

TRANSIENT RESPONSE OF CAPACITOR RC CIRCUITS

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TOPIC OUTLINE

Charging a Capacitor

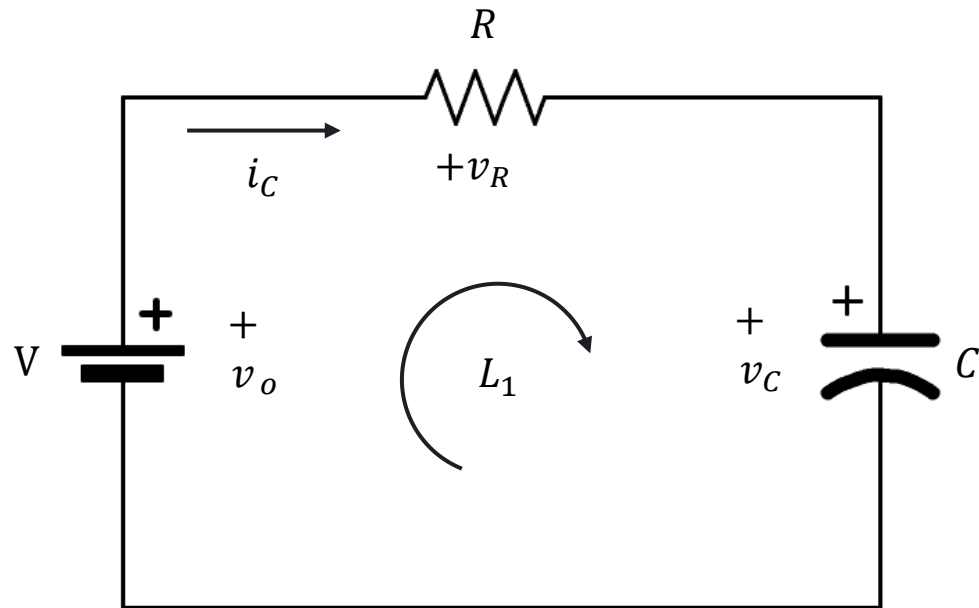
Discharging a Capacitor



CHARGING A CAPACITOR



RC CIRCUIT



KVL @ L_1

$$-v_o + v_R + v_C = 0$$

$$v_R + v_C = v_o$$

$$i_C R + v_C = v_o \quad ; i_C = C \frac{d}{dt} v_C$$

$$RC \frac{d}{dt} v_C + v_C = v_o$$

$$\frac{d}{dt} v_C + \frac{1}{RC} v_C = \frac{v_o}{RC}$$

... first-order ODE

$$v_C(t) = v_o \left(1 - e^{-\frac{t}{RC}}\right)$$



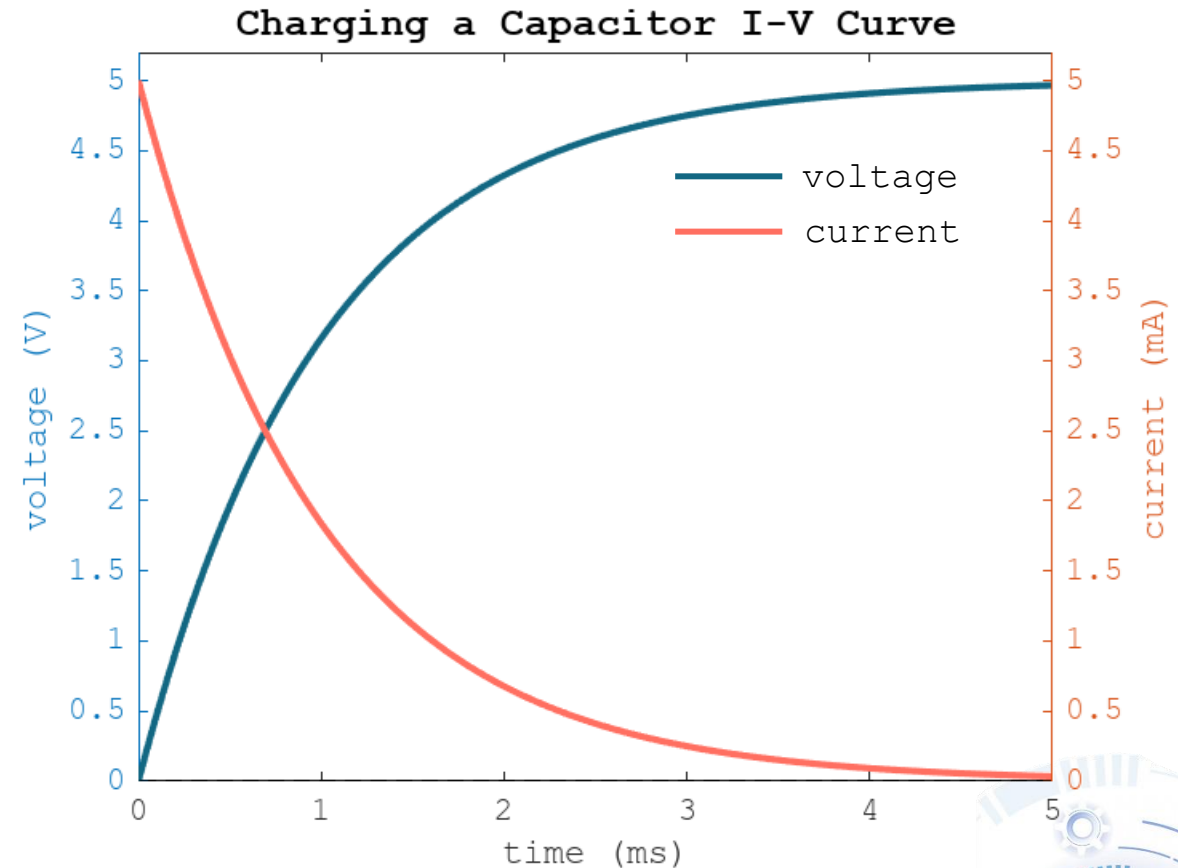
CAPACITOR VOLTAGE

Charging Equation

$$v_c(t) = v_o(1 - e^{-\frac{t}{\tau}})$$

where: $\tau = RC$

The voltage across the capacitor starts at zero and exponentially increases to its maximum voltage (v_o).



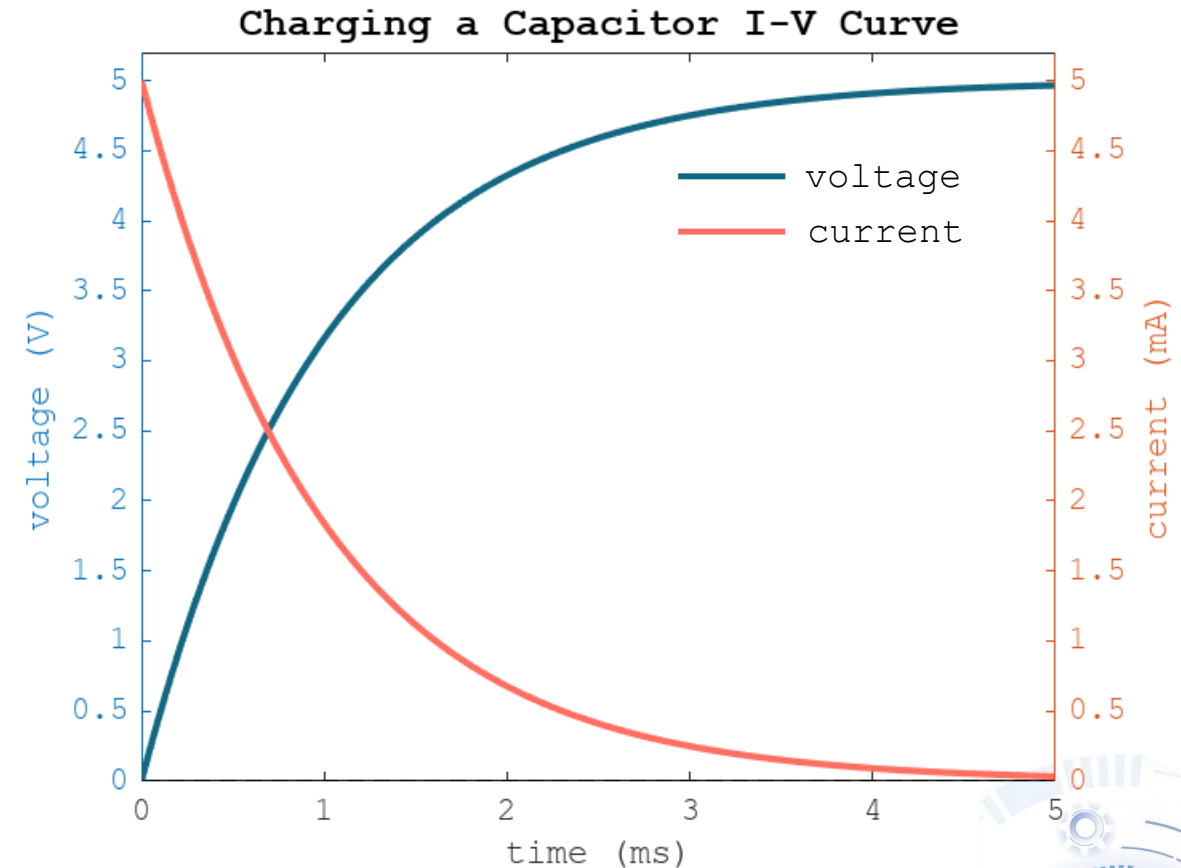
CAPACITOR CURRENT

Charging Equation

$$i_c(t) = \frac{v_o}{R} e^{-\frac{t}{\tau}}$$

where: $\tau = RC$

The current through the capacitor instantly jumps to its maximum value (v_o/R) amperes then decays exponentially to zero.



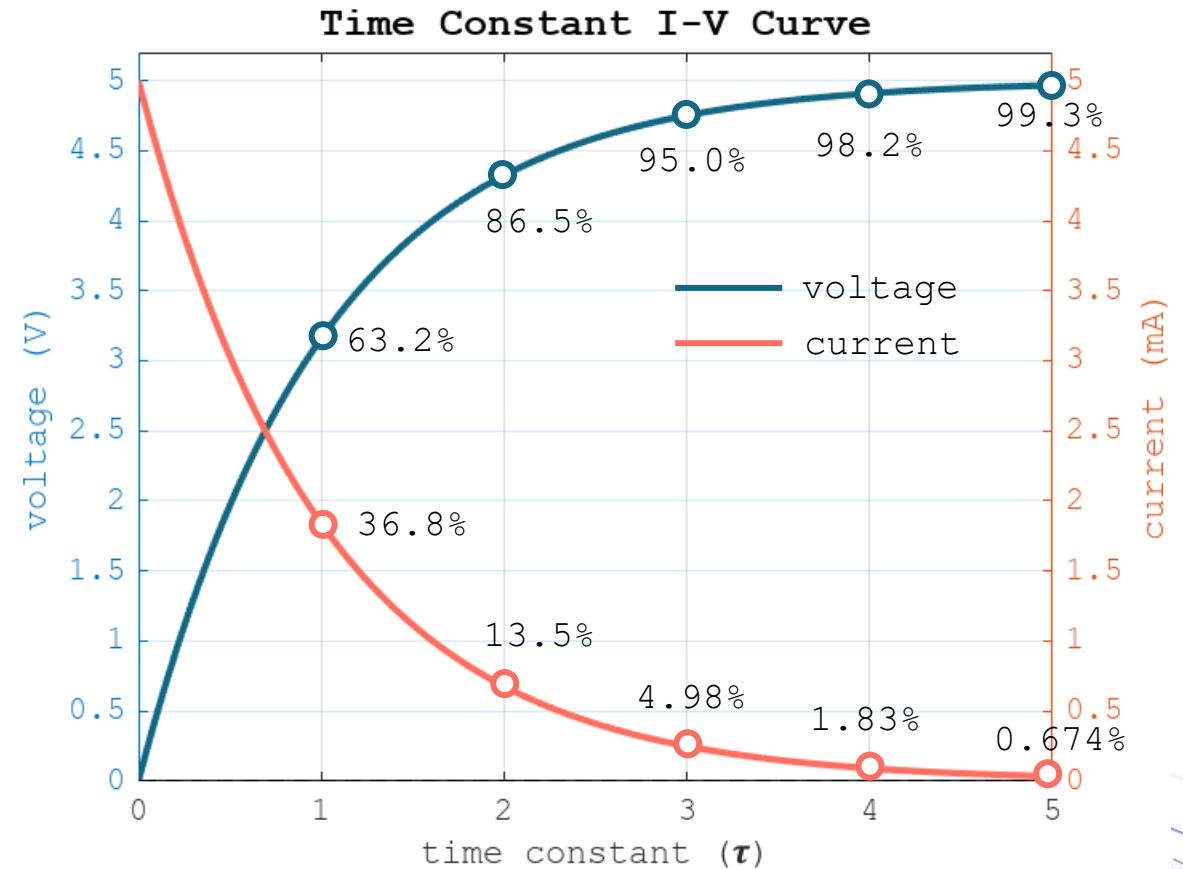
TIME CONSTANT

The time constant (τ) is a measure of how quickly a capacitor charges or discharges in an RC circuit.

Formula

$$\tau = RC$$

unit: second



EXERCISE

A $100\mu\text{F}$ capacitor is connected to a 12V DC power supply through a resistor of $1\text{K}\Omega$. Determine the time it takes for the capacitor to charge to 86.5% of its maximum voltage.

Solution



EXERCISE

A $100\mu\text{F}$ capacitor is connected to a 12V DC power supply through a resistor of $1\text{K}\Omega$. Determine the voltage across the capacitor after 200 *ms* of charging.

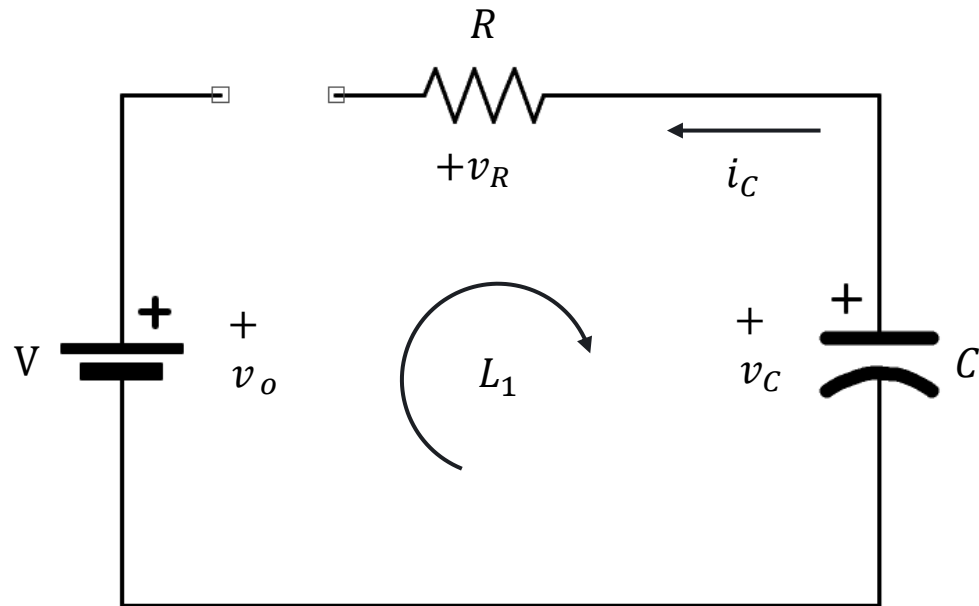
Solution



DISCHARGING A CAPACITOR



RC CIRCUIT



KVL @ L_1

$$v_R + v_C = 0$$

$$i_C R + v_C = 0 \quad ; i_C = C \frac{d}{dt} v_C$$

$$RC \frac{d}{dt} v_C + v_C = 0$$

$$\frac{d}{dt} v_C + \frac{1}{RC} v_C = 0$$

... first-order ODE

$$v_C(t) = v_o e^{-\frac{t}{RC}}$$



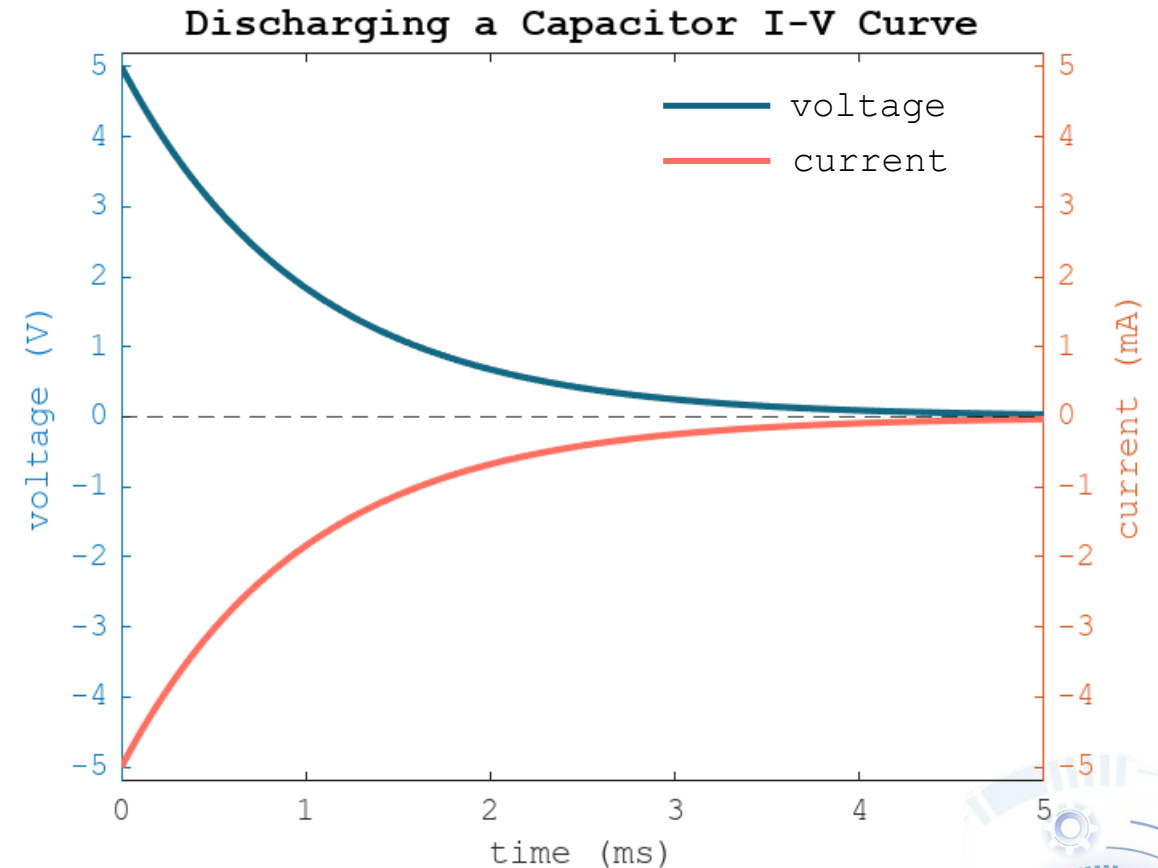
CAPACITOR VOLTAGE

Discharging Equation

$$v_c(t) = v_o e^{-\frac{t}{\tau}}$$

where: $\tau = RC$

The voltage across the capacitor starts at its maximum voltage (v_o) then decays exponentially to zero.



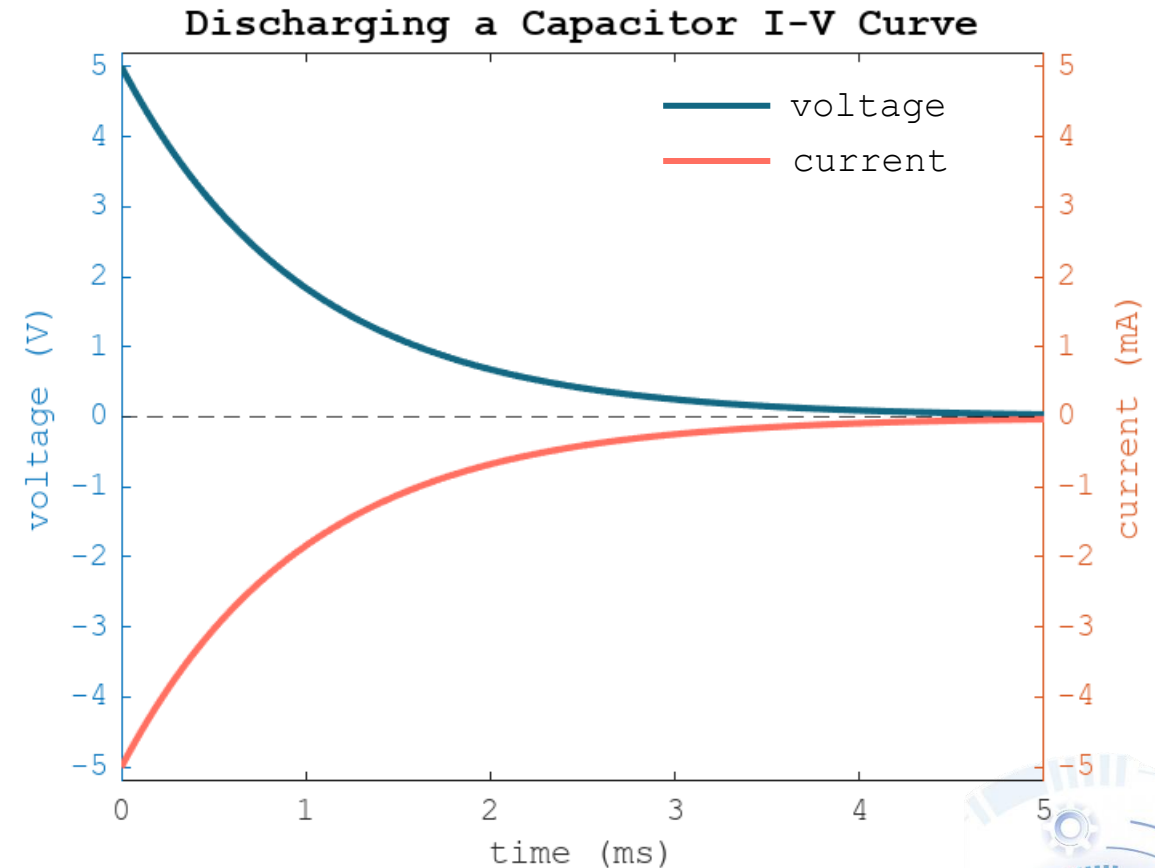
CAPACITOR CURRENT

Discharging Equation

$$i_c(t) = -\frac{v_o}{R} \left(e^{-\frac{t}{\tau}} \right)$$

where: $\tau = RC$

The current through the capacitor instantly jumps to its maximum value, but in opposite direction ($-v_o/R$) then decays exponentially to zero.



EXERCISE

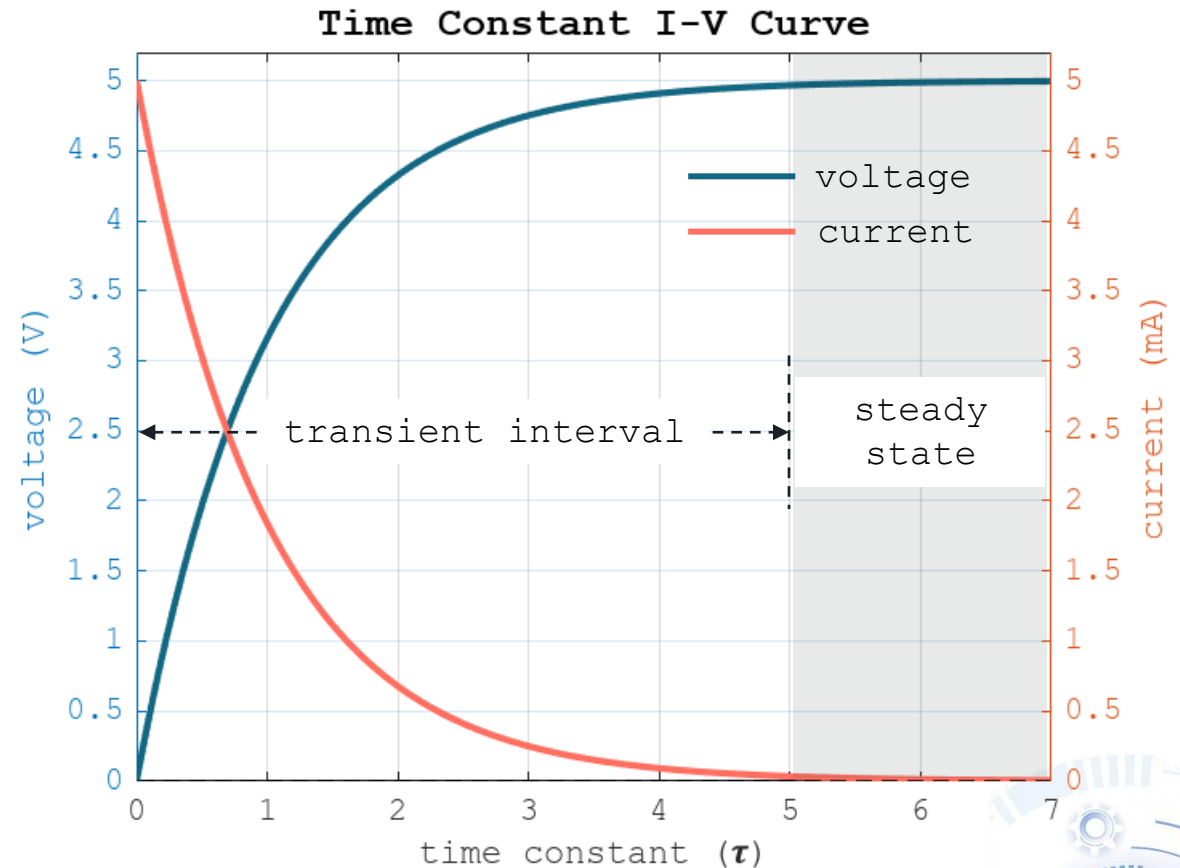
A $200\mu\text{F}$ capacitor is initially charged to 12V. It is then disconnected from the power supply and discharged a resistor of $1.5\text{K}\Omega$. Determine the voltage across the capacitor after 0.1s of discharging.

Solution



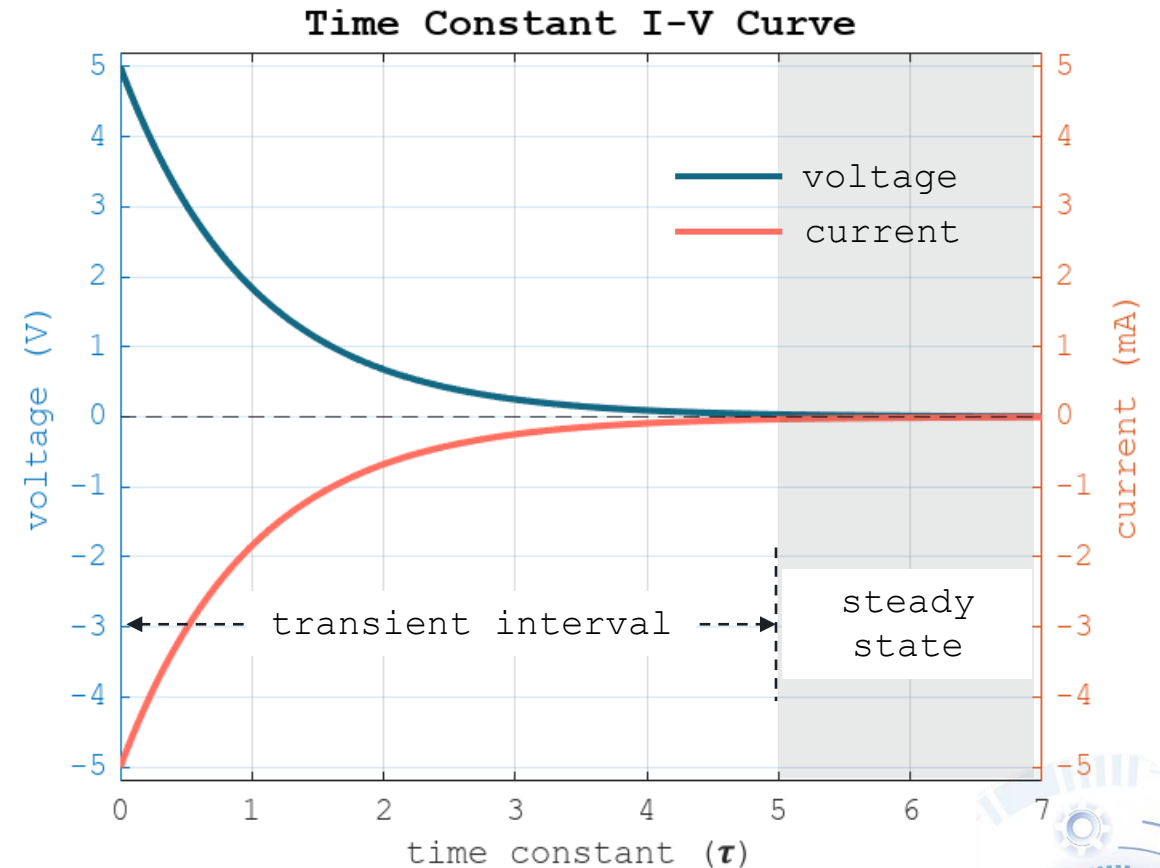
TRANSIENT RESPONSE

The transient response of a capacitor describes the time-dependent changes in voltage across the capacitor and the current through it. The transient phase is typically considered to last for approximately five time constants (5τ) after which the system is assumed to have reached steady-state conditions.



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LABORATORY

