Object Recognition via Convolution

An Example of Global Optimization

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Slide credit: Stanford CS 131

Recall convolutions...

12	3	19
25	10	1
9	7	17

*

1	2
3	4

=

?	?
?	?

$$f[n,m] * h[n,m] = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} f[k,l]\, h[n-k, m-l]$$

Recall convolutions…

$$\begin{bmatrix} 12 & 3 & 19 \\ 25 & 10 & 1 \\ 9 & 7 & 17 \end{bmatrix} * \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 133 & ? \\ ? & ? \end{bmatrix}$$

$$f[n,m] * h[n,m] = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} f[k,l]\, h[n-k, m-l]$$

Recall convolutions...

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=

133	75
?	?

$$f[n,m] * h[n,m] = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} f[k,l]\, h[n-k, m-l]$$

Recall convolutions...

12	3	19
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$*$

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$=$

133	75
100	?

$$f[n,m] * h[n,m] = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} f[k,l]\, h[n-k, m-l]$$

Recall convolutions...

12	3	19
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*

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3	4

=

133	75
100	101

$$f[n,m] * h[n,m] = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} f[k,l]\, h[n-k, m-l]$$

Recall convolutions...

$$\begin{bmatrix} 12 & 3 & 19 \\ 25 & 10 & 1 \\ 9 & 7 & 17 \end{bmatrix} * \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 133 & 75 \\ 100 & 101 \end{bmatrix}$$

$$f[n,m] * h[n,m] = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} f[k,l]\, h[n-k, m-l]$$

Recall convolutions…

$$\begin{array}{|c|c|}\hline 12 & 21 \\ \hline 18 & 31 \\ \hline \end{array} * \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 4 \\ \hline \end{array} = \boxed{?}$$

$$f[n,m] * h[n,m] = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} f[k,l]\, h[n-k, m-l]$$

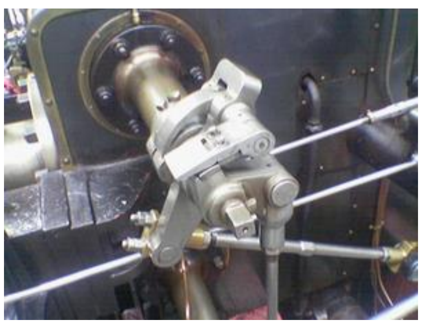
Recall convolutions...

$$\begin{bmatrix} 12 & 21 \\ 18 & 31 \end{bmatrix} * \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 232 \end{bmatrix}$$

$$f[n,m] * h[n,m] = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} f[k,l]\, h[n-k, m-l]$$

Why they are useful

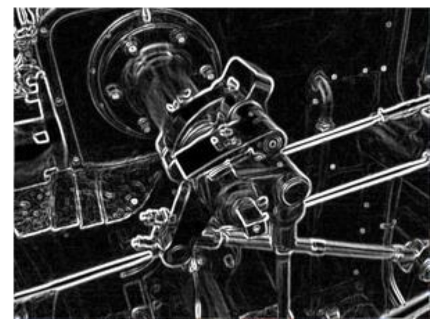
Allow us to find **interesting insights/features** from images!



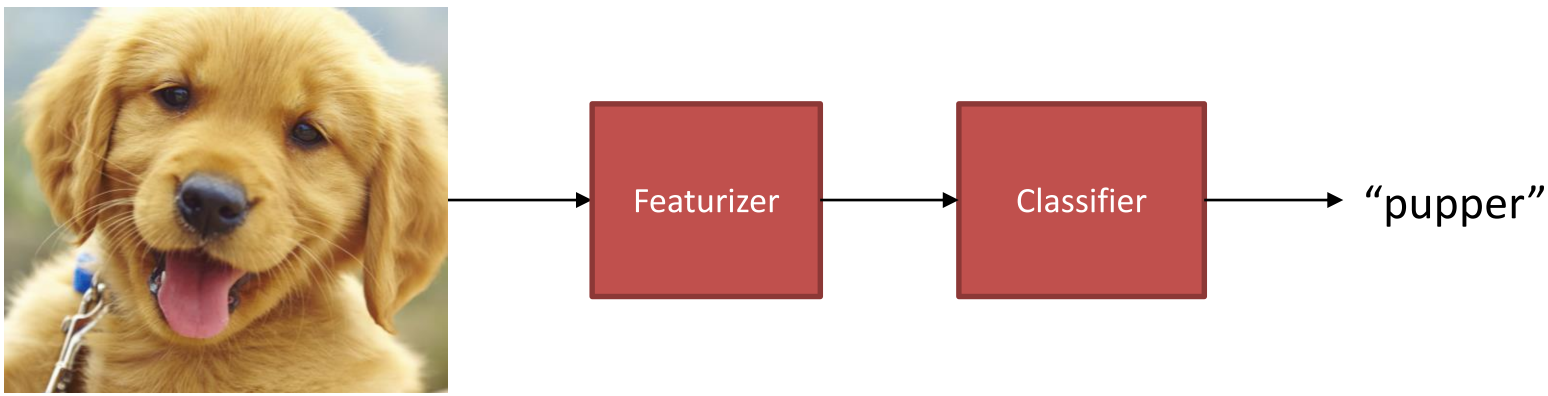
*

0	-½	0
0	0	0
0	½	0

=



Recall Image Classification…



Allow us to use features to put **images in categories**!

Wait a Minute...

Convolution = Image -> Features

Classification Algorithm = Features -> Category

Wait a Minute…

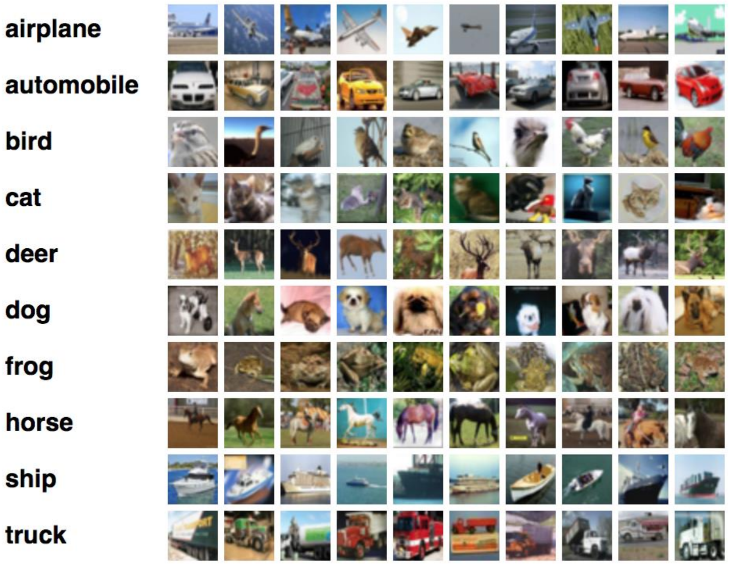
Convolution = Image -> Features

Classification Algorithm = Features -> Category

Let’s put ‘em together!

In Specific…

Let's build a **convolution-based** classification algorithm for the CIFAR-10 dataset (10 classes, 32x32 images):



Feature Extractor

*

32x32 “Airplane Filter”

=

Feature Extractor

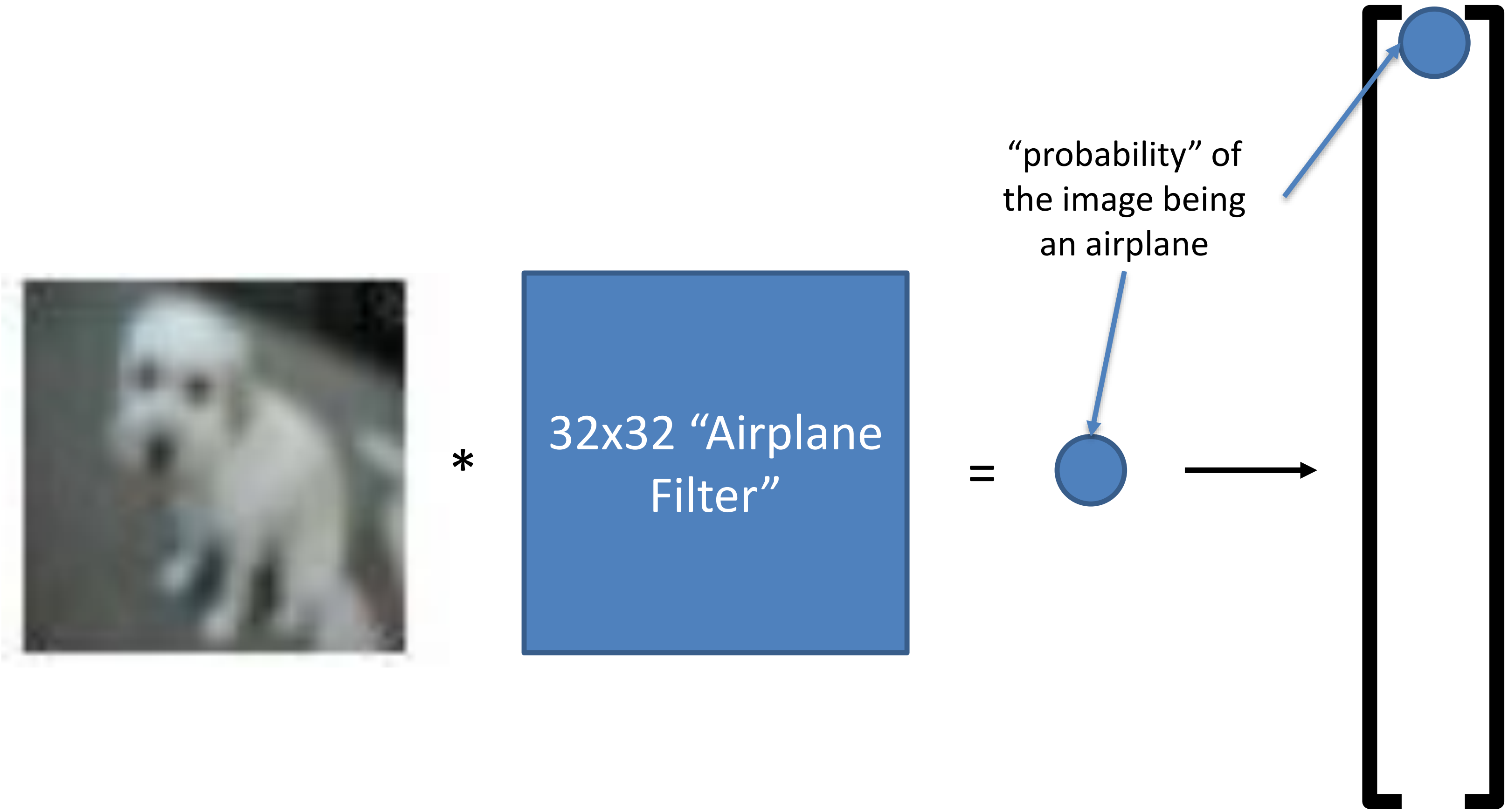
“probability” of
the image being
an airplane

*

32x32 “Airplane
Filter”

=

Feature Extractor



Feature Extractor

This is not really a probability but a score, because it can be less than 0 and greater than 1

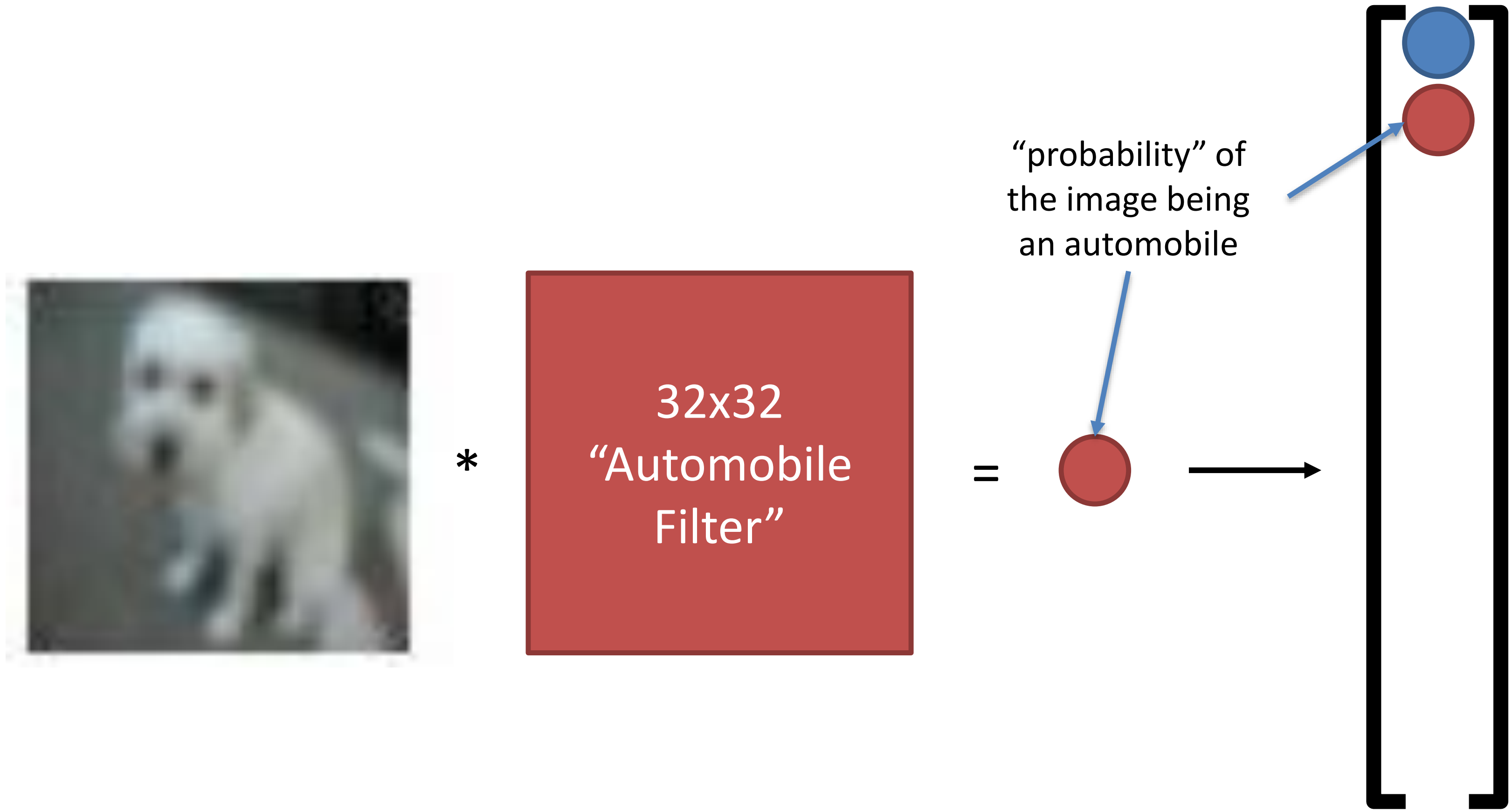
"probability" of the image being an airplane

*

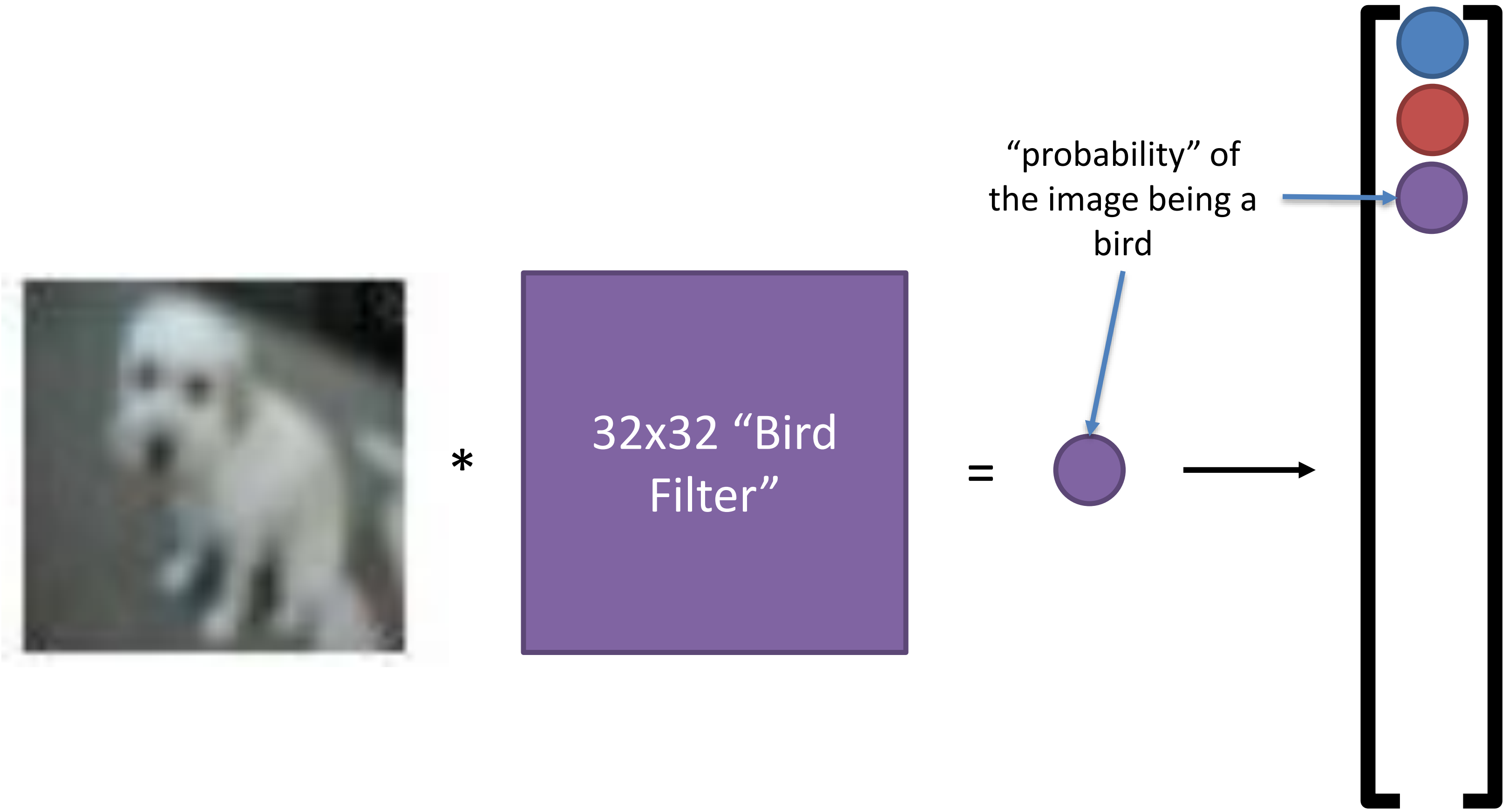
32x32 "Airplane Filter"

=

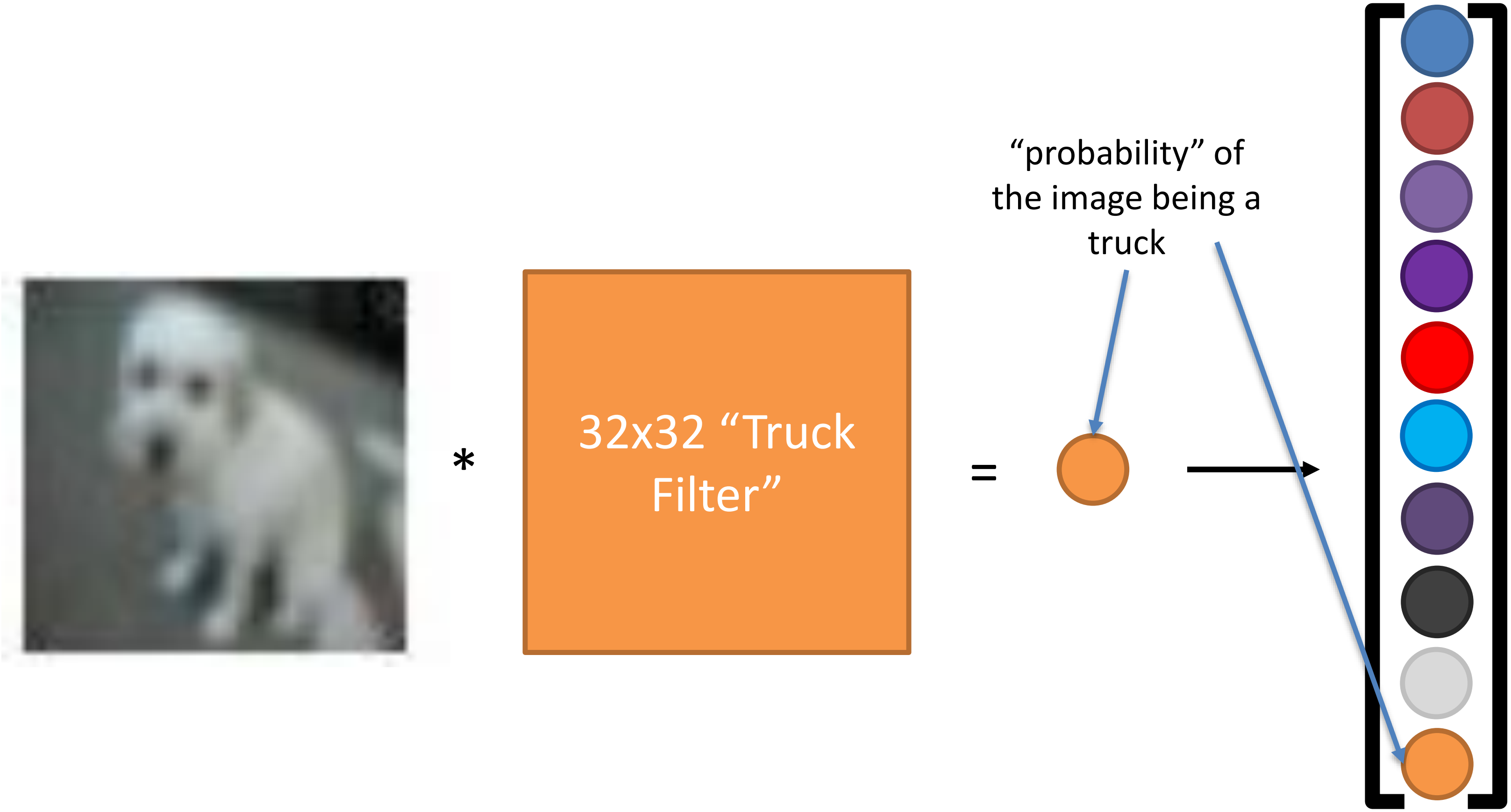
Feature Extractor



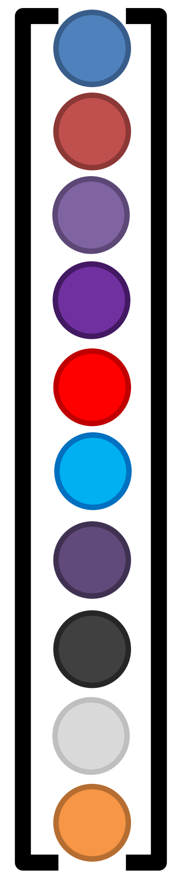
Feature Extractor



Feature Extractor

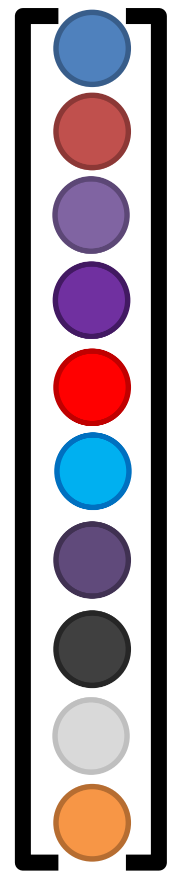


Classifier

$c_{pred} = \arg\max($  $)$

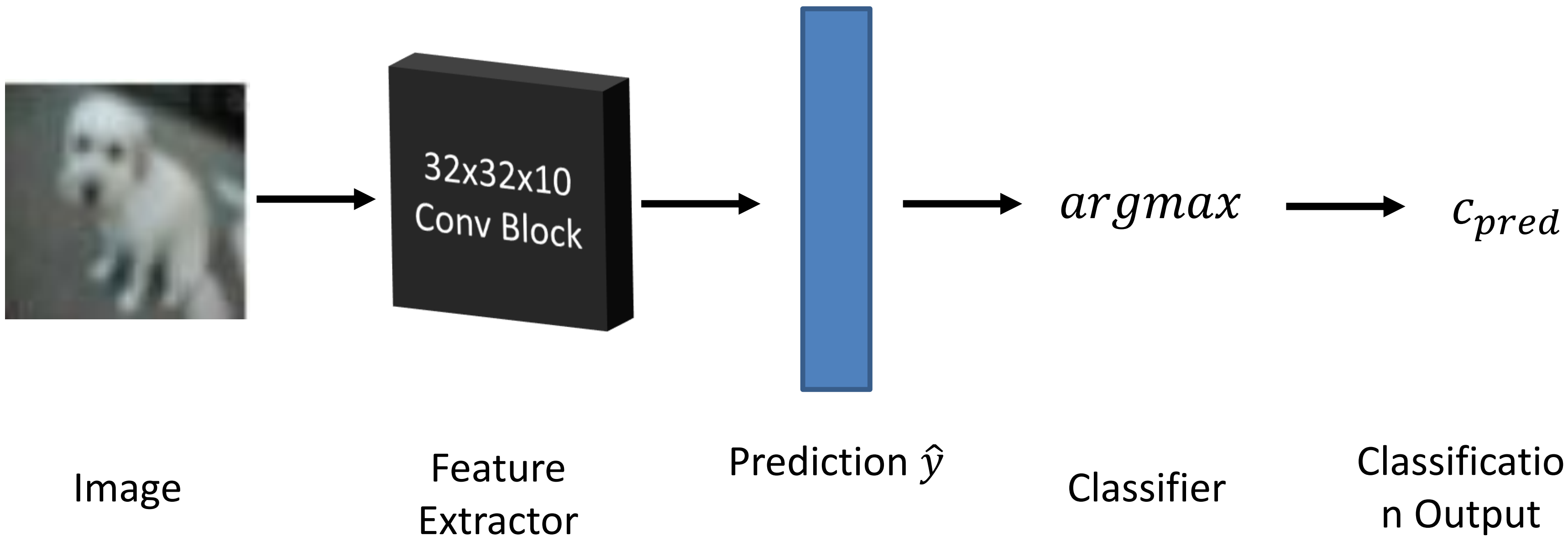
Classifier

$$c_{pred} = \arg\max(\quad)$$

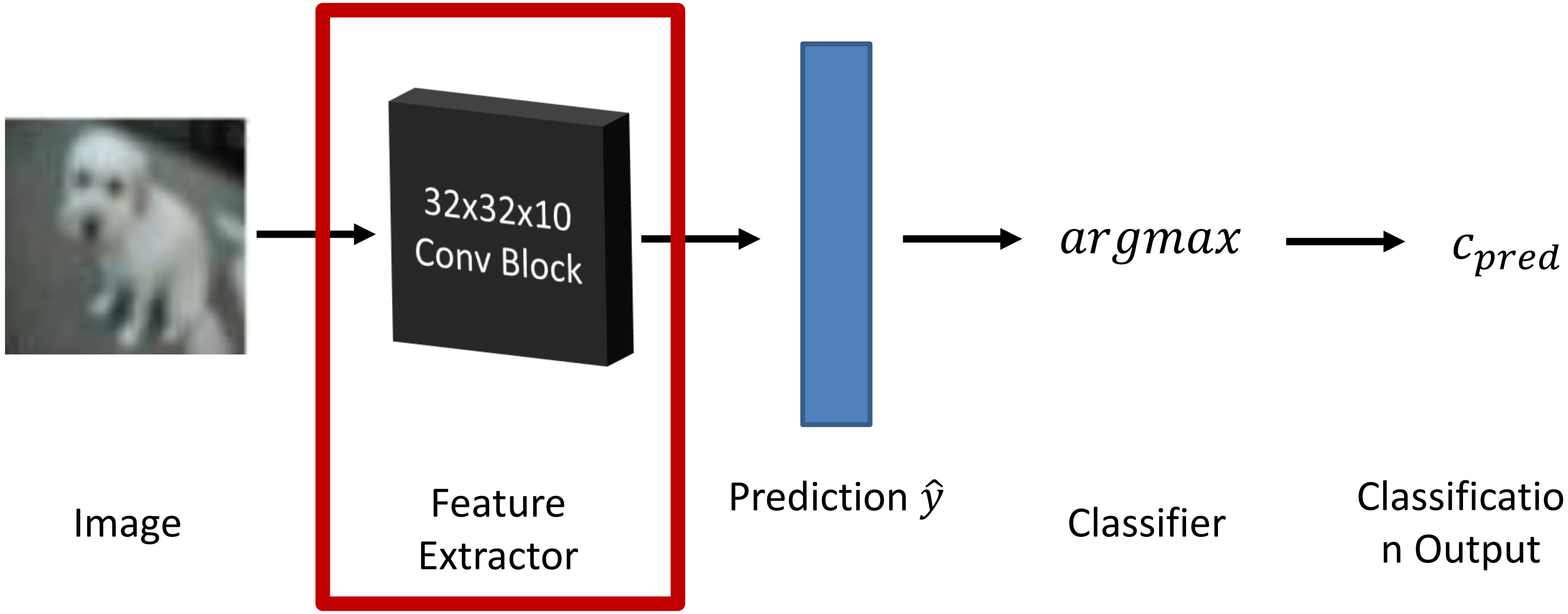


We predict the class that
has the highest probability!

The Whole Shebang



The Whole Shebang



The Whole Shebang

Image

Extractor

c_{pred}

Classificatio
n Output

Reframing convolution

12	21
18	31

*

1	2
3	4

Reframing convolution

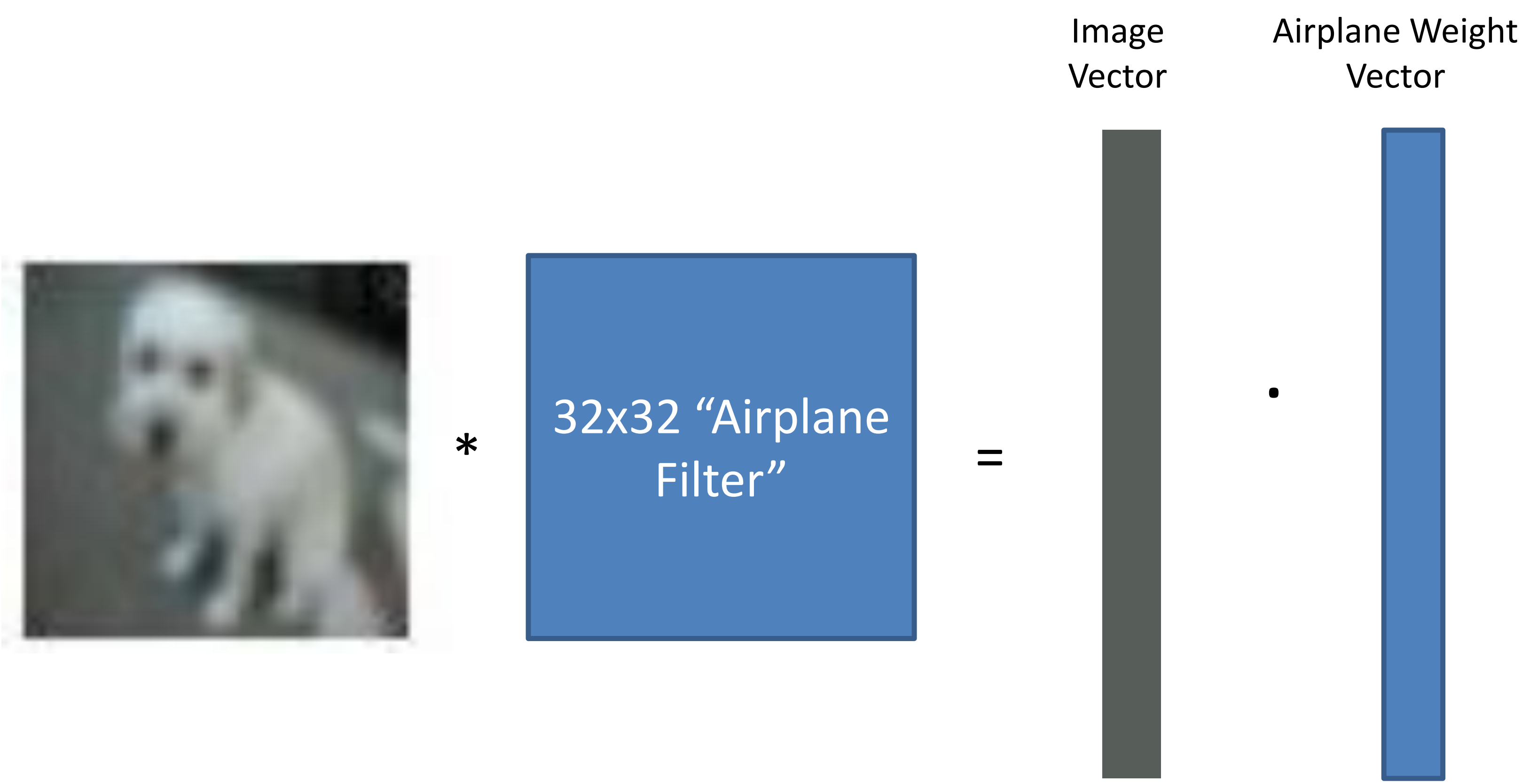
$$\begin{array}{|c|c|}\hline 12 & 21 \\ \hline 18 & 31 \\ \hline \end{array} * \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 4 \\ \hline \end{array} = \begin{bmatrix} 12 \\ 21 \\ 18 \\ 31 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

Reframed Feature Extractor

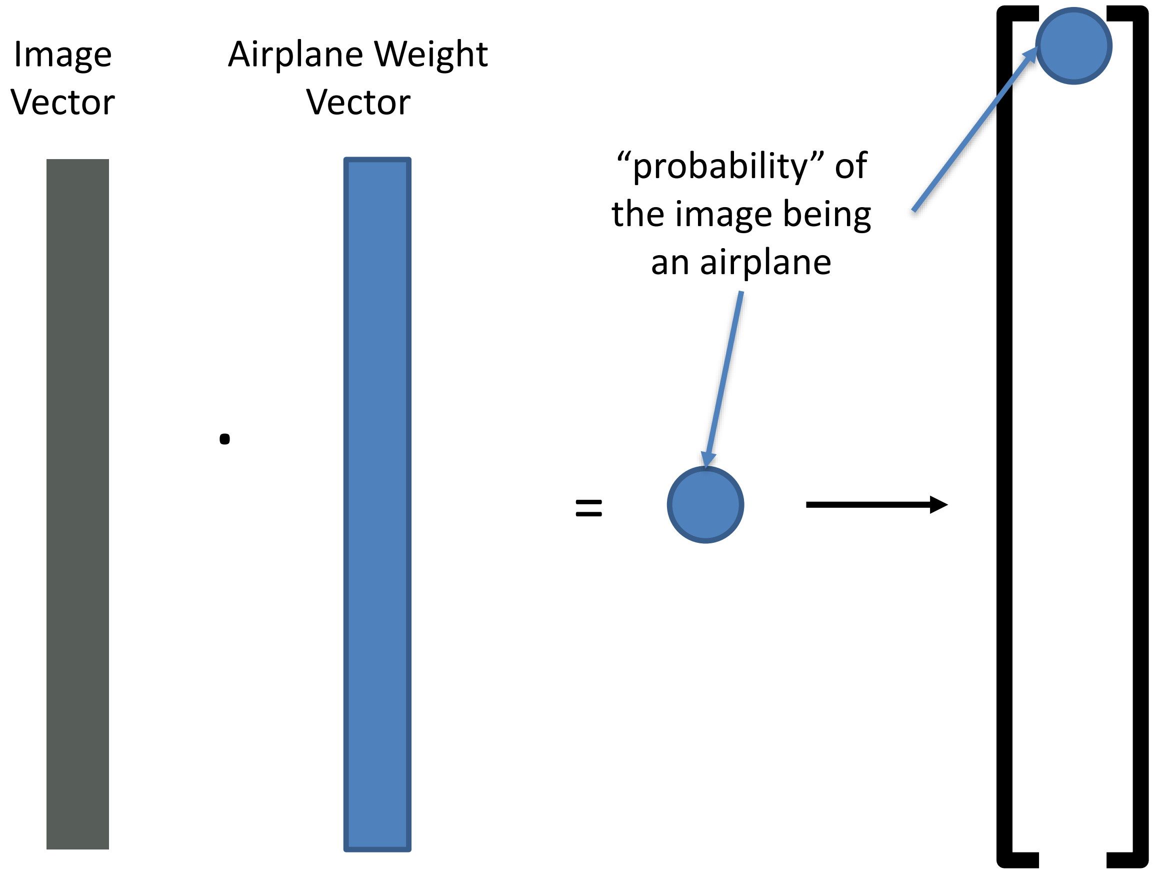
*

32x32 “Airplane Filter”

Reframed Feature Extractor



New Feature Extractor



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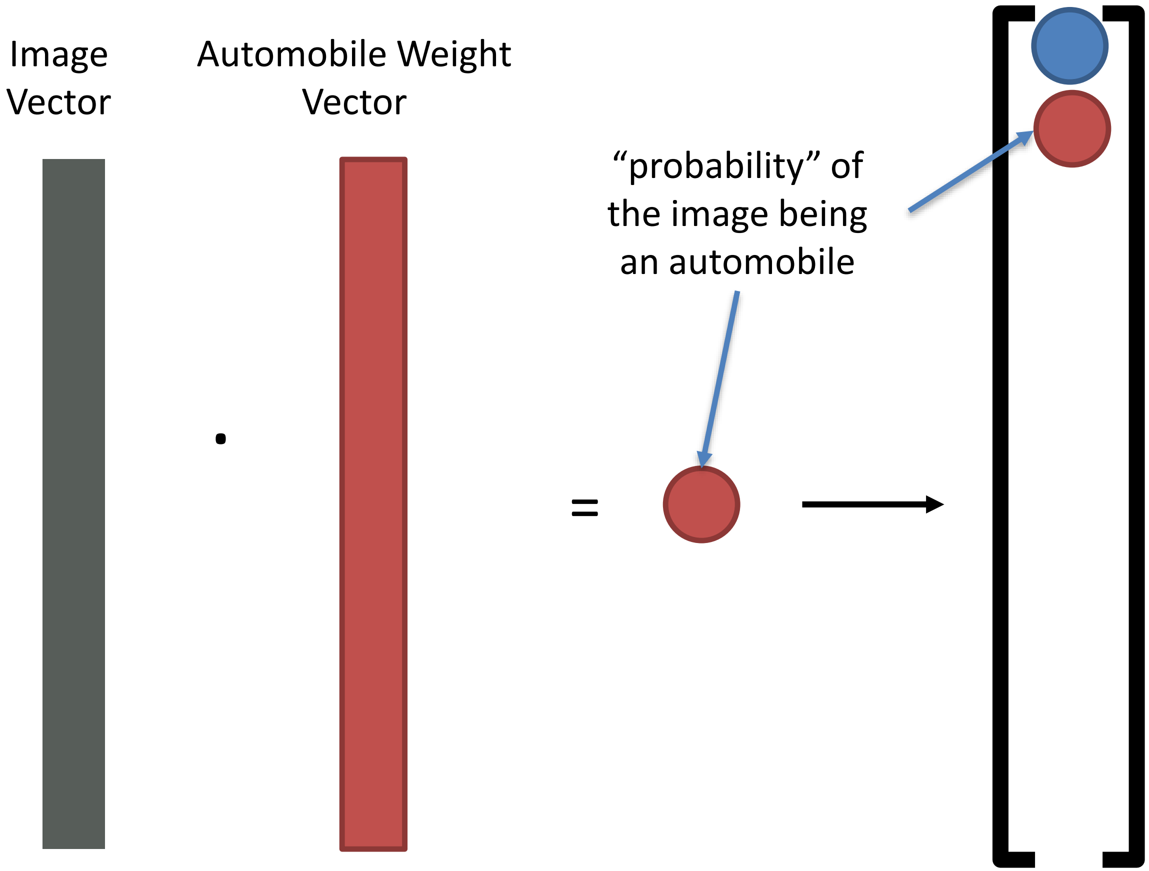

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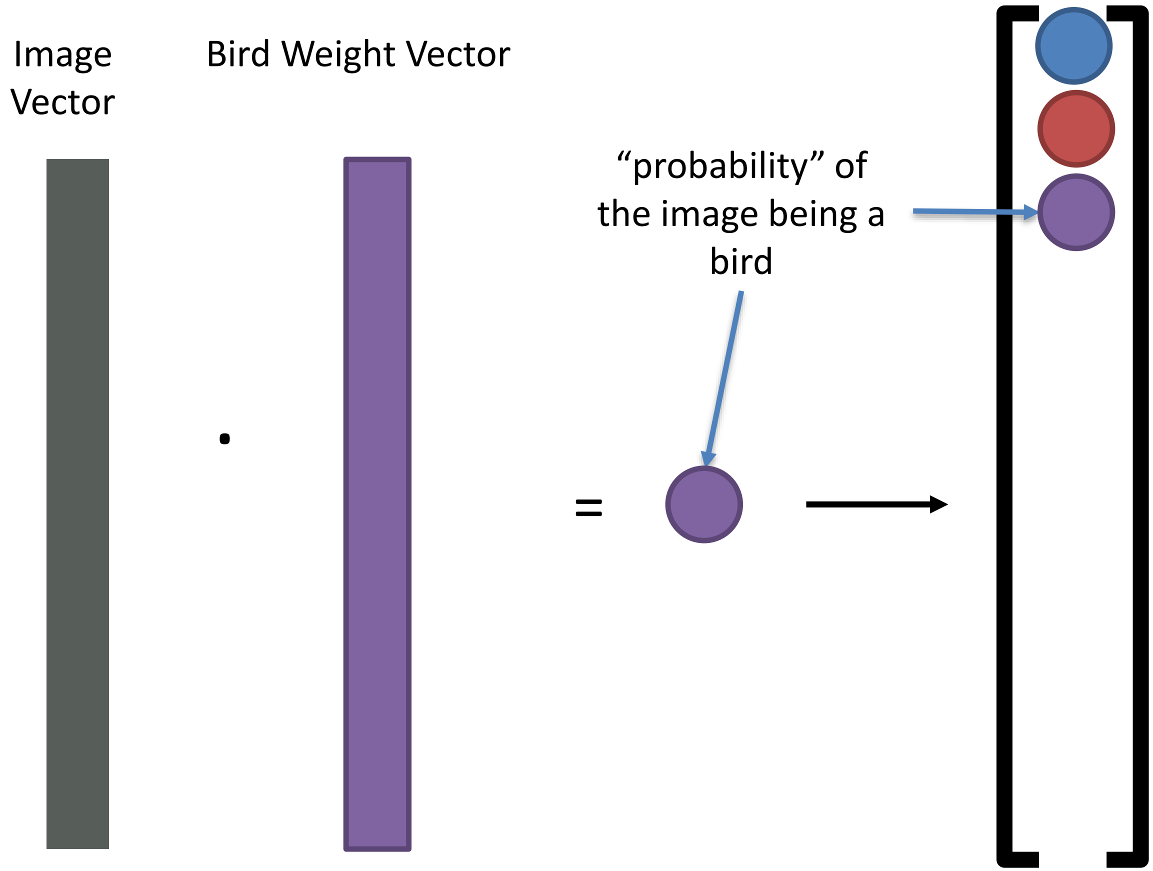

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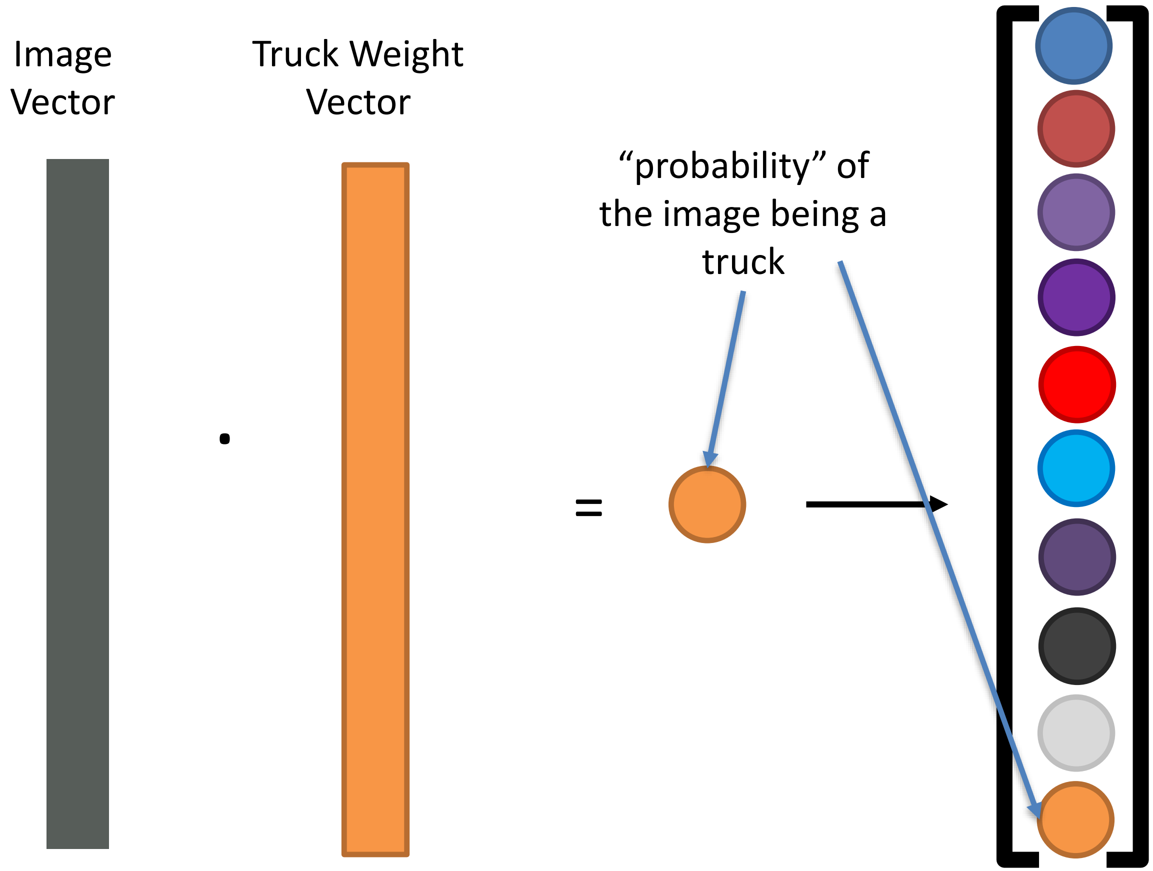

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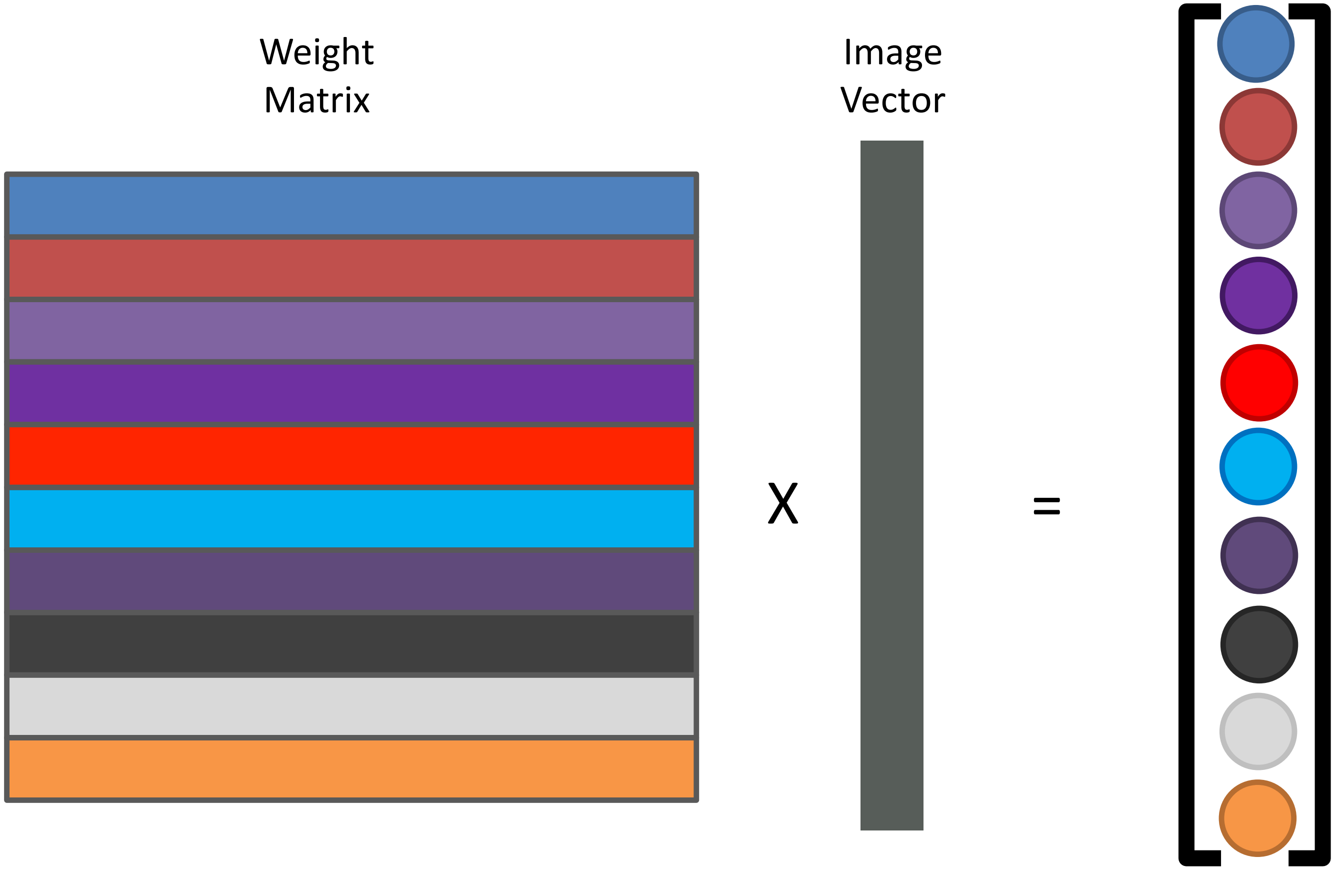


New Feature Extractor

$$Wx = \hat{y}$$

W: the (10x1024) matrix of weight vectors

x: the (1024x1) image vector

$\hat{y}$: the (10x1) vector of class "probabilities"

New Feature Extractor

This simple computation is called a *fully-connected layer*!

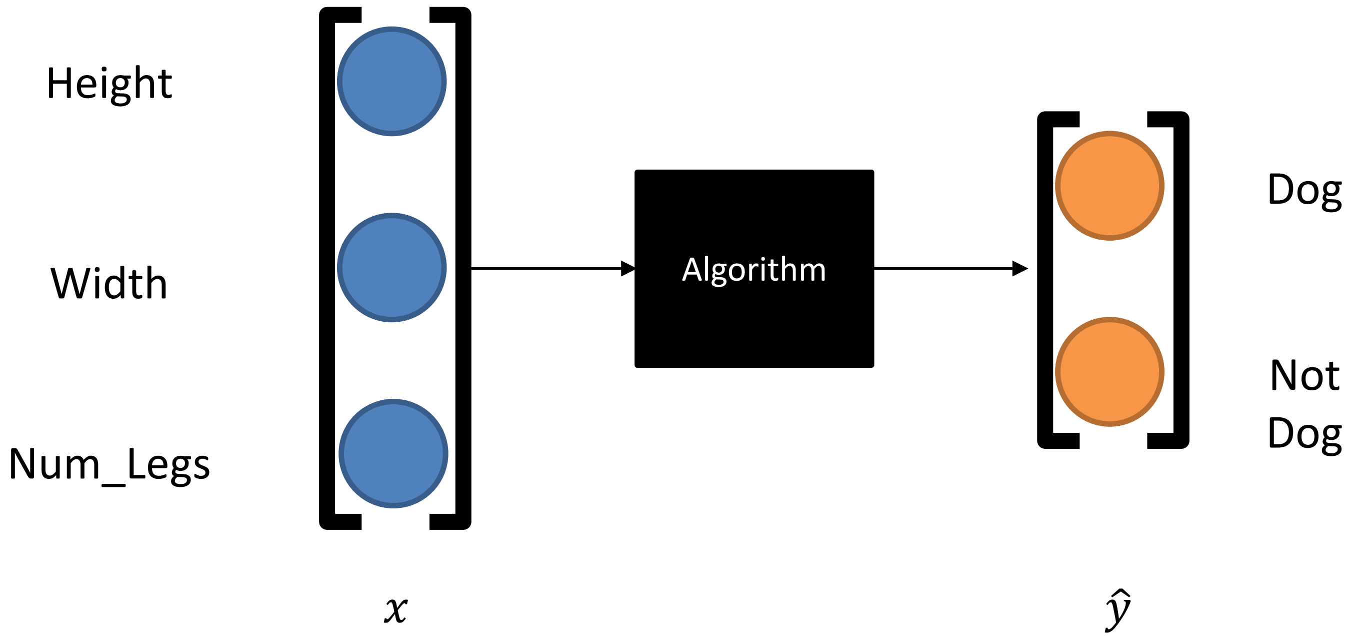
$$\boxed{Wx} = \hat{y}$$

W: the (10x1024) matrix of weight vectors

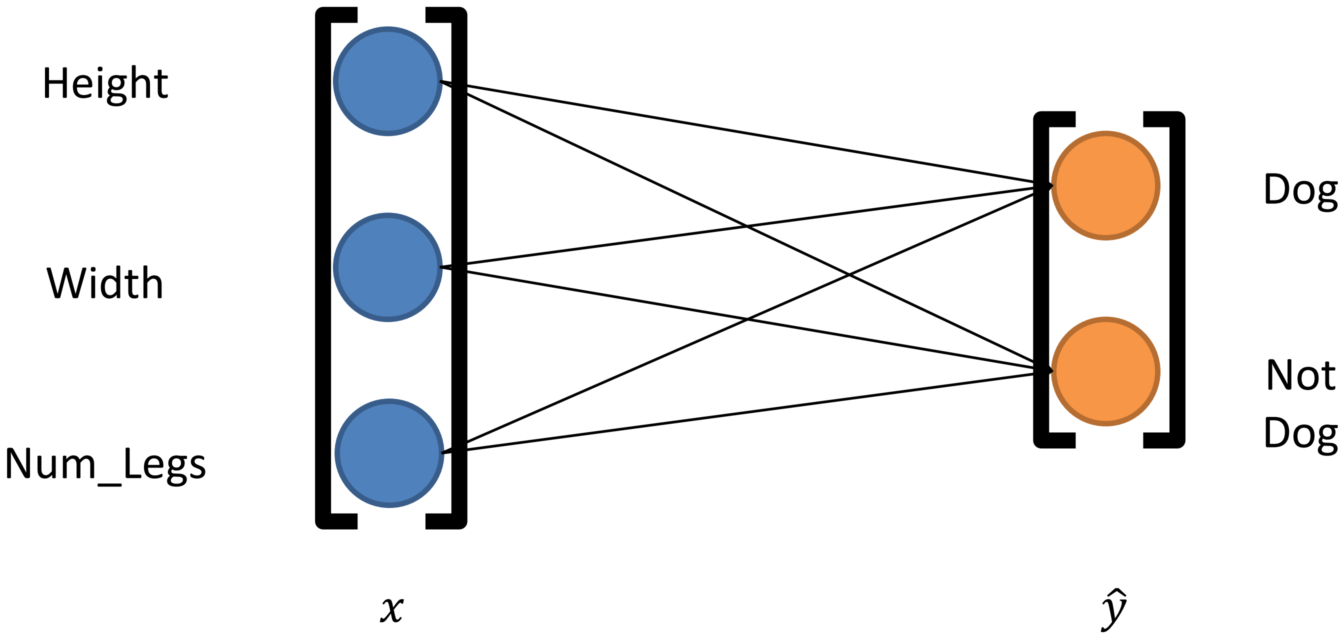
x: the (1024x1) image vector

$\hat{y}$: the (10x1) vector of class “probabilities”

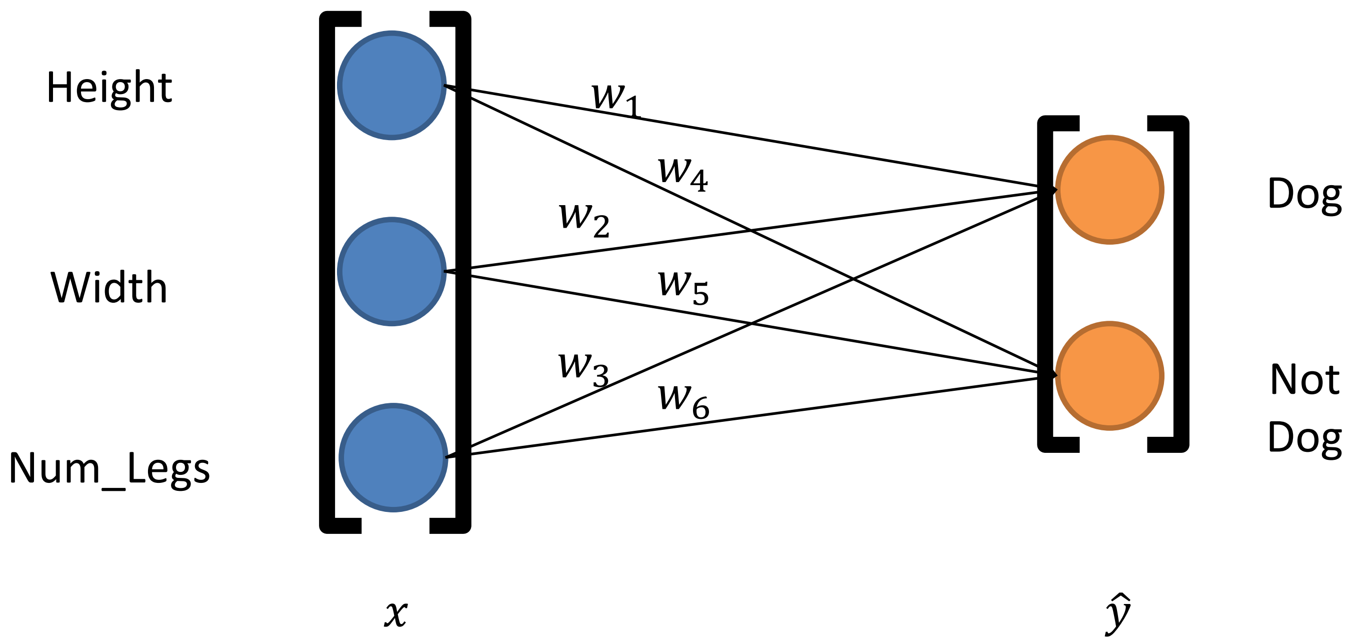
Aside: Fully-Connected Neural Networks



Aside: Fully-Connected Neural Networks

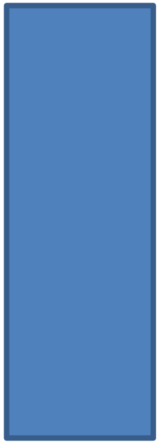


Aside: Fully-Connected Neural Networks

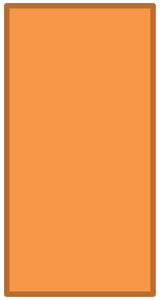


Aside: Fully-Connected Neural Networks

$$\begin{bmatrix} w_1 & w_2 & w_3 \\ w_4 & w_5 & w_6 \end{bmatrix} \cdot$$



$$=$$

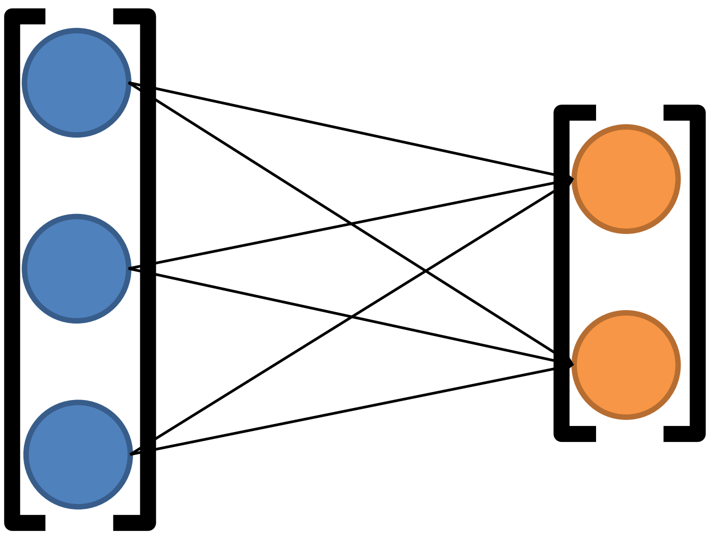


W $\quad$ x $\quad$ $\hat{y}$

$$Wx = \hat{y}$$

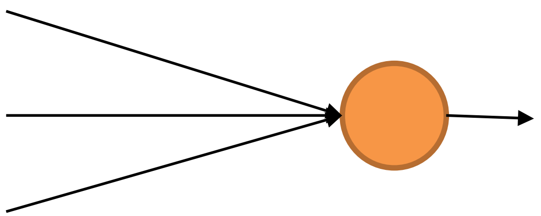
Aside: Fully-Connected Neural Networks

"Fully-Connected"



Every node is connected
to every other node

"Neural Network"



Kinda looks like a
neuron!

New Feature Extractor

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x: the (1024x1) image vector

$\hat{y}$: the (10x1) vector of class “probabilities”?

Class Probability Vector

- Must have values between 0 and 1
- Must sum to 1
- There’s no guarantee either requirement is satisfied!

$$\hat{y} = Wx$$

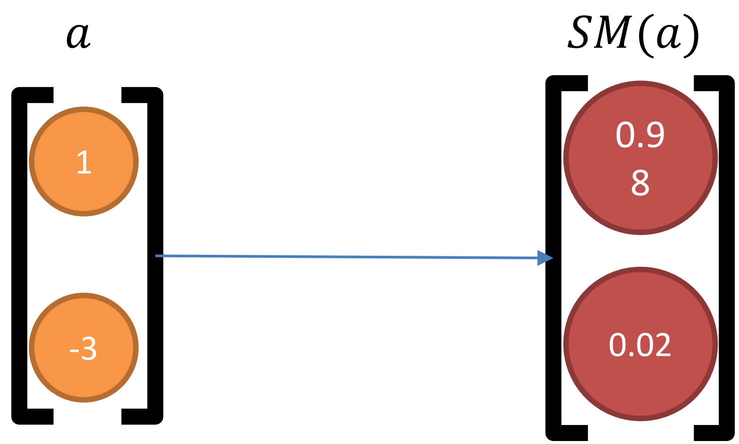
Softmax Function

a

$$\begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

Softmax: $a(x)_i = \frac{e^{x_i}}{\sum_j e^{x_j}}$

Softmax Function



Softmax: $a(x)_i = \frac{e^{x_i}}{\sum_j e^{x_j}}$

Class Probability Vector

- Must have values between 0 and 1
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$$\hat{y} = Wx$$

Class Probability Vector

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$$\hat{y} = SM(Wx)$$

System so far...

- Feature extractor:
- Classifier:

$$\hat{\mathrm{y}} = SM(Wx)$$

$$c_{pred} = \arg\max(\hat{y})$$

System so far…

- Feature extractor:
- Classifier:

$$\hat{y} = SM(\boxed{W}x)$$

$$c_{pred} = \arg\max(\hat{y})$$

System so far…

- Feature extractor:

- Classifier:

Using the label

Let's compare our prediction with the real answer!
For each image, we have the label y which tells us the true class:

X

y

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Dog class index

Key Insight:

We want:

$$\arg\max(\hat{y}) = \arg\max(y)$$

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Which we can accomplish by:

$$W^* = \arg\min_{W}\left(-\sum_{x,y}\log(p_c)\right)$$

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We want:

$$\arg\max(\hat{y}) = \arg\max(y)$$

Which we can accomplish by:

$$W^* = \arg\min_{W}\left(-\sum_{x,y}\log(p_c)\right)$$

Where p_c is the probability of the true class in $\hat{y}$

Cross-Entropy Loss

Our loss function represents *how bad we are currently doing*:

$$L = -\log(p_c)$$

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Examples:

$$p_c = 0 \rightarrow L = -\log(0) = \infty$$

$$p_c = 0.1 \rightarrow L = -\log(0.1) = 2.3$$

$$p_c = 0.9 \rightarrow L = -\log(0.9) = 0.1$$

$$p_c = 1 \rightarrow L = -\log(1) = 0$$

Cross-Entropy Loss

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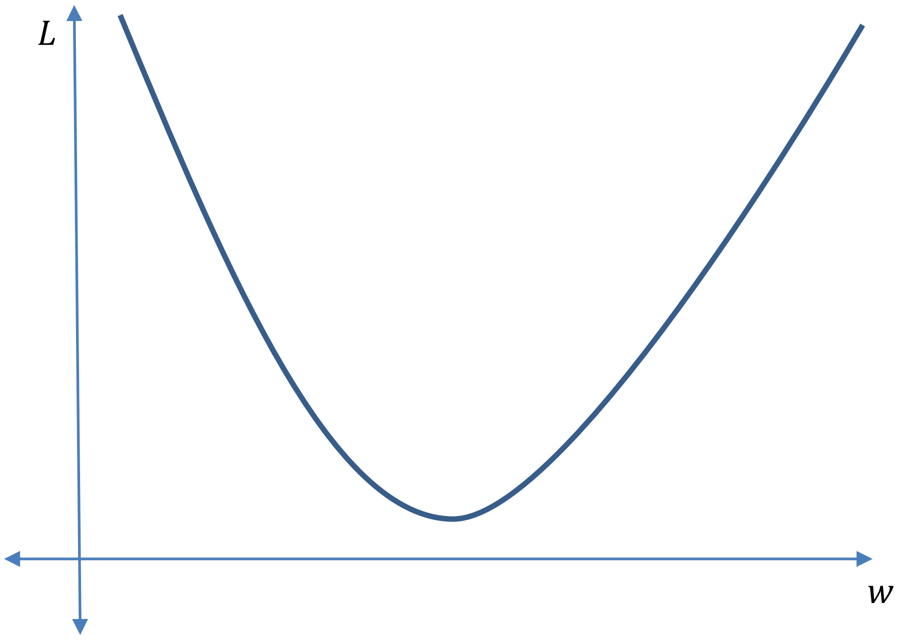
$$p_c = 0.9 \rightarrow L = -\log(0.9) = 0.1$$

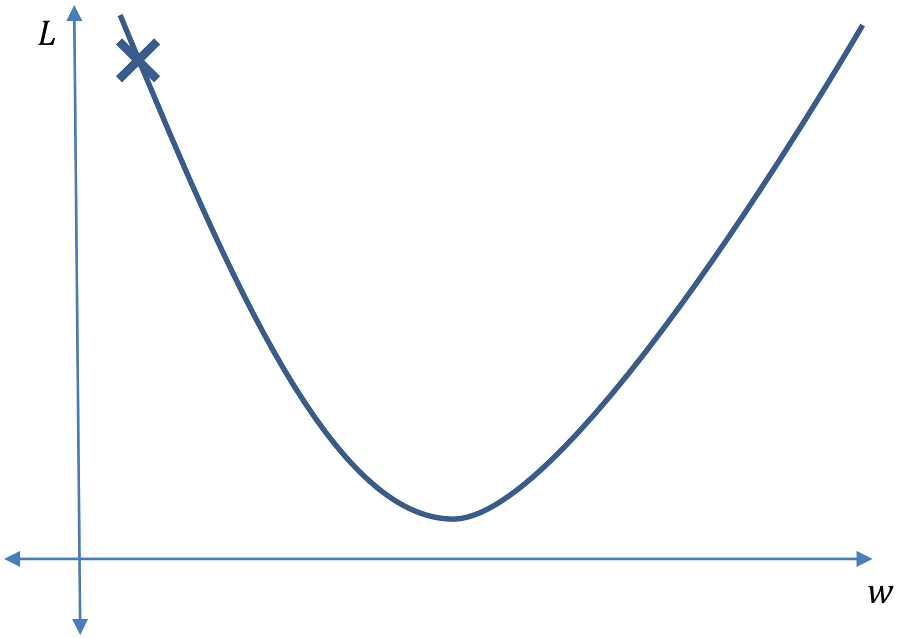
$$p_c = 1 \rightarrow L = -\log(1) = 0$$

The larger the loss, the worse our prediction.
We want to minimize L!

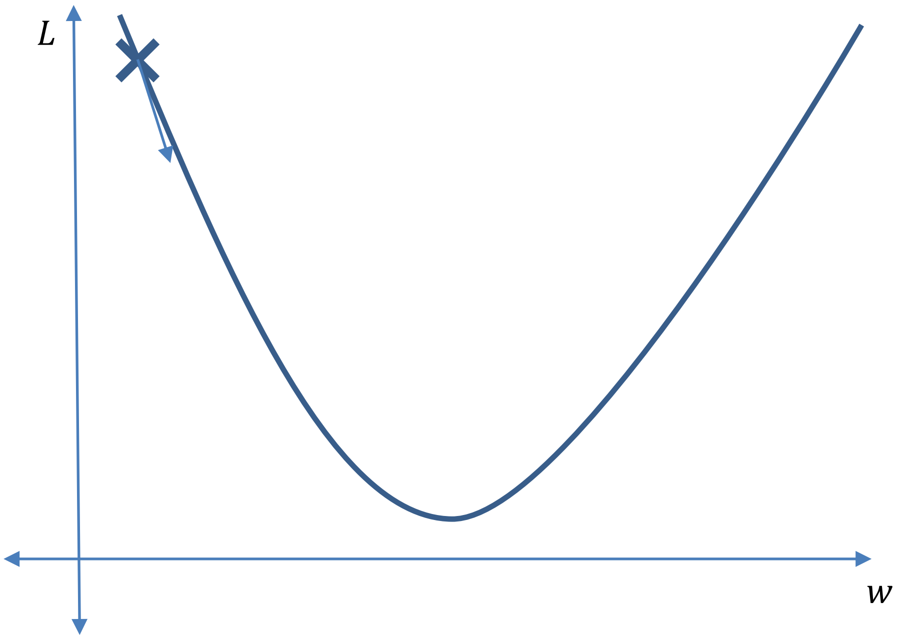
Minimizing Loss



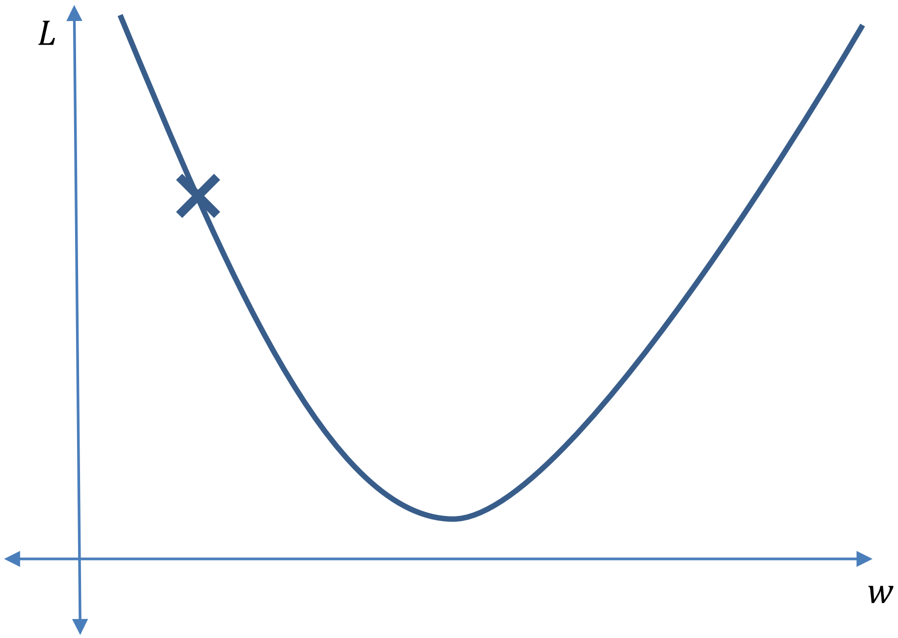
Minimizing Loss



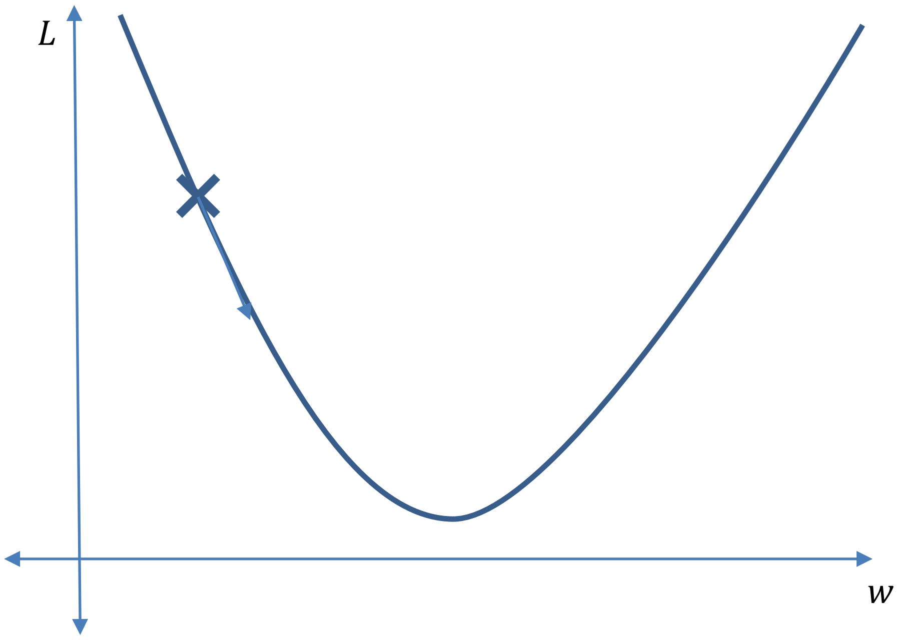
Minimizing Loss



Minimizing Loss



Minimizing Loss



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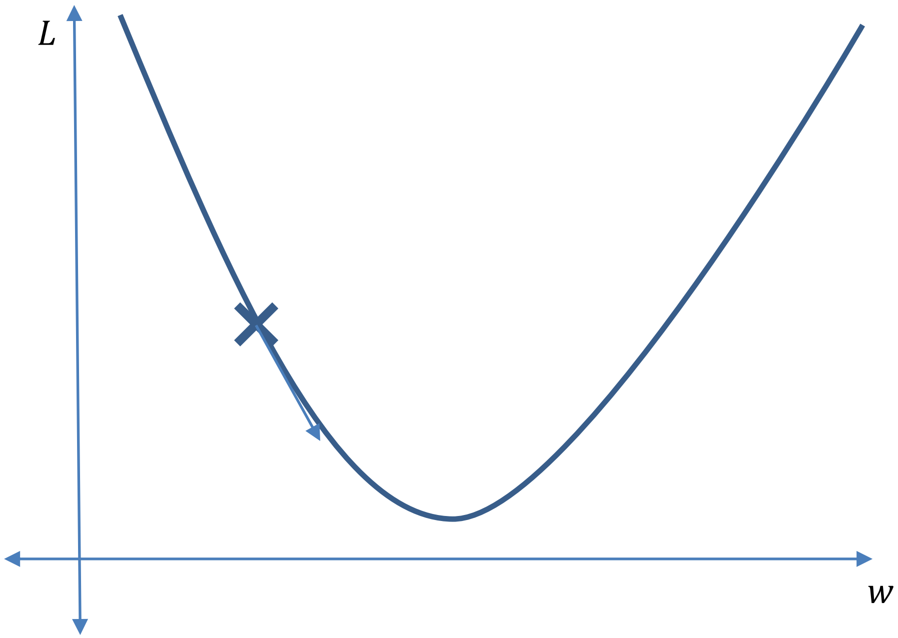

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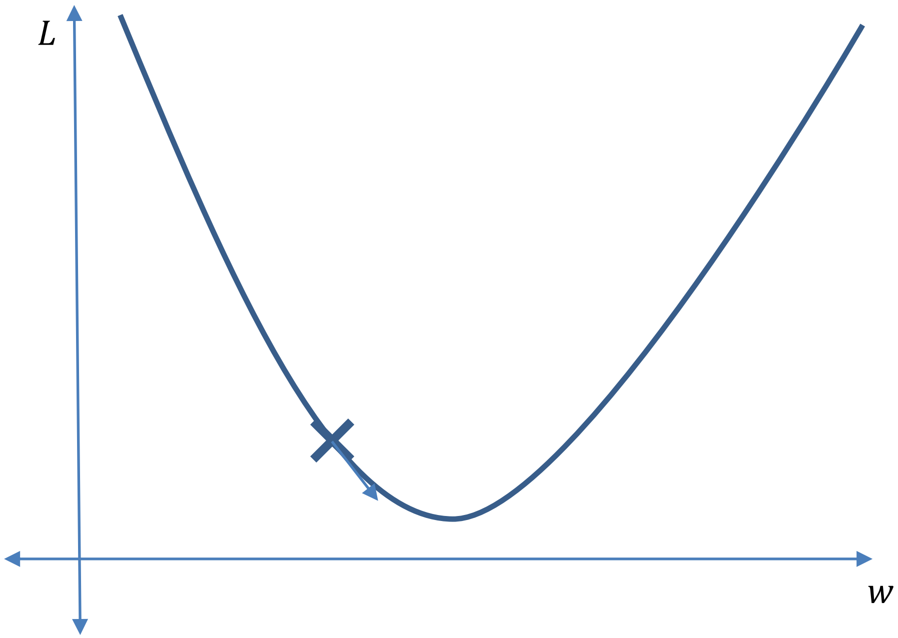

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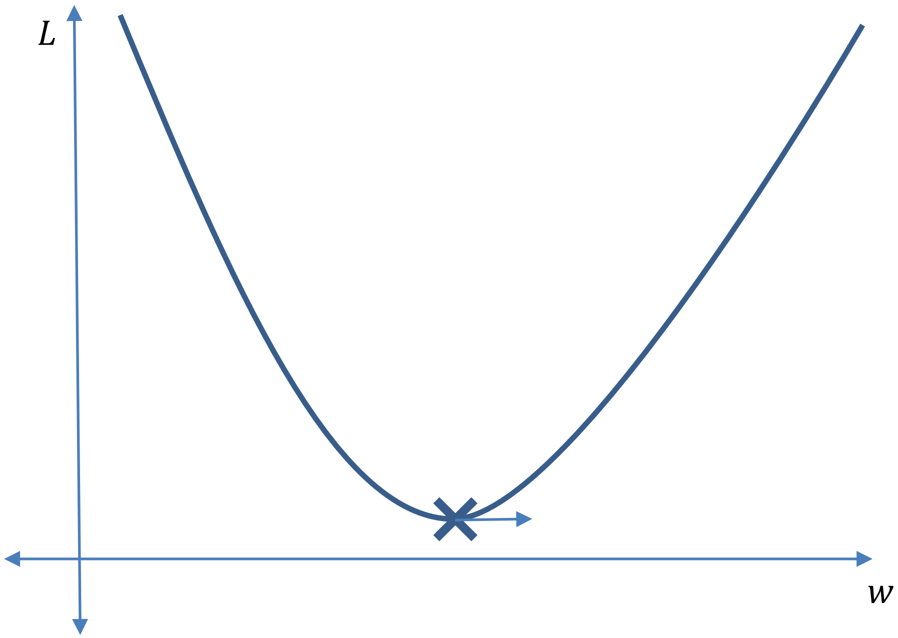


Minimizing Loss

Our Classification System

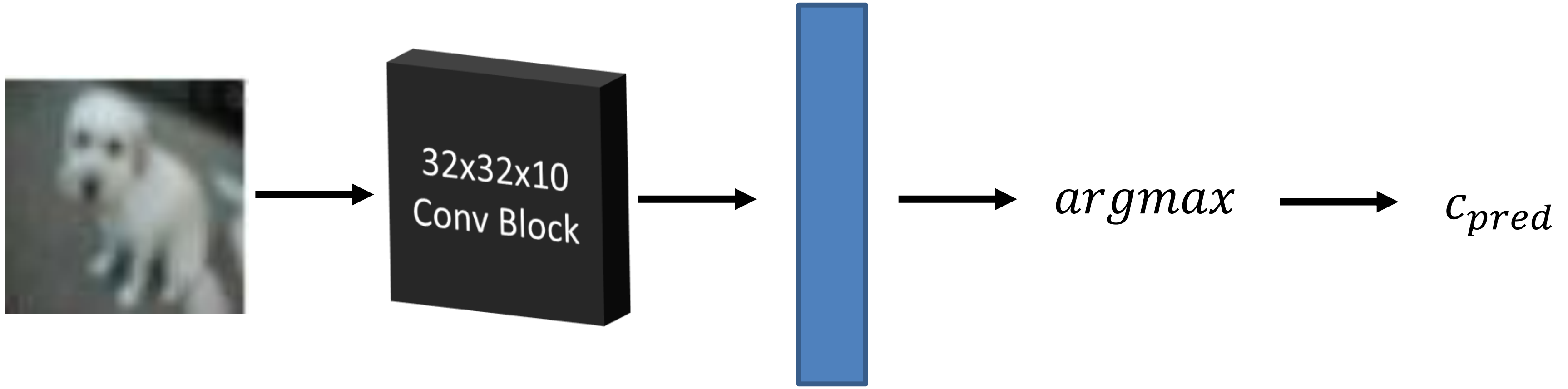
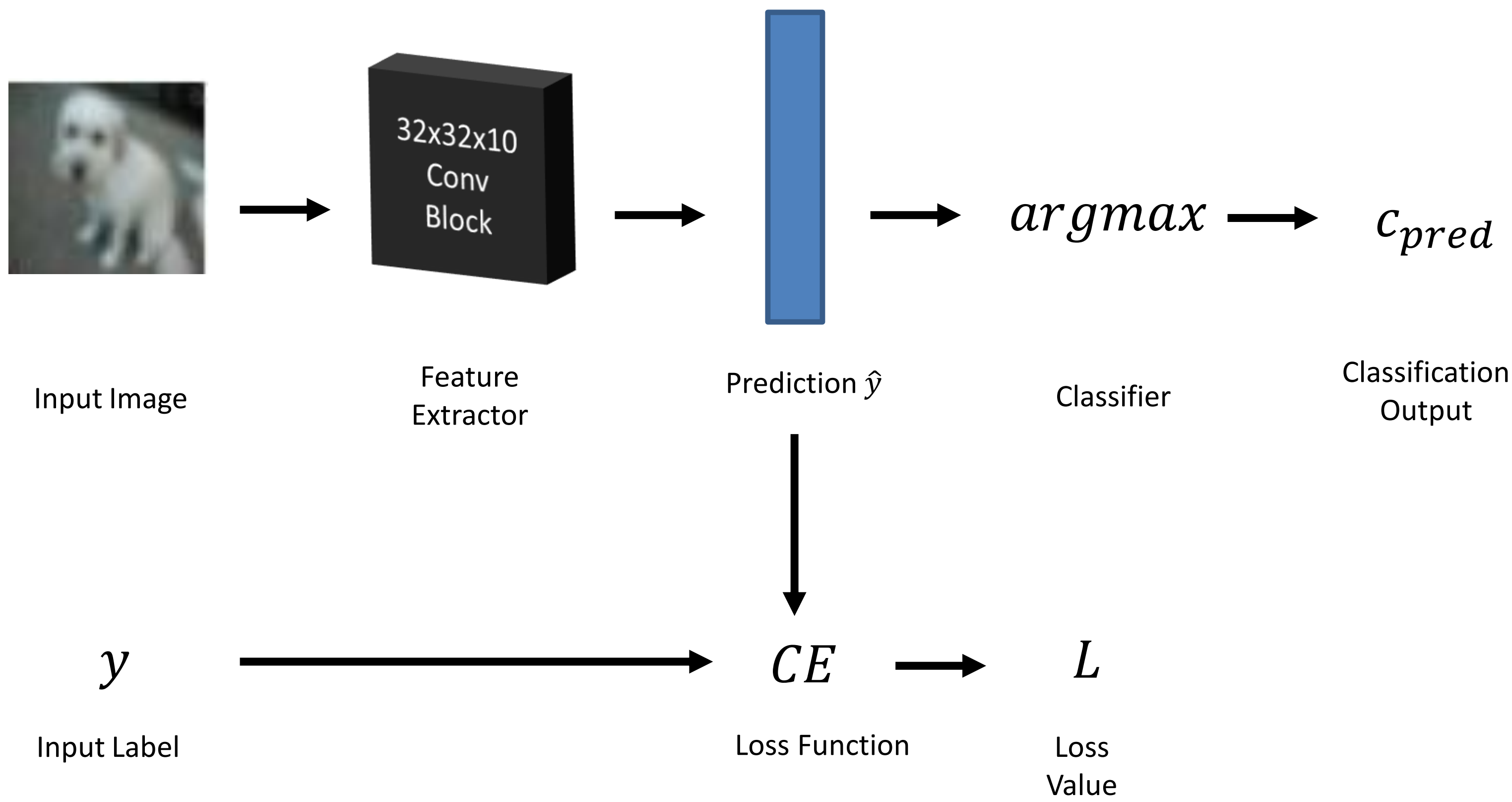


Image | Feature Extractor | Prediction $\hat{y}$ | Classifier | Classification Output

Our Classification System (modified)



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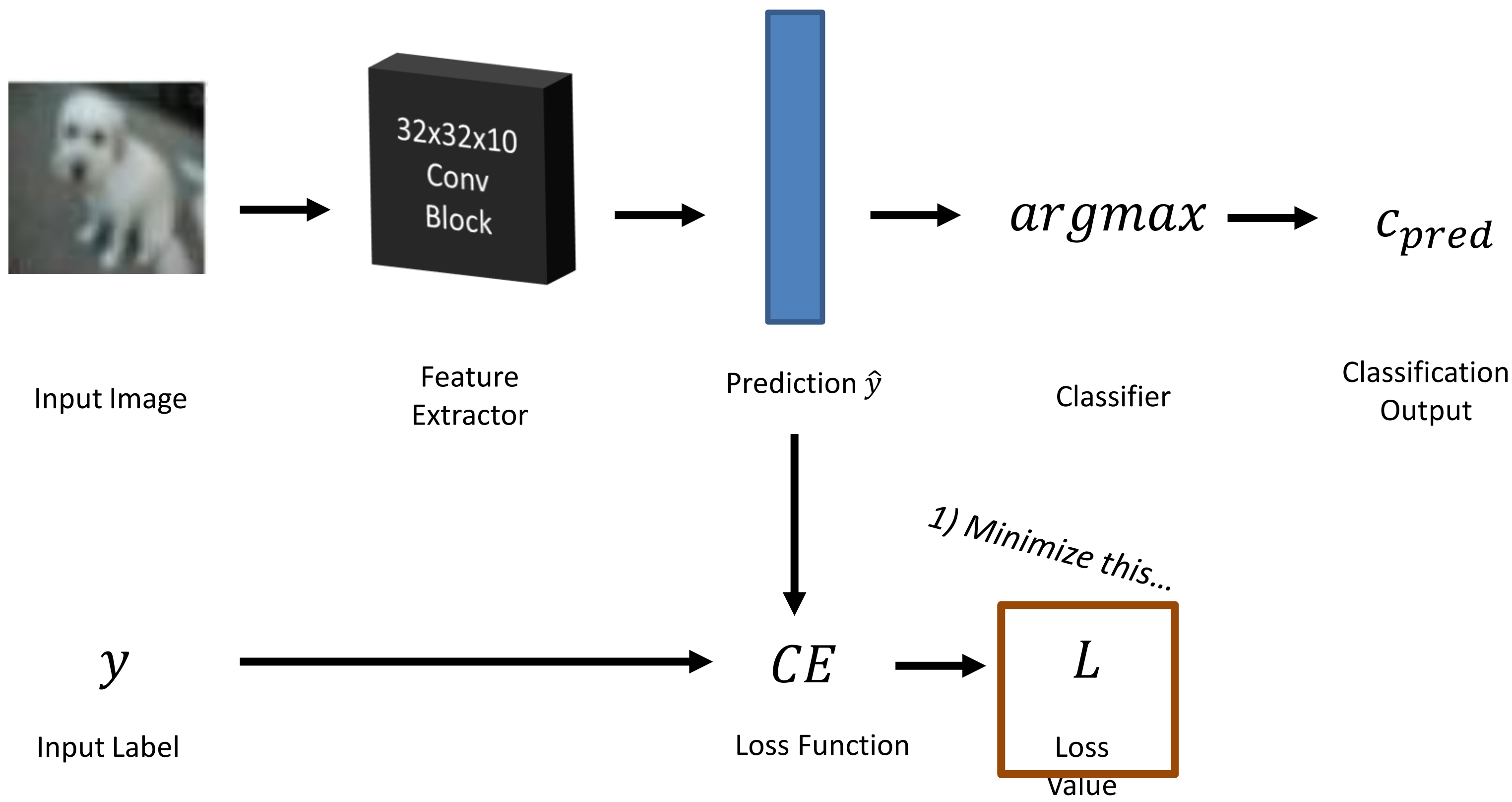

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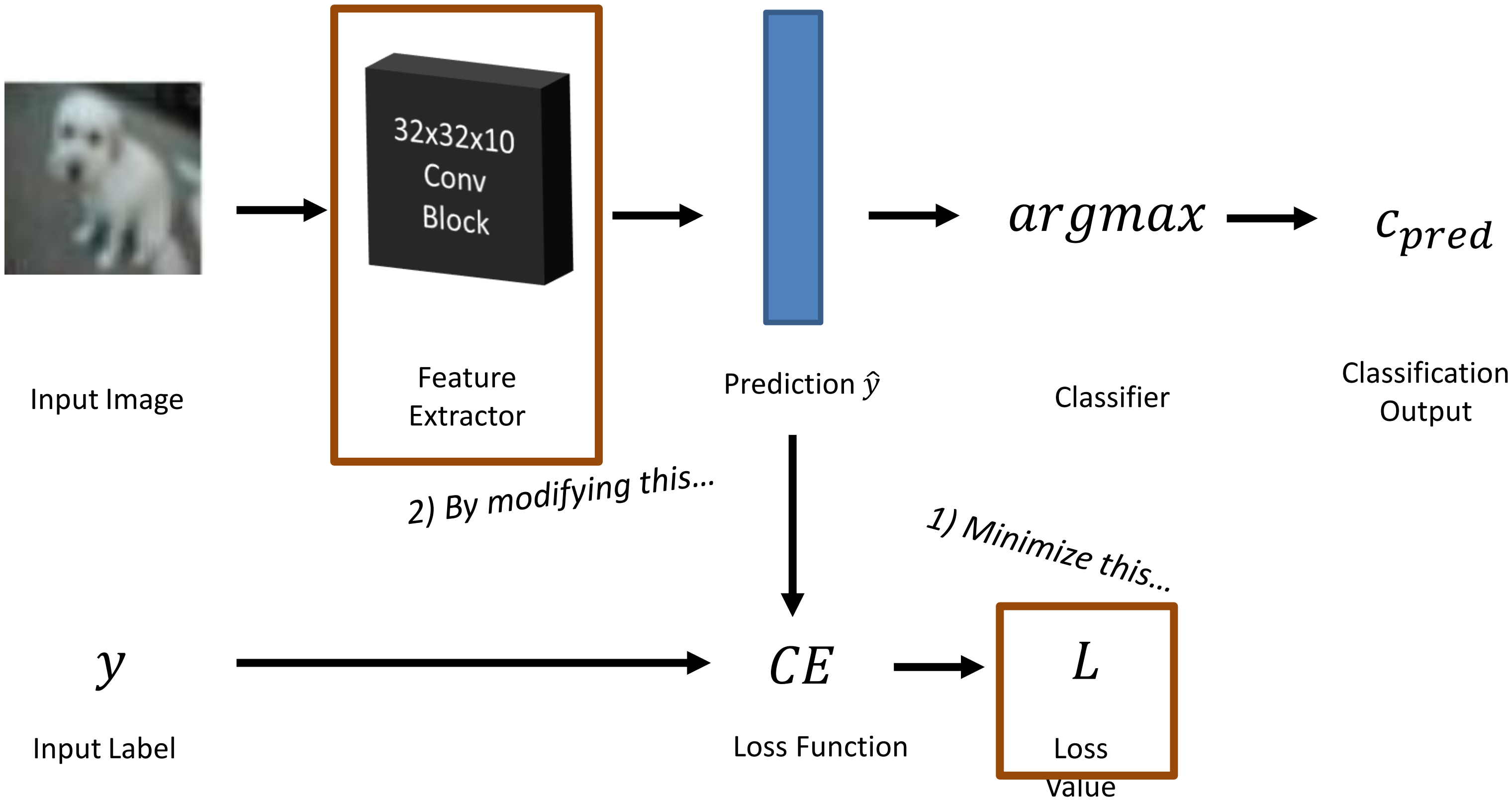

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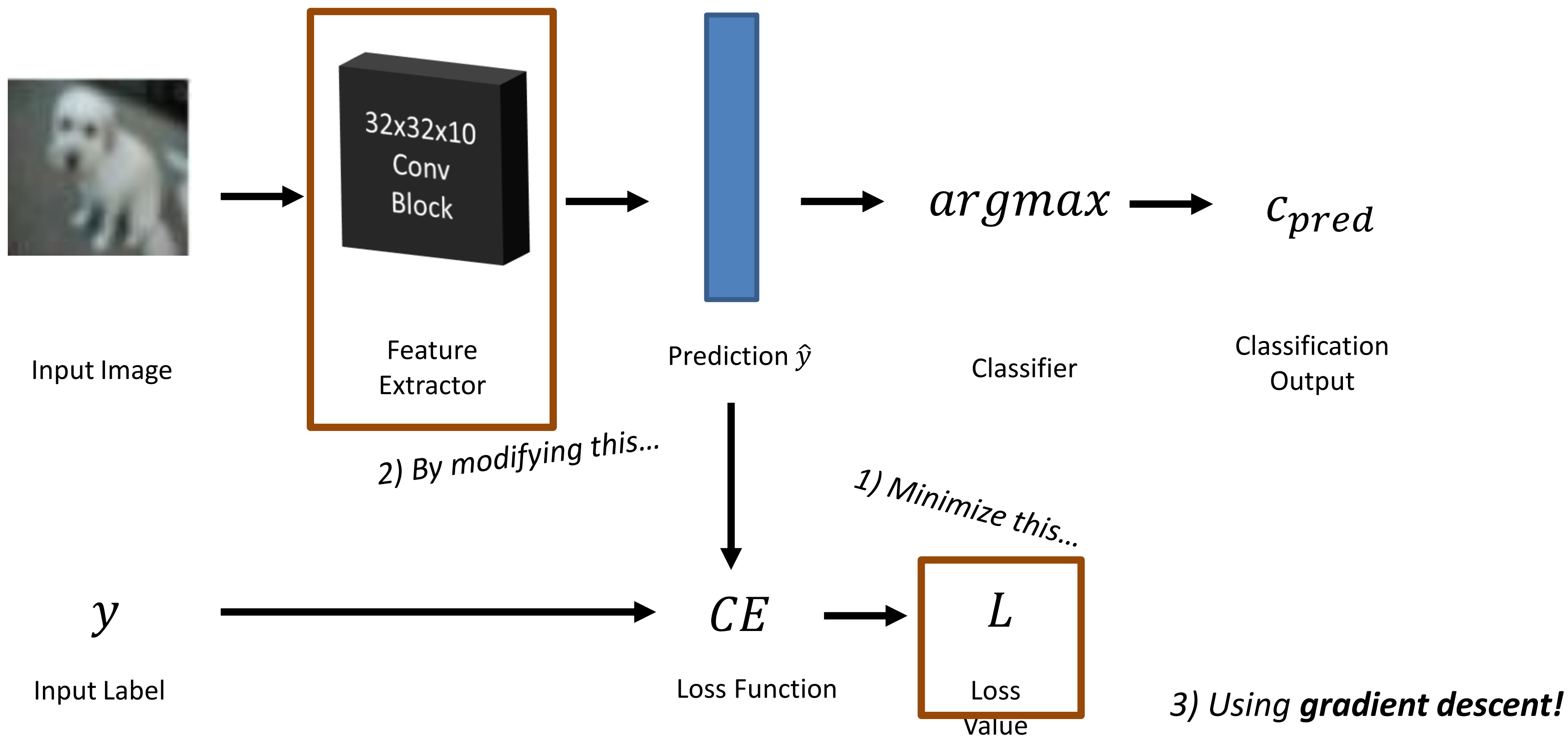

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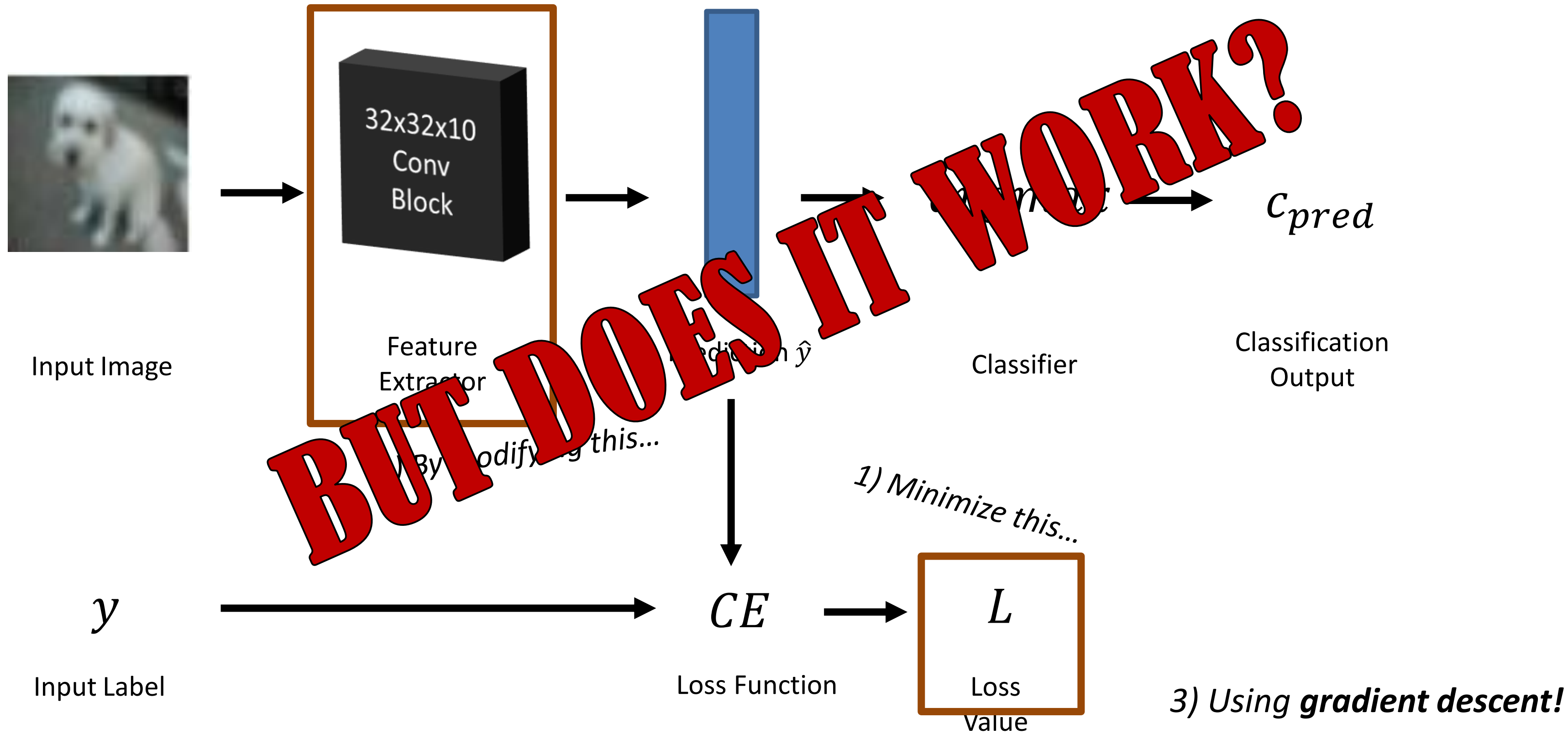


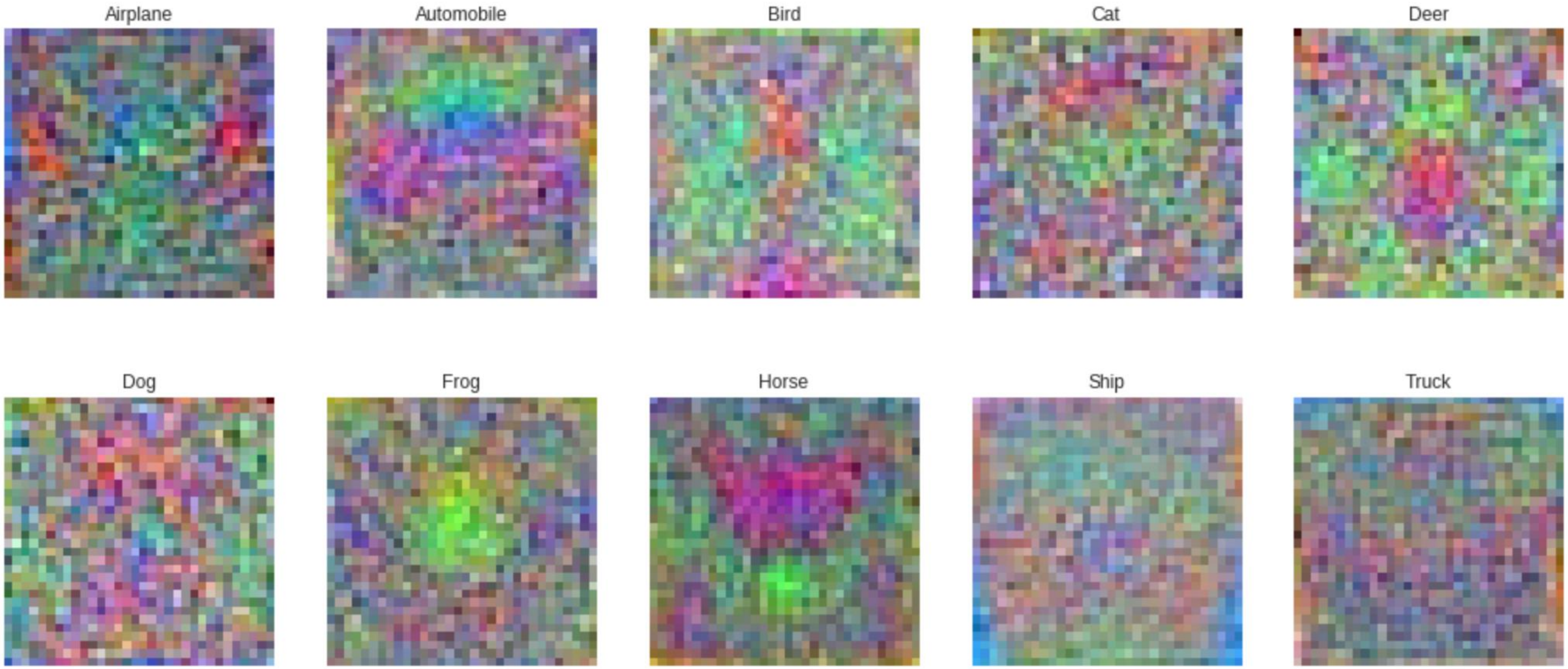
Our System’s Performance

- ~40% accuracy on CIFAR-10 test
  - Best class: Truck (~60%)
  - Worst class: Horse (~16%)

- Check out the model at: **https://tinyurl.com/cifar10**

- What about the filters? What do they look like?

Visualizing the Filters



Next Time...

Building a stronger convolution-based feature extractor

History of deep learning + computer vision (Convolutional Neural Nets!)

Applications of CNNs