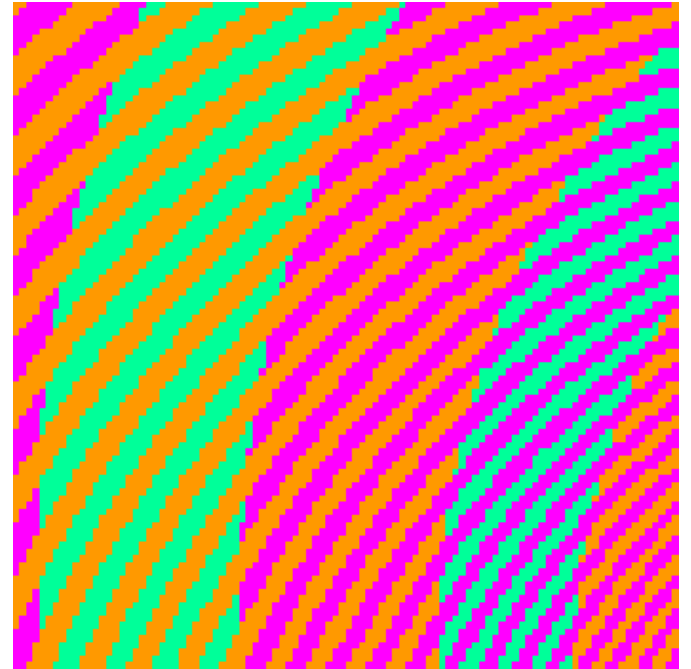
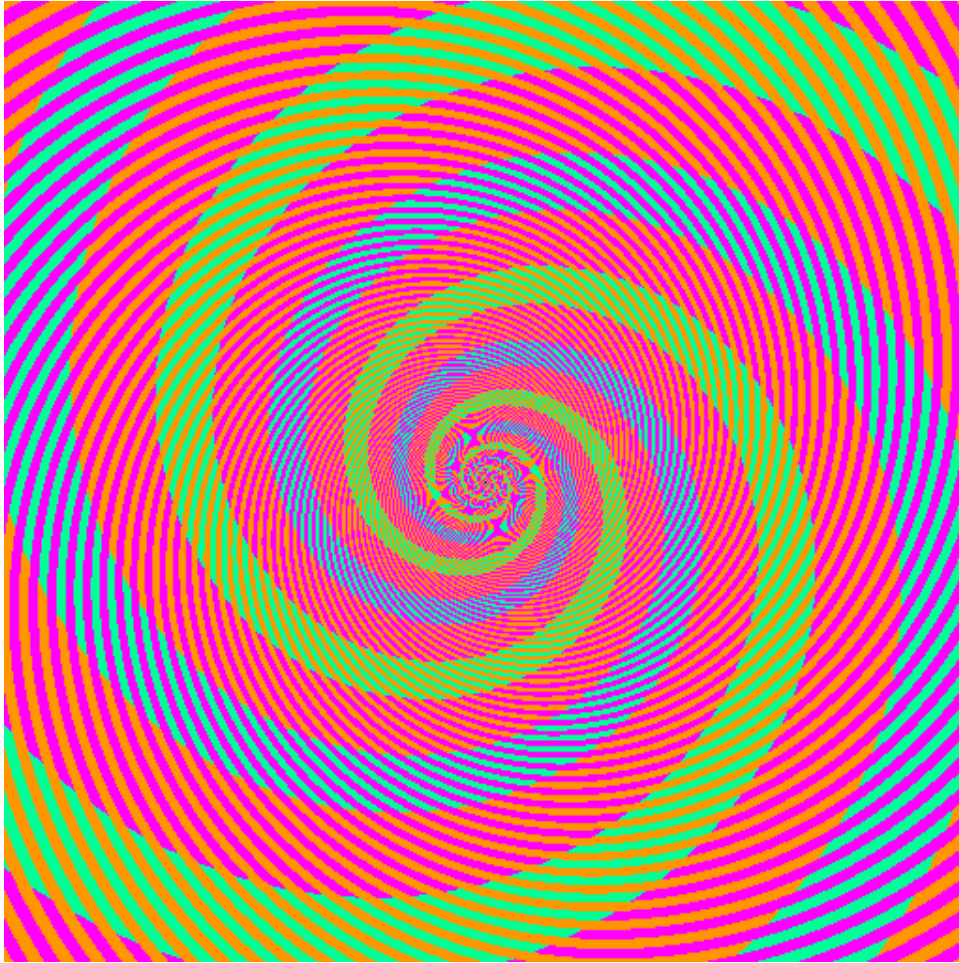

Edge Detection

边缘检测

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<http://blogs.discovermagazine.com/badastronomy/2009/06/24/the-blue-and-the-green/>

Outline

- 边缘及边缘检测概念
- 用模板实现卷积
- 基于一阶导数的边缘检测
- 基于二阶导数的边缘检测
 - Laplacian算子
 - LoG算子（Marr&Hildreth算子）
- **Canny**边缘检测

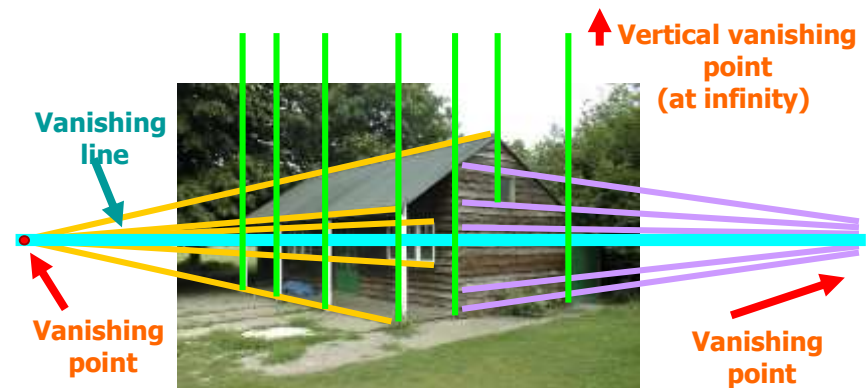
Edge detection

- **Goal:** Identify sudden changes (discontinuities) in an image
 - Intuitively, most **semantic** and **shape** information from the image can be encoded in the edges
 - More compact than pixels
- **Ideal:** artist's line drawing (but artist is also using object-level knowledge)

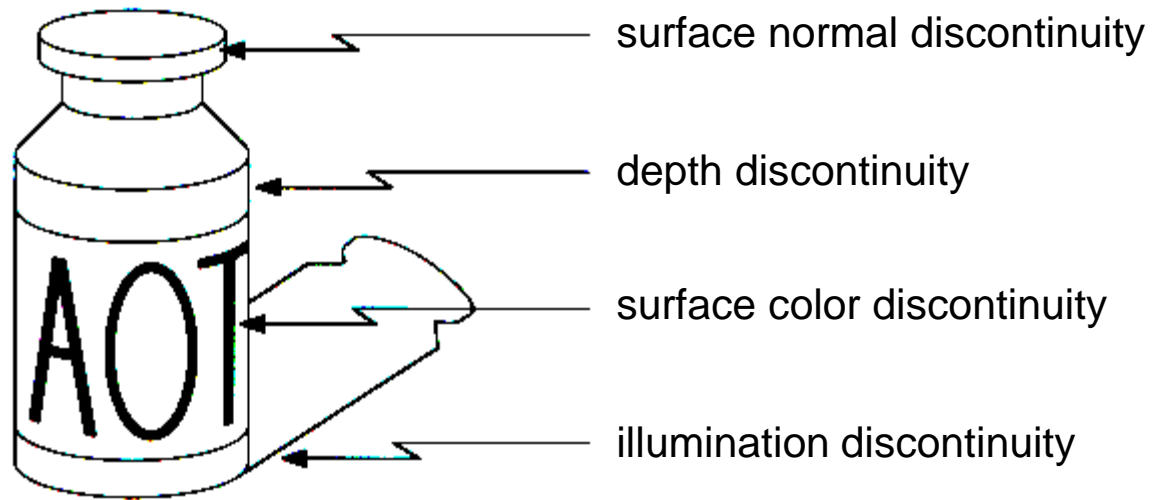


Why do we care about edges?

- Extract information, recognize objects
- Recover geometry and viewpoint



Origin of Edges

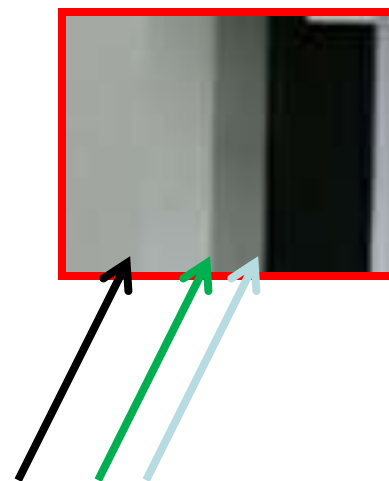


- Edges are caused by a variety of factors

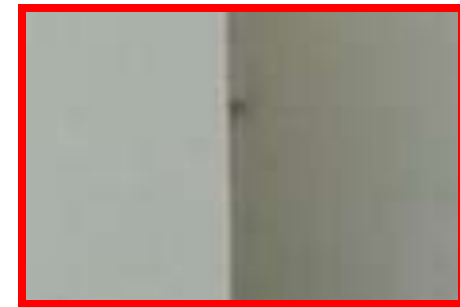
Closeup of edges



Closeup of edges



Closeup of edges

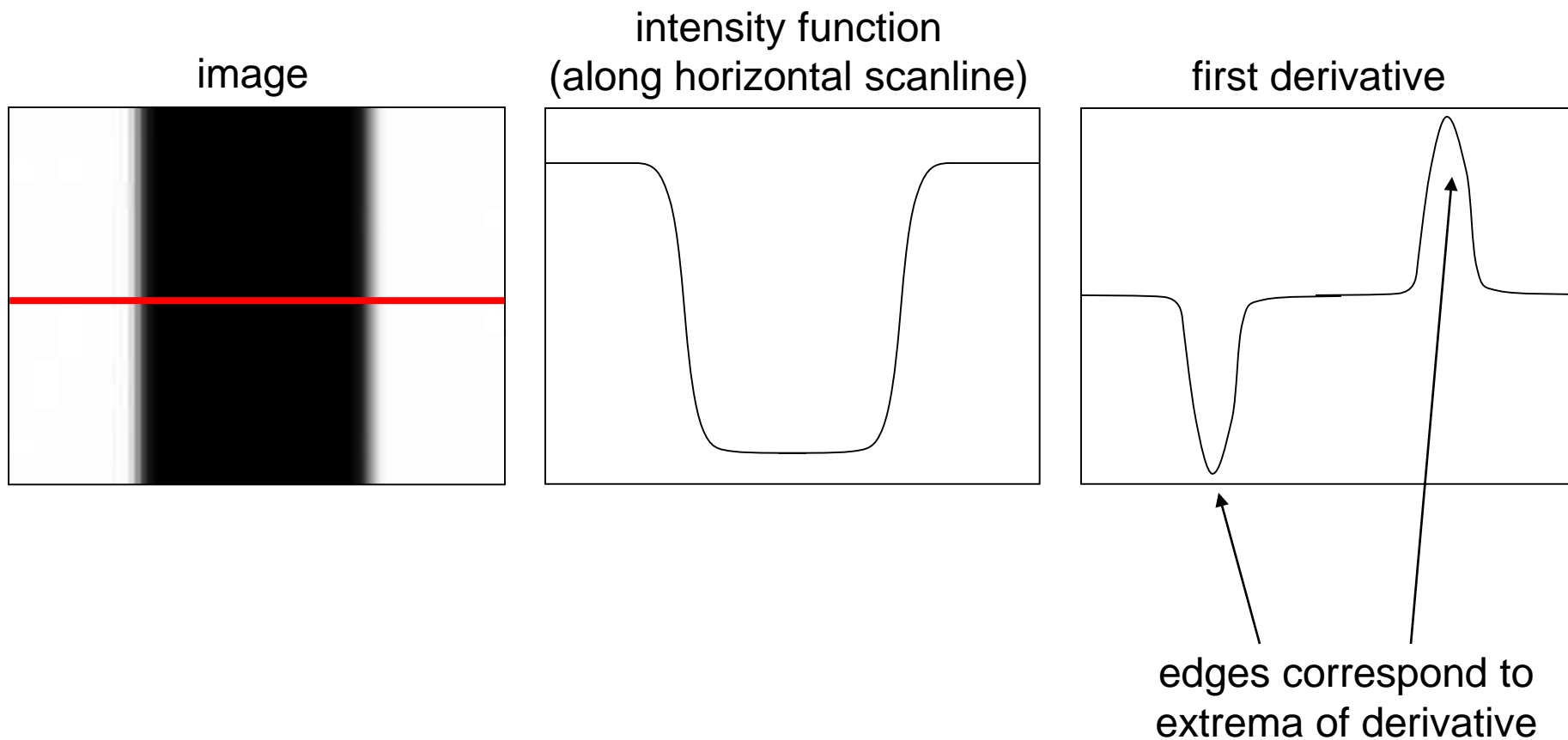


Closeup of edges



Characterizing edges

- An edge is a place of rapid change in the image intensity function



边缘检测

- **Goal:**

Identify sudden changes (discontinuities) in an image

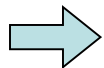
- **Solution**

Zero-crossing of second-order derivative

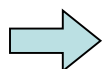
Local maxima of first-order derivative

示意图

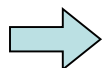
理论曲线



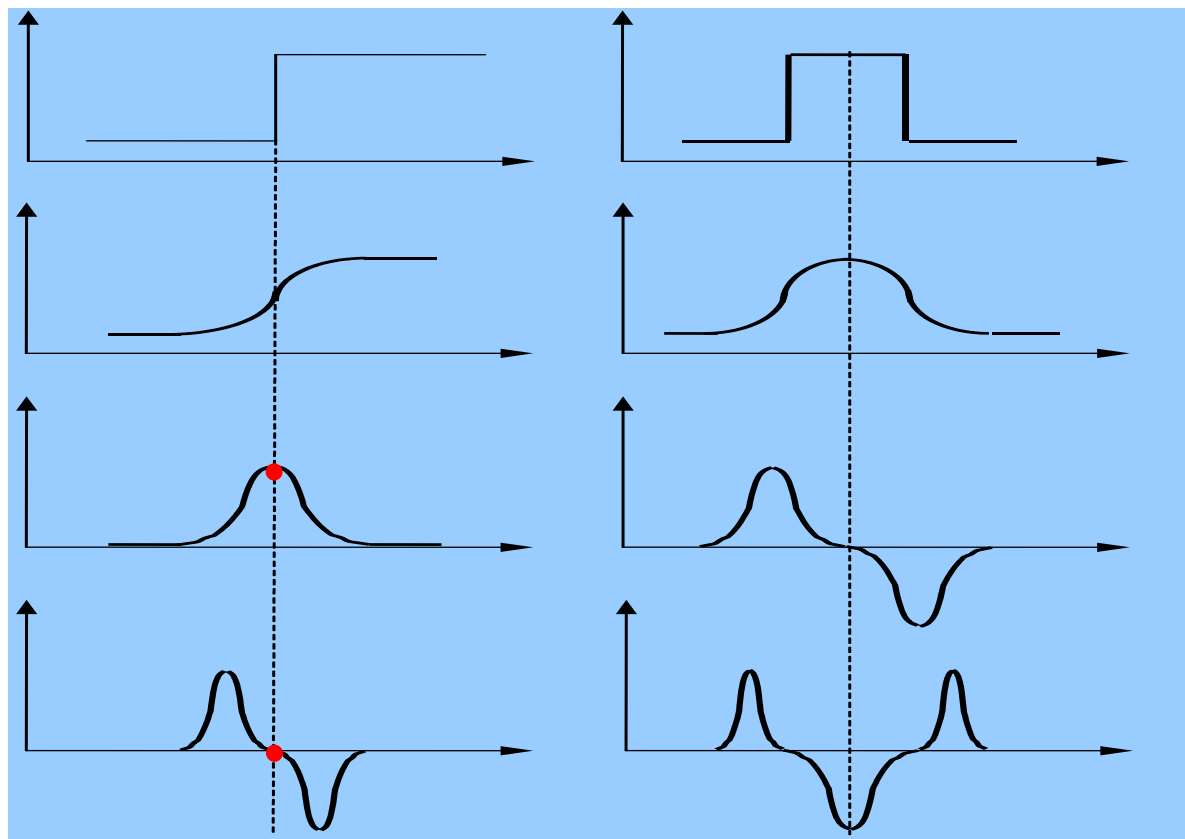
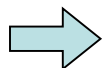
实际曲线



一阶导数



二阶导数



(a)阶跃函数

(b)线条函数

对噪音的敏感性比较

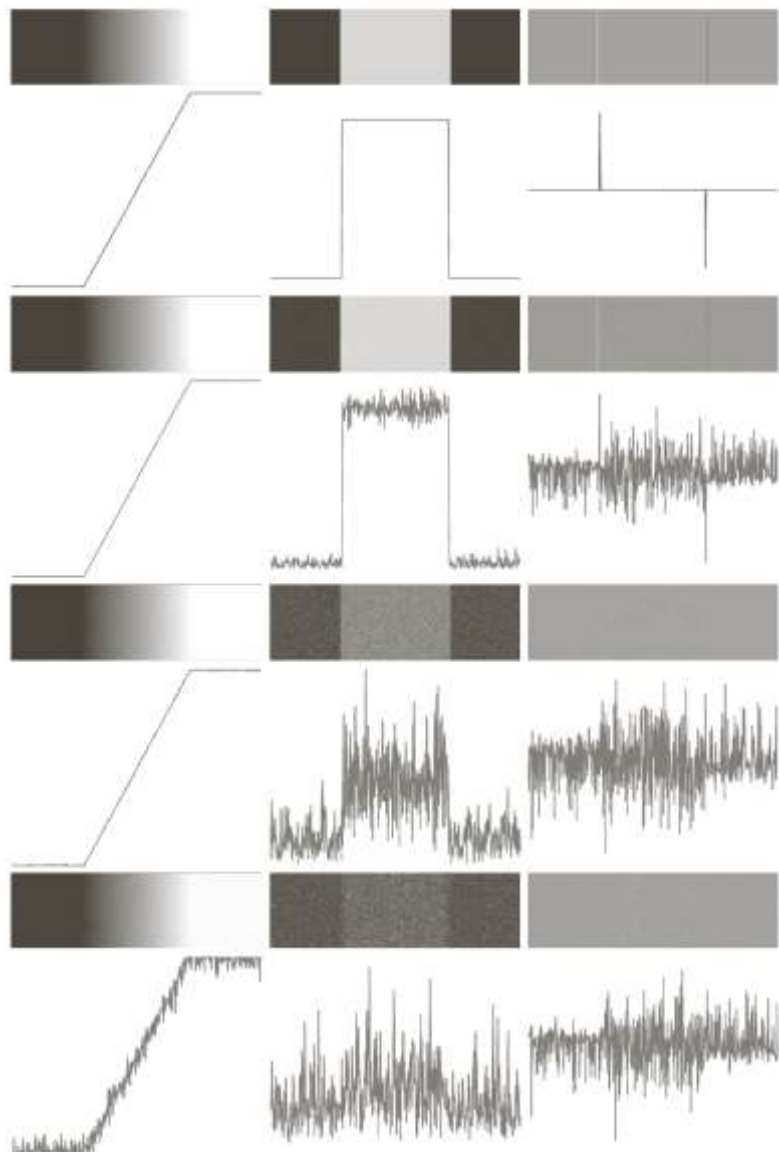
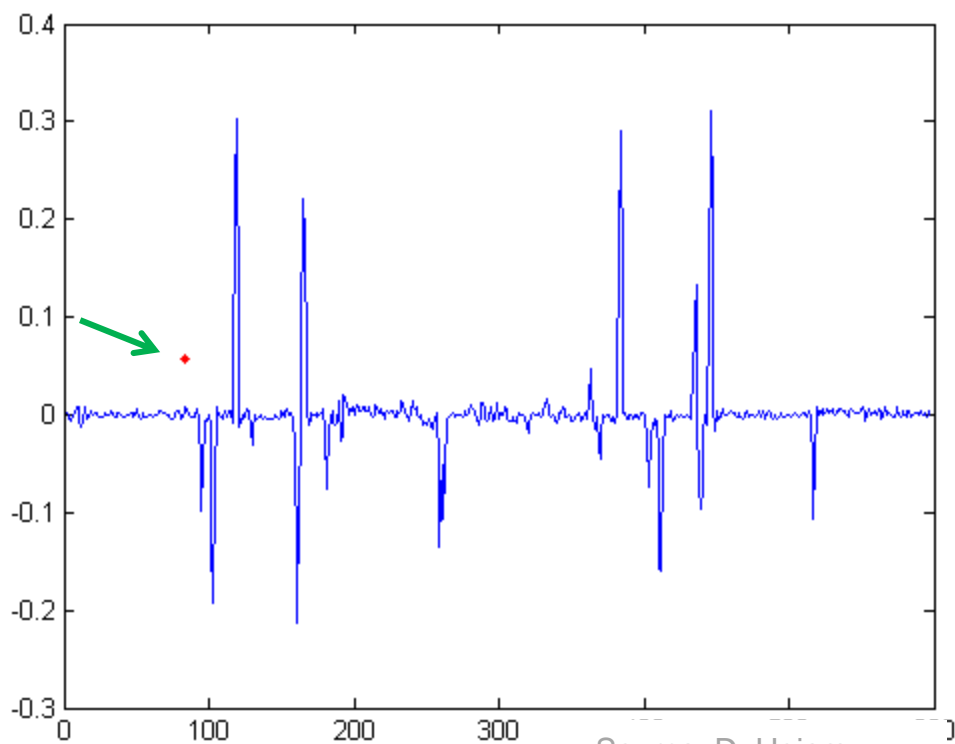
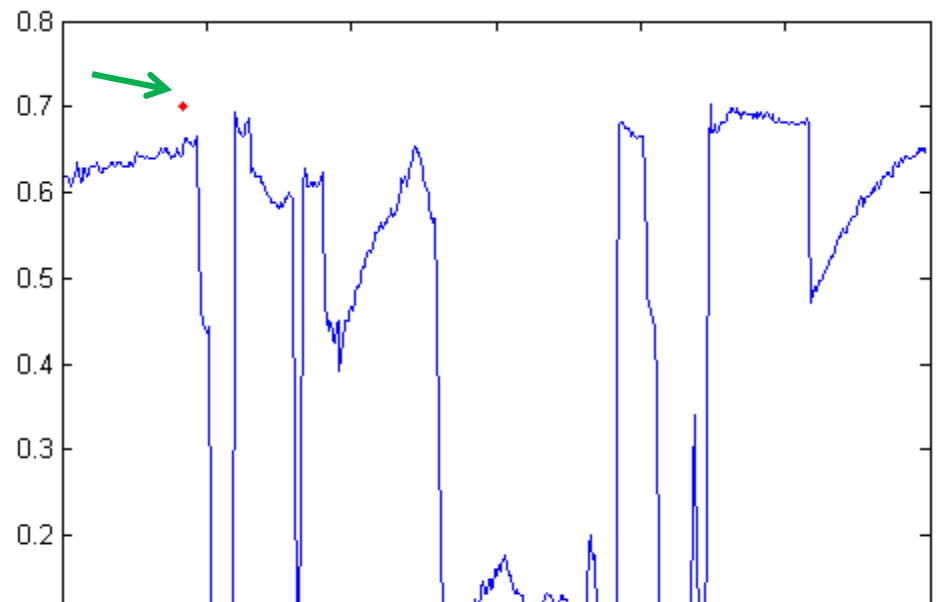
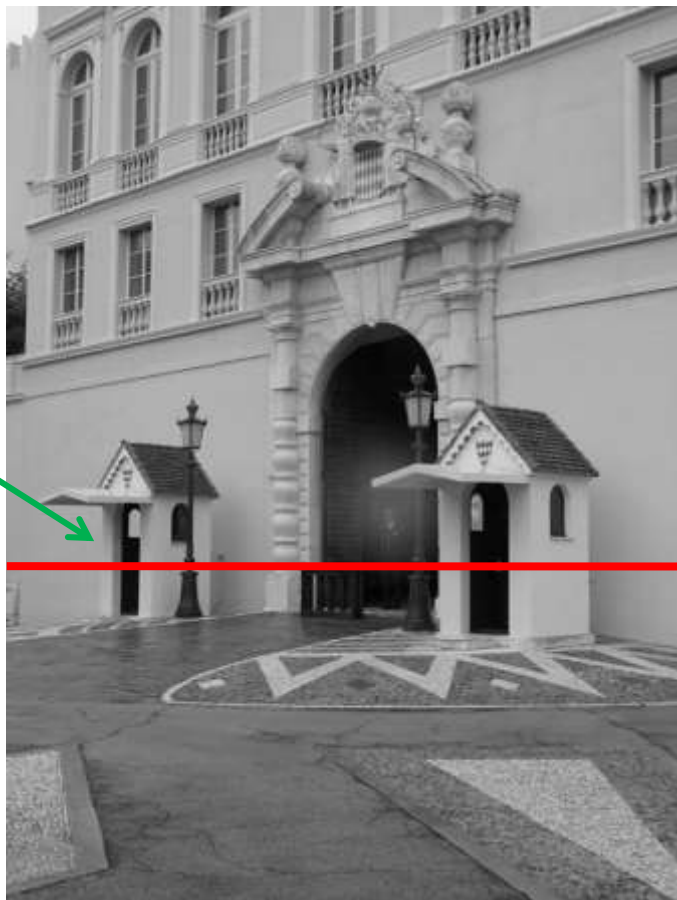
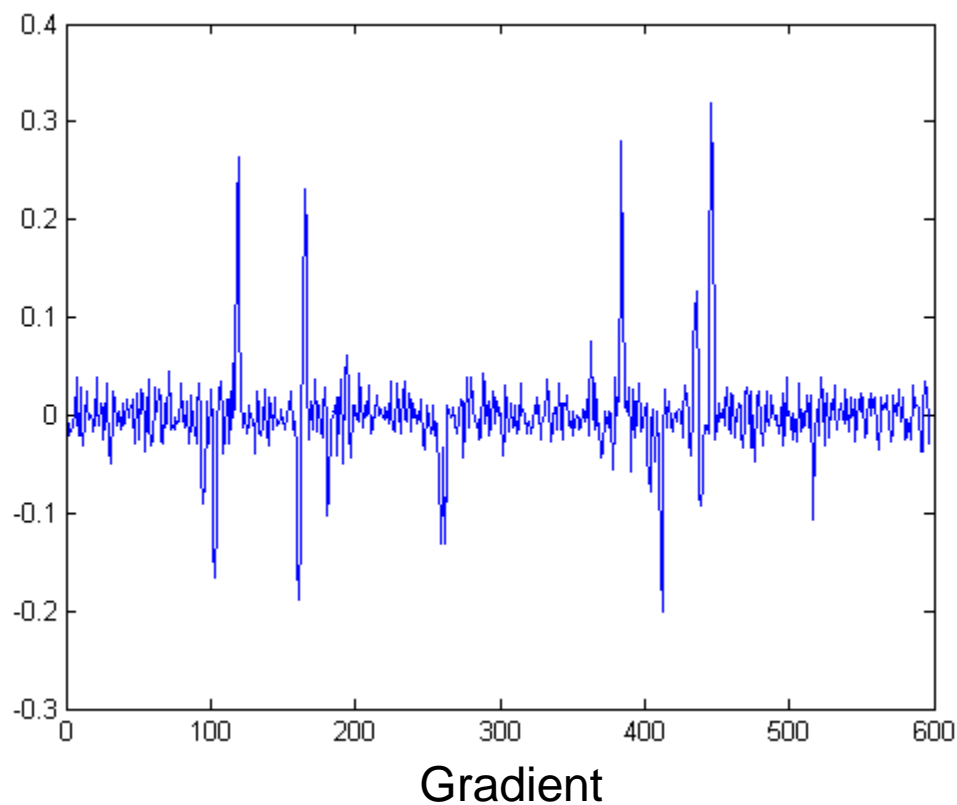
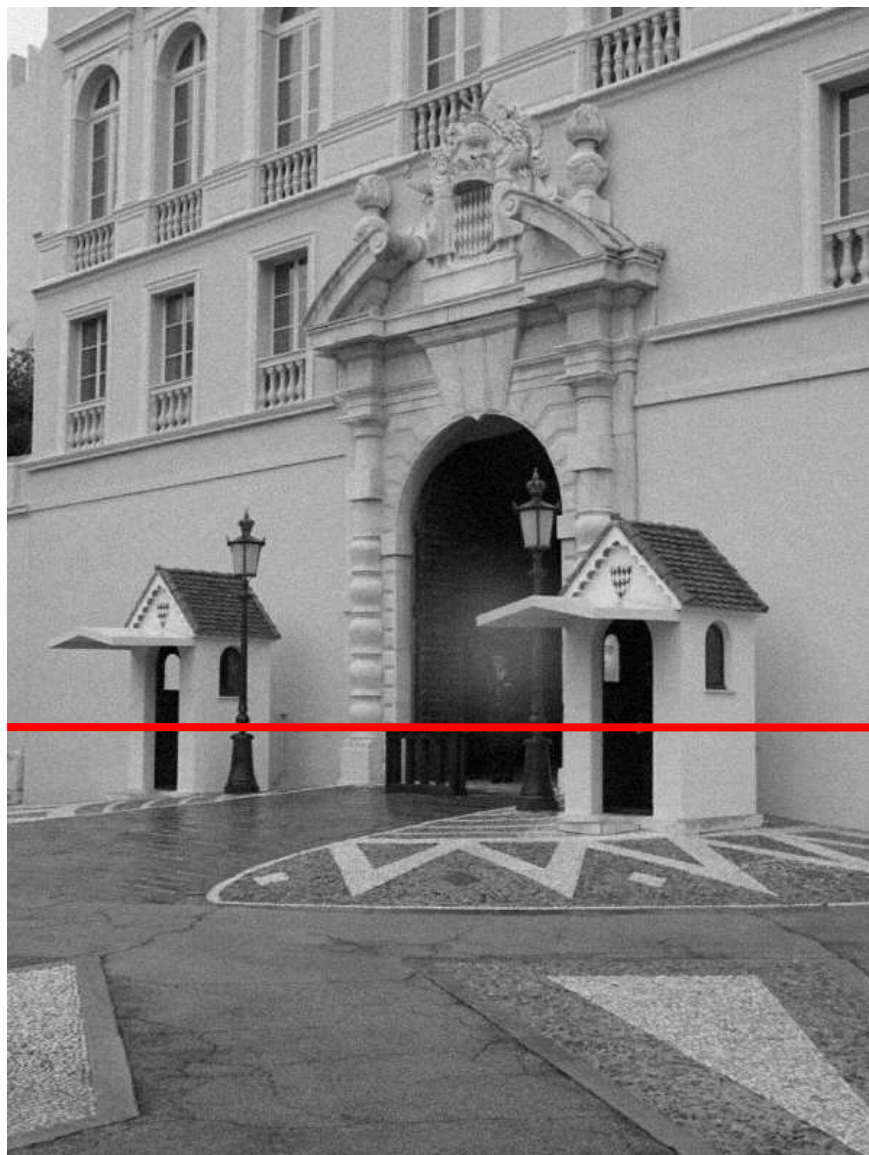


FIGURE 10.11 First column: Images and intensity profiles of a ramp edge corrupted by random Gaussian noise of zero mean and standard deviations of 0.0, 0.1, 1.0, and 10.0 intensity levels, respectively. Second column: First-derivative images and intensity profiles. Third column: Second-derivative images and intensity profiles.

Intensity profile



With a little Gaussian noise



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Convolution with Template/Kernel

- **模板(Template/Kernel)**: A matrix represents an operator. A convolution template centers on each pixel in an image and generates new output pixels.
- **卷积(Convolution)**: by using the template, the new pixel value is computed by multiplying each pixel value in the neighborhood with the corresponding **weight** in the convolution mask and **summing** these products.

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 4 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

Convolution with Template/Kernel

34	20	10	30	38	198	246
28	45	0	1	4	9	2
0	9	0	0	0	2	0
238	5	5	2	9	3	9
8	98	1	8	2	8	2
2	5	4	7	1	6	2
9	3	6	5	3	1	4

-1	1	-1
1	4	1
-1	1	-1

Convolution with Template/Kernel



Convolution with Template/Kernel

- $T(x,y)$ is a template ($n \times m$), $I(x,y)$ is an image ($M \times N$), then the convoluting of T with I is

$$T * I(X,Y) = \sum_{i=0}^{n-1} \sum_{j=0}^{m-1} T(i,j) I(X+i, Y+j)$$

- Usually, the template is not allowed to shift off the edge of the image, so the resulting image will **normally be smaller** than the original image.

1	0
0	1

 *

1	1	3	3	4
1	1	4	4	3
2	1	3	3	3
1	1	1	4	4

 =

2	5	7	6	*
2	4	7	7	*
3	2	7	7	*
*	*	*	*	*

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基于一阶导数的边缘检测

- **梯度(Gradient):** 是图像对应二维函数的一阶导数

$$G(x, y) = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

- 梯度的幅值: isotropic operator

anisotropic

$$|G(x, y)| = \sqrt{G_x^2 + G_y^2}$$



$$|G(x, y)| = |G_x| + |G_y|$$

$$|G(x, y)| \approx \max(|G_x|, |G_y|)$$

基于一阶导数的边缘检测

- 梯度方向:

$$\alpha(x, y) = \arctan(G_y / G_x)$$

梯度方向为函数最大变化率方向

- 图像中用差分近似偏导数

用差分近似偏导数

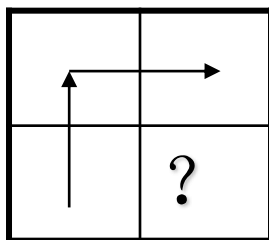
$$G_x = f[x+1, y] - f[x, y]$$

$$G_y = f[x, y] - f[x, y+1]$$

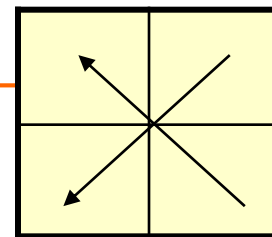
一般用卷积模板进行计算：

$$G_x = \begin{bmatrix} -1 & 1 \end{bmatrix} \quad G_y = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

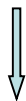
上述表示？



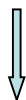
(1) Roberts交叉算子



$$G[i, j] = |G_x| + |G_y|$$



$$G[i, j] = |f[i, j] - f[i + 1, j + 1]| + |f[i + 1, j] - f[i, j + 1]|$$



$$G_x = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$G_y = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

2X2梯度算子？

3X3梯度算子！

(2) Sobel算子

$$G_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

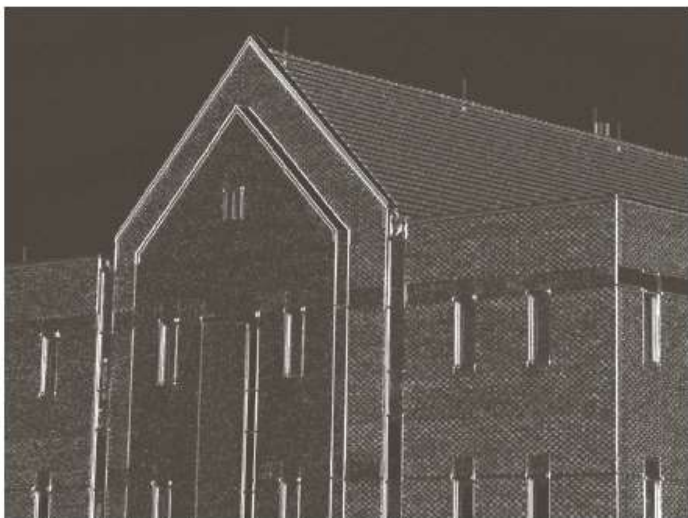
$$G_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

(3) Prewitt算子：运算较快

$$G_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$G_y = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

Sobel算子实例



均值差分：一定邻域内灰度平均值之差

3×3邻域加权

$$G_x = (a_2 + ca_3 + a_4) - (a_0 + ca_7 + a_6)$$

$$G_y = (a_0 + ca_1 + a_2) - (a_6 + ca_5 + a_4)$$

a_0	a_1	a_2
a_7	$[i,j]$	a_3
a_6	a_5	a_4

C=1: Prewitt算子

C=2: Sobel算子

C=3: Sethi算子

OpenCV函数

```
void cvSobel( const CvArr* src, CvArr* dst,  
              int xorder, int yorder, int aperture_size=3 );
```

aperture_size: -1, 1, 3, 5, 7

X方向: xorder=1, yorder=0, aperture_size=3

Y方向: xorder=0, yorder=1, aperture_size=3

注意: **8bit depth 溢出! 用 cvAbs cvConvertScale**
dst: IPL_DEPTH_16S or IPL_DEPTH_32F

```
void cvFilter2D( const CvArr* src, CvArr* dst,  
                 const CvMat* kernel,  
                 CvPoint anchor=cvPoint(-1,-1));
```

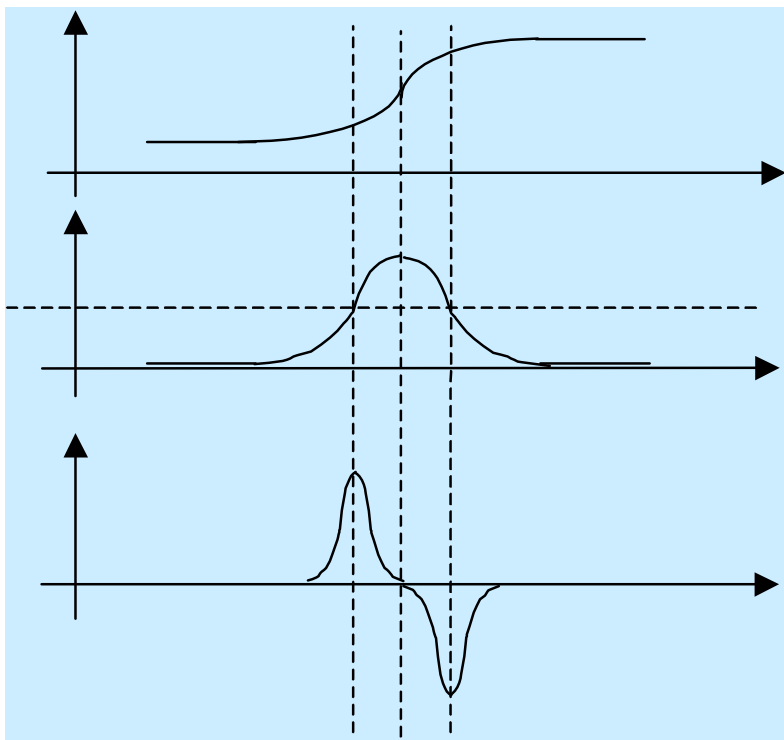
```
float k[9]={ 1, 2, 1,  
            0, 0, 0,  
            -1, -2, -1 };
```

```
CvMat kernel=cvMat(3,3,CV_32FC1, k); 【kernel必须为浮点类型】
```

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基于二阶导数的边缘检测



二阶微分算子

图像灰度二阶导数的过零点对应边缘点.

拉普拉斯 (Laplacian) 算子

拉普拉斯算子是二阶导数的二维等效式:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

用差分近似微分:

$$\begin{aligned}\frac{\partial^2 f}{\partial x^2} &= \frac{\partial G_x}{\partial x} \\ &= \frac{\partial (f[i, j+1] - f[i, j])}{\partial x} \\ &= \frac{\partial f[i, j+1]}{\partial x} - \frac{\partial f[i, j]}{\partial x} \\ &= (f[i, j+1] - f[i, j]) - (f[i, j] - f[i, j-1])\end{aligned}$$

$$\frac{\partial^2 f}{\partial x^2} = (f[i, j+1] - 2f[i, j]) + f[i, j-1]$$

$$\frac{\partial^2 f}{\partial y^2} = (f[i+1, j] - 2f[i, j]) + f[i-1, j]$$

表示为卷积模板：

$$\nabla^2 \approx \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

邻域中心点具有更大权值的近似算子：

$$\nabla^2 \approx \begin{bmatrix} 1 & 4 & 1 \\ 4 & -20 & 4 \\ 1 & 4 & 1 \end{bmatrix}$$

LoG边缘检测算法

LoG = Laplacian of Gaussian

高斯滤波+拉普拉斯边缘检测

基本特征：

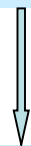
- 平滑滤波器是高斯滤波器.
- 采用拉普拉斯算子计算二阶导数.
- 边缘检测判据是二阶导数零交叉点并对应一阶导数的较大峰值.
- 使用线性内插方法在子像素分辨率水平上估计边缘的位置.

(Marr & Hildreth 80)

LoG算子

$$h(x, y) = \nabla^2 [g(x, y) * f(x, y)]$$

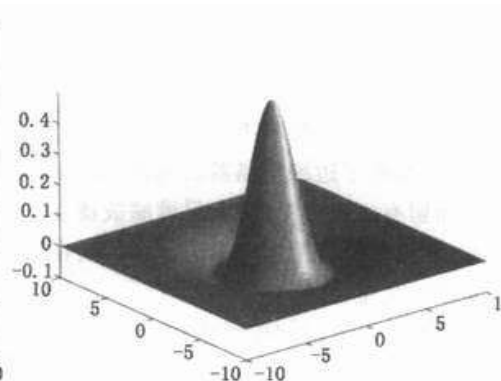
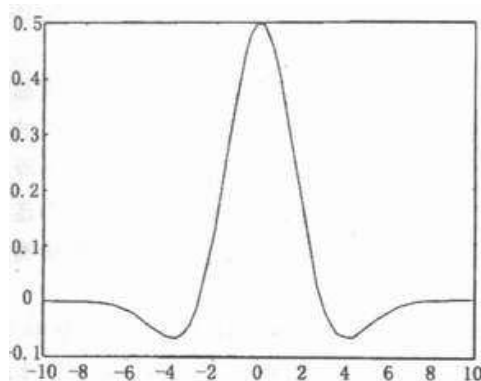
根据卷积求导法



$$h(x, y) = [\nabla^2 g(x, y)] * f(x, y)$$

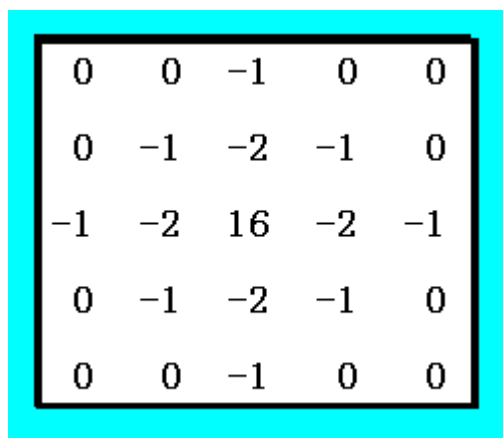
$$\nabla^2 g(x, y) = \left(\frac{x^2 + y^2 - 2\sigma^2}{\sigma^4} \right) e^{-\frac{x^2 + y^2}{2\sigma^2}}$$

墨西哥草帽算子:



两种等效计算方法

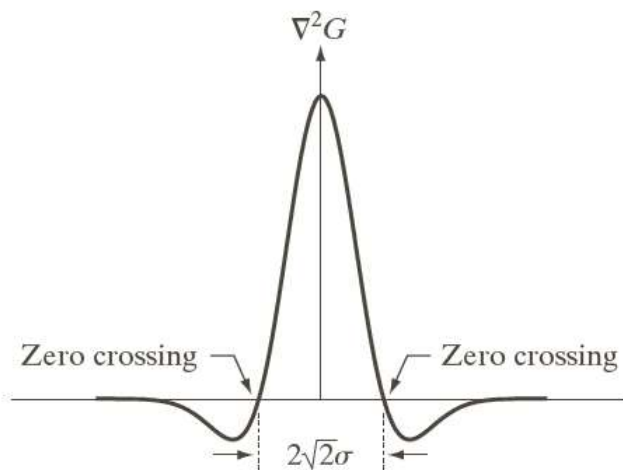
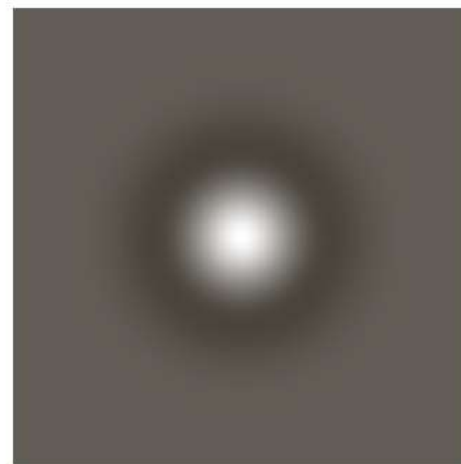
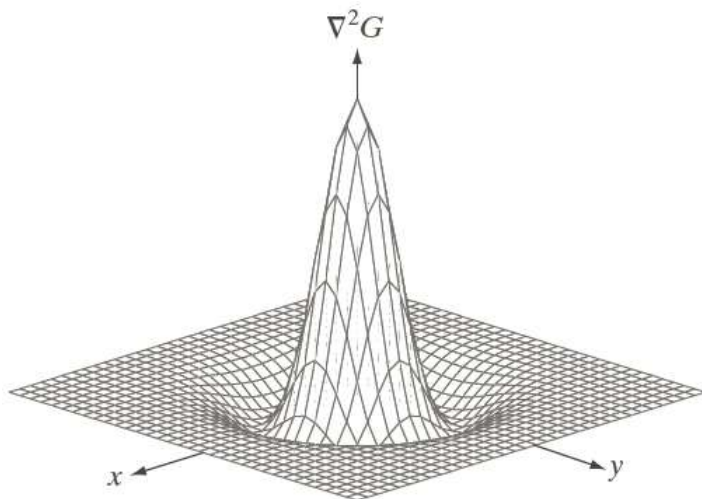
1. 图像与高斯函数卷积，再求卷积的拉普拉斯微分
2. 求高斯函数的拉普拉斯微分，再与图像卷积



0	0	-1	0	0
0	-1	-2	-1	0
-1	-2	16	-2	-1
0	-1	-2	-1	0
0	0	-1	0	0

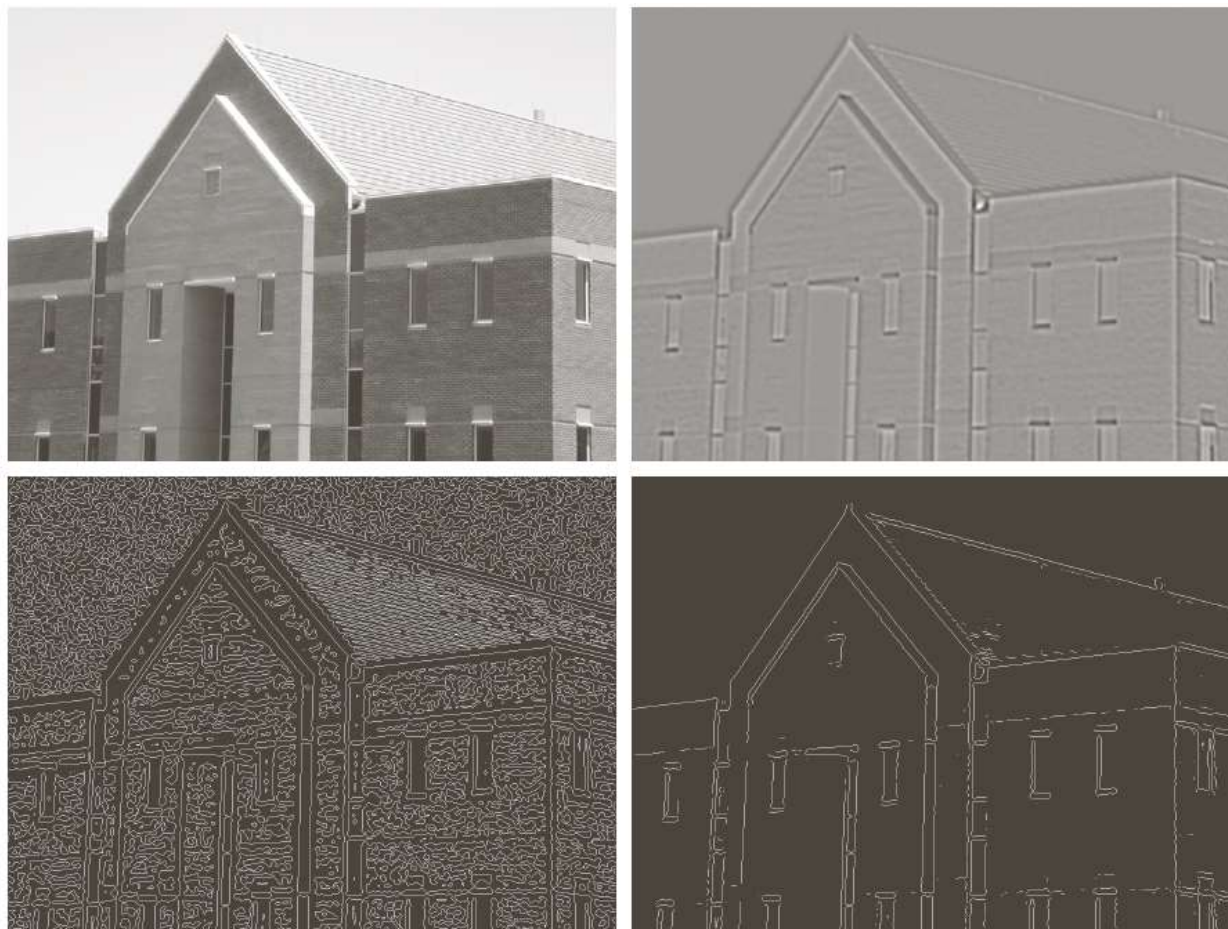
5X5拉普拉斯高斯模板

LoG算子



0	0	-1	0	0
0	-1	-2	-1	0
-1	-2	16	-2	-1
0	-1	-2	-1	0
0	0	-1	0	0

LoG算子



a	b
c	d

FIGURE 10.22

(a) Original image of size 834×1114 pixels, with intensity values scaled to the range $[0, 1]$. (b) Results of Steps 1 and 2 of the Marr-Hildreth algorithm using $\sigma = 4$ and $n = 25$. (c) Zero crossings of (b) using a threshold of 0 (note the closed-loop edges). (d) Zero crossings found using a threshold equal to 4% of the maximum value of the image in (b). Note the thin edges.

OpenCV相关函数

- **void cvLaplace**

(CvArr *src, CvArr *dst, int aperture_size=3)

– If aperture_size=1

$$\nabla^2 \approx \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

– 不进行Scale变换

- **cvFilter2D**

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A computational approach to edge detection

J Canny - IEEE Transactions on pattern analysis and machine ..., 1986 - ieeexplore.ieee.org

This paper describes a computational approach to edge detection. The success of the approach depends on the definition of a comprehensive set of goals for the computation of edge points. These goals must be precise enough to delimit the desired behavior of the ...

☆ 77 Cited by 33096 Related articles All 22 versions

Canny 边缘检测器



算法步骤:

1. 用高斯滤波器平滑图像.
2. 用一阶偏导有限差分计算**梯度幅值和方向**.
3. 对梯度幅值进行**非极大值抑制** (NMS) .
4. 用**双阈值**算法检测和连接边缘.

Why 高斯滤波器?

平滑去噪和边缘检测是一对矛盾，应用高斯函数的一阶导数，在二者之间获得最佳的平衡。

[Canny86]

步1. 图像与高斯平滑滤波器卷积:

$$S[i, j] = G[i, j; \sigma] * I[i, j]$$

步2a. 使用一阶有限差分计算偏导数阵列P与Q:

$$G_x[i, j] \approx (S[i, j+1] - S[i, j] + S[i+1, j+1] - S[i+1, j]) / 2$$

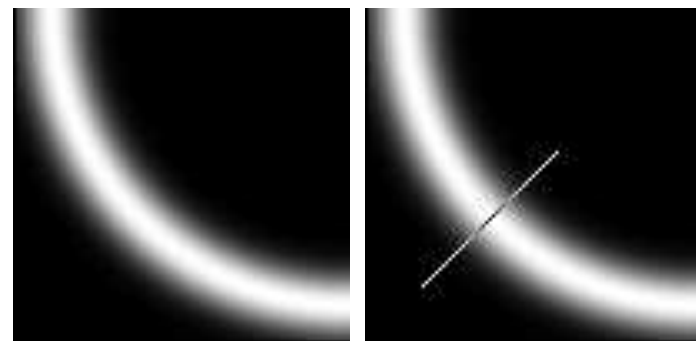
$$G_y[i, j] \approx (S[i, j] - S[i+1, j] + S[i, j+1] - S[i+1, j+1]) / 2$$

步2b. 计算梯度幅值与方向角:

$$M[i, j] = \sqrt{G_x[i, j]^2 + G_y[i, j]^2}$$

$$\theta[i, j] = \arctan(G_y[i, j] / G_x[i, j])$$

Before Non-max Suppression



After non-max suppression



步3. 非极大值抑制 (NMS) :

去掉幅值局部变化非极大的点.

* 将梯度角离散为圆周的四个扇区之一, 以便使用 3×3 的窗口作抑制运算

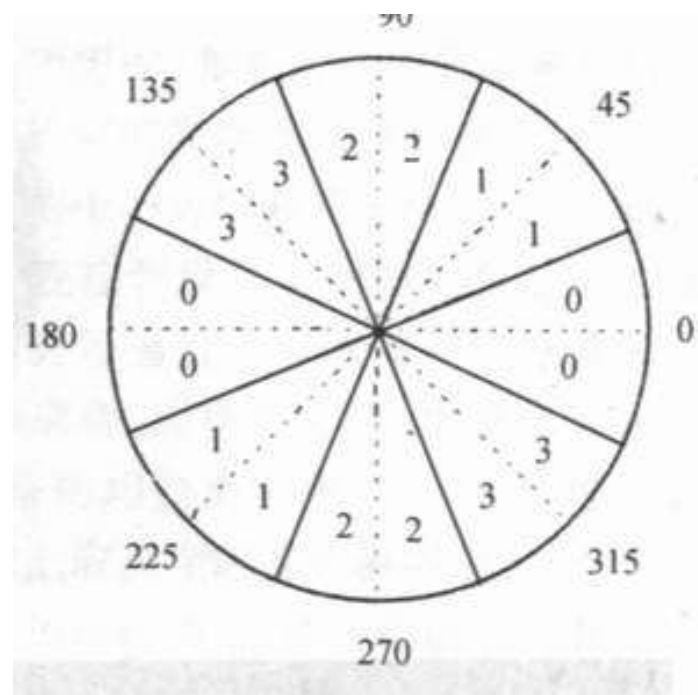
* **方向角**离散化:

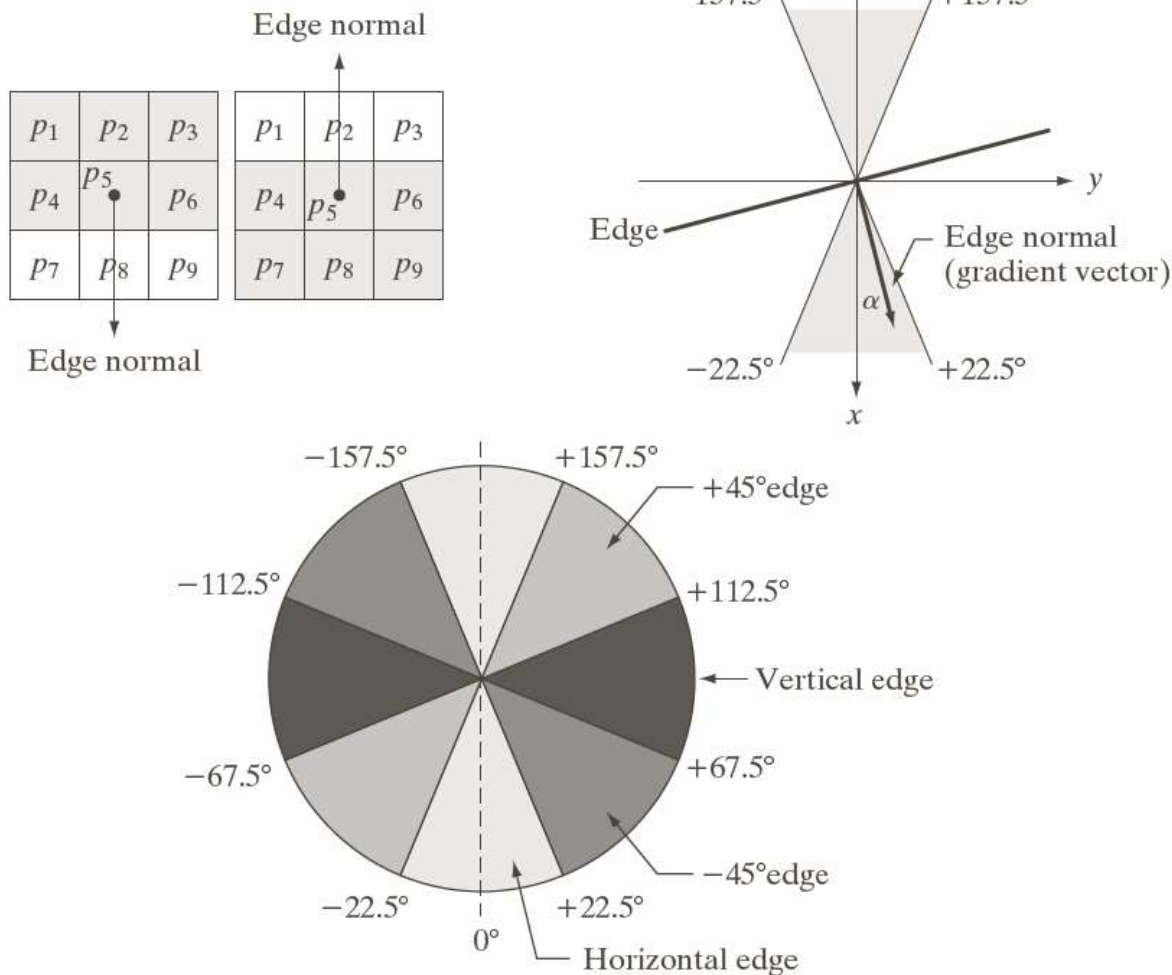
$$\zeta[i, j] = \text{Sector}(\theta[i, j])$$

* 抑制, 得到新幅值图:

$$N[i, j] = \text{NMS}(M[i, j], \zeta[i, j])$$

How抑制? 若 $M[i, j]$ 不比沿梯度线方向上的两个相邻点幅值大, 则 $N[i, j]=0$





a b
c

FIGURE 10.24

(a) Two possible orientations of a horizontal edge (in gray) in a 3×3 neighborhood. (b) Range of values (in gray) of α , the direction angle of the edge normal, for a horizontal edge. (c) The angle ranges of the edge normals for the four types of edge directions in a 3×3 neighborhood. Each edge direction has two ranges, shown in corresponding shades of gray.

After non-max suppression



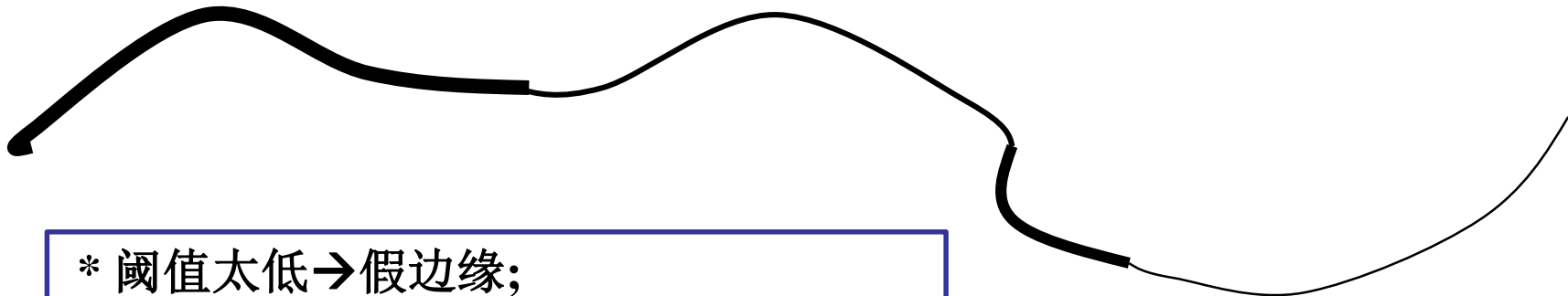
步4. 双阈值化并边缘链接

(a) 取高低两个阈值(T_2, T_1)作用于新幅值图 $N[i,j]$, 得到两个边缘图: 高阈值和低阈值边缘图。

高阈值图: $N[i,j] > T_2$;

低阈值图: $N[i,j] > T_1$

(b) 连接高阈值边缘图, 出现断点时, 在低阈值边缘图中的8邻点域搜寻边缘点。



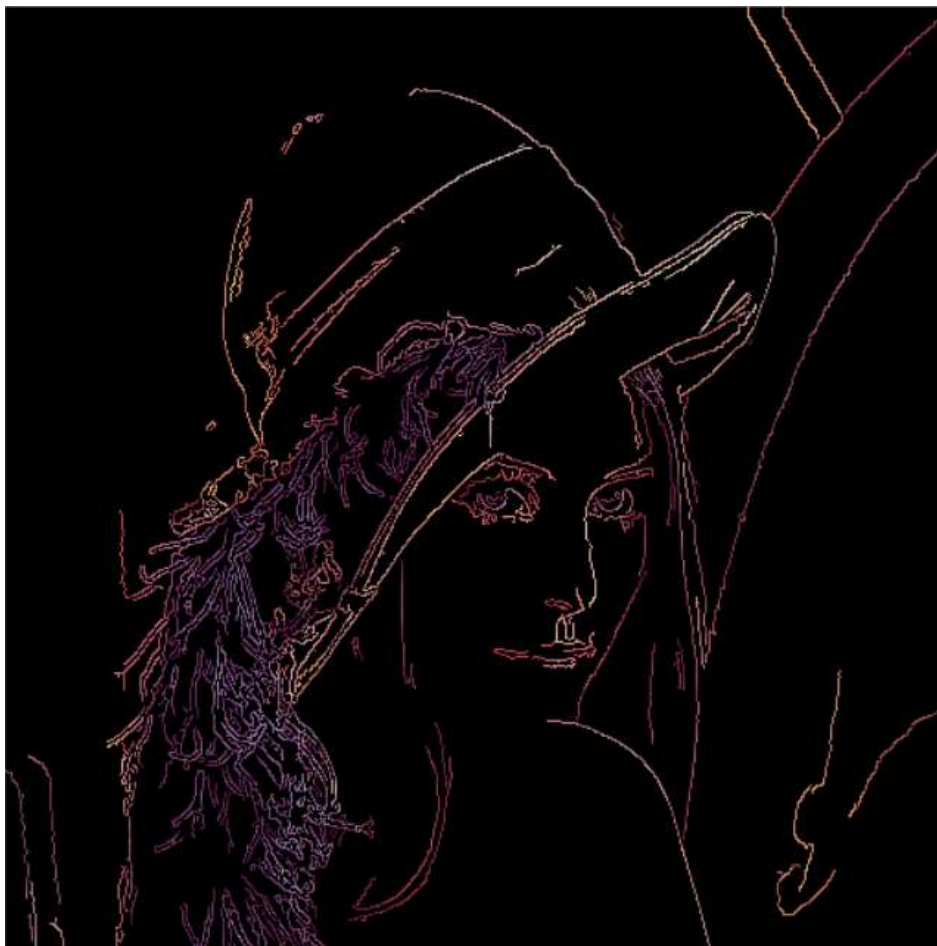
- * 阈值太低→假边缘;
- * 阈值太高→部分轮廓丢失.
- * 选用两个阈值: 更有效的阈值方案.

最终结果



提取彩色边缘

最终边缘结果图
(作为mask)
覆盖在原彩色图
像后, 可提取出
彩色边缘图



Canny 边缘检测器



算法步骤:

1. 用高斯滤波器平滑图像.
2. 用一阶偏导有限差分计算**梯度幅值和方向**.
3. 对梯度幅值进行**非极大值抑制** (NMS) .
4. 用**双阈值**算法检测和连接边缘.

[Canny86]

Effect of σ (Gaussian kernel spread/size)



original



Canny with $\sigma = 1$

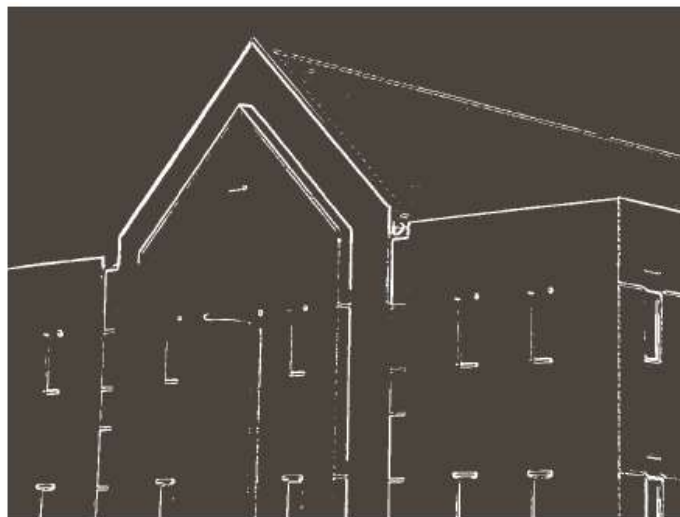


Canny with $\sigma = 2$

The choice of σ depends on desired behavior

- large σ detects large scale edges
- small σ detects fine features

Source: S. Seitz



a	b
c	d

FIGURE 10.25

(a) Original image of size 834×1114 pixels, with intensity values scaled to the range $[0, 1]$.

(b) Thresholded gradient of smoothed image.

(c) Image obtained using the Marr-Hildreth algorithm.

(d) Image obtained using the Canny algorithm. Note the significant improvement of the Canny image compared to the other two.



a	b
c	d

FIGURE 10.26

(a) Original head CT image of size 512×512 pixels, with intensity values scaled to the range $[0, 1]$.
(b) Thresholded gradient of smoothed image.
(c) Image obtained using the Marr-Hildreth algorithm.
(d) Image obtained using the Canny algorithm.
(Original image courtesy of Dr. David R. Pickens, Vanderbilt University.)

OpenCV相关函数

- **cvCanny**

(CvArr* src, CvArr* dst,
double threshold1, double threshold2
int aperture_size=3)

- threshold2 \approx (2 or 3) * threshold1
- src **must**: grayscale; dst **must**: grayscale
- aperture_size: for cvSobel()

Review

- 边缘及边缘检测概念
- 用模板实现卷积
- 基于一阶的边缘检测
- 基于二阶的边缘检测
 - Laplacian算子
 - LoG算子（Marr&Hildreth算子）
- **Canny**边缘检测