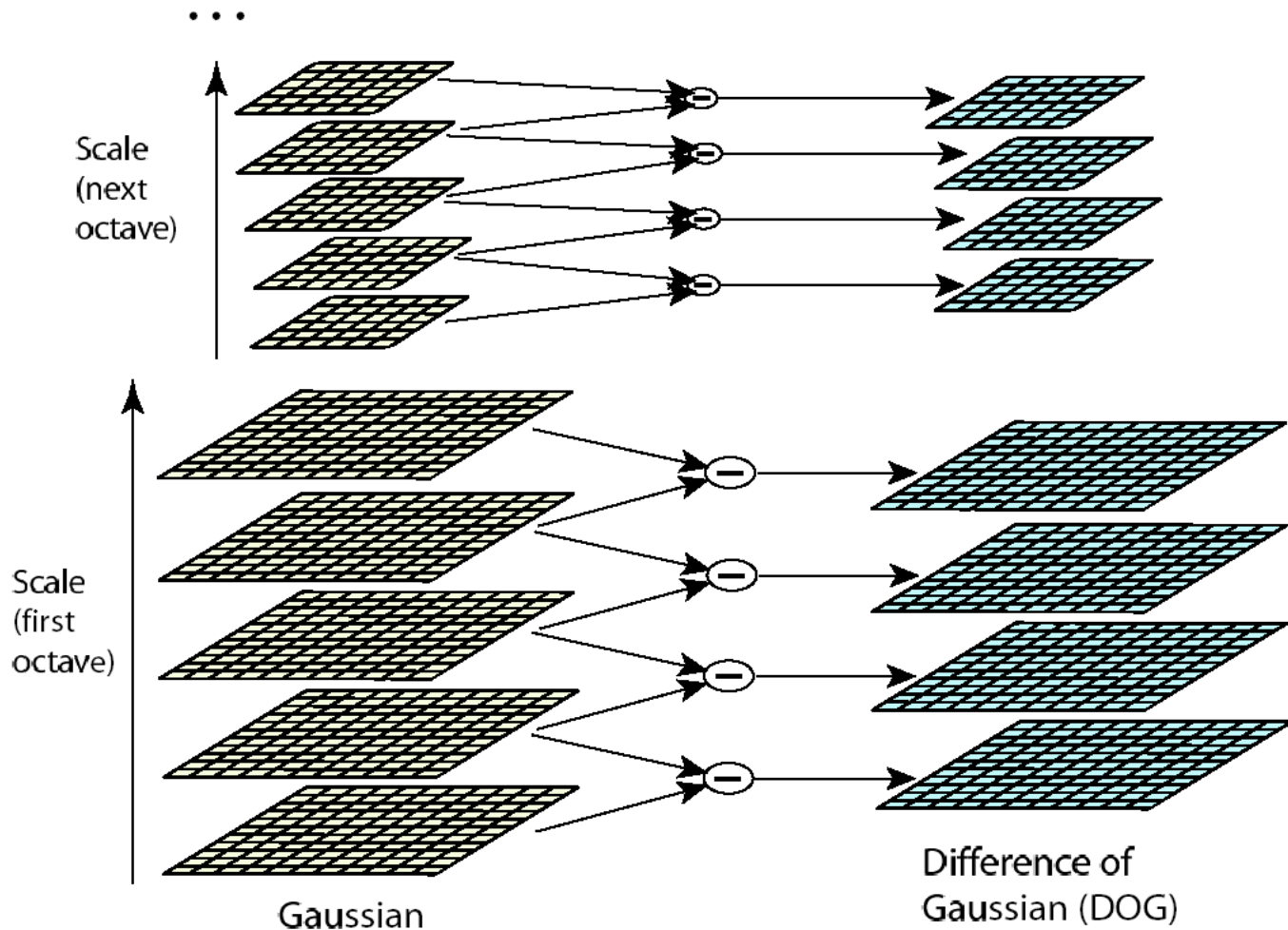
- Procedure:
  - Detect feature points in both images
  - Find corresponding pairs
  - Use these pairs to align the images

# Main Flow



- Detect key points

# Detect Key Points

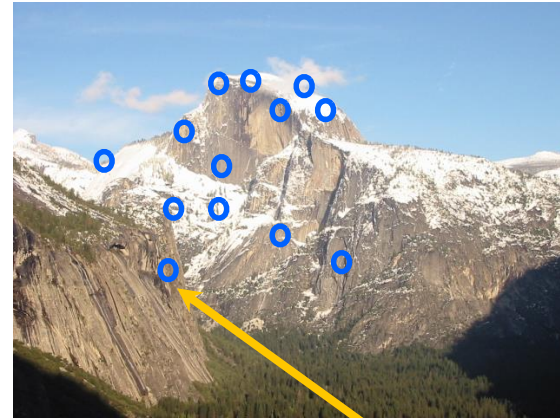




# Main Flow



$(u_1, u_2, \dots, u_{128})$



$(v_1, v_2, \dots, v_{128})$

- Detect key points
- Build the SIFT descriptors



# Build the SIFT Descriptors

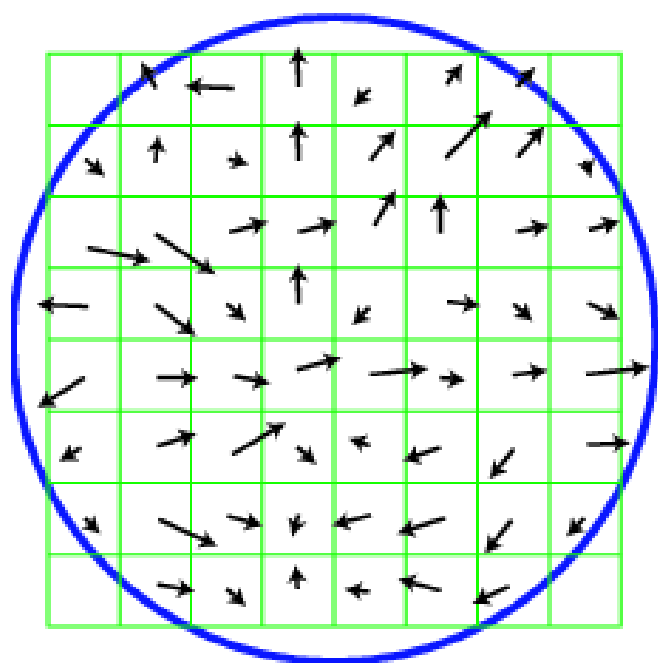
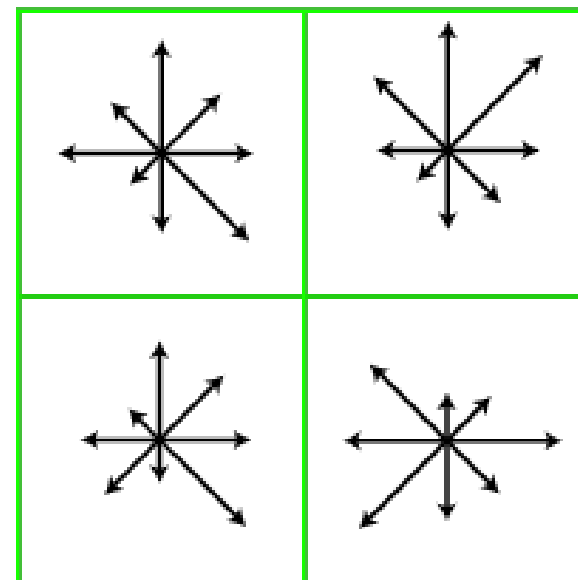


Image gradients

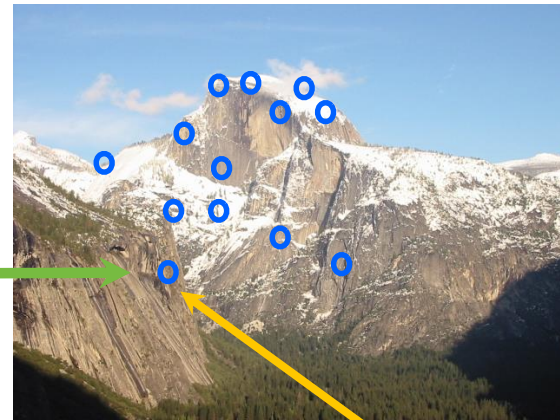


Keypoint descriptor

# Main Flow



$(u_1, u_2, \dots, u_{128})$

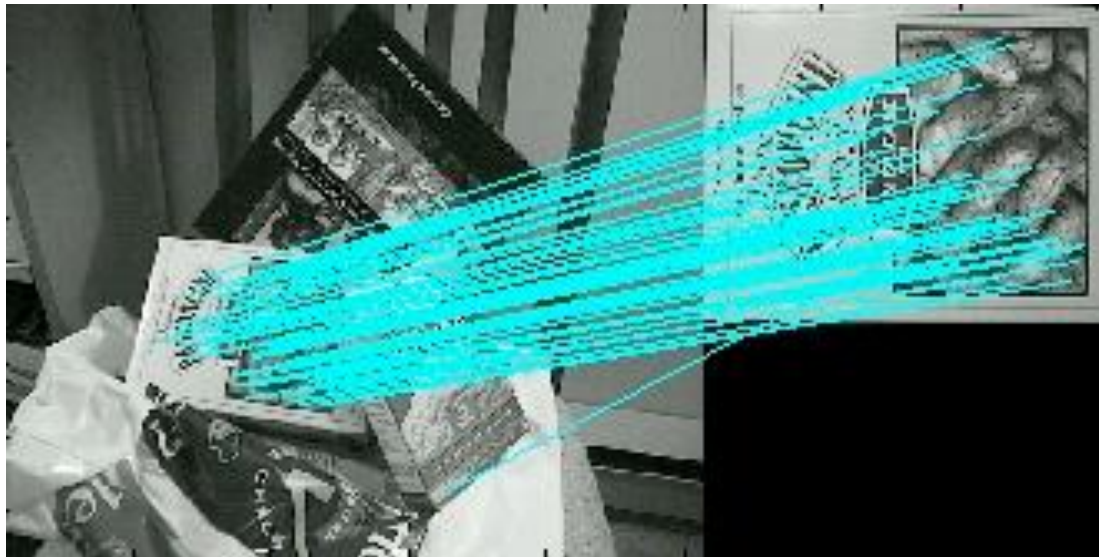


$(v_1, v_2, \dots, v_{128})$

- Detect key points
- Build the SIFT descriptors
- Match SIFT descriptors

# Match SIFT Descriptors

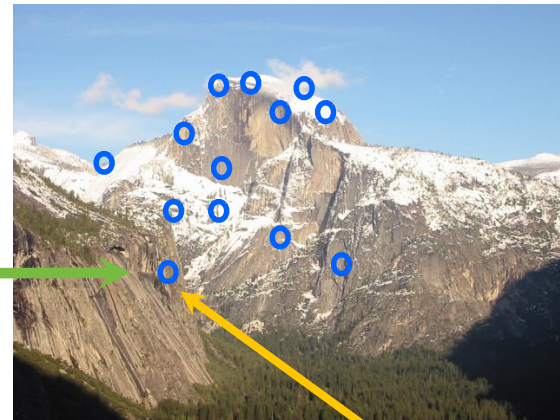
- Euclidean distance between descriptors



# Main Flow



$(u_1, u_2, \dots, u_{128})$



$(v_1, v_2, \dots, v_{128})$

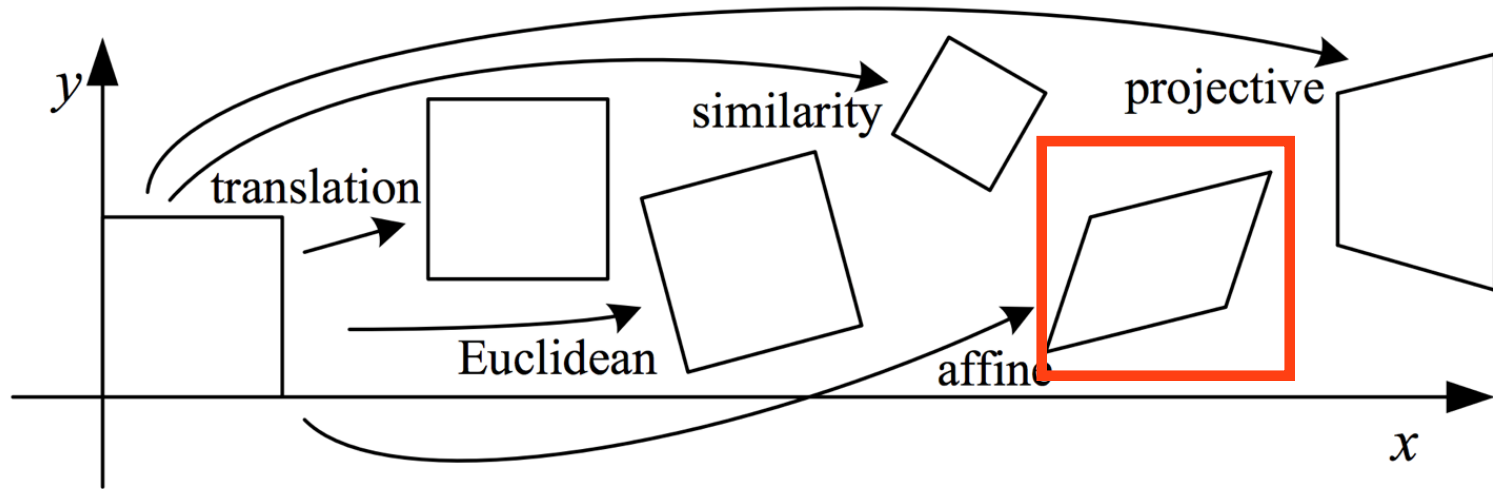
- Detect key points
- Build the SIFT descriptors
- Match SIFT descriptors
- Fitting the transformation

$$T = \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ 0 & 0 & 1 \end{bmatrix}$$



# Fitting the transformation

- 2D transformations



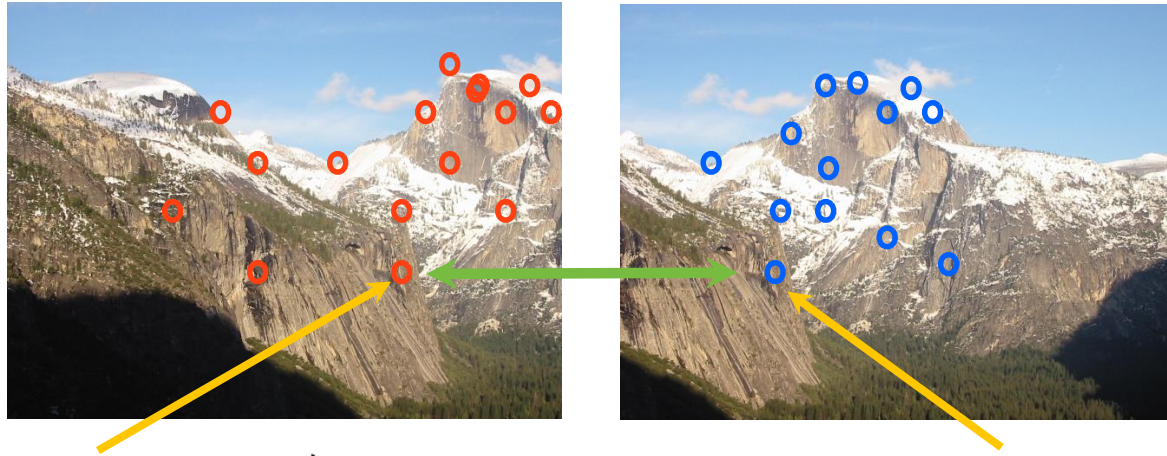
# Skeleton Code

- Fit the transformation matrix

$$H = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

- Six variables
  - each point give two equations
  - at least three points
- Least squares

# Main Flow

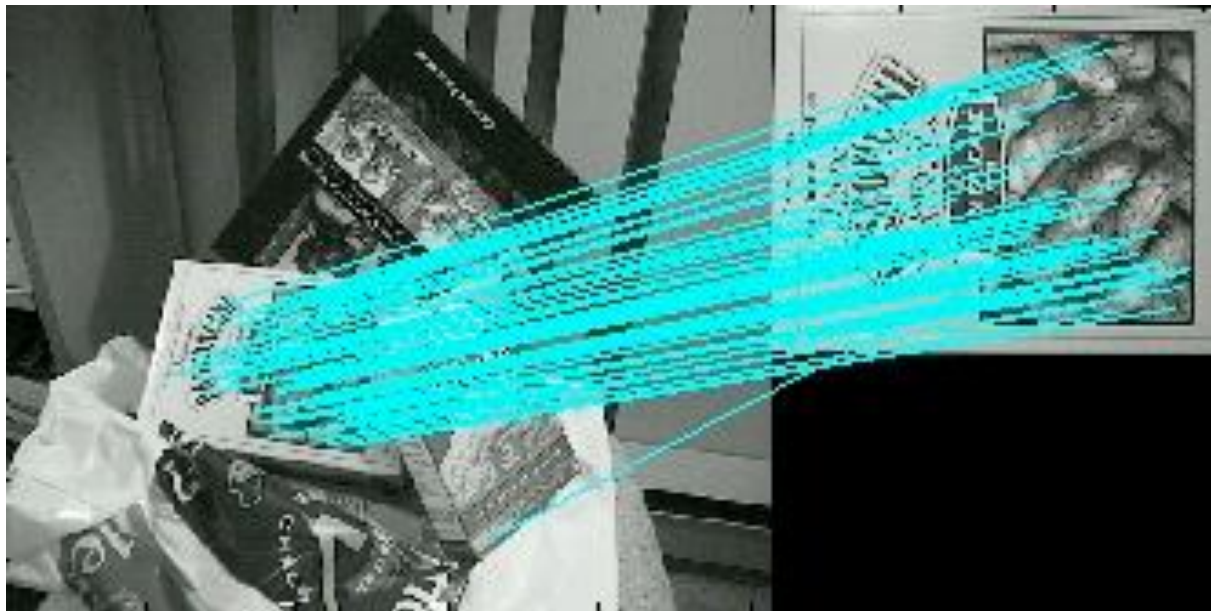


- $(u_1, u_2, \dots, u_{128})$ 
  - Detect key points
  - Build the SIFT descriptors
  - Match SIFT descriptors
  - Fitting the transformation
  - RANSAC

$(v_1, v_2, \dots, v_{128})$

# RANSAC

- A further refinement of matches





# RANSAC [Fischler & Bolles 1981]

- **RAN**dom **SA**mples **C**onsensus
- Approach: we want to avoid the impact of **outliers**, so let's look for “**inliers**”, and use only those.
- Intuition: if an outlier is chosen to compute the current fit, then the resulting line won't have much **support from rest of the points**.

# RANSAC [Fischler & Bolles 1981]

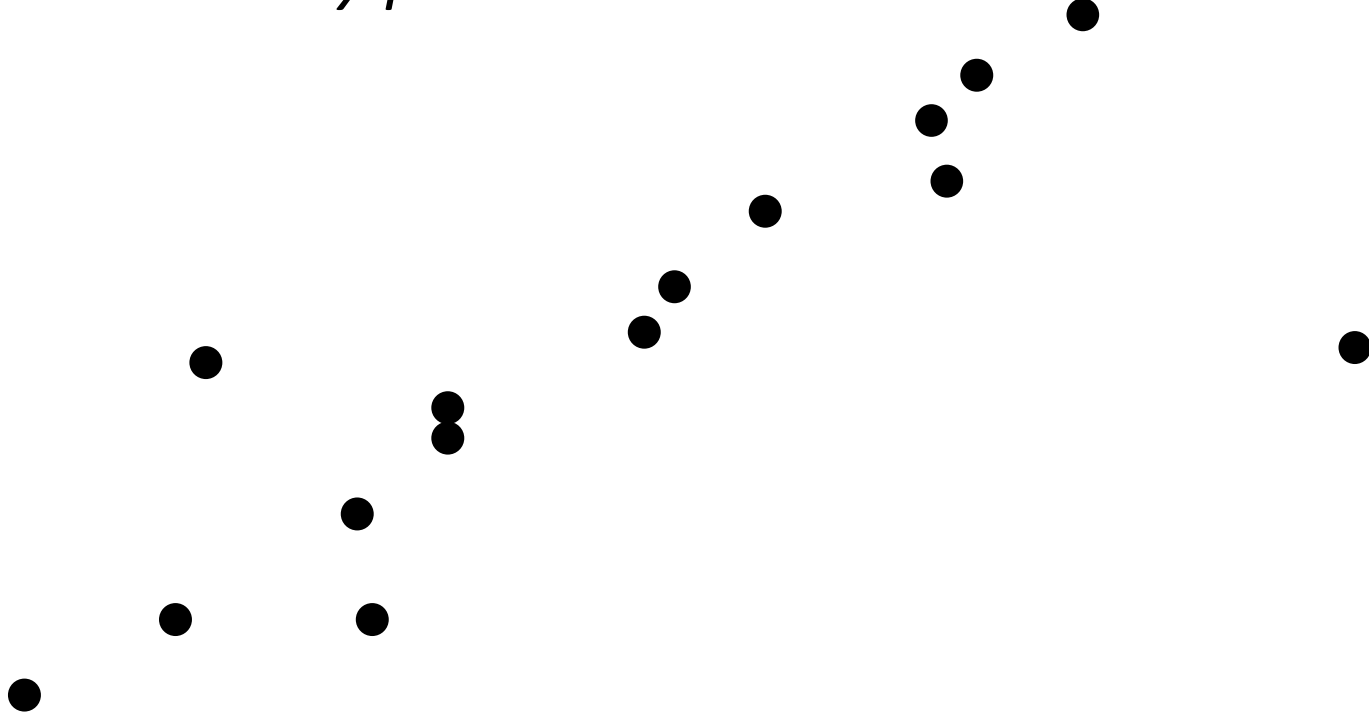
## RANSAC loop:

1. Randomly select a *seed group* of points on which to base transformation estimate (e.g., a group of matches)
2. Compute transformation from seed group
3. Find *inliers* to this transformation
4. If the number of inliers is sufficiently large, **re-compute** least-squares estimate of transformation on all of the inliers

**Keep** the transformation with the largest number of inliers

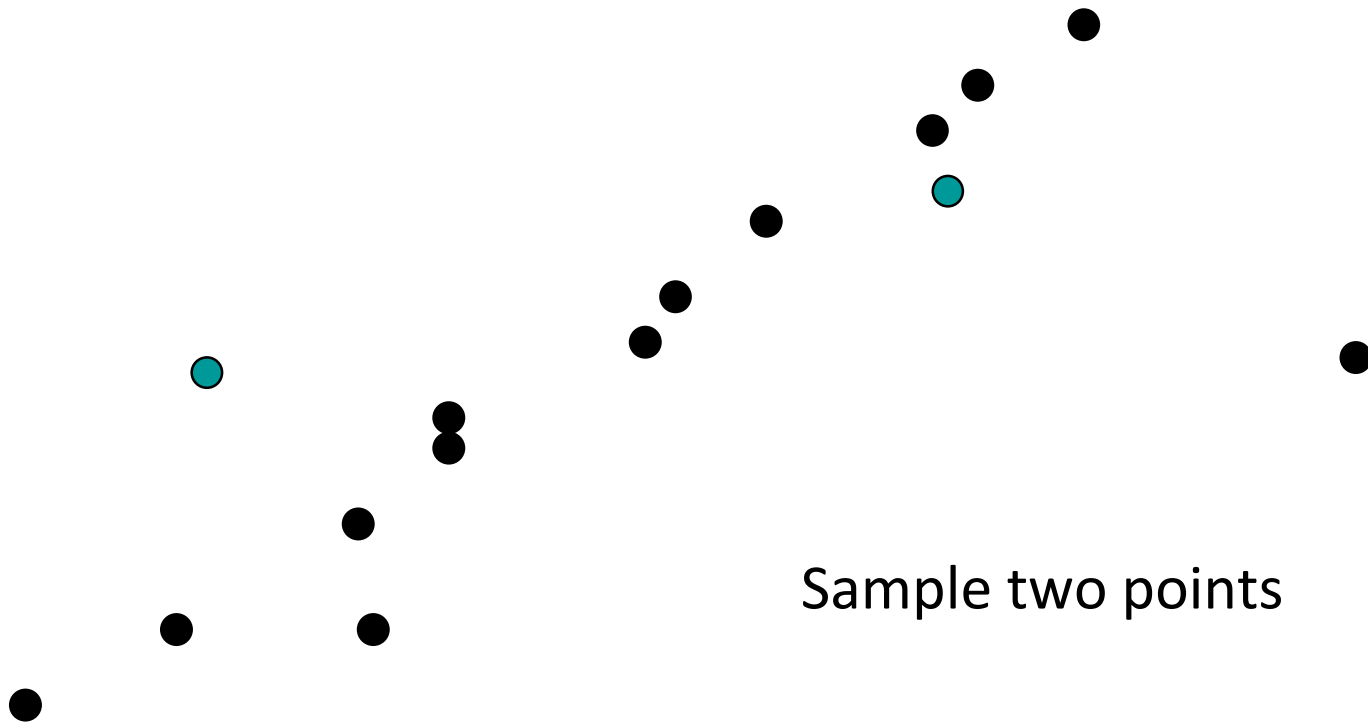
# RANSAC Line Fitting Example

- Task: Estimate the best line
  - *How many points do we need to estimate the line?*



# RANSAC Line Fitting Example

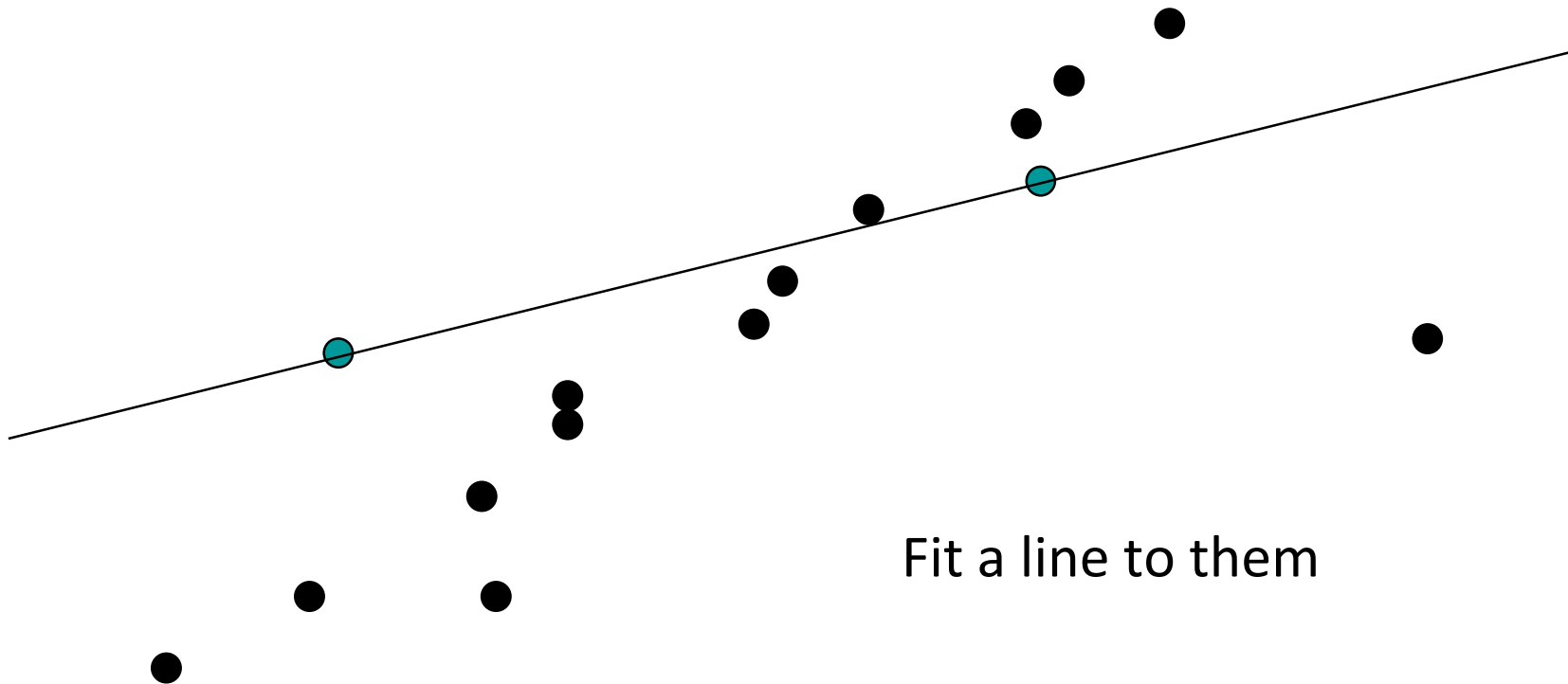
- Task: Estimate the best line





# RANSAC Line Fitting Example

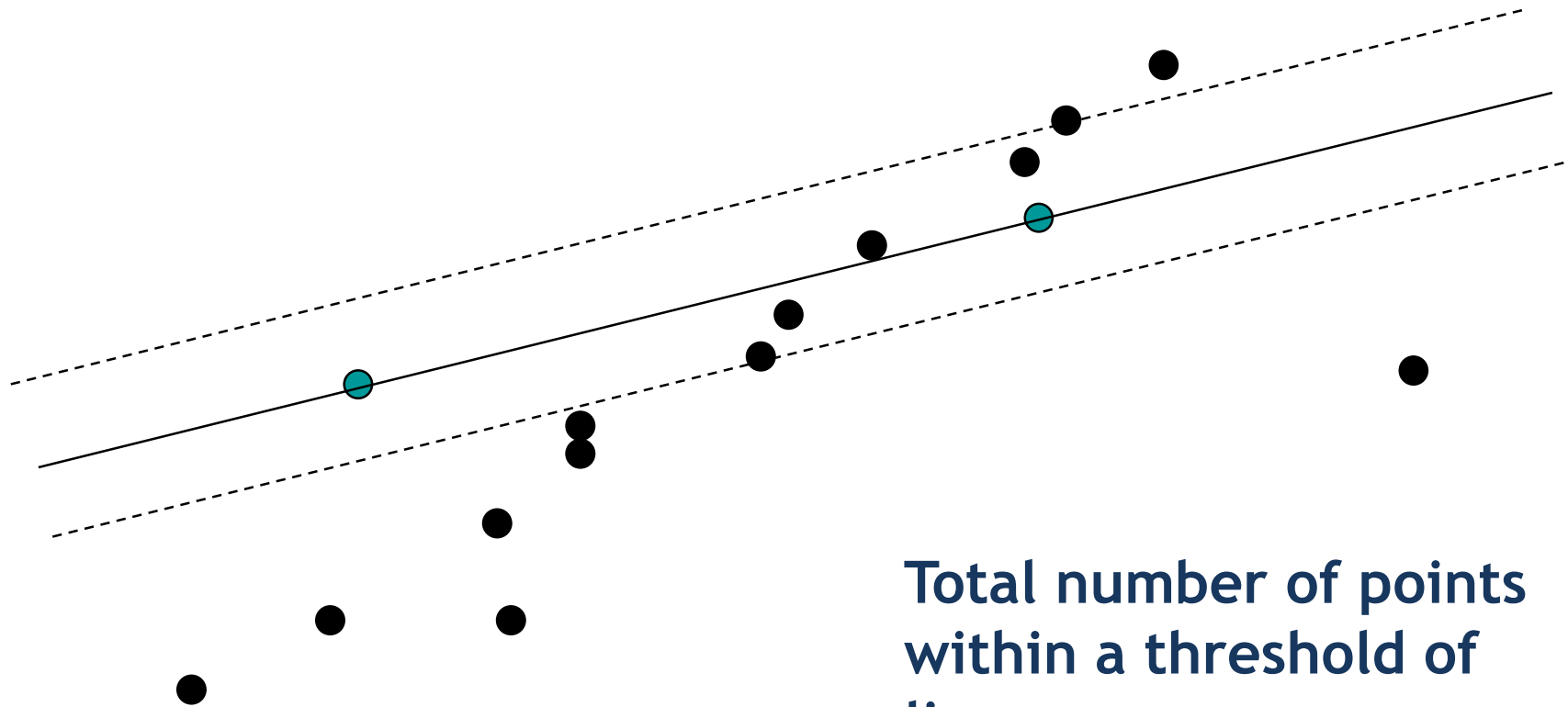
- Task: Estimate the best line



Fit a line to them

# RANSAC Line Fitting Example

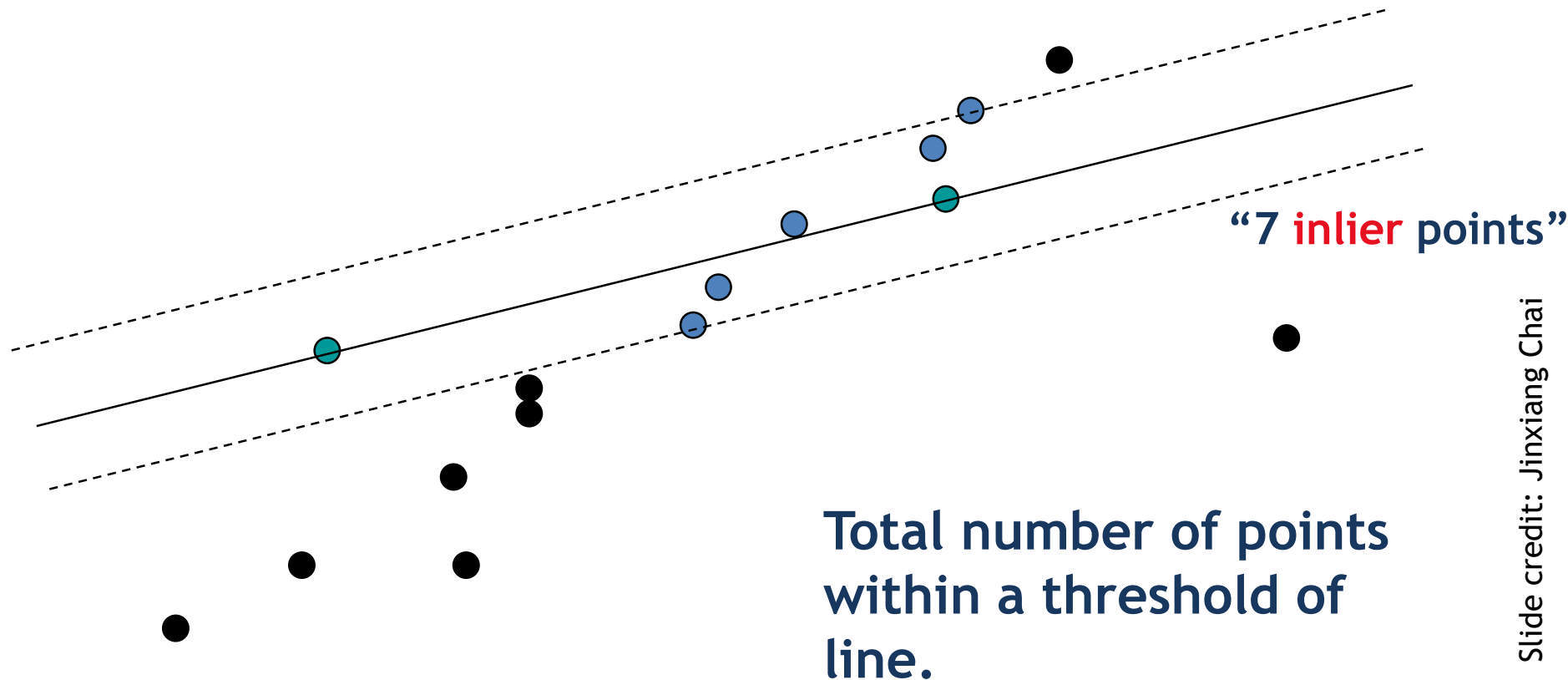
- Task: Estimate the best line



Total number of points  
within a threshold of  
line.

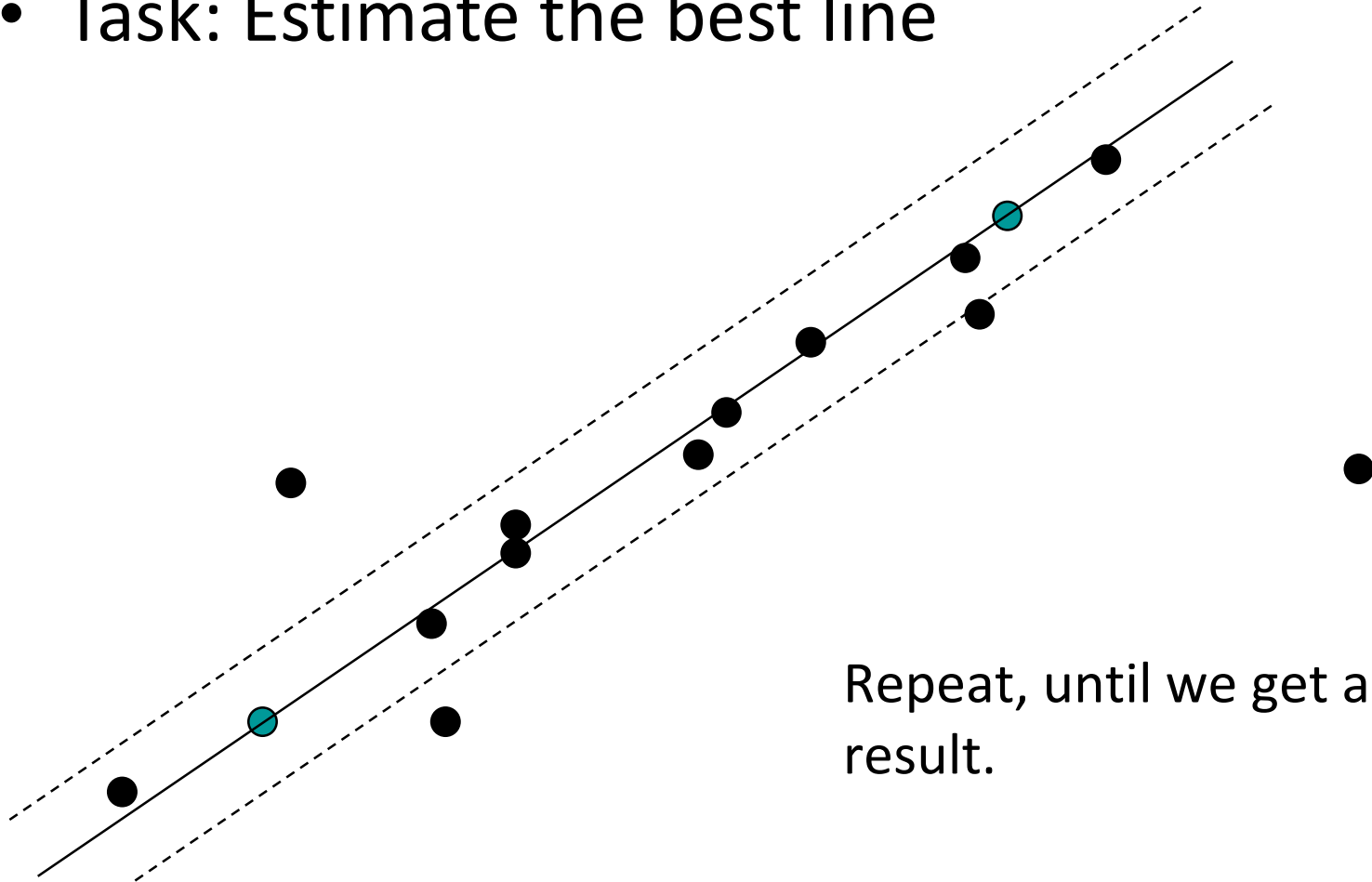
# RANSAC Line Fitting Example

- Task: Estimate the best line



# RANSAC Line Fitting Example

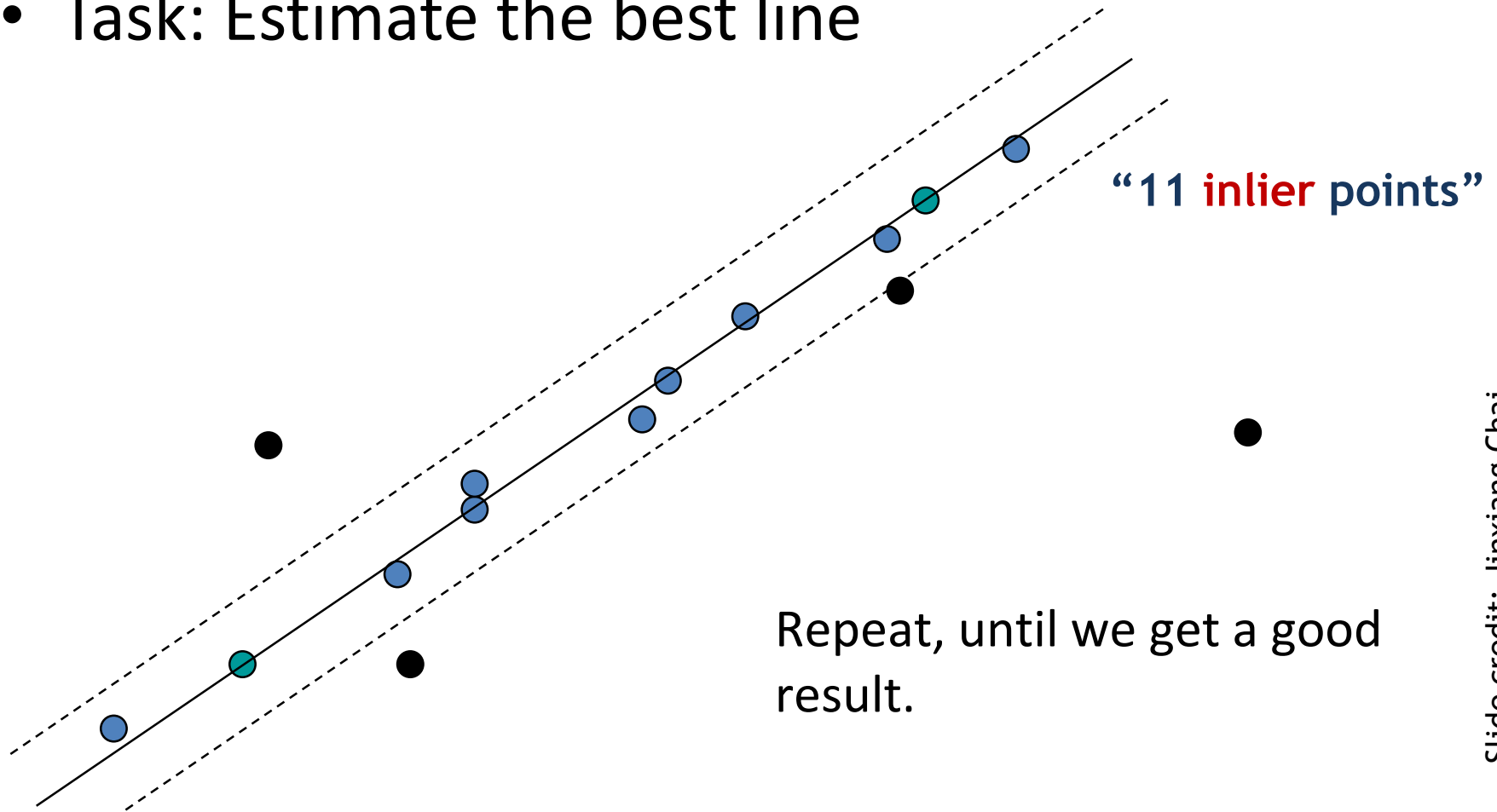
- Task: Estimate the best line





# RANSAC Line Fitting Example

- Task: Estimate the best line



### Algorithm 15.4: RANSAC: fitting lines using random sample consensus

Determine:

- $n$  — the smallest number of points required
- $k$  — the number of iterations required
- $t$  — the threshold used to identify a point that fits well
- $d$  — the number of nearby points required  
to assert a model fits well

Until  $k$  iterations have occurred

- Draw a sample of  $n$  points from the data  
uniformly and at random
- Fit to that set of  $n$  points
- For each data point outside the sample  
Test the distance from the point to the line  
against  $t$ ; if the distance from the point to the line  
is less than  $t$ , the point is close  
end
- If there are  $d$  or more points close to the line  
then there is a good fit. Refit the line using all  
these points.

end

Use the best fit from this collection, using the  
fitting error as a criterion

# RANSAC: How many samples?

- How many samples are needed?
    - Suppose  $w$  is fraction of inliers (points from line).
    - $n$  points needed to define hypothesis (2 for lines)
    - $k$  samples chosen.
  - Prob. that a single sample of  $n$  points is correct:  $w^n$
  - Prob. that all  $k$  samples fail is:  $(1 - w^n)^k$
- ⇒ Choose  $k$  high enough to keep this below desired failure rate.

Slide credit: David Lowe

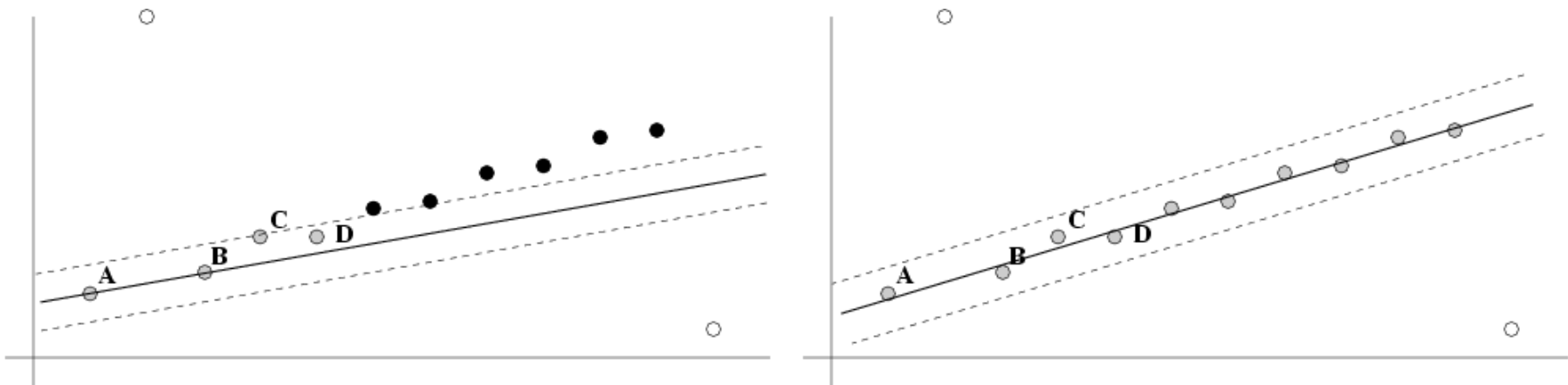
# RANSAC: Computed k ( $p=0.99$ )

Sample size n	Proportion of outliers						
	5%	10%	20%	25%	30%	40%	50%
2	2	3	5	6	7	11	17
3	3	4	7	9	11	19	35
4	3	5	9	13	17	34	72
5	4	6	12	17	26	57	146
6	4	7	16	24	37	97	293
7	4	8	20	33	54	163	588
8	5	9	26	44	78	272	1177

Slide credit: David Lowe

# After RANSAC

- RANSAC divides data into inliers and outliers and yields estimate computed from minimal set of inliers.
- Improve this initial estimate with estimation over all inliers (e.g. with standard least-squares minimization).
- But this may change inliers, so alternate fitting with re-classification as inlier/outlier.



# RANSAC: Pros and Cons

- **Pros:**
  - **General method** suited for a wide range of model fitting problems
  - **Easy** to implement and easy to calculate its failure rate
- **Cons:**
  - Only handles a moderate percentage of outliers without cost blowing up
  - Many real problems have high rate of outliers (but sometimes selective choice of random subsets can help)
- **A voting strategy**, The Hough transform, can handle high percentage of outliers



# Skeleton Code

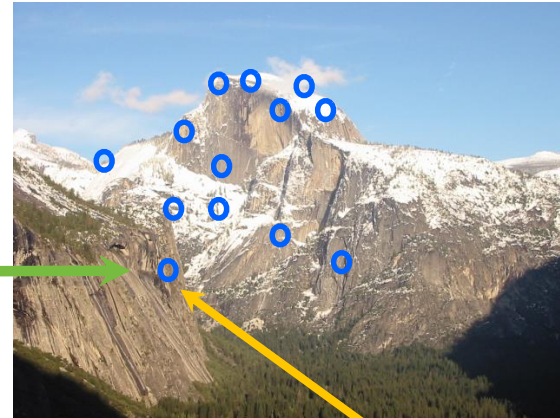
- RANSAC
  - ComputeError

$$\left\| \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} - H \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} \right\|_2$$

# Main Flow



$(u_1, u_2, \dots, u_{128})$

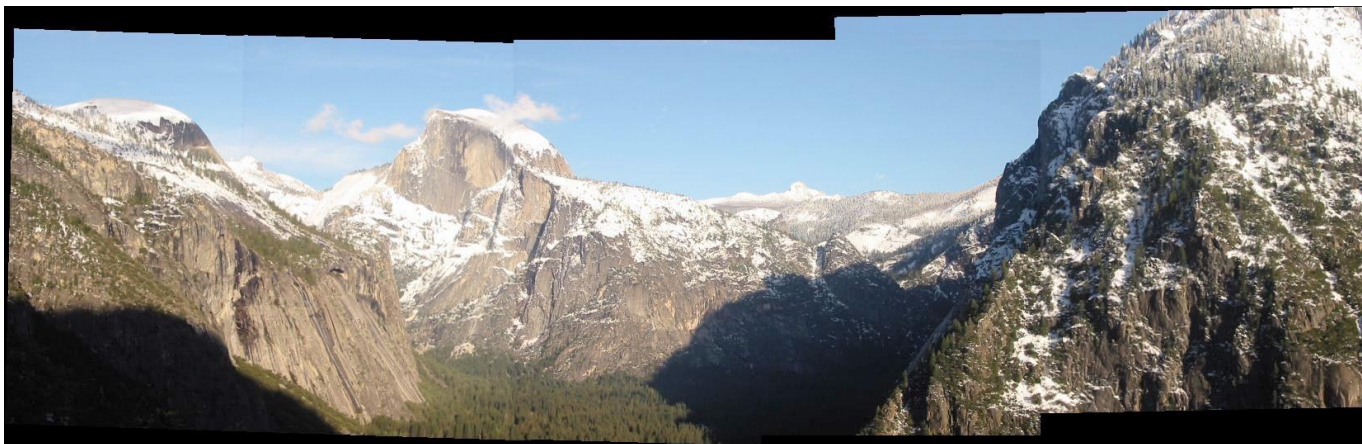


$(v_1, v_2, \dots, v_{128})$

- Detect key points
- Build the SIFT descriptors
- Match SIFT descriptors
- Fitting the transformation
- RANSAC

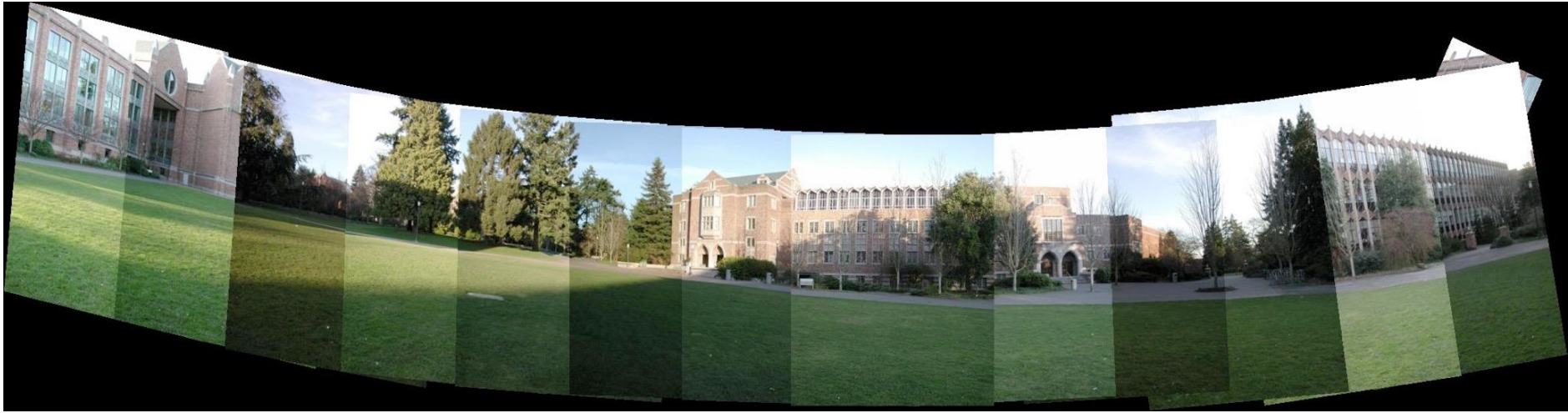


# Results



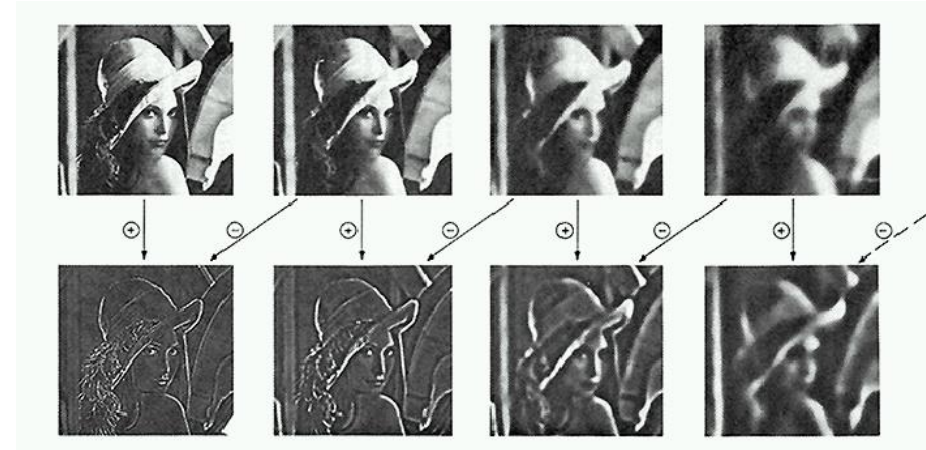
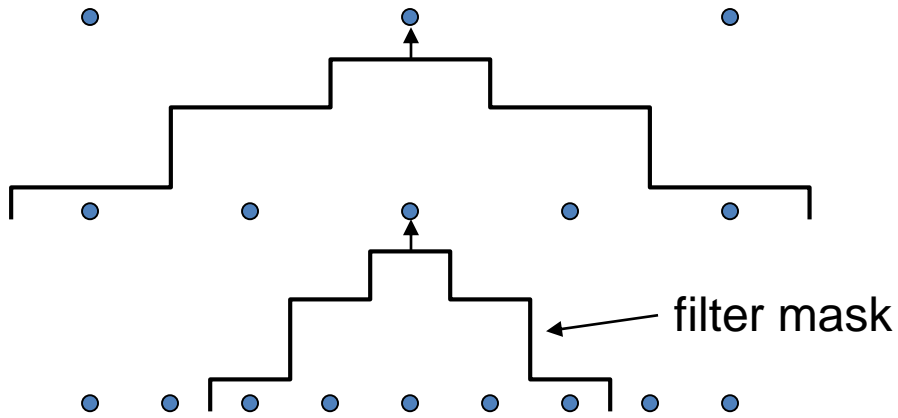


# Results

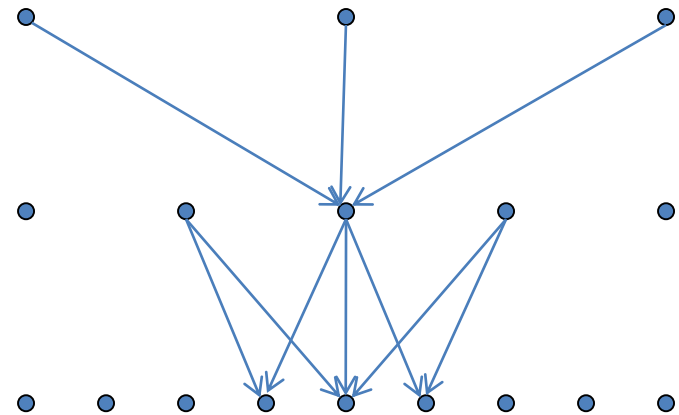


# Image Blending

# Pyramid Creation



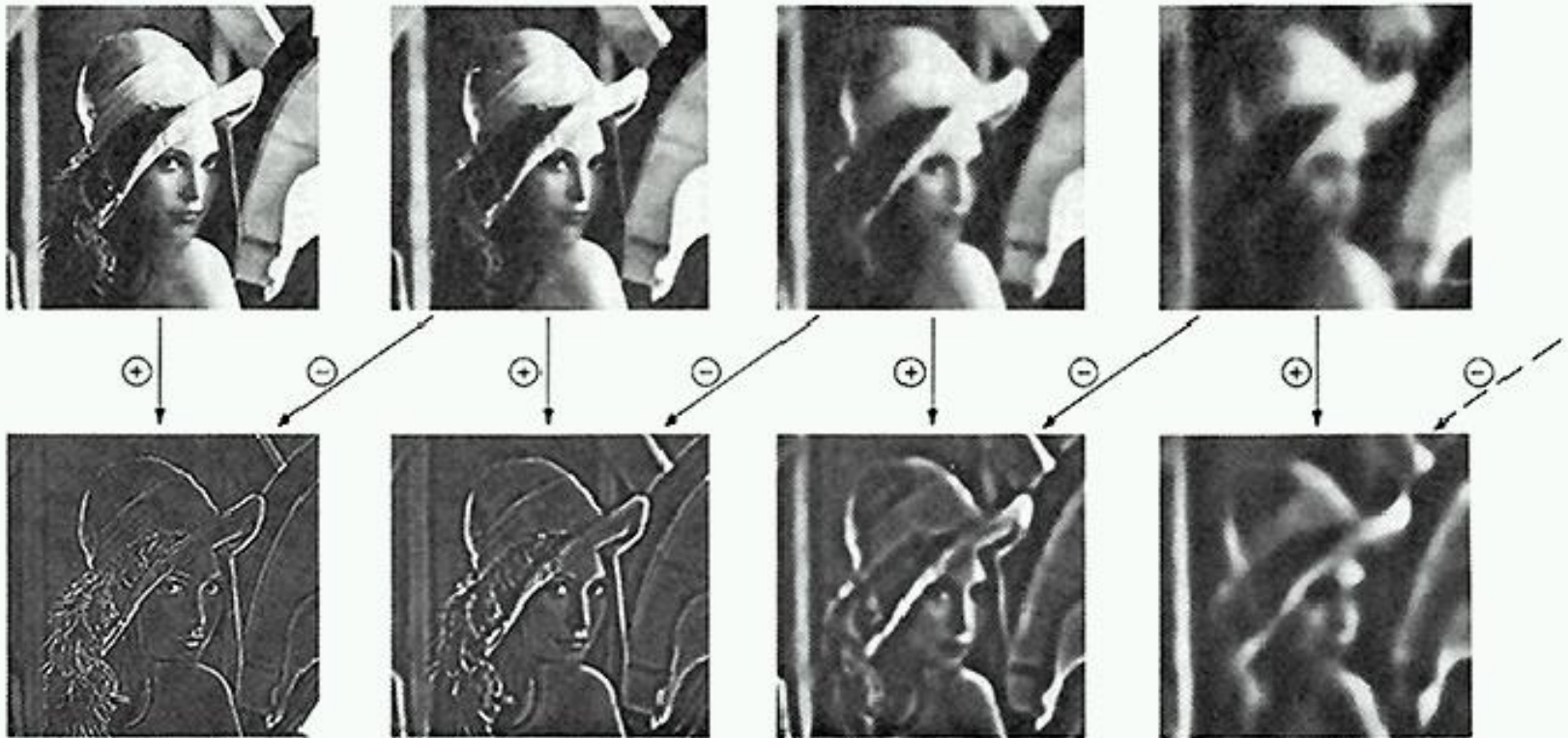
- “Gaussian” Pyramid
- “Laplacian” Pyramid
  - Created from Gaussian pyramid by subtraction
  - $L_l = G_l - \text{expand}(G_{l+1})$





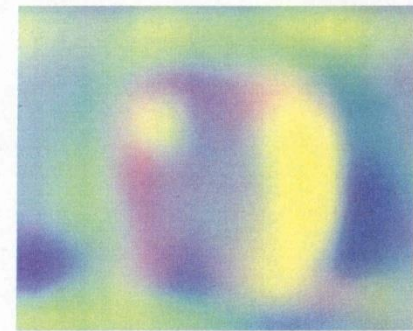
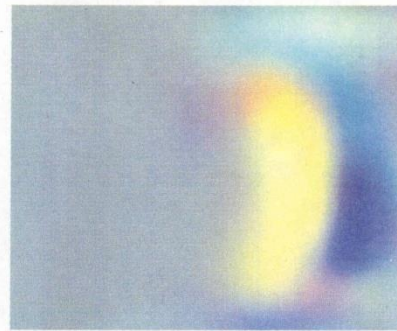
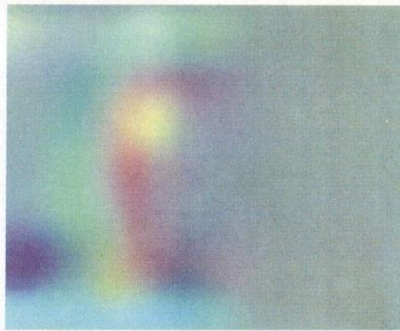
# Octaves in the Spatial Domain

## Lowpass Images

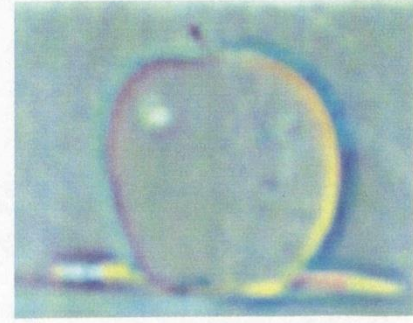
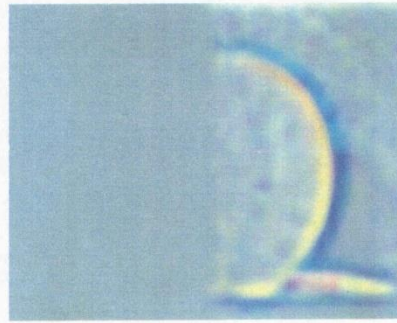
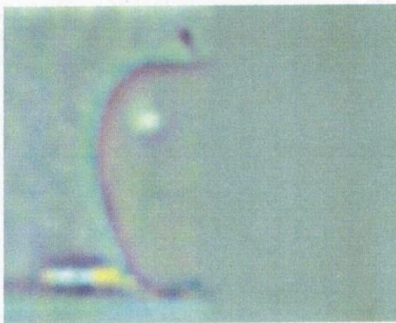


- Bandpass Images

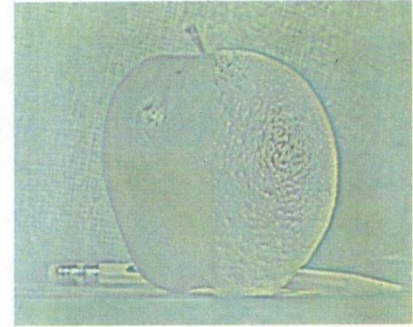
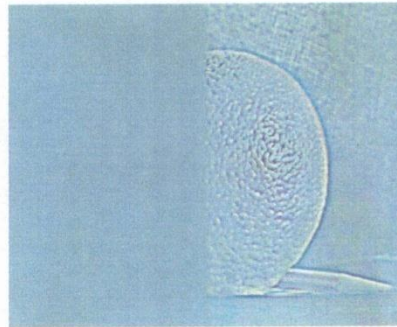
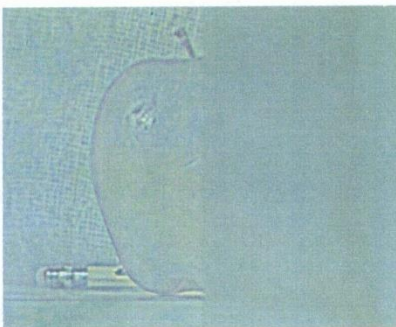
Laplacian  
level  
4



Laplacian  
level  
2



Laplacian  
level  
0



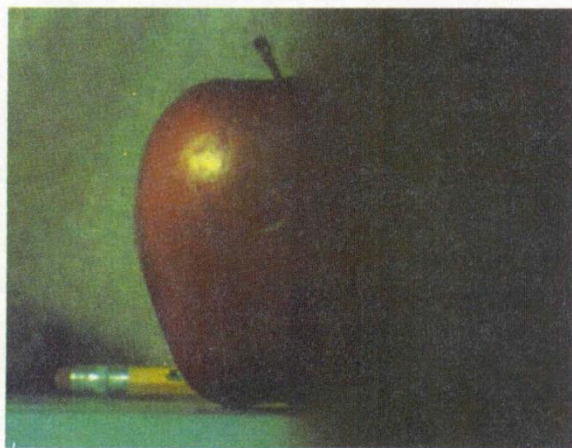
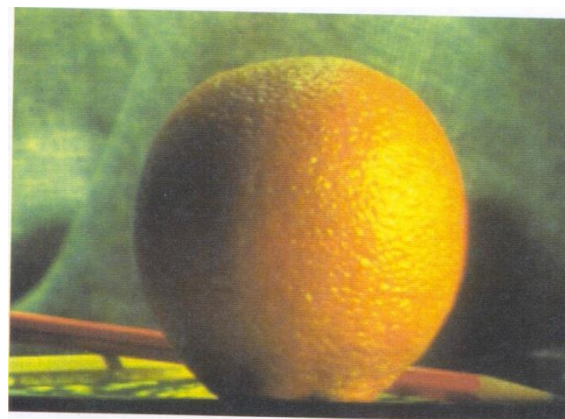
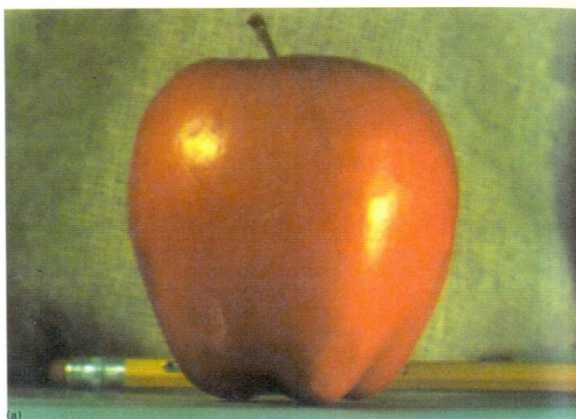
left pyramid

right pyramid

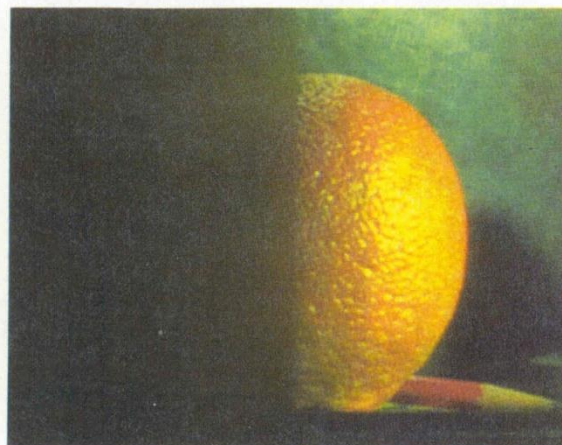
blended pyramid



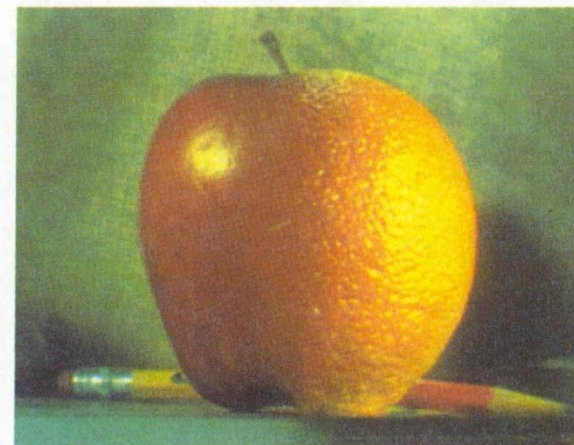
# Pyramid Blending



(d)



(h)



(l)

# Main Flow

- Detect key points
- Build the SIFT descriptors
- Match SIFT descriptors
- Fitting the transformation
- RANSAC
- Image Blending