NODE ANALYSIS METHOD

BASIC CIRCUIT ANALYSIS METHOD

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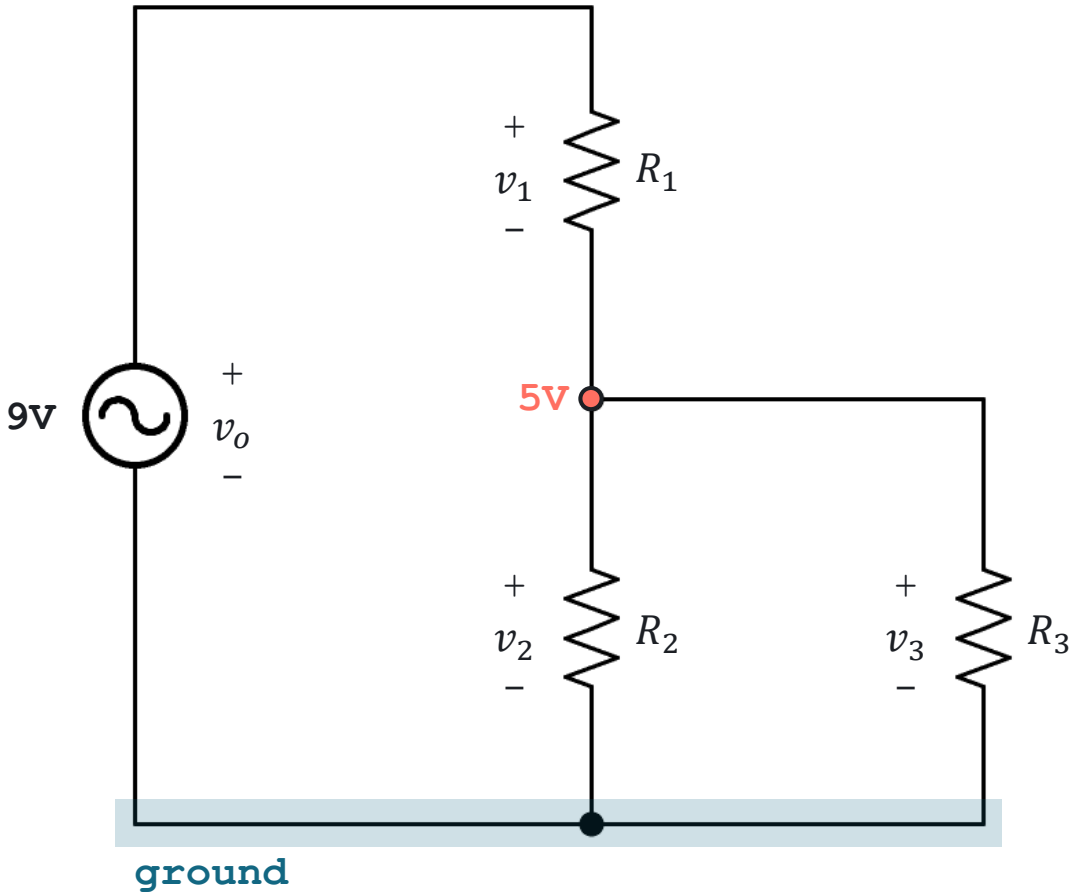
TOPIC OUTLINE

Node Voltage

Node Analysis Method

NODE ANALYSIS METHOD

NODE VOLTAGE



Node Voltage is the electrical potential difference at a specific node in a circuit relative to a reference node.

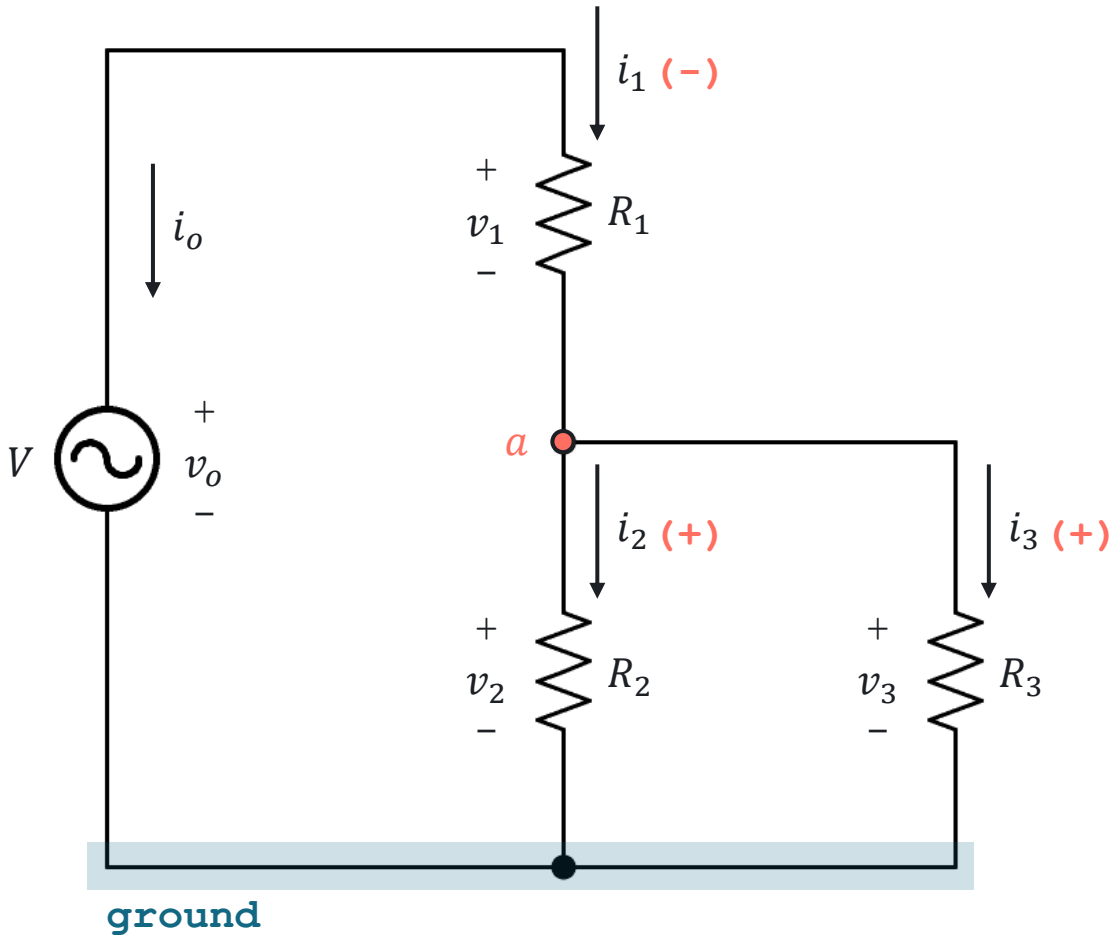
example

$v_1 = 9\text{V} - 5\text{V}$

$v_2 = 5\text{V} - 0$

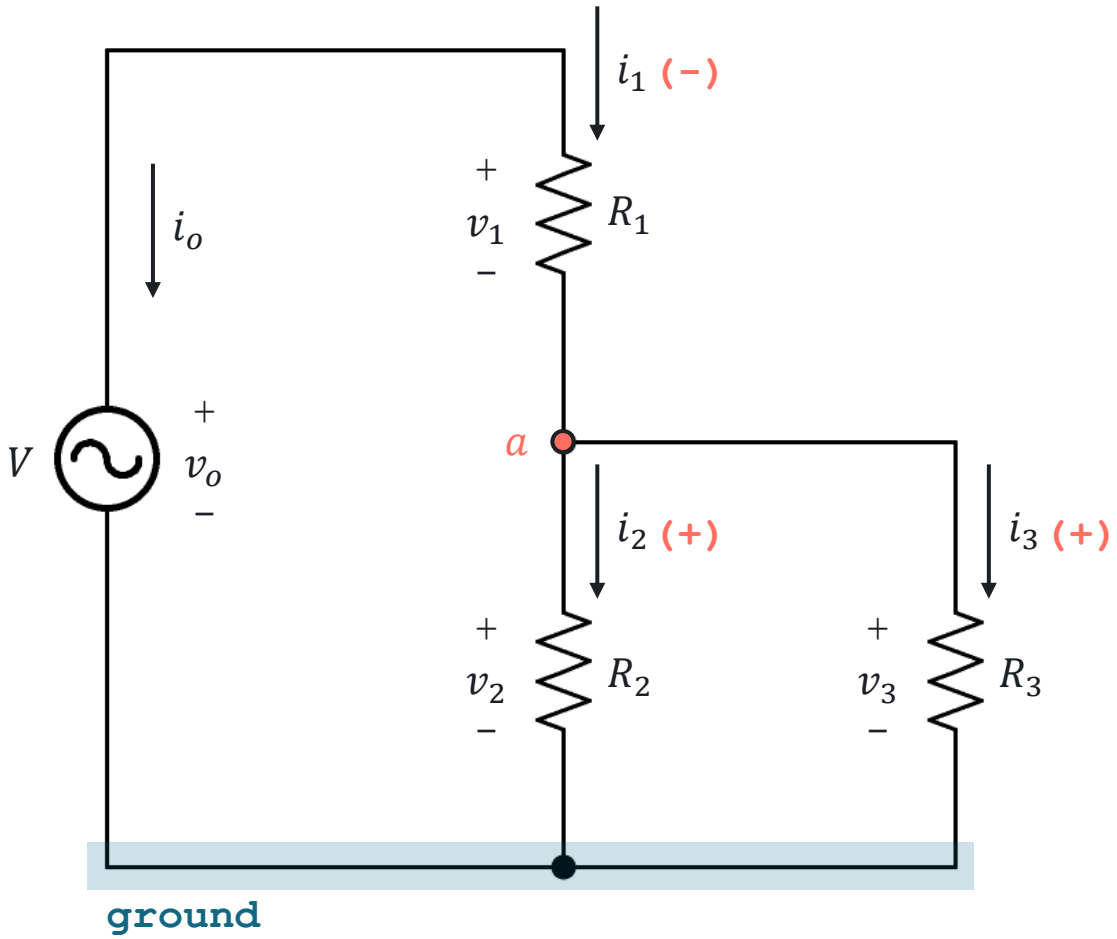
$v_3 = 5\text{V} - 0$

NODE ANALYSIS METHOD



The **node analysis method** is based on Kirchhoff's current law (KCL), which is implicitly applied to establish voltage-current relationships forming a system of equations to solved for the **unknown node voltages**.

NODE ANALYSIS METHOD



KCL @a

$$-i_1 + i_2 + i_3 = 0$$

$$-\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3} = 0$$

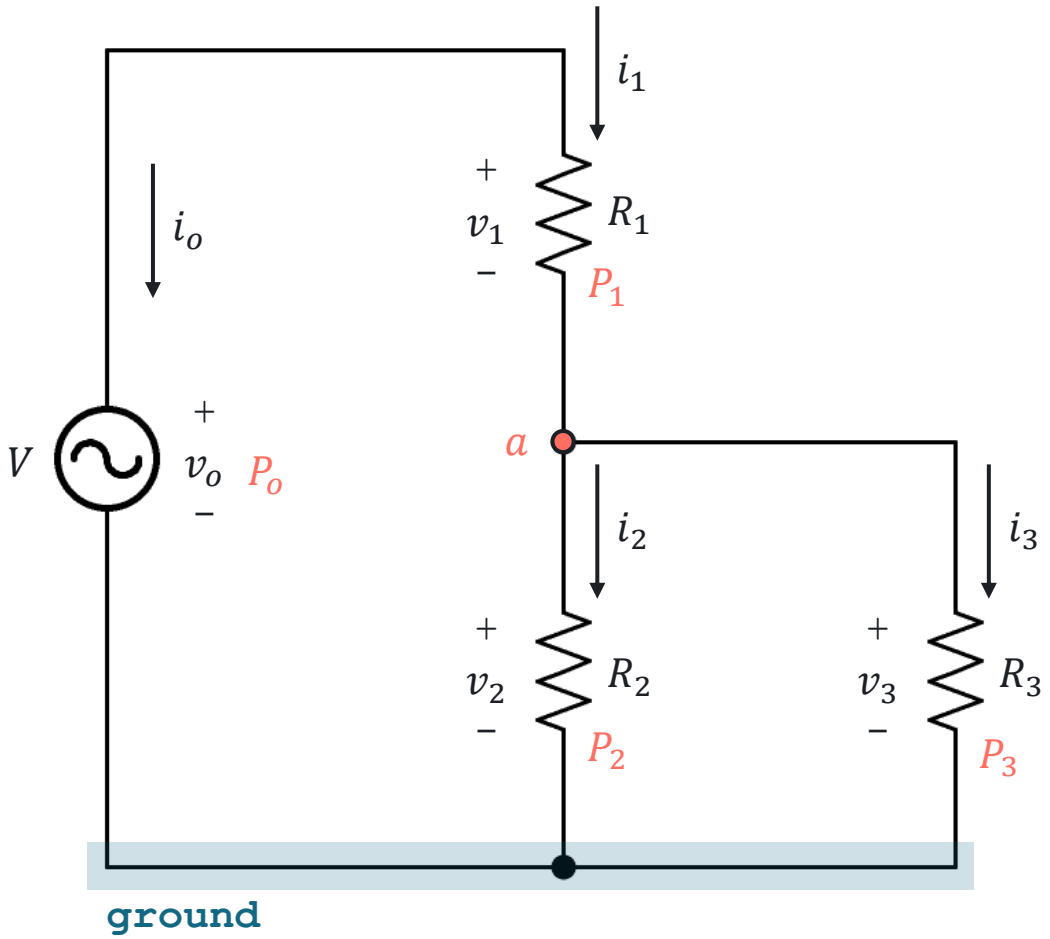
$$-\frac{v_o - v_a}{R_1} + \frac{v_a - 0}{R_2} + \frac{v_a - 0}{R_3} = 0$$

$$-v_o G_1 + v_a G_1 + v_a G_2 + v_a G_3 = 0$$

$$v_a(G_1 + G_2 + G_3) = v_o G_1$$

$$v_a = \frac{v_o G_1}{G_1 + G_2 + G_3}$$

ELECTRICAL POWER



Electrical power refers to the rate at which electrical energy is converted per unit time (joules/second).

Formulas

$$P = vi$$

$$P = i^2R$$

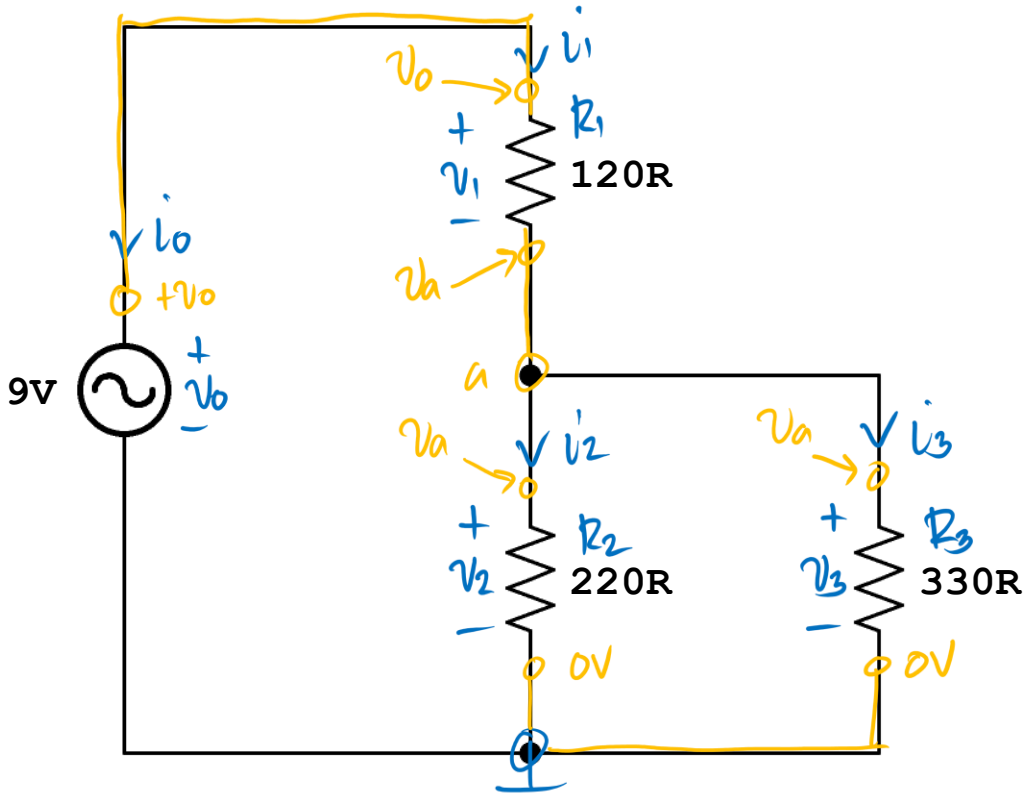
$$P = \frac{v^2}{R}$$

$$P_o = P_1 + P_2 + P_3 + \cdots P_n$$

unit: Watt (W)

EXERCISE

Determine the current through and voltage across each resistor, as well as the total power of the given circuit.



Solution

KCL@a

$$-i_1 + i_2 + i_3 = 0$$

$$-\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} = 0$$

$$-V_1 G_1 + V_2 G_2 + V_3 G_3 = 0$$

$$-(V_o - V_a) G_1 + (V_a - 0) G_2 + (V_a - 0) G_3 = 0$$

$$-V_o G_1 + V_a G_1 + V_a G_2 + V_a G_3 = 0$$

factor out V_a

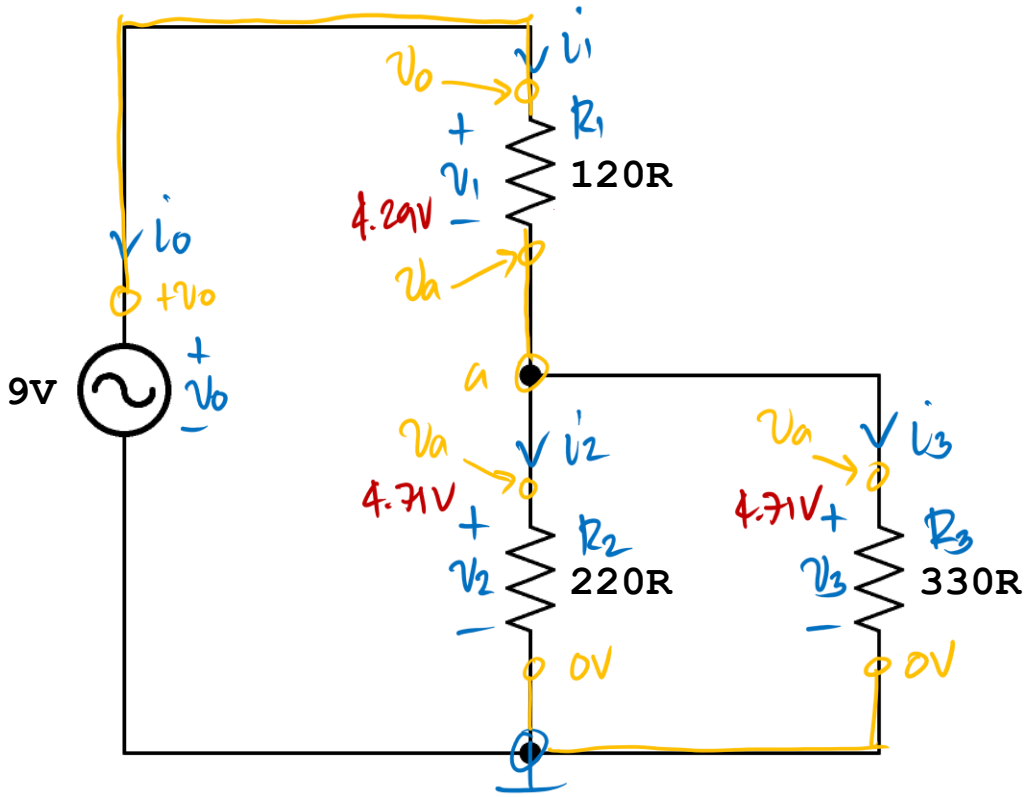
$$V_a (G_1 + G_2 + G_3) = V_o G_1$$

$$\frac{V_a (G_1 + G_2 + G_3)}{G_1 + G_2 + G_3} = \frac{V_o G_1}{G_1 + G_2 + G_3}$$

$$V_a = \frac{V_o G_1}{G_1 + G_2 + G_3}$$

EXERCISE

Determine the current through and voltage across each resistor, as well as the total power of the given circuit.



Solution

$$v_a = \frac{v_o G_1}{G_1 + G_2 + G_3}$$

$$v_a = \frac{9(1/120)}{1/120 + 1/220 + 1/330}$$

$$v_a = 4.71\ V$$

$$v_1 = v_o - v_a$$

$$v_1 = 9 - 4.71$$

$$v_1 = 4.29\ V$$ ans

$$v_2 = v_a - 0$$

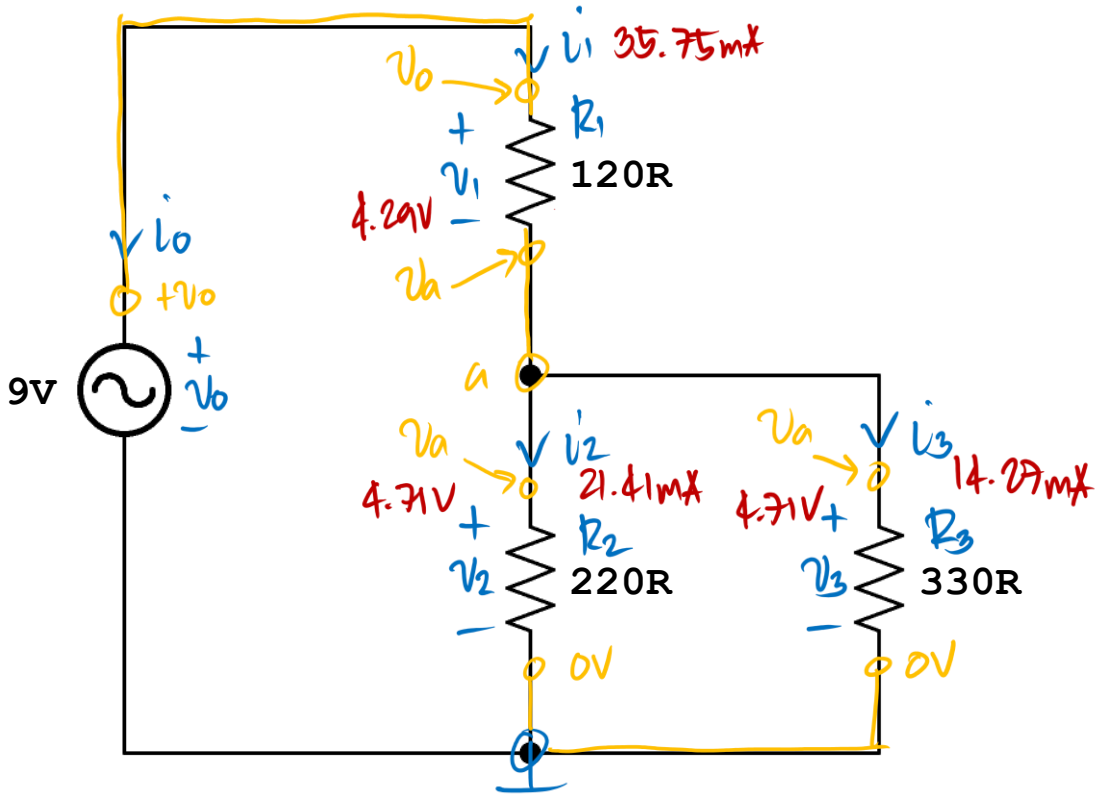
$$v_2 = 4.71\ V$$ ans

$$v_3 = v_a - 0$$

$$v_3 = 4.71\ V$$ ans

EXERCISE

Determine the current through and voltage across each resistor, as well as the total power of the given circuit.



Solution

$$i_1 = \frac{v_1}{R_1}$$

$$i_1 = \frac{4.29}{120}$$

$$i_1 = 35.75\ mA$$ ans

$$i_3 = \frac{v_3}{R_3}$$

$$i_3 = \frac{4.71}{330}$$

$$i_3 = 14.27\ mA$$ ans

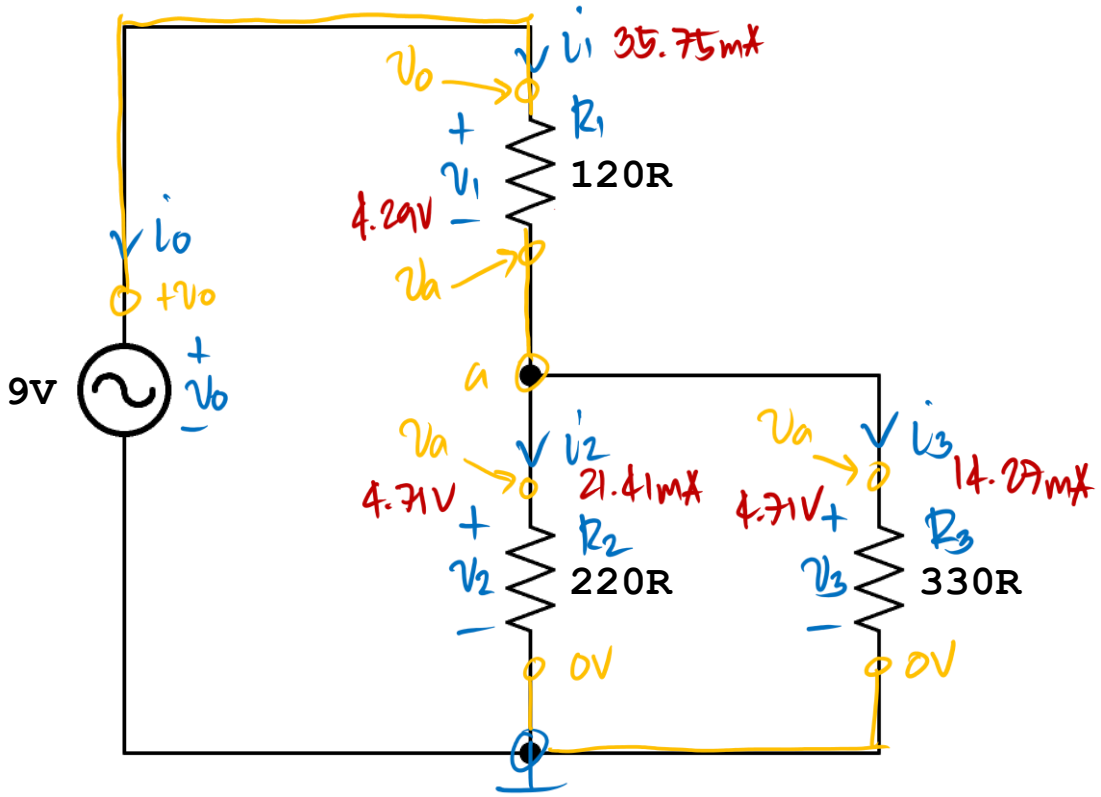
$$i_2 = \frac{v_2}{R_2}$$

$$i_2 = \frac{4.71}{220}$$

$$i_2 = 21.41\ mA$$ ans

EXERCISE

Determine the current through and voltage across each resistor, as well as the total power of the given circuit.



Solution

$P_1 = i_1 v_1$

$P_1 = 35.75m(4.29)$

$P_1 = 153.37\,mW$

$P_2 = i_2 v_2$

$P_2 = 21.41m(4.71)$

$P_2 = 100.84\,mW$

$P_3 = i_3 v_3$

$P_3 = 14.27m(4.71)$

$P_3 = 67.21\,mW$

Total Power

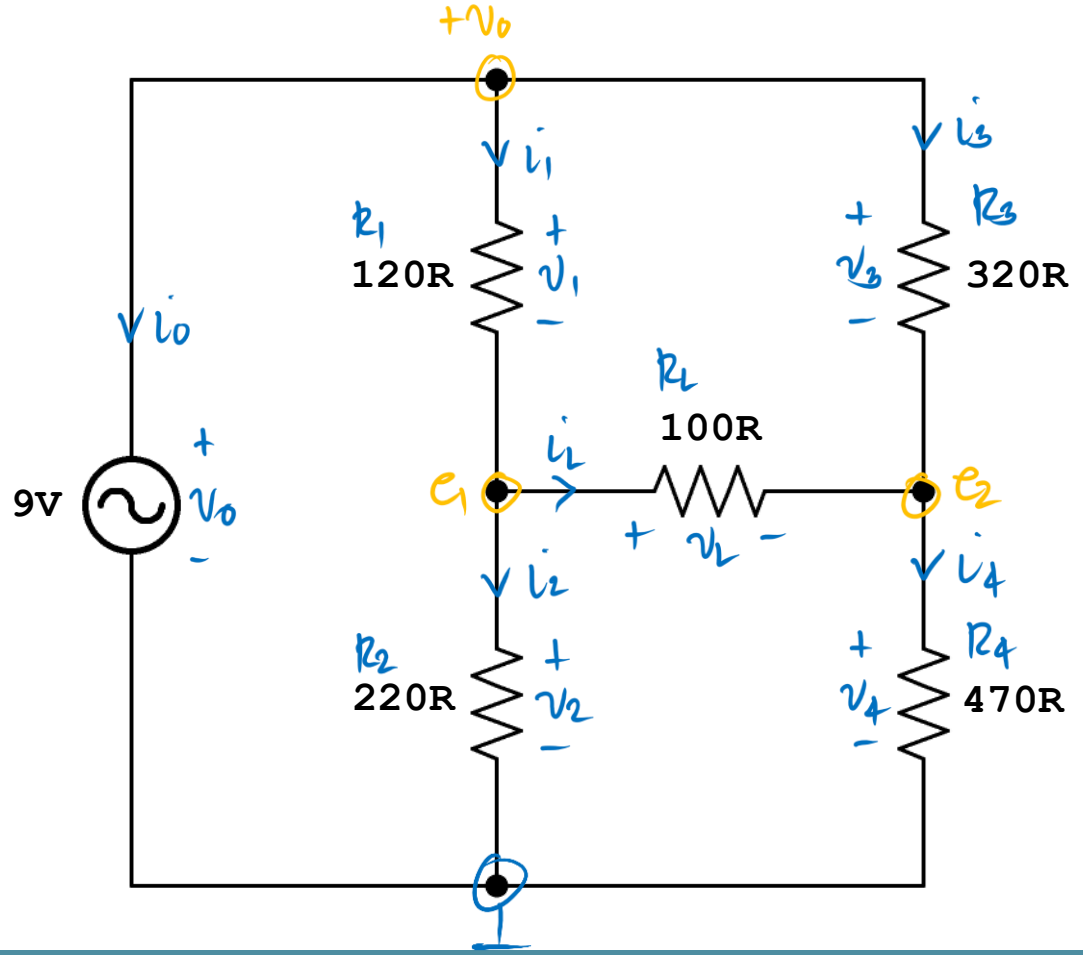
$P_o = P_1 + P_2 + P_3$

$P_o = 153.37 + 100.84 + 67.21$

$P_o = 321.42\,mW$ ans

EXERCISE

Determine the current through and voltage across each resistor, as well as the total power of the given circuit.



Solution

KCL @ e_1

$$-i_1 + i_2 + i_L = 0$$

$$-\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_L}{R_L} = 0$$

$$-\frac{v_o - e_1}{R_1} + \frac{e_1}{R_2} + \frac{e_1 - e_2}{R_L} = 0$$

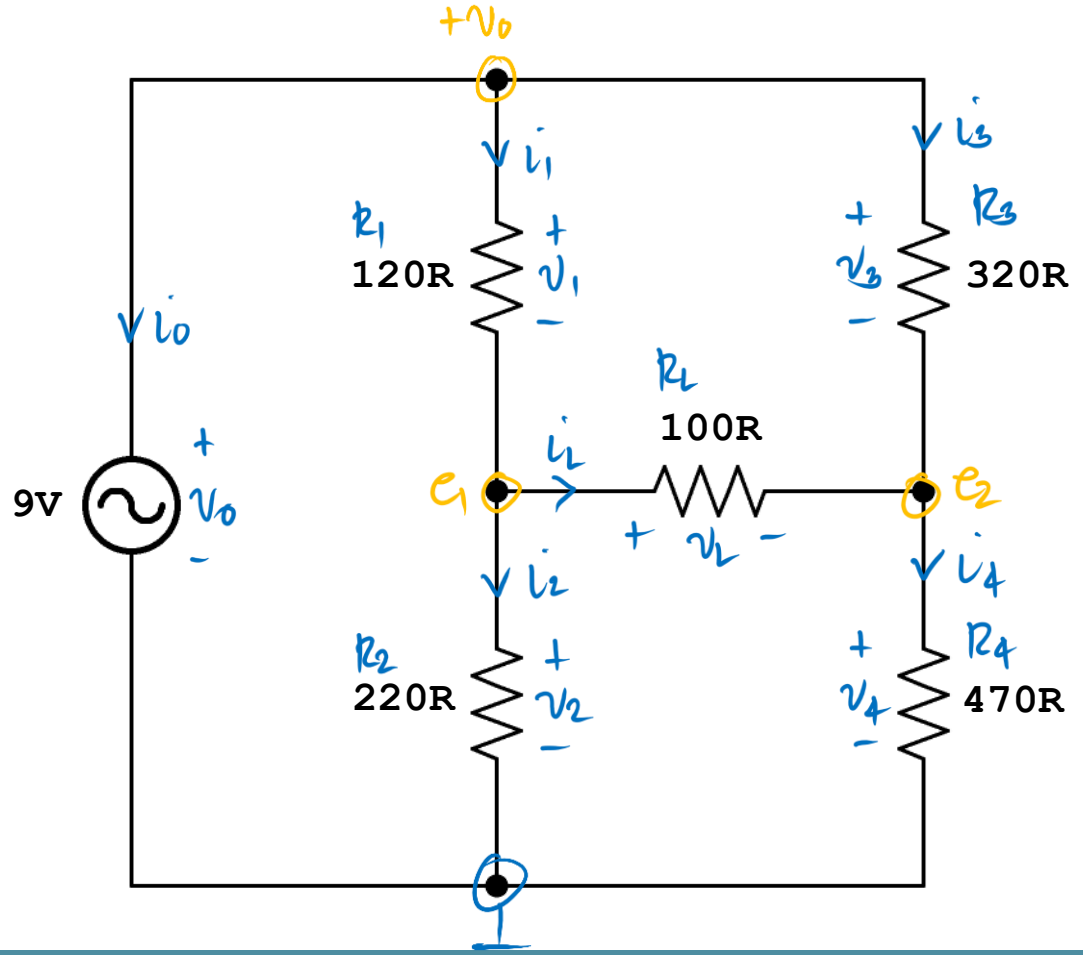
$$-\frac{v_o}{R_1} + \frac{e_1}{R_1} + \frac{e_1}{R_2} + \frac{e_1}{R_L} - \frac{e_2}{R_L} = 0$$

$$-v_o G_1 + e_1 G_1 + e_1 G_2 + e_1 G_L - e_2 G_L = 0$$

$$e_1(G_1 + G_2 + G_L) - e_2 G_L = v_o G_1 \quad \text{(eq. 1)}$$

EXERCISE

Determine the current through and voltage across each resistor, as well as the total power of the given circuit.



Solution

KCL @ e_2

$$-i_3 + i_4 - i_L = 0$$

$$-\frac{v_3}{R_3} + \frac{v_4}{R_4} - \frac{v_L}{R_L} = 0$$

$$-\frac{v_o - e_2}{R_3} + \frac{e_2}{R_4} - \frac{e_1 - e_2}{R_L} = 0$$

$$-\frac{v_o}{R_3} + \frac{e_2}{R_3} + \frac{e_2}{R_4} - \frac{e_1}{R_L} + \frac{e_2}{R_L} = 0$$

$$-v_o G_3 + e_2 G_3 + e_2 G_4 - e_1 G_L + e_2 G_L = 0$$

$$-e_1 G_L + e_2(G_3 + G_4 + G_L) = v_o G_3 \quad (eq.2)$$

EXERCISE

Gaussian Elimination Method

e_1	e_2		
$\frac{151}{66}$	-1	7.5	r_1
-1	$\frac{1147}{752}$	$\frac{45}{16}$	$r_2 \leftarrow \frac{66}{151} r_1 + r_2$

e_1	e_2		
$\frac{151}{66}$	-1	7.5	r_1
0	1.0882	6.091	r_2

System of Linear Equations

$$e_1(G_1 + G_2 + G_L) - e_2 G_L = v_0 G_1 \quad \text{(eq.1)}$$

$$-e_1 G_L + e_2(G_3 + G_4 + G_L) = v_0 G_3 \quad \text{(eq.2)}$$

$$100\left[\frac{151}{6600} e_1 - \frac{1}{100} e_2 = \frac{3}{40}\right] \quad \text{(eq.1)}$$

$$\frac{151}{66} e_1 - e_2 = 7.5$$

$$100\left[-\frac{1}{100} e_1 + \frac{1147}{75200} e_2 = \frac{9}{320}\right] \quad \text{(eq.2)}$$

$$-e_1 + \frac{1147}{752} e_2 = \frac{45}{16}$$

EXERCISE

Gaussian Elimination Method

e_1	e_2	
$\frac{151}{66}$	-1	7.5
-1	$\frac{1147}{752}$	$\frac{45}{16}$

r_1

$r_2 \leftarrow \frac{66}{151} r_1 + r_2$

e_1	e_2	
$\frac{151}{66}$	-1	7.5
0	1.0882	6.091

r_1

r_2

System of Linear Equations

from r_2

$$\frac{1.0882\, e_2}{1.0882} = \frac{6.091}{1.0882}$$

$$e_2 = 5.597\ V$$

from r_1

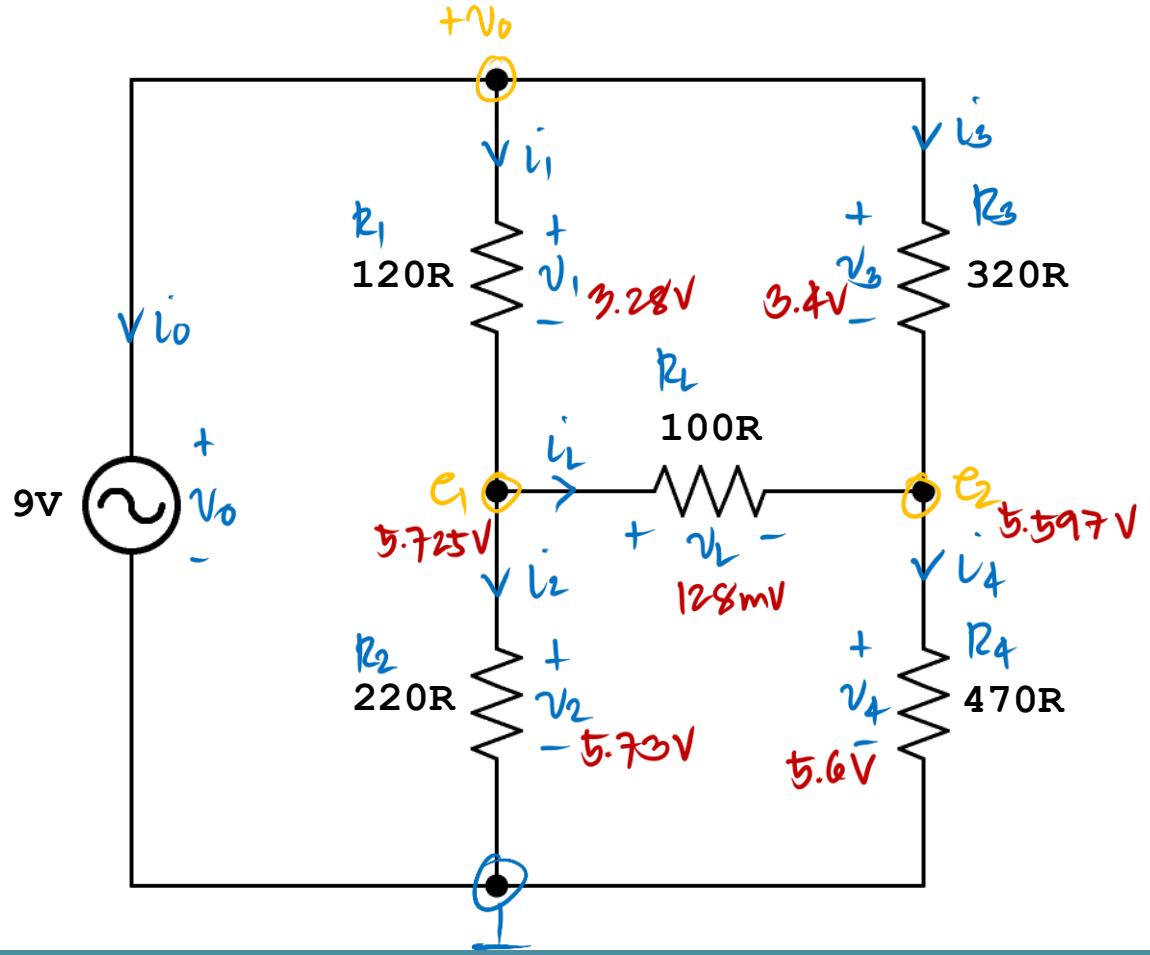
$$\frac{151}{66} e_1 - e_2 = 7.5$$

$$e_1 = (7.5 + 5.597)\frac{66}{151}$$

$$e_1 = 5.725\ V$$

EXERCISE

Determine the current through and voltage across each resistor, as well as the total power of the given circuit.



Solution

$v_L = e_1 - e_2$

$v_L = 5.725 - 5.597$

$v_L = 128\,mV$ ans

$v_1 = v_0 - e_1$

$v_1 = 9 - 5.725$

$v_1 = 3.28\,V$ ans

$v_2 = e_1 - 0$

$v_2 = 5.73\,V$ ans

$v_3 = v_0 - e_2$

$v_3 = 9 - 5.597$

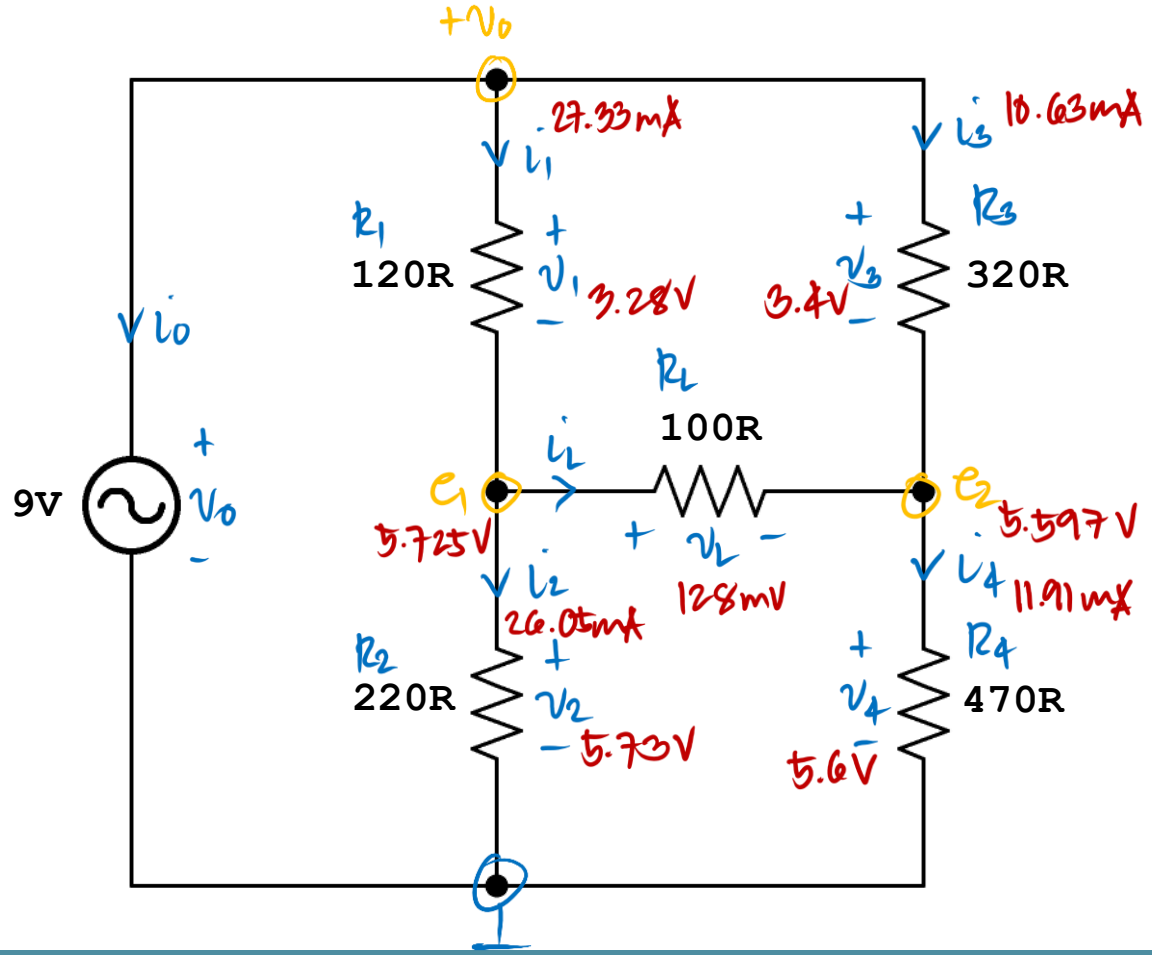
$v_3 = 3.4\,V$ ans

$v_4 = e_2 - 0$

$v_4 = 5.6\,V$ ans

EXERCISE

Determine the current through and voltage across each resistor, as well as the total power of the given circuit.



Solution

$$i_1 = \frac{V_1}{R_1}$$

$$i_1 = \frac{3.28}{120}$$

$$i_1 = 27.33\,mA$$ ans

$$i_3 = \frac{V_3}{R_3}$$

$$i_3 = \frac{3.4}{320}$$

$$i_3 = 10.63\,mA$$ ans

$$i_2 = \frac{V_2}{R_2}$$

$$i_2 = \frac{5.73}{220}$$

$$i_2 = 26.05\,mA$$ ans

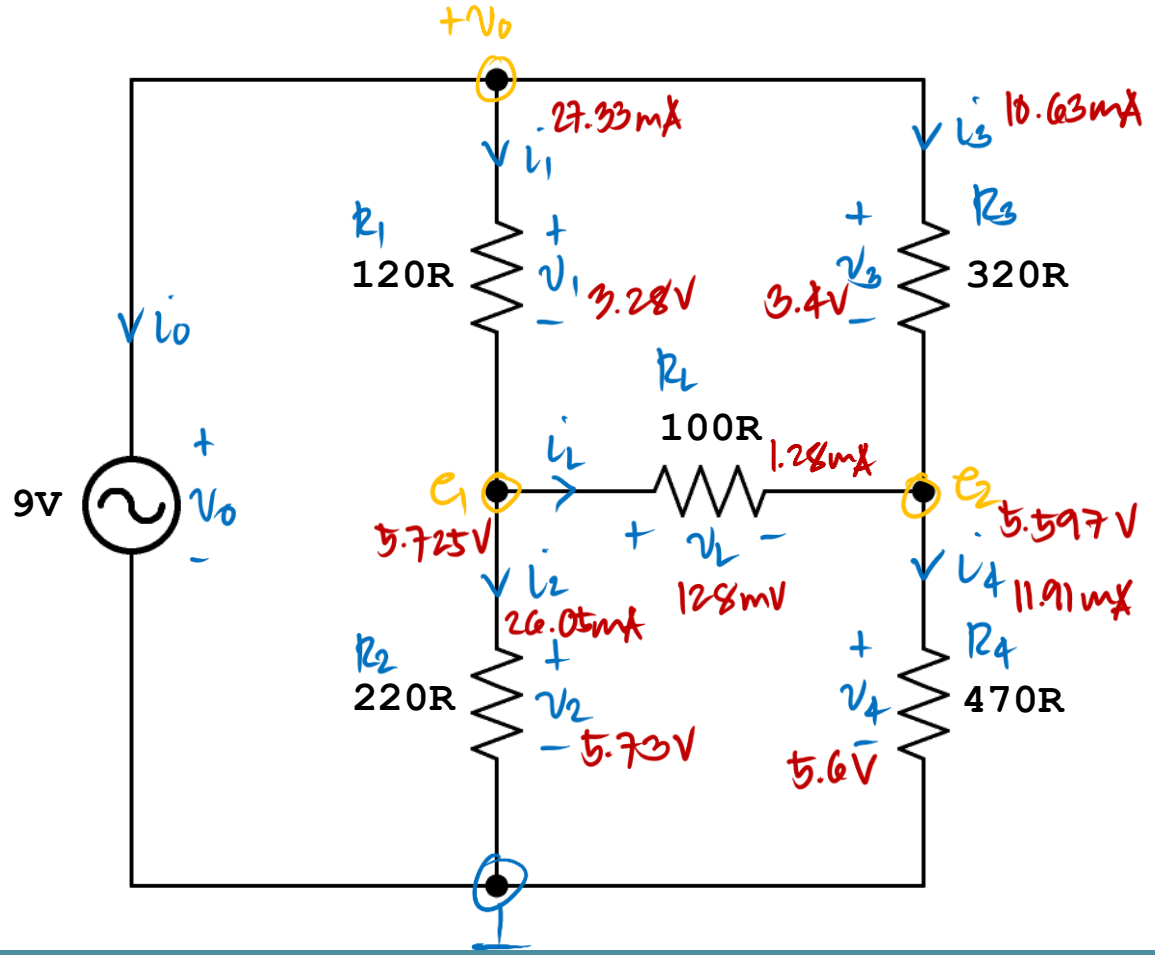
$$i_4 = \frac{V_4}{R_4}$$

$$i_4 = \frac{5.6}{470}$$

$$i_4 = 11.91\,mA$$ ans

EXERCISE

Determine the current through and voltage across each resistor, as well as the total power of the given circuit.



Solution

$$i_L = \frac{V_L}{R_L}$$

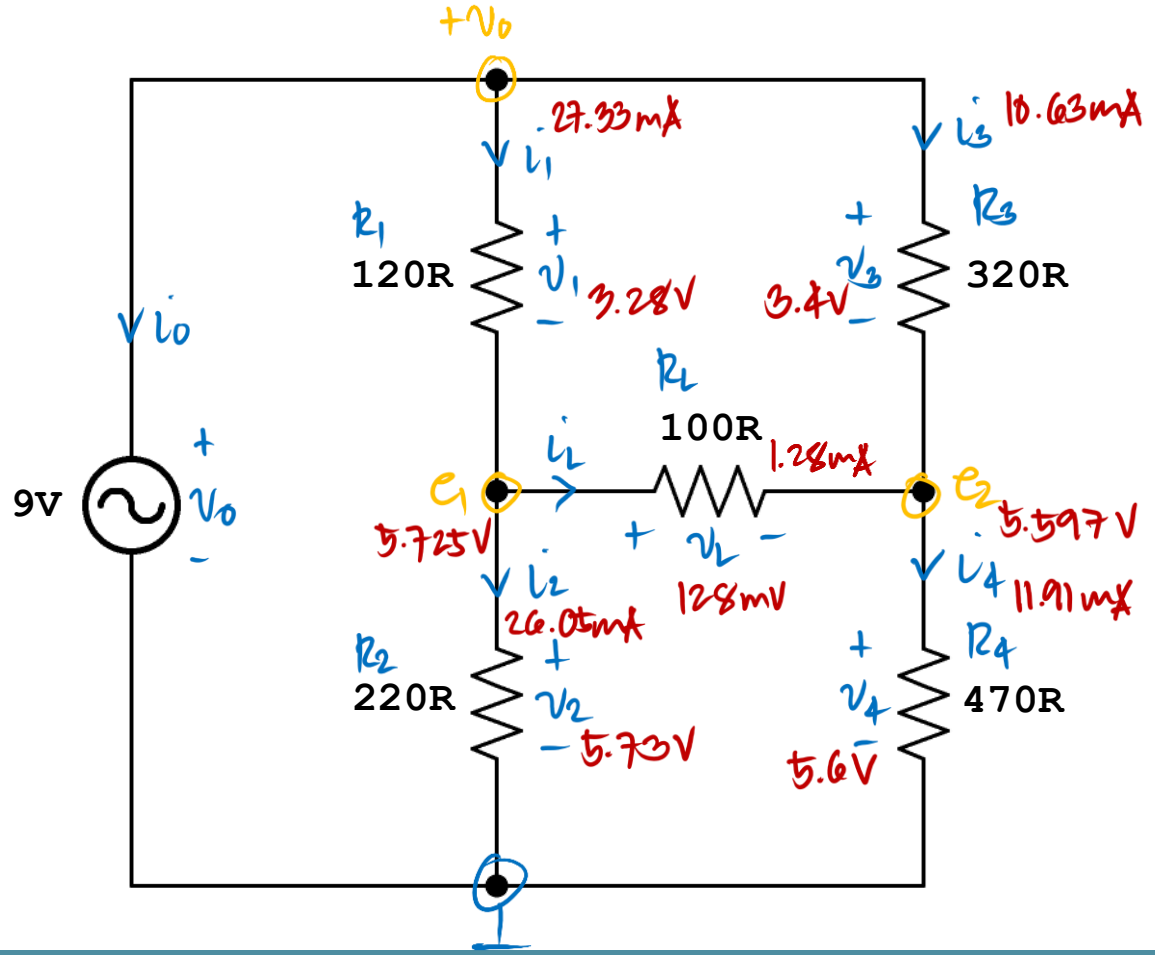
$$i_L = \frac{128m}{100}$$

$$i_L = 1.28\,mA$$

ans

EXERCISE

Determine the current through and voltage across each resistor, as well as the total power of the given circuit.



Solution

$$P_o = P_1 + P_2 + P_3 + P_4 + P_L$$

$$P_o = i_1 V_1 + i_2 V_2 + i_3 V_3 + i_4 V_4 + i_L V_L$$

$$P_o = \underbrace{27.33m(3.28)}_{P_1} + \underbrace{26.05m(5.73)}_{P_2} + \underbrace{10.63m(3.4)}_{P_3} + \underbrace{11.91m(5.6)}_{P_4} + \underbrace{1.28m(128m)}_{P_L}$$

$$\boxed{P_o = 341.91\,mW}$$

ans

LABORATORY