



# **CHAIN RULE**

## **DIFFERENTIAL CALCULUS**

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# TOPIC OUTLINE

## Chain Rule

- Leibniz Notation

# CHAIN RULE

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If  $g$  is differentiable at  $x$  and  $f$  is differentiable at  $g(x)$ , then the composite function defined by

$$F(x) = f(g(x))$$

is differentiable at  $x$  and  $F'$  is given by product

$$F'(x) = f'(g(x)) \cdot g'(x)$$

Differentiate the function  $y = \sin(2x)$ .

Differentiate the function  $y = \cos(\sqrt{x})$ .



## **EXERCISE**

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Differentiate the function  $y = \sqrt{x^2 + 1}$ .

Differentiate the function  $y = \tan(5x^3 + 2x)$ .



## LEIBNIZ NOTATION

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Differentiate the function  $y = \sin(2x)$ .

If  $y = f(u)$  and  $u = g(x)$  are both differentiable functions, then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Differentiate the function  $y = \cos(\sqrt{x})$ .



## **EXERCISE**

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Differentiate  $y = \sin x^2$ .

Differentiate  $y = x \sin \frac{1}{x}$ .



## **EXERCISE**

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Differentiate  $y = \frac{x^3}{\sqrt{1+x^2}}$ .

Differentiate  $g(t) = \left(\frac{t-2}{2t+1}\right)^9$ .



## **EXERCISE**

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The displacement of a particle on a vibrating string is given by the equation

$$s(t) = 10 + \frac{1}{4} \sin(10\pi t)$$

where  $s$  is measured in centimeters and  $t$  in seconds.

Find the velocity of the particle in  $t$  seconds.

Solution



## **EXERCISE**

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Differentiate  $y = \frac{\cos \pi x}{\sin \pi x + \cos \pi x}$ .

Differentiate  $y = [x + (x + \sin^2 x)^3]^4$ .



## **EXERCISE**

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A Cepheid variable star is a star whose brightness alternately increases and decreases. The most easily visible such star is Delta Cephei, for which the interval between times of maximum brightness is 5.4 days. The average brightness of this star is 4.0 and its brightness changes by  $\pm 0.35$ . In view of these data, the brightness of Delta Cephei at time  $t$ , where  $t$  is measured in days, has been modeled by the function

$$B(t) = 4.0 + 0.35 \sin\left(\frac{2\pi t}{5.4}\right)$$

Find the rate of change of the brightness after  $t$  days.

Solution



# LABORATORY