CHAIN RULE

DIFFERENTIAL CALCULUS

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TOPIC OUTLINE

Chain Rule

- **Leibniz Notation**

CHAIN RULE

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If g is differentiable at x and f is differentiable at $g(x)$, then the composite function defined by

$$F(x) = f(g(x))$$

is differentiable at x and F' is given by product

$$F'(x) = f'(g(x)) \cdot g'(x)$$

Differentiate the function $y = \sin(2x)$.

CHAIN RULE

Differentiate the function $y = \cos(\sqrt{x})$.

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EXERCISE

Differentiate the function $y = \sqrt{x^2 + 1}$.

Solution

EXERCISE

Differentiate the function $y = \tan(5x^3 + 2x)$.

Solution

LEIBNIZ NOTATION

If $\boldsymbol{y} = \boldsymbol{f}(\boldsymbol{u})$ and $\boldsymbol{u} = \boldsymbol{g}(\boldsymbol{x})$ are both differentiable functions, then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Differentiate the function $y = \sin(2x)$.

LEIBNIZ NOTATION

Differentiate the function $y = \cos(\sqrt{x})$.

If $\boldsymbol{y} = \boldsymbol{f}(\boldsymbol{u})$ and $\boldsymbol{u} = \boldsymbol{g}(\boldsymbol{x})$ are both differentiable functions, then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

EXERCISE

Differentiate $y = \sin x^2$.

Solution

EXERCISE

Differentiate $y = x \sin \frac{1}{x}$.

Solution

EXERCISE

Differentiate $y = \frac{x^3}{\sqrt{1+x^2}}$.

Solution

EXERCISE

Differentiate $g(t) = \left(\frac{t-2}{2t+1}\right)^9$.

Solution

EXERCISE

The displacement of a particle on a vibrating string is given by the equation

$$\boldsymbol{s(t) = 10 + \frac{1}{4} sin(10\pi t)}$$

where s is measured in centimeters and t in seconds. Find the velocity of the particle in t seconds.

Solution

EXERCISE

Differentiate $y = \dfrac{\cos \pi x}{\sin \pi x + \cos \pi x}$.

Solution

EXERCISE

Differentiate $y = [x + (x + \sin^2 x)^3]^4$.

Solution

EXERCISE

A Cepheid variable star is a star whose brightness alternately increases and decreases. The most easily visible such star is Delta Cephei, for which the interval between times of maximum brightness is 5.4 days. The average brightness of this star is 4.0 and its brightness changes by ± 0.35. In view of these data, the brightness of Delta Cephei at time t, where t is measured in days, has been modeled by the function

$$\boldsymbol{B(t) = 4.0 + 0.35\, sin\left(\frac{2\pi t}{5.4}\right)}$$

Find the rate of change of the brightness after t days.

Solution

LABORATORY