DIFFERENTIATION FORMULAS

DIFFERENTIAL CALCULUS

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TOPIC OUTLINE

Differentiation Formulas

- Constant Function Rule
- Power Rule
- Constant Multiple Rule
- Sum and Difference Rule
- Product Rule
- Quotient Rule

DIFFERENTIATION FORMULAS

CONSTANT FUNCTION RULE

$$\frac{d}{dx}(c) = 0$$

Determine the derivative of $y = 5$.

$$dy/dx = d/dx\,(5)$$

$$\boxed{dy/dx = 0}$$

ans

Differentiate the function $z = 1000$.

$$y' = d/dx\,(1000)$$

$$\boxed{y' = 0}$$

ans

POWER RULE

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Determine the derivative of $y = x^4$.

$$dy/dx = 4x^{4-1}$$

$$\boxed{dy/dx = 4x^3}$$

ans

Differentiate the function $q = t^{-7}$.

$$q' = -7t^{-7-1}$$

$$\boxed{q' = -7t^{-8}}$$

ans

CONSTANT MULTIPLE RULE

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$$

Determine the derivative of $y = 3x^4$.

$$y' = 3\,\frac{d}{dx}(x^4)$$

$$y' = 3(4x^{4-1})$$

$$\boxed{y' = 12x^3}$$

ans

Differentiate the function $m = 5n^{-3}$.

$$\frac{dm}{dn} = 5\,\frac{d}{dn}(n^{-3})$$

$$\frac{dm}{dn} = 5(-3n^{-3-1})$$

$$\boxed{\frac{dm}{dn} = -15n^{-4}}$$

ans

SUM AND DIFFERENCE RULE

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

$$\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

Determine the derivative of $q = t^3 + t^{-2} - 2t^5$.

$$dq/dt = d/dt(t^3) + d/dt(t^{-2}) - 2\, d/dt(t^5)$$

$$\boxed{dq/dt = 3t^2 - 2t^{-3} - 10t^4}$$

ans

Differentiate the function

$y = 8x^7 + 60x^4 - 16x^3 + 30x^2 - \cancel{6}.$ 0

$$\boxed{y' = 56x^6 + 240x^3 - 48x^2 + 60x}$$

ans

PRODUCT RULE

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$$

$$d(uv) = u\,dv + v\,du$$

Determine the derivative of $y = 2x^2\underline{x^3}$.

$$dy/dx = 2x^2 \, d/dx(x^3) + x^3 \, d/dx(2x^2)$$

$$dy/dx = 2x^2(3x^2) + x^3(4x)$$

$$dy/dx = 6x^4 + 4x^4 \rightarrow \boxed{dy/dx = 10x^4}$$

ans

Differentiate the function $a = b^2\cancel{\sqrt{b}}$. $\rightarrow b^{\frac{1}{2}}$

$$da/db = b^2 \, d/db\left(b^{\frac{1}{2}}\right) + b^{\frac{1}{2}} \, d/db\left(b^2\right)$$

$$da/db = b^2\left(\frac{1}{2}b^{-\frac{1}{2}}\right) + b^{\frac{1}{2}}(2b)$$

$$da/db = \frac{1}{2}b^{\frac{3}{2}} + 2b^{\frac{3}{2}}$$

$$\boxed{da/db = \frac{5}{2}\sqrt{b^3}}$$

ans

QUOTIENT RULE

$$v' = \frac{2w^2 + 4w - w^2 - 1}{(w+2)^2}$$

$$\boxed{v' = \frac{w^2 + 4w - 1}{(w+2)^2}}$$

ans

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{[g(x)]^2}$$

$$d\left(\frac{u}{v}\right) = \frac{v\,du - u\,dv}{v^2}$$

Determine the derivative of $y = \frac{x^2}{x+1}$.

$$y' = \frac{(x+1)\frac{d}{dx}(x^2) - x^2\frac{d}{dx}(x+1)}{(x+1)^2}$$

$2x^2+2x$

$$y' = \frac{(x+1)2x - x^2\cancel{(1+0)}^{1}}{(x+1)^2} \rightarrow \boxed{y' = \frac{x^2+2x}{(x+1)^2}}$$

ans

Differentiate the function $v = \frac{w^2+1}{w+2}$.

$$v' = \frac{(w+2)\frac{d}{dw}(w^2+1) - (w^2+1)\frac{d}{dw}(w+2)}{(w+2)^2}$$

$$v' = \frac{(w+2)(2w) - (w^2+1)(1+0)}{(w+2)^2}$$

EXERCISE

Differentiate the function $y = x^{-3}$.

Solution

$$dy/dx = d/dx\,(x^{-3})$$

$$dy/dx = -3x^{-4}$$

$$\boxed{dy/dx = -\frac{3}{x^4}}$$

ans

EXERCISE

If $y = \sqrt{x}$, find y'.

Solution

$$y' = \frac{d}{dx}(\sqrt{x})$$

$$y' = \frac{d}{dx}\left(x^{\frac{1}{2}}\right)$$

$$y' = \frac{1}{2}x^{-\frac{1}{2}}$$

$$\boxed{y' = \frac{1}{2\sqrt{x}}}$$

ans

EXERCISE

If $y = \frac{1}{x}$, find y'.

x^{-1}

$$y' = \frac{d}{dx}(x^{-1})$$

$$y' = -x^{-2}$$

$$\boxed{y' = -\frac{1}{x^2}}$$

ans

Solution

$$y' = \frac{x \cancel{\frac{d}{dx}(1)}^{0} - 1\frac{d}{dx}(x)}{x^2}$$

$$y' = \frac{\cancel{x(0)}^{0} - 1(1)}{x^2}$$

$$\boxed{y' = -\frac{1}{x^2}}$$

ans

EXERCISE

Differentiate the function $y = 6x^3 7x^4$.

Solution

$$dy/dx = 6x^3 \, d/dx(7x^4) + 7x^4 \, d/dx(6x^3)$$

$$dy/dx = 6x^3(28x^3) + 7x^4(18x^2)$$

$$dy/dx = 168x^6 + 126x^6$$

$$\boxed{dy/dx = 294x^6}$$

ans

EXERCISE

Differentiate the function $y = x^{3/2}x^4$.

$$\frac{3}{2} - 1 \cdot \frac{2}{2} = \frac{3}{2} - \frac{2}{2} = \frac{1}{2}$$

$$\frac{3}{2} + 3 \cdot \frac{2}{2} = \frac{3}{2} + \frac{6}{2} = \frac{9}{2}$$

$$4 \cdot \frac{2}{2} + \frac{1}{2} = \frac{8}{2} + \frac{1}{2} = \frac{9}{2}$$

$$4 \cdot \frac{2}{2} + \frac{3}{2} = \frac{8}{2} + \frac{3}{2} = \frac{11}{2}$$

Solution

$$y' = x^{\frac{3}{2}} \frac{d}{dx}(x^4) + x^4 \frac{d}{dx}\left(x^{\frac{3}{2}}\right)$$

$$y' = x^{\frac{3}{2}}(4x^3) + x^4\left(\frac{3}{2}x^{\frac{1}{2}}\right)$$

$$y' = 4x^{\frac{9}{2}} + \frac{3}{2}x^{\frac{9}{2}}$$

$$y' = \frac{11}{2}x^{\frac{9}{2}}$$

$$\boxed{y' = \frac{11}{2}\sqrt{x^9}}$$

ans

EXERCISE

Differentiate the function $y = \sqrt{x}\,(x-1)$.

$$1 \cdot \frac{2}{2} + \frac{1}{2} = \frac{2}{2} + \frac{1}{2} = \frac{3}{2}$$

Solution

$$y' = \sqrt{x}\,\frac{d}{dx}(x-1) + (x-1)\frac{d}{dx}(\cancel{\sqrt{x}}) \quad x^{\frac{1}{2}}$$

$$y' = x^{\frac{1}{2}}(1-0) + (x-1)\frac{1}{2}x^{-\frac{1}{2}}$$

$$y' = x^{\frac{1}{2}} + \frac{1}{2}x^{\frac{1}{2}} - \frac{1}{2}x^{-\frac{1}{2}}$$

$$y' = \frac{3}{2}x^{\frac{1}{2}} - \frac{1}{2}x^{-\frac{1}{2}}$$

$$\boxed{y' = \frac{3}{2}\sqrt{x} - \frac{1}{2\sqrt{x}}}$$

ans

EXERCISE

Differentiate the function $y = \frac{x^2+x-2}{x^3+6}$.

$$y' = \frac{(x^3+6)\frac{d}{dx}(x^2+x-2) - (x^2+x-2)\frac{d}{dx}(x^3+6)}{(x^3+6)^2}$$

$$y' = \frac{(x^3+6)(2x+1) - (x^2+x-2)(3x^2)}{(x^3+6)^2}$$

$$y' = \frac{(2x^4+x^3+12x+6) - (3x^4+3x^3-6x^2)}{(x^3+6)^2}$$

Solution

$$\boxed{y' = \frac{-x^4-2x^3+6x^2+12x+6}{(x^3+6)^2}}$$

ans

EXERCISE

Find the slope of the tangent line to the curve

$y = \frac{\sqrt{x}}{1+x^2}$ at the point $\left(1, \frac{1}{2}\right)$.

x, y

$$\frac{dy}{dx} = \frac{(1+x^2)\frac{d}{dx}(\sqrt{x}) - \sqrt{x}\frac{d}{dx}(1+x^2)}{(1+x^2)^2}$$

$x^{\frac{1}{2}}$

$$\frac{dy}{dx} = \frac{(1+x^2)\frac{1}{2}x^{-\frac{1}{2}} - x^{\frac{1}{2}}(2x)}{(1+x^2)^2}$$

$$\frac{dy}{dx} = \frac{\frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}x^{\frac{3}{2}} - 2x^{\frac{3}{2}}}{(1+x^2)^2}$$

Solution

$$\frac{dy}{dx} = \frac{\frac{1}{2\sqrt{x}} - \frac{3}{2}\sqrt{x^3}}{(1+x^2)^2}$$

@ $x=1$

$$\frac{dy}{dx} = \frac{\frac{1}{2\sqrt{1}} - \frac{3}{2}\sqrt{1^3}}{(1+1^2)^2}$$

$$\frac{dy}{dx} = \frac{\frac{1}{2} - \frac{3}{2}}{4}$$

$$\boxed{\frac{dy}{dx} = -\frac{1}{4}}$$

ans

EXERCISE

Differentiate the function $s = \sqrt{p} - p$.

Solution

$$\frac{ds}{dp} = \frac{d}{dp}\left(p^{\frac{1}{2}}\right) - \frac{d}{dp}(p)$$

$$\frac{ds}{dp} = \frac{1}{2}p^{-\frac{1}{2}} - 1$$

rationalize the denominator

$$\frac{ds}{dp} = \frac{1}{2\sqrt{p}} \cdot \frac{\sqrt{p}}{\sqrt{p}} - 1$$

$$\boxed{\frac{ds}{dp} = \frac{\sqrt{p}}{2p} - 1}$$

ans

EXERCISE

Differentiate the function $v = \left(\sqrt{x} + \frac{1}{\sqrt[3]{x}}\right)^2$.

$$v = x + 2\frac{\sqrt{x}}{\sqrt[3]{x}} + \frac{1}{(\sqrt[3]{x})^2}$$

$$v = x + 2\frac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}} + \frac{1}{(x^{\frac{1}{3}})^2}$$

$$x$$

$$\frac{1}{2}\cdot\frac{3}{3} - \frac{1}{3}\cdot\frac{2}{2} = \frac{3}{6} - \frac{2}{6} = \frac{1}{6}$$

$$v = x + 2x^{\frac{1}{6}} + \cancel{\frac{1}{x^{\frac{2}{3}}}} \rightarrow x^{-\frac{2}{3}}$$

Solution

$$\frac{1}{6} - 1\cdot\frac{6}{6} = -\frac{5}{6} \qquad -\frac{2}{3} - 1\cdot\frac{3}{3} = -\frac{5}{3}$$

$$dv/dx = 1 + 2x^{-\frac{5}{6}} - \frac{2}{3}x^{-\frac{5}{3}}$$

$$\boxed{dv/dx = 1 + \frac{1}{2x^{5/6}} - \frac{2}{3x^{5/3}}}$$

ans

EXERCISE

Find the slope of a tangent line to the curve

$y = \frac{2x}{x+1}$ at the point $(\overset{x,\,y}{1}, 1)$.

$$y' = \frac{(x+1)\frac{d}{dx}(2x) - 2x\frac{d}{dx}(x+1)}{(x+1)^2}$$

$$y' = \frac{(x+1)(2) - 2x(1)}{(x+1)^2}$$

$$y' = \frac{\cancel{2x}^{\,0} + 2 - \cancel{2x}^{\,0}}{(x+1)^2}$$

Solution

$$y' = \frac{2}{(x+1)^2}$$

@ $x=1$

$$y' = \frac{2}{(1+1)^2}$$

$$y' = \frac{2}{4}$$

$$\boxed{y' = \frac{1}{2}}$$

ans

LABORATORY