THE Z-DISTRIBUTION

INFERENTIAL STATISTICS

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TOPIC OUTLINE

Point Estimate

Confidence Interval

z-Distribution

POINT ESTIMATE

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A **point estimate** is a **single** value (statistic) derived from sample data that serves as the "best guess" for an unknown population parameter.

$\bar{x}$ is a point estimate of μ

s^2 is a point estimate of σ^2

example

A factory produces resistors labeled as $\mathbf{100\ \Omega}$, but due to manufacturing variations, the actual resistance varies. An engineer takes a random samples of 30 resistors and calculated the average resistance of the sample, $\bar{x} = \mathbf{101.2\ \Omega}$.

The point estimate for the true mean resistance μ of all resistors produced is $\mathbf{101.2\ \Omega}$.

CONFIDENCE INTERVAL

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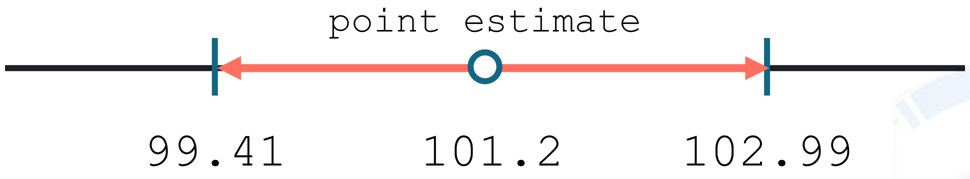
A **confidence interval** is a **range** of values, derived from sample data, that is likely to contain the true value of an unknown population parameter (e.g., $\boldsymbol{\mu}, \boldsymbol{\sigma}^2$).

example

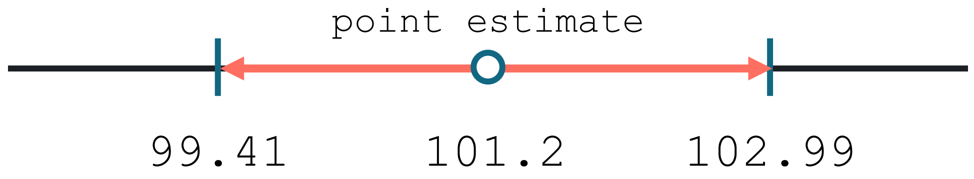
A factory produces resistors labeled as $\mathbf{100\ \Omega}$, but due to manufacturing variations, the actual resistance varies. An engineer takes a random samples of 30 resistors and calculated a 95% confidence interval for the true mean resistance, $\mathbf{95\%\ CI} = [\mathbf{99.41\ \Omega}, \mathbf{102.99\ \Omega}]$



CONFIDENCE INTERVAL

A **confidence interval** is a **range** of values, derived from sample data, that is likely to contain the true value of an unknown population parameter (e.g., $\boldsymbol{\mu}, \boldsymbol{\sigma}^2$).

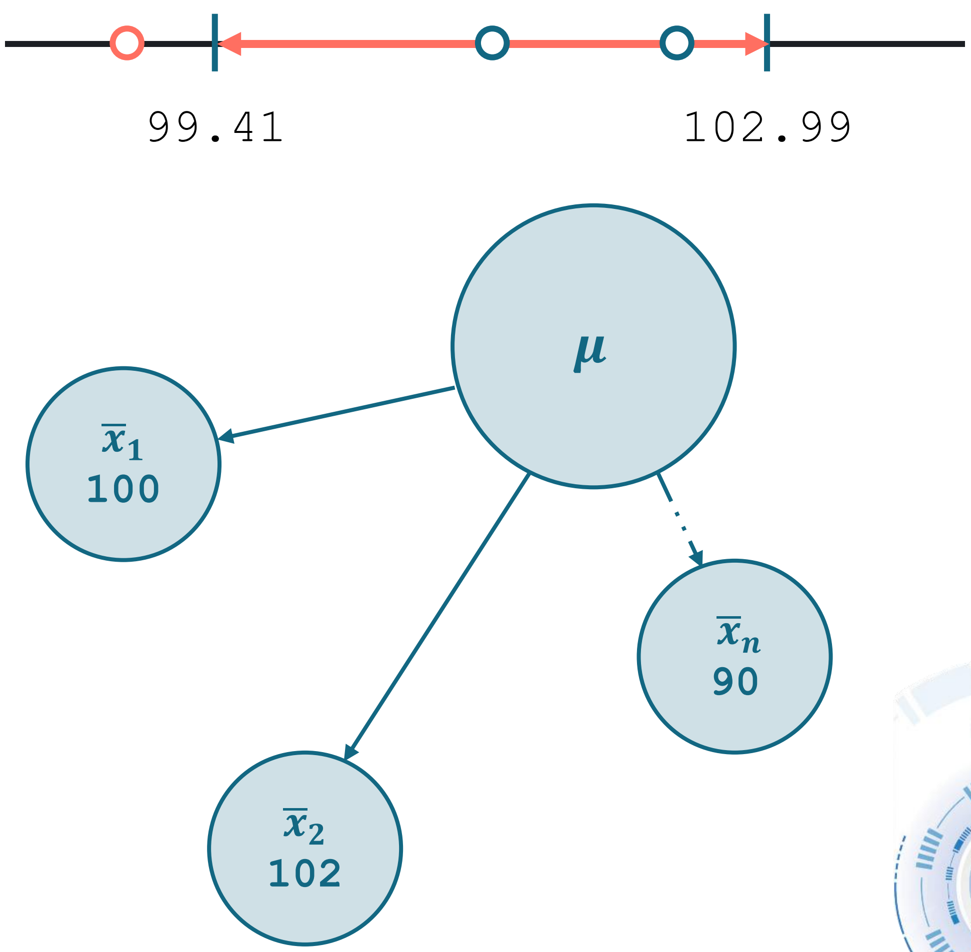
example



We are 95% confident that the true mean $\boldsymbol{\mu}$ of all resistors produced lies between 99.41 Ω and 102.99 Ω.

CONFIDENCE LEVEL

example



Confidence levels (e.g., 90%, 95%, 99%) describe the method's reliability over many samples.

A 95% confidence level means that if the same sampling process were **repeated** many times, approximately 95% of the calculated CIs would contain the true population parameter (e.g., $\boldsymbol{\mu}, \boldsymbol{\sigma}^2$).

CONFIDENCE LEVEL

Confidence levels (e.g., 90%, 95%, 99%) describe the method's reliability over many samples.

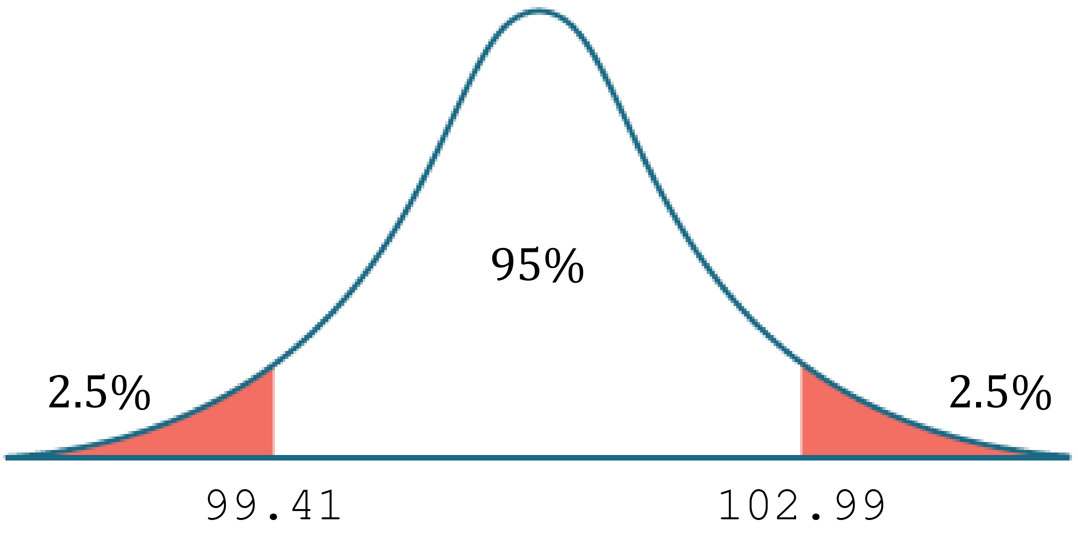
Formula

$$\textbf{confidence level} = \mathbf{1} - \boldsymbol{\alpha}$$

where

$$0 \leq \alpha \leq 1$$

95% Confidence Level



$\alpha = 0.05$

CONFIDENCE LEVEL

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Formula

$$\textbf{confidence level} = \mathbf{1} - \boldsymbol{\alpha}$$

where

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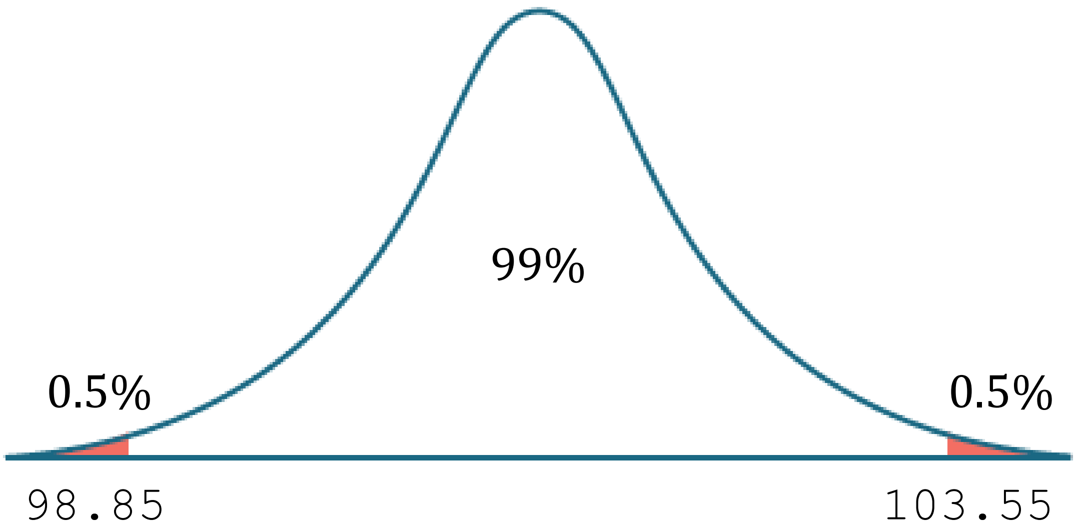
99% Confidence Level



$\alpha = 0.01$

CONFIDENCE LEVEL

Confidence levels (e.g., 90%, 95%, 99%) describe the method's reliability over many samples.

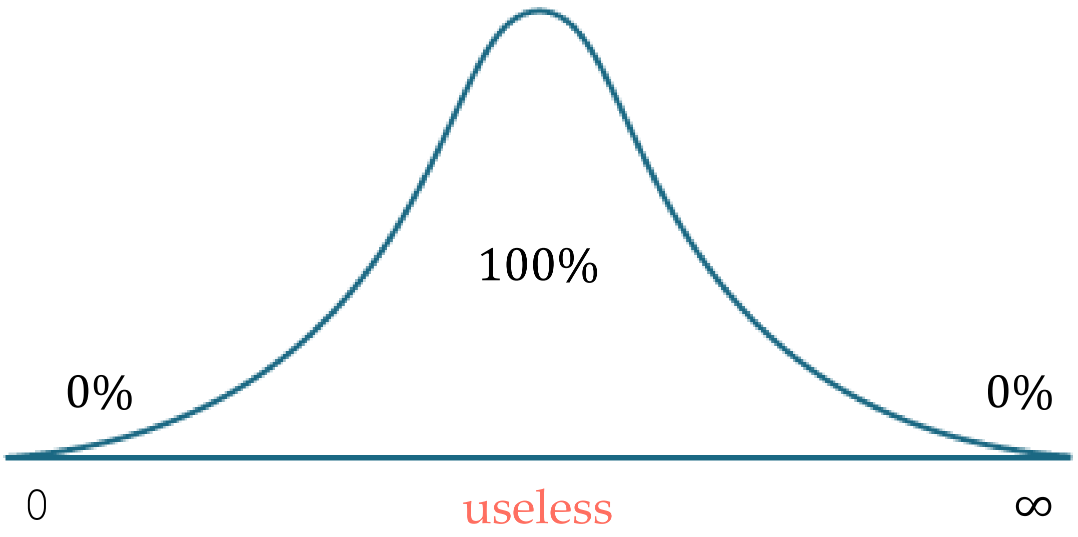
Formula

$$\textbf{confidence level} = \mathbf{1} - \boldsymbol{\alpha}$$

where

$$0 \le \alpha \le 1$$

100% Confidence Level



$\alpha = 0$

Z-DISTRIBUTION

Z-DISTRIBUTION

The **z-distribution** is used to calculate the confidence interval when:

1. The population variance (σ^2) is **known**.

2. The sample size is **large** ($n \geq 30$).

Formula

$$CI = \bar{x} \pm z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

where

$\bar{x}$ = sample mean

$z_{\alpha/2}$ = z-statistic

$\frac{\sigma}{\sqrt{n}}$ = standard error

z-table

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

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Formula

$$\boldsymbol{CI = \bar{x} \pm z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)}$$

where

$\bar{x}$ = sample mean

$z_{\alpha/2}$ = z-statistic

$\frac{\sigma}{\sqrt{n}}$ = standard error

syntax

Percent point function `norm.ppf()` returns the critical **z** value for a given probability `(1−alpha)`.

```
from scipy import stats
```

Two-Tailed Test

```
z_critical = stats.norm.ppf(1-alpha/2)
```

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z_critical = stats.norm.ppf(1-alpha)
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syntax

Interval function `norm.interval()` returns a tuple (lower_limit, upper_limit) representing the confidence interval for z-interval.

```
from scipy import stats
ci_lower, ci_upper =
stats.norm.interval(
    confidence = confidence_level,
    loc = sample_mean,
    scale = standard_error
)
```

EXERCISE

A power company measures the voltage output (in volts) of a batch of transformers. The population standard deviation ($\boldsymbol{\sigma}$) is known to be $\mathbf{0.5}$ volts. A random sample of **30** transformers is tested, and their voltage outputs are recorded in "transformer-voltage" data . Calculate a **95% confidence interval** for the true mean voltage output ($\boldsymbol{\mu}$) of all transformers.

solution

LABORATORY