Problem Set 1

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Contents

Problem Set 1

Exercise 1: 1/N portfolio

Compute the return time series of the 1/N portfolio between January 1977 and December 2022.

```
#Implement the 1/N strategy
df$`1/N` = apply(df[,c(2:31)],1,mean)
```

a) the first six elements of the return time series

```
ex1a = df[(df$X>=197701)&(df$X<=197706),][,c('X','1/N')]
ex1a$`1/N` = round(ex1a$`1/N`/100,5)
names(ex1a) = c('Time','Return of 1/N portfolio')
ex1a
```

```
##       Time Return of 1/N portfolio
## 601 197701                -0.03675
## 602 197702                -0.01153
## 603 197703                -0.00251
## 604 197704                 0.00916
## 605 197705                -0.01369
## 606 197706                 0.05596
```

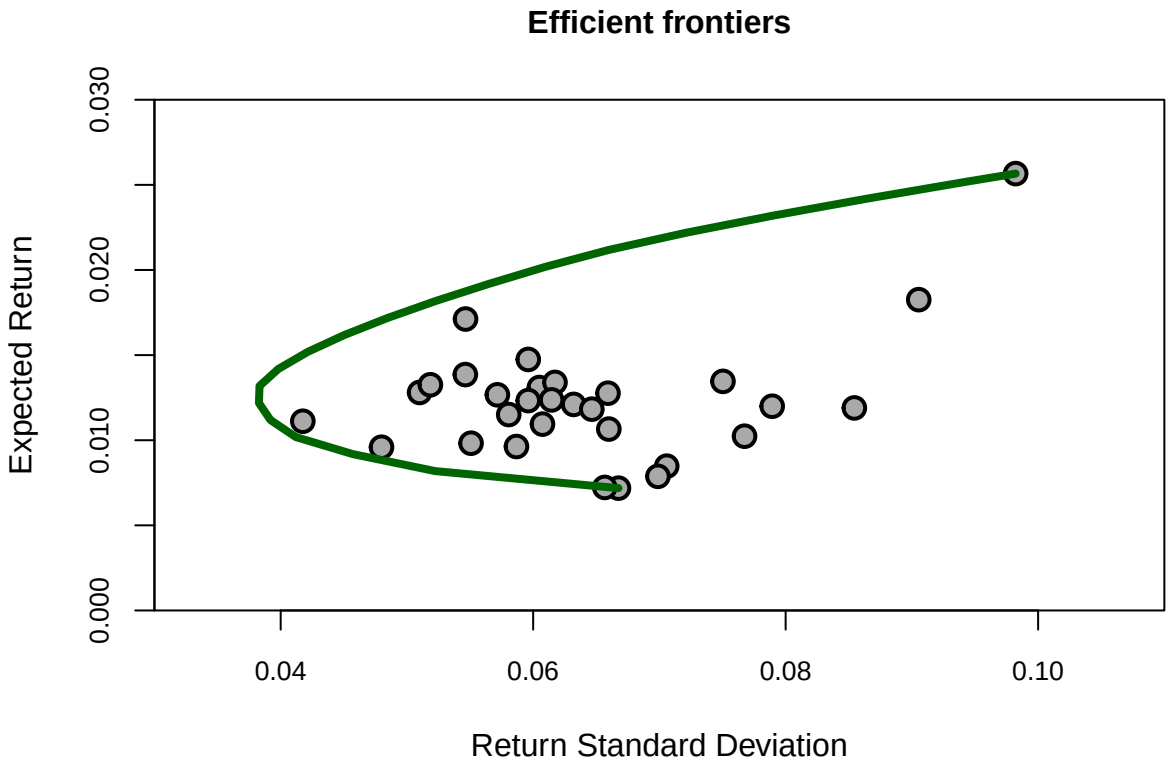
b) the out-of-sample Sharpe Ratio of the 1/N portfolio

```
df$RE = df$`1/N` - df$RF
mu = mean(df[(df$X>=197701),]$RE); std = sd(df[(df$X>=197701),]$RE)
SR = round(mu/std,4)
print(paste("the out-of-sample Sharpe Ratio of the 1/N portfolio =",SR))
```

```
## [1] "the out-of-sample Sharpe Ratio of the 1/N portfolio = 0.2245"
```

Exercise 2: Markowitz and Tobin

Run a Markowitz optimization



a) the covariance matrix used for the Markowitz optimization

```
SIGMA
```

```
##              Food        Beer        Smoke       Games       Books       Hshld
## Food  0.002601498 0.002385778 0.0017829890 0.003420135 0.002697657 0.002416654
## Beer  0.002385778 0.003443157 0.0015739657 0.003837598 0.002913154 0.002539312
## Smoke 0.001782989 0.001573966 0.0029860352 0.002497530 0.001731709 0.001641898
## Games 0.003420135 0.003837598 0.0024975305 0.007300538 0.004769026 0.003783803
## Books 0.002697657 0.002913154 0.0017317086 0.004769026 0.004456612 0.002879060
## Hshld 0.002416654 0.002539312 0.0016418977 0.003783803 0.002879060 0.003033802
## Clths 0.003068632 0.003076946 0.0021076249 0.005645663 0.004383850 0.002929714
## Hlth  0.002333215 0.002309704 0.0018277727 0.003065395 0.002337875 0.002657972
## Chems 0.002157432 0.002325142 0.0017800756 0.003653098 0.002848397 0.002350944
## Txtls 0.002564554 0.002687793 0.0018067542 0.004645972 0.003672870 0.002524928
## Cnstr 0.002854221 0.003120823 0.0020057486 0.004842258 0.003799607 0.002785879
## Steel 0.002131137 0.002488441 0.0017873634 0.003837003 0.003004874 0.002038494
## FabPr 0.002587555 0.002713547 0.0019596241 0.004150445 0.003264210 0.002677050
## ElcEq 0.002696153 0.002854647 0.0020667303 0.004443863 0.003438911 0.002981376
## Autos 0.002195446 0.002234757 0.0016289996 0.004104327 0.003177339 0.002194898
## Carry 0.002680459 0.002832157 0.0019408200 0.004996923 0.003759327 0.002833170
## Mines 0.002065396 0.002244264 0.0018006376 0.003351062 0.002529901 0.002048332
```

```
## Coal  0.002819518 0.002641886 0.0027137848 0.004229850 0.002934597 0.002222491
## Oil   0.001654921 0.001578634 0.0013272581 0.002129935 0.001866432 0.001541415
## Util  0.001762132 0.001706822 0.0011461766 0.002237399 0.001937206 0.001462091
## Telcm 0.001407693 0.001376908 0.0009024926 0.002060563 0.001648774 0.001306297
## Servs 0.003239476 0.003348483 0.0023899536 0.005703382 0.004262835 0.003231612
## BusEq 0.002396032 0.002599266 0.0016643900 0.004207088 0.003069657 0.002961799
## Paper 0.002274907 0.002333552 0.0015661847 0.003509280 0.002889727 0.002435381
## Trans 0.002708028 0.002719694 0.0020611289 0.004690575 0.003649873 0.002725360
## Whlsl 0.003008443 0.003078338 0.0021911213 0.005467739 0.004141137 0.003010468
## Rtail 0.002719100 0.002744466 0.0019162700 0.004496497 0.003470346 0.002758424
## Meals 0.003723009 0.003908460 0.0026343997 0.006773906 0.004909285 0.004072410
## Fin   0.002654444 0.002675904 0.0019084396 0.003994563 0.003182334 0.002519394
## Other 0.002783095 0.002808395 0.0021064318 0.004634442 0.003555631 0.002762212
##             Clths        Hlth       Chems       Txtls       Cnstr       Steel
## Food  0.003068632 0.002333215 0.002157432 0.002564554 0.002854221 0.002131137
## Beer  0.003076946 0.002309704 0.002325142 0.002687793 0.003120823 0.002488441
## Smoke 0.002107625 0.001827773 0.001780076 0.001806754 0.002005749 0.001787363
## Games 0.005645663 0.003065395 0.003653098 0.004645972 0.004842258 0.003837003
## Books 0.004383850 0.002337875 0.002848397 0.003672870 0.003799607 0.003004874
## Hshld 0.002929714 0.002657972 0.002350944 0.002524928 0.002785879 0.002038494
## Clths 0.006230818 0.002097507 0.003120910 0.004692168 0.004520846 0.003549862
## Hlth  0.002097507 0.003371638 0.002121018 0.001926523 0.002436275 0.001682398
## Chems 0.003120910 0.002121018 0.003267847 0.002953719 0.003075707 0.002901996
## Txtls 0.004692168 0.001926523 0.002953719 0.004355450 0.003829728 0.003121379
## Cnstr 0.004520846 0.002436275 0.003075707 0.003829728 0.004346594 0.003392726
## Steel 0.003549862 0.001682398 0.002901996 0.003121379 0.003392726 0.003995086
## FabPr 0.003551857 0.002556699 0.002961530 0.003112793 0.003559499 0.003173898
## ElcEq 0.003699056 0.002740434 0.002868317 0.003294771 0.003621855 0.002743063
## Autos 0.003883703 0.001557743 0.002500787 0.003205316 0.003234972 0.002594231
## Carry 0.004439554 0.002449165 0.003039918 0.003550930 0.003925350 0.003472730
## Mines 0.002813196 0.002010068 0.002383754 0.002540151 0.002865732 0.002908679
## Coal  0.003817827 0.002550413 0.003201064 0.003146001 0.003714417 0.003826132
## Oil   0.001796900 0.001667308 0.001875812 0.001652450 0.002112726 0.001849780
## Util  0.002331667 0.001278937 0.001583707 0.001842193 0.002143717 0.001761085
## Telcm 0.001934203 0.001202347 0.001398477 0.001533494 0.001832510 0.001417574
## Servs 0.005015626 0.002905240 0.003188285 0.004007992 0.004482539 0.003584962
## BusEq 0.003348272 0.002677638 0.002398151 0.002845929 0.003074444 0.002430026
## Paper 0.002994797 0.002336751 0.002676018 0.002770028 0.003108937 0.002629757
## Trans 0.004191598 0.002327832 0.003172802 0.003498283 0.003899019 0.003534841
## Whlsl 0.004952231 0.002741932 0.003178663 0.004026296 0.004305222 0.003576900
## Rtail 0.004113888 0.002191071 0.002491462 0.003295985 0.003507726 0.002532814
## Meals 0.005826595 0.003533020 0.003667325 0.004602827 0.004864420 0.003727156
## Fin   0.003638042 0.002410710 0.002636791 0.003031354 0.003613862 0.002798336
## Other 0.004073992 0.002577682 0.002917480 0.003464783 0.003819137 0.003106333
##             FabPr       ElcEq       Autos       Carry       Mines        Coal
## Food  0.002587555 0.002696153 0.002195446 0.002680459 0.002065396 0.002819518
## Beer  0.002713547 0.002854647 0.002234757 0.002832157 0.002244264 0.002641886
## Smoke 0.001959624 0.002066730 0.001629000 0.001940820 0.001800638 0.002713785
## Games 0.004150445 0.004443863 0.004104327 0.004996923 0.003351062 0.004229850
## Books 0.003264210 0.003438911 0.003177339 0.003759327 0.002529901 0.002934597
## Hshld 0.002677050 0.002981376 0.002194898 0.002833170 0.002048332 0.002222491
## Clths 0.003551857 0.003699056 0.003883703 0.004439554 0.002813196 0.003817827
## Hlth  0.002556699 0.002740434 0.001557743 0.002449165 0.002010068 0.002550413
## Chems 0.002961530 0.002868317 0.002500787 0.003039918 0.002383754 0.003201064
```

```
## Txtls 0.003112793 0.003294771 0.003205316 0.003550930 0.002540151 0.003146001
## Cnstr 0.003559499 0.003621855 0.003234972 0.003925350 0.002865732 0.003714417
## Steel 0.003173898 0.002743063 0.002594231 0.003472730 0.002908679 0.003826132
## FabPr 0.003659231 0.003333163 0.002594415 0.003650915 0.002885176 0.003718076
## ElcEq 0.003333163 0.004179260 0.002777743 0.003620495 0.002673775 0.003568910
## Autos 0.002594415 0.002777743 0.003553156 0.003119823 0.001844685 0.002244429
## Carry 0.003650915 0.003620495 0.003119823 0.004981799 0.003145408 0.003924134
## Mines 0.002885176 0.002673775 0.001844685 0.003145408 0.003553753 0.003612208
## Coal  0.003718076 0.003568910 0.002244429 0.003924134 0.003612208 0.009647550
## Oil   0.002143911 0.002020746 0.001458453 0.002187226 0.002048458 0.003004005
## Util  0.001821827 0.001788874 0.001765711 0.002069353 0.001512801 0.002148033
## Telcm 0.001627390 0.001517933 0.001472976 0.001809020 0.001172183 0.001609201
## Servs 0.003983364 0.004069572 0.003401587 0.004546304 0.003319516 0.004403696
## BusEq 0.002956164 0.003154839 0.002383870 0.003412509 0.002416471 0.003035672
## Paper 0.002876331 0.002815490 0.002247666 0.002888441 0.002305310 0.002668143
## Trans 0.003627911 0.003436176 0.002984212 0.004191095 0.002877118 0.003916311
## Whlsl 0.003899270 0.003803823 0.003122514 0.004465239 0.003307310 0.004238786
## Rtail 0.002825066 0.003090084 0.003062504 0.003350739 0.002158043 0.002522575
## Meals 0.004315947 0.004479025 0.003791024 0.004976482 0.003406301 0.004369106
## Fin   0.003157696 0.003175848 0.002641034 0.003305785 0.002506945 0.003223976
## Other 0.003502615 0.003496704 0.002810507 0.004008332 0.002979719 0.003897094
##               Oil        Util        Telcm       Servs       BusEq       Paper
## Food  0.001654921 0.001762132 0.0014076928 0.003239476 0.002396032 0.002274907
## Beer  0.001578634 0.001706822 0.0013769077 0.003348483 0.002599266 0.002333552
## Smoke 0.001327258 0.001146177 0.0009024926 0.002389954 0.001664390 0.001566185
## Games 0.002129935 0.002237399 0.0020605631 0.005703382 0.004207088 0.003509280
## Books 0.001866432 0.001937206 0.0016487738 0.004262835 0.003069657 0.002889727
## Hshld 0.001541415 0.001462091 0.0013062968 0.003231612 0.002961799 0.002435381
## Clths 0.001796900 0.002331667 0.0019342029 0.005015626 0.003348272 0.002994797
## Hlth  0.001667308 0.001278937 0.0012023472 0.002905240 0.002677638 0.002336751
## Chems 0.001875812 0.001583707 0.0013984772 0.003188285 0.002398151 0.002676018
## Txtls 0.001652450 0.001842193 0.0015334936 0.004007992 0.002845929 0.002770028
## Cnstr 0.002112726 0.002143717 0.0018325096 0.004482539 0.003074444 0.003108937
## Steel 0.001849780 0.001761085 0.0014175740 0.003584962 0.002430026 0.002629757
## FabPr 0.002143911 0.001821827 0.0016273903 0.003983364 0.002956164 0.002876331
## ElcEq 0.002020746 0.001788874 0.0015179333 0.004069572 0.003154839 0.002815490
## Autos 0.001458453 0.001765711 0.0014729764 0.003401587 0.002383870 0.002247666
## Carry 0.002187226 0.002069353 0.0018090197 0.004546304 0.003412509 0.002888441
## Mines 0.002048458 0.001512801 0.0011721834 0.003319516 0.002416471 0.002305310
## Coal  0.003004005 0.002148033 0.0016092014 0.004403696 0.003035672 0.002668143
## Oil   0.002689287 0.001548263 0.0011989563 0.002384509 0.001803157 0.001919911
## Util  0.001548263 0.002302818 0.0014858939 0.002356738 0.001498545 0.001624299
## Telcm 0.001198956 0.001485894 0.0017434794 0.001941428 0.001467703 0.001467637
## Servs 0.002384509 0.002356738 0.0019414284 0.005889783 0.003759695 0.003202721
## BusEq 0.001803157 0.001498545 0.0014677029 0.003759695 0.003689356 0.002586389
## Paper 0.001919911 0.001624299 0.0014676368 0.003202721 0.002586389 0.002984327
## Trans 0.002143107 0.002074697 0.0019457330 0.004308122 0.003012624 0.002936445
## Whlsl 0.002099220 0.002060240 0.0018024584 0.005287523 0.003509346 0.003169299
## Rtail 0.001597863 0.001890881 0.0016588596 0.003894240 0.002816905 0.002514304
## Meals 0.002318596 0.002227372 0.0020016341 0.005946390 0.004397330 0.003809226
## Fin   0.002194117 0.002226346 0.0017989678 0.004039802 0.002693562 0.002765477
## Other 0.002230187 0.002042418 0.0018073555 0.004405954 0.003210612 0.002895167
##             Trans       Whlsl       Rtail       Meals         Fin       Other
## Food  0.002708028 0.003008443 0.002719100 0.003723009 0.002654444 0.002783095
```

```
## Beer  0.002719694 0.003078338 0.002744466 0.003908460 0.002675904 0.002808395
## Smoke 0.002061129 0.002191121 0.001916270 0.002634400 0.001908440 0.002106432
## Games 0.004690575 0.005467739 0.004496497 0.006773906 0.003994563 0.004634442
## Books 0.003649873 0.004141137 0.003470346 0.004909285 0.003182334 0.003555631
## Hshld 0.002725360 0.003010468 0.002758424 0.004072410 0.002519394 0.002762212
## Clths 0.004191598 0.004952231 0.004113888 0.005826595 0.003638042 0.004073992
## Hlth  0.002327832 0.002741932 0.002191071 0.003533020 0.002410710 0.002577682
## Chems 0.003172802 0.003178663 0.002491462 0.003667325 0.002636791 0.002917480
## Txtls 0.003498283 0.004026296 0.003295985 0.004602827 0.003031354 0.003464783
## Cnstr 0.003899019 0.004305222 0.003507726 0.004864420 0.003613862 0.003819137
## Steel 0.003534841 0.003576900 0.002532814 0.003727156 0.002798336 0.003106333
## FabPr 0.003627911 0.003899270 0.002825066 0.004315947 0.003157696 0.003502615
## ElcEq 0.003436176 0.003803823 0.003090084 0.004479025 0.003175848 0.003496704
## Autos 0.002984212 0.003122514 0.003062504 0.003791024 0.002641034 0.002810507
## Carry 0.004191095 0.004465239 0.003350739 0.004976482 0.003305785 0.004008332
## Mines 0.002877118 0.003307310 0.002158043 0.003406301 0.002506945 0.002979719
## Coal  0.003916311 0.004238786 0.002522575 0.004369106 0.003223976 0.003897094
## Oil   0.002143107 0.002099220 0.001597863 0.002318596 0.002194117 0.002230187
## Util  0.002074697 0.002060240 0.001890881 0.002227372 0.002226346 0.002042418
## Telcm 0.001945733 0.001802458 0.001658860 0.002001634 0.001798968 0.001807355
## Servs 0.004308122 0.005287523 0.003894240 0.005946390 0.004039802 0.004405954
## BusEq 0.003012624 0.003509346 0.002816905 0.004397330 0.002693562 0.003210612
## Paper 0.002936445 0.003169299 0.002514304 0.003809226 0.002765477 0.002895167
## Trans 0.004883890 0.004302208 0.003271412 0.004692152 0.003533501 0.003995235
## Whlsl 0.004302208 0.005630311 0.003608472 0.005773394 0.003792042 0.004436814
## Rtail 0.003271412 0.003608472 0.003810500 0.004582978 0.003021310 0.003172330
## Meals 0.004692152 0.005773394 0.004582978 0.008198435 0.004141772 0.004756776
## Fin   0.003533501 0.003792042 0.003021310 0.004141772 0.003777324 0.003519508
## Other 0.003995235 0.004436814 0.003172330 0.004756776 0.003519508 0.004312600
```

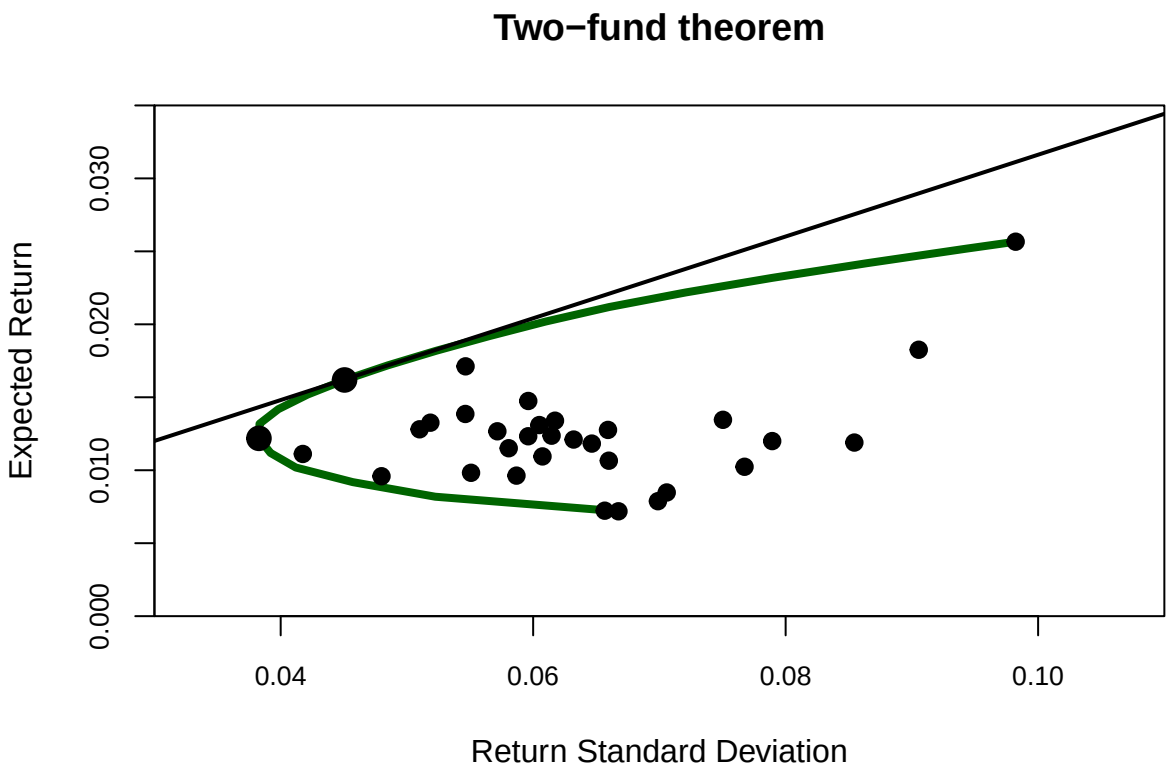
b) the weights for the first six portfolios on the efficient frontier.

```
# the minimum variance portfolio and the five following portfolios
minpos = sort(sigmaP.SSC,index.return=TRUE)$ix[1]
ex2b = data.frame(w.SSC[,minpos:(minpos+5)])
rownames(ex2b) = colnames(df)[2:31]; colnames(ex2b) = 1:6
ex2b
```

```
##                   1             2             3             4             5
## Food   6.659146e-17  4.499598e-18  4.500654e-17  4.068147e-17  6.506770e-17
## Beer   1.435800e-18 -2.235290e-17 -2.761221e-17 -3.683652e-17 -4.715410e-17
## Smoke  1.705888e-01  2.845191e-01  3.796667e-01  4.236809e-01  4.655487e-01
## Games -2.480498e-16 -8.823911e-18 -4.472479e-18 -3.470744e-18 -2.373726e-18
## Books -1.173218e-17  4.170448e-17  6.668004e-17  9.075838e-17  1.153076e-16
## Hshld  4.036568e-02 -7.225451e-17 -1.175782e-16 -1.618358e-16 -2.036629e-16
## Clths -6.338577e-17 -3.767838e-17  5.374399e-17  1.231197e-16  1.909542e-16
## Hlth   2.277521e-02 -1.284618e-16 -1.237987e-16 -1.138656e-16 -1.014865e-16
## Chems -1.168916e-16  1.107194e-16  1.350952e-16  1.159414e-16  9.544867e-17
## Txtls -2.049508e-17  1.627214e-16  1.748652e-16  1.923605e-16  2.105801e-16
## Cnstr  2.174830e-16  1.764941e-16  9.422782e-18 -1.275220e-16 -2.593932e-16
## Steel  2.542315e-17  1.351885e-16  1.771325e-16  2.484213e-16  2.978093e-16
## FabPr -9.057791e-18  3.047538e-16  2.956569e-16  2.969146e-16  2.991335e-16
## ElcEq  4.606157e-17  1.500060e-17  3.953700e-17  6.983221e-17  1.008852e-16
## Autos -6.192250e-18  3.582872e-17  1.819795e-17 -1.006571e-18 -2.260700e-17
```

```
## Carry -3.284118e-16  2.719319e-16  2.665662e-16  2.715963e-16  2.217199e-16
## Mines  1.612653e-17  1.022800e-02  9.965880e-03  2.281334e-18 -7.908407e-18
## Coal  -1.574670e-17  0.000000e+00  3.241741e-02  9.075133e-02  1.481785e-01
## Oil    1.120652e-01  1.518183e-01  1.322714e-01  9.651016e-02  5.603362e-02
## Util   9.862808e-02 -1.797656e-18  3.284437e-17  3.878266e-17  4.648875e-17
## Telcm  5.555770e-01  5.534346e-01  4.456787e-01  3.890576e-01  3.302392e-01
## Servs -3.242905e-17 -2.838961e-17 -1.033164e-16 -1.435497e-16 -1.807348e-16
## BusEq  1.602669e-17  1.404067e-17  3.488156e-18 -4.799566e-18 -1.427047e-17
## Paper -3.659030e-17 -4.918513e-18 -2.980989e-17 -7.472418e-18  1.343642e-17
## Trans -1.359401e-16 -3.734793e-17 -1.002051e-16 -1.627819e-16 -1.708068e-16
## Whlsl -4.278166e-17  9.897651e-19  1.769978e-17  1.554756e-17  1.495052e-17
## Rtail -8.480025e-18 -3.774160e-17 -3.614227e-17 -3.016021e-17 -2.798645e-17
## Meals  1.967893e-16 -6.456607e-17 -3.668422e-17 -1.251355e-17  1.349866e-17
## Fin    3.754423e-16 -1.421006e-17 -1.447719e-17 -3.393611e-18  1.269759e-17
## Other  5.416060e-17 -2.804909e-16 -3.564120e-16 -3.763000e-16 -3.929202e-16
##                   6
## Food   8.945392e-17
## Beer  -5.747167e-17
## Smoke  5.074165e-01
## Games -1.276709e-18
## Books  1.398568e-16
## Hshld -2.454901e-16
## Clths  2.587887e-16
## Hlth  -8.910748e-17
## Chems  7.495593e-17
## Txtls  2.287996e-16
## Cnstr -3.912644e-16
## Steel  3.471974e-16
## FabPr  3.013524e-16
## ElcEq  1.874492e-16
## Autos -4.420743e-17
## Carry  2.828657e-16
## Mines  8.537269e-18
## Coal   2.056056e-01
## Oil    1.555707e-02
## Util   5.419483e-17
## Telcm  2.714208e-01
## Servs -2.179200e-16
## BusEq -2.374138e-17
## Paper -8.673617e-19
## Trans -2.898540e-16
## Whlsl  1.435347e-17
## Rtail -2.581268e-17
## Meals  5.338866e-17
## Fin    2.878880e-17
## Other -4.095404e-16
```

Tobin's two-fund theorem



c) the weights of the tangency portfolio

```
ex2c = data.frame(t(w.SSC[,root]))
colnames(ex2c) = colnames(df)[2:31]; ex2c
```

```
##          Food         Beer     Smoke         Games        Books         Hshld
## 1 6.50677e-17 -4.71541e-17 0.4655487 -2.373726e-18 1.153076e-16 -2.036629e-16
##          Clths          Hlth        Chems        Txtls         Cnstr
## 1 1.909542e-16 -1.014865e-16 9.544867e-17 2.105801e-16 -2.593932e-16
##          Steel        FabPr        ElcEq       Autos        Carry         Mines
## 1 2.978093e-16 2.991335e-16 1.008852e-16 -2.2607e-17 2.217199e-16 -7.908407e-18
##         Coal         Oil         Util     Telcm         Servs         BusEq
## 1 0.1481785 0.05603362 4.648875e-17 0.3302392 -1.807348e-16 -1.427047e-17
##          Paper         Trans        Whlsl         Rtail        Meals
## 1 1.343642e-17 -1.708068e-16 1.495052e-17 -2.798645e-17 1.349866e-17
##            Fin         Other
## 1 1.269759e-17 -3.929202e-16
```

d) the out-of-sample return (for January 1977) of the tangency portfolio

```
# compute the out-of-sample return (for January 1977) of the tangency portfolio
R = sum(df[df$X == 197701,][,2:31]*w.SSC[,root])
# compare the return
print(paste("the out-of-sample return in January 1977 of the tangency portfolio:",round(R,4),"%"))
```

```
## [1] "the out-of-sample return in January 1977 of the tangency portfolio: -2.0268 %"
```

```
print(paste("the return of the 1/N portfolio in January 1977:",df[df$X == 197701,]$`1/N`,"%"))
```

```
## [1] "the return of the 1/N portfolio in January 1977: -3.675 %"
```

Thoughts:

- The out-of-sample return (for January 1977) of the tangency portfolio is higher than the return of the 1/N portfolio, which means **Tobin's method** works **better** than naïve diversification for January 1977.

Exercise 3: Optimal vs. Naïve Diversification

*Create a **function (called out_of_sample)** with 2 input (Date: such as 197701, window: such as 120) and 1 output which is the out-of-sample returns of the tangency portfolio (see file **"Problem_set_1_yanjue.html"** for the complete code)*

Compute the time series of out-of-sample returns of the tangency portfolio from January 1977 to December 2022, based on a rolling estimation window of *120* months.

```
D = df[df$X >= 197701,]$X
# the time series of out-of-sample returns
R120 = rep(NA,length(D))
# the time series of out-of-sample excess returns
RE120 = rep(NA,length(D))
for(i in 1:length(D)){
  R120[i]=out_of_sample(D[i],120)
  RE120[i]=R120[i] - df[df$X==D[i],]$RF
}
```

a) the first six out-of-sample returns (January 1977 to June 1977) for the tangency portfolio.

```
R120[1:6]
```

```
## [1] -2.0267909  0.7776566 -1.0779955  0.6582210 -0.8818662  3.7262269
```

b) the out-of-sample Sharpe Ratio for the tangency portfolio, based on the full time series of out-of-sample returns (i.e., January 1977 - December 2022)

```
mu2 = mean(RE120)
std2 = sd(RE120)
SR2 = mu2/std2
print(paste("the out-of-sample Sharpe Ratio for rolling window of 120:",round(SR2,4)))
```

```
## [1] "the out-of-sample Sharpe Ratio for rolling window of 120: 0.2216"
```

Compute the time series of out-of-sample returns of the tangency portfolio , based on a rolling estimation window of *480* months.

```
D = df[df$X >= 197701,]$X
# the time series of out-of-sample returns
R480 = rep(NA,length(D))
# the time series of out-of-sample excess returns
RE480 = rep(NA,length(D))
for(i in 1:length(D)){
  R480[i]=out_of_sample(D[i],480)
  RE480[i]=R480[i] - df[df$X==D[i],]$RF
}
mu3 = mean(RE480)
std3 = sd(RE480)
SR3 = mu3/std3
print(paste("the out-of-sample Sharpe Ratio for rolling window of 480:",round(SR3,4)))
```

```
## [1] "the out-of-sample Sharpe Ratio for rolling window of 480: 0.2285"
```

Compute the time series of out-of-sample returns of the tangency portfolio , based on a rolling estimation window of *600* months.

```
D = df[df$X >= 197701,]$X
# the time series of out-of-sample returns
R600 = rep(NA,length(D))
# the time series of out-of-sample excess returns
RE600 = rep(NA,length(D))
for(i in 1:length(D)){
  R600[i]=out_of_sample(D[i],600)
  RE600[i]=R600[i] - df[df$X==D[i],]$RF
}
mu4 = mean(RE600)
std4 = sd(RE600)
SR4 = mu4/std4
print(paste("the out-of-sample Sharpe Ratio for rolling window of 600:",round(SR4,4)))
```

```
## [1] "the out-of-sample Sharpe Ratio for rolling window of 600: 0.2387"
```

Compare *the Sharpe Ratios* for tangency portfolios with the three different rolling estimation windows to that of the 1/N portfolio from Exercise 1.

```
## [1] "the out-of-sample Sharpe Ratio for rolling window of 120: 0.2216"
```

```
## [1] "the out-of-sample Sharpe Ratio for rolling window of 480: 0.2285"
```

```
## [1] "the out-of-sample Sharpe Ratio for rolling window of 600: 0.2387"
```

```
## [1] "the out-of-sample Sharpe Ratio of 1/N portfolio: 0.2245"
```

Thoughts:

- Sharpe Ratio is a measure of risk-adjusted return which can measure the performance of a portfolio.

- The tangency portfolio with a rolling window of **120** performs **the worst**, even worse than the 1/N portfolio, and the tangency portfolio with a rolling window of **600** performs **the best**. So we can't say definitely that the optimal diversification is better than the naïve method, it depends.
- Comparing the 3 optimal diversification, we can see that the **longer** estimation windows we use, the better the tangency portfolio performs, which means **collecting more historical data** will contribute to the risk-adjusted return of the tangency portfolio.