THE DERIVATIVES

DIFFERENTIAL CALCULUS

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TOPIC OUTLINE

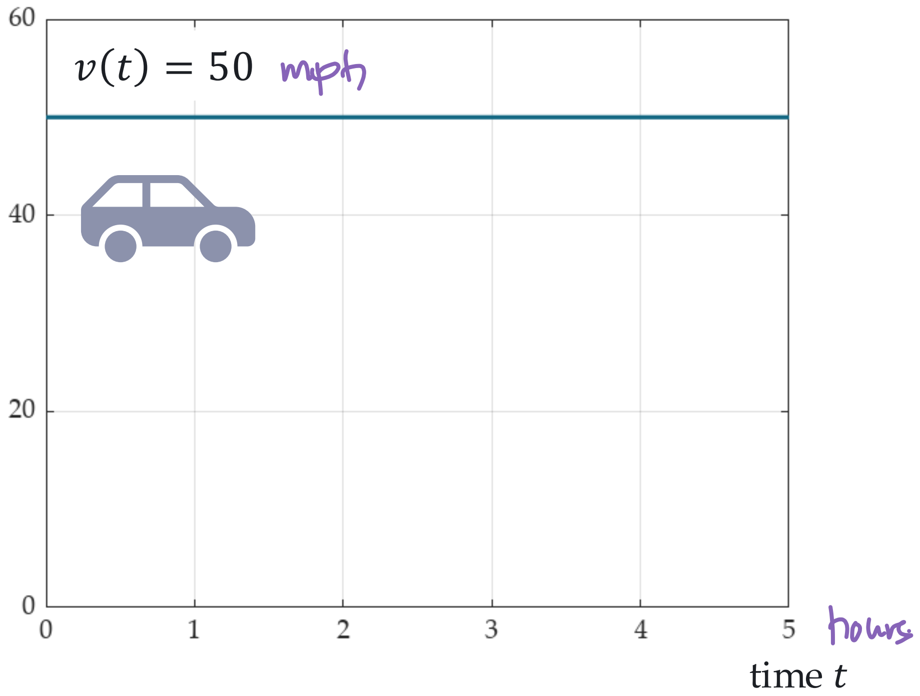
The Slope of a Line

The Derivatives

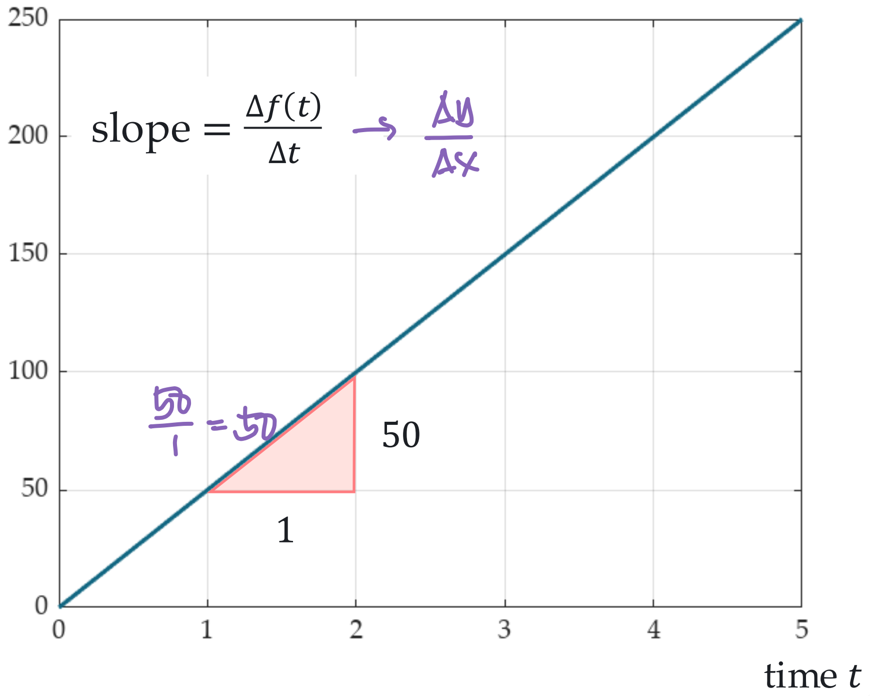
THE SLOPE OF A LINE

CONSTANT VELOCITY

velocity $v(t)$ / speed $\leftarrow \frac{\text{distance}}{\text{time}}$

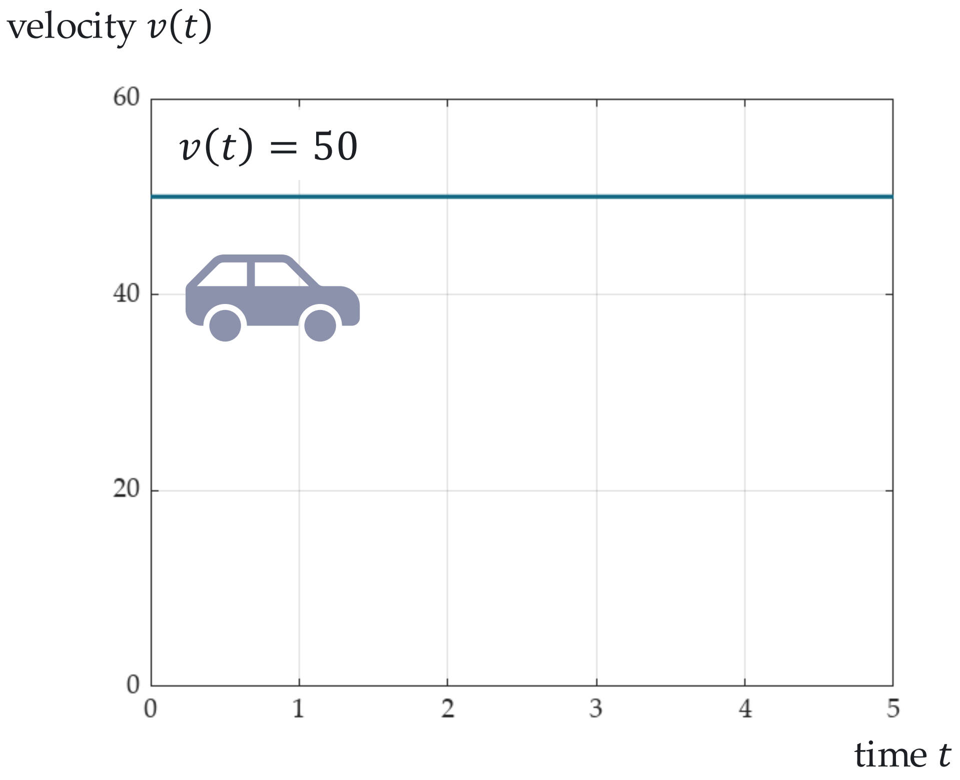


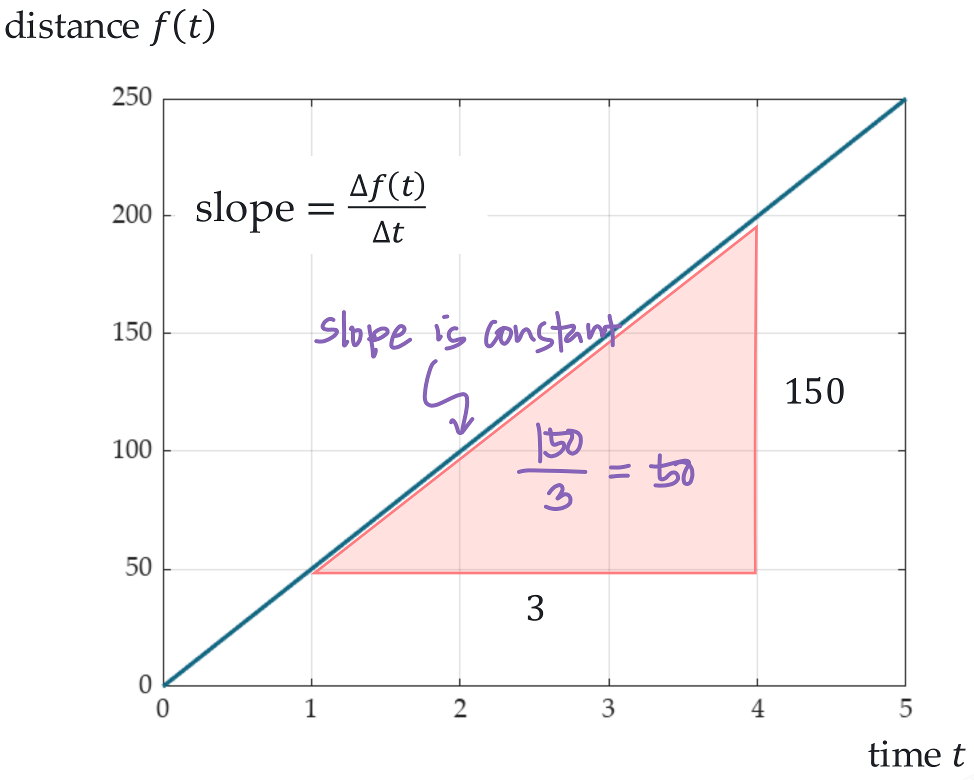
distance $f(t)$



Linearly increasing distance
$f(t) = 50t$

CONSTANT VELOCITY

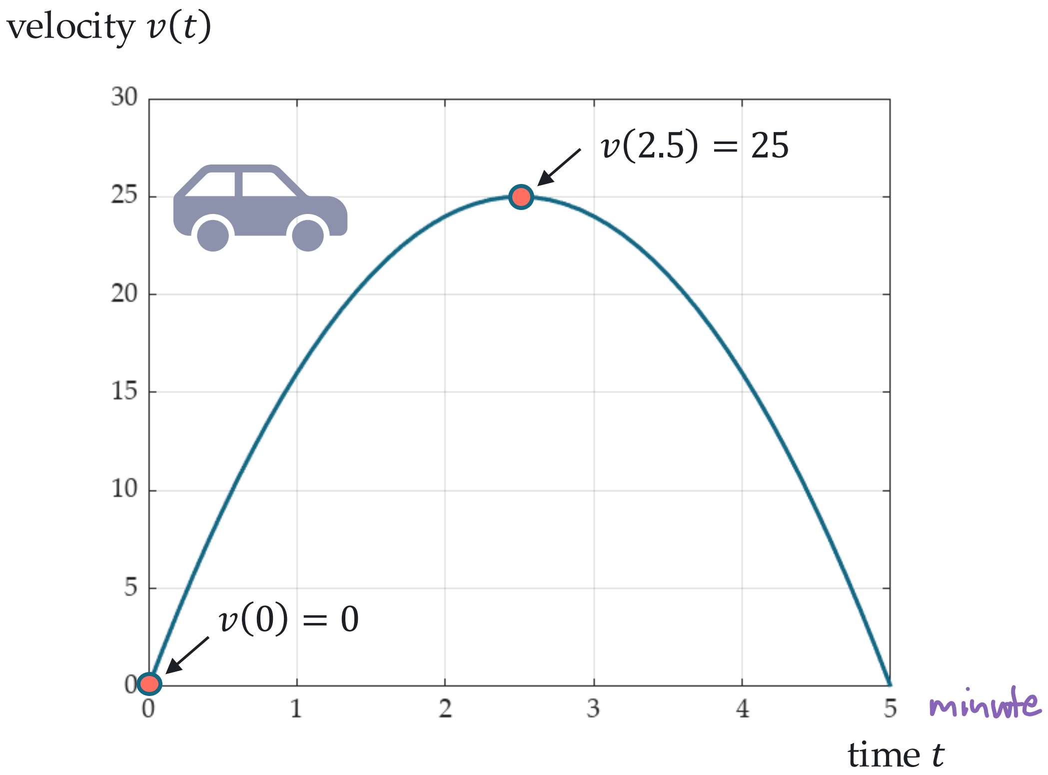




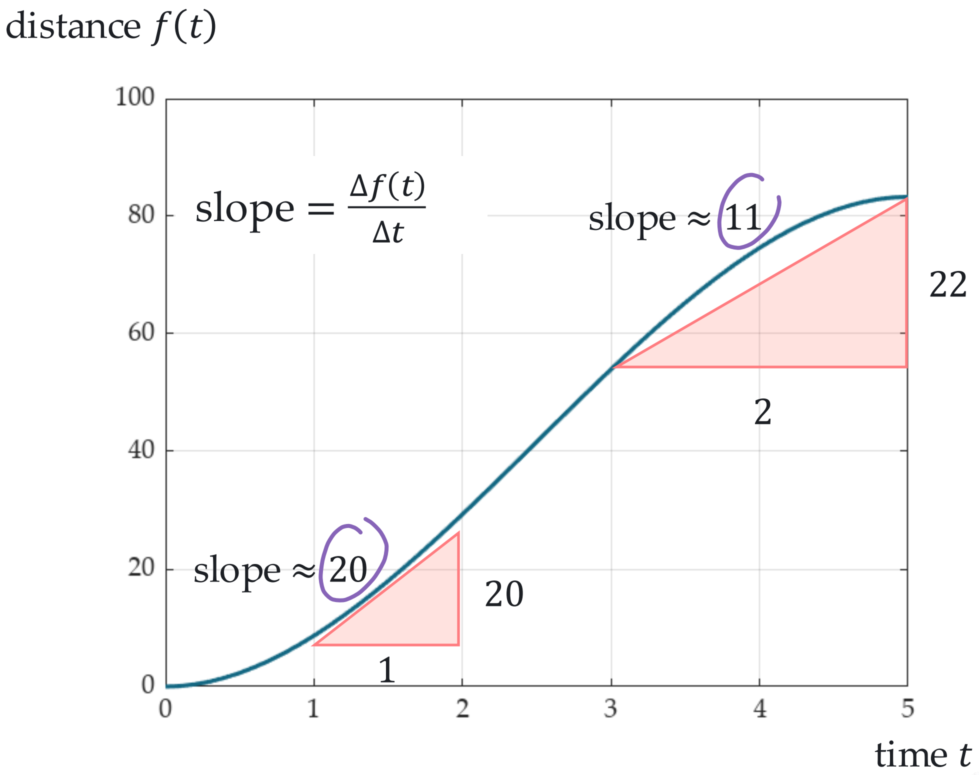
Linearly increasing distance

$f(t) = 50t$

INSTANTENEOUS VELOCITY

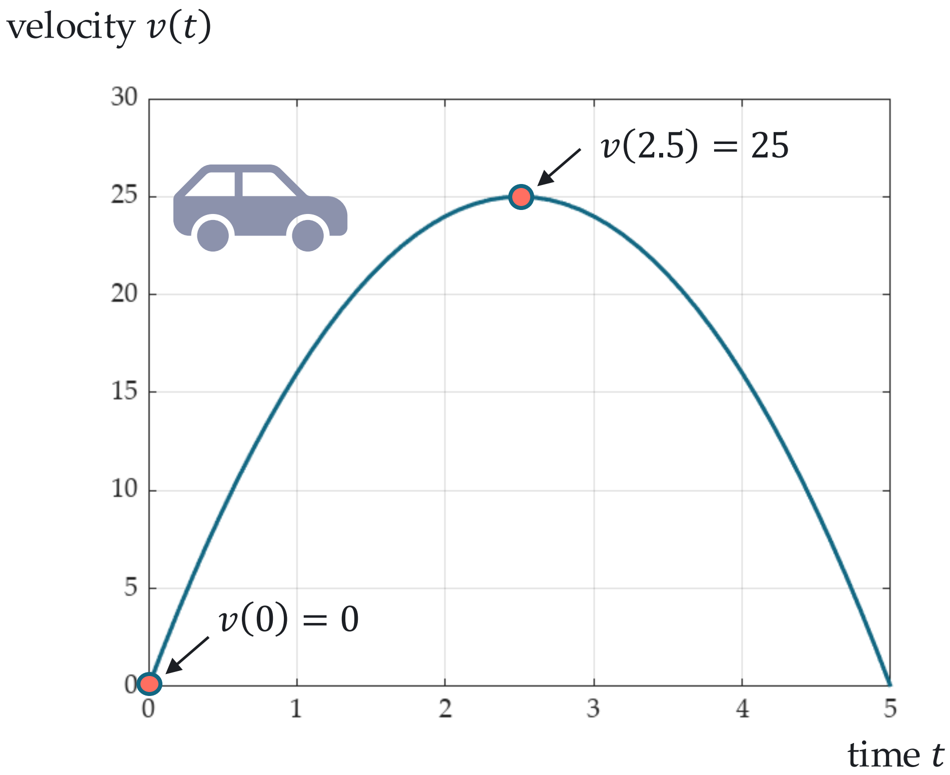


Real-life problems are nonlinear



Slopes (velocity) are not equal

INSTANTENEOUS VELOCITY



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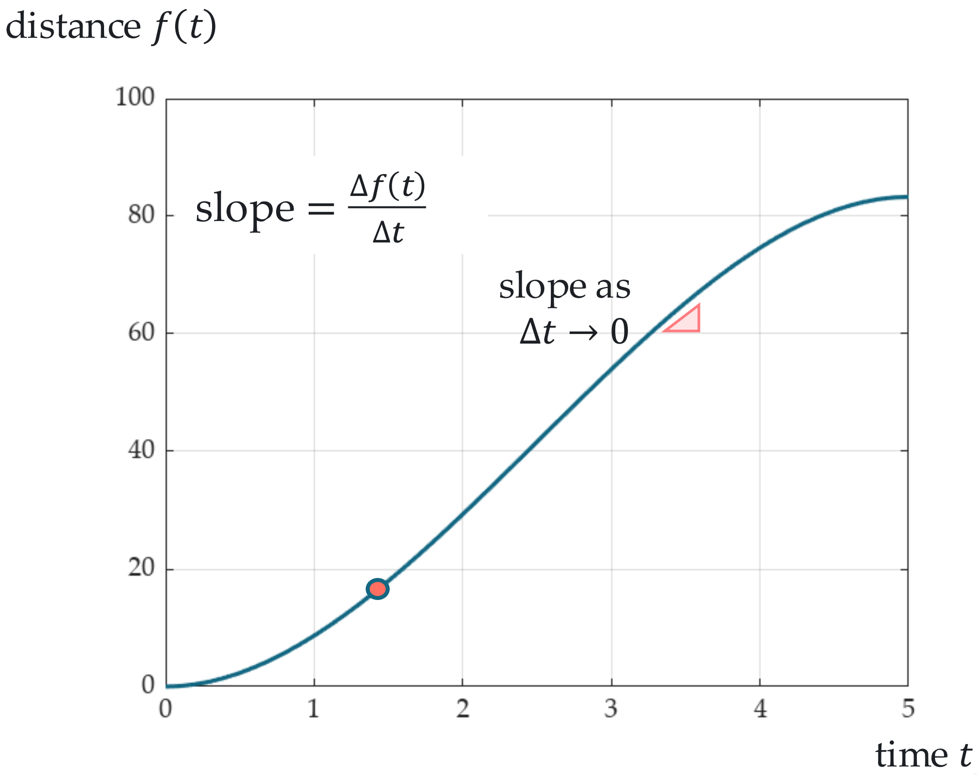


Let Δt approaches zero

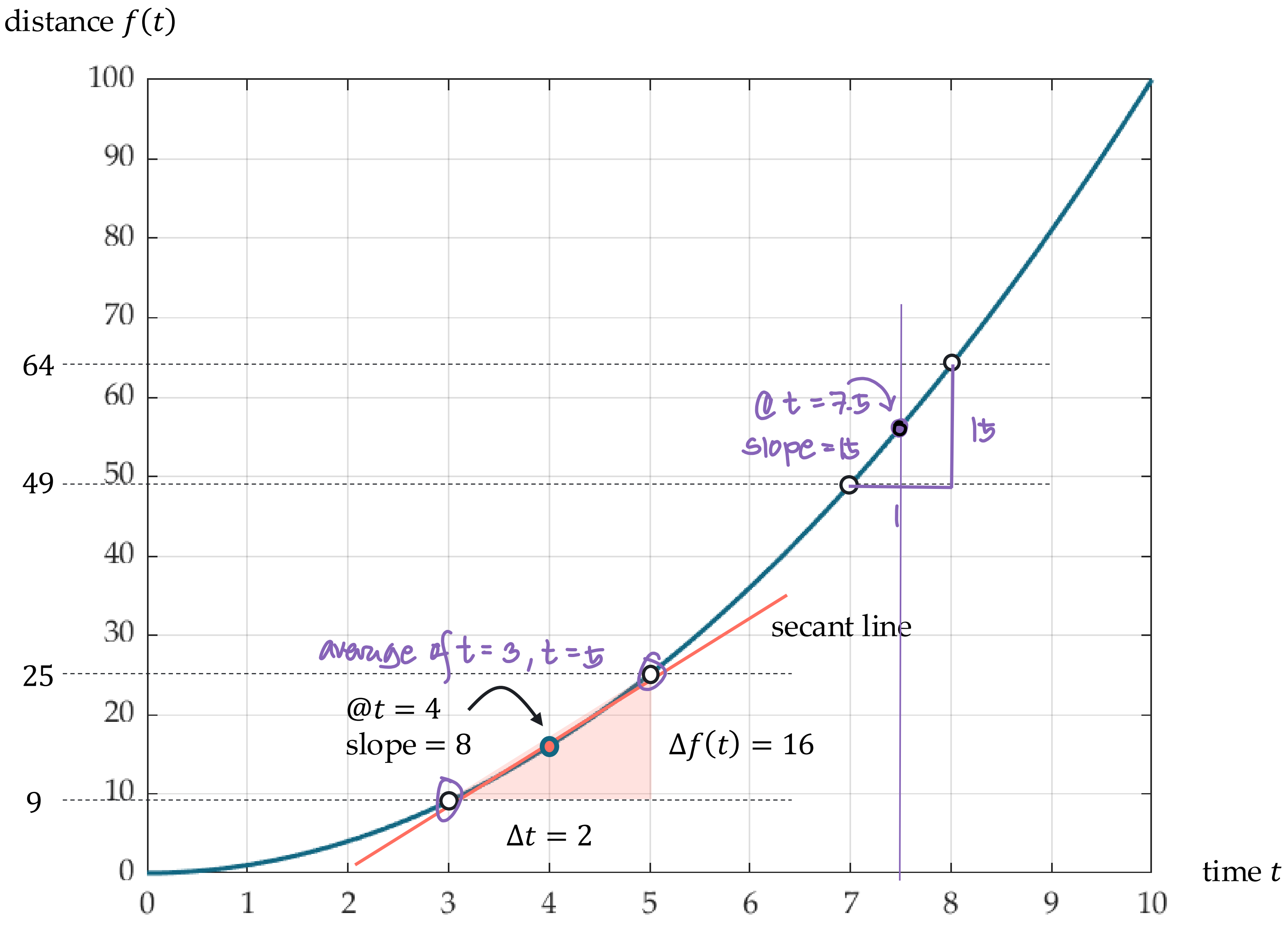
VELOCITY AT AN INSTANT

distance

$$f(t) = t^2$$

average velocity

$$\frac{\Delta f(t)}{\Delta t} = 2t$$



VELOCITY AT AN INSTANT

<u>distance</u>

$$f(t) = t^2$$

<u>average velocity</u>

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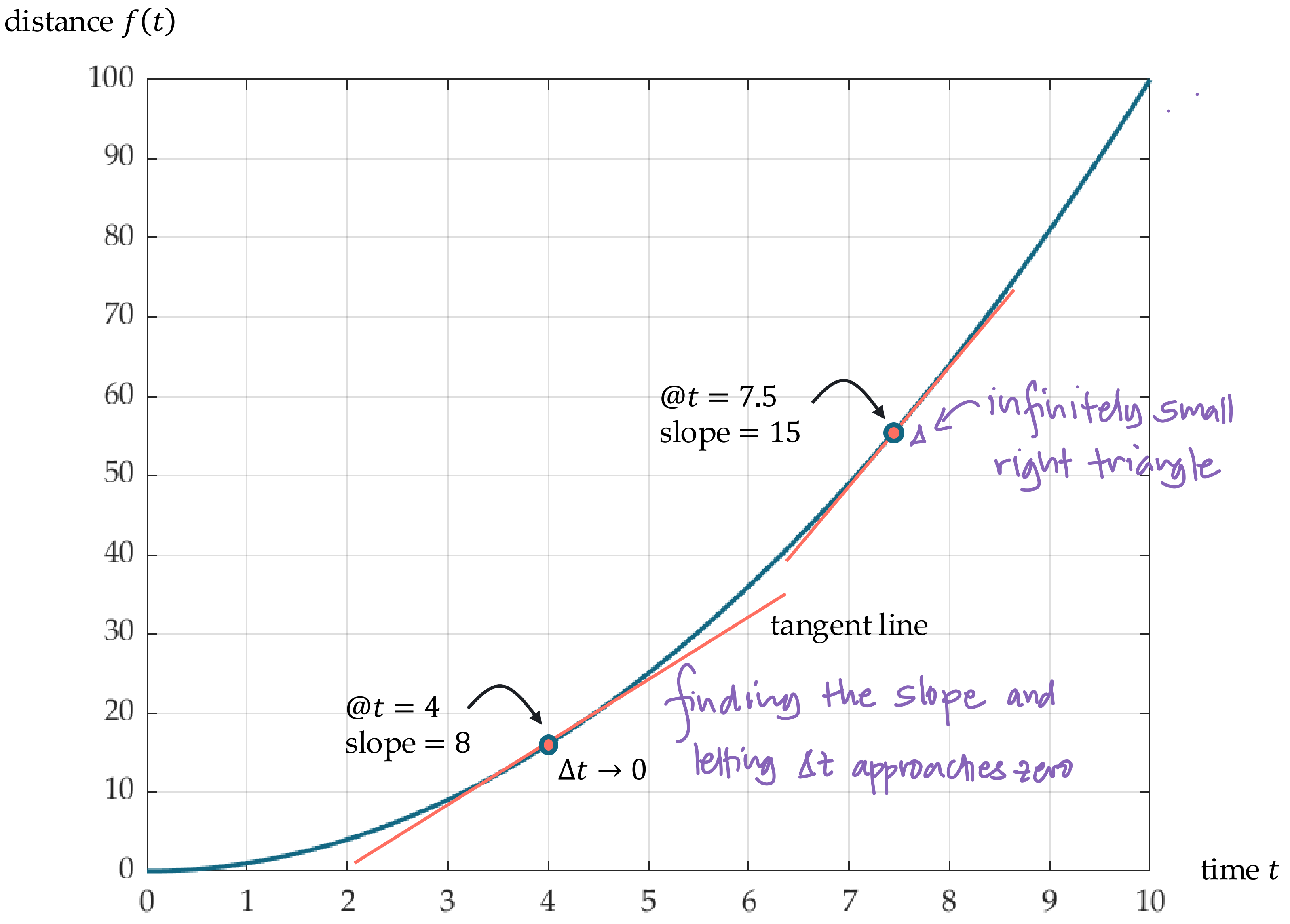
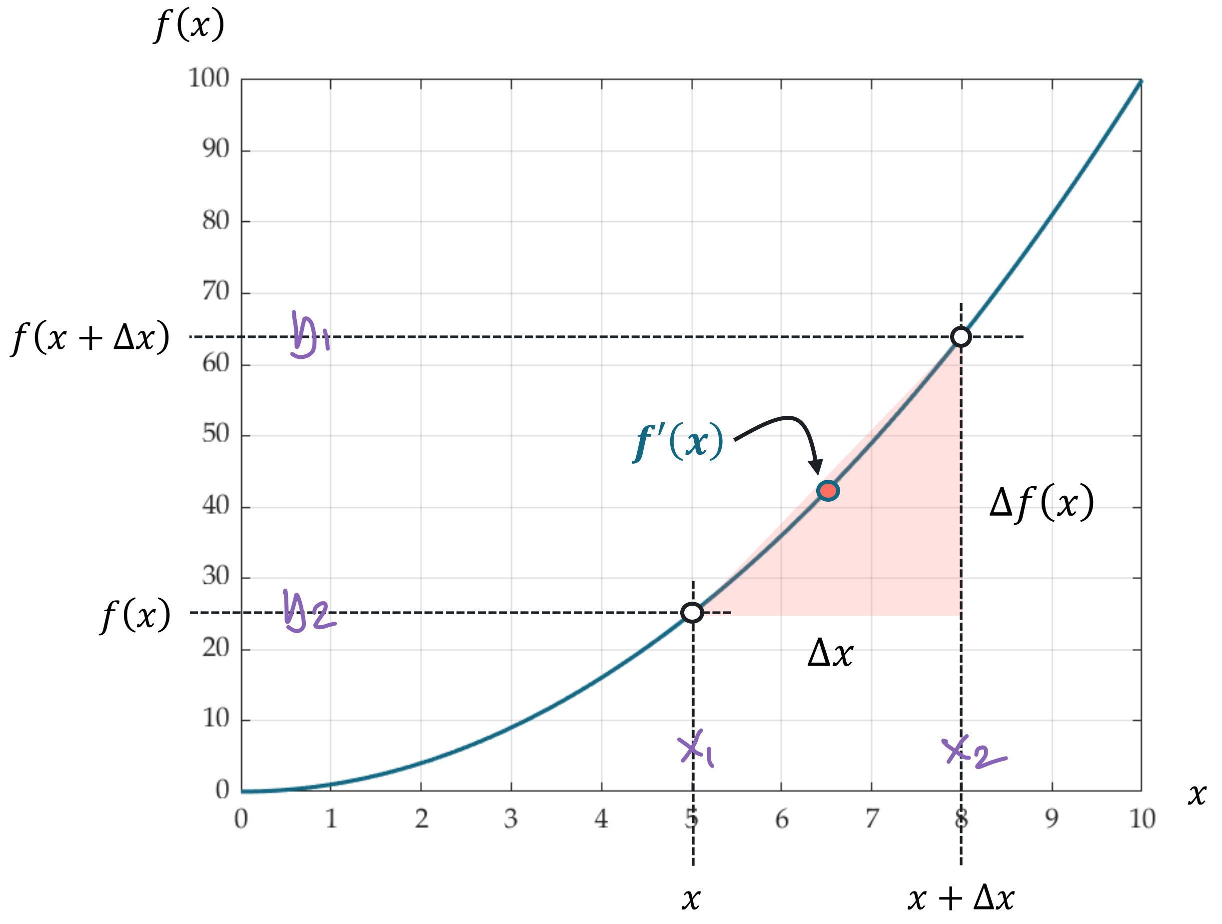


THE DERIVATIVES

DERIVATIVE OF A FUNCTION



Difference Quotient

slope

$$\frac{\Delta \boldsymbol{f}(\boldsymbol{x})}{\Delta \boldsymbol{x}} = \frac{\boldsymbol{f}(\boldsymbol{x}+\Delta \boldsymbol{x}) - \boldsymbol{f}(\boldsymbol{x})}{\Delta \boldsymbol{x}} \rightarrow \frac{y_2 - y_1}{x_2 - x_1}$$

The Derivative of $f(x)$

$$\boldsymbol{f}'(\boldsymbol{x}) = \lim_{\Delta x \to 0} \frac{\boldsymbol{f}(\boldsymbol{x}+\Delta \boldsymbol{x}) - \boldsymbol{f}(\boldsymbol{x})}{\Delta \boldsymbol{x}}$$

EXERCISE

Find the derivative of the function

$$f(x) = 2x$$

(use the difference quotient formula).

note

for $f(x+\Delta x)$, replace every x w/ $(x+\Delta x)$

$$f(\underline{x}) = \cancel{2x} \rightarrow 2(x+\Delta x)$$

$(x+\Delta x)$

Solution

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{2(x+\Delta x) - 2x}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\cancelto{0}{2x} + \cancelto{1}{2\Delta x} - \cancelto{0}{2x}}{\cancel{\Delta x}}$$

$$f'(x) = \lim_{\Delta x \to 0} (2)$$

$$\boxed{f'(x) = 2}$$

ans

EXERCISE

Find the derivative of the function

$f(x) = 2x^2$ let $\Delta x = h$

(use the difference quotient formula).

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{2(x+h)^2 - 2x^2}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{2(x^2 + 2xh + h^2) - 2x^2}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancelto{0}{2x^2} + 4xh + 2h^2 - \cancelto{0}{2x^2}}{h}$$

Solution

$$f'(x) = \lim_{h \to 0} \frac{\cancelto{1}{h}(4x + 2h)}{\cancelto{1}{h}}$$

$$f'(x) = \lim_{h \to 0} (4x + \cancelto{0}{2h})$$

$$\boxed{f'(x) = 4x}$$

ans

EXERCISE

Find the derivative of the function

Solution

$$f(x) = x^3 - x$$

(use the difference quotient formula).

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^3 - (x+h) - (x^3 - x)}{h} \rightarrow (x+h)^3 = (x+h)^2(x+h)$$

$$f'(x) = \lim_{h \to 0} \frac{(x^2+2xh+h^2)(x+h) - (x+h) - (x^3-x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{(\cancel{x^3} + 2x^2h + xh^2 + x^2h + 2h^2 + h^3) - (\cancel{x}+h) - (\cancel{x^3} - \cancel{x})}{h}$$

EXERCISE

Solution

$$f'(x) = \lim_{h \to 0} \frac{\cancel{h}^{\,1}\left(2x^2 + \cancel{xh}^{\to 0} + x^2 + \cancel{2h}^{\to 0} + \cancel{h^2}^{\to 0} - 1\right)}{\cancel{h}^{\,1}}$$

$$f'(x) = \lim_{h \to 0} \left(3x^2 - 1\right)$$

$$\boxed{f'(x) = 3x^2 - 1}$$

ans

EXERCISE

Find the derivative of the function

$$f(x) = x^2 - 8x + 9$$

(use the difference quotient formula).

Solution

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - 8(x+h) + 9 - (x^2 - 8x + 9)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancelto{0}{x^2} + 2xh + h^2 - \cancelto{0}{8x} - 8h + \cancelto{0}{9} - \cancelto{0}{x^2} + \cancelto{0}{8x} - \cancelto{0}{9}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancelto{1}{h}(2x + h - 8)}{\cancelto{1}{h}}$$

$$f'(x) = \lim_{h \to 0} (2x + \cancelto{0}{h} - 8)$$

$$\boxed{f'(x) = 2x - 8}$$

ans

EXERCISE

Find the derivative of the function

$$f(x) = x^{-2} \rightarrow f(x) = \frac{1}{x^2}$$

(use the difference quotient formula).

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{(x+h)^2} \cdot \frac{x^2}{x^2} - \frac{1}{x^2} \cdot \frac{(x+h)^2}{(x+h)^2}}{h}$$

$$\frac{1}{2} - \frac{2}{3} \rightarrow \frac{1}{2} \cdot \frac{3}{3} - \frac{2}{3} \cdot \frac{2}{2} = \frac{3-4}{6}$$

Solution

$$f'(x) = \lim_{h \to 0} \frac{\frac{x^2 - (x+h)^2}{x^2(x+h)^2}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{x^2}^{\,0} - (\cancel{x^2}^{\,0} + 2xh + h^2)}{x^2(x+h)^2 h}$$

$$f'(x) = \lim_{h \to 0} \frac{-\cancel{h}^{\,1}(2x+h)}{x^2(x+h)^2 \cancel{h}^{\,1}}$$

$$f'(x) = \lim_{h \to 0} \frac{-2x + \cancel{h}^{\,0}}{x^2(x+\cancel{h})^2}$$

EXERCISE

Solution

$$f'(x) = \frac{-2x}{x^4}$$

$$\boxed{f'(x) = -\frac{2}{x^3}}$$

ans

LABORATORY