T-DISTRIBUTION

INFERENTIAL STATISTICS

prepared by:

Gyro A. Madrona

Electronics Engineer

TOPIC OUTLINE

t-Distribution

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The **t-distribution** (Student's t-Distribution) is used to calculate the confidence interval when:

1. The population variance ($\boldsymbol{\sigma^2}$) is **unknown**.

2. The sample size is **small** ($\boldsymbol{n < 30}$).

Formula

$$CI = \bar{x} \pm t_{n-1,\, \alpha/2}\left(\frac{s}{\sqrt{n}}\right)$$

where

$\bar{x}$ = sample mean

$t_{n-1,\, \alpha/2}$ = t-statistic

$\frac{s}{\sqrt{n}}$ = standard error

t-table

d.f. / α	0.1	0.05	0.025	0.01	0.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
35	1.306	1.690	2.030	2.438	2.724
40	1.303	1.684	2.021	2.423	2.704
50	1.299	1.676	2.009	2.403	2.678
60	1.296	1.671	2.000	2.390	2.660
120	1.289	1.658	1.980	2.358	2.617
inf.	1.282	1.645	1.960	2.326	2.576
CI*	80%	90%	95%	98%	99%

z-statistic

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syntax

Percent point function `t.ppf()` returns the critical **t** value for a given probability `(1−alpha)`.

```
from scipy import stats
```

Two-Tailed Test

```
t_critical = stats.t.ppf(1-alpha/2,dof)
```

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syntax

Interval function `t.interval()` returns a tuple (lower_limit, upper_limit) representing the confidence interval for t-interval.

```
from scipy import stats
ci_lower, ci_upper = stats.t.interval(
    confidence = confidence_level,
    df = degrees of freedom (n-1),
    loc = sample_mean,
    scale = standard_error
)
```

EXERCISE

A power company measures the voltage output (in volts) of a batch of transformers. A random sample of **10** transformers is tested, and their voltage outputs are recorded in "transformer-voltage-data-10-sample" dataset . Calculate a **95% confidence interval** for the true mean voltage output ($\boldsymbol{\mu}$) of all transformers.

solution

LABORATORY