STANDARD NORMAL DISTRIBUTION

INFERENTIAL STATISTICS

prepared by:

Gyro A. Madrona

Electronics Engineer

TOPIC OUTLINE

Standard Normal Distribution

Central Limit Theorem

STANDARD NORMAL DISTRIBUTION

STANDARDIZATION

Standardization is the process of converting the distribution of a variable $\boldsymbol{X \sim (\mu, \sigma^2)}$ to a normal distribution $\boldsymbol{Z \sim N(0, 1)}$.

Formula

$$Z = \frac{X - u}{\sigma}$$

where

Z = z-score

X = random variable

example

Data	$X - \mu$	Z-score
1	-4	-1.46
2	-3	-1.095
3	-2	-0.73
4	-1	-0.365
5	0	0
6	1	0.365
7	2	0.73
8	3	1.095
9	4	1.46

$\bar{x} = 5.0$ $s = 2.74$

STANDARDIZATION

Standardization is the process of converting the distribution of a variable $\boldsymbol{X}\sim(\boldsymbol{\mu}, \boldsymbol{\sigma}^2)$ to a normal distribution $\boldsymbol{Z}\sim\boldsymbol{N}(\mathbf{0}, \mathbf{1})$.

Formula

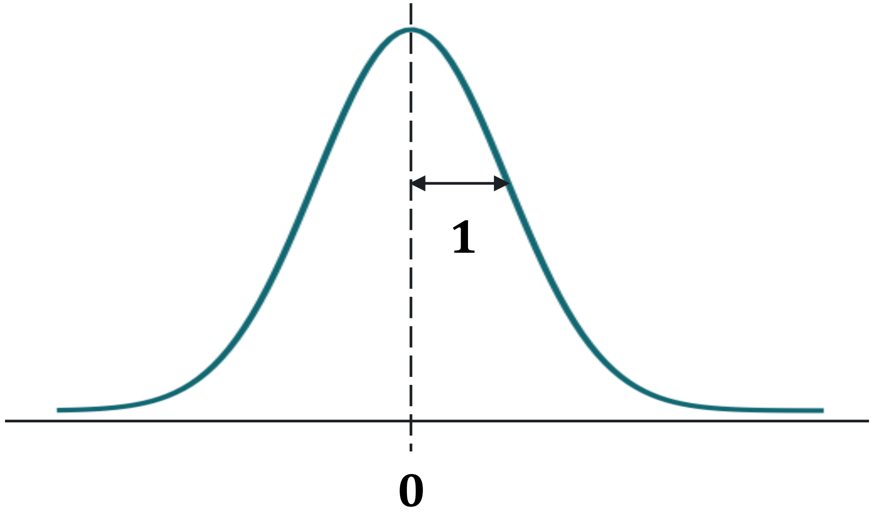
$$\boldsymbol{Z} = \frac{\boldsymbol{X} - \boldsymbol{u}}{\boldsymbol{\sigma}}$$

where

Z = z-score

X = random variable

Normal Distribution



STANDARD NORMAL DISTRIBUTION

When we standardize the **normal distribution** $\boldsymbol{X \sim N(\mu, \sigma^2)}$, the result is a **standard normal distribution** $\boldsymbol{Z \sim N(0, 1)}$.

Formula

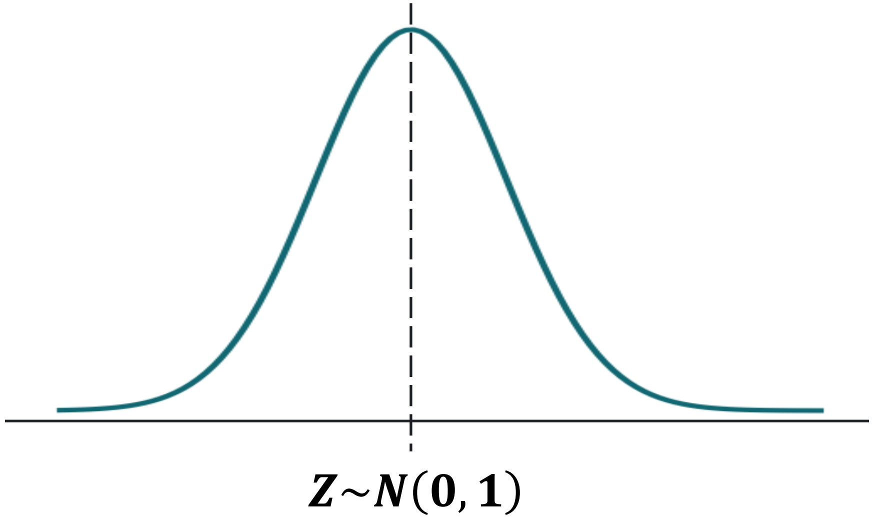
$$Z = \frac{X - u}{\sigma}$$

where

Z = z-score

X = random variable

Standard Normal Distribution



EXERCISE

Convert the given dataset into a **standard normal distribution** $\boldsymbol{N(0, 1)}$ by computing the **z-score** for each data point.

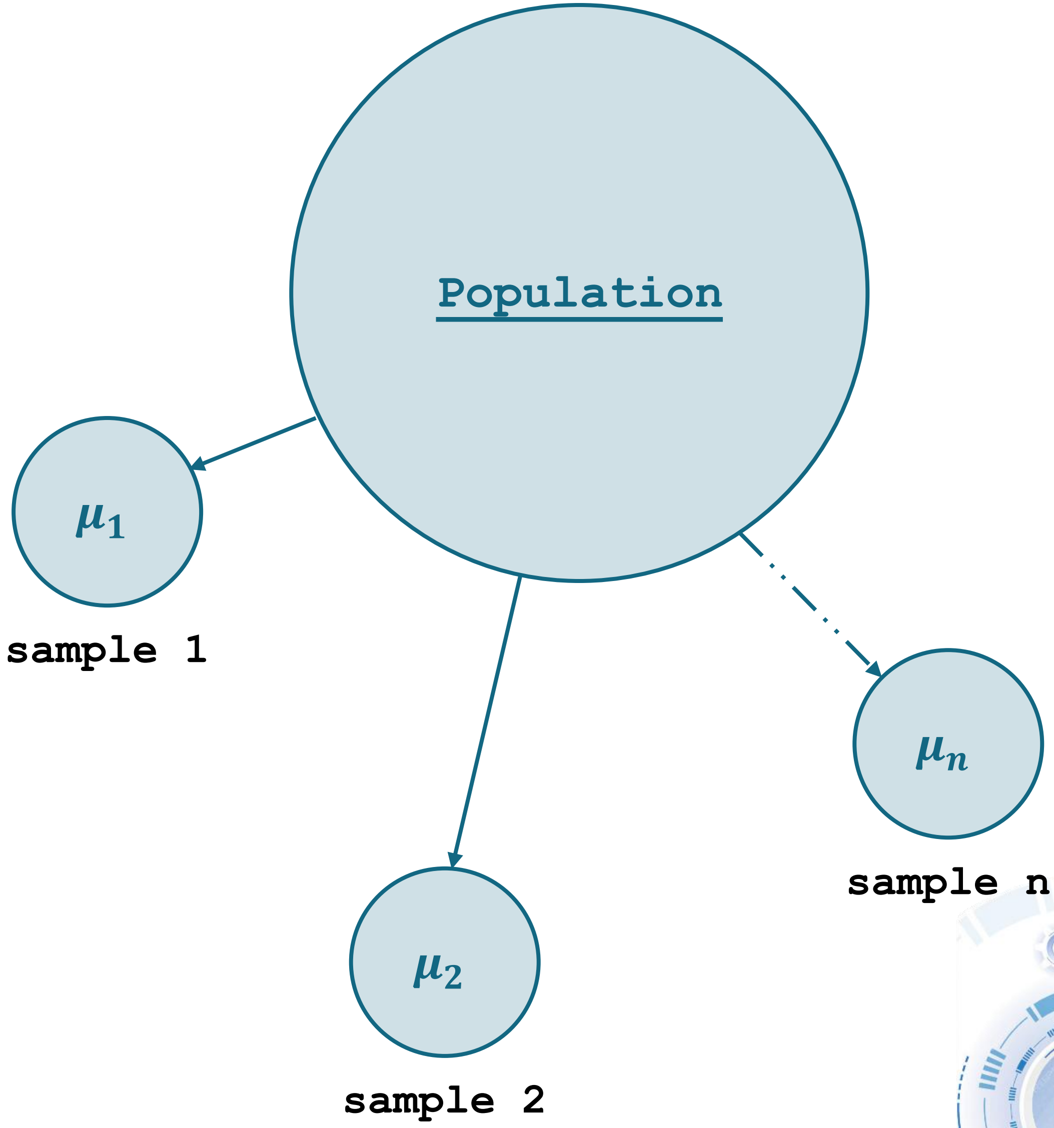
solution

Data
1
2
2
3
3
3
4
4
5

CENTRAL LIMIT THEOREM

CENTRAL LIMIT THEOREM

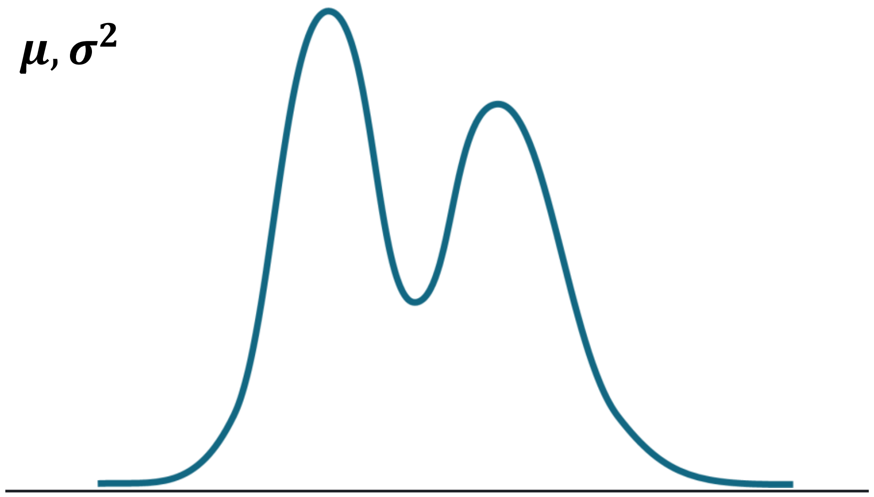
The Central Limit Theorem (CLT) states that the sampling distribution of the **sample mean** will be normally distributed, regardless of the shape of the original population distribution.



CENTRAL LIMIT THEOREM

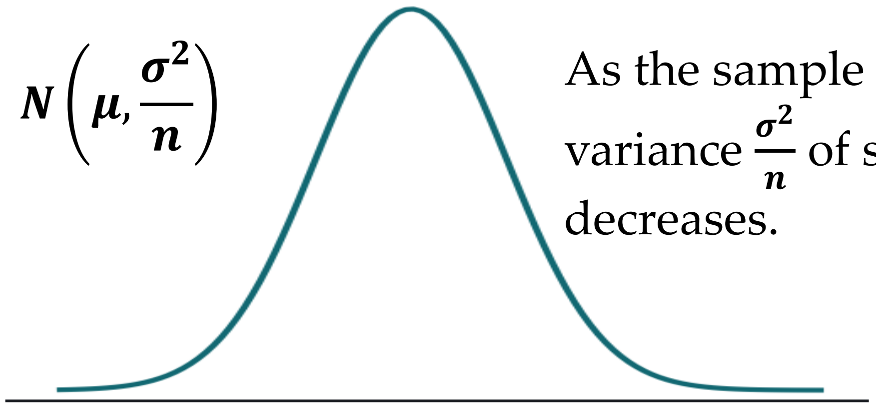
Original Population Distribution

μ, σ^2

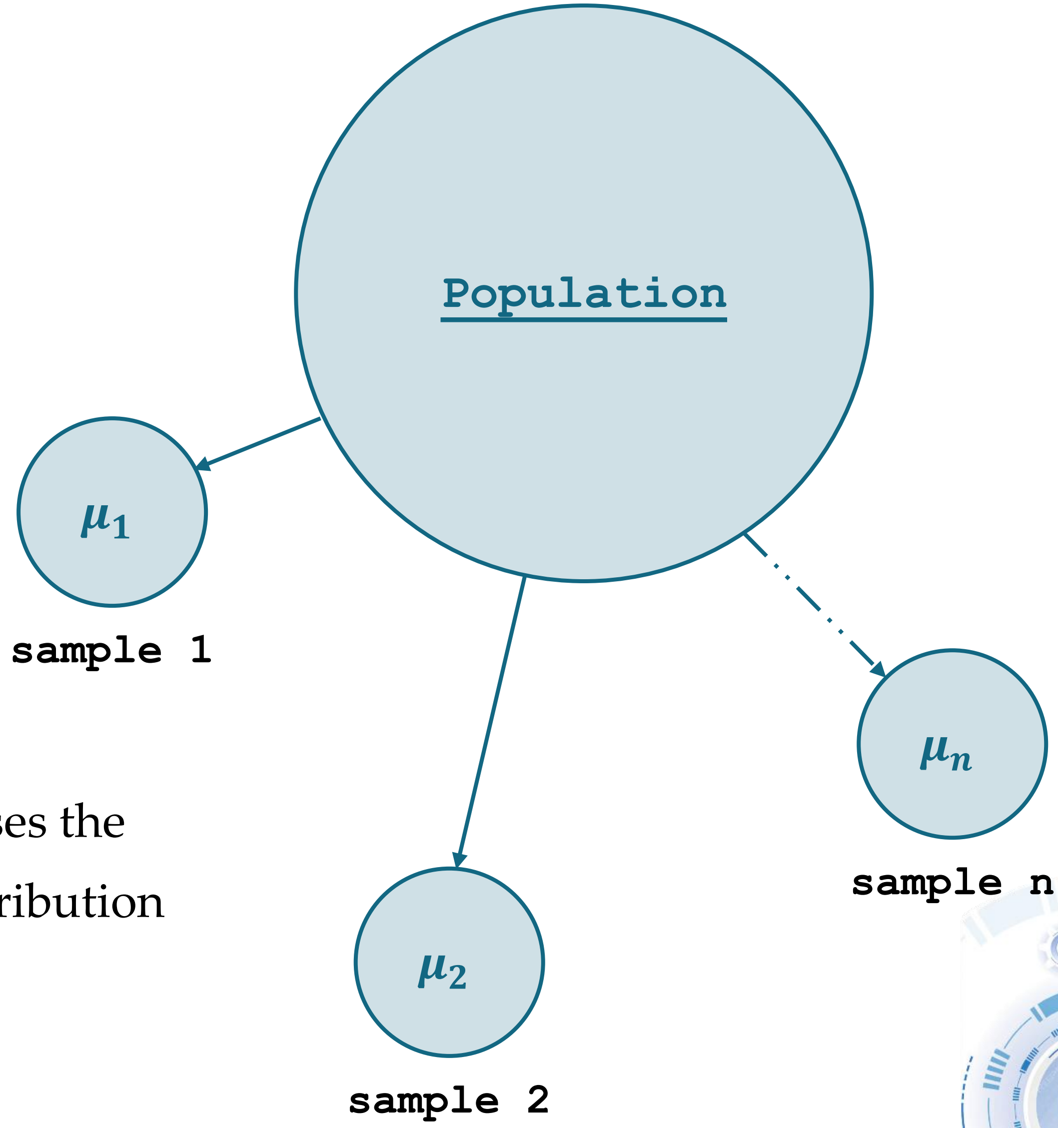


Sampling Distribution

$N\left(\mu, \frac{\sigma^2}{n}\right)$



As the sample size n increases the variance $\frac{\sigma^2}{n}$ of sampling distribution decreases.



SAMPLING DISTRIBUTION

A **sampling distribution** is the probability distribution of a **statistic** (e.g., $\boldsymbol{\mu}, \boldsymbol{\sigma}^2$) obtained from a large number of samples drawn from a specific population.

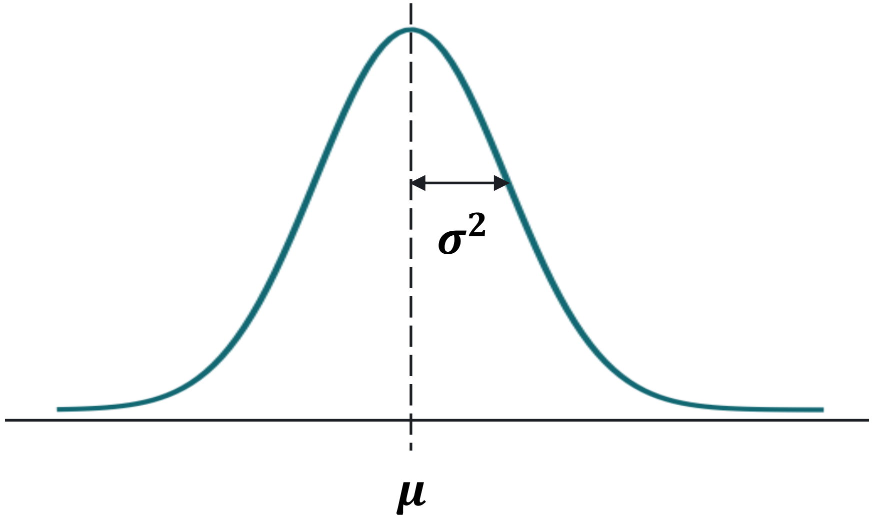
Denoted by

$$N\left(\mu, \frac{\sigma^2}{n}\right) \quad , n > 30$$

where

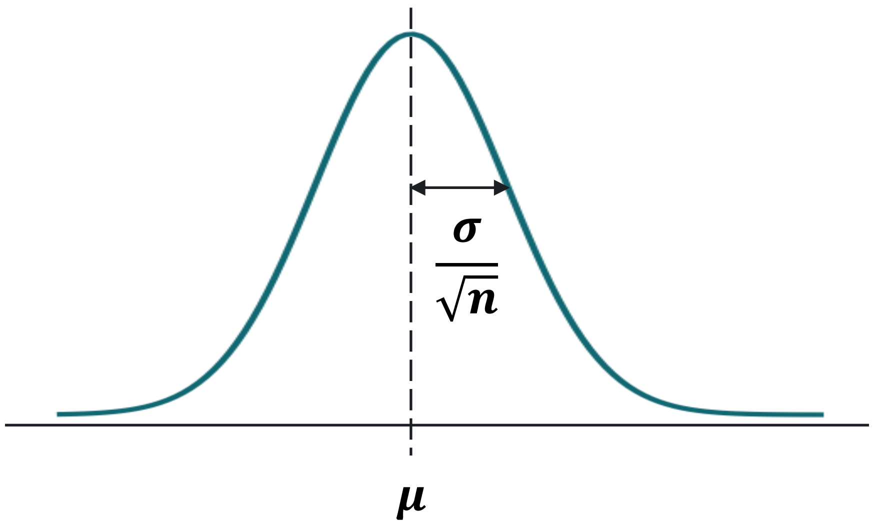
$\frac{\sigma^2}{n}$ = variance of the sampling distribution

Sampling Distribution



STANDARD ERROR

Sampling Distribution



Standard error is the **standard deviation** of the distribution formed by the **sample means**.

Formula

$$SE = \frac{\sigma}{\sqrt{n}}$$

where

σ = sampling standard deviation

n = number of observations

LABORATORY