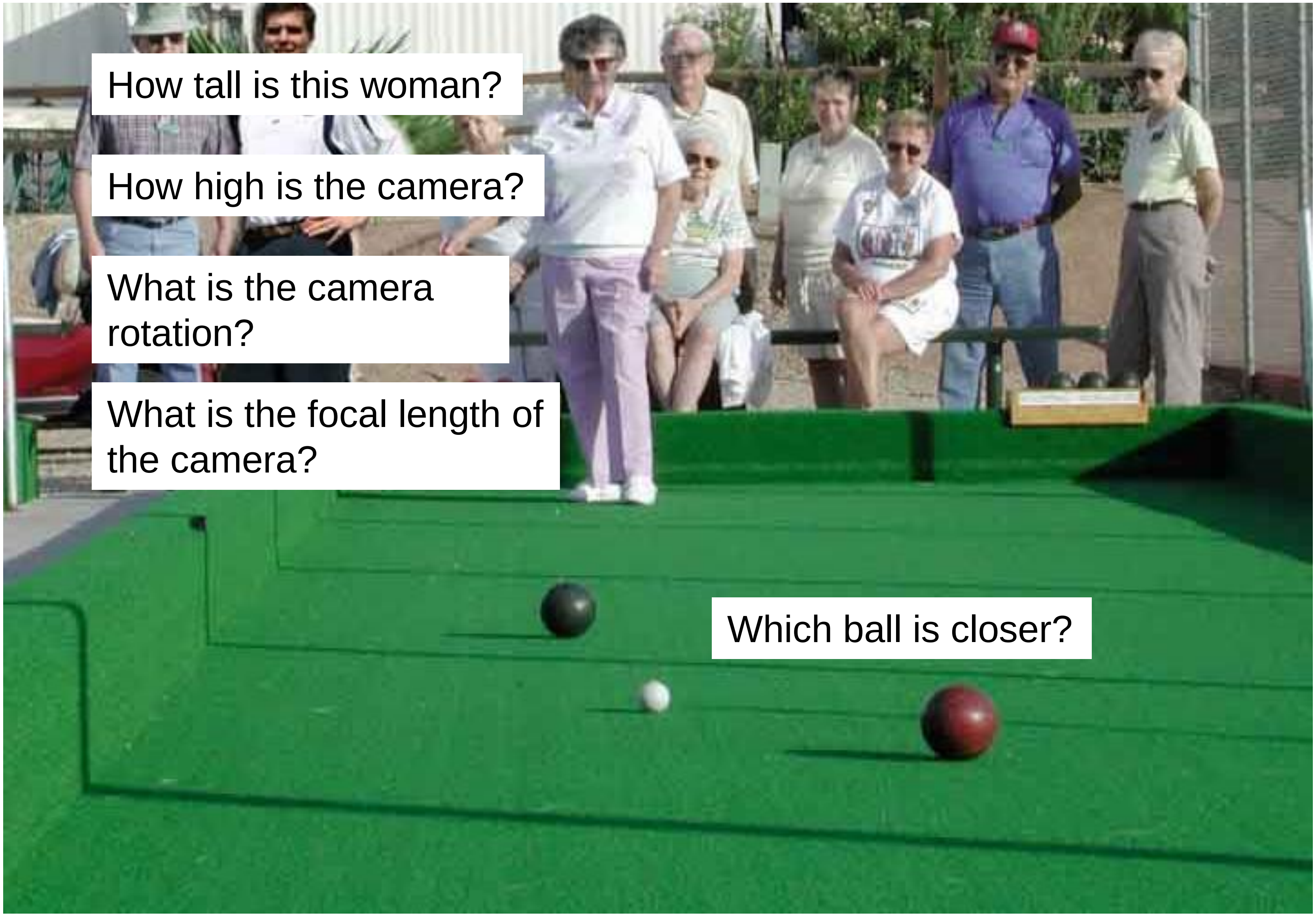
Camera Model

Gang Pan
Zhejiang University


How tall is this woman?
How high is the camera?
What is the camera rotation?
What is the focal length of the camera?
Which ball is closer?

The Camera

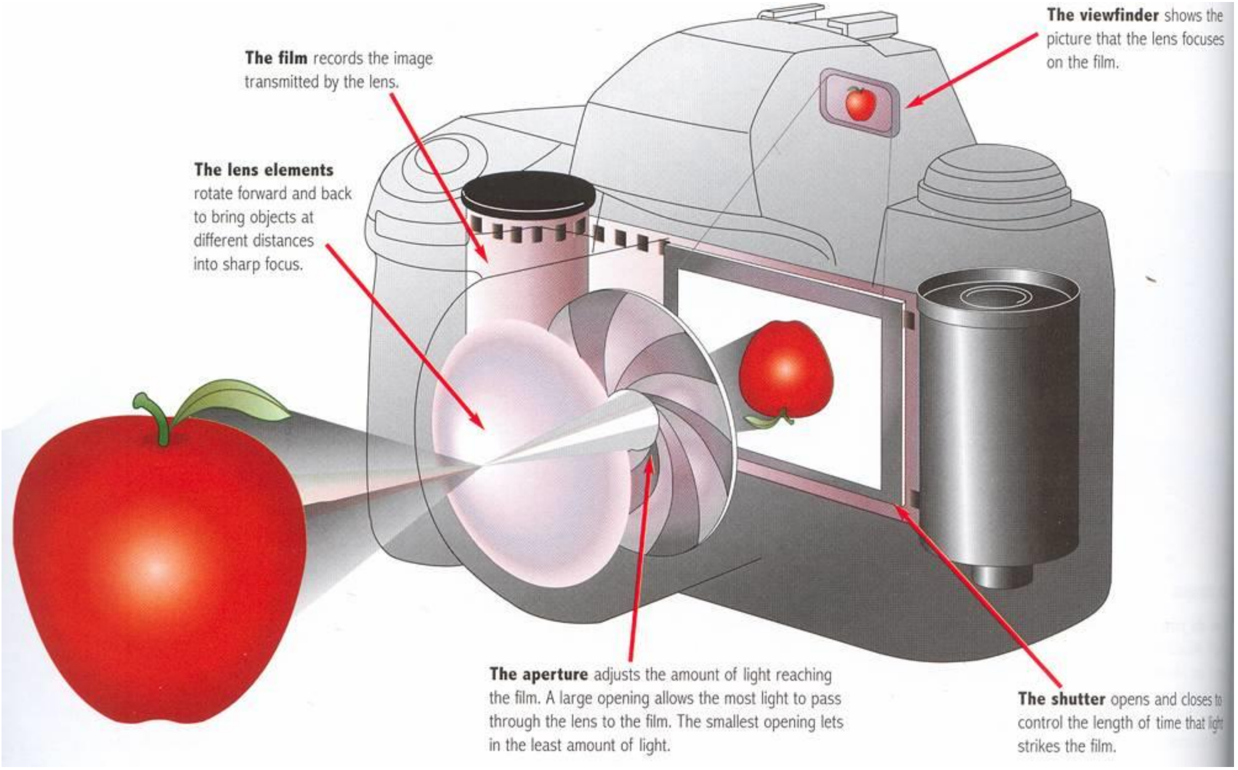
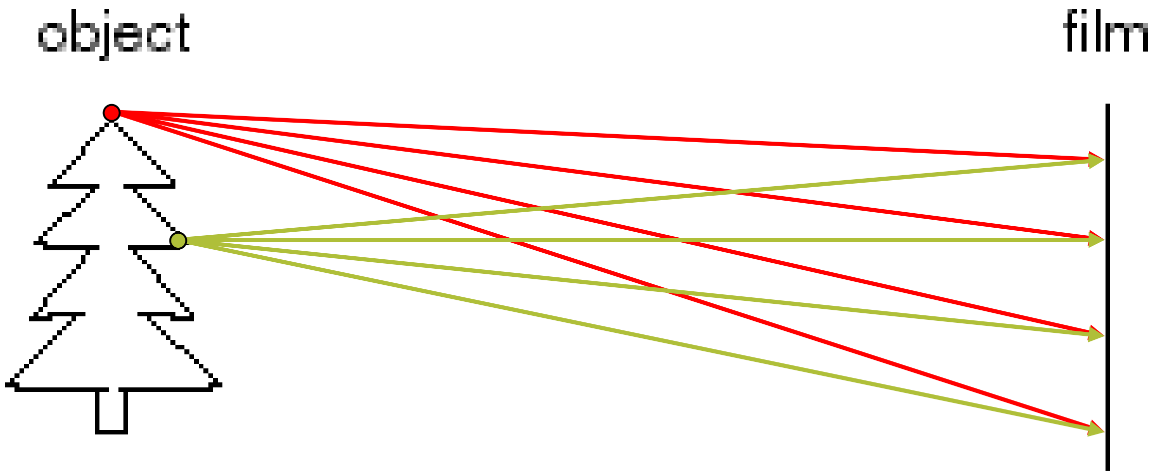
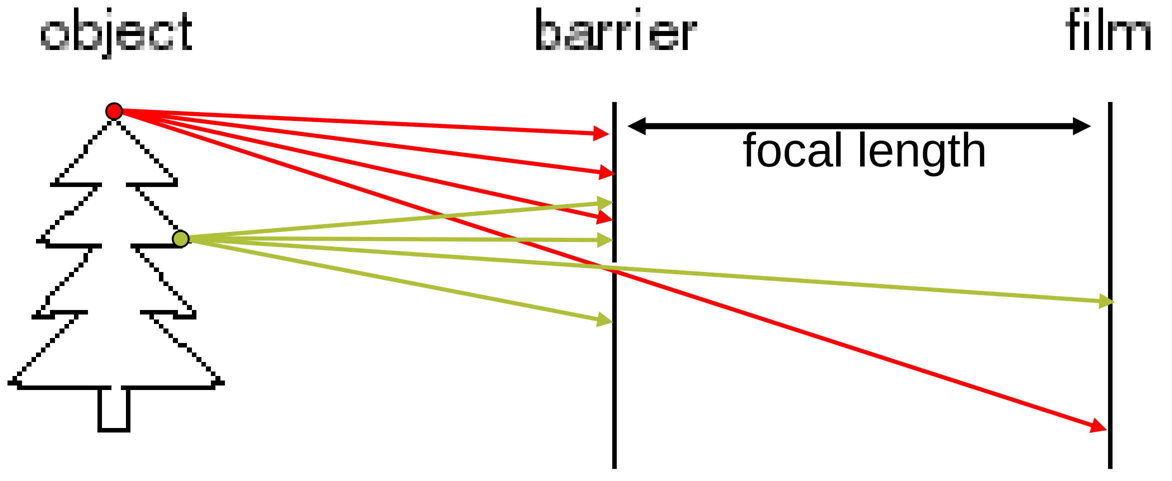

The viewfinder shows the picture that the lens focuses on the film.
The film records the image transmitted by the lens.
The lens elements rotate forward and back to bring objects at different distances into sharp focus.
The aperture adjusts the amount of light reaching the film. A large opening allows the most light to pass through the lens to the film. The smallest opening lets in the least amount of light.
The shutter opens and closes to control the length of time that light strikes the film.

Image formation



- Let's design a camera
  - Idea 1: put a piece of film in front of an object
  - Do we get a reasonable image?

Pinhole camera



- Add a barrier to block off most of the rays
  - This reduces blurring
  - The opening known as the **aperture**
  - How does this transform the image?

Camera obscura: the pre-camera

- Known during classical period in China and Greece (e.g. Mozi, China, 470BC to 390BC)

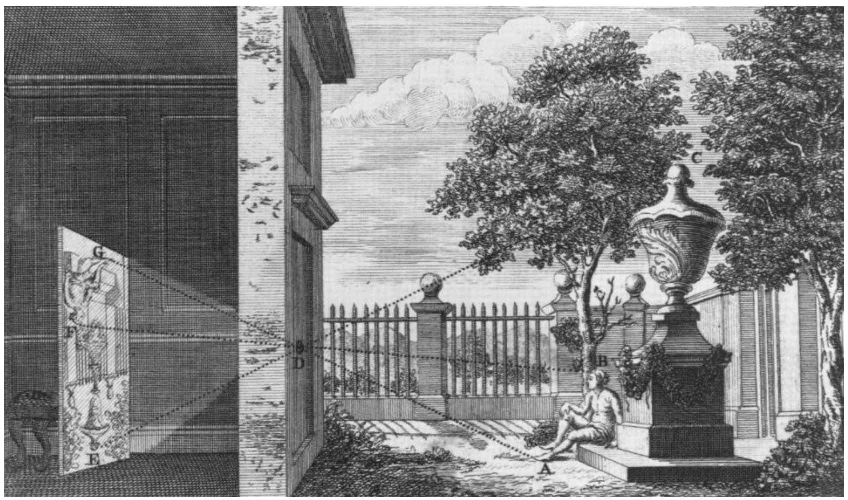


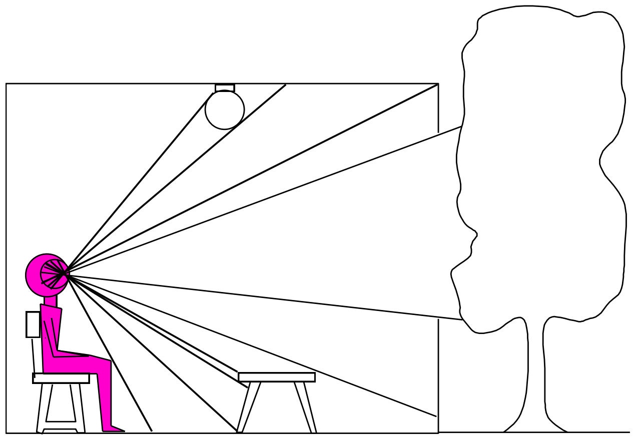
Illustration of Camera Obscura

Freestanding camera obscura at UNC Chapel Hill

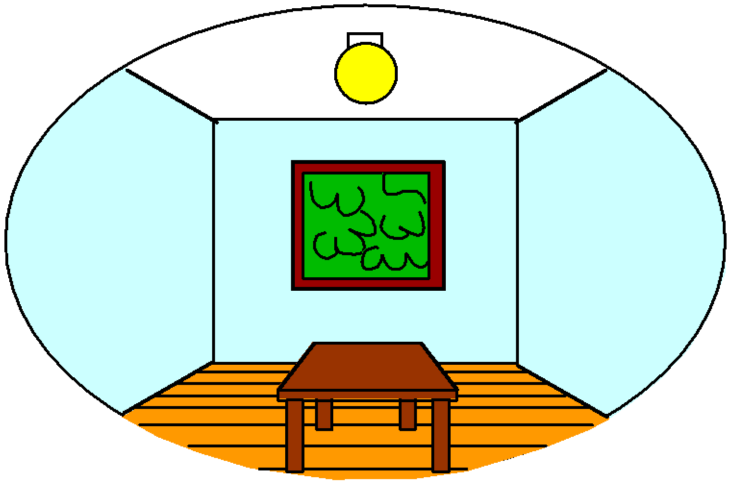
Dimensionality Reduction Machine (3D to 2D)

3D world

2D image



Point of observation

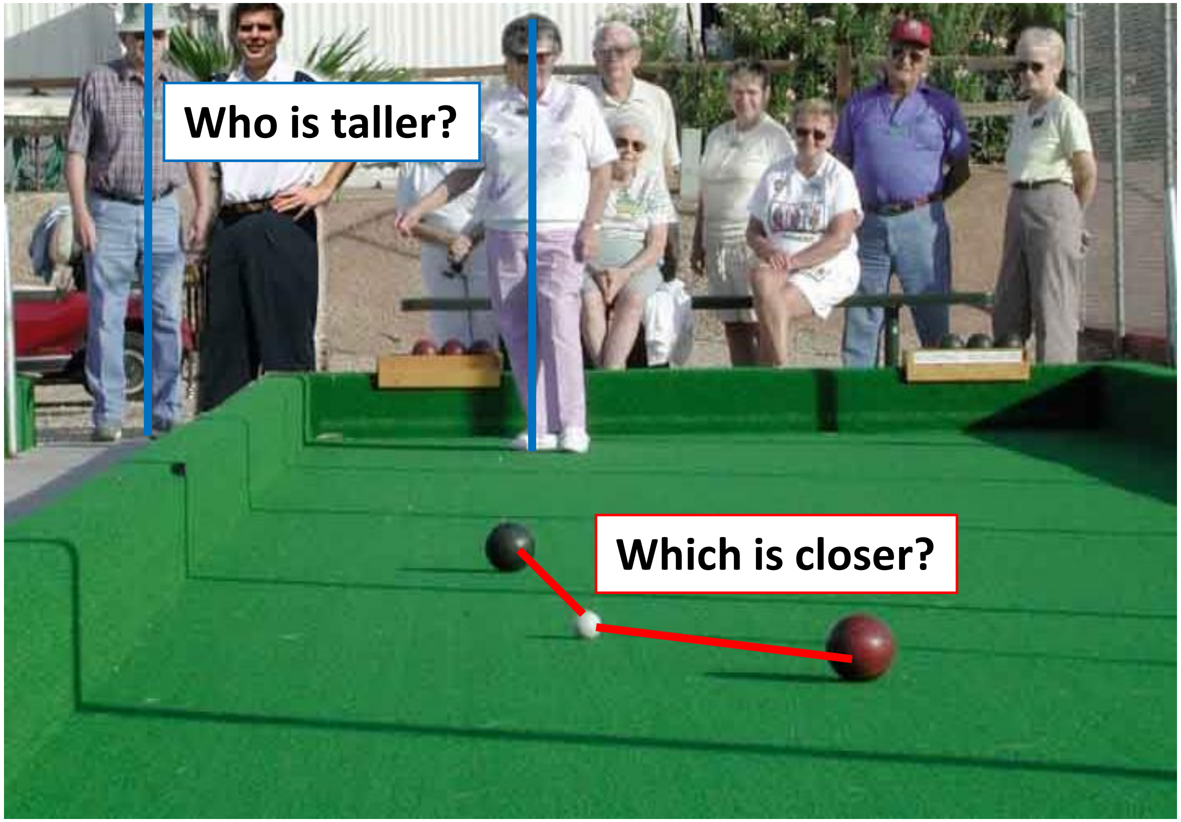


Projection can be tricky…

Projective Geometry

What is lost?

- Length



Length is not preserved

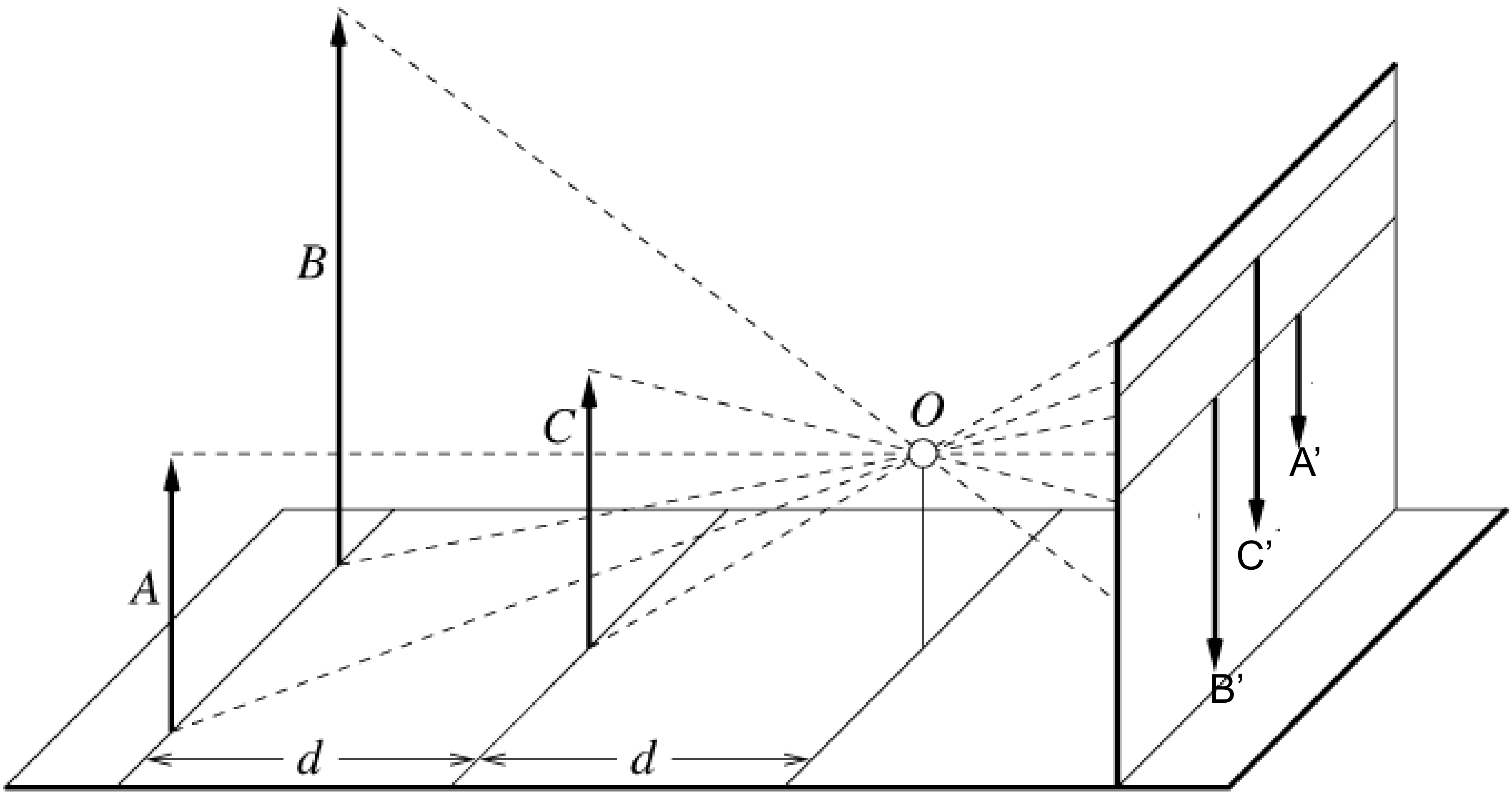
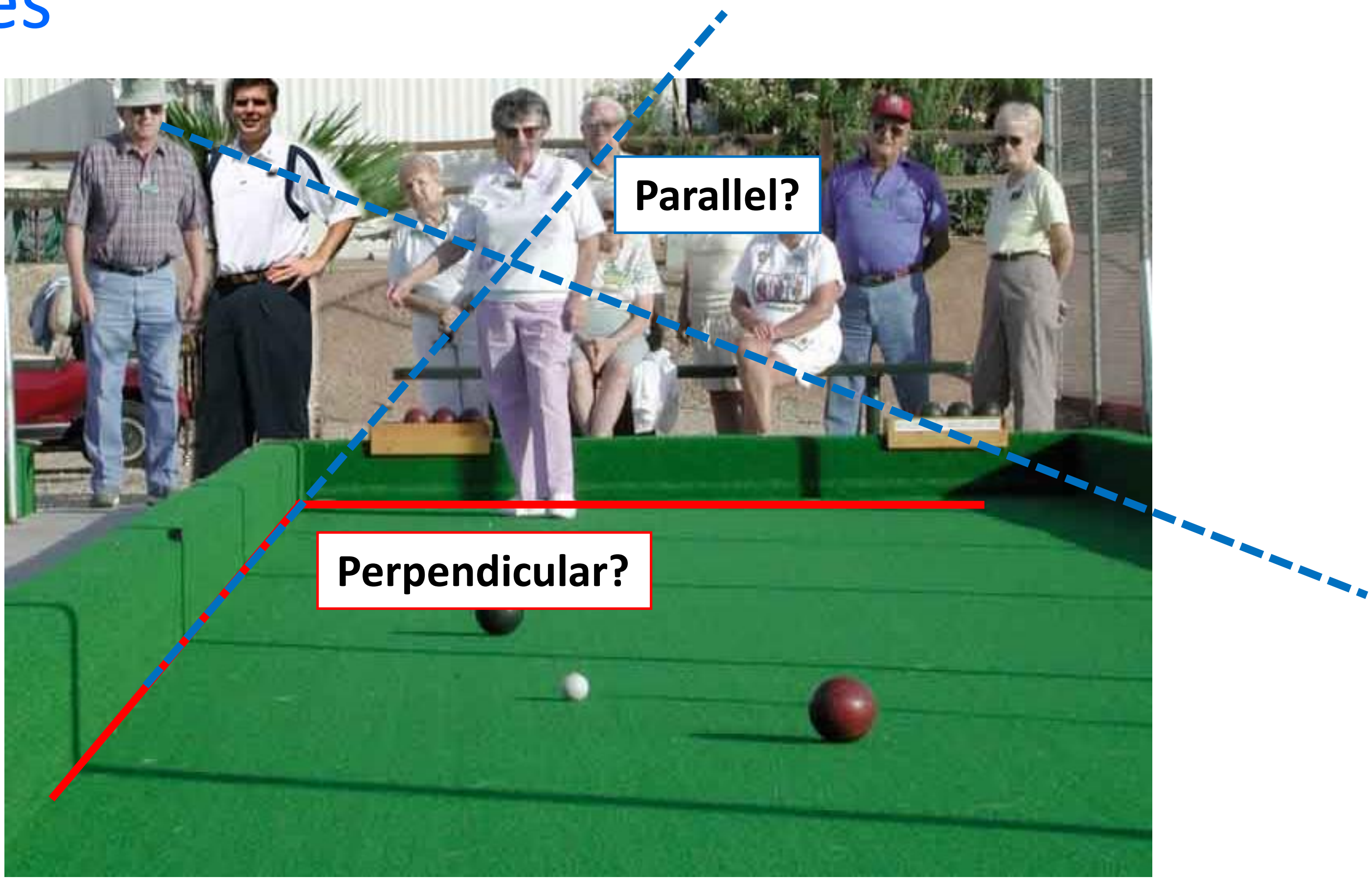


Figure by David Forsyth

Projective Geometry

What is lost?

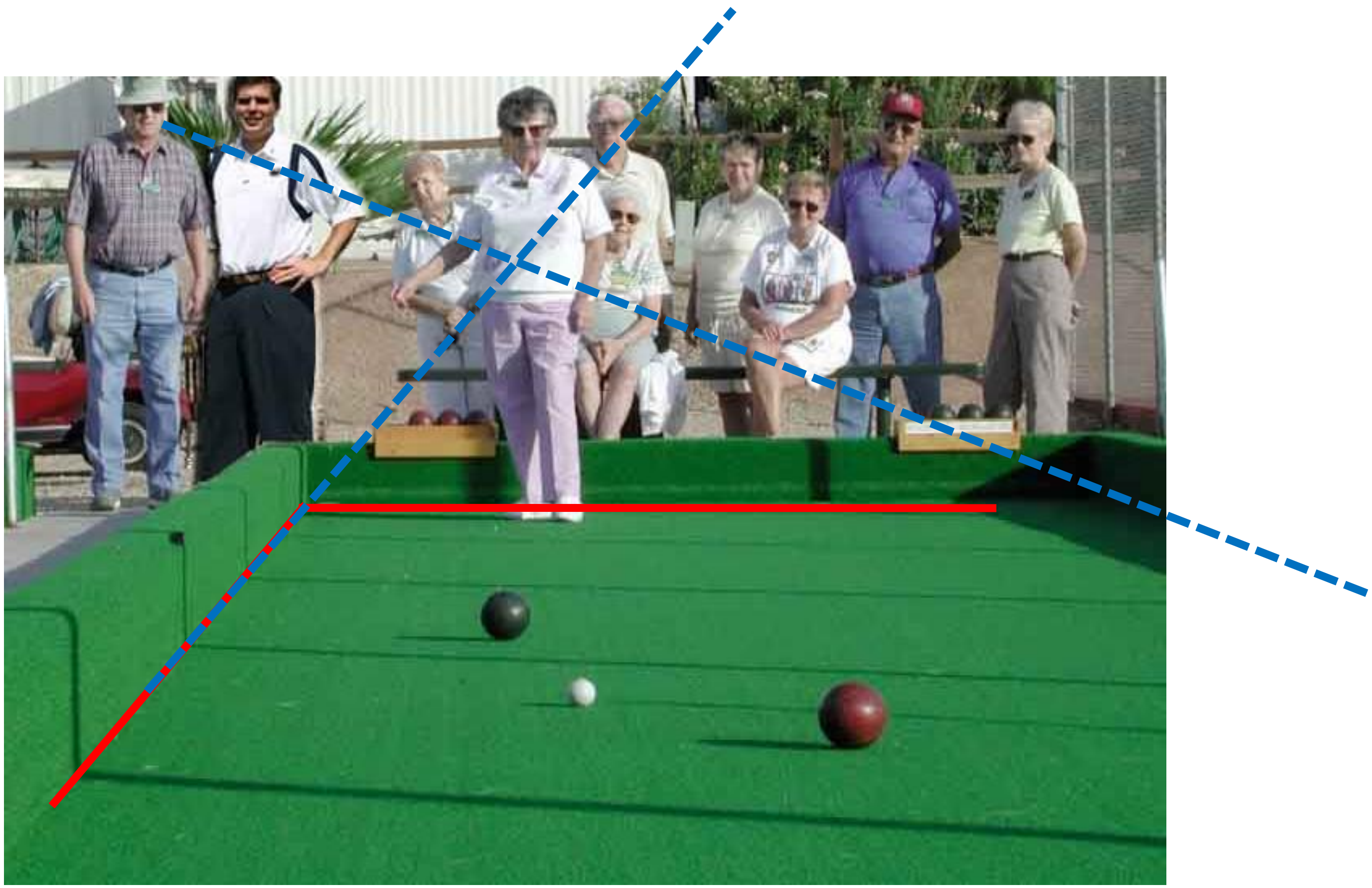
- Length
- Angles



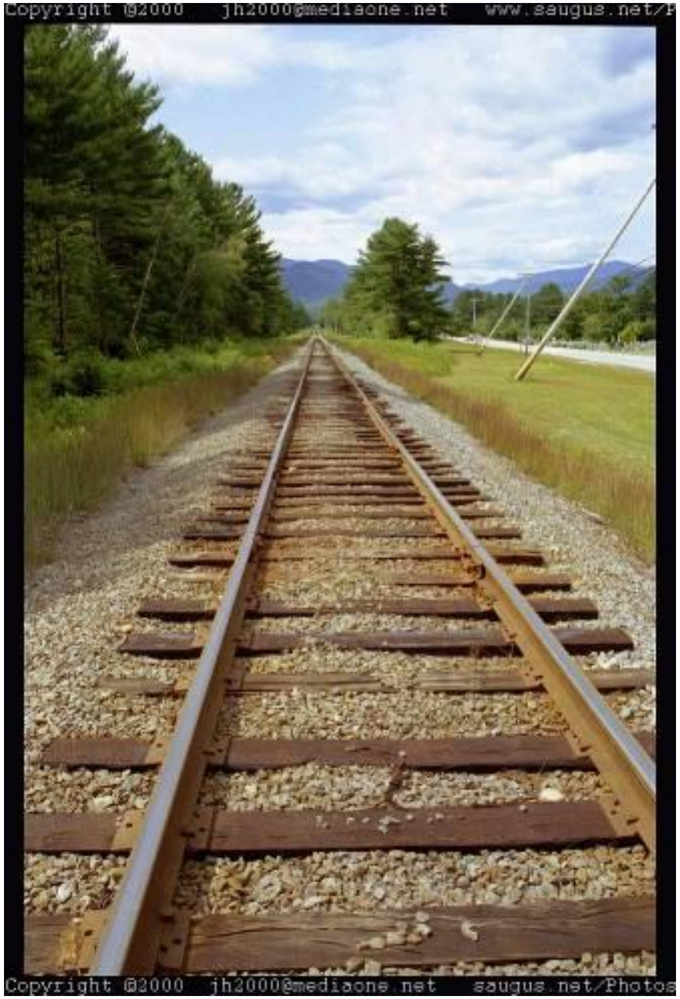
Projective Geometry

What is preserved?

- Straight lines are still straight (colinearity)



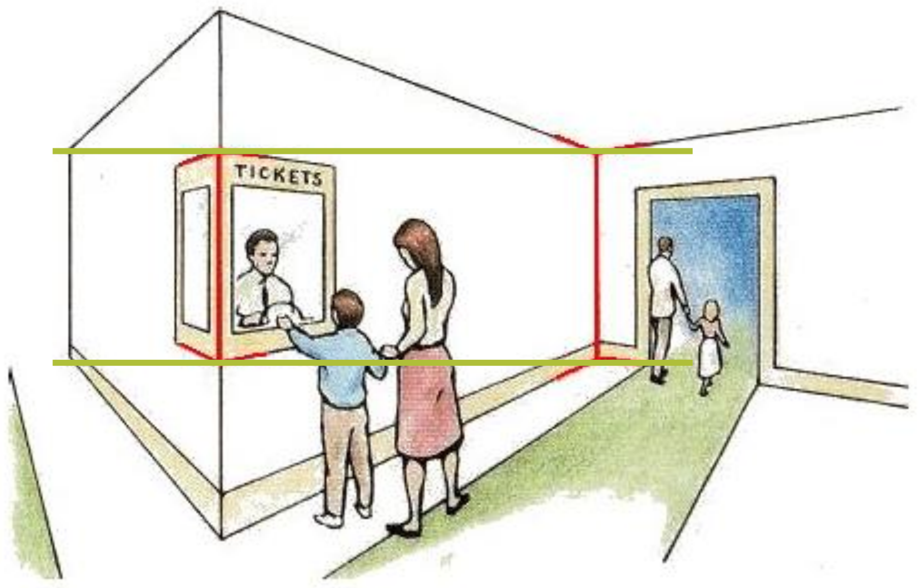
Parallel lines in images



Four geometries

	Euclidean	similarity	affine	projective
Transformations				
rotation	X	X	X	X
translation	X	X	X	X
uniform scaling		X	X	X
nonuniform scaling			X	X
shear			X	X
perspective projection				X
composition of projections				X
Invariants				
length	X			
angle	X	X		
ratio of lengths	X	X		
parallelism	X	X	X	
incidence	X	X	X	X
cross ratio	X	X	X	X

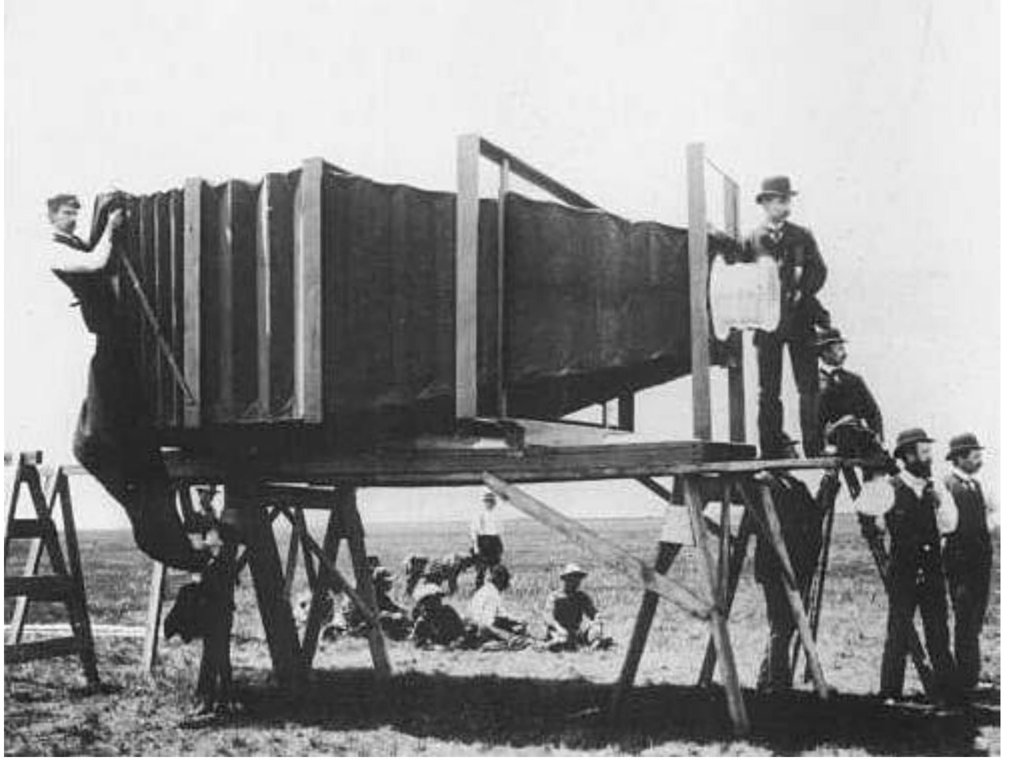
Müller-Lyer Illusion



http://www.michaelbach.de/ot/sze_muelue/index.html

Building a real camera

The largest camera (1900)



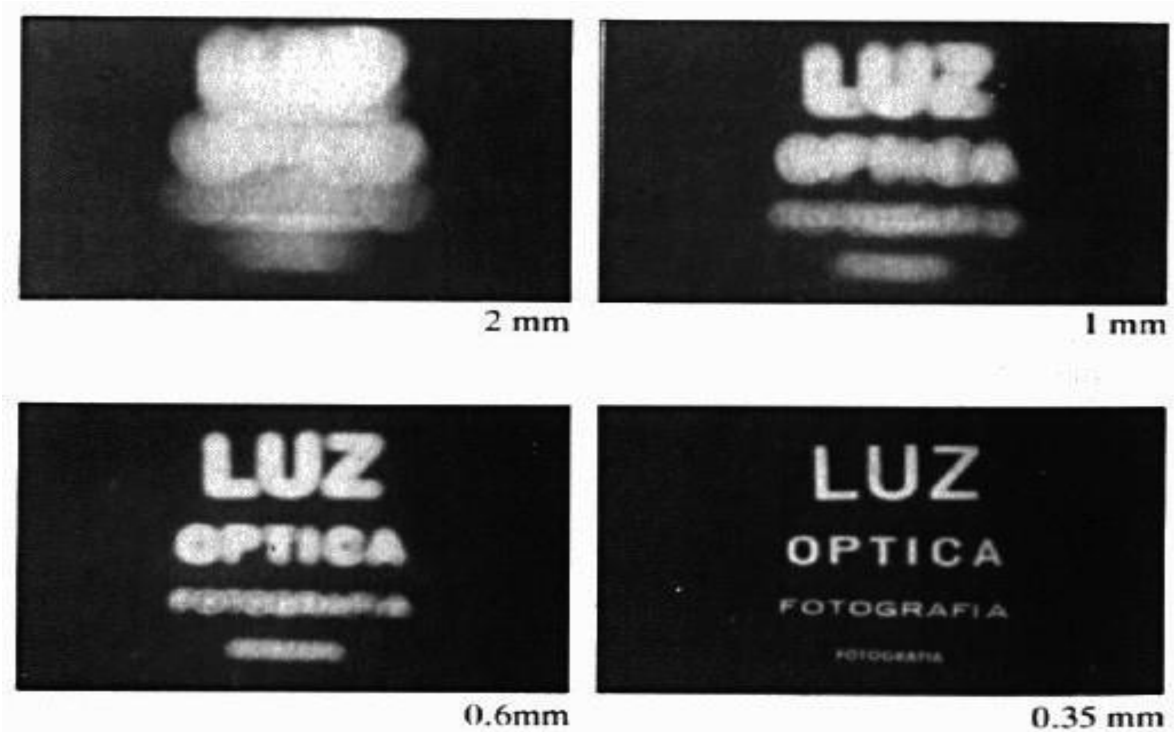
- In 1900 the Chicago & Alton Railroad Train co. , commissioned Lawrence with the manufacture of the largest camera ever made and the largest photo ever shot in order to promote a new train.

Home-made pinhole camera

Why so blurry?

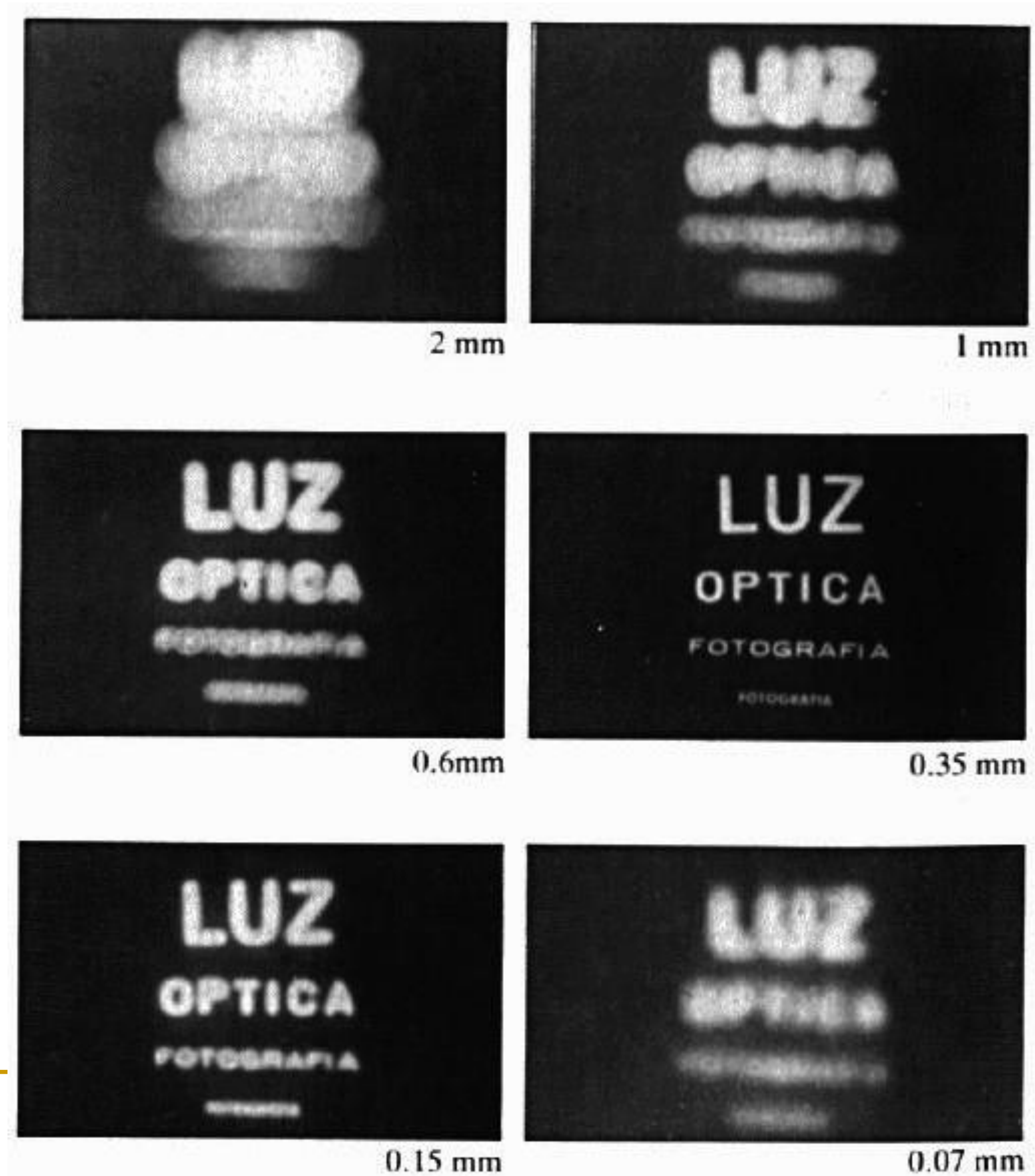
http://www.debevec.org/Pinhole/

Shrinking the aperture

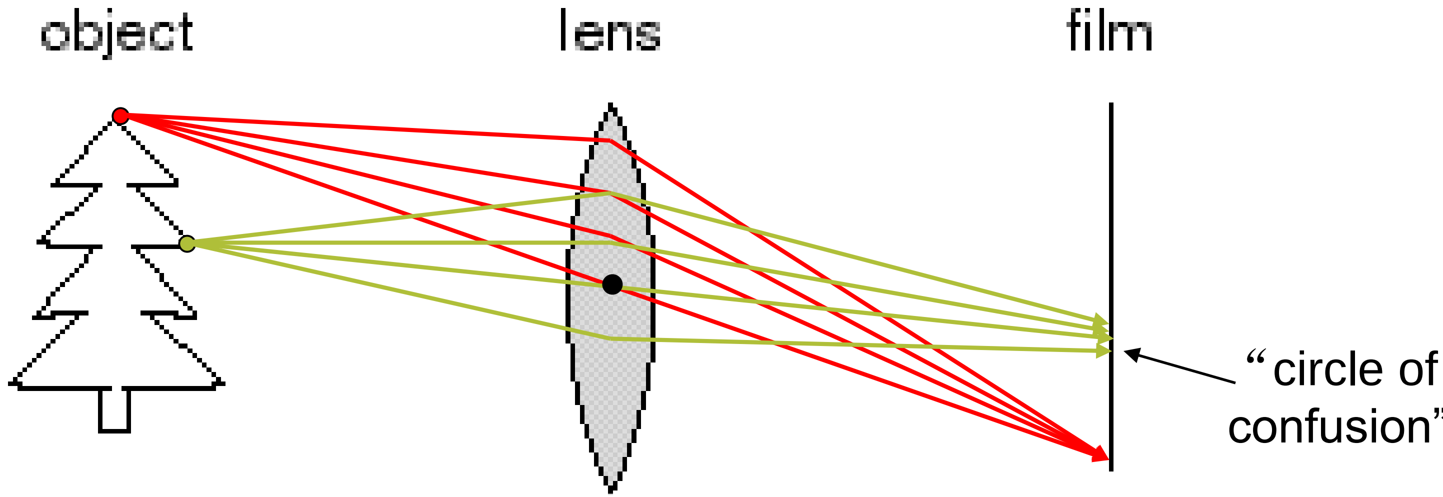


- Why not make the aperture as small as possible?
  - Less light gets through
  - *Diffraction* effects

Shrinking the aperture

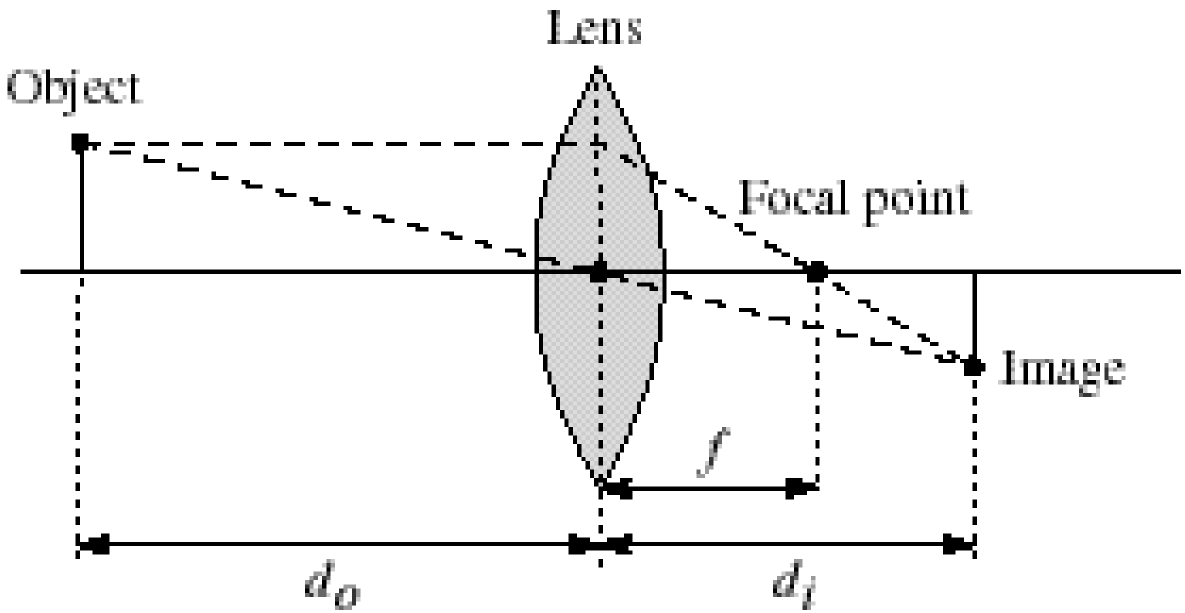


Adding a lens helps



- A lens focuses light onto the film
  - There is a specific distance at which objects are “in focus”
    - other points project to a “circle of confusion” in the image
  - Changing the shape of the lens changes this distance

Thin lenses



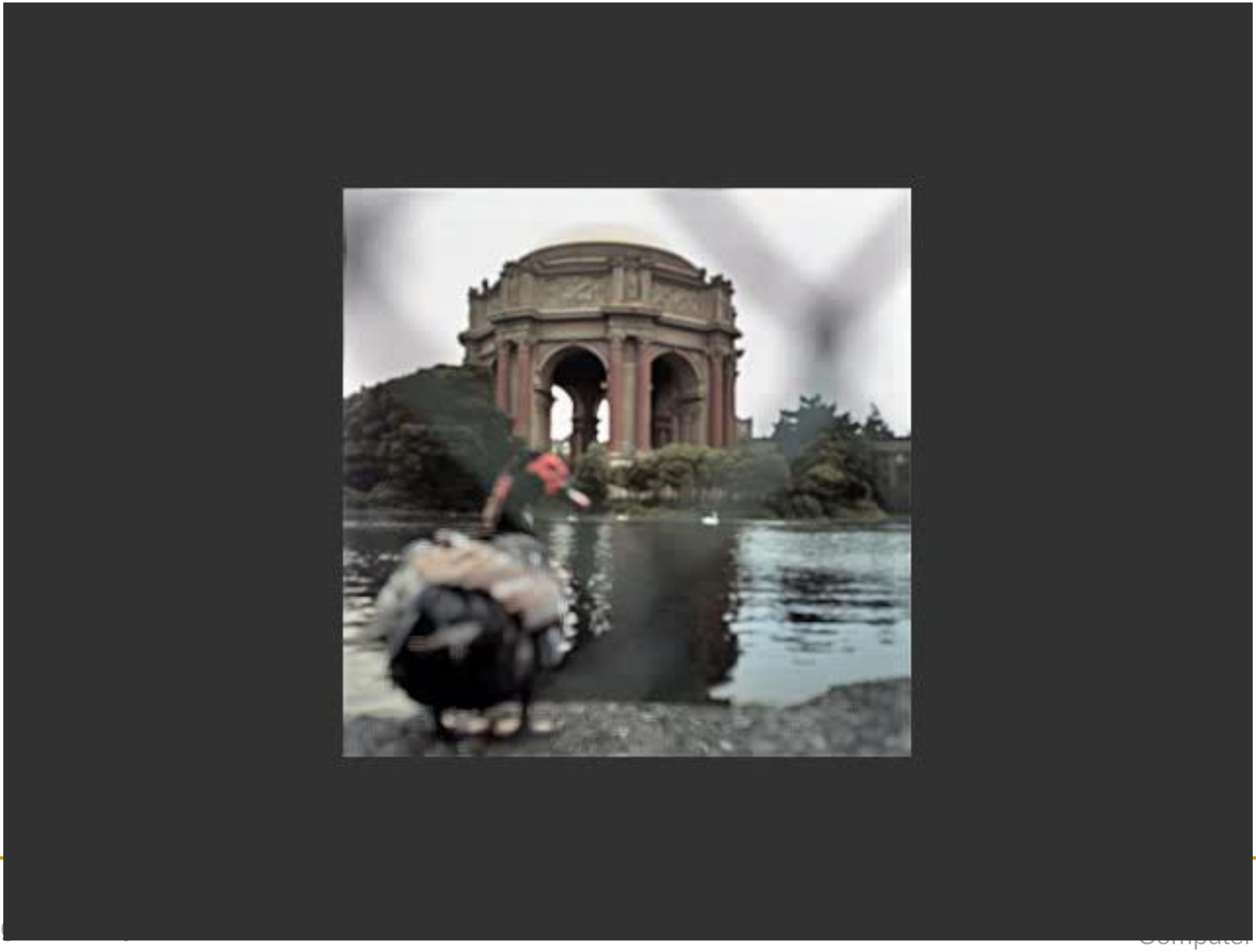
- Thin lens equation:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

  - Any object point satisfying this equation is in focus
  - How can we change the focus region?

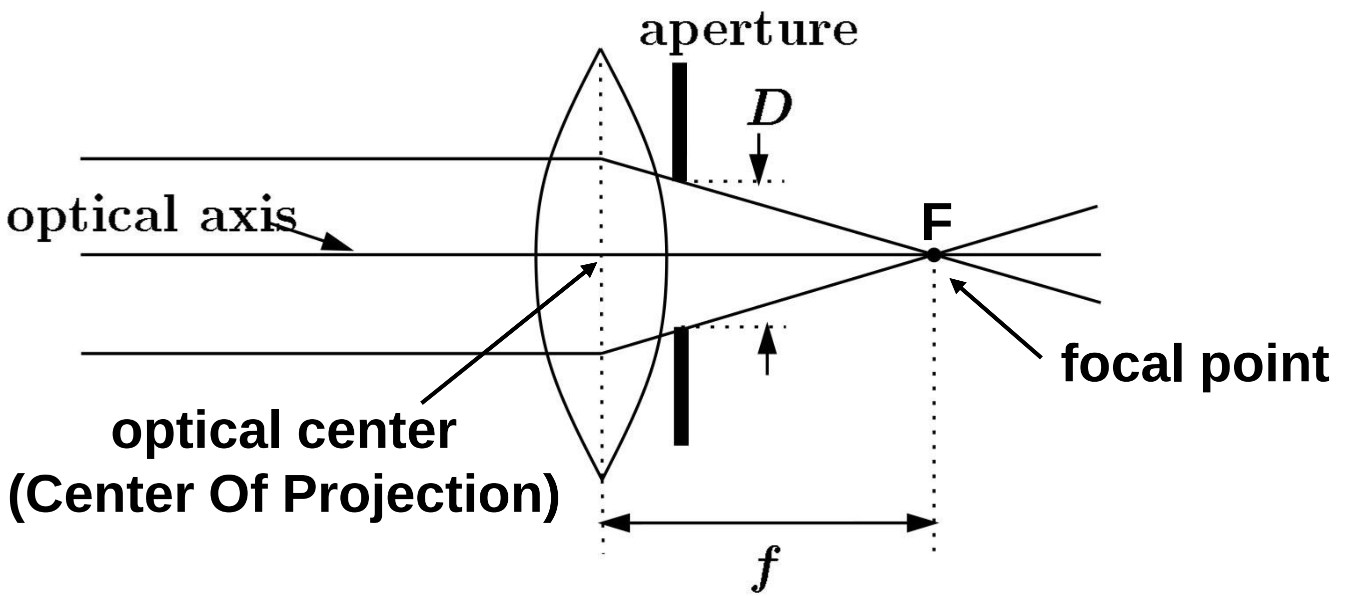
Thin lens applet: http://www.phy.ntnu.edu.tw/java/Lens/lens_e.html

Varying Focus



Ren Ng

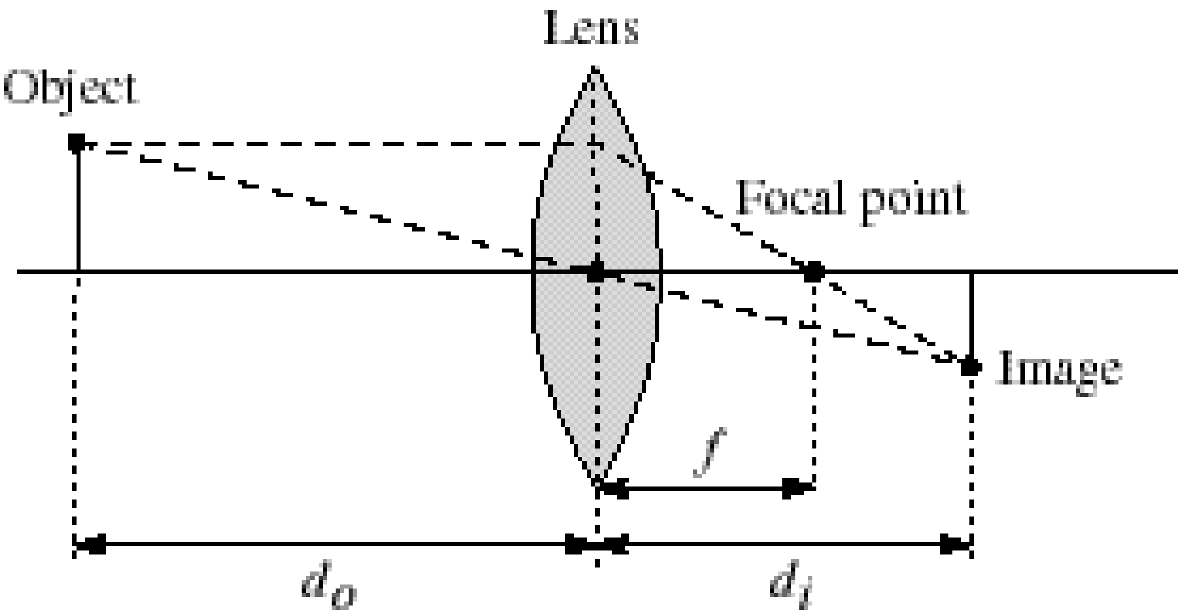
Lenses



- A lens focuses parallel rays onto a single focal point
  - focal point at a distance *f* beyond the plane of the lens
    - *f* is a function of the shape and index of refraction of the lens
  - Aperture of diameter D restricts the range of rays
    - aperture may be on either side of the lens

Depth of Field

We have known



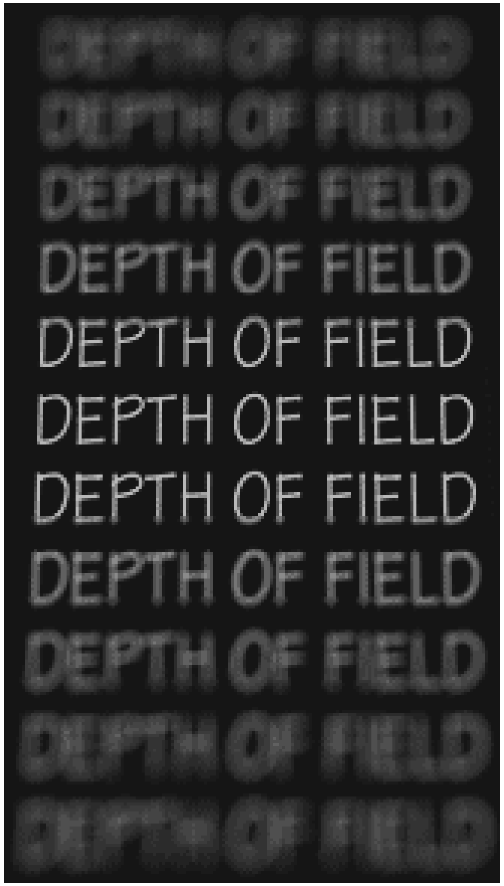
- Thin lens equation:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

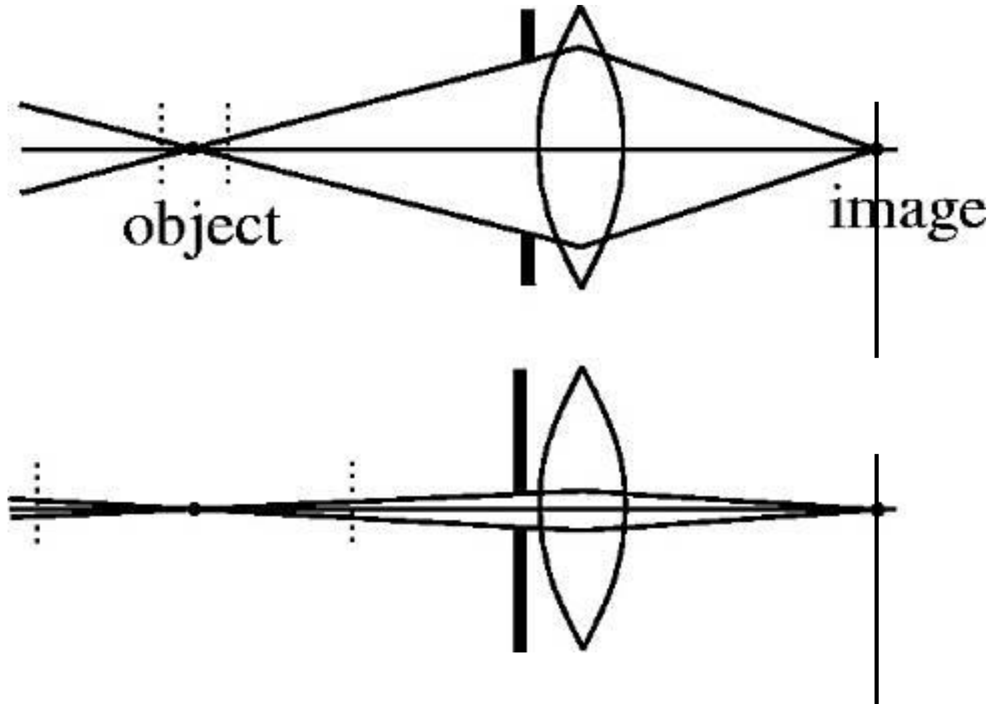
  - Any object point satisfying this equation is in focus

But …

- It is not always correct in practice.



Aperture controls Depth of Field



- Changing the aperture size affects depth of field
  - A smaller aperture increases the range in which the object is approximately in focus
  - But small aperture reduces amount of light – need to increase exposure

Varying the aperture

f/2.8

Large apeture = small DOF

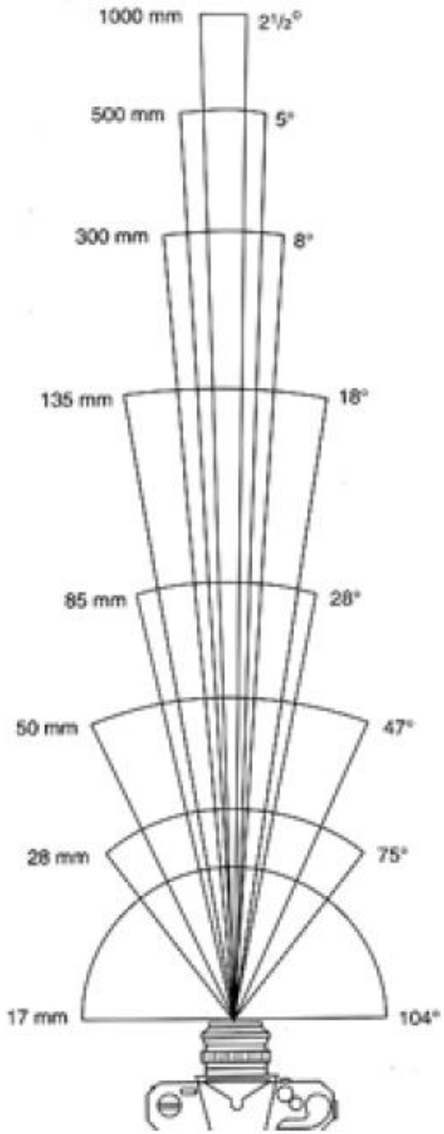
f/22

Small apeture = large DOF

Nice Depth of Field effect

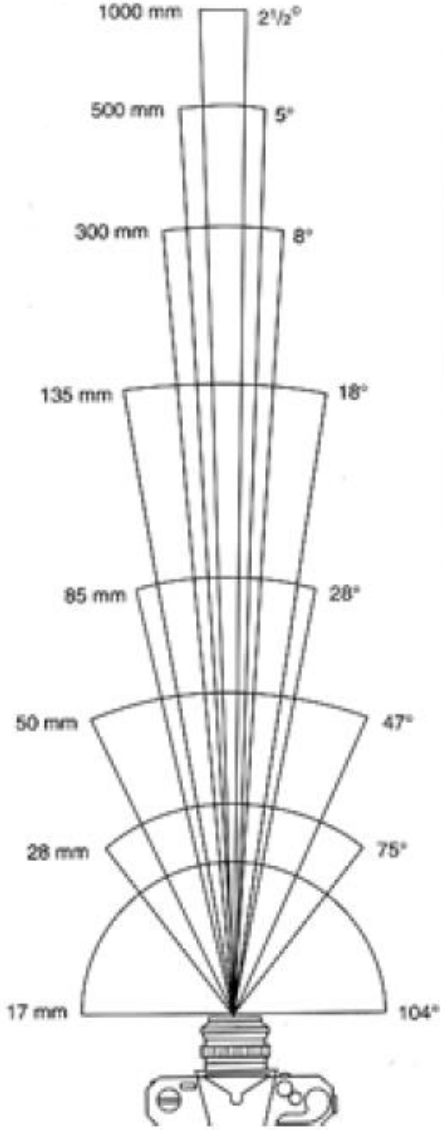
Field of View (Zoom)

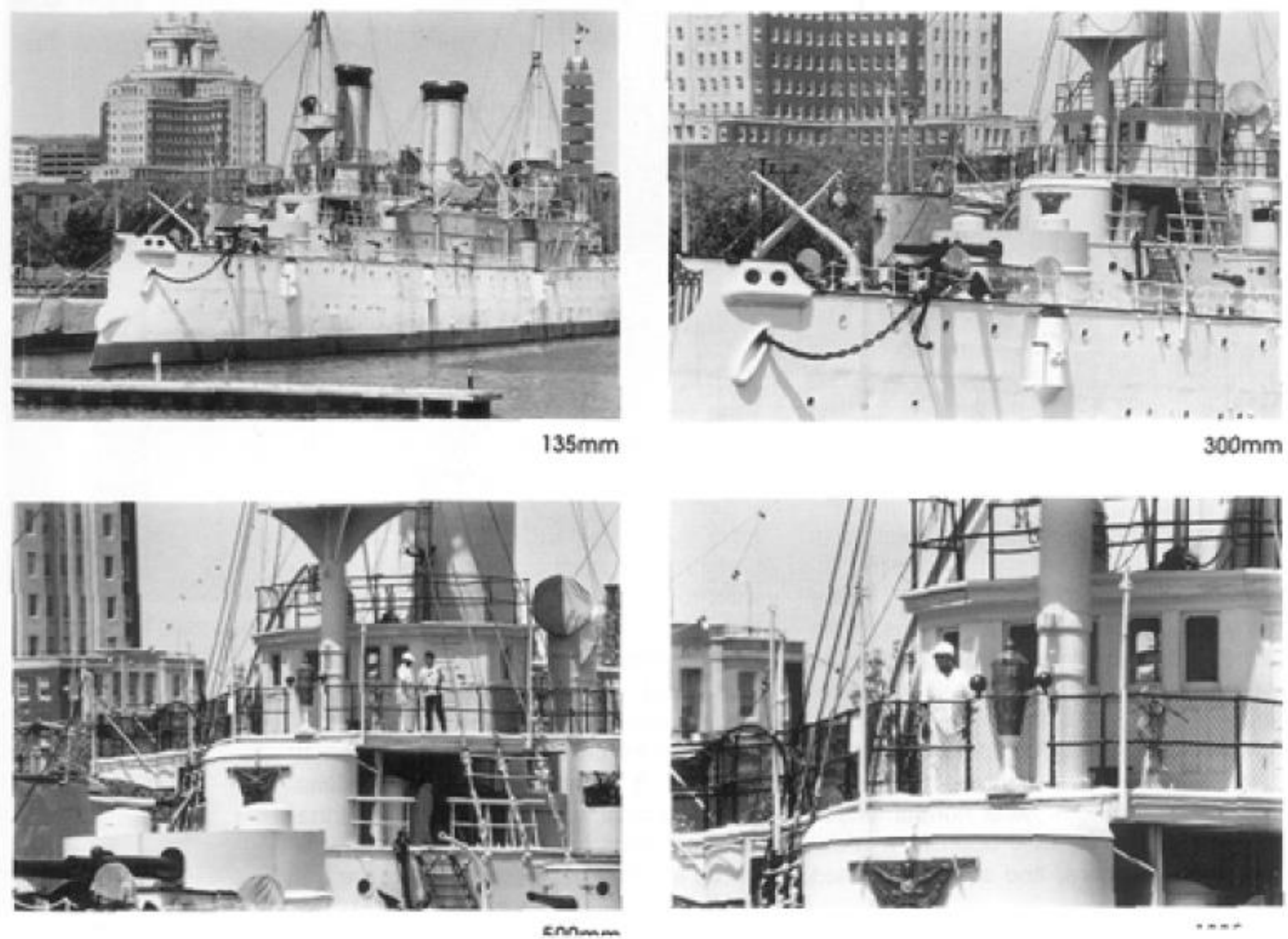
Field of View (Zoom)



From London and Upton

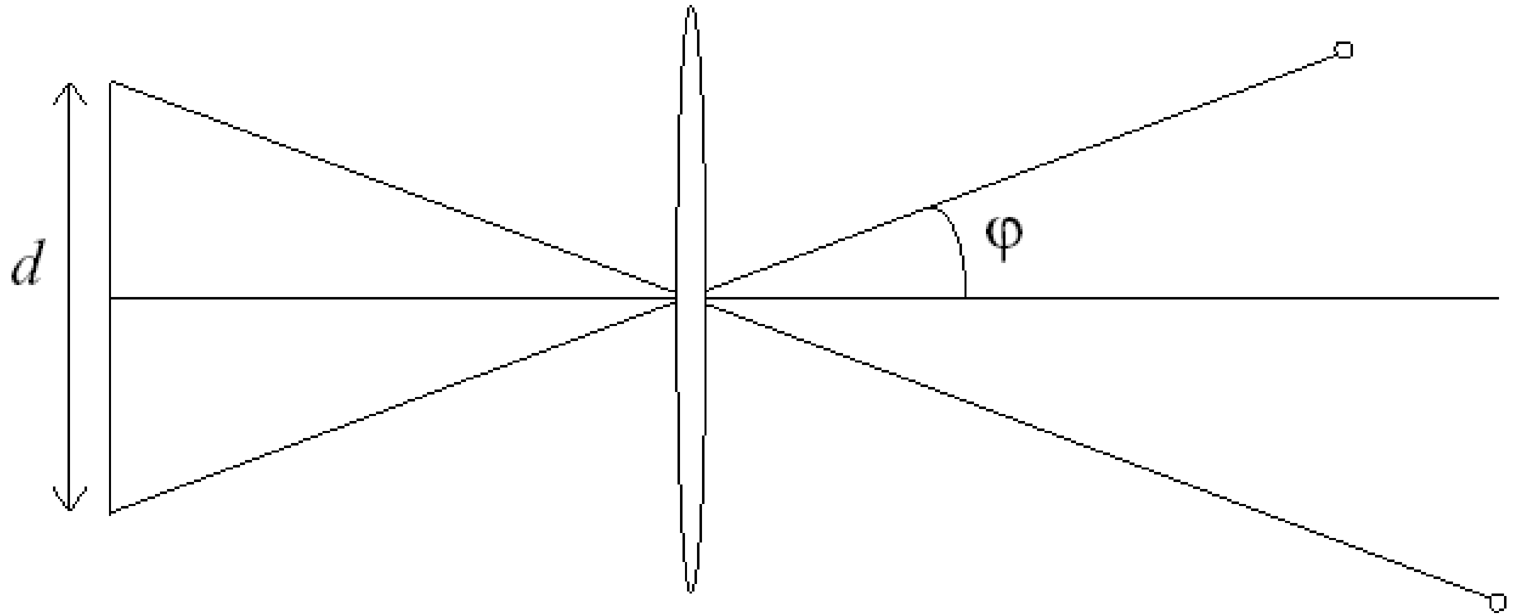
Field of View (Zoom)





From London and Upton

FOV depends of Focal Length



Size of field of view governed by size of the camera retina:

$$\varphi = \tan^{-1}\left(\frac{d}{2f}\right)$$

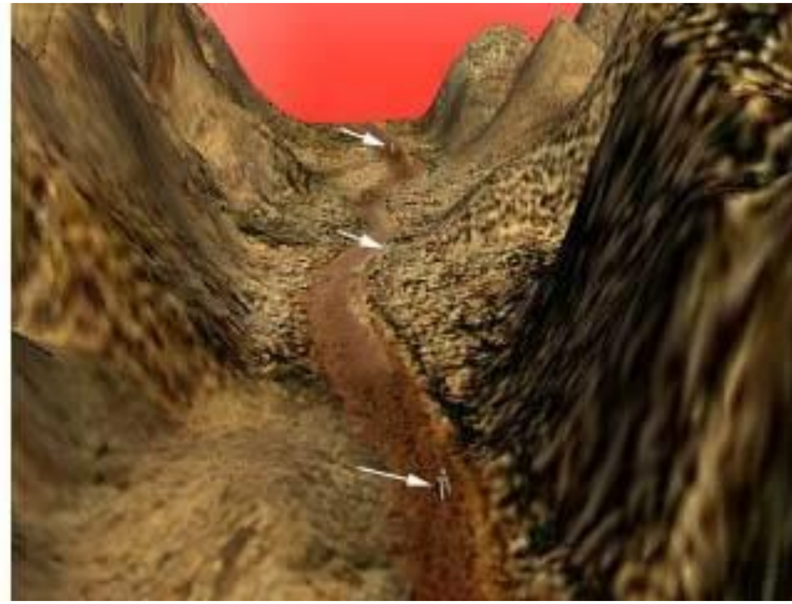
Smaller FOV = larger Focal Length

Field of View / Focal Length

Large FOV
Camera close to car

Small FOV
Camera far from the car

Fun with Focal Length (Jim Sherwood)



http://www.hash.com/users/jsherwood/tutes/focal/Zoomin.mov

Figure 5.1

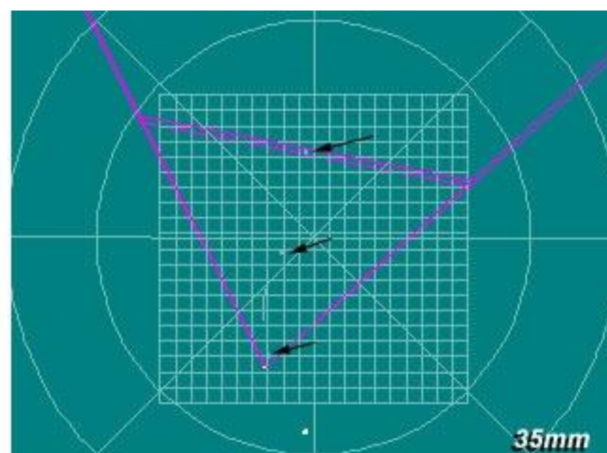
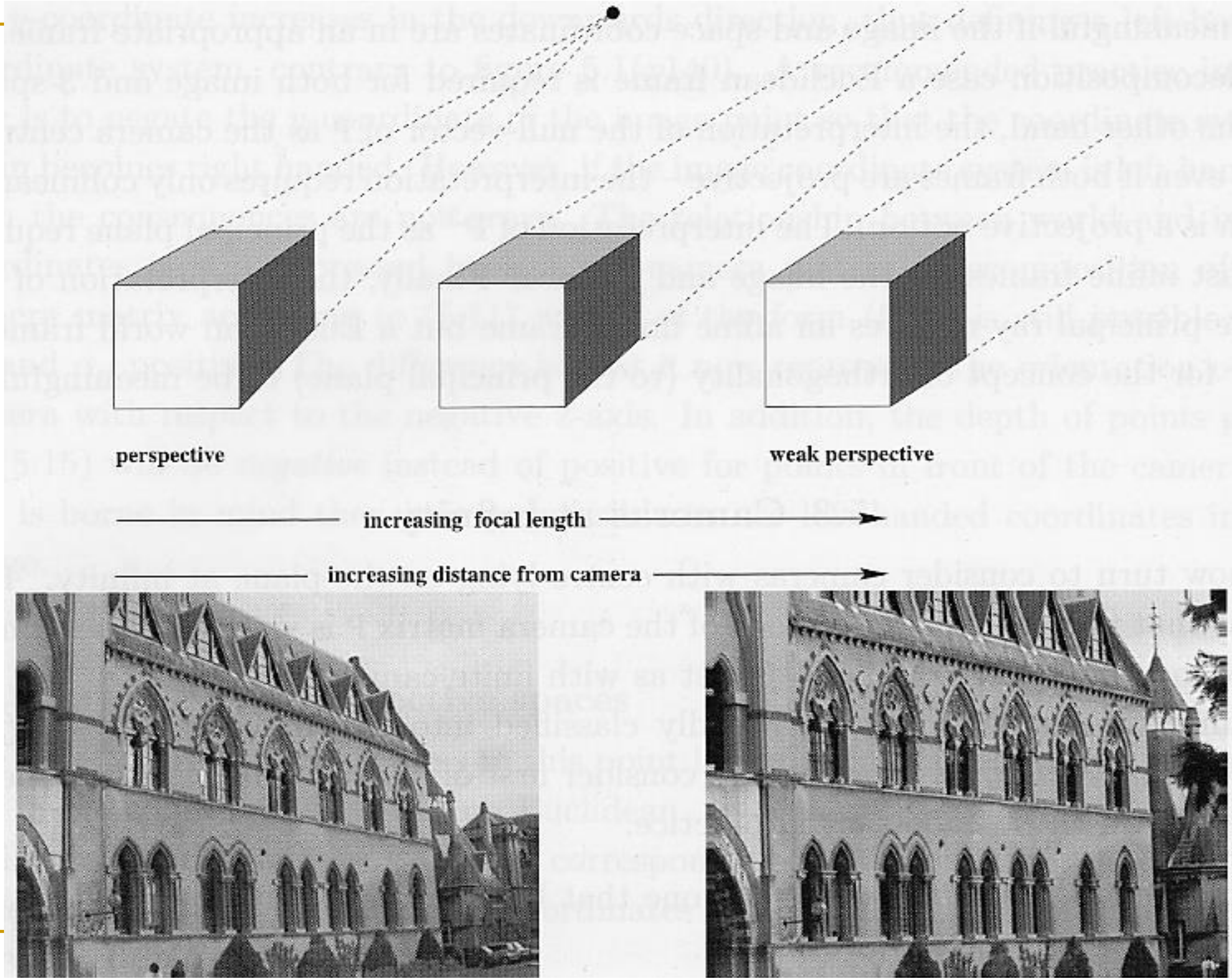


Figure 5.2

Large Focal Length compresses depth

400 mm 200 mm 100 mm 50 mm 28 mm 17 mm

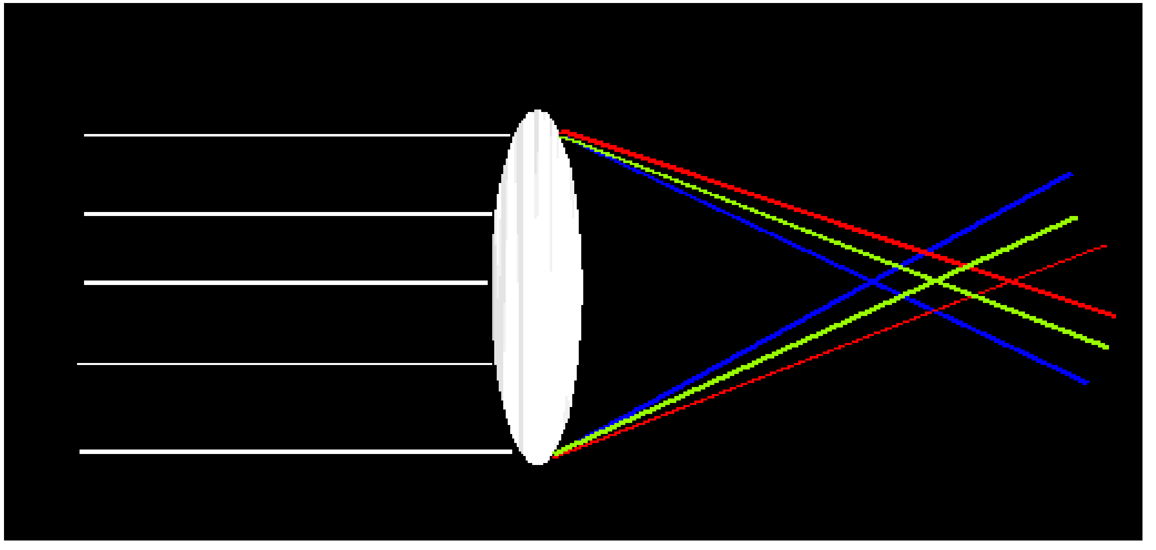


From Zisserman & Hartley

Lens Flaws

Lens Flaws: Chromatic Aberration

- Dispersion: wavelength-dependent refractive index
  - (enables prism to spread white light beam into rainbow)
- Modifies ray-bending and lens focal length: f(λ)



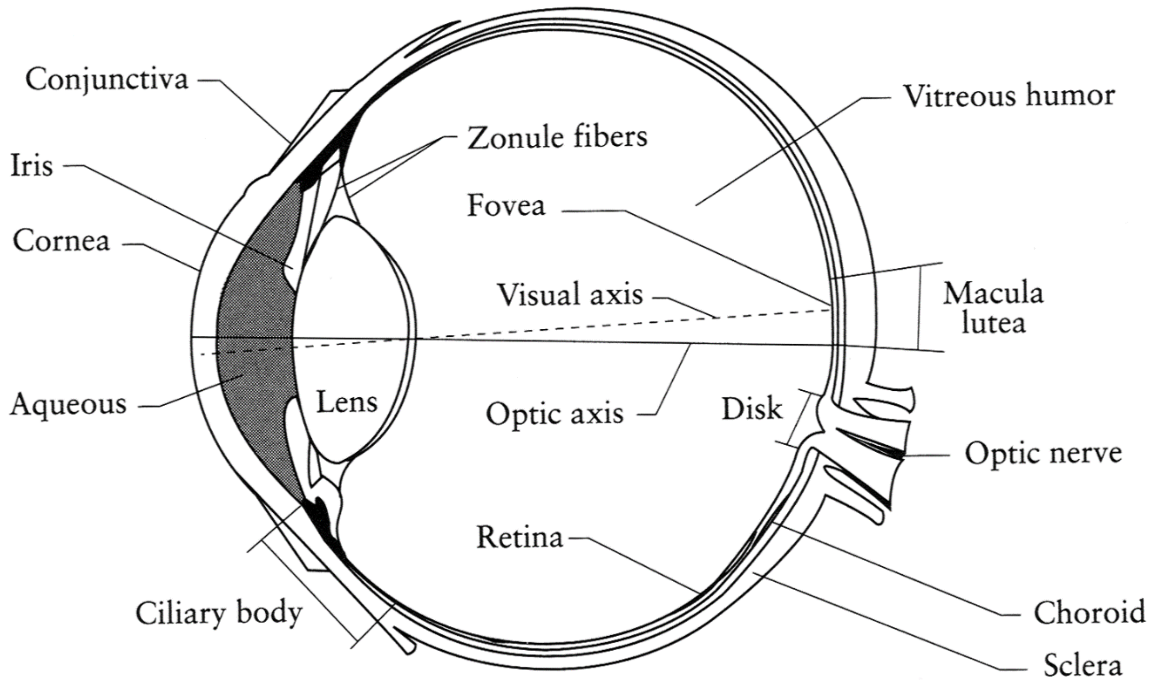
- color fringes near edges of image
- Corrections: add ‘doublet’ lens of flint glass, etc.

Chromatic Aberration

Near Lens Center

Near Lens Outer Edge

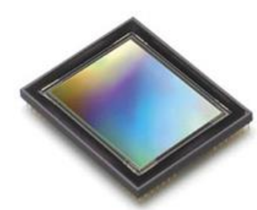
The eye

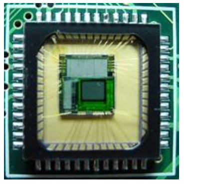


- The human eye is a camera
  - **Iris** - colored annulus with radial muscles
  - **Pupil** - the hole (aperture) whose size is controlled by the iris
  - What's the "film"?
- Photoreceptor cells (rods and cones) in the **retina**

Digital camera

- A digital camera replaces film with a sensor array
  - Each cell in the array is light-sensitive diode that converts photons to electrons
  - Two common types
    - Charge Coupled Device (CCD)
    - CMOS



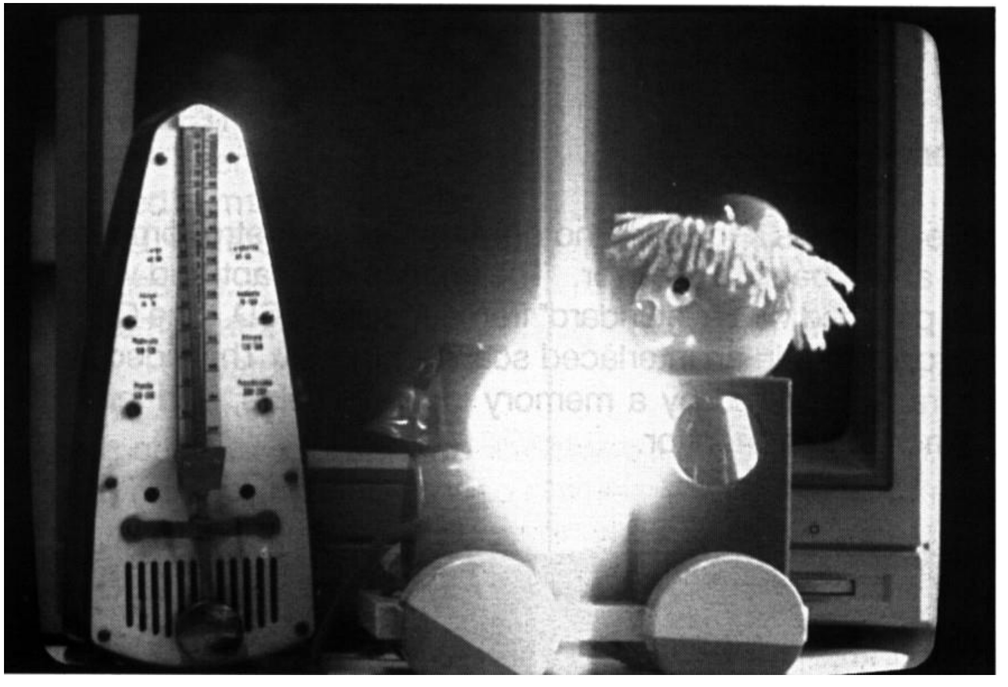


  - http://electronics.howstuffworks.com/digital-camera.htm

Digital camera issues

- Some things that affect digital cameras
  - blooming
  - color issues
  - interlace scanning

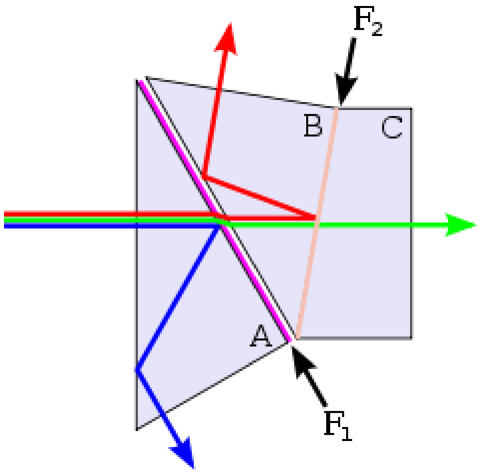
Blooming

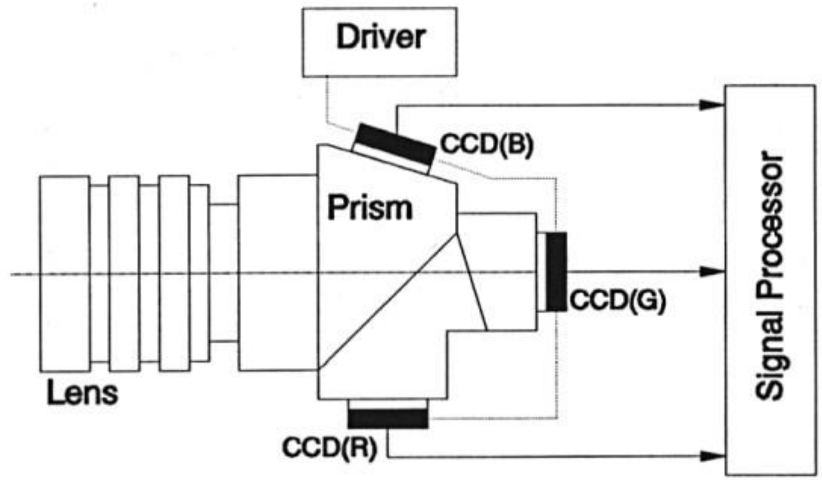


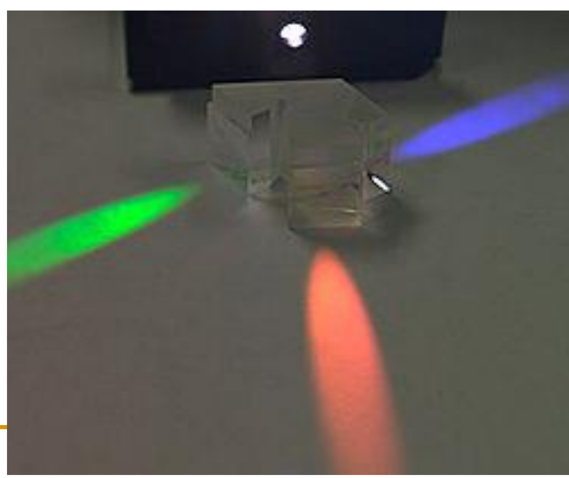
Theuseissen 1995

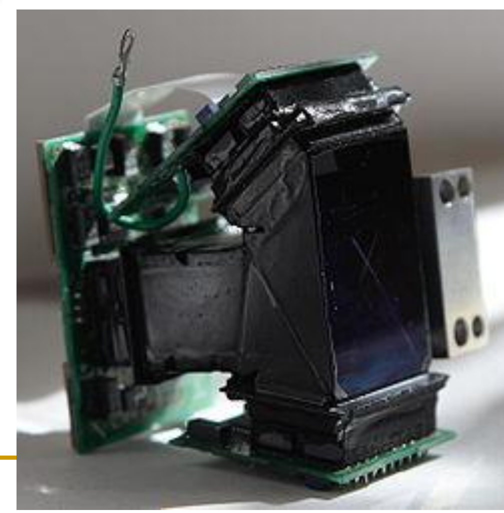
- Light is converted into an electrical charge.
- CCD limit to charge each pixel.
- If there is too much charge for one pixel, it will overflow to its neighboring pixel.

Handling Color: 3-chip cameras

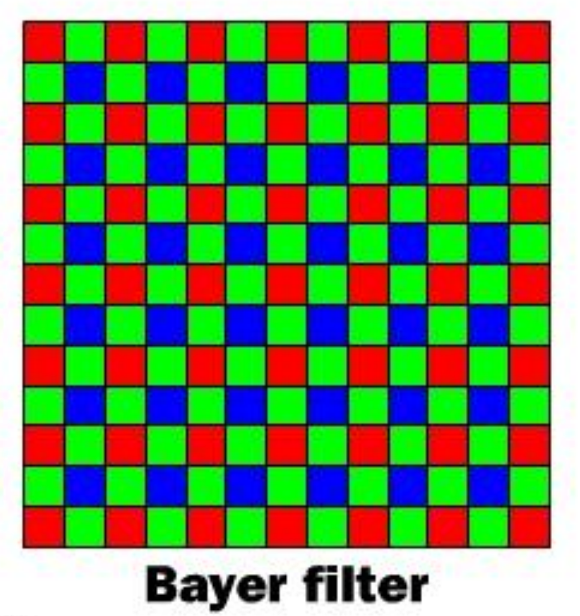


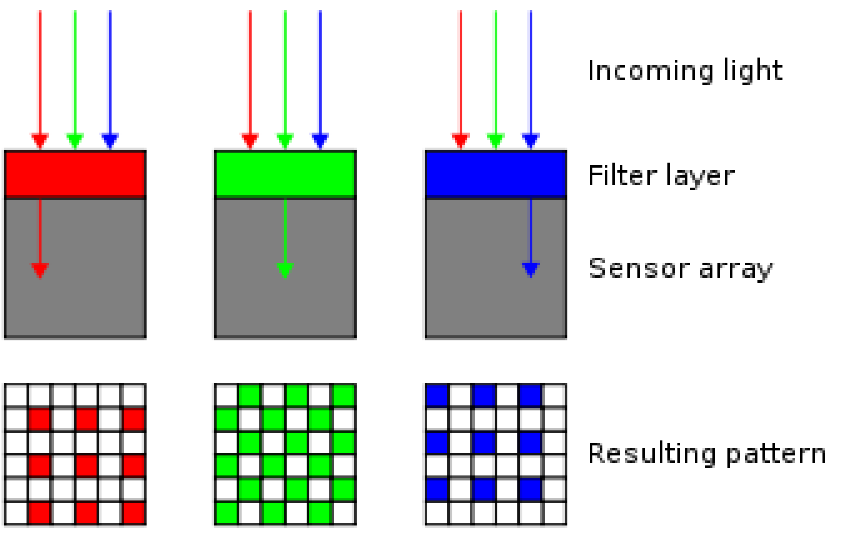


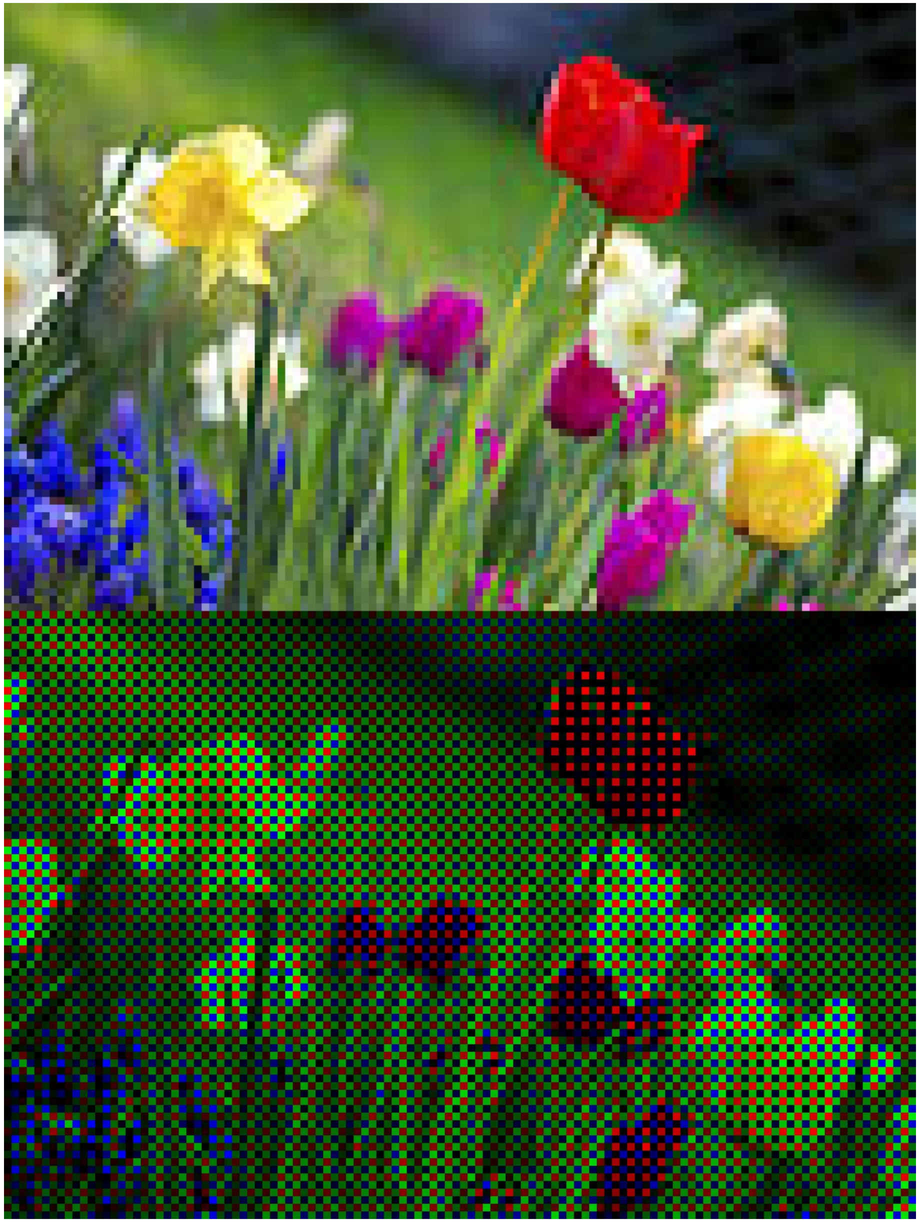




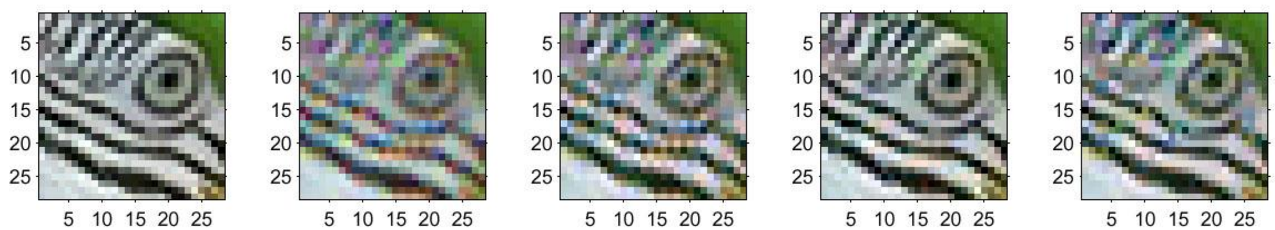
Handling Color: Mosaicing and Demosaicing





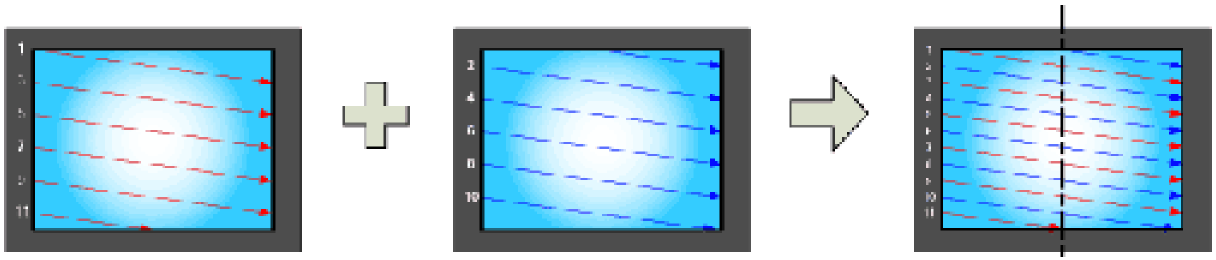


Issue in Mosaicing and Demosaicing



Input *Bilinear* *Cok* *Freeman* *LaRoche*

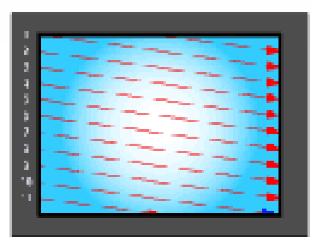
Interlace vs. progressive scan



1st field: Odd field

2nd field: Even field

One complete frame
using interlaced scanning



One complete frame
using progressive scanning

http://www.axis.com/products/video/camera/progressive_scan.htm

Progressive scan

http://www.axis.com/products/video/camera/progressive_scan.htm

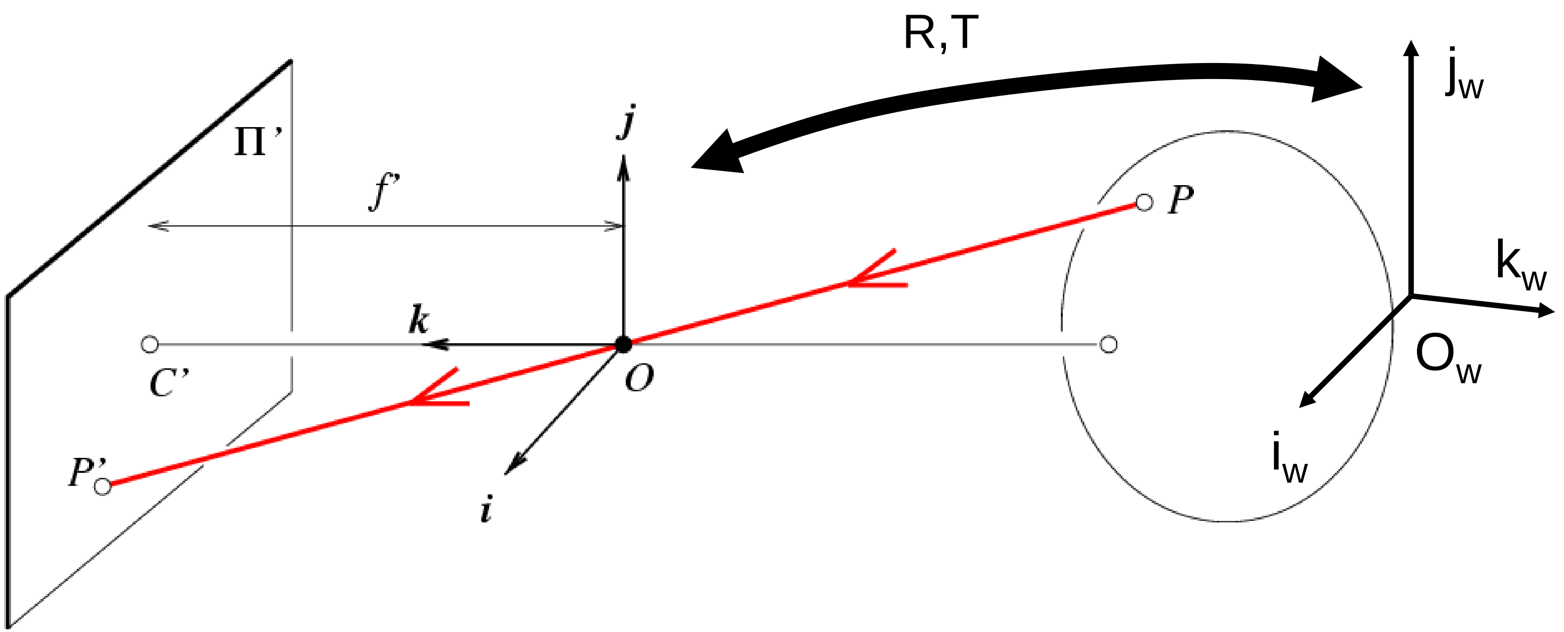
Interlace

http://www.axis.com/products/video/camera/progressive_scan.htm

Projections

Slide Credit: Saverese

Projection



$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

x: Image coordinates: (u,v,1)
K: Intrinsic matrix (3x3)
R: Rotation (3x3)
t: Translation (3x1)
X: World coordinates: (X,Y,Z,1)

Why does this matter?

Relating multiple views

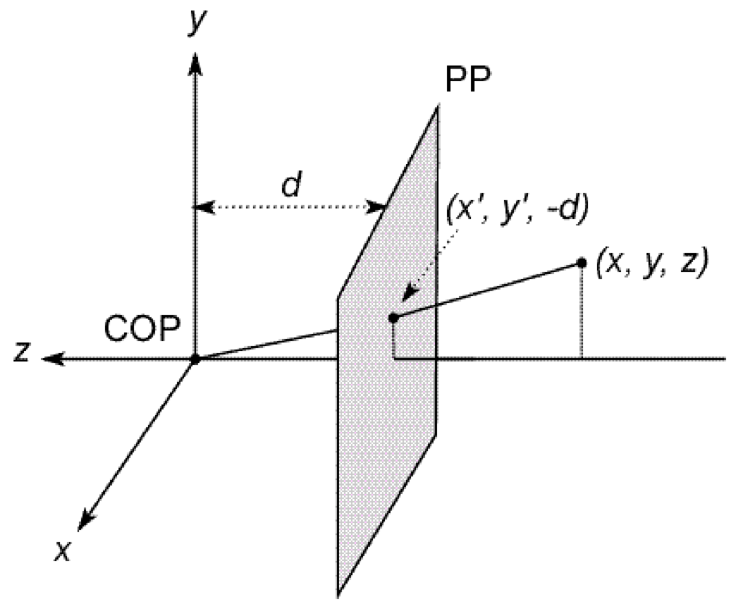
Object Recognition (CVPR 2006)

Inserting photographed objects into images (SIGGRAPH 2007)

Original

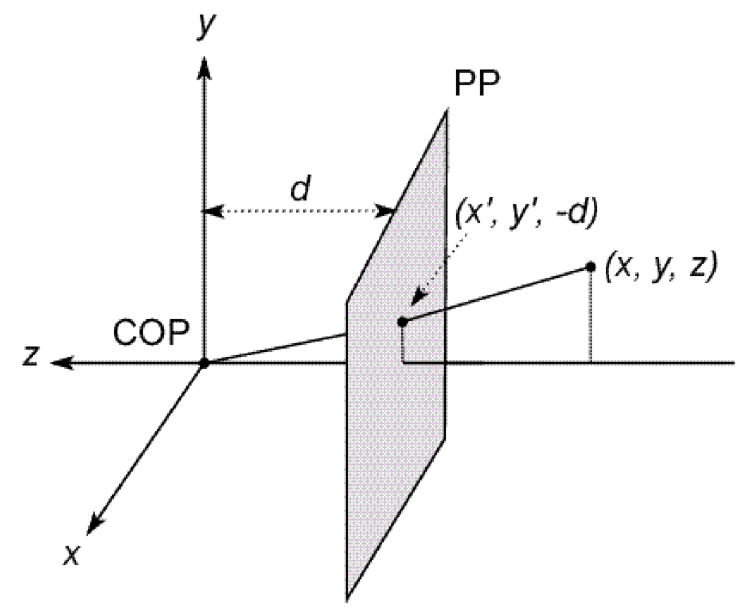
Created

Modeling projection



- The coordinate system
  - We will use the pin-hole model as an approximation
  - Put the optical center (**C**enter **O**f **P**rojection) at the origin
  - Put the image plane (**P**rojection **P**lane) *in front* of the COP
  - The camera looks down the *negative* z axis
    - we need this if we want right-handed-coordinates

Modeling projection



- Projection equations
  - Compute intersection with PP of ray from (x,y,z) to COP
  - Derived using similar triangles (on board)

$$(x, y, z) \rightarrow (-d\frac{x}{z},\ -d\frac{y}{z},\ -d)$$

  - We get the projection by throwing out the last coordinate:

$$(x, y, z) \rightarrow (-d\frac{x}{z},\ -d\frac{y}{z})$$

(Fundamental Equations)

Homogeneous coordinates (齐次坐标)

- Is this a linear transformation? $(x, y, z) \rightarrow (-d\frac{x}{z}, \; -d\frac{y}{z})$
  - no—division by z is nonlinear
- Trick: add one more coordinate:

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene coordinates

- Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

Basic geometry in homogeneous coordinates

- Line equation: ax + by + c = 0

$$line_i = \begin{bmatrix} a_i \\ b_i \\ c_i \end{bmatrix}$$

- Append 1 to pixel coordinate to get homogeneous coordinate

$$p_i = \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix}$$

- Line given by cross product of two points

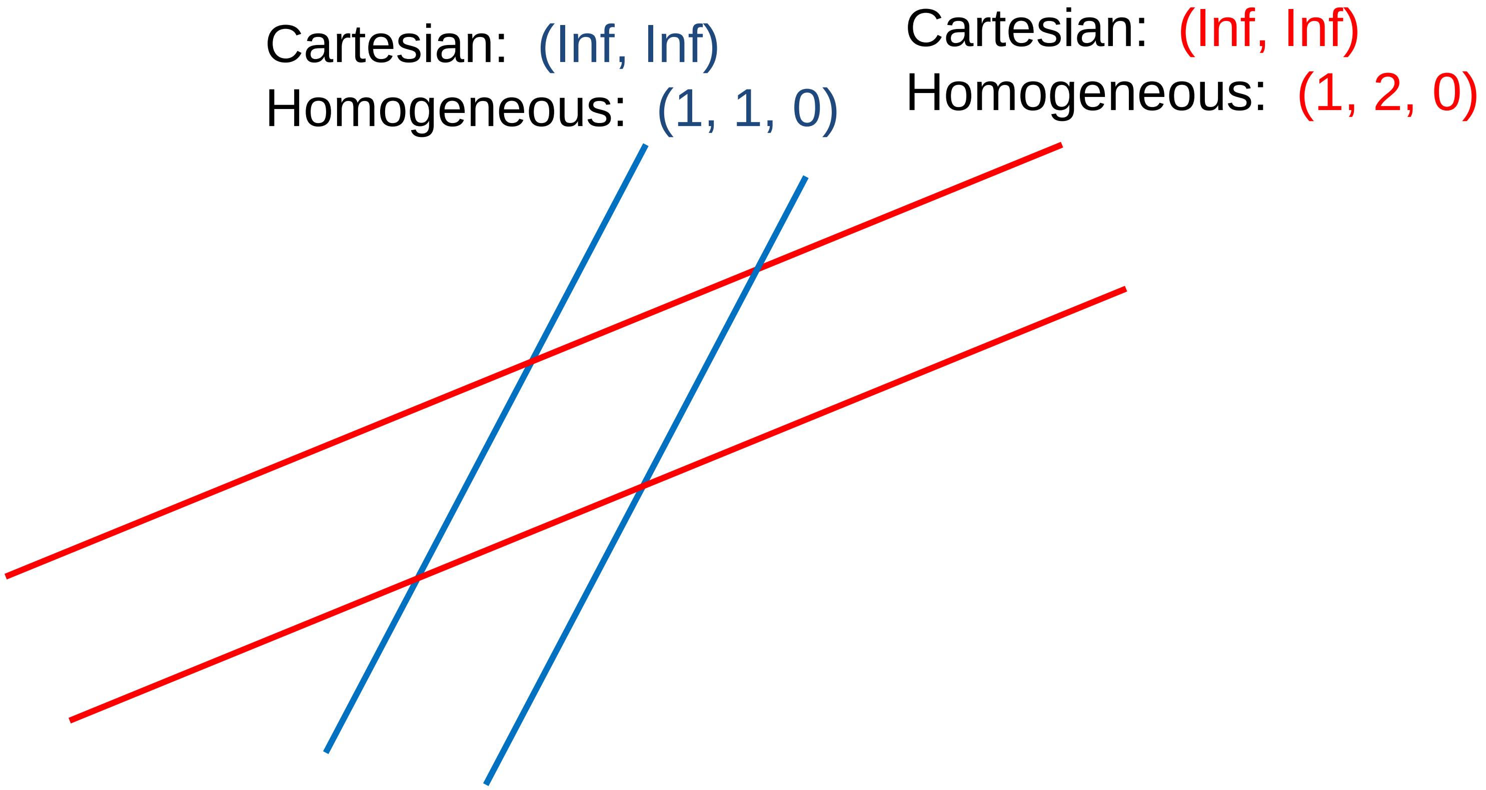
$$line_{ij} = p_i \times p_j$$

- Intersection of two lines given by cross product of the lines

$$q_{ij} = line_i \times line_j$$

Another problem solved by homogeneous coordinates

Intersection of parallel lines



Homogeneous coordinates

point	$\mathbf{p} = (X, Y, W)$
incidence	$\mathbf{p}^T \mathbf{u} = 0$
collinearity	$\lvert \mathbf{p}_1 \quad \mathbf{p}_2 \quad \mathbf{p}_3 \rvert = 0$
join of 2 points	$\mathbf{u} = \mathbf{p}_1 \times \mathbf{p}_2$
ideal points	$(X, Y, 0)$

(a)

line	$\mathbf{u} = (a, b, c)$
incidence	$\mathbf{p}^T \mathbf{u} = 0$
concurrence	$\lvert \mathbf{u}_1 \quad \mathbf{u}_2 \quad \mathbf{u}_3 \rvert = 0$
intersection of 2 lines	$\mathbf{p} = \mathbf{u}_1 \times \mathbf{u}_2$
ideal line	$(0, 0, c)$

(b)

Perspective Projection (透视投影)

- Projection is a matrix multiply using homogeneous coordinates:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ -z/d \end{bmatrix} \Rightarrow (-d\frac{x}{z}, \quad -d\frac{y}{z})$$

divide by third coordinate

- This is known as **perspective projection**
  - The matrix is the **projection matrix**
  - Can also formulate as a 4x4 or a 3x4 matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ -z/d \end{bmatrix} \Rightarrow (-d\frac{x}{z}, \quad -d\frac{y}{z})$$

divide by the fourth coordinate

Perspective Projection

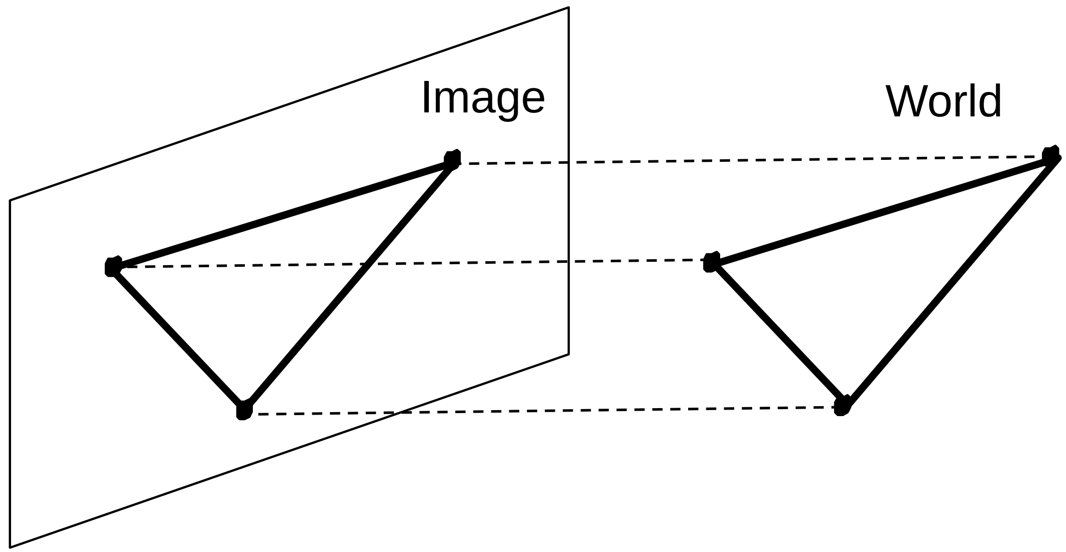
- How does scaling the projection matrix change the transformation?

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ -z/d \end{bmatrix} \Rightarrow (-d\frac{x}{z}, \quad -d\frac{y}{z})$$

$$\begin{bmatrix} -d & 0 & 0 & 0 \\ 0 & -d & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} -dx \\ -dy \\ z \end{bmatrix} \Rightarrow (-d\frac{x}{z}, \quad -d\frac{y}{z})$$

Orthographic projection 正交投影

- Special case of perspective projection
  - Distance *d* from the COP to the PP is infinite
  - *z = -d-k => -d/z = 1+k/z = 1*



  - Also called "parallel projection": (x, y, z) → (x, y)
  - What's the projection matrix?

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow (x, y)$$

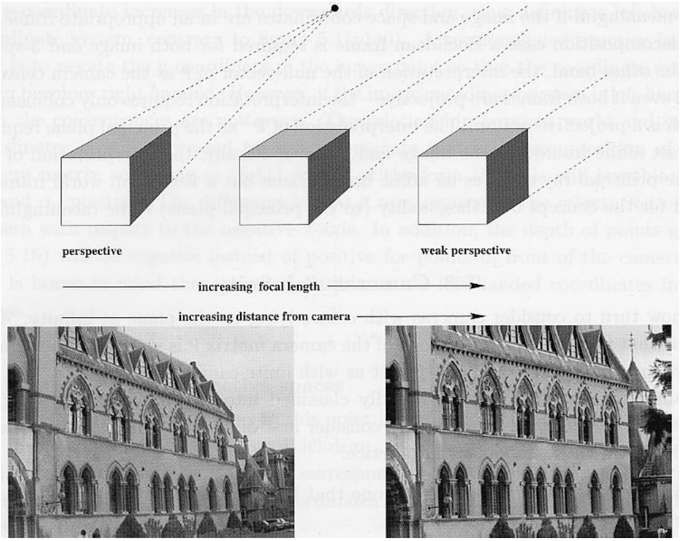
Other types of projection

- Scaled orthographic $\quad when\,\Delta z << z \quad (\Delta z < \bar{z}/20)$
  - Also called "weak perspective"

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1/d \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1/d \end{bmatrix} \Rightarrow (dx, dy)$$

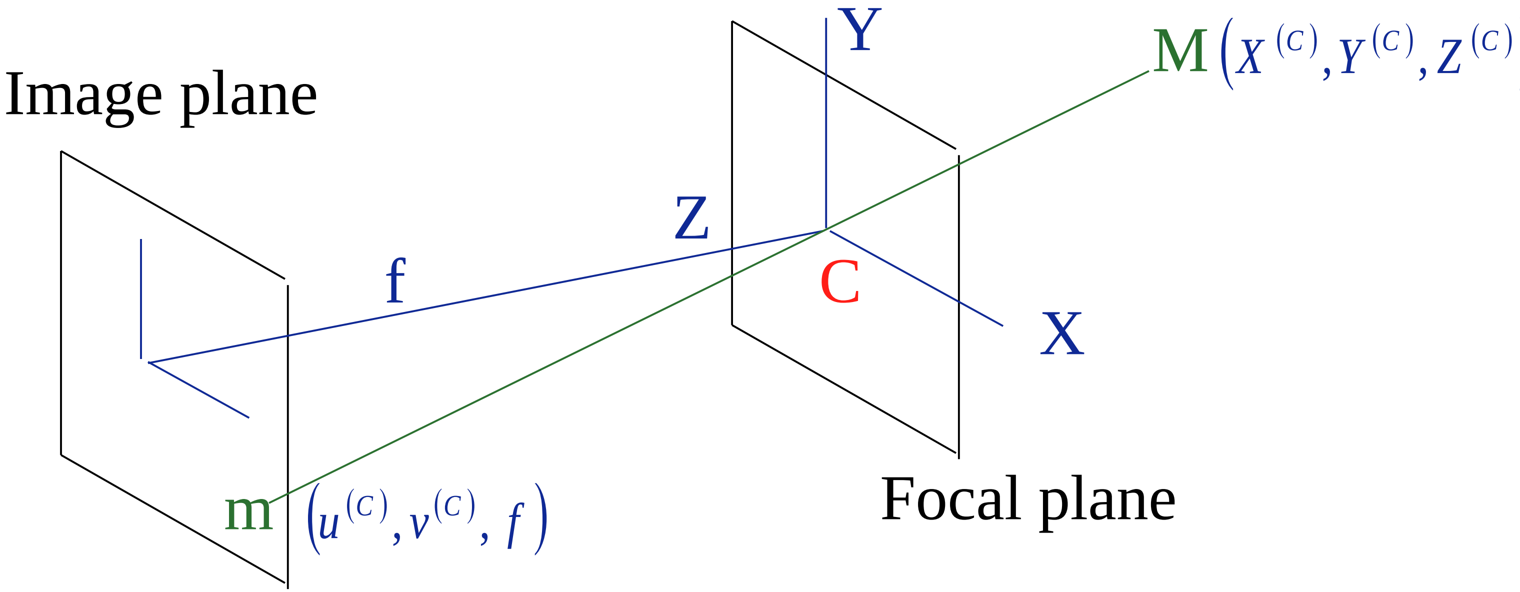
- **Affine projection**
  - Also called "paraperspective"

$$\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



Camera Parameters

Camera Parameters



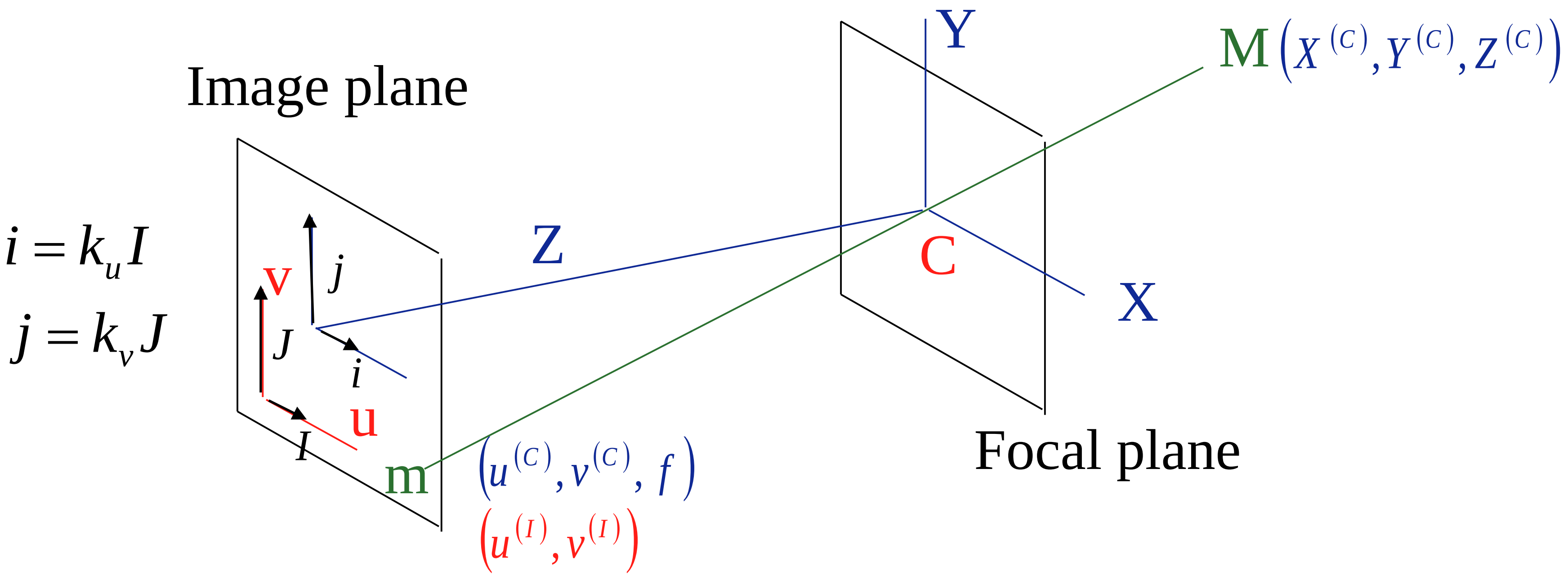
From Perspective Projection

$$u^{(C)} = -f\,\frac{X^{(C)}}{Z^{(C)}} = \frac{U}{S}$$

$$v^{(C)} = -f\,\frac{Y^{(C)}}{Z^{(C)}} = \frac{V}{S}$$

$$\Rightarrow \begin{bmatrix} U \\ V \\ S \end{bmatrix} = \begin{bmatrix} -f & 0 & 0 & 0 \\ 0 & -f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X^{(C)} \\ Y^{(C)} \\ Z^{(C)} \\ 1 \end{bmatrix}$$

Camera Parameters



$$\begin{bmatrix} u^{(I)} \\ v^{(I)} \\ 1 \end{bmatrix} = \begin{bmatrix} k_u & 0 & 0 \\ 0 & k_v & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u^{(c)} \\ v^{(c)} \\ 1 \end{bmatrix} + \begin{bmatrix} u_0 \\ v_0 \\ 0 \end{bmatrix} = \begin{bmatrix} k_u & 0 & u_0 \\ 0 & k_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u^{(c)} \\ v^{(c)} \\ 1 \end{bmatrix}$$

Camera Parametersc

$$\begin{bmatrix} u^{(I)} \\ v^{(I)} \\ 1 \end{bmatrix} = \begin{bmatrix} k_u & 0 & u_0 \\ 0 & k_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u^{(c)} \\ v^{(c)} \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} U^{(new)} \\ V^{(new)} \\ S \end{bmatrix} = \begin{bmatrix} k_u & 0 & u_0 \\ 0 & k_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} U \\ V \\ S \end{bmatrix}$$

Equation 1

$$u^{(I)} = \frac{U^{(new)}}{S} \qquad u^{(C)} = \frac{U}{S}$$

$$v^{(I)} = \frac{V^{(new)}}{S} \qquad v^{(C)} = \frac{V}{S}$$

Camera Parameters

Equation 2: $$\begin{bmatrix} U \\ V \\ S \end{bmatrix} = \begin{bmatrix} -f & 0 & 0 & 0 \\ 0 & -f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X^{(C)} \\ Y^{(C)} \\ Z^{(C)} \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} U^{(new)} \\ V^{(new)} \\ S \end{bmatrix} = \begin{bmatrix} k_u & 0 & u_0 \\ 0 & k_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} U \\ V \\ S \end{bmatrix} = \begin{bmatrix} k_u & 0 & u_0 \\ 0 & k_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -f & 0 & 0 & 0 \\ 0 & -f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X^{(C)} \\ Y^{(C)} \\ Z^{(C)} \\ 1 \end{bmatrix}$$

Camera Parameters

$$\begin{bmatrix} U^{(new)} \\ V^{(new)} \\ S \end{bmatrix} = \begin{bmatrix} -fk_u & 0 & u_0 & 0 \\ 0 & -fk_v & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X^{(C)} \\ Y^{(C)} \\ Z^{(C)} \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} U^{(new)} \\ V^{(new)} \\ S \end{bmatrix} = \begin{bmatrix} -f_u & 0 & u_0 & 0 \\ 0 & -f_v & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X^{(C)} \\ Y^{(C)} \\ Z^{(C)} \\ 1 \end{bmatrix}$$

Equation 3

$$f_u = fk_u$$
$$f_v = fk_v$$

Camera Parameters

Intrinsic Parameters (Do not depend on camera position):

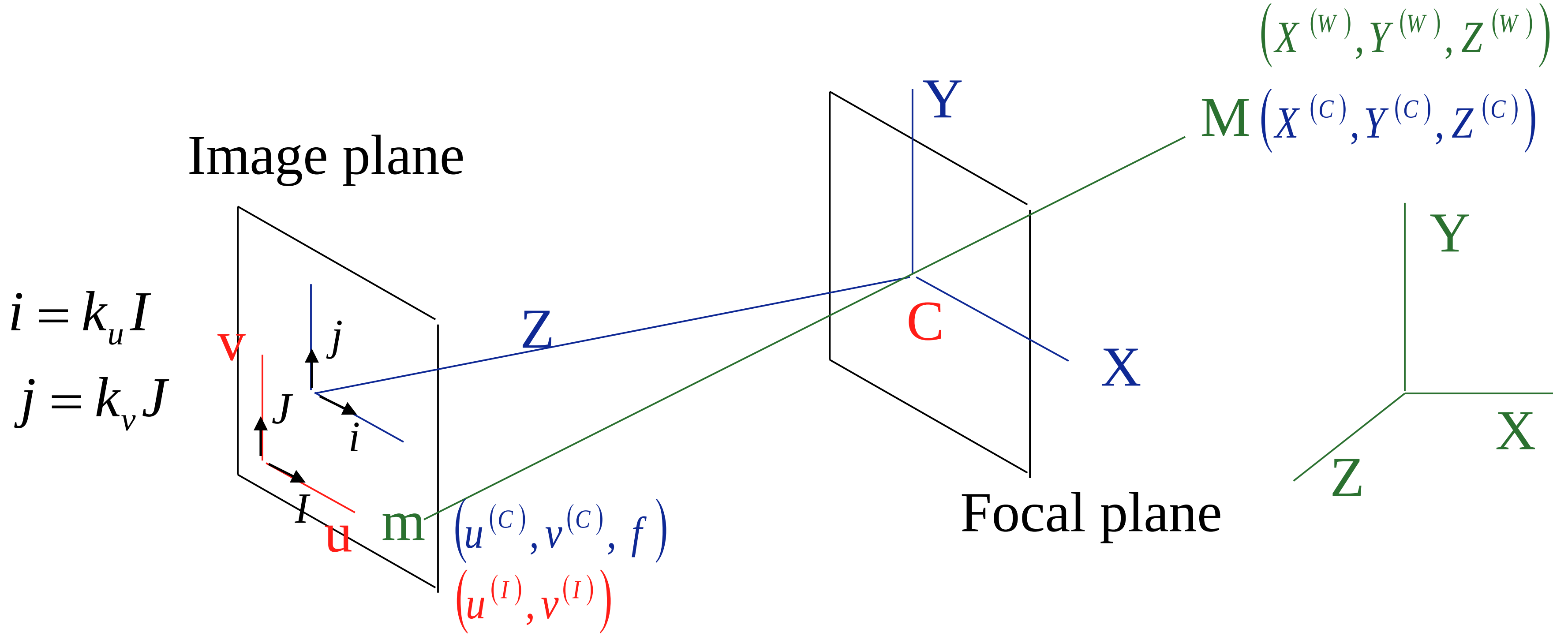
1. $f_u = fk_u$
2. $f_v = fk_v$
3. u_0
4. v_0

Intrinsic Parameters

$$\begin{bmatrix} U^{(new)} \\ V^{(new)} \\ S \end{bmatrix} = \begin{bmatrix} -f_u & 0 & u_0 & 0 \\ 0 & -f_v & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X^{(C)} \\ Y^{(C)} \\ Z^{(C)} \\ 1 \end{bmatrix}$$

$$m^{(I)} = PM^{(C)} = \begin{bmatrix} Q_1^T \\ Q_2^T \\ Q_3^T \end{bmatrix} M^{(C)} = \begin{bmatrix} Q_1^T M^{(C)} \\ Q_2^T M^{(C)} \\ Q_3^T M^{(C)} \end{bmatrix}$$

Extrinsic Parameters



By Rigid Body Transformation:

$$\begin{bmatrix} X^{(C)} \\ Y^{(C)} \\ Z^{(C)} \\ 1 \end{bmatrix} = \begin{bmatrix} R_{3\times 3} & T_{3\times 1} \\ 0_{1\times 3} & 1 \end{bmatrix} \begin{bmatrix} X^{(W)} \\ Y^{(W)} \\ Z^{(W)} \\ 1 \end{bmatrix} \Rightarrow M^{(C)} = DM^{(W)}$$

Camera Model

$$m^{(I)} = PM^{(C)}, M^{(C)} = DM^{(W)} \Rightarrow m^{(I)} = PDM^{(W)}$$

$$\begin{bmatrix} U^{(new)} \\ V^{(new)} \\ S \end{bmatrix} = \begin{bmatrix} -f_u & 0 & u_0 & 0 \\ 0 & -f_v & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} R_{3\times 3} & T_{3\times 1} \\ 0_{1\times 3} & 1 \end{bmatrix} \begin{bmatrix} X^{(W)} \\ Y^{(W)} \\ Z^{(W)} \\ 1 \end{bmatrix}$$

$$\text{Let } R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \text{ and } T = \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix}$$

Camera Model

$$u^{(I)} - u_0 = -f_u \frac{r_{11}X^{(W)} + r_{12}Y^{(W)} + r_{13}Z^{(W)} + T_X}{r_{31}X^{(W)} + r_{32}Y^{(W)} + r_{33}Z^{(W)} + T_Z}$$

$$v^{(I)} - v_0 = -f_v \frac{r_{21}X^{(W)} + r_{22}Y^{(W)} + r_{23}Z^{(W)} + T_Y}{r_{31}X^{(W)} + r_{32}Y^{(W)} + r_{33}Z^{(W)} + T_Z}$$

$$u^{(I)} = \frac{U^{(new)}}{S}$$

$$v^{(I)} = \frac{V^{(new)}}{S}$$

Suggested Reading

- Chapter 3, Olivier Faugeras, "Three Dimensional Computer Vision", MIT Press, 1993
- Chapter 2, David A. Forsyth and Jean Ponce, "Computer Vision: A Modern Approach”