BOOLEAN ALGEBRA

LOGIC MINIMIZATION

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TOPIC OUTLINE

Laws of Boolean Algebra

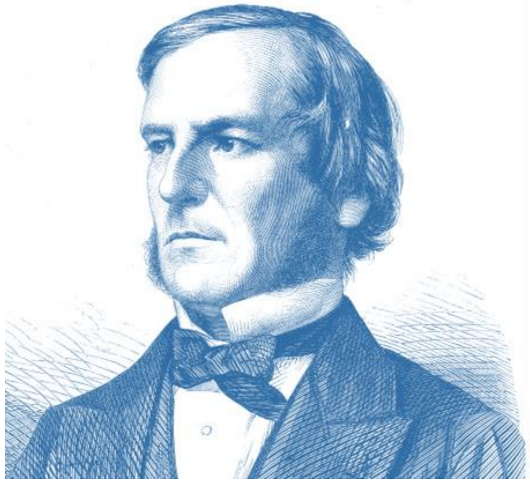
Rules of Boolean Algebra

DeMorgan's Theorem

LAWS OF BOOLEAN ALGEBRA

BOOLEAN ALGEBRA

Boolean algebra is the mathematics of digital logic. It was formulated in 1874 by George Boole.

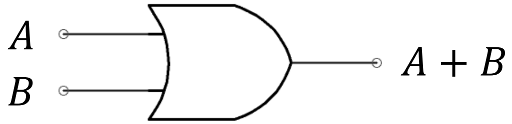


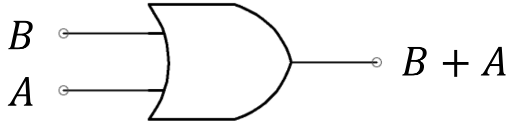
George Boole

COMMUTATIVE LAWS

Commutative law of addition

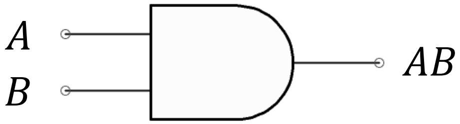
$$A + B = B + A$$

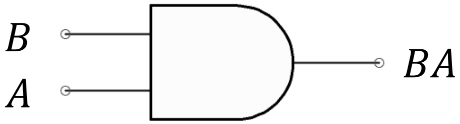




Commutative law of multiplication

$$AB = BA$$

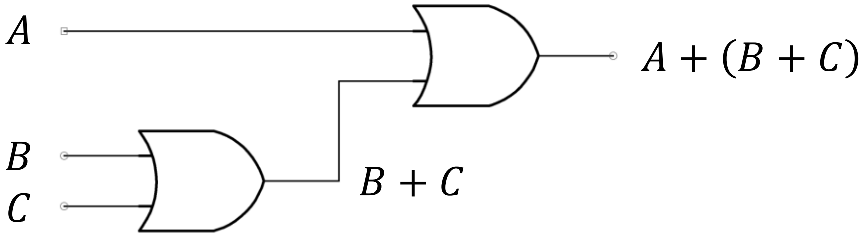


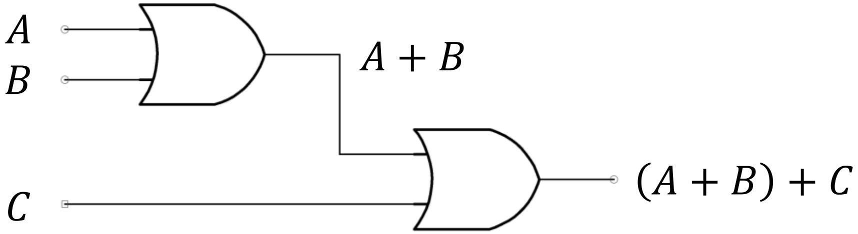


ASSOCIATIVE LAWS

Associative law of addition

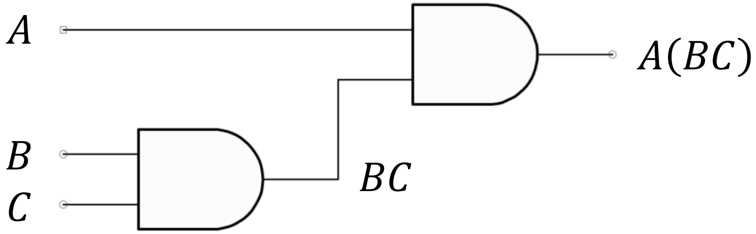
$$A + (B + C) = (A + B) + C$$

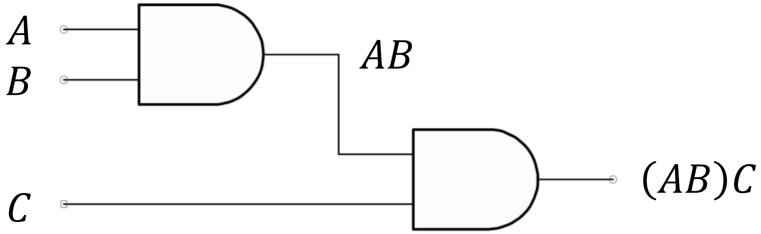




Associative law of multiplication

$$A(BC) = (AB)C$$

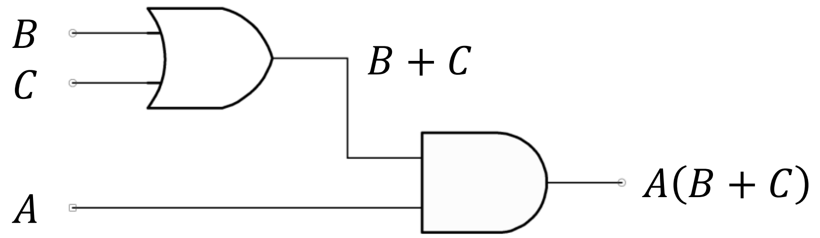


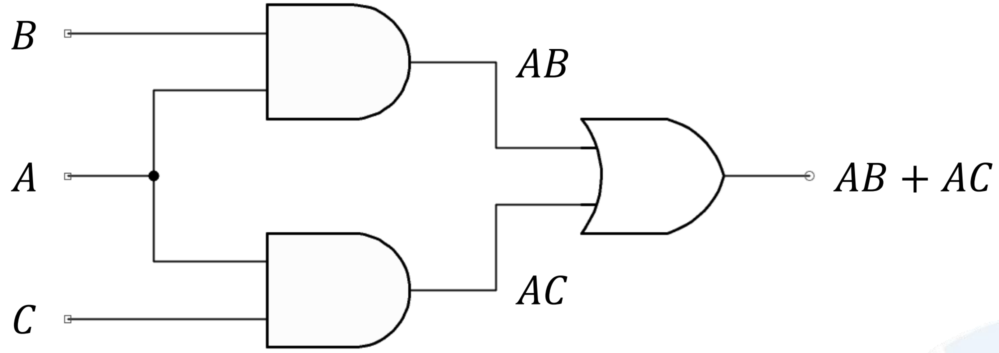


DISTRIBUTIVE LAW

Distributive law

$$A(B+C) = AB + AC$$





RULES OF BOOLEAN ALGEBRA

BASIC RULES OF BOOLEAN ALGEBRA

Basic rules of Boolean algebra are useful in manipulating and simplifying Boolean expressions.

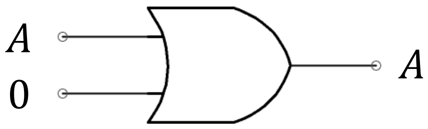
Basic rules of Boolean algebra

1. $A + 0 = A$
2. $A + 1 = 1$
3. $A \cdot 0 = 0$
4. $A \cdot 1 = A$
5. $A + A = A$
6. $A + \bar{A} = 1$
7. $A \cdot A = A$
8. $A \cdot \bar{A} = 0$
9. $\bar{\bar{A}} = A$
10. $A + AB = A$
11. $A + \bar{A}B = A + B$
12. $(A + B)(A + C) = A + BC$

RULE 1 AND 2

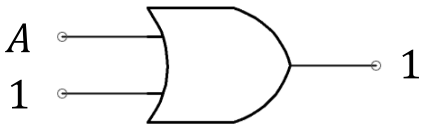
Rule 1: $\mathbf{A + 0 = A}$

A variable Ored with 0 is always equal to the variable.



Rule 2: $\mathbf{A + 1 = 1}$

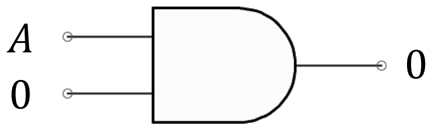
A variable Ored with 1 is always equal to 1.



RULE 3 AND 4

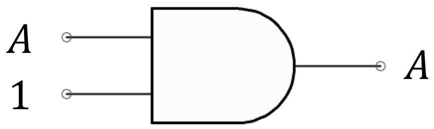
Rule 3: $\mathbf{A \cdot 0 = 0}$

A variable ANDed with 0 is always equal to 0



Rule 4: $\mathbf{A \cdot 1 = A}$

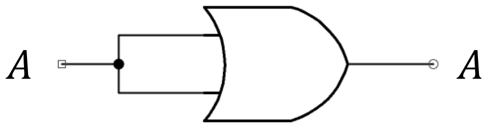
A variable ANDed with 1 is always equal to the variable.



RULE 5 AND 6

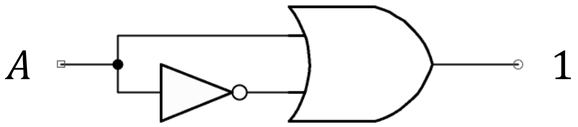
Rule 5: $\mathbf{A + A = A}$

A variable ORed with itself is always equal to the variable.



Rule 6: $\mathbf{A + \bar{A} = 1}$

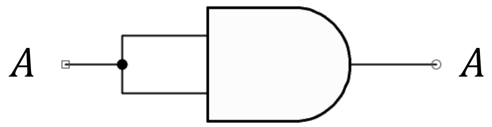
A variable ORed with its complement is always equal to 1.



RULE 7 AND 8

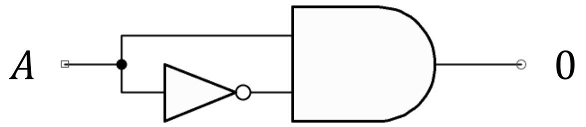
Rule 7: $\mathbf{A \cdot A = A}$

A variable ANDed with itself is always equal to the variable.



Rule 8: $\mathbf{A \cdot \overline{A} = 0}$

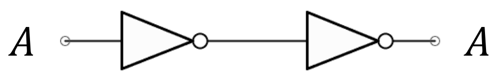
A variable ANDed with its complement is always equal to 0.



RULE 9 AND 10

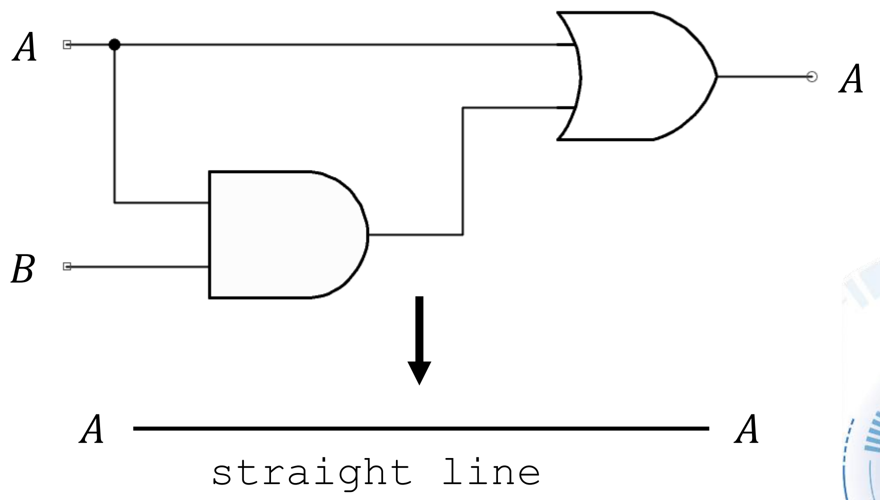
Rule 9: $\overline{\overline{\mathbf{A}}} = \mathbf{A}$

A double complement of a variable is always equal to the variable.



Rule 10: $\mathbf{A + AB = A}$

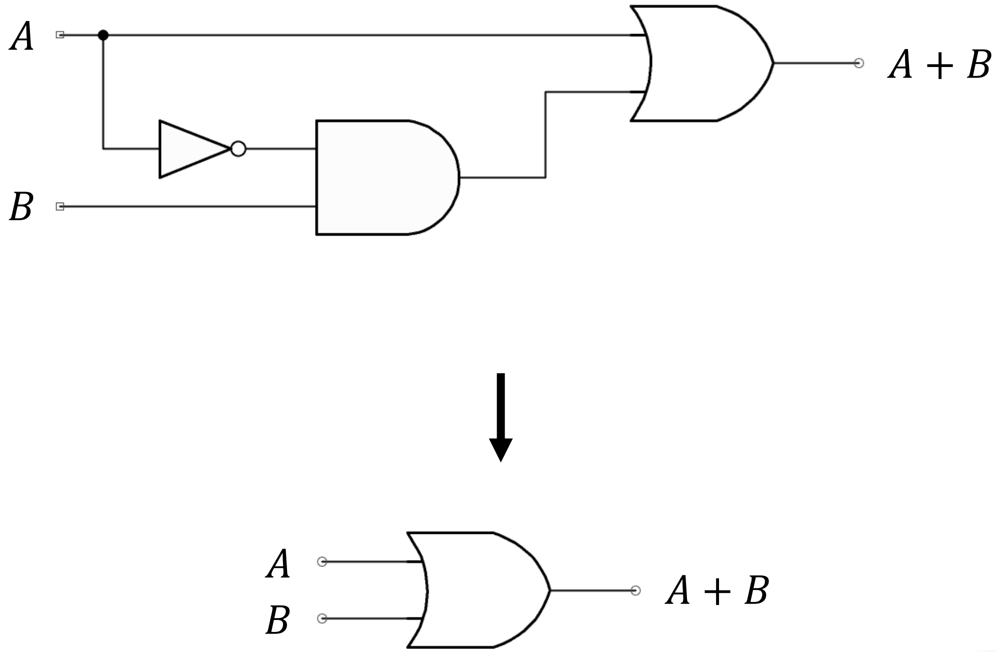
$$\begin{aligned} A + AB &= A \cdot 1 + AB \\ &= A(1 + B) \\ &= A(1) \\ &= A \end{aligned}$$



RULE 11

Rule 11: $\mathbf{A + \overline{A}B = A + B}$

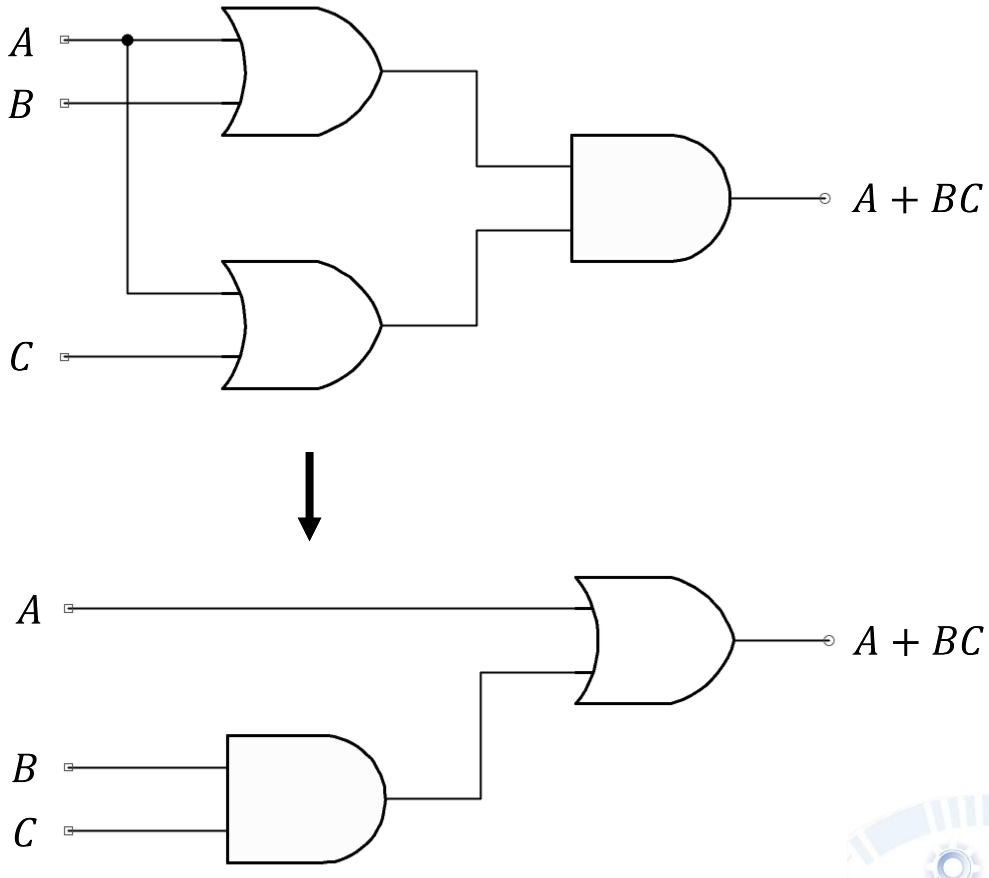
$$\begin{aligned}
A + \overline{A}B &= (A + AB) + \overline{A}B \\
&= (AA + AB) + \overline{A}B \\
&= AA + AB + A\overline{A} + \overline{A}B \\
&= (A + \overline{A})(A + B) \\
&= 1 \cdot (A + B) \\
&= A + B
\end{aligned}$$



RULE 12

Rule 12: **$(A + B)(A + C) = A + BC$**

$$(A + B)(A + C) = AA + AC + AB + BC$$
$$= A + AC + AB + BC$$
$$= A + AB + BC$$
$$= A + BC$$



DEMORGAN'S THEOREMS

FIRST THEOREM

Truth Table

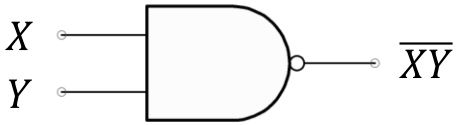
X	Y	$\overline{XY}$	$\bar{X} + \bar{Y}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

DeMorgan's first theorem states that the complement of a product of variables is equal to the sum of the complements of the variables.

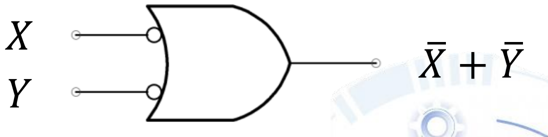
Logic Expression

$$\boldsymbol{\overline{XY} = \bar{X} + \bar{Y}}$$

NAND



Negative-OR



SECOND THEOREM

Truth Table

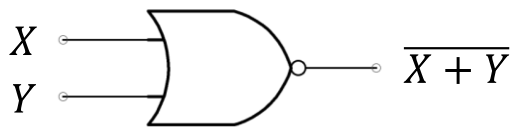
X	Y	$\overline{X+Y}$	$\bar{X} \cdot \bar{Y}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

DeMorgan's second theorem states that the complement of a sum of variables is equal to the product of the complements of the variables.

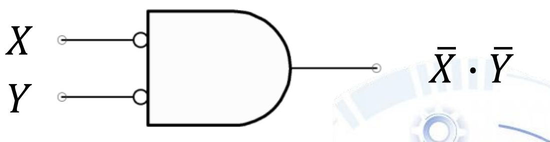
Logic expression

$$\boldsymbol{\overline{X+Y} = \bar{X} \cdot \bar{Y}}$$

NOR



Negative-AND



EXERCISE

Apply DeMorgan's theorems to the expression:

$f = \overline{(A + B)C}$

Solution

$f = (\overline{A+B}) \cdot \overline{C}$

$\boxed{f = \overline{A}\,\overline{B}\,\overline{C}}$

ans

EXERCISE

Simplify the Boolean expression:

$$f = AB + A(B + C) + B(B + C)$$

$$f = \underbrace{AB + \cancel{AB}}_{X+X=X} + AC + \underbrace{\overset{\to B}{\cancel{BB}}}_{X\cdot X=X} + BC$$

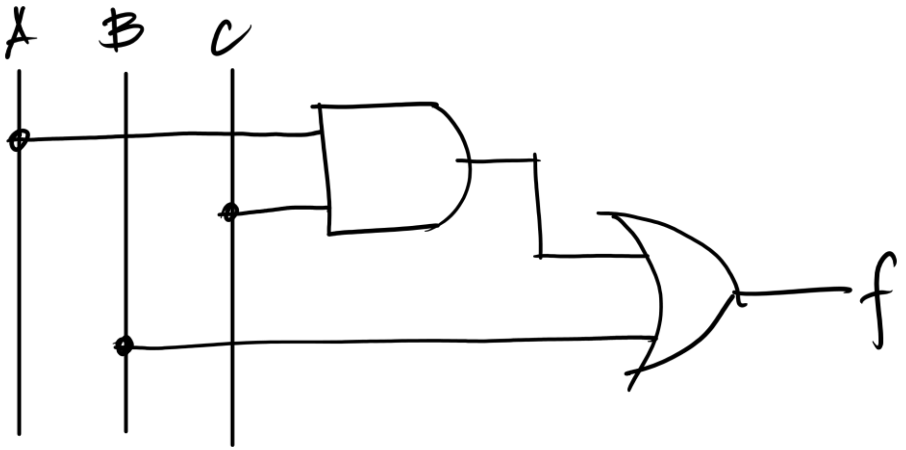
$$f = AB + AC + \underbrace{B + \cancel{BC}}_{X+XY=X}$$

$$f = \underbrace{\cancel{AB} + B}_{X+XY=X} + AC$$

$$\boxed{f = B + AC}$$

ans

Solution



EXERCISE

Simplify the Boolean expression:

$$f = [A\bar{B}(C + BD) + \bar{A}\bar{B}]C$$

$$f = [A\bar{B}C + \cancelto{0}{A\bar{B}BD} + \bar{A}\bar{B}]C$$

$$f = [A\bar{B}C + \bar{A}\bar{B}]C$$

$$f = A\bar{B}\cancelto{C}{CC} + \bar{A}\bar{B}C$$

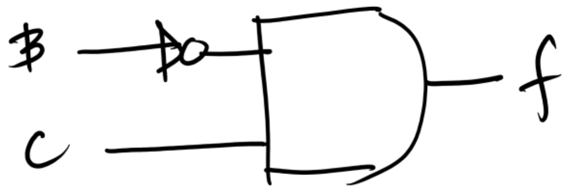
$$f = \underline{A\bar{B}C} + \underline{\bar{A}\bar{B}C}$$

$$f = \bar{B}C(\cancelto{1}{A + \bar{A}})$$

Solution

$$\boxed{f = \bar{B}C}$$

ans



EXERCISE

Apply DeMorgan's theorems to the expression:

$$f = \overline{(\bar{A} + B) + CD}$$

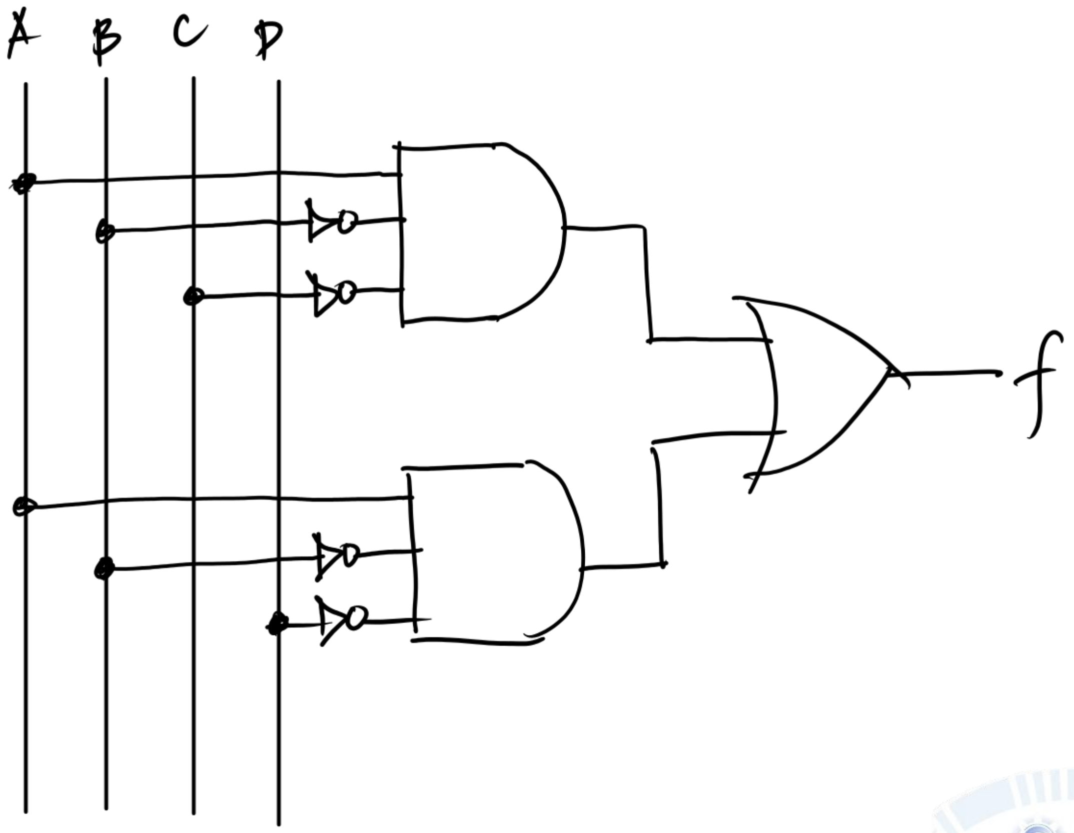
$$f = \overline{(\bar{A} + B)} \cdot \overline{CD}$$

$$f = \bar{\bar{A}} \cdot \bar{B} \cdot [\bar{C} + \bar{D}]$$

$$\boxed{f = A\bar{B}\bar{C} + A\bar{B}\bar{D}}$$

ans

Solution



EXERCISE

Simplify the Boolean expression:

Solution

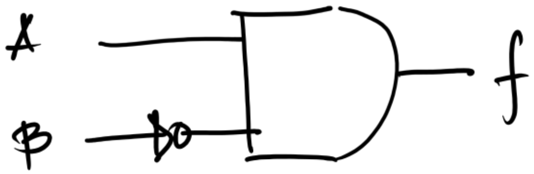
$$f = A\bar{B} + A(\overline{B + C}) + B(\overline{B + C})$$

$$f = A\bar{B} + A\bar{B}\bar{C} + \cancel{B\bar{B}}^{\,0}\,\bar{C}$$

$$X + XY = X$$

$$\boxed{f = A\bar{B}}$$

ans



LABORATORY