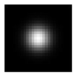
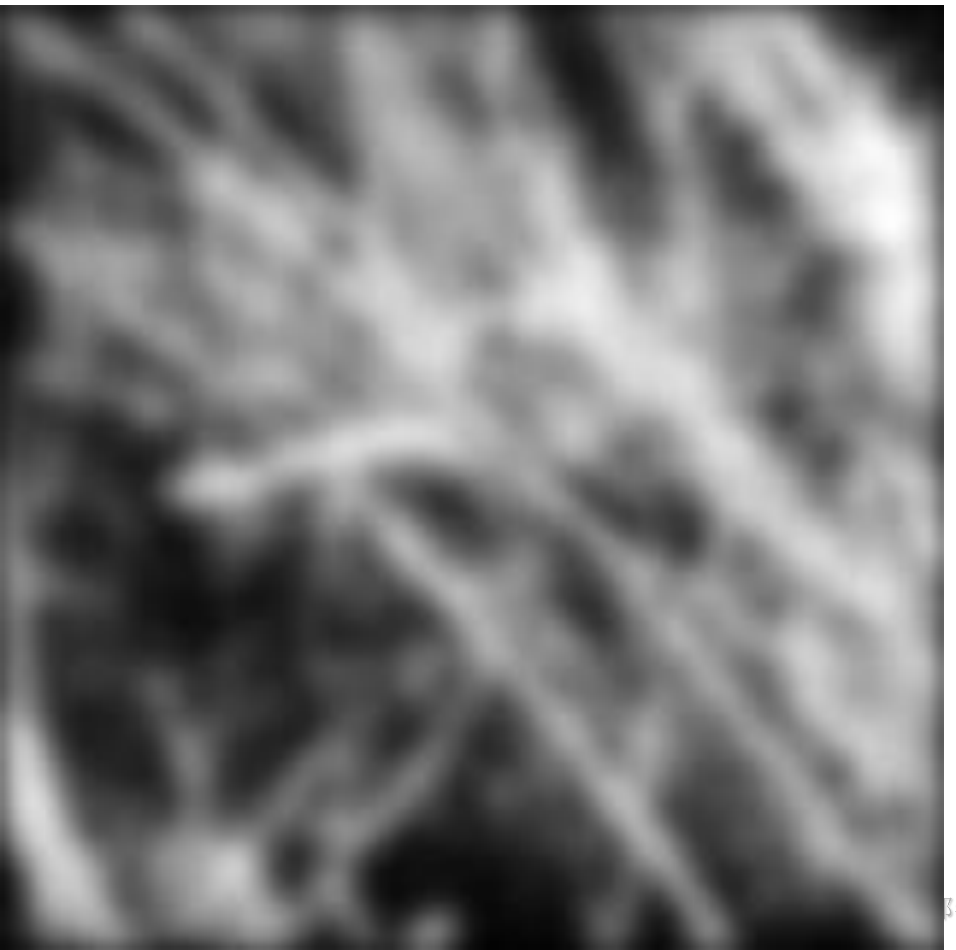
Frequency of Images

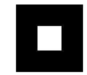
Gang Pan
Zhejiang University

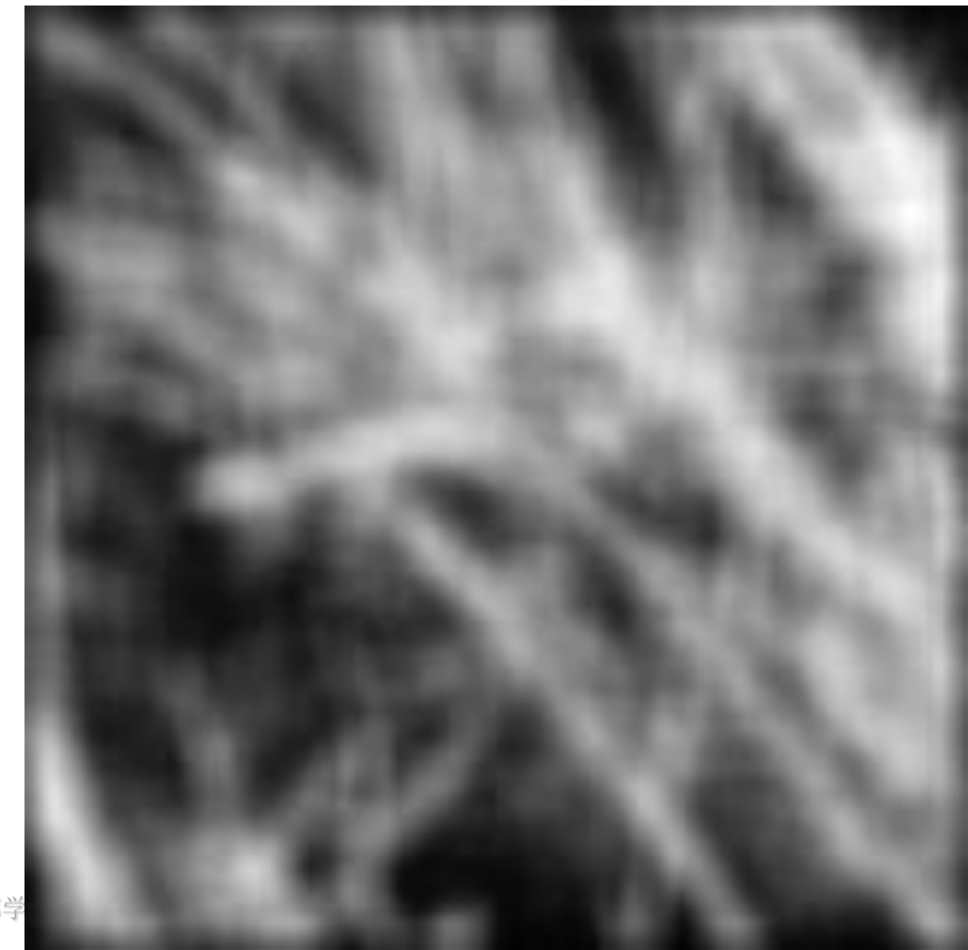
Slides: James Hays, Hoiem, Efros, and others

Why does the Gaussian give a nice smooth image, but the square filter give edgy artifacts?

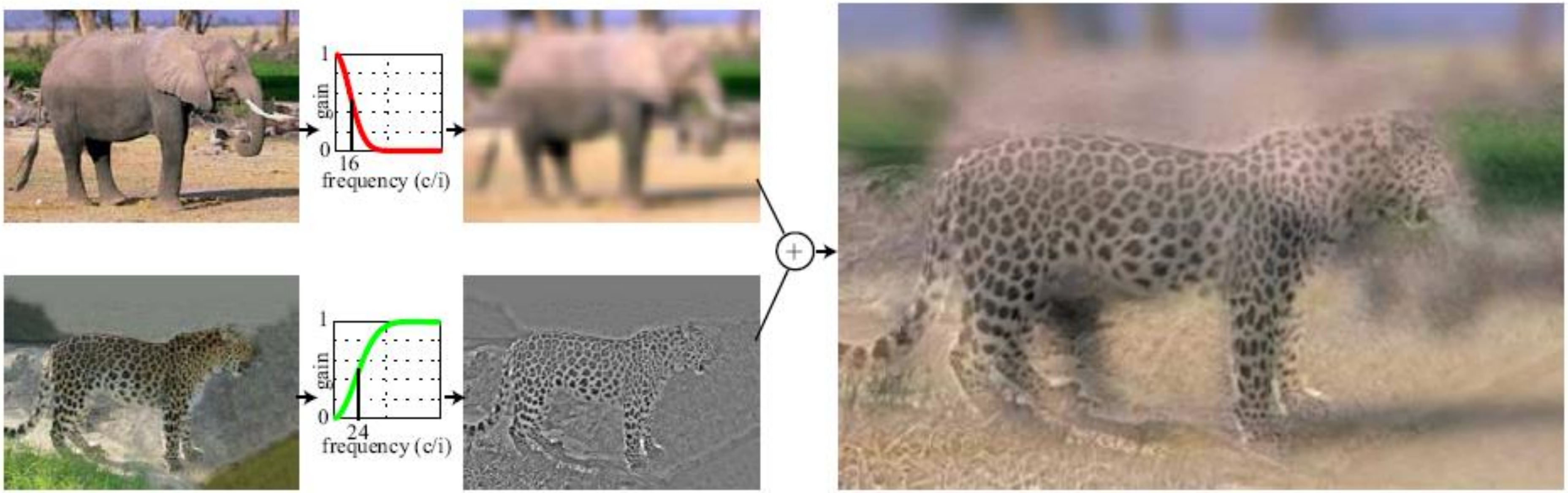
Gaussian 



Box filter 



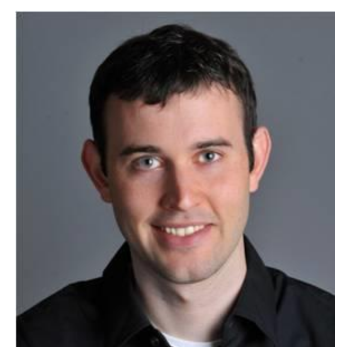
Hybrid Images



A. Oliva, A. Torralba, P.G. Schyns,
[“Hybrid Images,”]() SIGGRAPH 2006

Why do we get different, distance-dependent interpretations of hybrid images?

?



Slide: Hoiem

Why does a lower resolution image still make sense to us? What do we lose?

How is it that a 4MP image can be compressed to a few hundred KB without a noticeable change?

Image: http://www.flickr.com/photos/igorms/136916757/

Slide: Hoiem

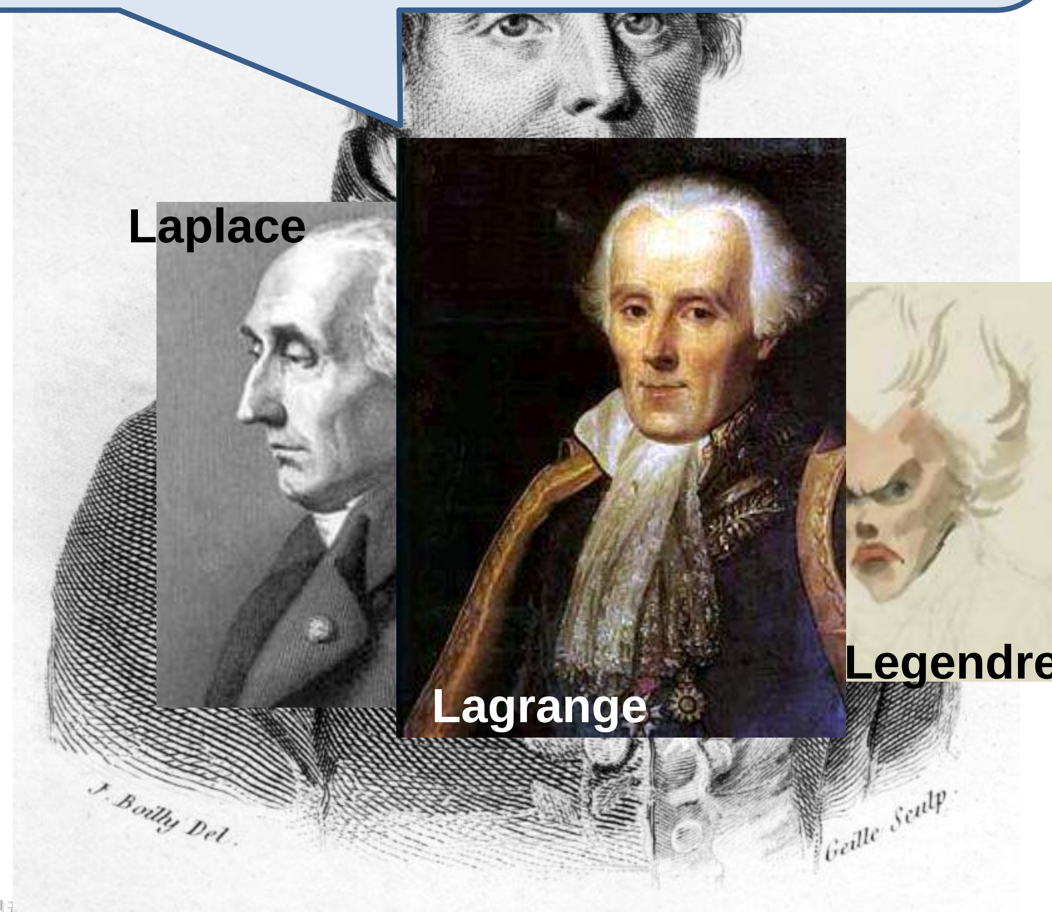
Jean Baptiste Joseph Fourier (1768-1830)

had crazy idea (1807):

> ***Any*** *univariate function can rewritten as a weighted sum sines and cosines of different frequencies.*

...the manner in which the author arrives at these equations is not exempt of difficulties and...his analysis to integrate them still leaves something to be desired on the score of generality and even rigour.

- Don't believe it?
  - Neither did Lagrange, Laplace, Poisson and other big wigs
  - Not translated into English until 1878!
- But it's (mostly) true!
  - called Fourier Series
  - there are some subtle restrictions



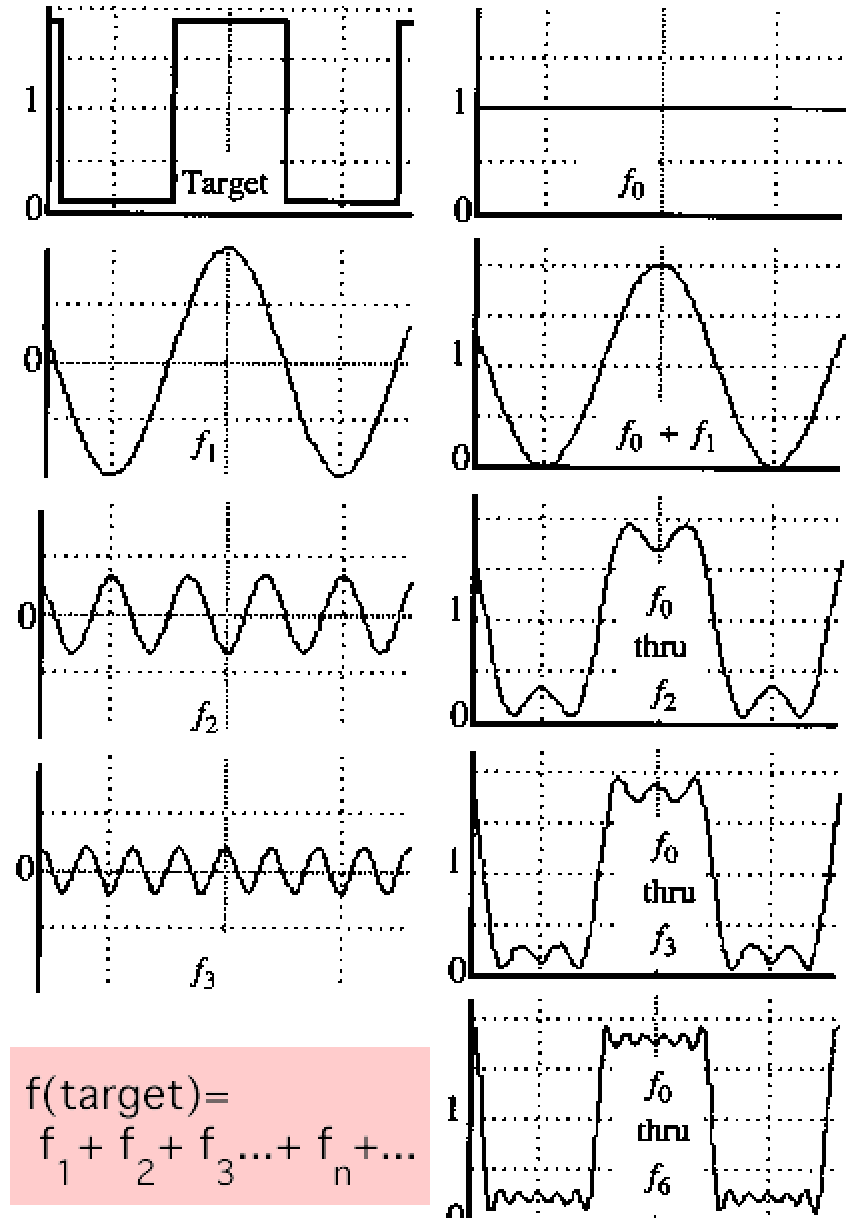
http://www-history.mcs.st-and.ac.uk/Biographies/Fourier.html

A sum of sines

Our building block:

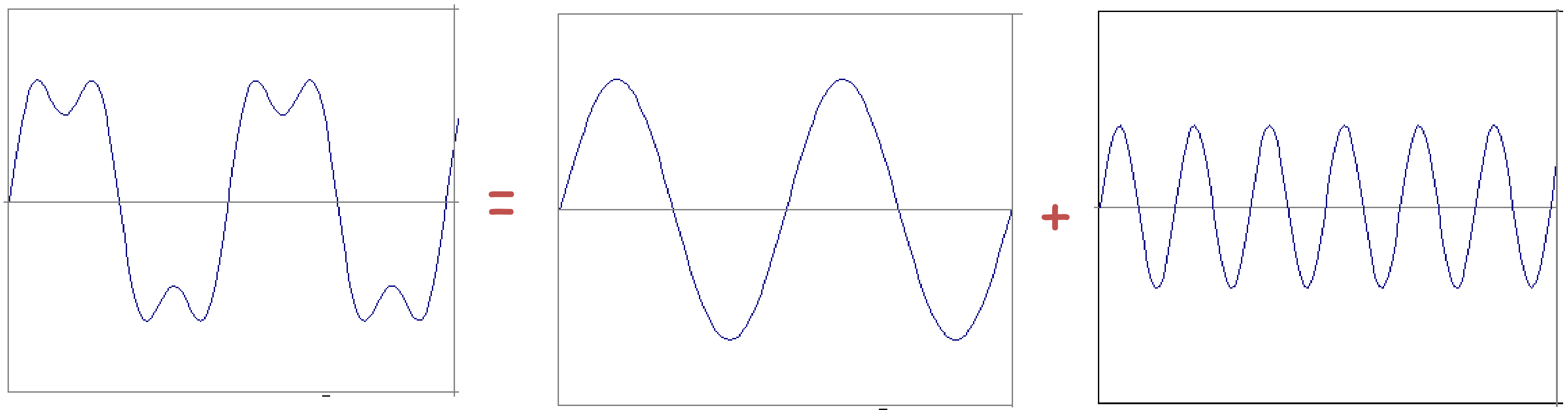
$$A\sin(\omega x + \phi)$$

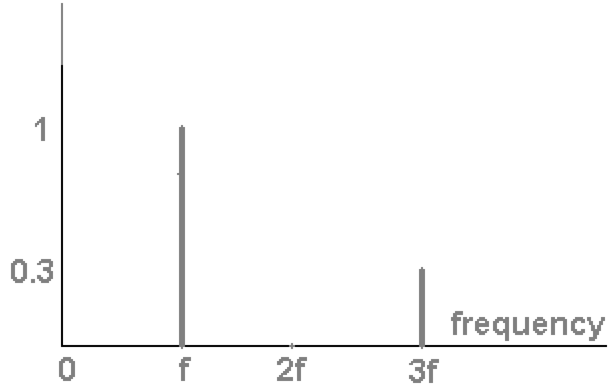
Add enough of them to get any signal *g(x)* you want!



Frequency Spectra

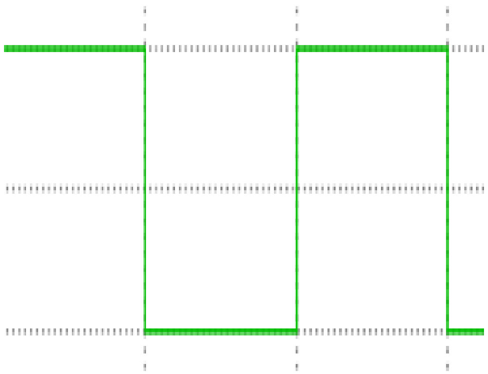
- example : $g(t) = \sin(2\pi f\, t) + (1/3)\sin(2\pi(3f)\, t)$



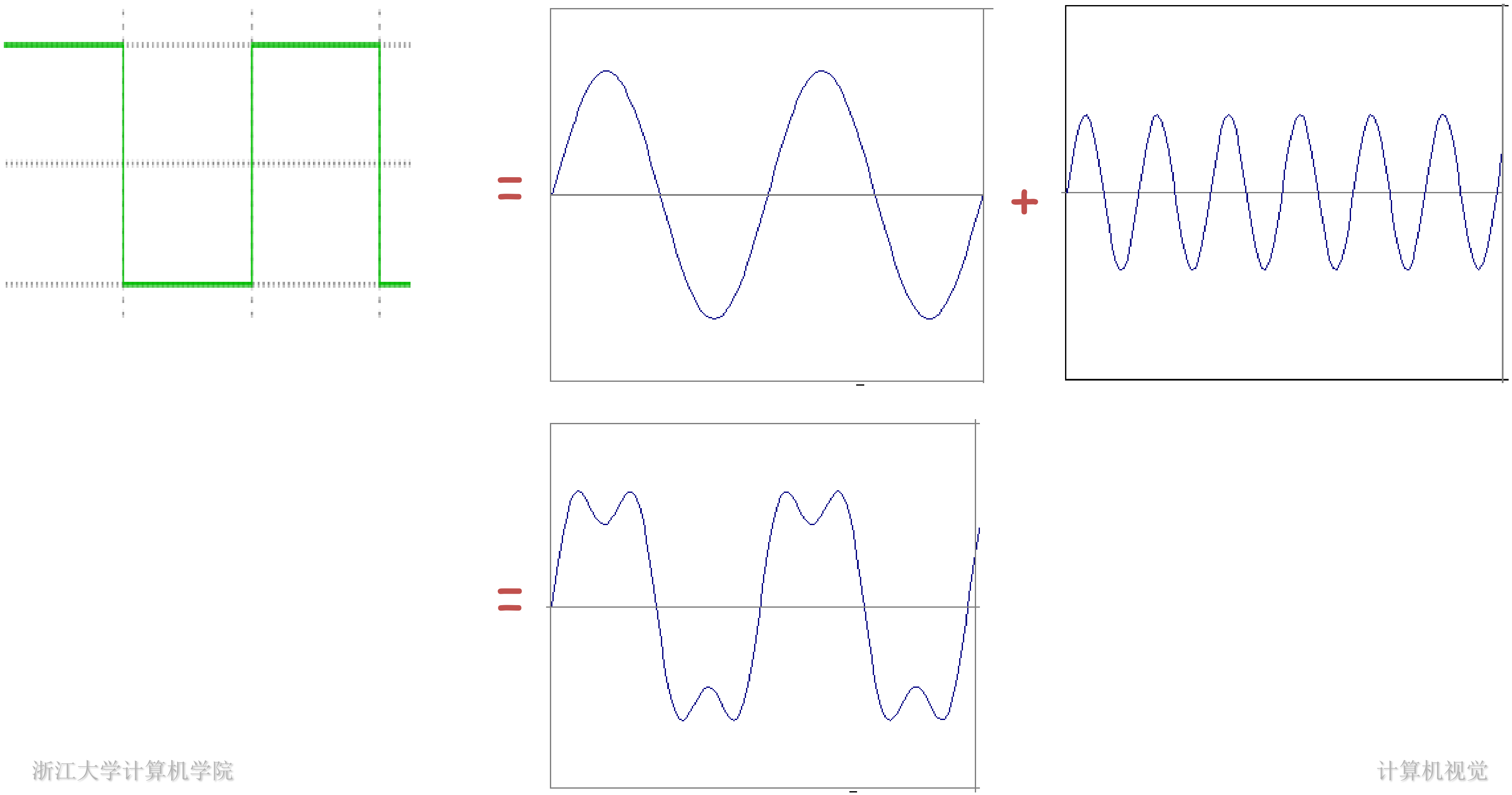


Slides: Efros

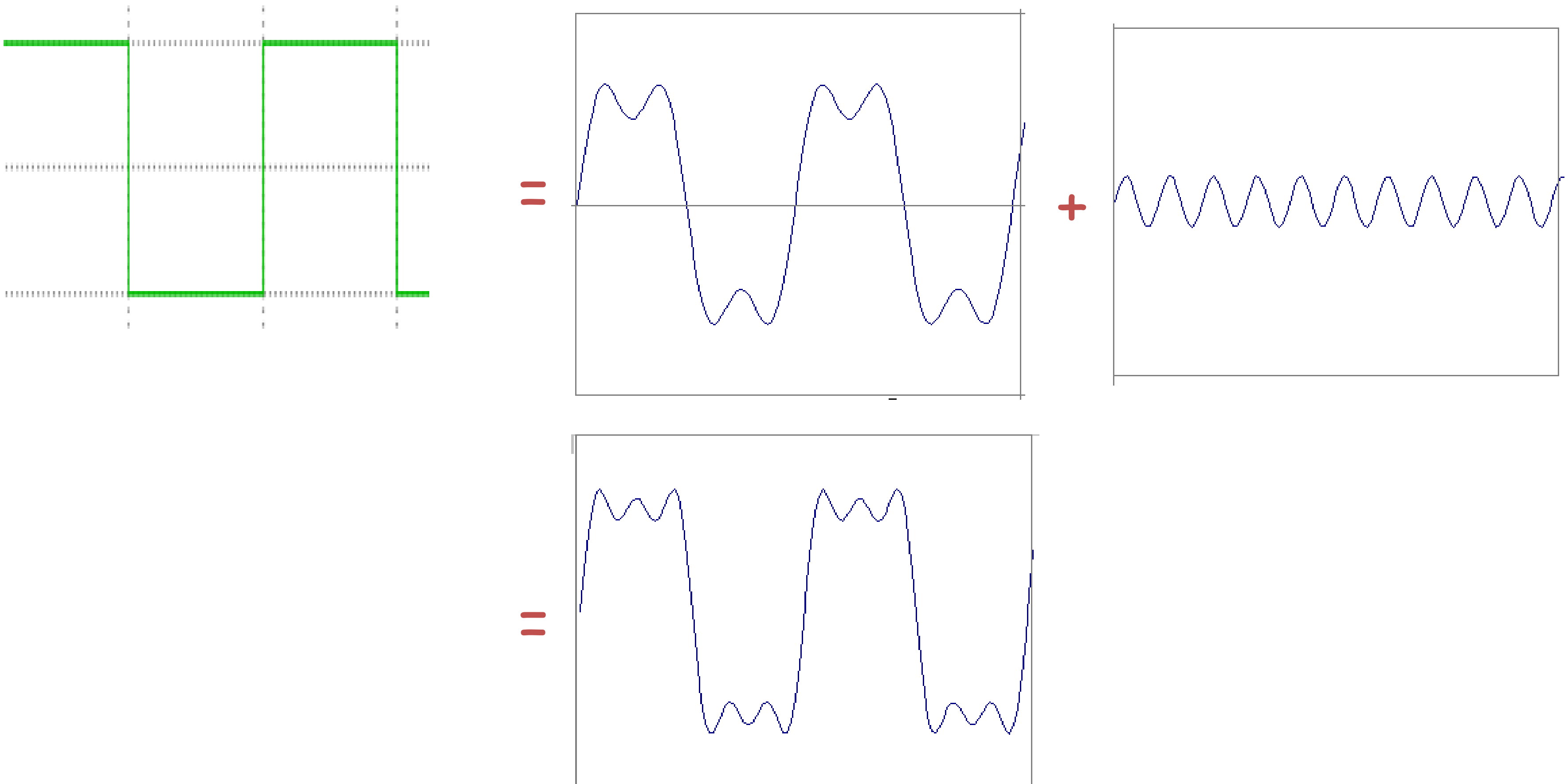
Frequency Spectra



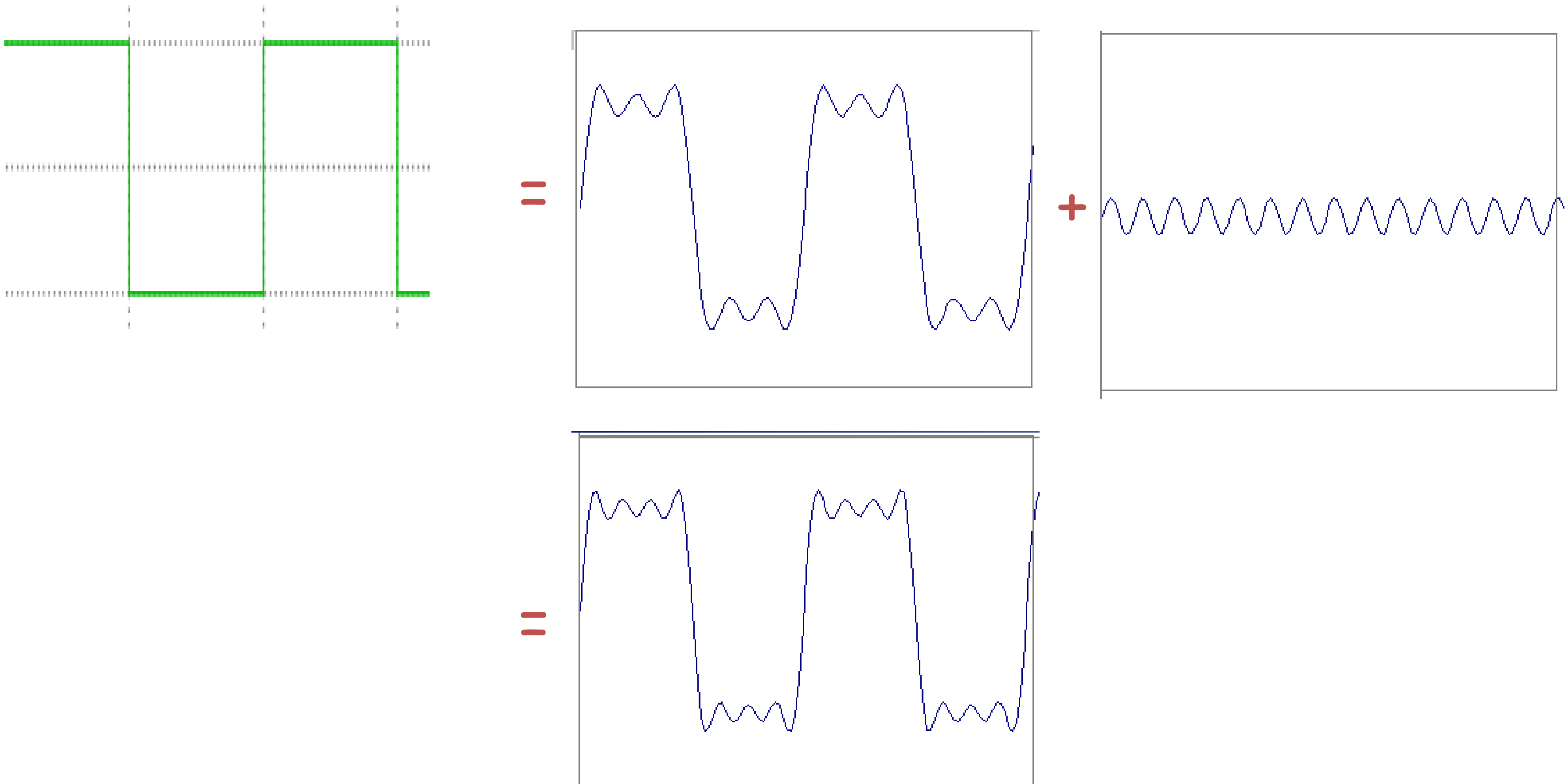
Frequency Spectra



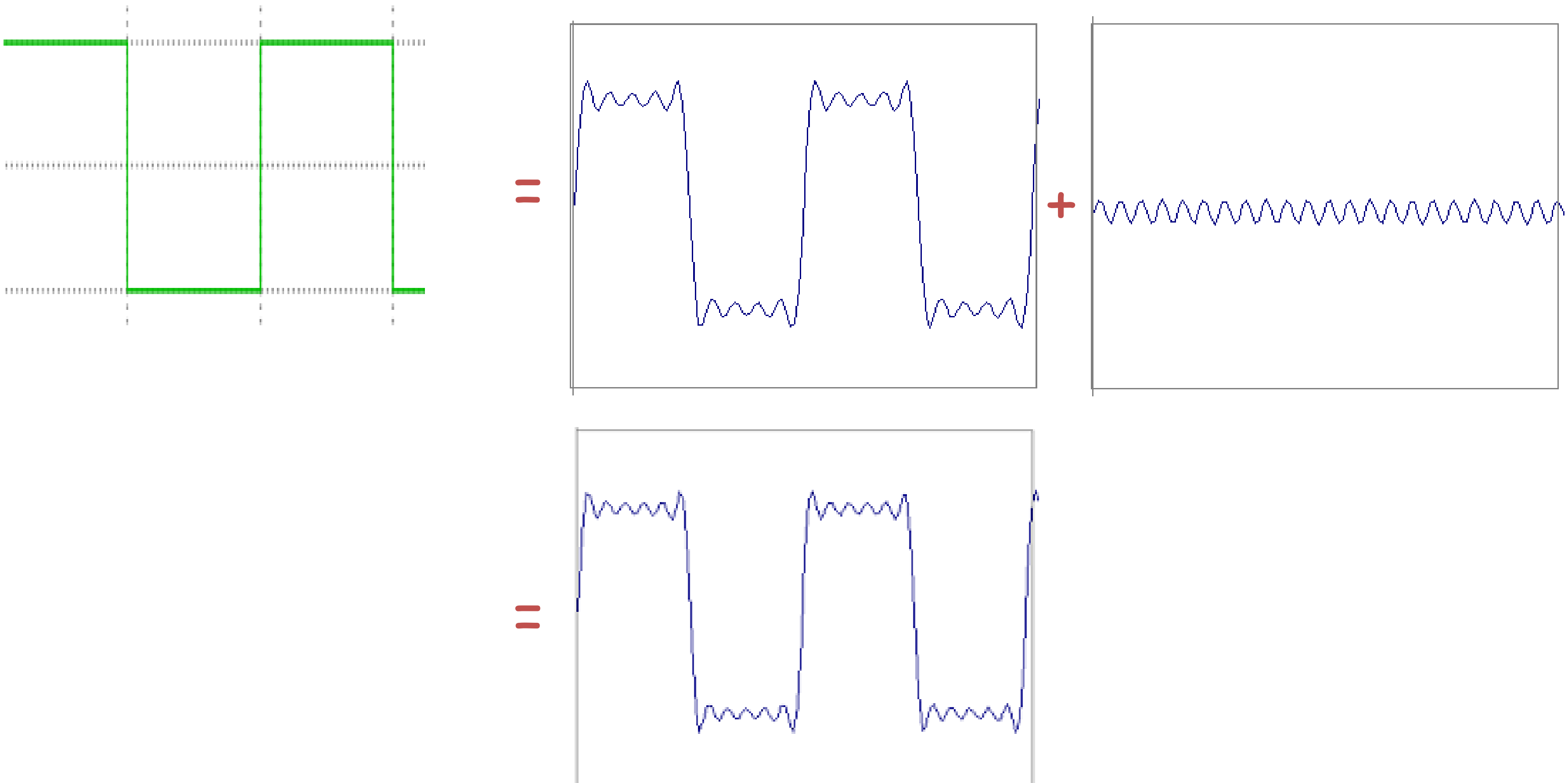
Frequency Spectra



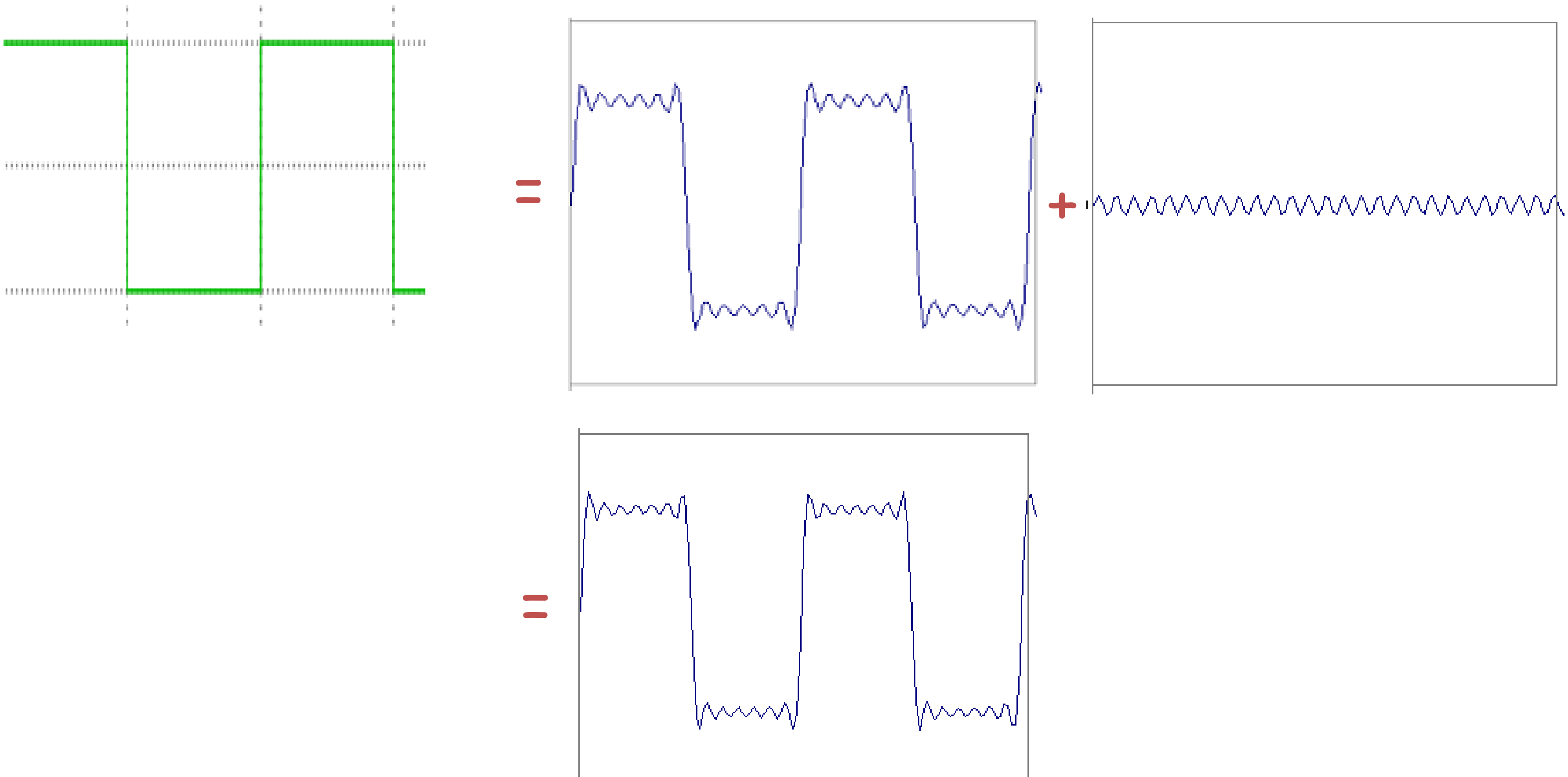
Frequency Spectra



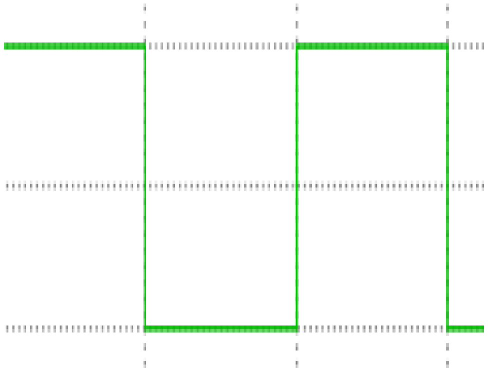
Frequency Spectra



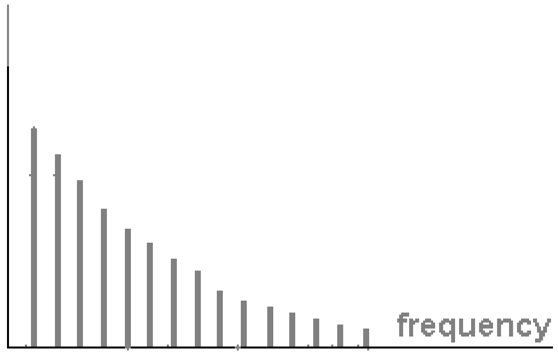
Frequency Spectra



Frequency Spectra

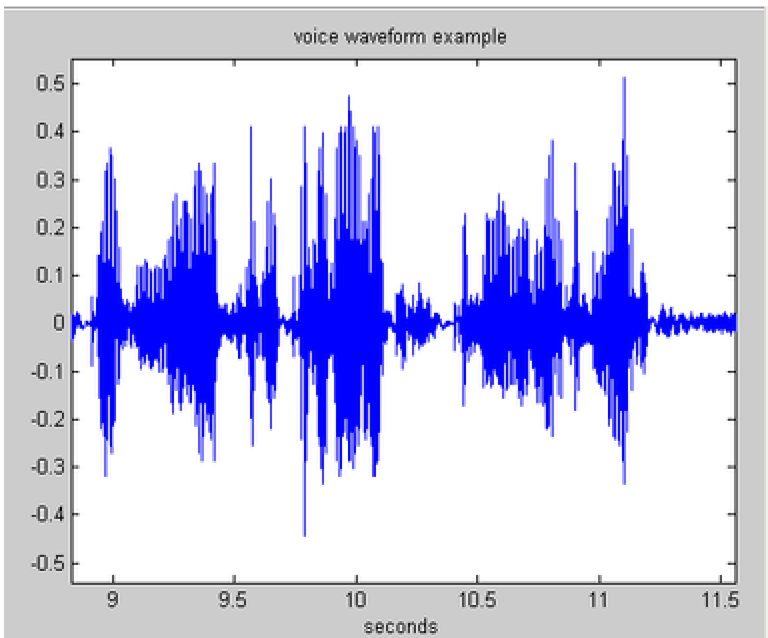


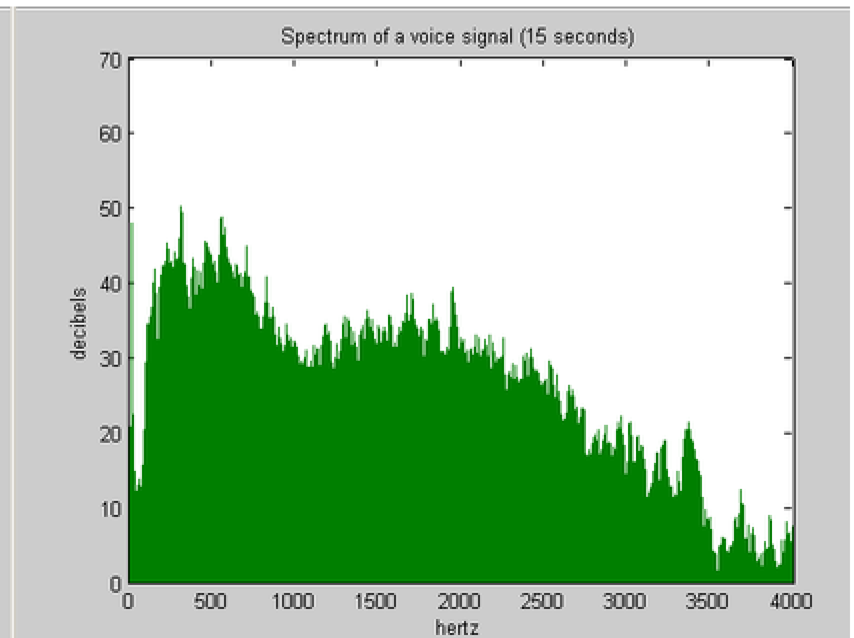
$$= A\sum_{k=1}^{\infty}\frac{1}{k}\sin(2\pi kt)$$



Example: Music

- We think of music in terms of frequencies at different magnitudes

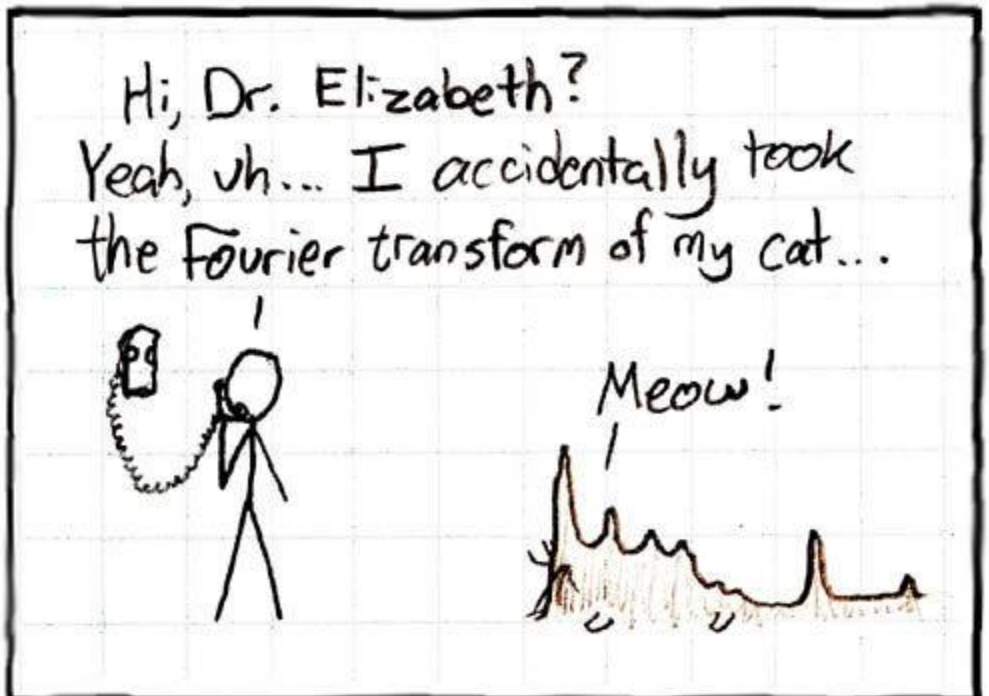




Slide: Hoiem

Other signals

- We can also think of all kinds of other signals the same way

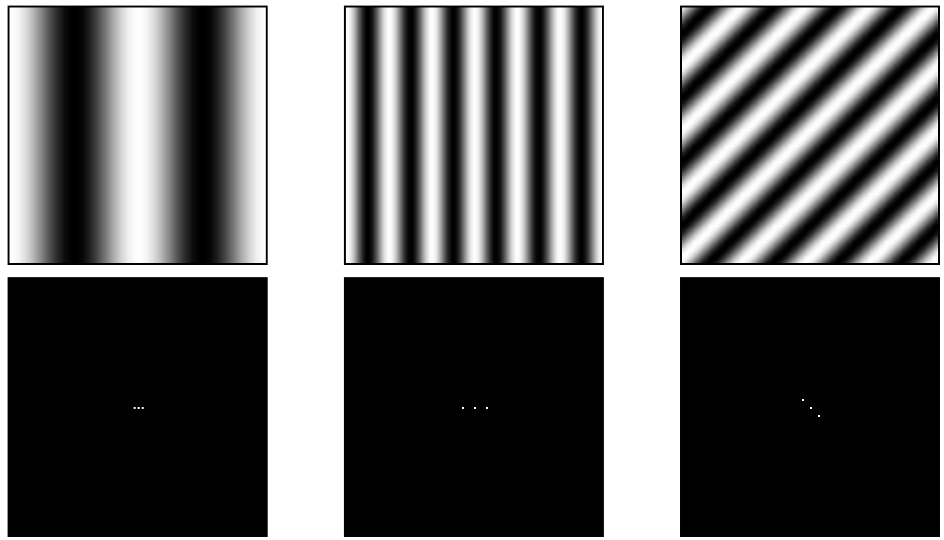


xkcd.com

Fourier analysis in images

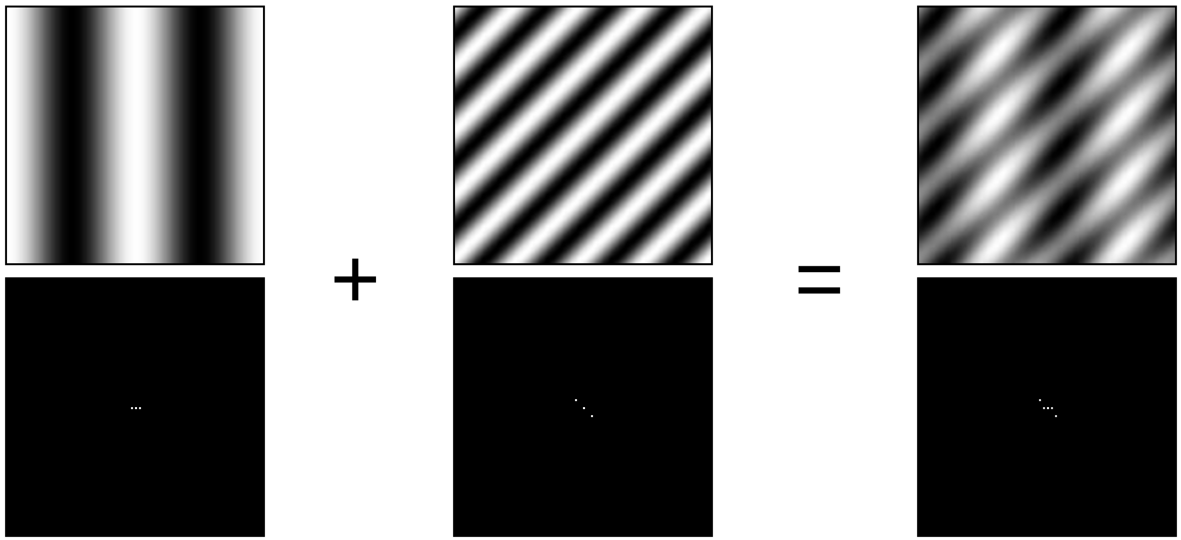
Intensity Image

Fourier Image



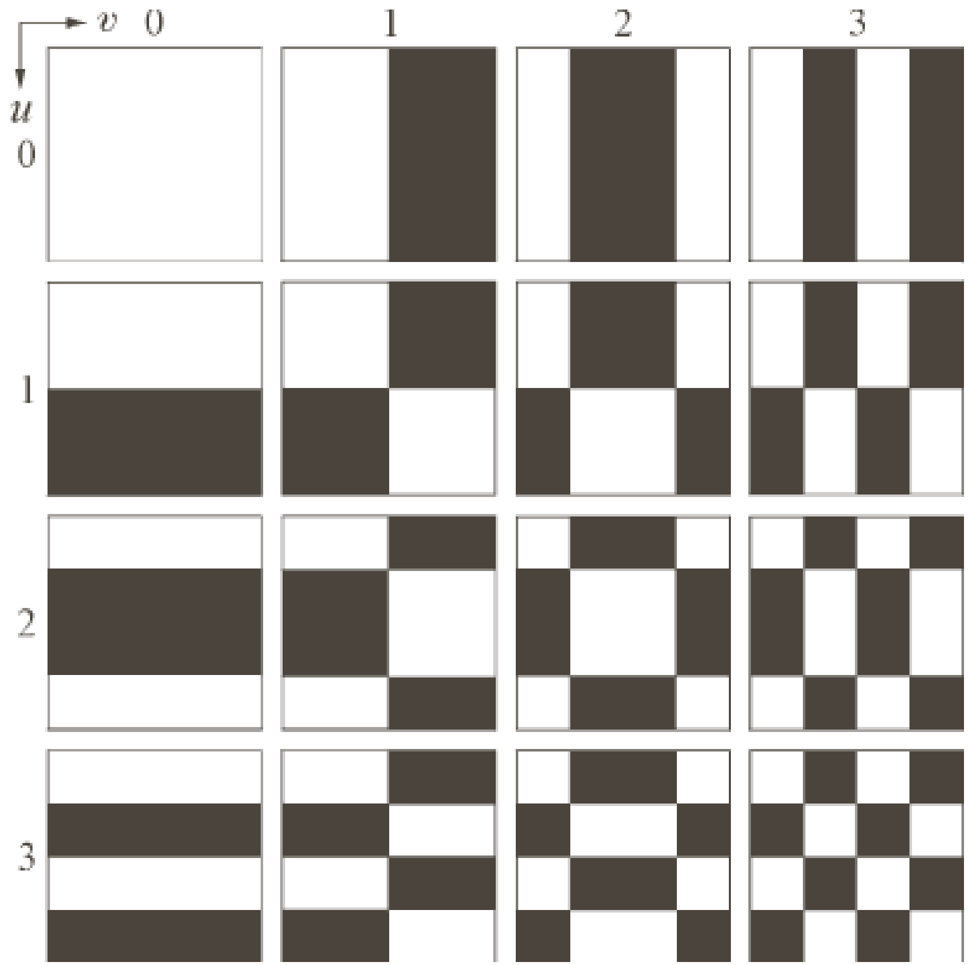
http://sharp.bu.edu/~slehar/fourier/fourier.html#filtering

Signals can be composed

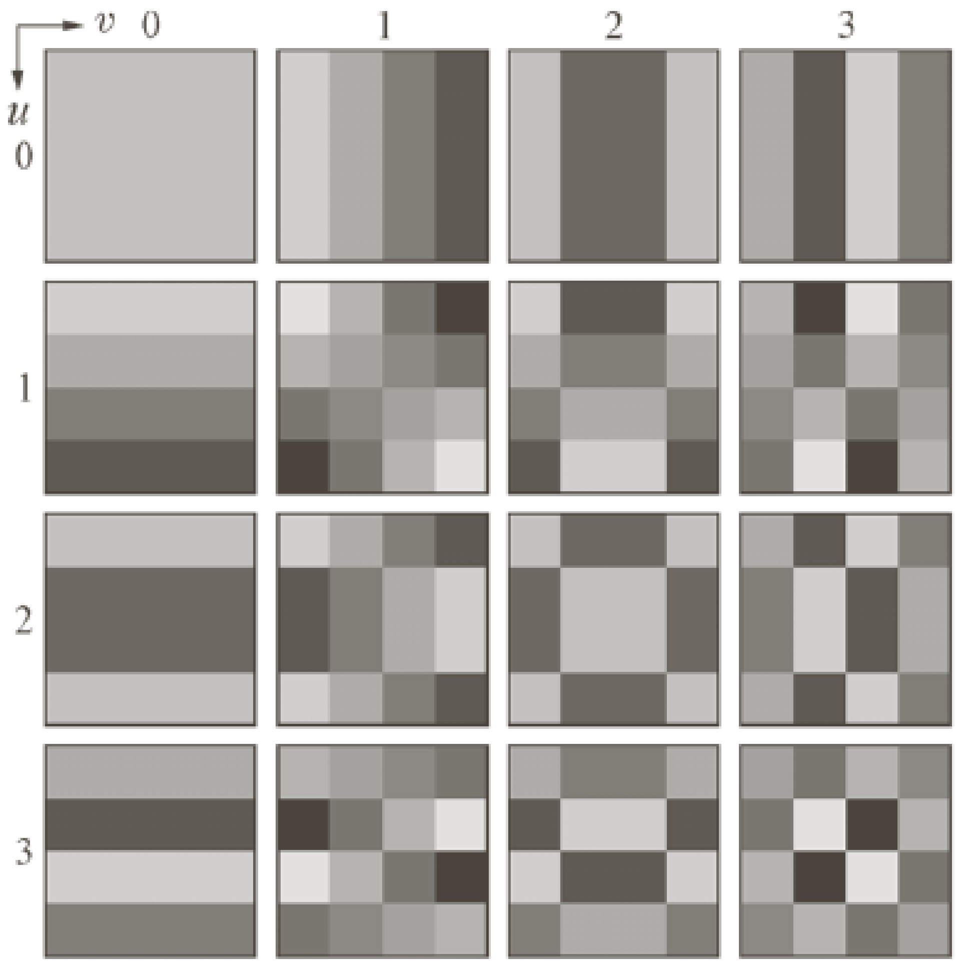


http://sharp.bu.edu/~slehar/fourier/fourier.html#filtering
More: http://www.cs.unm.edu/~brayer/vision/fourier.html

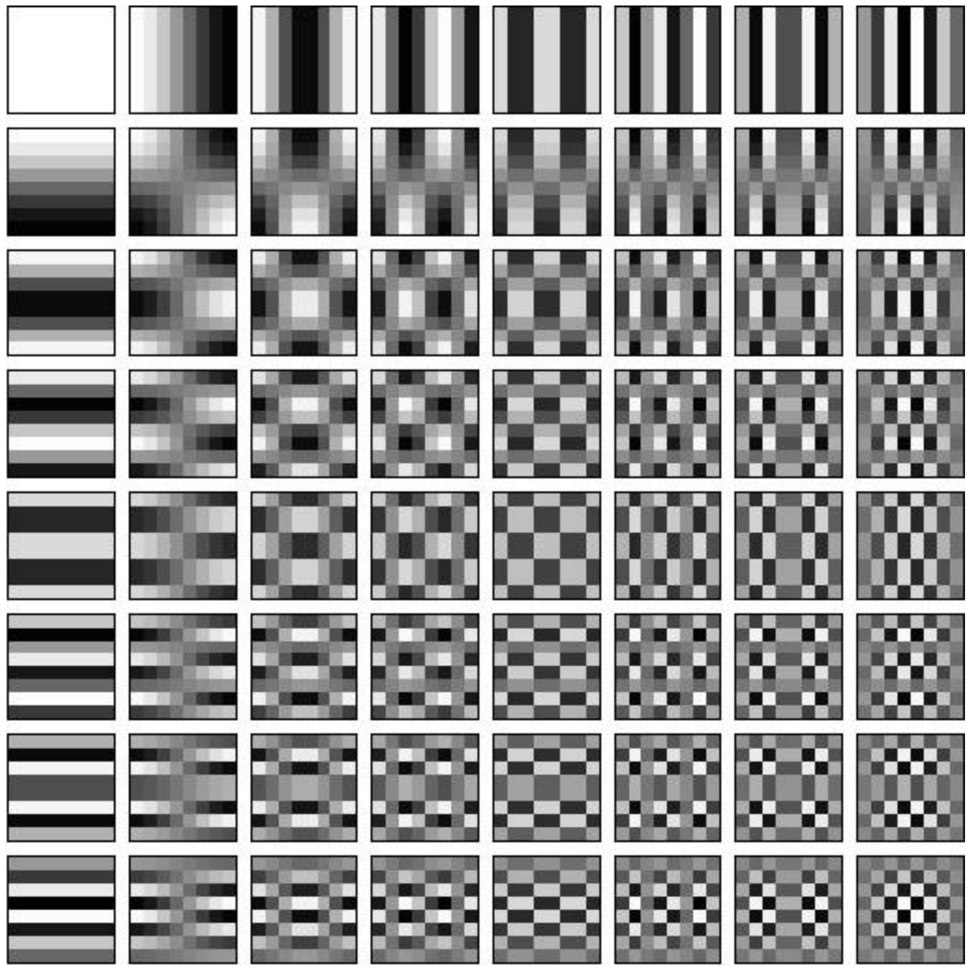
Hadamard变换（基图像）N=4



DCT的基函数（基图像）N=4

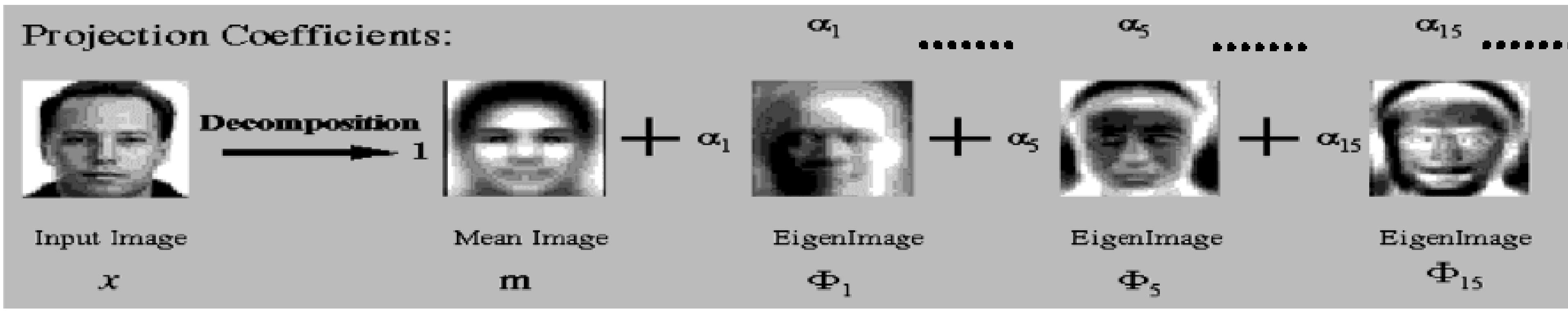


DCT的基函数（基图像）N=8



Eigenface

Eigenface



Projection Coefficients:
α_1 .... α_5 .... α_{15} ....
Decomposition
1
$+ \alpha_1$
$+ \alpha_5$
$+ \alpha_{15}$
Input Image
x
Mean Image
m
EigenImage
Φ_1
EigenImage
Φ_5
EigenImage
Φ_{15}

Fourier Transform

- Fourier transform stores the magnitude and phase at each frequency
  - Magnitude encodes how much signal there is at a particular frequency
  - Phase encodes spatial information (indirectly)
  - For mathematical convenience, this is often notated in terms of real and complex numbers

Amplitude: $A = \pm\sqrt{R(\omega)^2 + I(\omega)^2}$

Phase: $\phi = \tan^{-1}\frac{I(\omega)}{R(\omega)}$

The Convolution Theorem

- The Fourier transform of the convolution of two functions is the product of their Fourier transforms

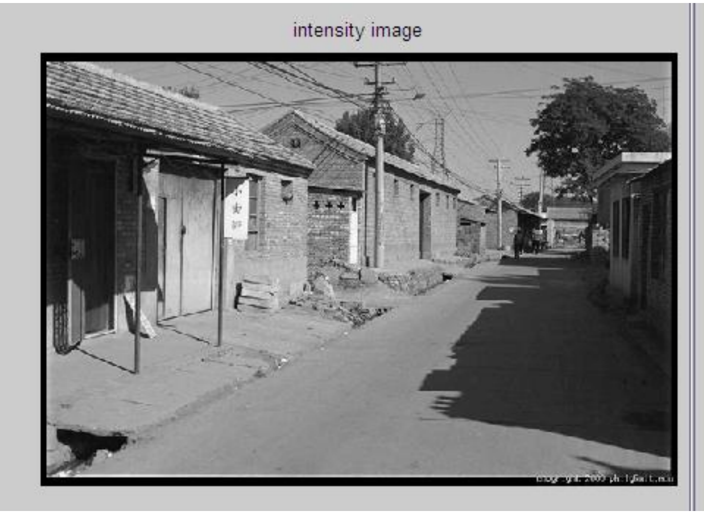
$$\mathrm{F}[g * h] = \mathrm{F}[g]\,\mathrm{F}[h]$$

- **Convolution** in spatial domain is equivalent to **multiplication** in frequency domain!

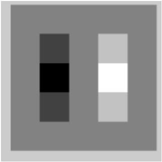
$$g * h = \mathrm{F}^{-1}[\mathrm{F}[g]\,\mathrm{F}[h]]$$

Filtering in spatial domain

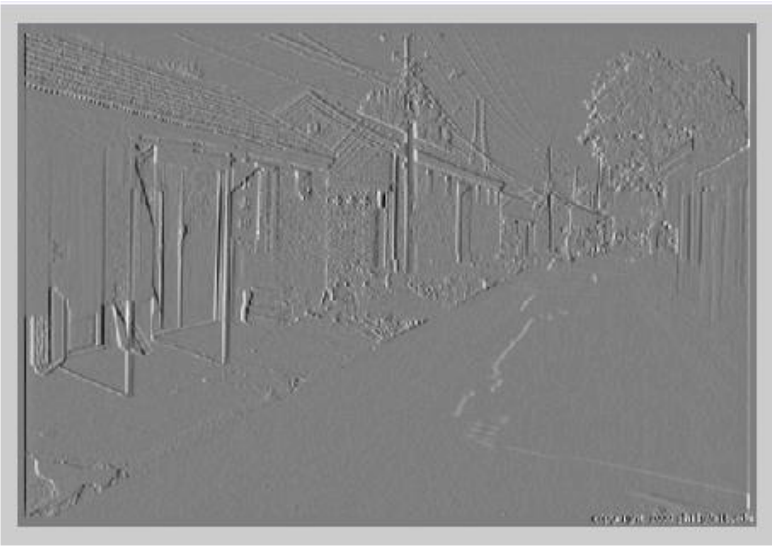
1	0	-1
2	0	-2
1	0	-1



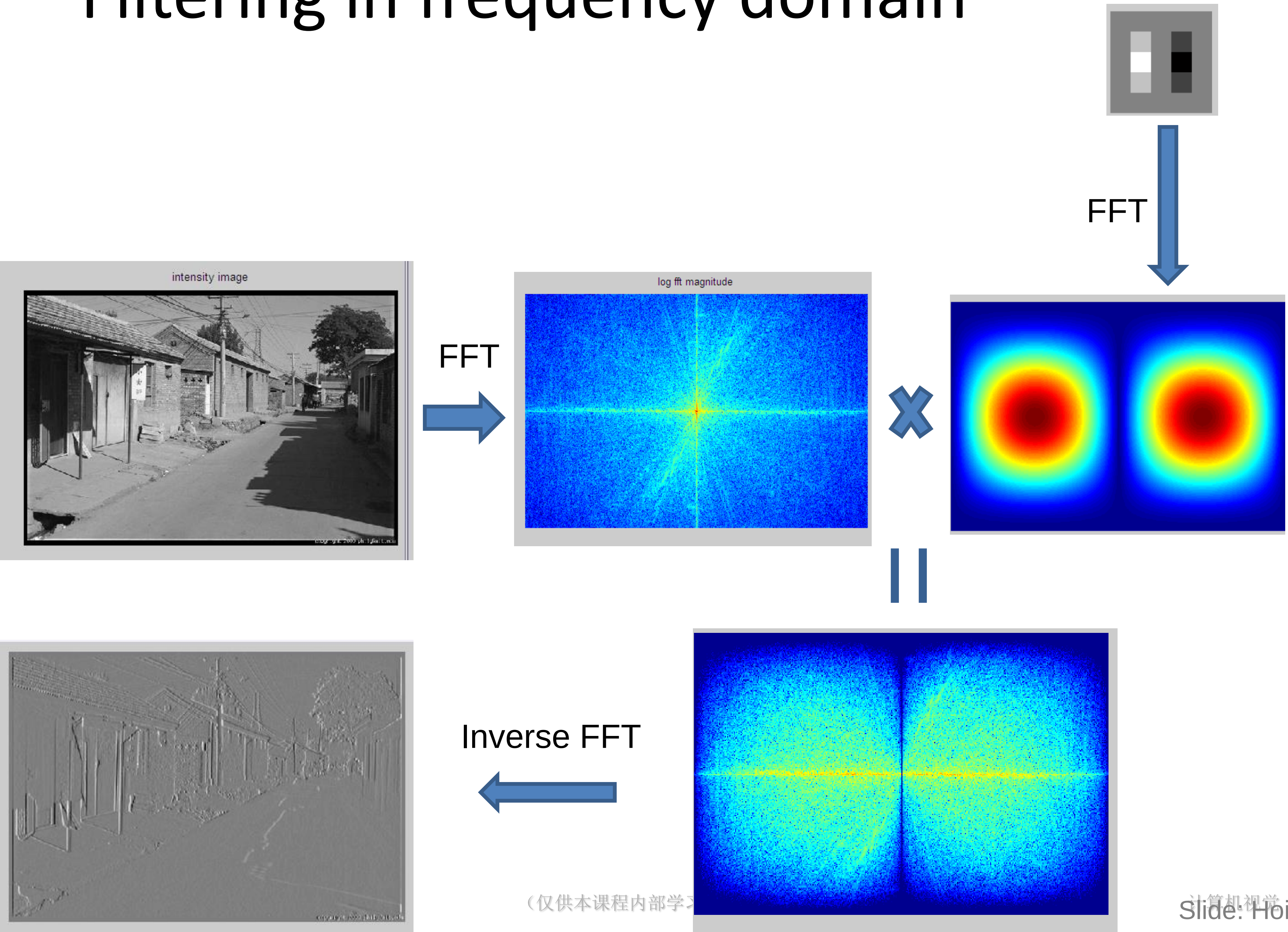
*



=



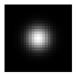
Filtering in frequency domain

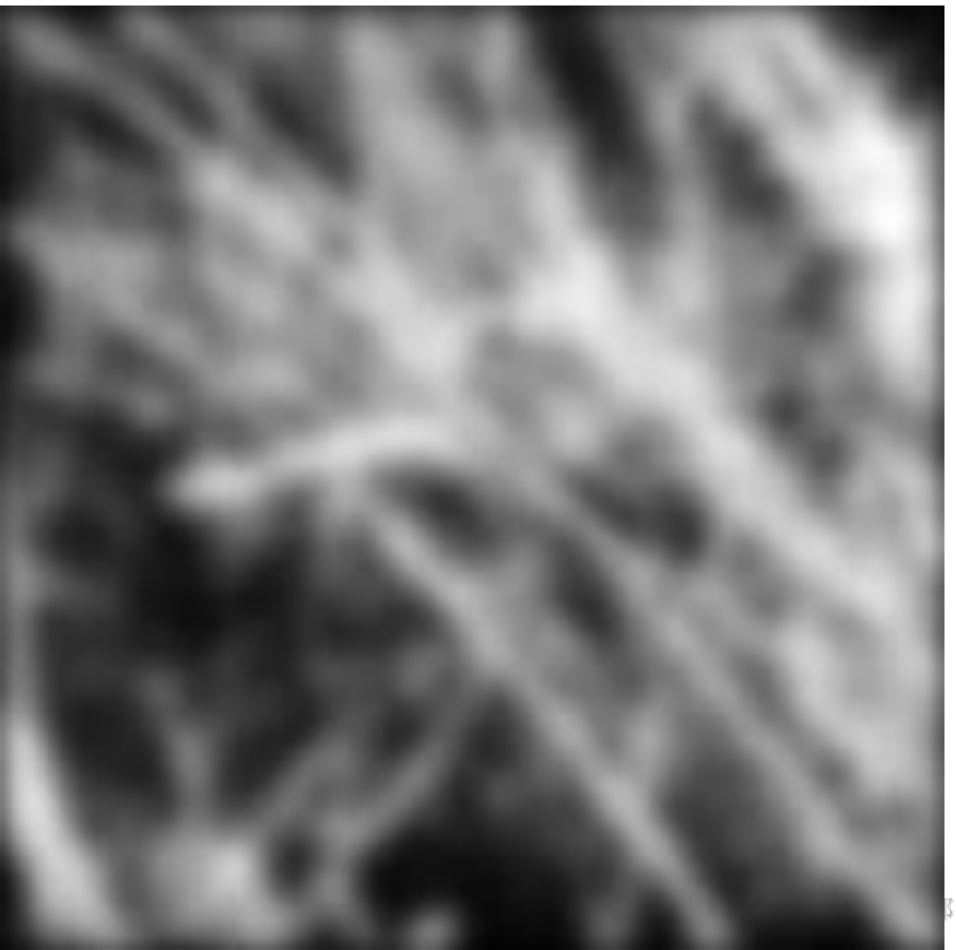


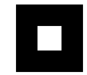
Slide: Hoiem

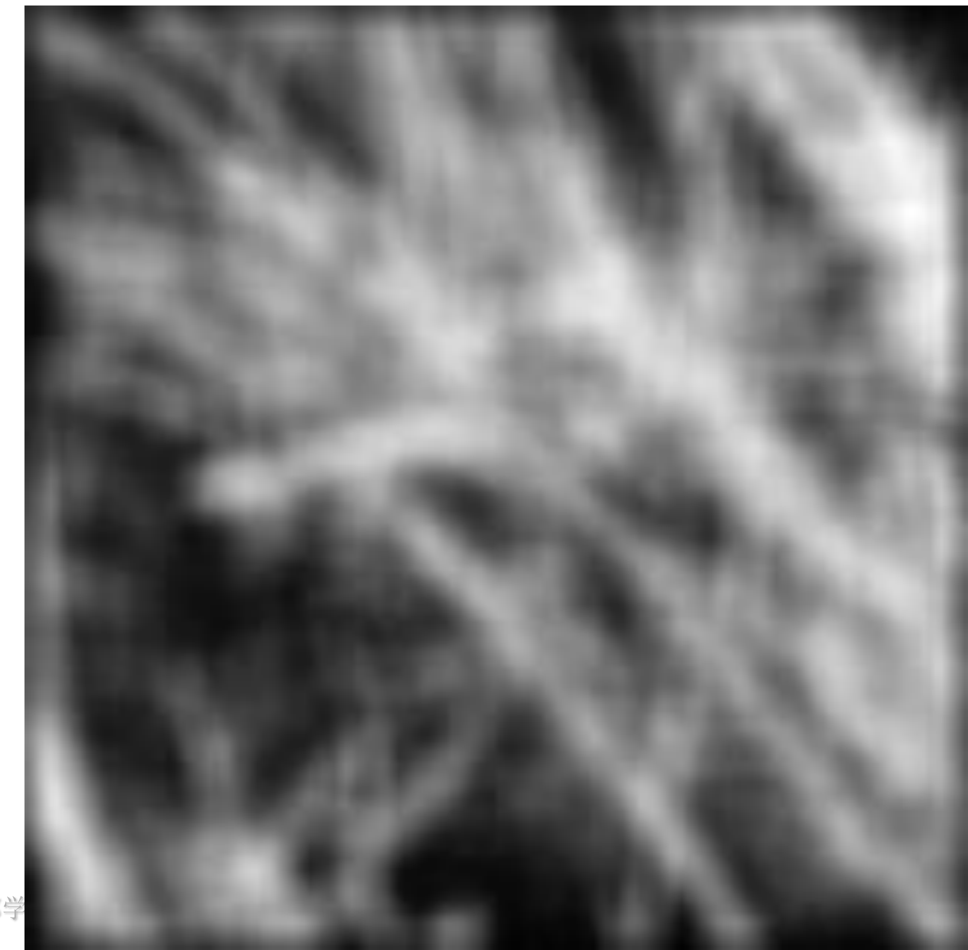
Filtering

Why does the Gaussian give a nice smooth image, but the square filter give edgy artifacts?

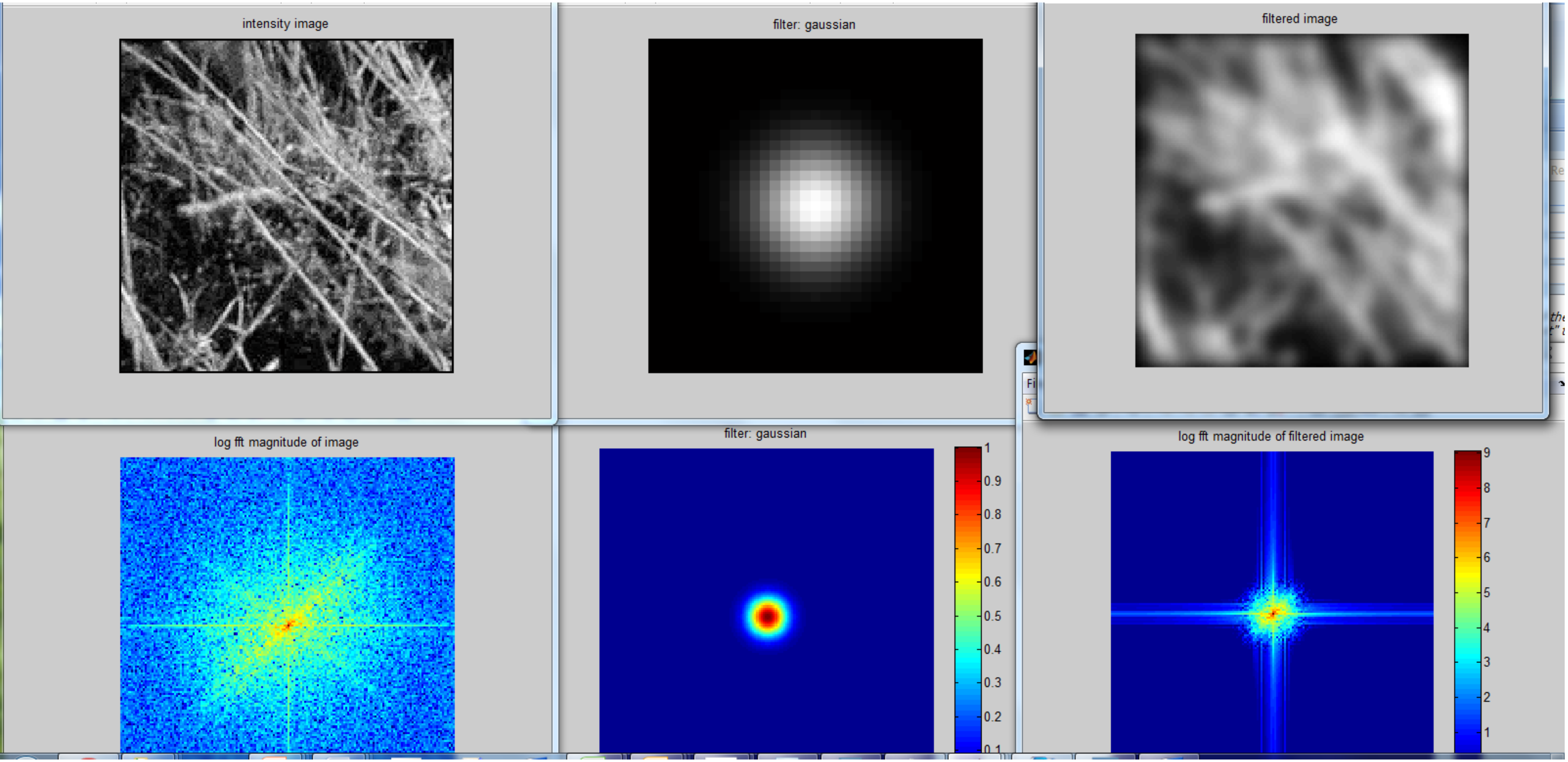
Gaussian 



Box filter 



Gaussian



intensity image
filter: gaussian
filtered image
log fft magnitude of image
filter: gaussian
log fft magnitude of filtered image

Box Filter

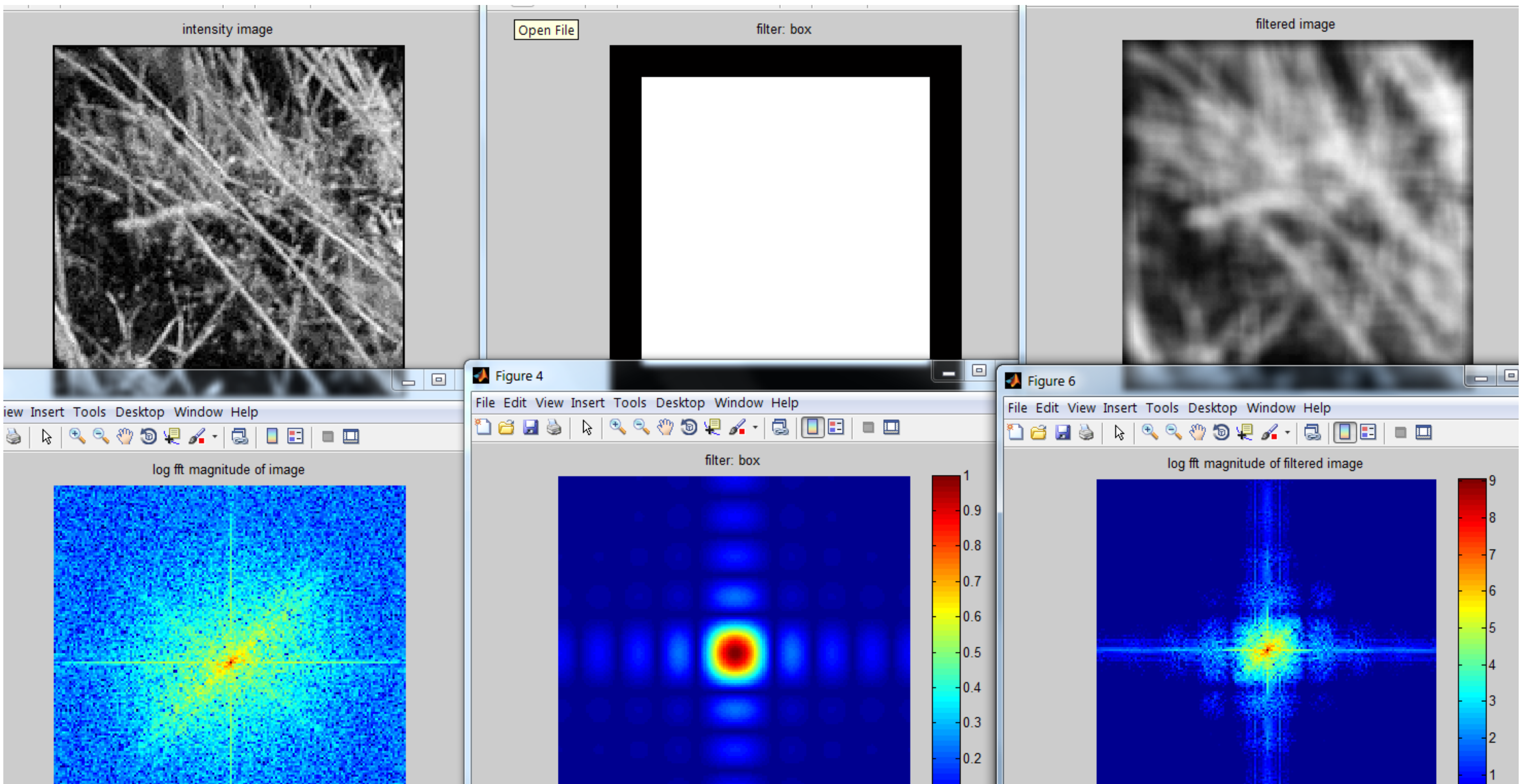
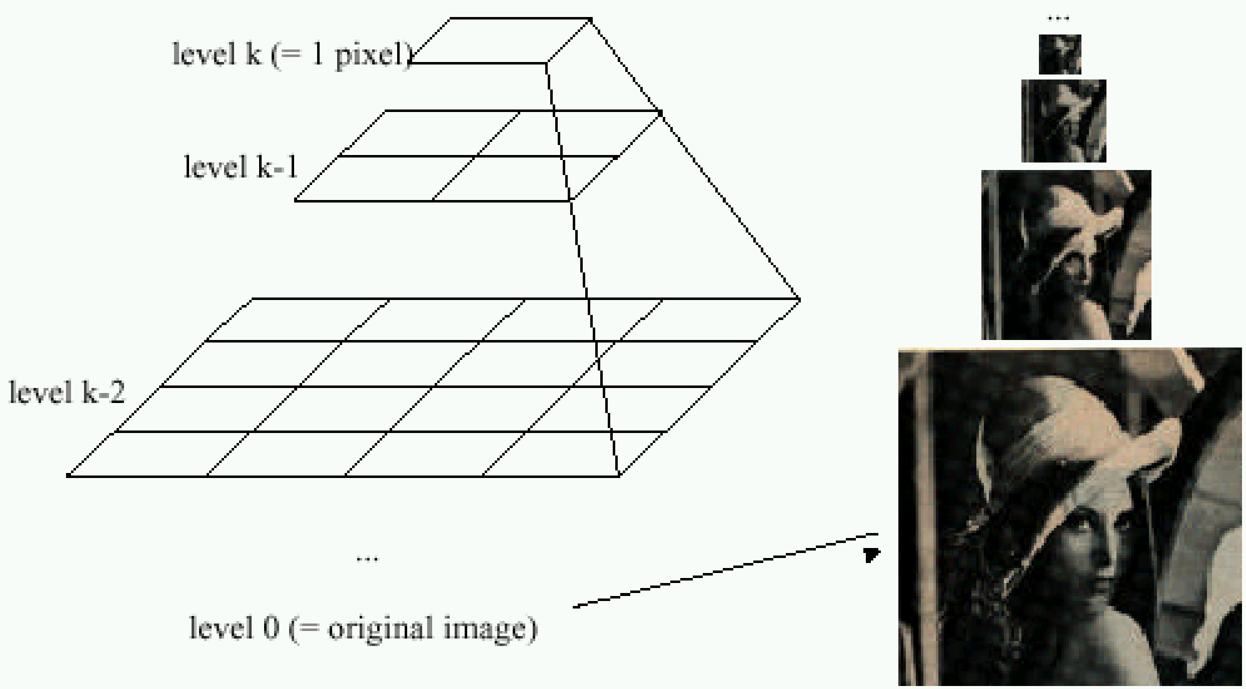

intensity image
Open File
filter: box
filtered image
Figure 4
Figure 6
File Edit View Insert Tools Desktop Window Help
log fft magnitude of image
filter: box
log fft magnitude of filtered image

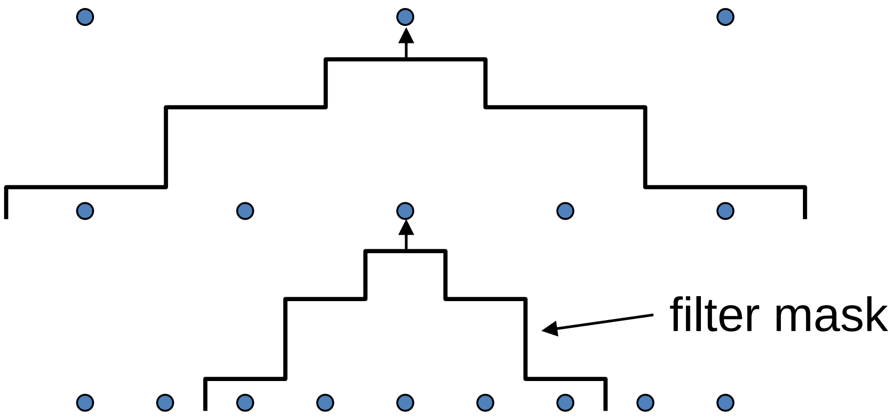
Image Pyramids

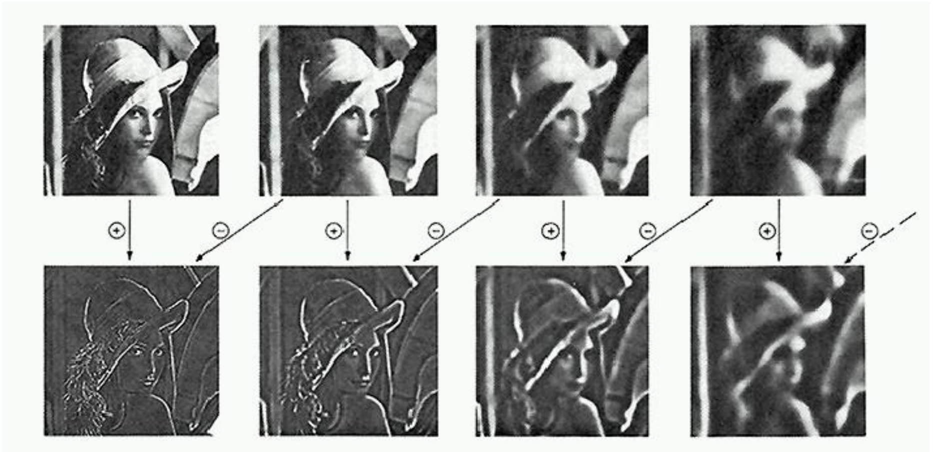
Image Pyramids

Idea: Represent NxN image as a “pyramid” of 1×1, 2×2, 4×4,…, $2^k\times2^k$ images (assuming $N=2^k$)

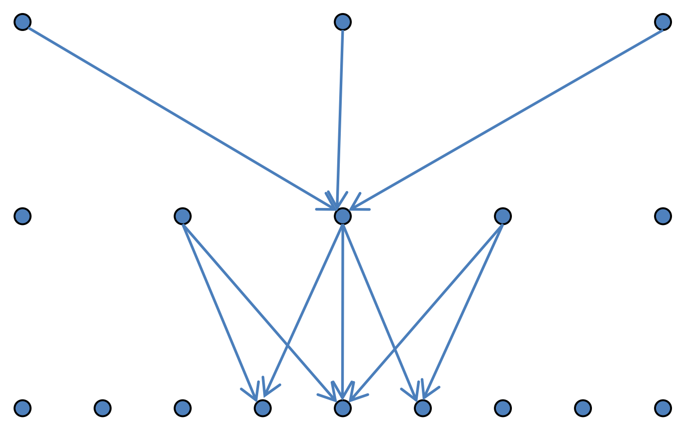


Pyramid Creation

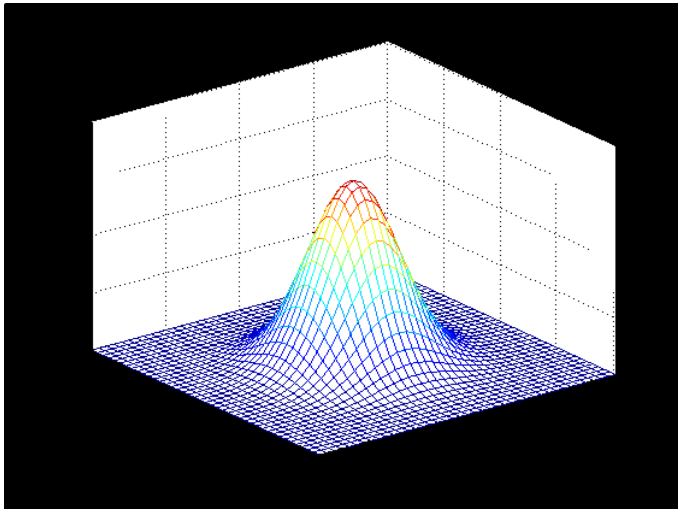




- "Gaussian" Pyramid
- "Laplacian" Pyramid
  - Created from Gaussian pyramid by subtraction
    $L_l = G_l - \text{expand}(G_{l+1})$



Gaussian Filter



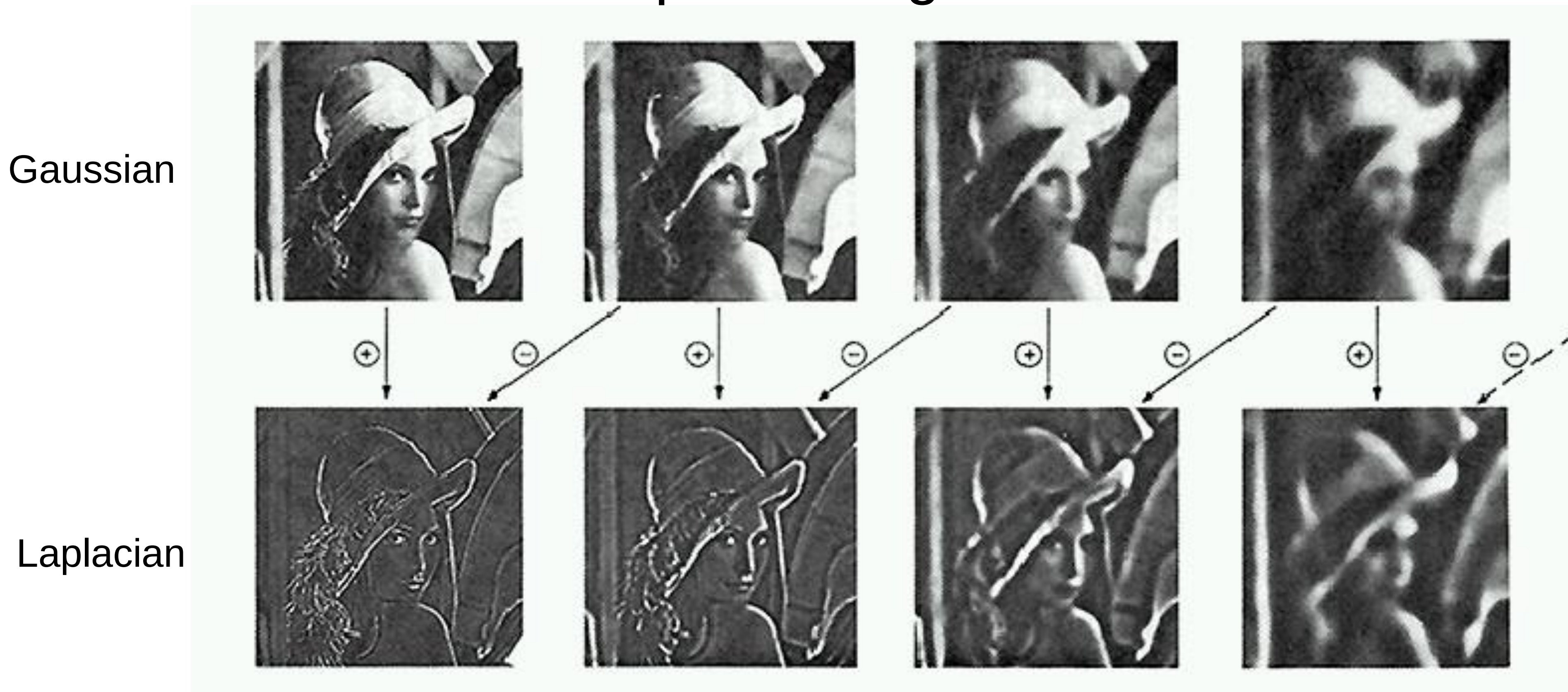
$$G_\sigma(x, y) = \frac{1}{2\pi\sigma^2} \exp\left(- \frac{\left(x^2 + y^2\right)}{2\sigma^2} \right)$$

$$H(i, j) = \frac{1}{2\pi\sigma^2} \exp\left(- \frac{\left((i-k-1)^2 + (j-k-1)^2\right)}{2\sigma^2} \right)$$

where $H(i, j)$ is $(2k+1) \times (2k+1)$ array

Octaves in the Spatial Domain

Lowpass Images



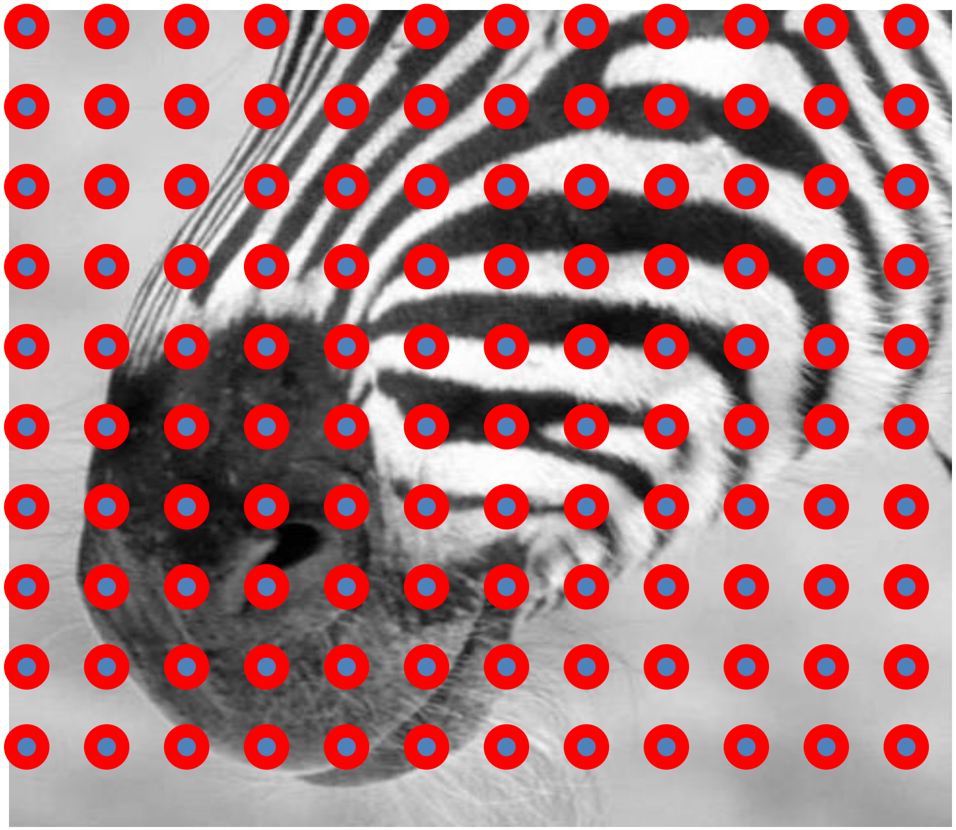
- Bandpass Images

Sampling

Why does a lower resolution image still make sense to us? What do we lose?

Image: http://www.flickr.com/photos/igorms/136916757/

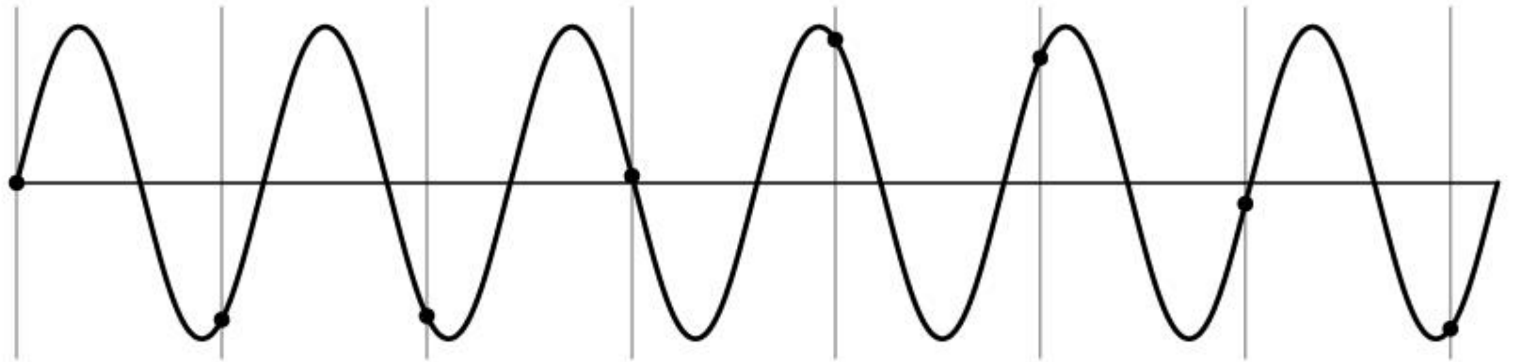
Subsampling by a factor of 2



Throw away every other row and column to create a 1/2 size image

Aliasing problem

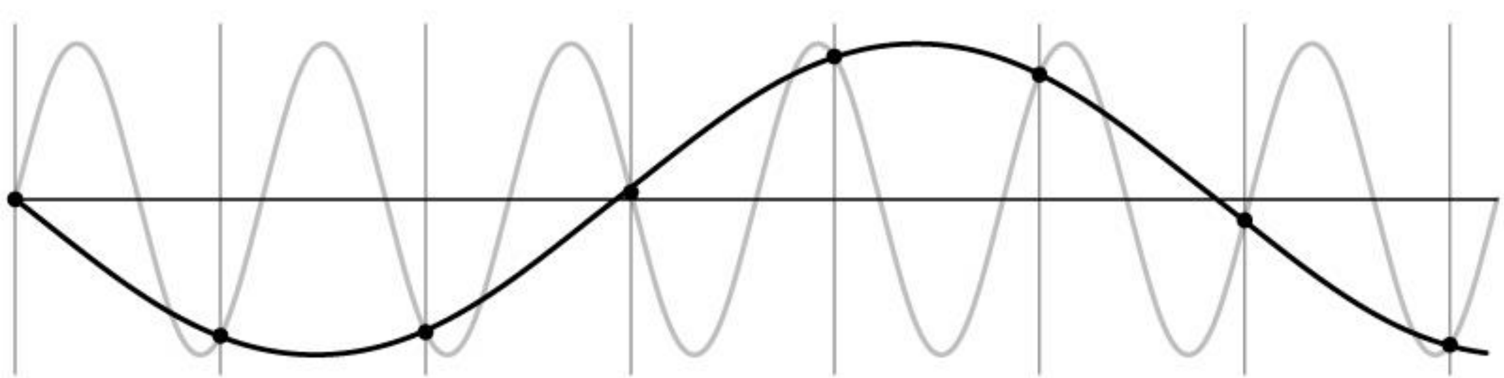
- 1D example (sinewave):



Source: S. Marschner

Aliasing problem

- 1D example (sinewave):



Source: S. Marschner

Aliasing problem

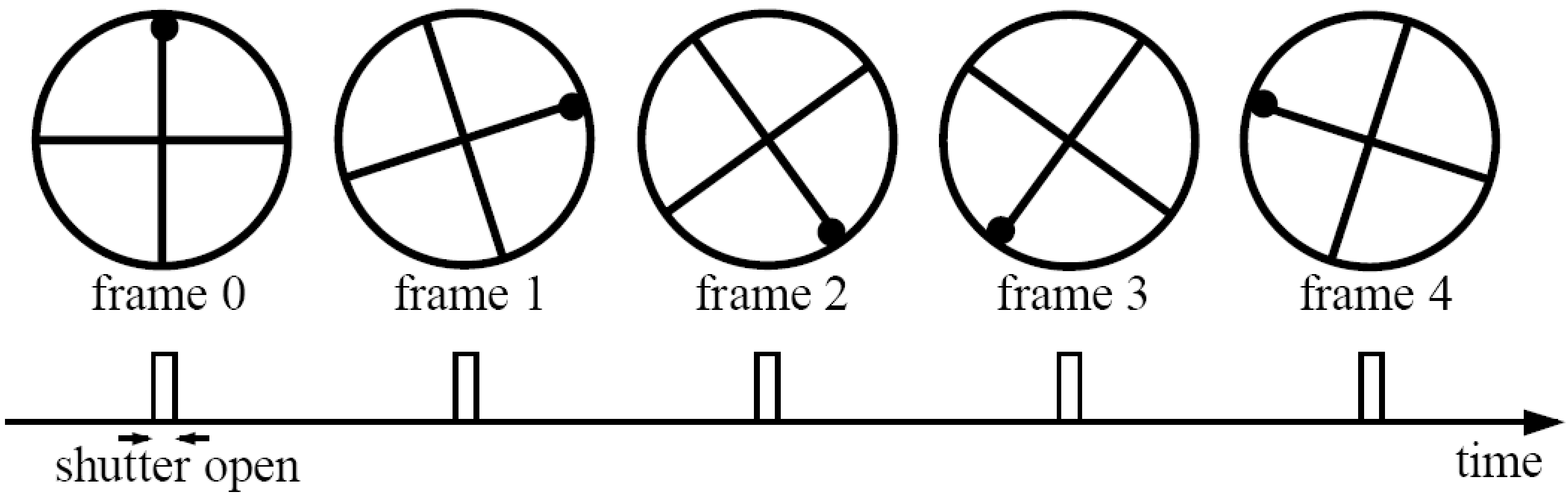
- Sub-sampling may be dangerous….
- Characteristic errors may appear:
  - “Wagon wheels rolling the wrong way in movies”
  - “Checkerboards disintegrate in ray tracing”
  - “Striped shirts look funny on color television”

Source: D. Forsyth

Aliasing in video

Imagine a spoked wheel moving to the right (rotating clockwise). Mark wheel with dot so we can see what's happening.

If camera shutter is only open for a fraction of a frame time (frame time = 1/30 sec. for video, 1/24 sec. for film):



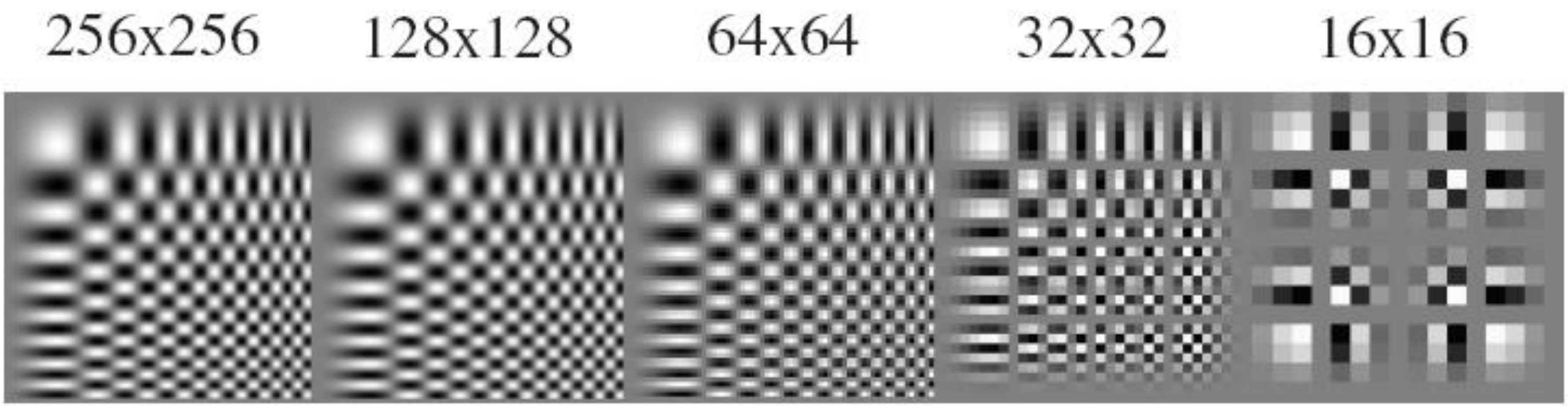
Without dot, wheel appears to be rotating slowly backwards! (counterclockwise)

Slide by Steve Seitz

Aliasing in graphics

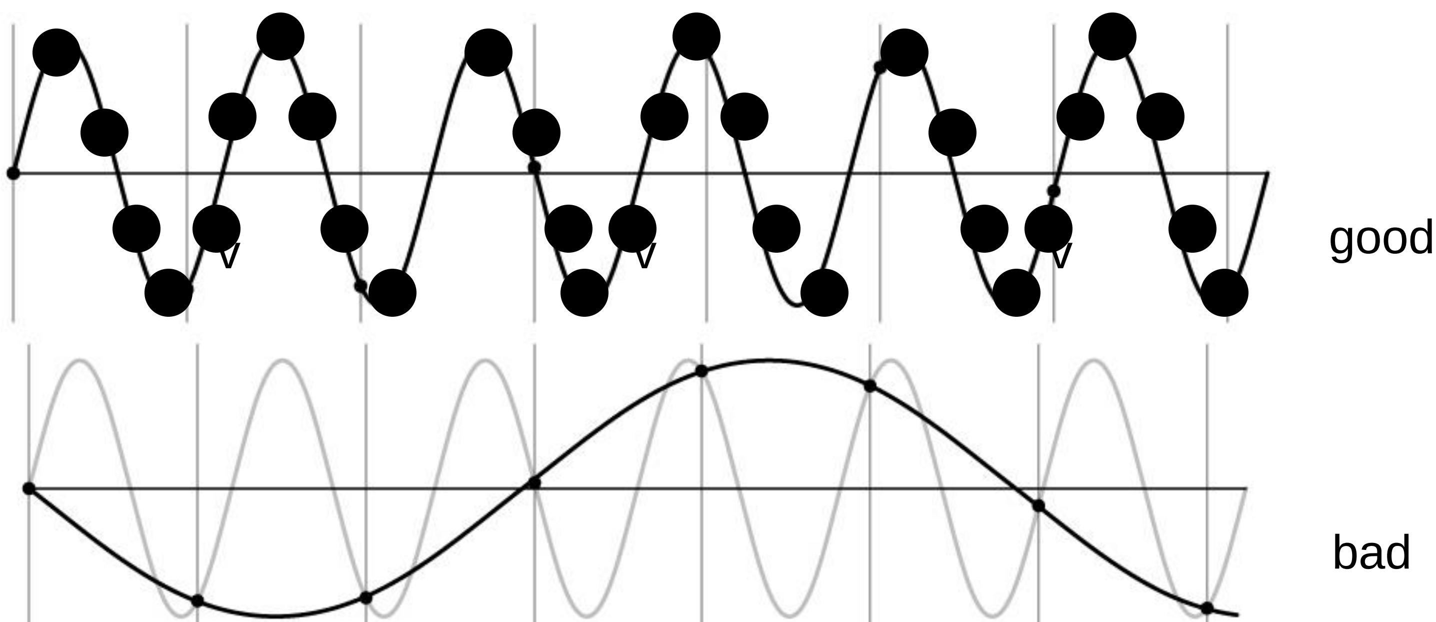
Source: A. Efros

Sampling and aliasing



Nyquist-Shannon Sampling Theorem

- When sampling a signal at discrete intervals, the sampling frequency must be $\geq 2 \times f_{max}$
- f_{max} = max frequency of the input signal
- This will allows to reconstruct the original perfectly from the sampled version



Anti-aliasing

Solutions:

- Sample more often

- Get rid of all frequencies that are greater than half the new sampling frequency
  - Will lose information
  - But it’s better than aliasing
  - Apply a smoothing filter

Algorithm for downsampling by factor of 2

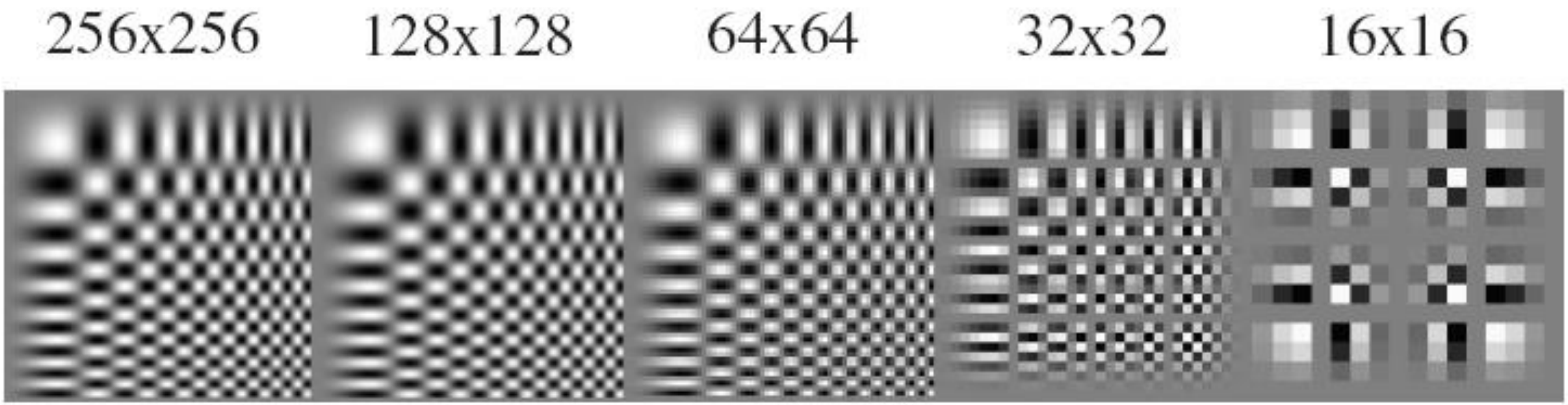
1. Start with image(h, w)
2. Apply low-pass filter

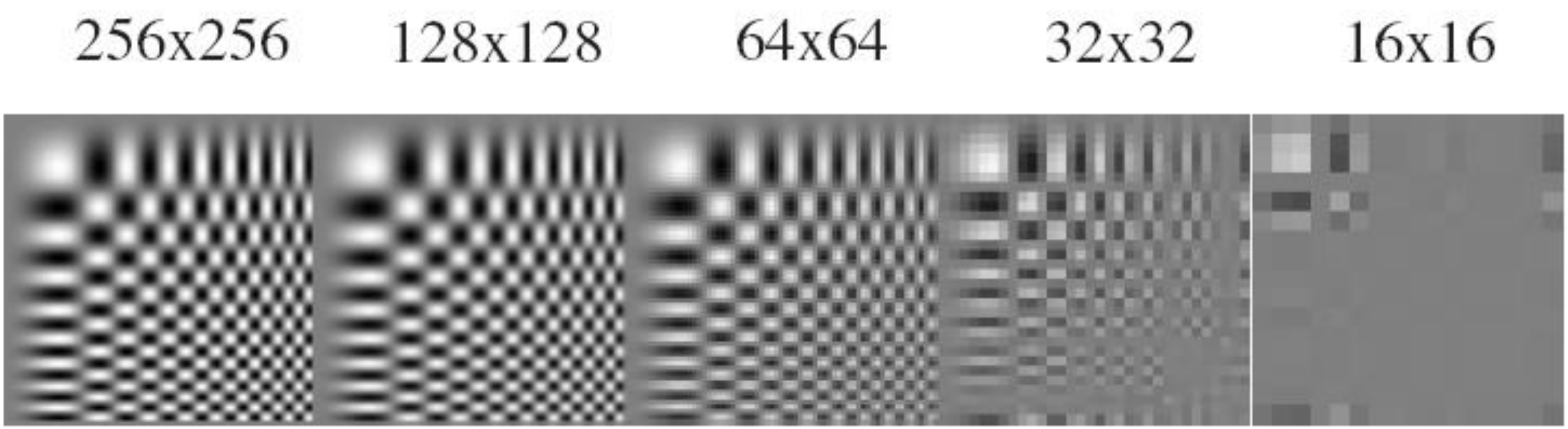
   ```
   im_blur = imfilter(image, fspecial('gaussian', 7, 1))
   ```

3. Sample every other pixel

   ```
   im_small = im_blur(1:2:end, 1:2:end);
   ```

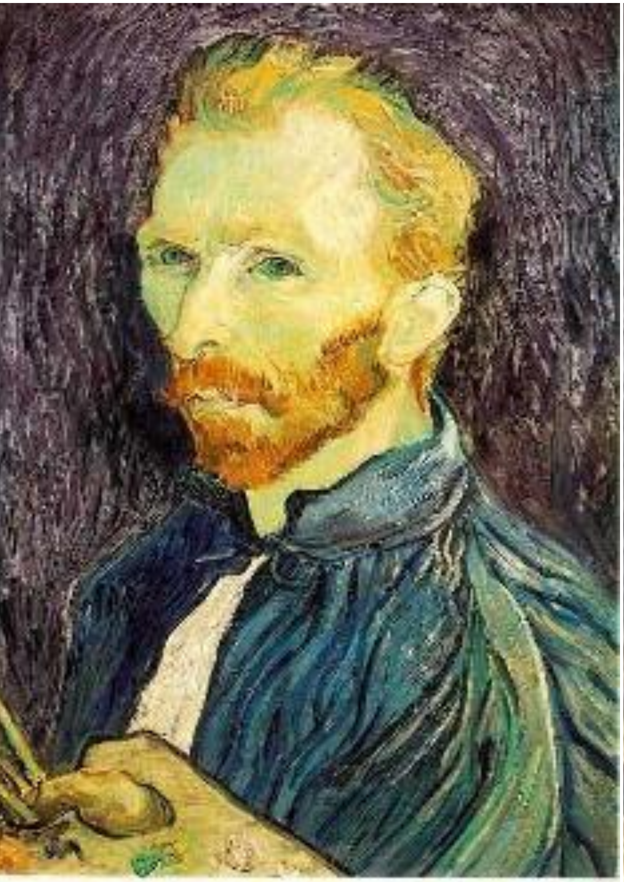
Anti-aliasing





Forsyth and Ponce 2002

Subsampling without pre-filtering



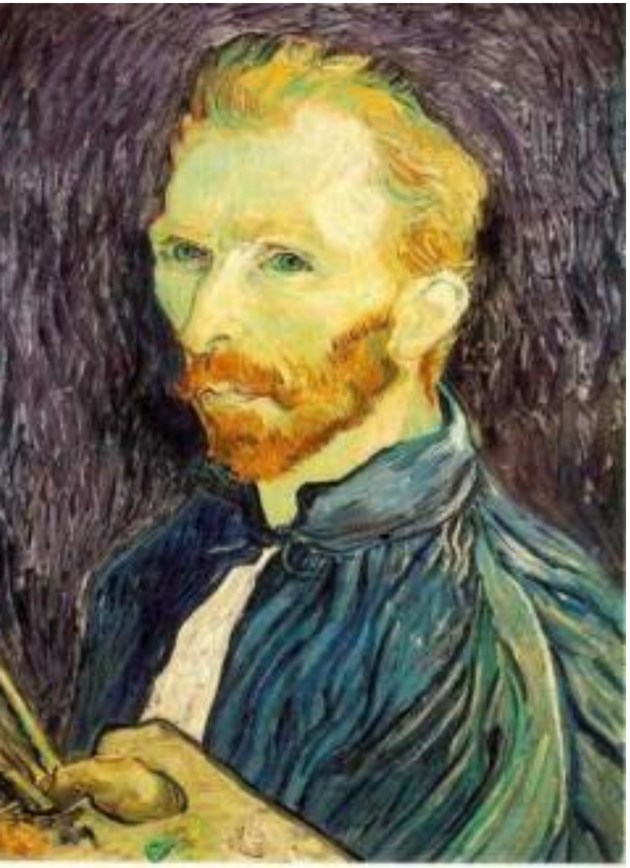
1/2

1/4 (2x zoom)

1/8 (4x zoom)

Slide by Steve Seitz

Subsampling with Gaussian pre-filtering



Gaussian 1/2

G 1/4

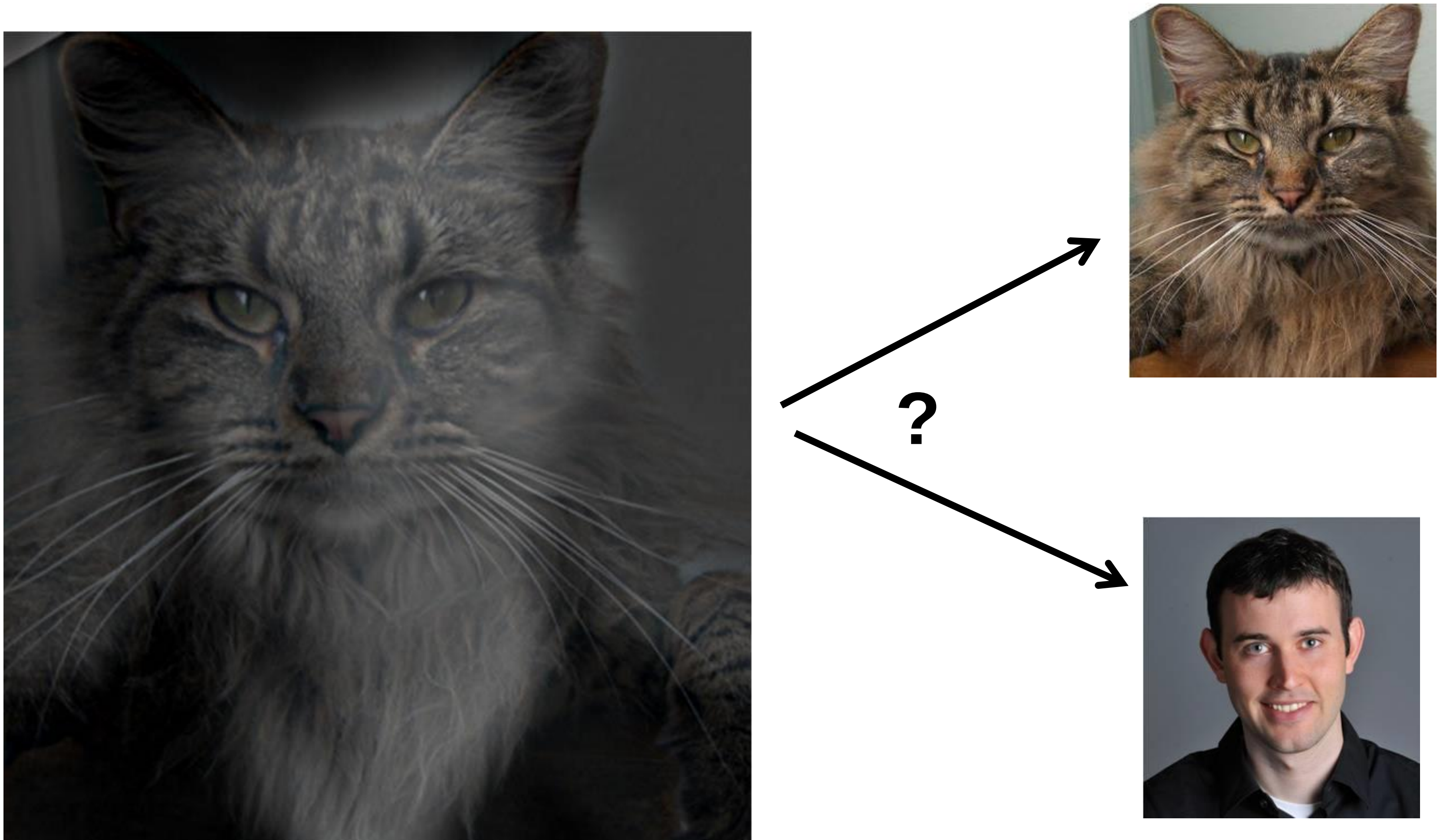
G 1/8

Slide by Steve Seitz

Why does a lower resolution image still make sense to us? What do we lose?

Image: http://www.flickr.com/photos/igorms/136916757/

Why do we get different, distance-dependent interpretations of hybrid images?



Salvador Dali invented Hybrid Images?

Salvador Dali
"Gala Contemplating the Mediterranean Sea, which at 30 meters becomes the portrait of Abraham Lincoln", 1976