HYPOTHESIS TESTING

INFERENTIAL STATISTICS

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TOPIC OUTLINE

Hypothesis Test

Rejection Region

Critical Value and Z-score

p-Value

HYPOTHESIS TEST

HYPOTHESIS

A **hypothesis** is an initial **assumption** formed before collecting data, and it serves as a statement about a **population** parameter rather than about the sample data.

HYPOTHESIS TEST

A **hypothesis test** is simply comparing reality to an assumption and asking, "**Did things change?**"

Null Hypothesis (H_o)

Represents **no change**, no effect, or the status quo.

Alternative Hypothesis (H_a)

Represents the possibility that things did change or that there is a **significant difference**.

IS YOUR DATA GUILTY?

Hypothesis testing is like a legal system where the defendant is assumed **innocent** until proven guilty.

H_o: Innocent	The Truth: Innocent	The Truth: Guilty
Accept	✓	✗
Reject	✗	✓

TYPES OF ERROR

1. **Type I Error** :

Rejecting the null hypothesis when it is actually true. The risk of making type I error is denoted by $\boldsymbol{\alpha}$ (e.g., 0.05).

2. **Type II Error** :

Failing to reject the null hypothesis when it is actually false. The risk of making a type II error is denoted by $\boldsymbol{\beta}$ (e.g. 0.20)

H_0: Innocent	The Truth: Innocent	The Truth: Guilty
Accept	✓	Type II Error (False negative)
Reject	Type I Error (False positive)	✓

EXERCISE

A fitness tracker company claims their device measures heart rate with **95%** accuracy compared to medical-grade monitors. An independent lab wants to verify this claim.

Null Hypothesis

$$H_o: \mu_o = 95$$

The average accuracy is 95%.

Alternative Hypothesis

$$H_a: \mu_o \neq 95$$

The average accuracy differs from 95%.

EXERCISE

A manufacturer claims that their new energy-efficient LED bulbs have an average lifespan of **at least 25,000 hours**. A consumer group suspects that the actual lifespan is shorter and decides to test this claim.

Null Hypothesis

$$H_o: \mu_o = 25,000$$

The average lifespan of the LED is 25,000 hours.

Alternative Hypothesis

$$H_a: \mu_o < 25,000$$

The average lifespan of the LED is less than 25,000 hours.

EXERCISE

A study suggests that storing apples in a controlled atmosphere **extends** their shelf life beyond **30 days**. A food scientist wants to verify if this method truly increases shelf life compared to conventional storage.

Null Hypothesis

$$H_o: \mu_o = 30$$

Controlled-atmosphere storage shelf life is 30 days.

Alternative Hypothesis

$$H_a: \mu_o > 30$$

Controlled-atmosphere storage increases shelf life beyond 30 days.

REJECTION REGION

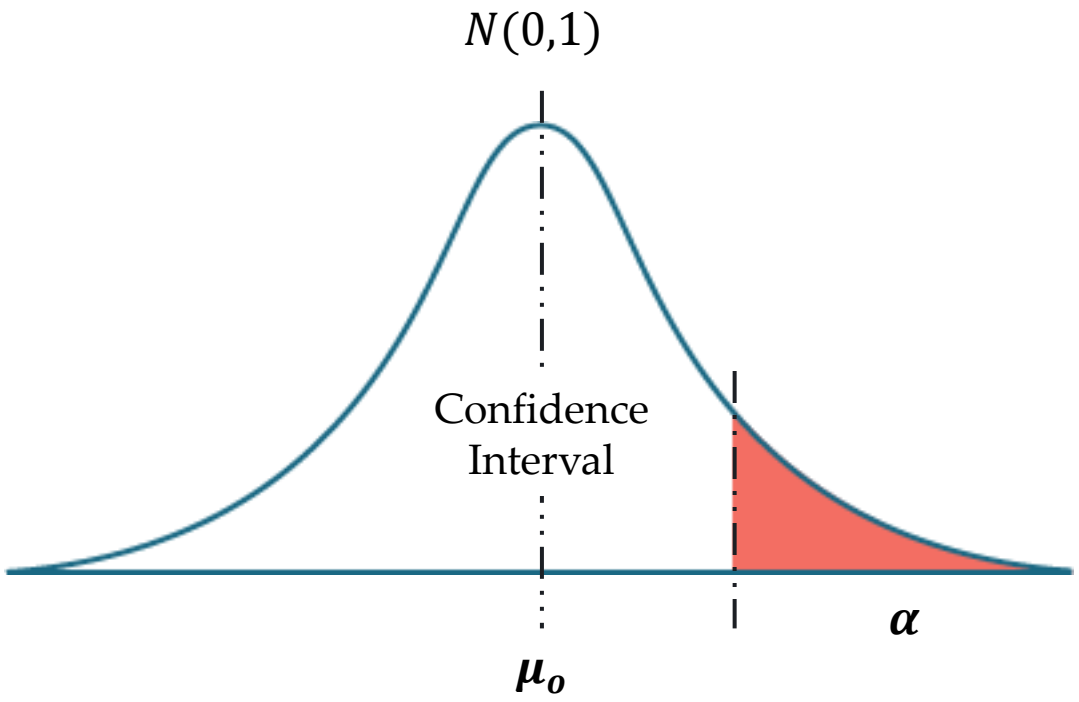
SIGNIFICANCE LEVEL

The **significance level** (α) determines the threshold for deciding whether to **reject** the null hypothesis (H_o).

Typical values for α:

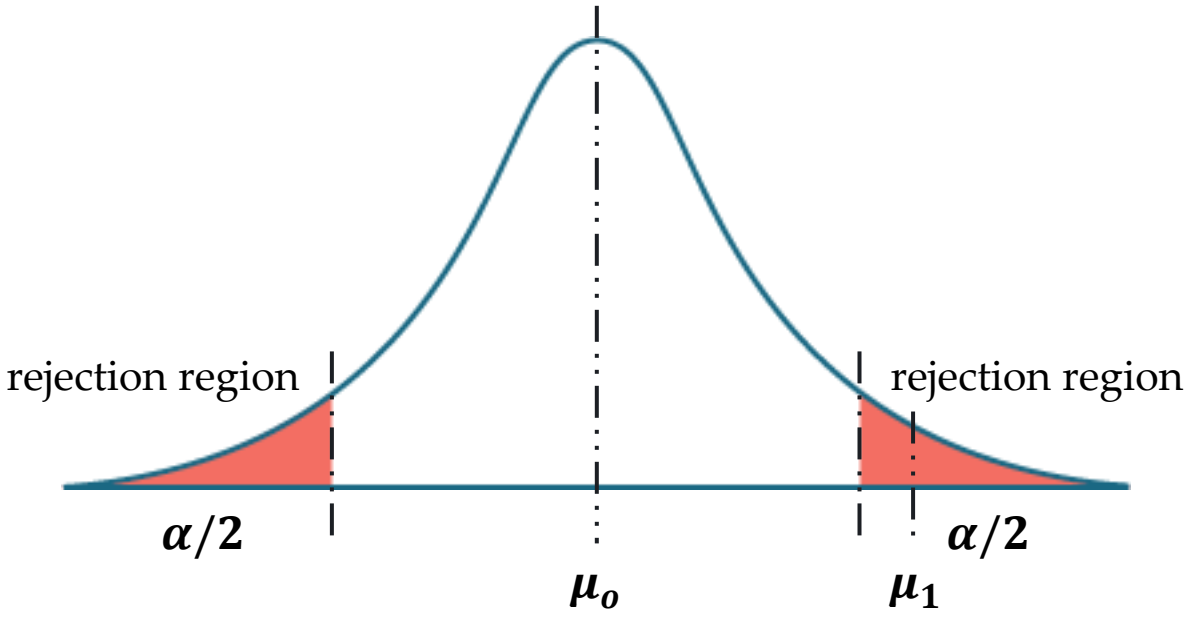
- **0.01**
- **0.05**
- **0.1**

Standard Normal Distribution



REJECTION REGION

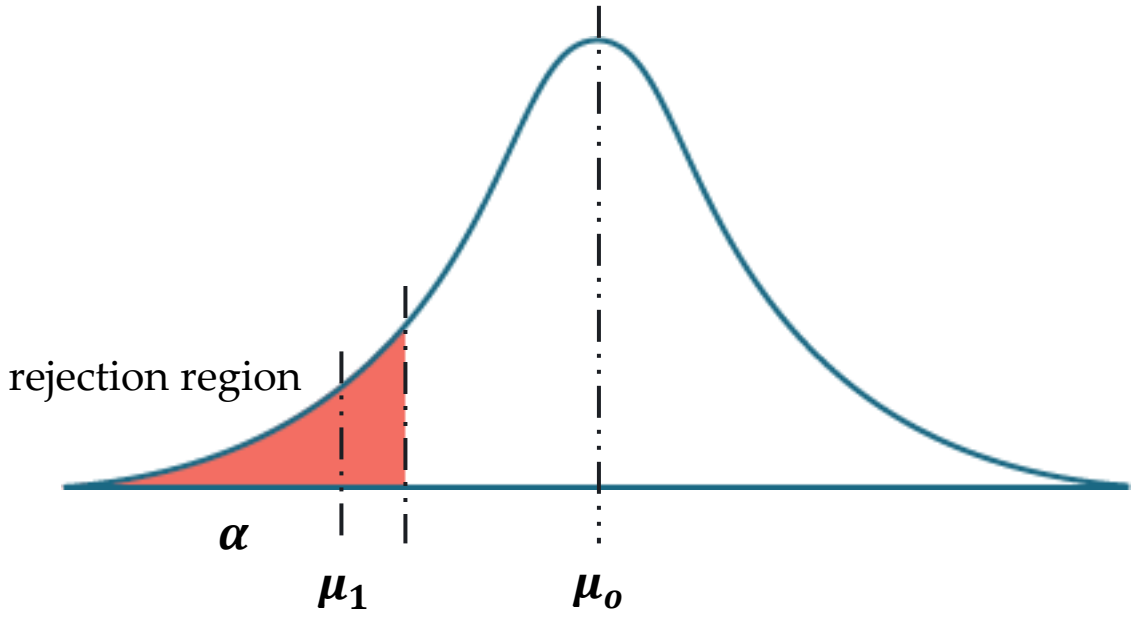
Two-sided Test



H_o: $\mu_1 = \mu_o$

H_a: $\mu_1 \neq \mu_o$

One-sided Test



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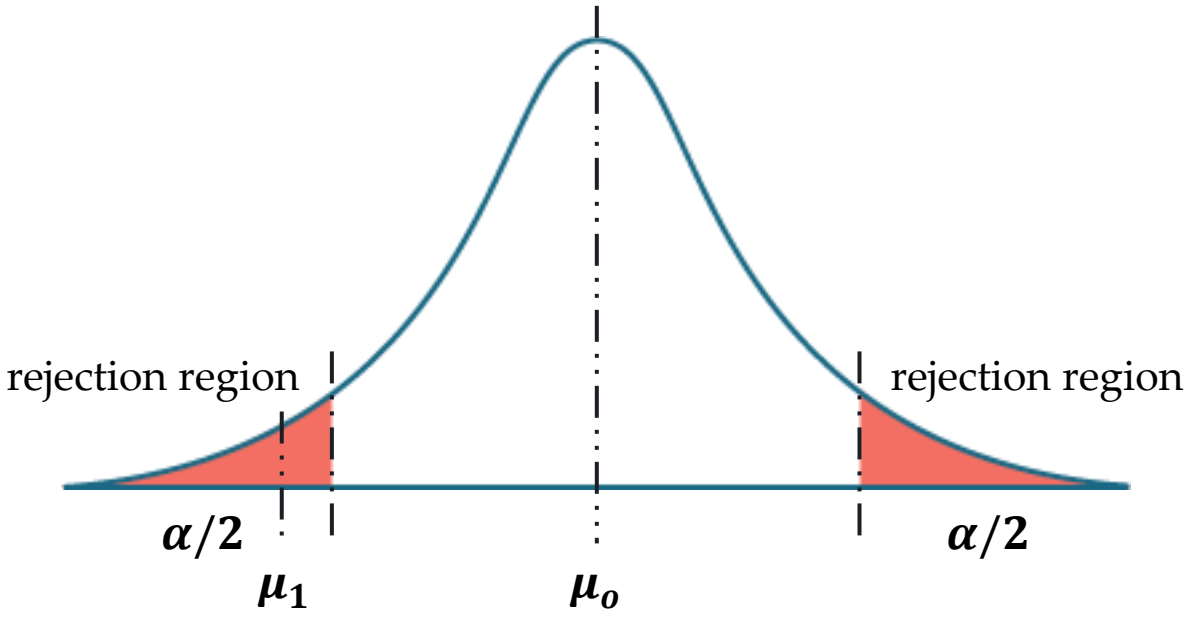
H_a: $\mu_1 < \mu_o$

REJECTION REGION

Two-sided Test



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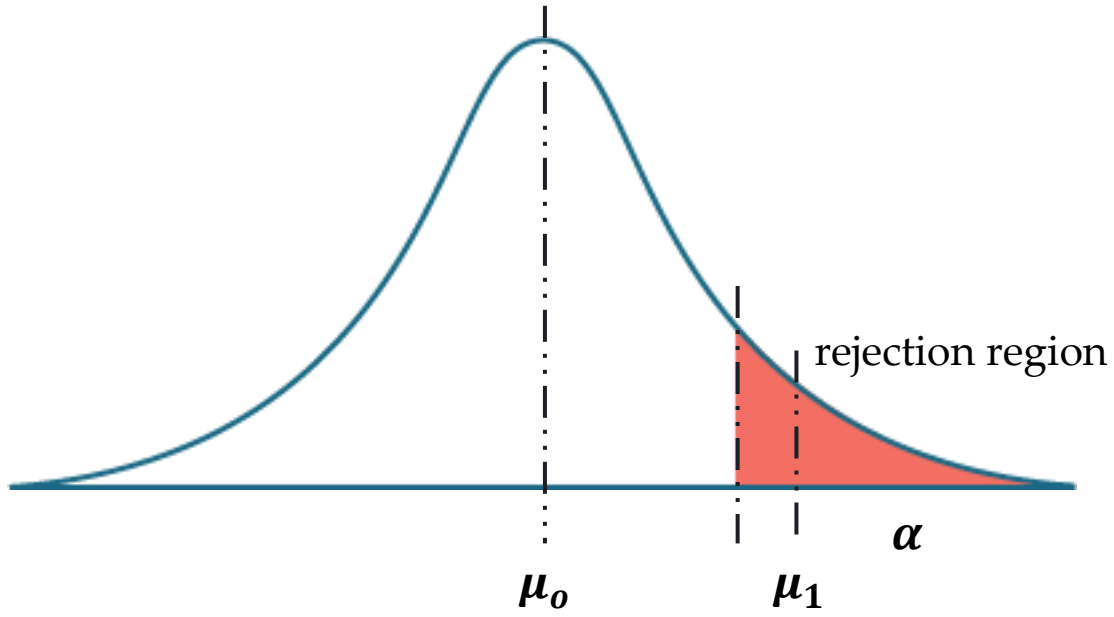
One-sided Test



H_o: $\mu_1 = \mu_o$

H_a: $\mu_1 > \mu_o$

CRITICAL VALUE AND Z-SCORE

CRITICAL VALUE AND Z-SCORE

lowercase z

z refers to the **critical value** obtained from the standard normal distribution table (z-table).

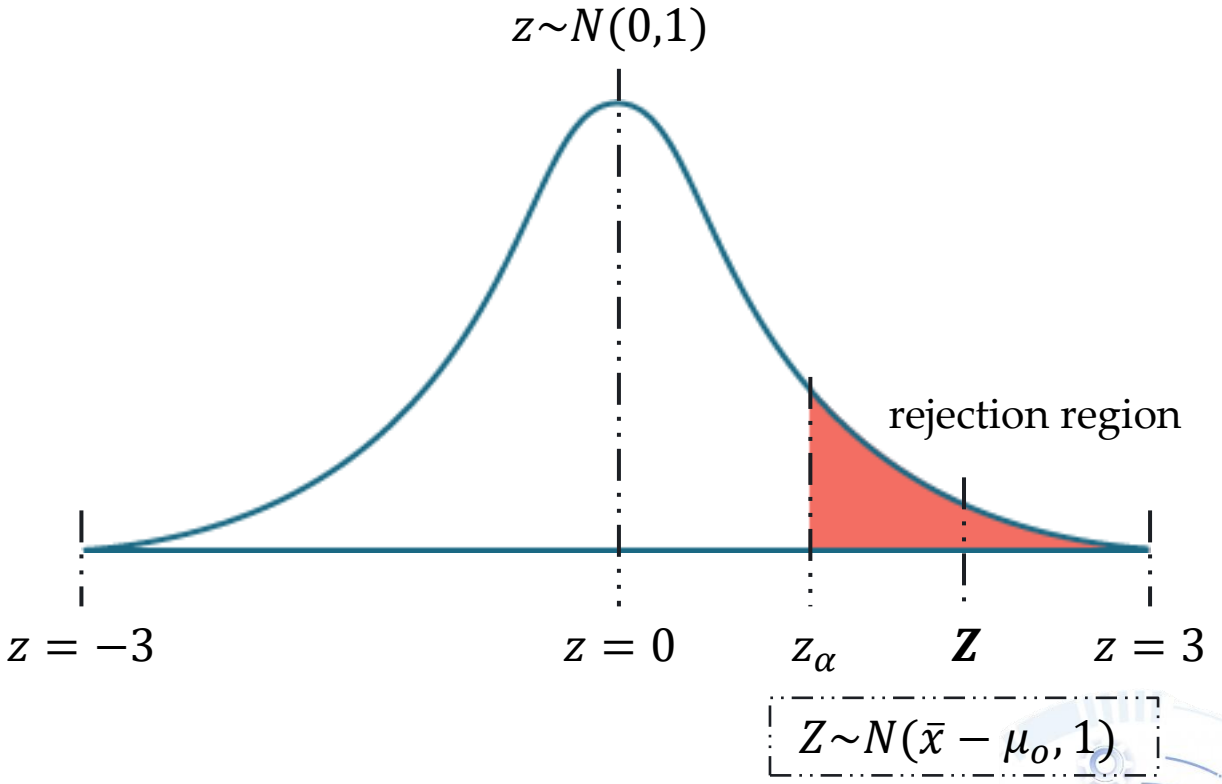
uppercase Z

Z is a standardized variable associated with the test called the **Z-score**.

Formula:

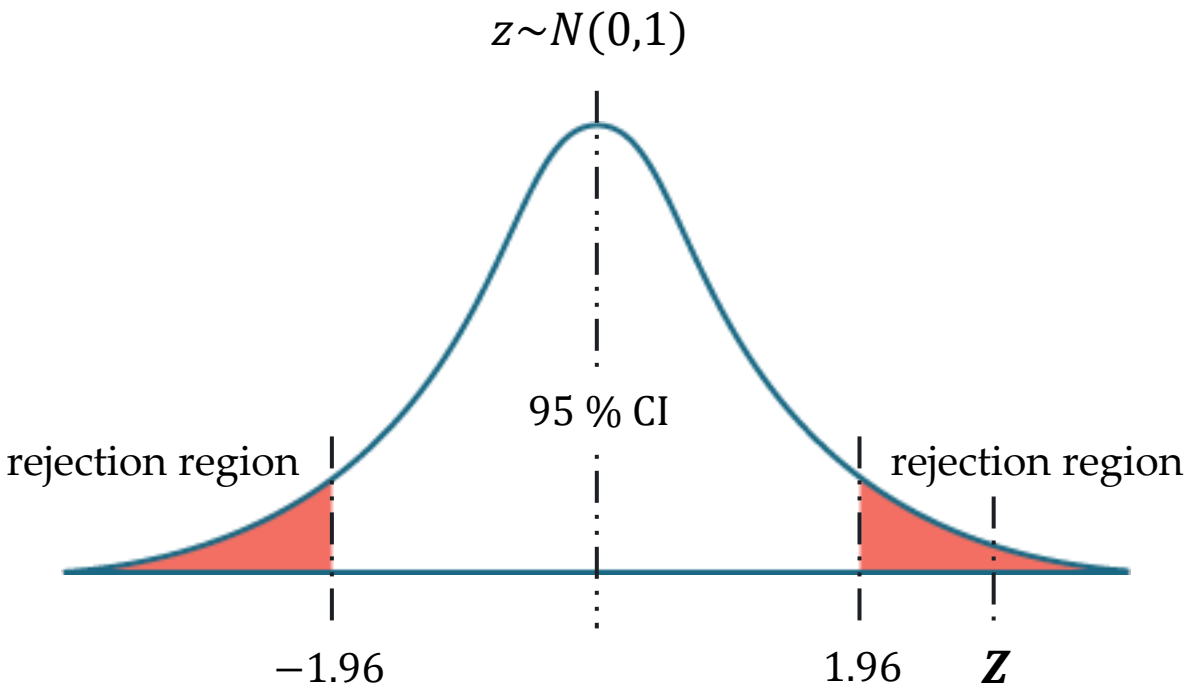
$$Z = \frac{\bar{x} - \mu_o}{\sigma/\sqrt{n}}$$

One-sided Test



EXERCISE

Two-sided Test



$\alpha = 0.05$

$z_{0.025} = 1.96$

Null Hypothesis

$\boldsymbol{H_o: \mu_o = 95}$

The average accuracy is 95%.

Alternative Hypothesis

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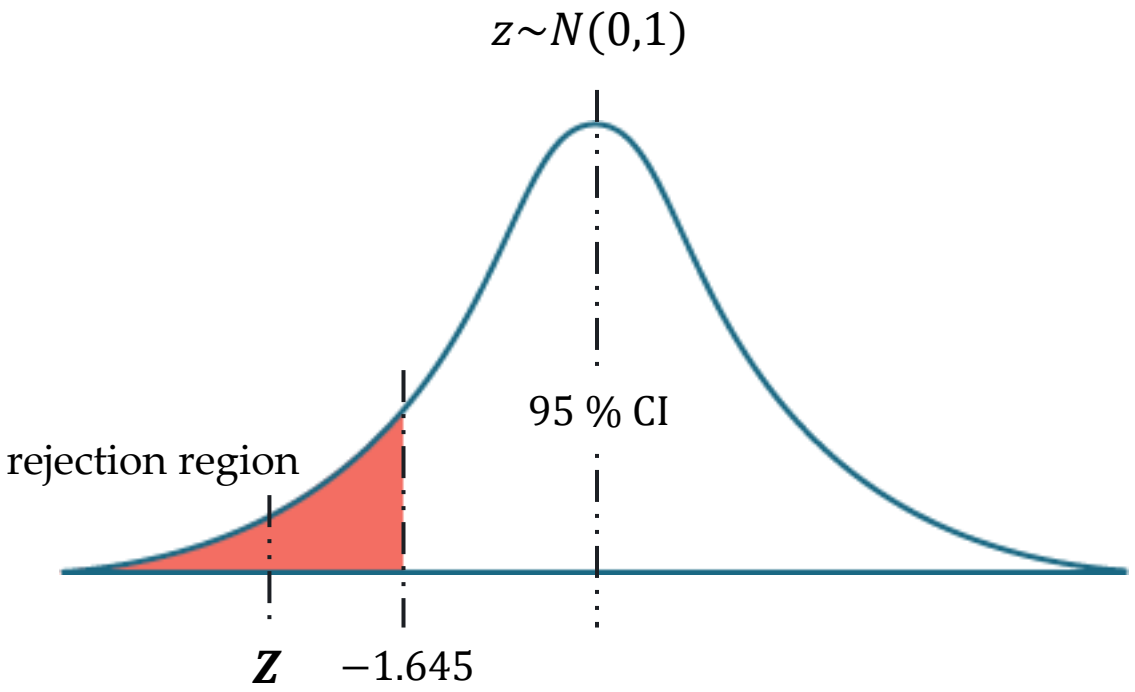
The average accuracy differs from 95%.

EXERCISE

One-sided Test



$\alpha = 0.05$

$z_{0.05} = 1.645$

Null Hypothesis

$\boldsymbol{H_o: \mu_o = 25,000}$

The average lifespan of the LED is 25,000 hours.

Alternative Hypothesis

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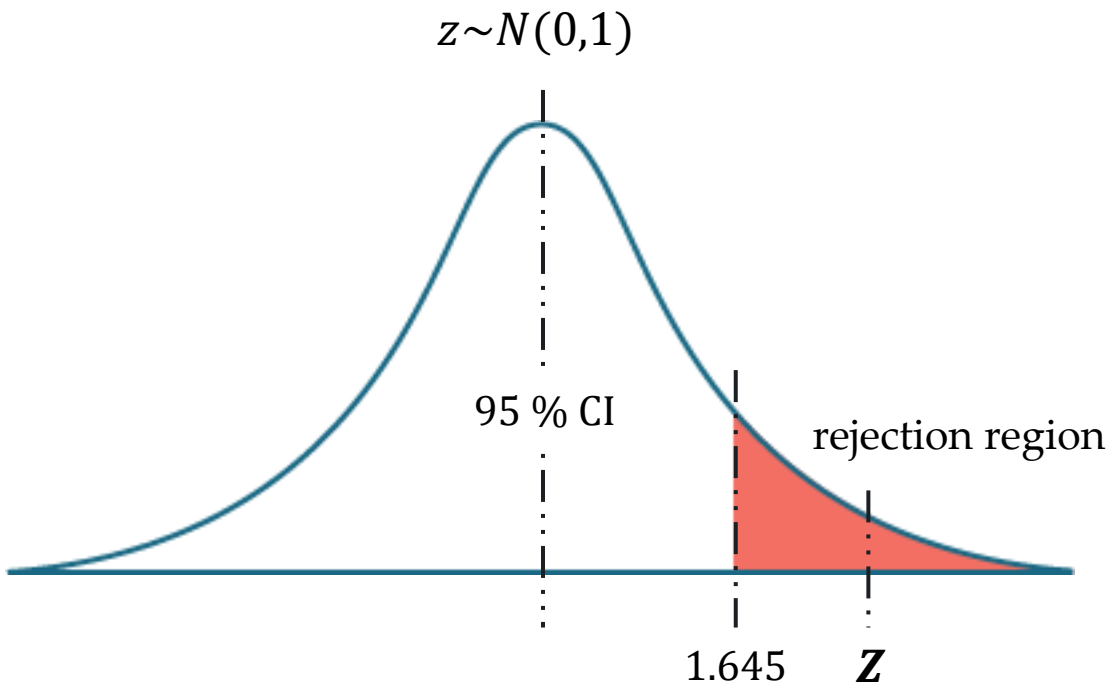
The average lifespan of the LED is less than 25,000 hours.

EXERCISE

One-sided Test



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$z_{0.05} = 1.645$

Null Hypothesis

$\boldsymbol{H_o: \mu_o = 30}$

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Alternative Hypothesis

$\boldsymbol{H_a: \mu_o > 30}$

Controlled-atmosphere storage increases shelf life beyond 30 days.

EXERCISE

A manufacturing process is claimed to have an average defect rate of **10.32** units, with a known standard deviation of **3.17** units. The Statistical Process Control (SPC) department suspects this claim may no longer be valid and collects a **random sample** of **30** production units to test whether the true average **defect rate differs** significantly from **10.32**.

Dataset

defects-data-30-samples.csv

Solution

P-VALUE

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The p-value (probability value) is the **smallest level of significance** at which we can still reject the null hypothesis, given the observed sample statistic.

One-sided Test

p-value = 1 − value from the table

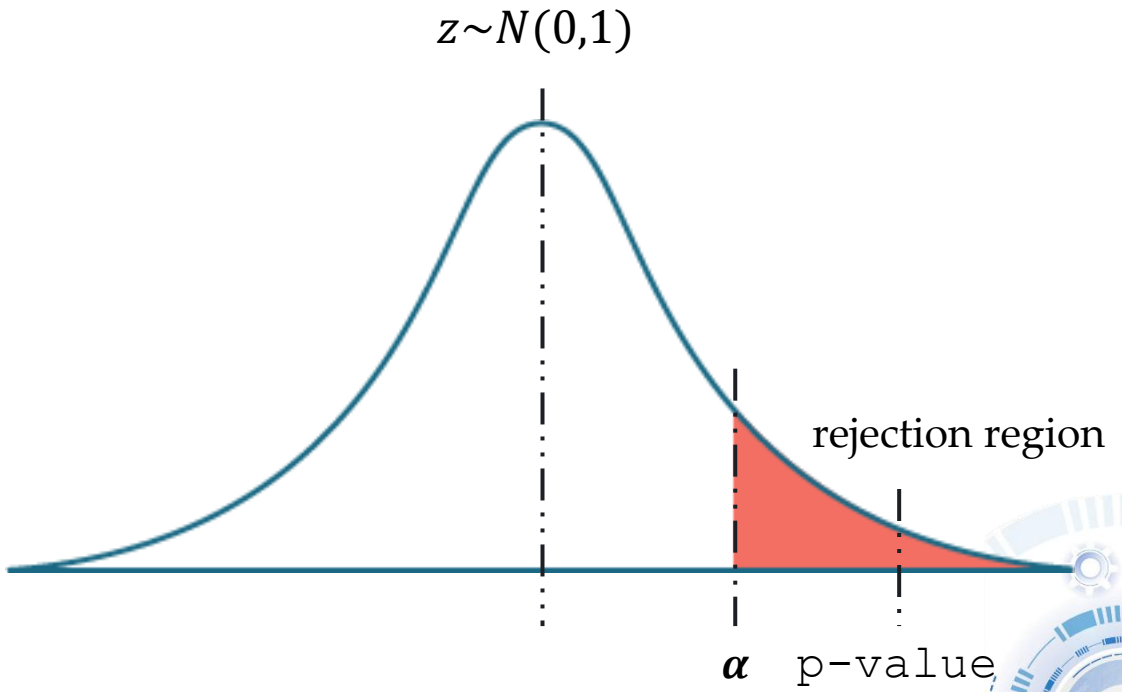
Two-sided Test

p-value = (1 − value from the table**) ×2**

Hypothesis Test

Reject H_o if **p-value $\leq \alpha$**

Fail to reject H_o if p-value $> \alpha$



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Cumulative Distribution Function (CDF)

`cdf()` returns the probability that a random variable ***Z*** (Z-score) from a standard normal distribution is less than or equal to a given crtitical value (***z***).

Syntax

One-sided Test

```
p_value = 1-stats.norm.cdf(Z_score)
```

Two-sided Test

```
p_value = 2*(1-stats.norm.cdf(Z_score))
```

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Solution

LABORATORY