MEASURES OF VARIABILITY

DESCRIPTIVE STATISTICS

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TOPIC OUTLINE

Measures of Variability

Range and Interquartile Range

Variance and Standard Deviation

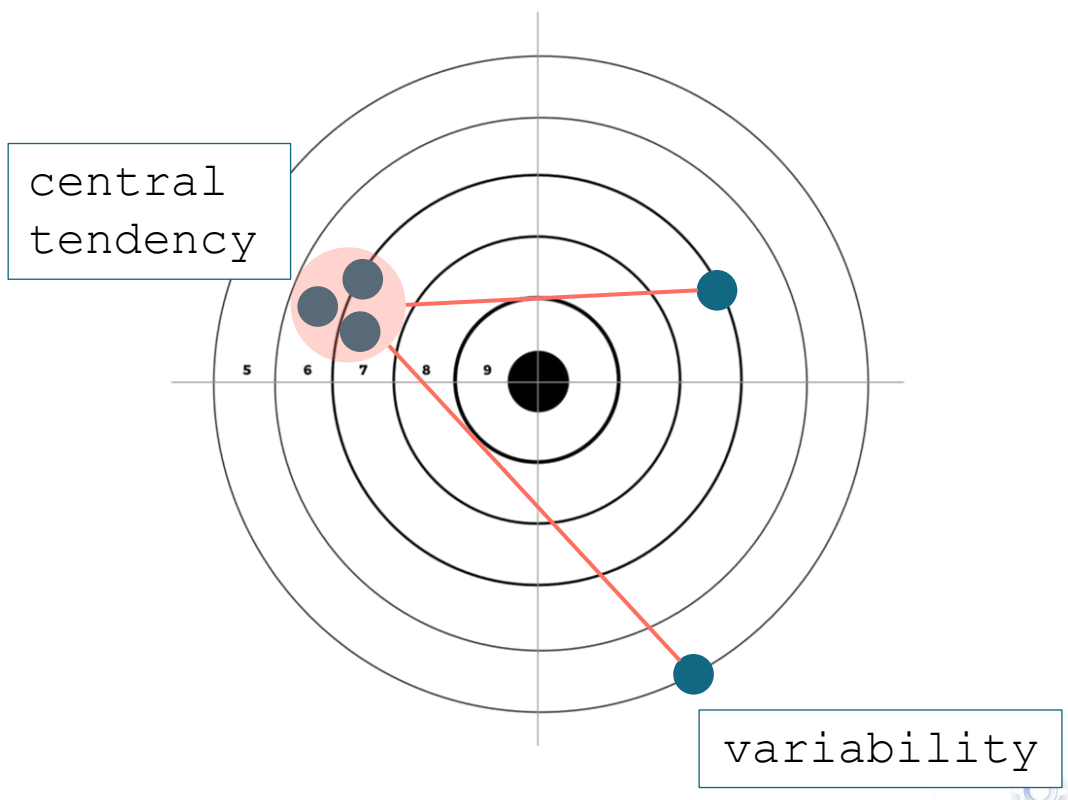
Coefficient of Variation

MEASURES OF VARIABILITY

MEASURES OF VARIABILITY

Measures of variability (or dispersion) describe how **spread out** or scattered the data points relative to the central tendency (mean, median, or mode).

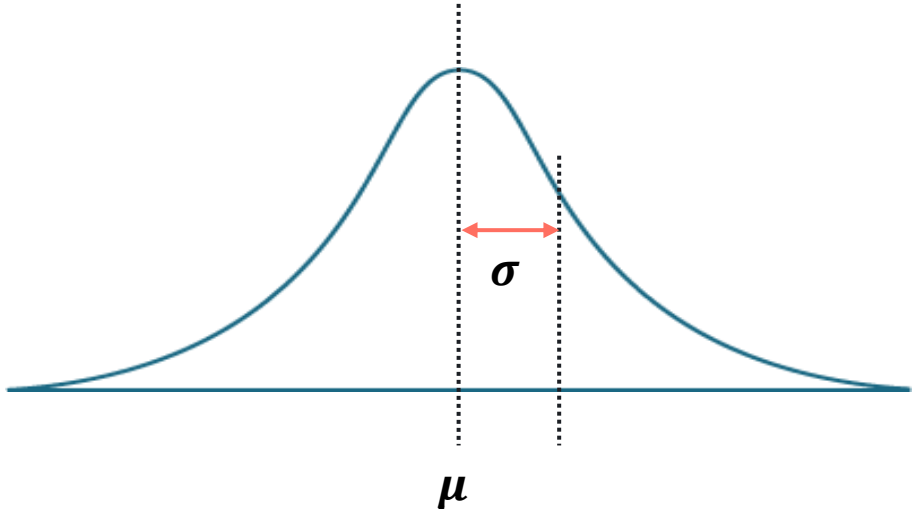
Dartboard Analogy



MEASURES OF VARIABILITY

Measures of variability (or dispersion) describe how **spread out** or scattered the data points relative to the central tendency (mean, median, or mode).

Normal Distribution

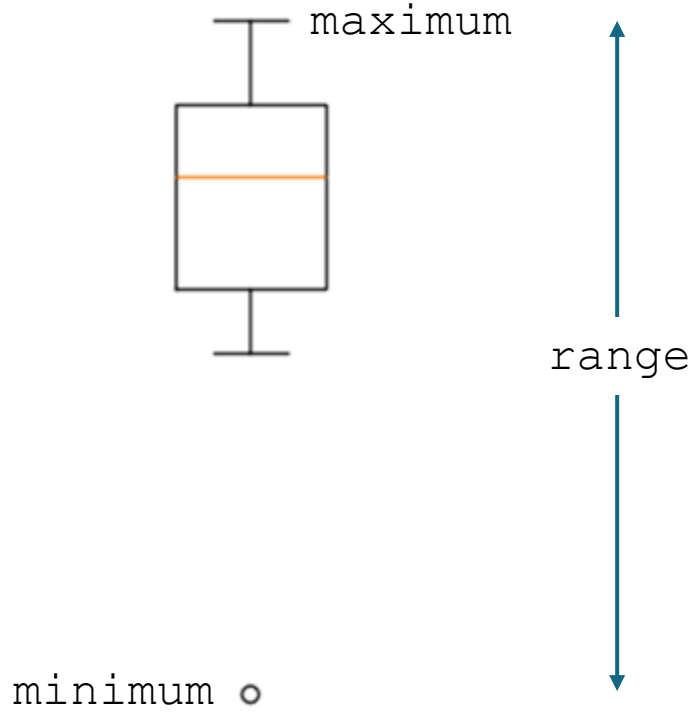


RANGE AND INTERQUARTILE

RANGE

RANGE

Boxplot



Range is the simplest measure of variability and is calculated as the difference between the **maximum** and **minimum** values in a dataset.

Formula

$$\text{range} = \text{maximum value} - \text{minimum value}$$

INTERQUARTILE RANGE

The **interquartile range (IQR)** measures the spread of the middle 50% of the data, reducing the influence of outliers.

Formula

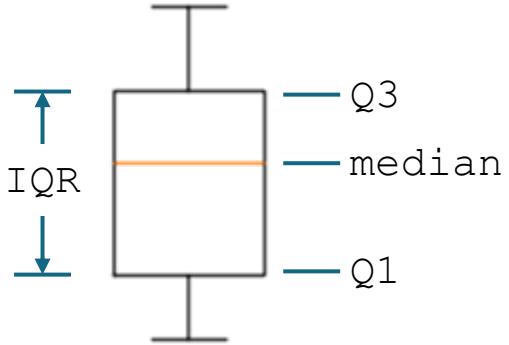
$$IQR = Q_3 - Q_1$$

where

Q_1(first quartile) is the median of the lower half of the data (25%).

Q_3(third quartile) is the median of the upper half of the data (75%).

Boxplot



OUTLIERS

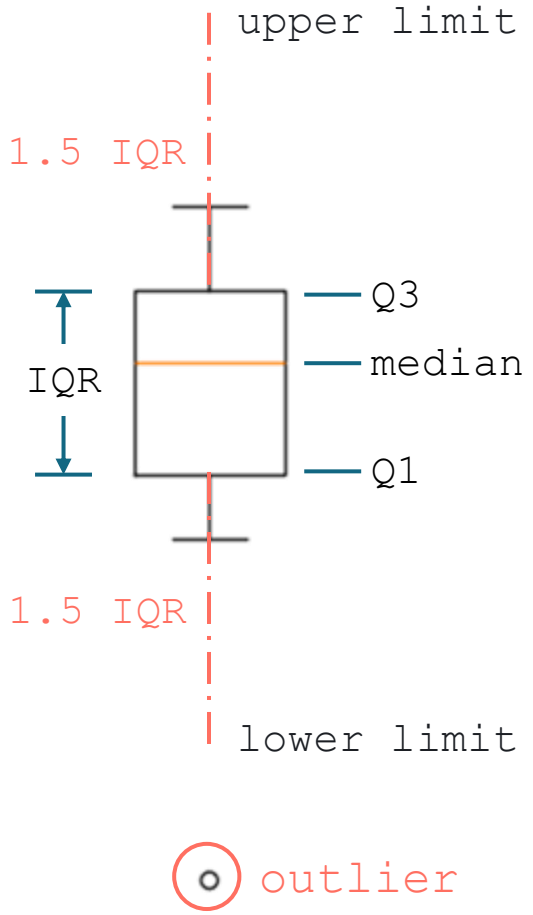
Outliers are data points that lie significantly **outside** the typical range of the rest of the dataset.

Formula

$$\textbf{Lower limit} = Q_1 - 1.5\,IQR$$

$$\textbf{Upper limit} = Q_3 + 1.5\,IQR$$

Boxplot



LINEAR INTERPOLATION

For a given sorted dataset $\boldsymbol{x} = [\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_n}]$, the pth quantile position is :

$$\mathbf{pos} = \boldsymbol{p}(\boldsymbol{n} - \mathbf{1}) + \mathbf{1}$$

where

$p = 0.25$ for Q_1

$p = 0.5$ for Q_2(median)

$p = 0.75$ for Q_3

$n =$ number of samples

then, the quantile $\boldsymbol{Q}$ is :

$$\boldsymbol{Q} = \boldsymbol{x_i} + \boldsymbol{f}(\boldsymbol{x_{i+1}} - \boldsymbol{x_i})$$

where

$x =$ data point

$i =$ integer part of the position

$f =$ fractional part of the position

EXERCISE

The dataset provided contains the exam grades of 12 students. Calculate the **range** and **interquartile range** (IQR) to analyze the spread and variability of the grades.

solution

Exam Performance

Student	Grade
1	3.5
2	6.7
3	7
4	7.4
5	7.8
6	8.2
7	8.5
8	8.8
9	9
10	9.1
11	9.4
12	9.8

VARIANCE AND STANDARD DEVIATION

VARIANCE

Variance measures the **average squared deviation** of each data point from the mean.

Population Variance

$$\sigma^2 = \frac{\sum_{i=1}^{N}(x_i - \mu)^2}{N}$$

Sample Variance

$$s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$$

STANDARD DEVIATION

The **standard deviation** is the **square root** of variance.

Population Standard Deviation

$$\sigma = \sqrt{\sigma^2}$$

Sample Standard Deviation

$$s = \sqrt{s^2}$$

EXERCISE

The dataset provided contains the sugar content (in grams) per serving for 10 popular breakfast cereals. Calculate the **variance** and **standard deviation** to measure the spread or variability in the sugar content across these cereals.

Breakfast Cereal

Brand	Sugar
A	12
B	9
C	15
D	8
E	10
F	11
G	13
H	7
I	14
J	6

Solution

POOLED STANDARD DEVIATION

Pooled standard deviation is a weighted **average** of the standard deviations from two or more groups.

Formula

$$\bar{\sigma}_{pooled} = \sqrt{\frac{\sigma_1^2 + \sigma_2^2 + \cdots \sigma_k^2}{k}}$$

where

k = number of groups

Variances add

$$\sigma_{total}^2 = \sigma_1^2 + \sigma_2^2 + \cdots \sigma_k^2$$

Standard deviations do not

$$\sigma_{total} \neq \sigma_1 + \sigma_2 + \cdots \sigma_k$$

EXERCISE

The dataset provided contains the battery life (in hours) for smartphones from different models. Calculate the **pooled standard deviation** to measure the combined variability in battery life across these models.

Solution

Battery Life

Model	Hours
A	12.5
A	12.8
A	12.7
A	13.3
A	12.6
B	13.5
B	14.1
B	13.9
B	14.3
B	13.7
C	11.8
C	11.9
C	12.1
C	12.2
C	11.6

COEFFICIENT OF VARIATION

COEFFICIENT OF VARIATION

Coefficient of variation (c_v) is a relative measure of variability, expressed as the **ratio** of the standard deviation to the mean.

Population Coefficient of Variation

$$c_v = \frac{\sigma}{\mu}$$

Sample Coefficient of Variation

$$\hat{c}_v = \frac{s}{\bar{x}}$$

EXERCISE

The provided dataset includes ice cream prices listed in both USD and PHP. Calculate the **standard deviation** and **coefficient of variation** for each currency to analyze the variability in prices.

Solution

Ice Cream Price List

Brand	Price (USD)	Price (PHP)
Brand A	3.5	203
Brand B	4	232
Brand C	3.75	217.5
Brand D	4.25	246.5
Brand E	3.9	226.2
Brand F	4.1	237.8
Brand G	3.6	208.8
Brand H	4.5	261
Brand I	3.8	220.4
Brand J	4.15	240.7

EXERCISE

The given dataset consists of test results from two machines, the Jaguar and Panther models, which produce **10 Ω** resistors with ±5% tolerance. Determine which machine performs better based on its **measures of variability** for resistance values.

Resistance Test

Test No.	Jaguar	Panther
1	10.6	10.1
2	9.1	11
3	9.3	9.1
4	9.8	20
5	10.5	9.2
6	10.4	10.8
7	9.5	9.9
8	11	9.2
9	10.4	9.1
10	3	9.1
11	9.8	

LABORATORY