



THE KARNAUGH MAP

LOGIC MINIMIZATION

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TOPIC OUTLINE

SOP Minimization

POS Minimization



SOP_MINIMIZATION



KARNAUGH MAP

A Karnaugh map is an array of cells in which each cell represents a binary value of the input variables. It provides a systematic method for simplifying Boolean expressions.

3-variable K-map

AB \ C	C	
	0	1
00	$\bar{A}\bar{B}\bar{C}$	$\bar{A}\bar{B}C$
01	$\bar{A}B\bar{C}$	$\bar{A}BC$
11	$AB\bar{C}$	ABC
10	$A\bar{B}\bar{C}$	$A\bar{B}C$



KARNAUGH MAP

A Karnaugh map is an array of cells in which each cell represents a binary value of the input variables. It provides a systematic method for simplifying Boolean expressions.

4-variable K-map

$AB \backslash CD$		00	01	11	10
00	$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}\bar{B}\bar{C}D$	$\bar{A}\bar{B}CD$	$\bar{A}\bar{B}C\bar{D}$	
01	$\bar{A}B\bar{C}\bar{D}$	$\bar{A}B\bar{C}D$	$\bar{A}BCD$	$\bar{A}BC\bar{D}$	
11	$AB\bar{C}\bar{D}$	$AB\bar{C}D$	$ABCD$	$ABC\bar{D}$	
10	$A\bar{B}\bar{C}\bar{D}$	$A\bar{B}\bar{C}D$	$A\bar{B}CD$	$A\bar{B}C\bar{D}$	



MAPPING A STANDARD SOP EXPRESSION

A 1 is placed on the K-map cell that corresponds to the value of a product term in the expression.

3-variable K-map

$$f = \bar{A}\bar{B}\bar{C} + \bar{A}BC + AB\bar{C} + ABC$$

000 011 110 111

		C	
		0	1
AB	00	1	
	01		1
	11	1	1
	10		



GROUPING 1s

A group must contain either 1, 2, 4, 8, or 16 cells, which are all powers of two.

AB \ C	C	
	0	1
00	1	
01		1
11	1	1
10		

AB \ C	C	
	0	1
00	1	1
01	1	
11		1
10	1	1

AB \ CD	CD			
	00	01	11	10
00	1	1		
01	1	1	1	1
11				
10		1	1	

AB \ CD	CD			
	00	01	11	10
00	1			1
01	1	1		1
11	1	1		1
10	1		1	1

SOP MINIMIZATION

The process that results in an expression containing the fewest possible terms with the fewest possible variables is called minimization.

AB \ C	C	
	0	1
00	1	
01		1
11	1	1
10		

$$f = \bar{A}\bar{B}\bar{C} + AB + BC$$

AB \ C	C	
	0	1
00	1	1
01	1	
11		1
10	1	1

$$f = \bar{B} + \bar{A}\bar{C} + AC$$

AB \ CD	CD			
	00	01	11	10
00	1	1		
01	1	1	1	1
11				
10		1	1	

$$f = \bar{A}\bar{C} + \bar{A}B + A\bar{B}D$$

AB \ CD	CD			
	00	01	11	10
00	1			1
01	1	1		1
11	1	1		1
10	1		1	1

$$f = \bar{D} + B\bar{C} + A\bar{B}C$$

Variables that occur both complemented and uncomplemented within the group are eliminated. These are called contradictory variables.

EXERCISE

Use a Karnaugh map to minimize the given standard SOP expression.

$$f = \sum m(2, 4, 6)$$

$$f = \overline{A}B\overline{C} + A\overline{B}\overline{C} + AB\overline{C}$$

$\overset{2}{/}$ $\overset{4}{/}$ $\overset{6}{/}$
 010 100 110

Solution

$\begin{matrix} A \backslash B \\ C \end{matrix}$	<u>0</u>	1
00		
<u>01</u>	1	
<u>11</u>	1	
<u>10</u>	1	

$$f = \underline{B\overline{C}} + \underline{A\overline{C}}$$

$$f = \overline{C}(A+B)$$

ans

EXERCISE

Use a Karnaugh map to minimize the given standard SOP expression.

$$f = \sum m(13, 14, 15)$$

$$f = \overset{13}{AB\bar{C}D} + \overset{14}{AB\bar{C}\bar{D}} + \overset{15}{ABCD}$$

$\begin{matrix} 1101 & 1110 & 1111 \end{matrix}$

Solution

AB \ CD	00	01	11	10
00				
01				
<u>11</u>		1	1	1
10				

$$f = \underline{ABD} + \underline{ABC}$$

$$f = AB(C+D)$$

ans



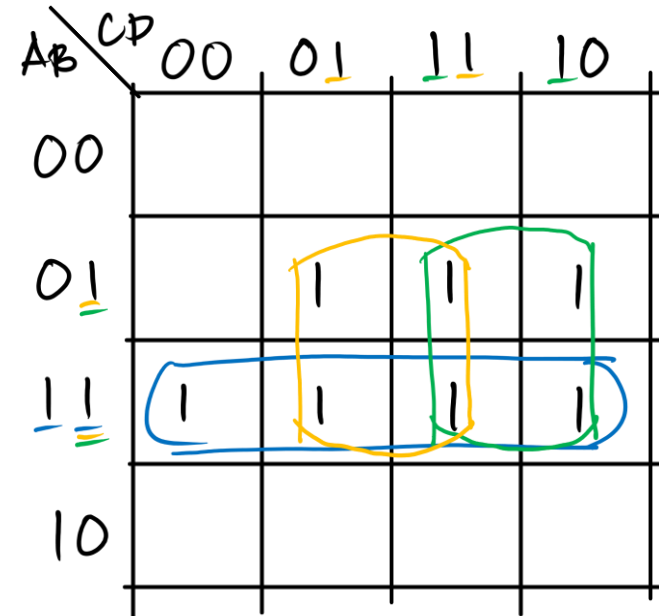
EXERCISE

Use a Karnaugh map to minimize the given standard SOP expression.

$$f = \sum m(5, 6, 7, 12, 13, 14, 15)$$

$$\begin{aligned} f = & \overset{5}{\bar{A}\bar{B}\bar{C}D} + \overset{6}{\bar{A}B\bar{C}\bar{D}} + \overset{7}{\bar{A}BCD} + \overset{12}{A\bar{B}\bar{C}\bar{D}} + \\ & \overset{13}{AB\bar{C}\bar{D}} + \overset{14}{AB\bar{C}D} + \overset{15}{ABCD} \end{aligned}$$

Solution



$$f = \underline{AB} + \underline{BD} + \underline{BC}$$

$$f = B(A + C + D)$$

ans

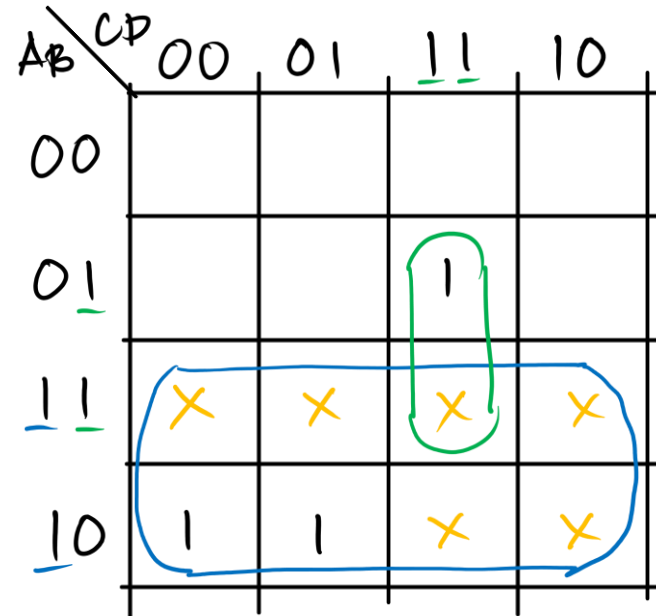
EXERCISE

Map the given table to a Karnaugh map and generate the minimized expression.

A	B	C	D	f
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1

A	B	C	D	f
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

Solution



$$f = \underline{A} + \underline{BCD}$$

ans

note

X represents “don’t care” term – it can be treated as either 1 or 0.



POS MINIMIZATION



KARNAUGH MAP

3-variable K-map

$AB \backslash C$	0	1
00	$A + B + C$	$A + B + \bar{C}$
01	$A + \bar{B} + C$	$A + \bar{B} + \bar{C}$
11	$\bar{A} + \bar{B} + C$	$\bar{A} + \bar{B} + \bar{C}$
10	$\bar{A} + B + C$	$\bar{A} + B + \bar{C}$

4-variable K-map

$AB \backslash CD$	00	01	11	10
00	$A + B + C + D$	$A + B + C + \bar{D}$	$A + B + \bar{C} + \bar{D}$	$A + B + \bar{C} + D$
01	$A + \bar{B} + C + D$	$A + \bar{B} + C + \bar{D}$	$A + \bar{B} + \bar{C} + \bar{D}$	$A + \bar{B} + \bar{C} + D$
11	$\bar{A} + \bar{B} + C + D$	$\bar{A} + \bar{B} + C + \bar{D}$	$\bar{A} + \bar{B} + \bar{C} + \bar{D}$	$\bar{A} + \bar{B} + \bar{C} + D$
10	$\bar{A} + B + C + D$	$\bar{A} + B + C + \bar{D}$	$\bar{A} + B + \bar{C} + \bar{D}$	$\bar{A} + B + \bar{C} + D$



MAPPING A STANDARD POS EXPRESSION

A 0 is placed on the K-map cell that corresponds to the value of a sum term in the expression.

3-variable K-map

$$f = (A + B + C)(A + \bar{B} + \bar{C})(\bar{A} + \bar{B} + C)(\bar{A} + \bar{B} + \bar{C})$$

		000	011	110	111
AB \ C	0	0			
	1		0		
00					
01			0		
11	0	0			
10					



GROUPING 0s

A group must contain either 1, 2, 4, 8, or 16 cells,
which are all **powers of two**.

AB \ C	C	
	0	1
00	0	
01		0
11	0	0
10		

AB \ C	C	
	0	1
00	0	0
01	0	
11		0
10	0	0

AB \ CD	CD			
	00	01	11	10
00	0	0		
01	0	0	0	0
11				
10		0	0	

AB \ CD	CD			
	00	01	11	10
00	0			0
01	0	0		0
11	0	0		0
10	0		0	0

POS MINIMIZATION

The process that results in an expression containing the fewest possible terms with the fewest possible variables is called minimization.

AB \ C	C	
	0	1
00	0	
01		0
11	0	0
10		

$$f = (A + B + C)(\bar{A} + \bar{B})$$
$$(\bar{B} + \bar{C})$$

AB \ C	C	
	0	1
00	0	0
01	0	
11		0
10	0	0

$$f = (B)(A + C)(\bar{A} + \bar{C})$$

AB \ CD	CD			
	00	01	11	10
00	0	0		
01	0	0	0	0
11				
10		0	0	

$$f = (A + C)(A + \bar{B})(\bar{A} + B + \bar{D})$$

AB \ CD	CD			
	00	01	11	10
00	0			0
01	0	0		0
11	0	0		0
10	0		0	0

$$f = (D)(\bar{B} + C)(\bar{A} + B + \bar{C})$$

Variables that occur both complemented and uncomplemented within the group are eliminated. These are called contradictory variables.

EXERCISE

Use a Karnaugh map to minimize the given standard POS expression.

$$f = \prod M(0, 1, 3, 5, 7)$$

$$f = (A+B+C) (A+B+\bar{C}) (A+\bar{B}+\bar{C}) (\bar{A}+B+\bar{C}) (\bar{A}+\bar{B}+\bar{C})$$

000 001 011 101 111

Solution

$A \backslash B \backslash C$	0	1
00	0	0
01		0
10		0
11		0

$$f = \bar{C} (A+B)$$

ans

EXERCISE

Use a Karnaugh map to minimize the given standard POS expression.

$$f = \prod M(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12)$$

$$f = (A + \overset{0}{B} + \overset{1}{C} + \overset{2}{D})(A + \overset{1}{B} + \overset{2}{C} + \bar{D})(A + \overset{2}{B} + \bar{C} + D)$$

$$\begin{array}{ccc} 0000 & 0001 & 0010 \\ \downarrow 3 & \downarrow 4 & \downarrow 5 \\ (A + \bar{B} + \bar{C} + \bar{D})(A + \bar{B} + C + D)(A + \bar{B} + C + \bar{D}) \end{array}$$

$$\begin{array}{ccc} 0011 & 0100 & 0101 \\ \downarrow 6 & \downarrow 7 & \downarrow 8 \\ (A + \bar{B} + \bar{C} + D)(A + \bar{B} + \bar{C} + \bar{D})(\bar{A} + \bar{B} + C + D) \end{array}$$

$$\begin{array}{ccc} 0110 & 0111 & 1000 \\ \downarrow 9 & \downarrow 10 & \downarrow 11 \\ (\bar{A} + \bar{B} + C + \bar{D})(\bar{A} + \bar{B} + C + D)(\bar{A} + \bar{B} + \bar{C} + \bar{D}) \end{array}$$

$$\begin{array}{ccc} 1001 & 1010 & 1011 \\ \downarrow 12 & & \\ (\bar{A} + \bar{B} + C + D) & & \end{array}$$

$$1100$$

Solution

AB \ CD	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0			
10	0	0	0	0

$$f = \underline{A} (\underline{C + D}) \underline{B}$$

$$\boxed{f = AB(C + D)}$$

ans

EXERCISE

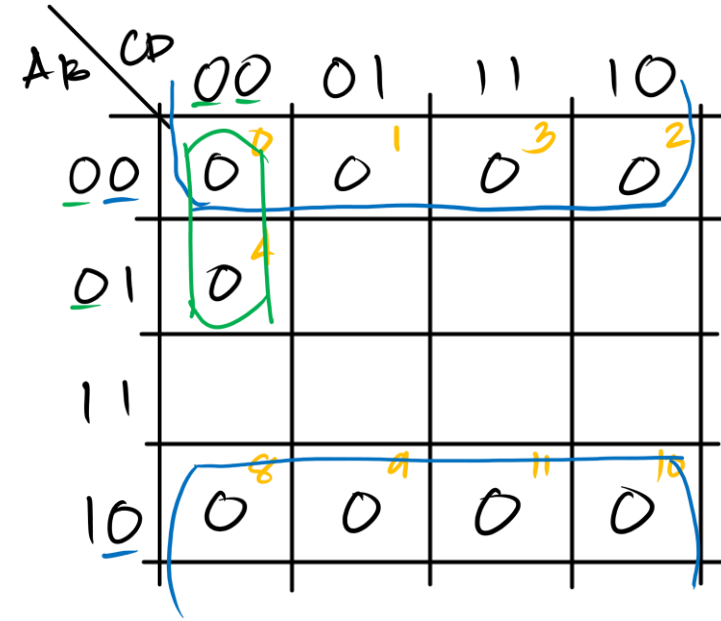
Use a Karnaugh map to minimize the given standard SOP expression.

$$f = \prod M(0, 1, 2, 3, 4, 8, 9, 10, 11)$$

$$f = \underline{B} (\underline{A + C + D})$$

ans

Solution



EXERCISE

Map the given table to a Karnaugh map and generate the minimized expression.

A	B	C	D	f
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1

A	B	C	D	f
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

Solution

AB \ CD	00	01	11	10
00	0	0	0	0
01	0	0		0
11	X	X	X	X
10			X	X

$$f = (A + C)(A + B)(\bar{C} + D)$$

Ans

note

X represents "don't care" term – it can be treated as either 1 or 0.

LABORATORY

