



SPECIAL RIGHT TRIANGLES

RIGHT TRIANGLES

prepared by:

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TOPIC OUTLINE

The Unit Circle

45-45-90 Triangle

30-60-90 Triangle



UNIT CIRCLE



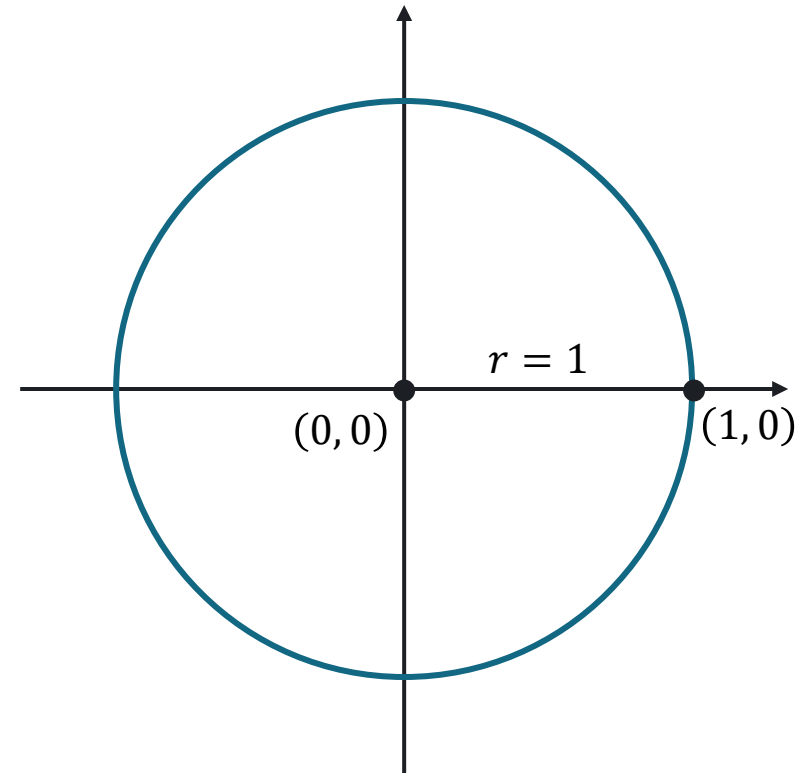
UNIT CIRCLE

A unit circle is a circle with radius 1 unit, centered at the origin of the xy -plane.

Equation

$$x^2 + y^2 = 1$$

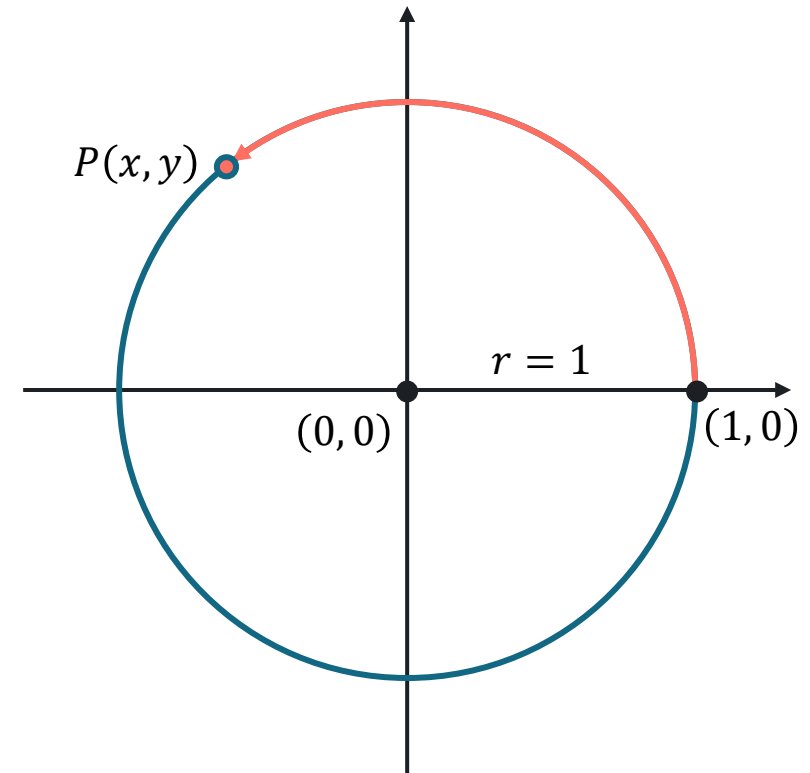
Unit Circle



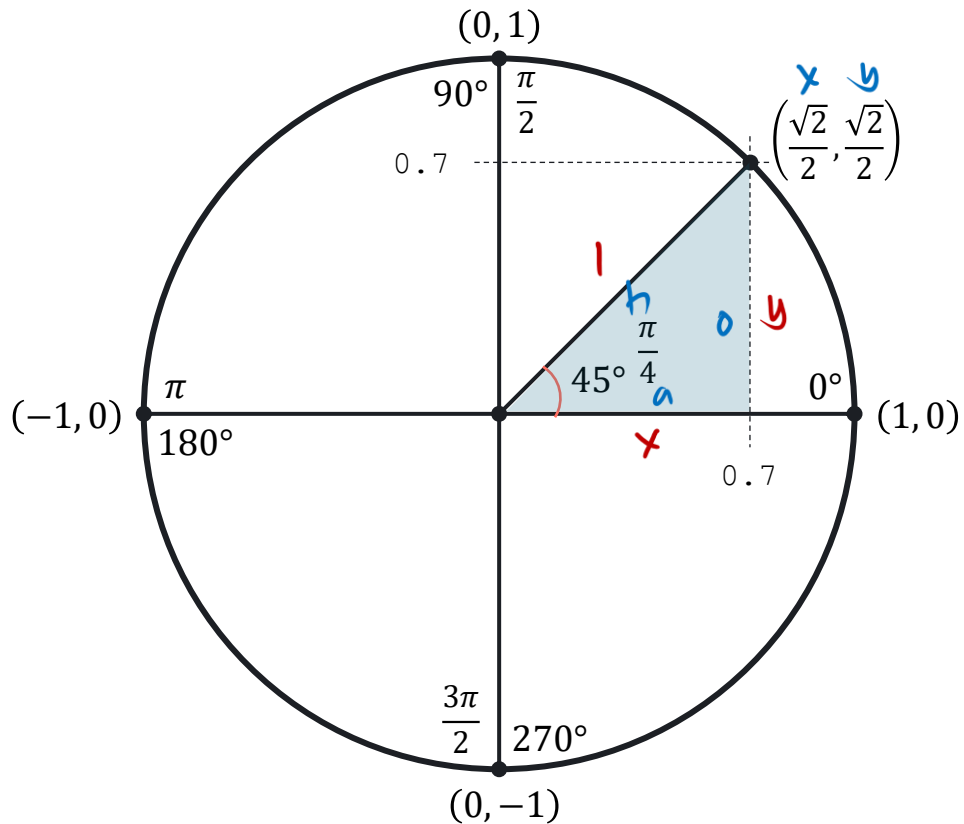
TERMINAL POINTS

The terminal point is a point on the unit circle that corresponds to a given angle measured from the positive x -axis.

Terminal Point on the Unit Circle



45° ANGLE



Trigonometric function values for 45°

$$\sin \theta = \frac{o}{h}$$

$$\cos \theta = \frac{a}{h}$$

$$\tan \theta = \frac{o}{a}$$

$$\sin \theta = \frac{y}{1}$$

$$\cos \theta = \frac{x}{1}$$

$$\tan \theta = \frac{y}{x}$$

$$\sin \theta = y$$

$$\cos \theta = x$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin 45^\circ = \frac{\sqrt{2}}{2}$$

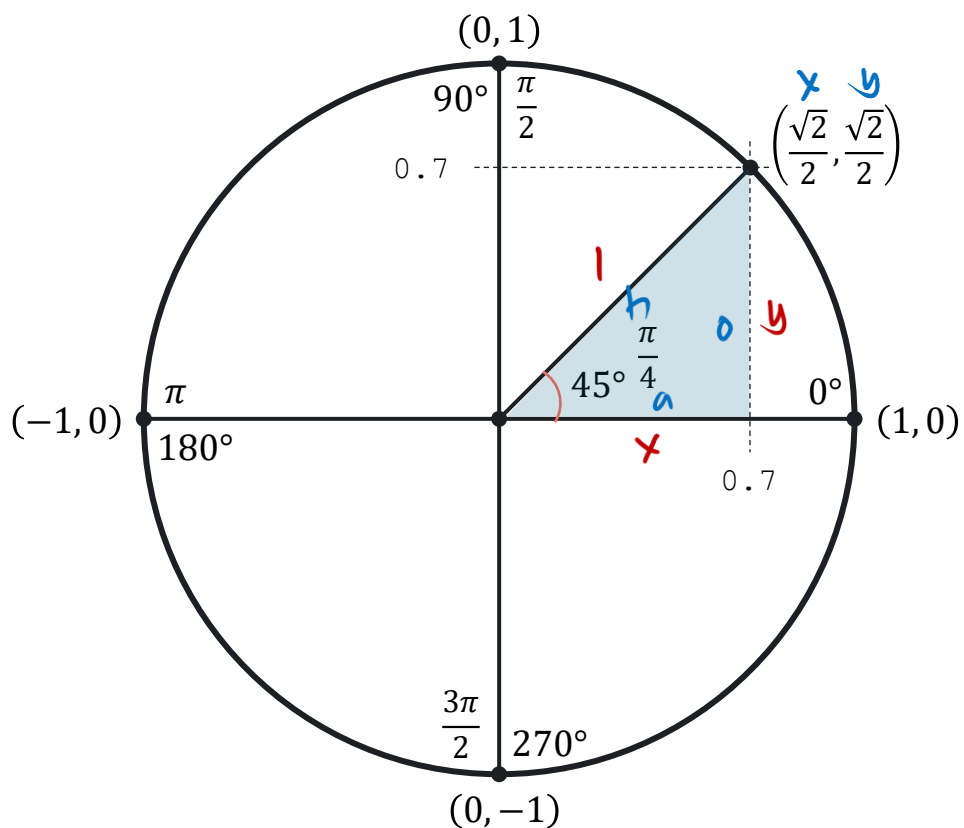
ans

$$\csc 45^\circ = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$\csc 45^\circ = \frac{\sqrt{2}}{1}$$

ans

45° ANGLE



Trigonometric function values for 45°

$$\sin \theta = \frac{o}{h}$$

$$\cos \theta = \frac{a}{h}$$

$$\tan \theta = \frac{o}{a}$$

$$\sin \theta = \frac{y}{1}$$

$$\cos \theta = \frac{x}{1}$$

$$\tan \theta = \frac{y}{x}$$

$$\underline{\sin \theta = y}$$

$$\underline{\cos \theta = x}$$

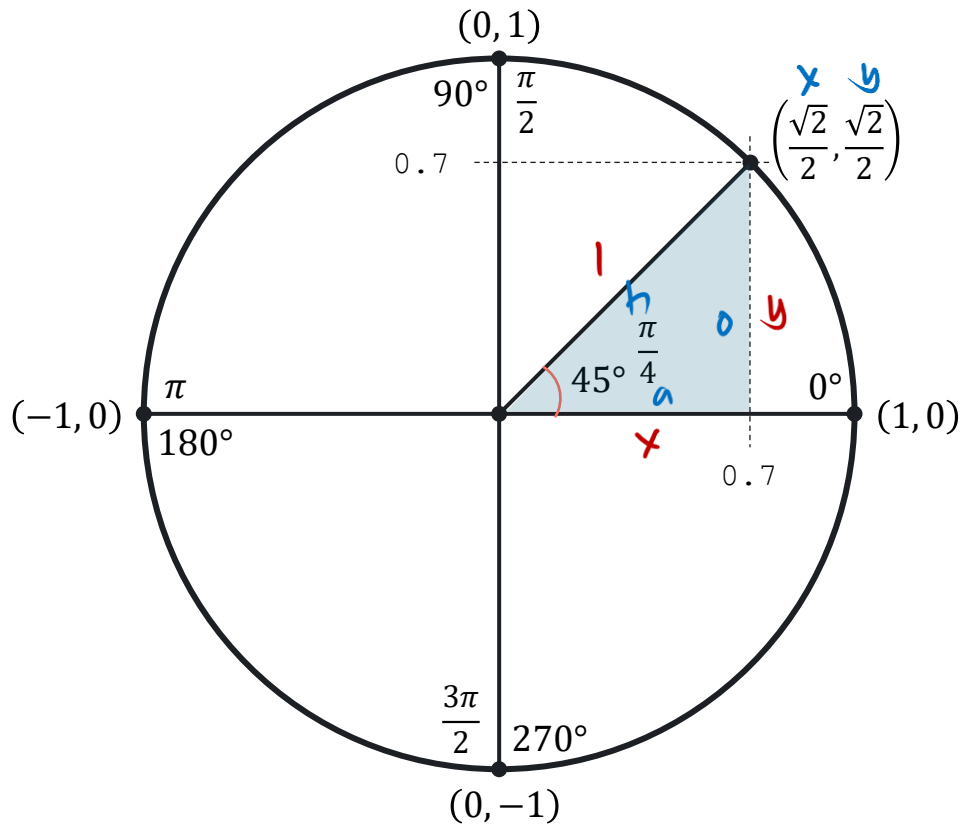
$$\underline{\tan \theta = \frac{\sin \theta}{\cos \theta}}$$

$$\boxed{\cos 45^\circ = \frac{\sqrt{2}}{2}} \quad \text{ans}$$

$$\sec 45^\circ = \frac{2}{\cancel{\sqrt{2}}} \cdot \frac{\sqrt{2}}{\cancel{\sqrt{2}}}$$

$$\boxed{\sec 45^\circ = \frac{\sqrt{2}}{2}} \quad \text{ans}$$

45° ANGLE



Trigonometric function values for 45°

$$\sin \theta = \frac{o}{h}$$

$$\cos \theta = \frac{a}{h}$$

$$\tan \theta = \frac{o}{a}$$

$$\sin \theta = \frac{y}{1}$$

$$\cos \theta = \frac{x}{1}$$

$$\tan \theta = \frac{y}{x}$$

$$\sin \theta = y$$

$$\cos \theta = x$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\tan 45^\circ = \frac{\cancel{\sqrt{2}}/2}{\cancel{\sqrt{2}}/2}$$

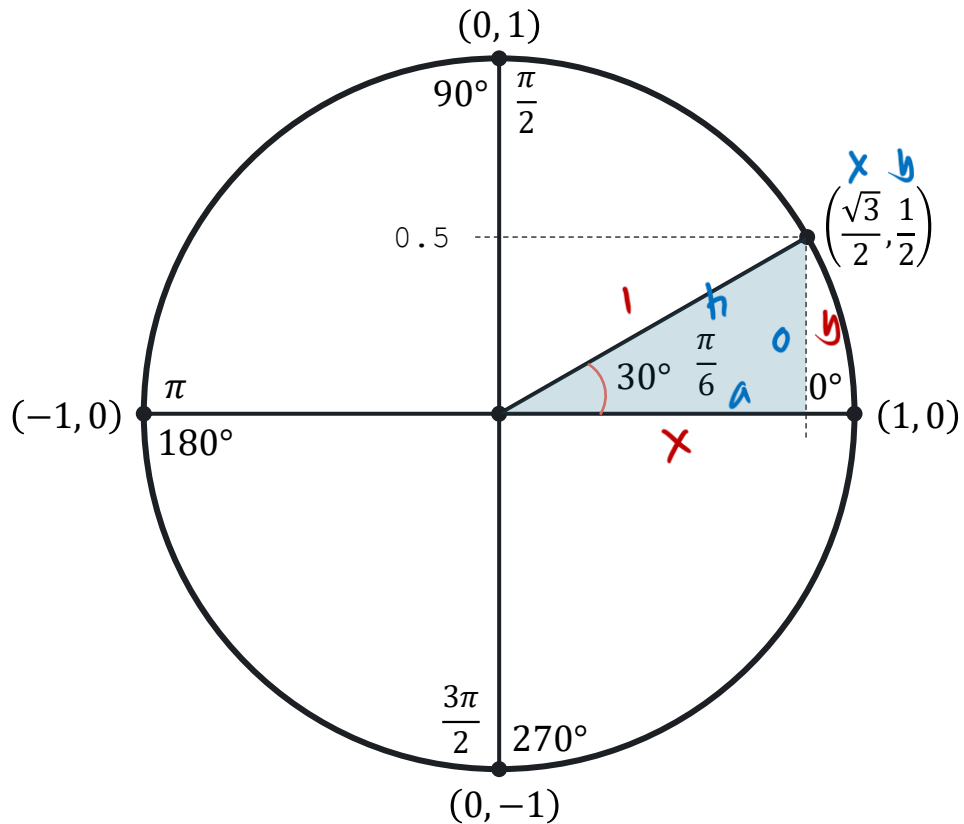
$$\boxed{\tan 45^\circ = 1}$$

ans

$$\boxed{\cot 45^\circ = 1}$$

ans

30° ANGLE



Trigonometric function values for 30°

$$\sin \theta = \frac{o}{h}$$

$$\cos \theta = \frac{a}{h}$$

$$\tan \theta = \frac{o}{a}$$

$$\sin \theta = \frac{y}{1}$$

$$\cos \theta = \frac{x}{1}$$

$$\tan \theta = \frac{y}{x}$$

$$\underline{\sin \theta = y}$$

$$\underline{\cos \theta = x}$$

$$\underline{\tan \theta = \frac{\sin \theta}{\cos \theta}}$$

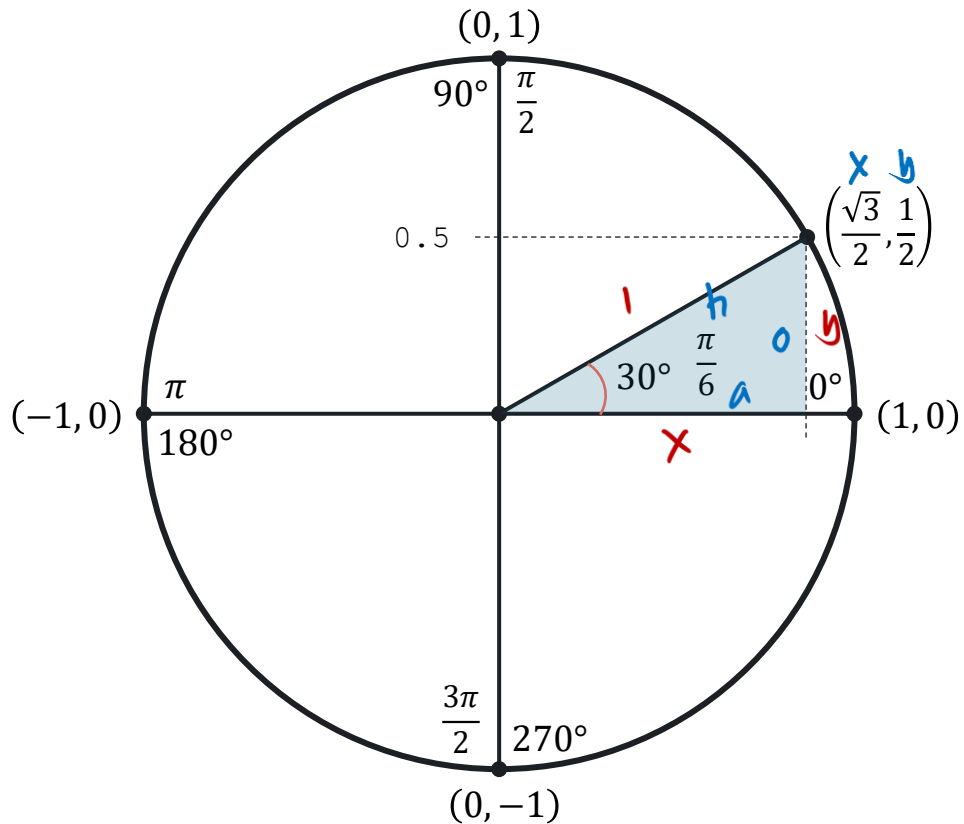
$$\boxed{\sin 30^\circ = \frac{1}{2}}$$

ans

$$\boxed{\csc 30^\circ = 2}$$

ans

30° ANGLE



Trigonometric function values for 30°

$$\sin \theta = \frac{o}{h}$$

$$\cos \theta = \frac{a}{h}$$

$$\tan \theta = \frac{o}{a}$$

$$\sin \theta = \frac{y}{1}$$

$$\cos \theta = \frac{x}{1}$$

$$\tan \theta = \frac{y}{x}$$

$$\sin \theta = y$$

$$\cos \theta = x$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

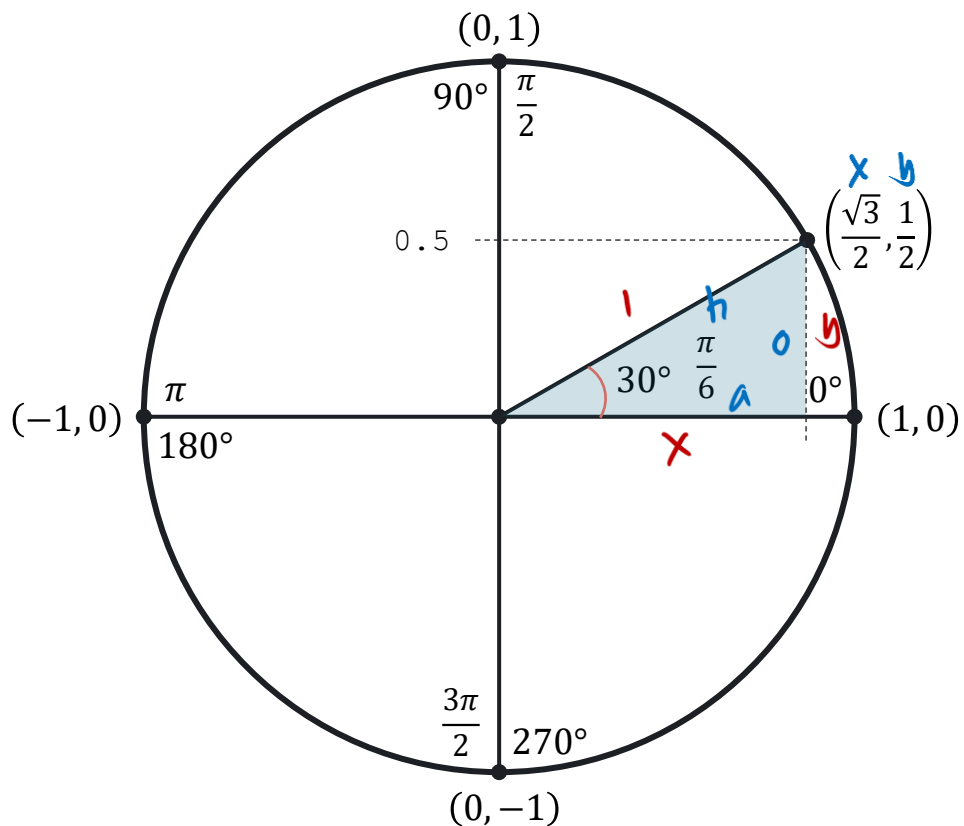
ans

$$\sec 30^\circ = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\sec 30^\circ = \frac{2\sqrt{3}}{3}$$

ans

30° ANGLE



Trigonometric function values for 30°

$$\sin \theta = \frac{o}{h}$$

$$\cos \theta = \frac{a}{h}$$

$$\tan \theta = \frac{o}{a}$$

$$\sin \theta = \frac{y}{1}$$

$$\cos \theta = \frac{x}{1}$$

$$\tan \theta = \frac{y}{x}$$

$$\sin \theta = y$$

$$\cos \theta = x$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\tan 30^\circ = \frac{1/2}{\sqrt{3}/2}$$

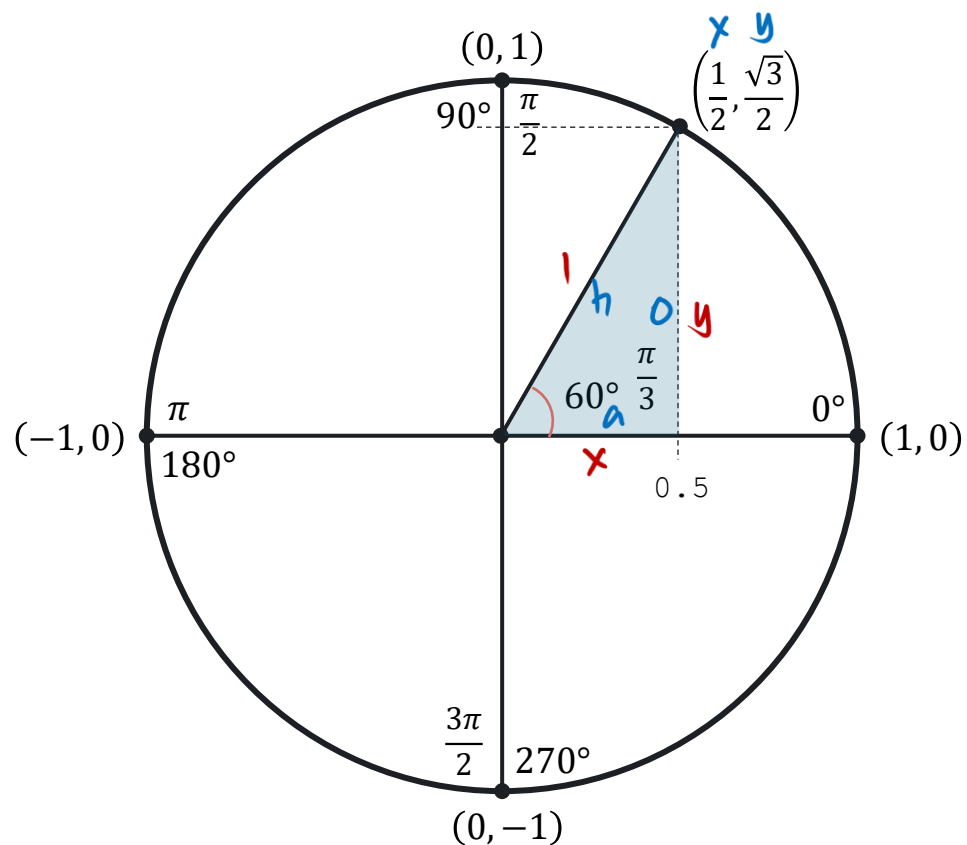
$$\tan 30^\circ = \frac{1}{2} \cdot \frac{2}{\sqrt{3}}$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\rightarrow \boxed{\tan 30^\circ = \frac{\sqrt{3}}{3}} \text{ ans}$$

$$\boxed{\tan 30^\circ = \frac{\sqrt{3}}{3}} \text{ ans}$$

60° ANGLE



Trigonometric function values for 60°

$$\sin \theta = \frac{o}{h}$$

$$\cos \theta = \frac{a}{h}$$

$$\tan \theta = \frac{o}{a}$$

$$\sin \theta = \frac{y}{1}$$

$$\cos \theta = \frac{x}{1}$$

$$\tan \theta = \frac{y}{x}$$

$$\sin \theta = y$$

$$\cos \theta = x$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

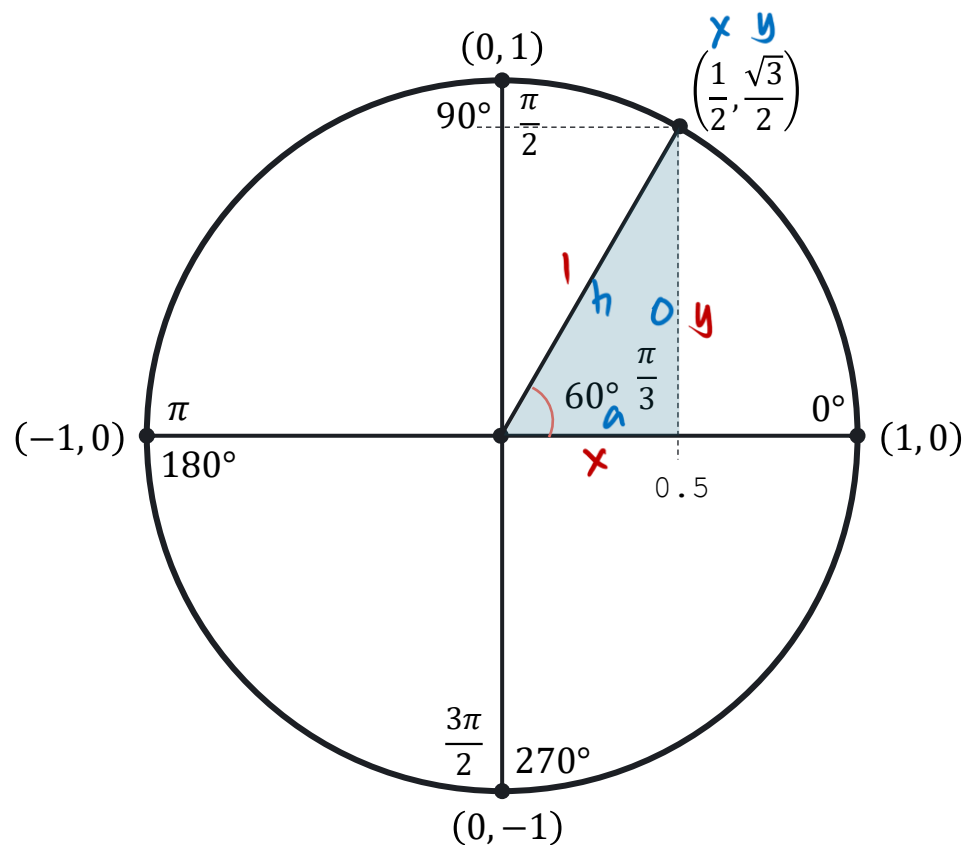
ans

$$\csc 60^\circ = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\csc 60^\circ = \frac{2\sqrt{3}}{3}$$

ans

60° ANGLE



Trigonometric function values for 60°

$$\sin \theta = \frac{o}{h}$$

$$\cos \theta = \frac{a}{h}$$

$$\tan \theta = \frac{o}{a}$$

$$\sin \theta = \frac{y}{1}$$

$$\cos \theta = \frac{x}{1}$$

$$\tan \theta = \frac{y}{x}$$

$$\underline{\sin \theta = y}$$

$$\underline{\cos \theta = x}$$

$$\underline{\tan \theta = \frac{\sin \theta}{\cos \theta}}$$

$$\boxed{\cos 60^\circ = \frac{1}{2}}$$

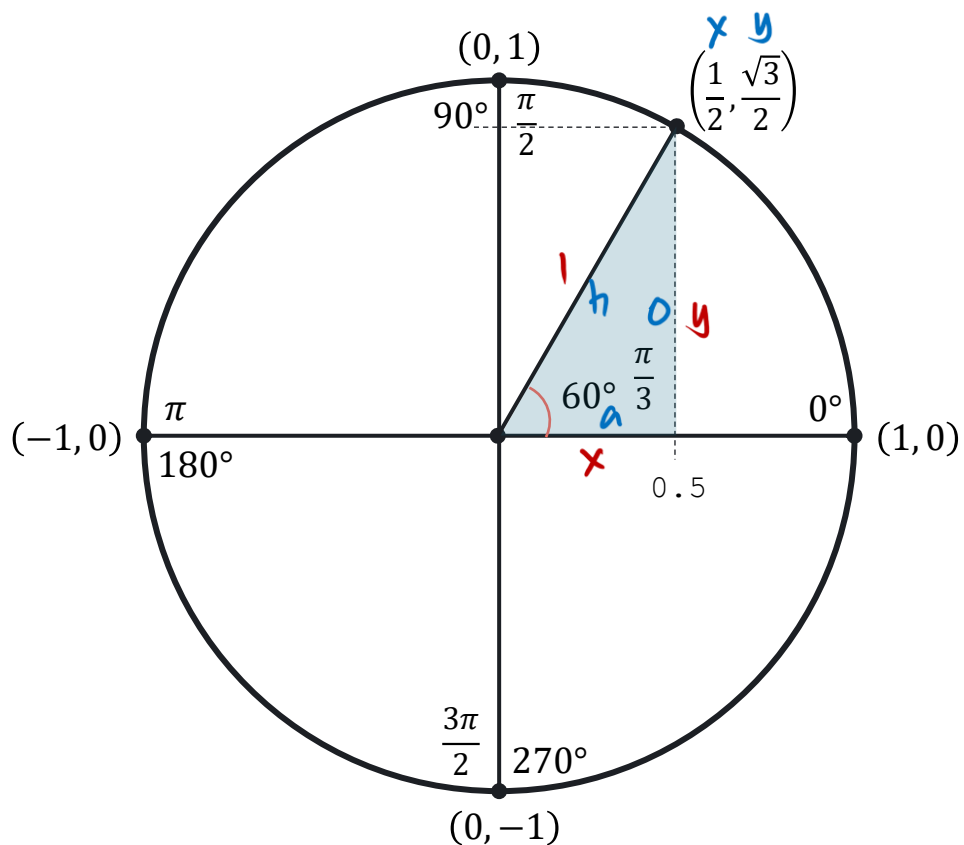
ans

$$\boxed{\sec 60^\circ = 2}$$

ans



60° ANGLE



Trigonometric function values for 60°

$$\sin \theta = \frac{o}{h}$$

$$\cos \theta = \frac{a}{h}$$

$$\tan \theta = \frac{o}{a}$$

$$\sin \theta = \frac{y}{1}$$

$$\cos \theta = \frac{x}{1}$$

$$\tan \theta = \frac{y}{x}$$

$$\sin \theta = y$$

$$\cos \theta = x$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\tan 60^\circ = \frac{\sqrt{3}/2}{1/2}$$

$$\cot 60^\circ = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\tan 60^\circ = \frac{\sqrt{3}}{2} \cdot \frac{2}{1}$$

$$\cot 60^\circ = \frac{\sqrt{3}}{3}$$

ans

$$\tan 60^\circ = \sqrt{3}$$

ans

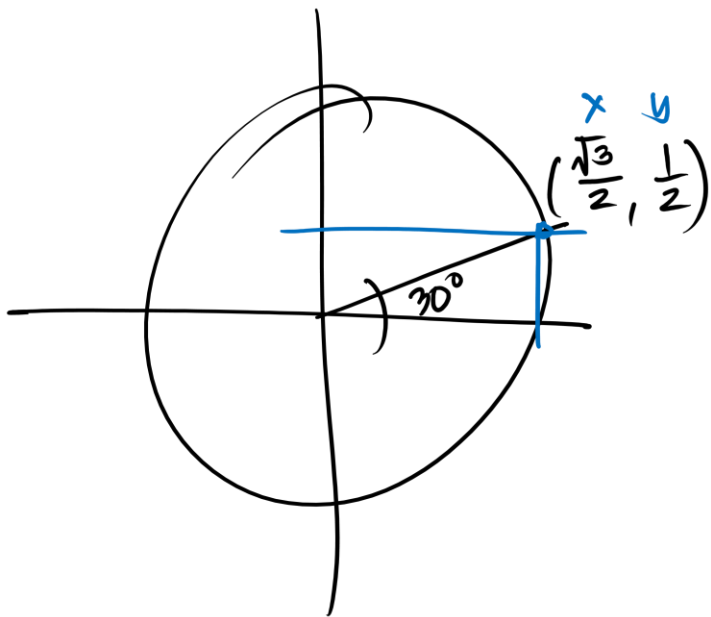
EXERCISE

Evaluate the expression without using a calculator.

a. $\sin 30^\circ \csc 30^\circ$

b. $\sin \frac{\pi}{6} + \cos \frac{\pi}{6}$

c. $\sin 30^\circ \cos 60^\circ + \sin 60^\circ \cos 30^\circ$



Solution

a. $\sin 30^\circ \csc 30^\circ$

$$= \sin 30^\circ \cdot \frac{1}{\sin 30^\circ}$$

$$= \frac{1}{2} \cdot \frac{1}{1/2}$$

$$= \frac{1}{2} \cdot 2$$

$$= \boxed{1}$$

ans



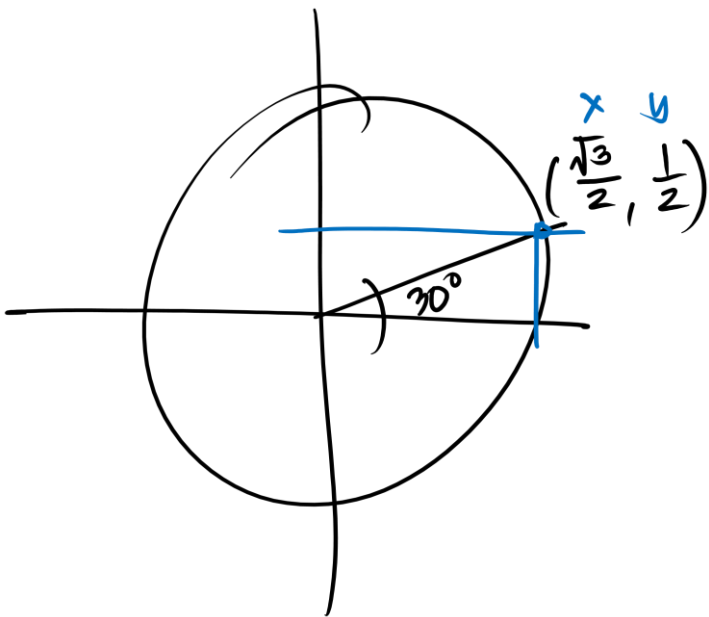
EXERCISE

Evaluate the expression without using a calculator.

a. $\sin 30^\circ \csc 30^\circ$

b. $\sin \frac{\pi}{6} + \cos \frac{\pi}{6} \rightarrow \frac{\pi}{6} \text{ rad} = 30^\circ$

c. $\sin 30^\circ \cos 60^\circ + \sin 60^\circ \cos 30^\circ$



Solution

b. $\sin \frac{\pi}{6} + \cos \frac{\pi}{6}$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2}$$

$$= \boxed{\frac{1 + \sqrt{3}}{2}} \text{ ans}$$



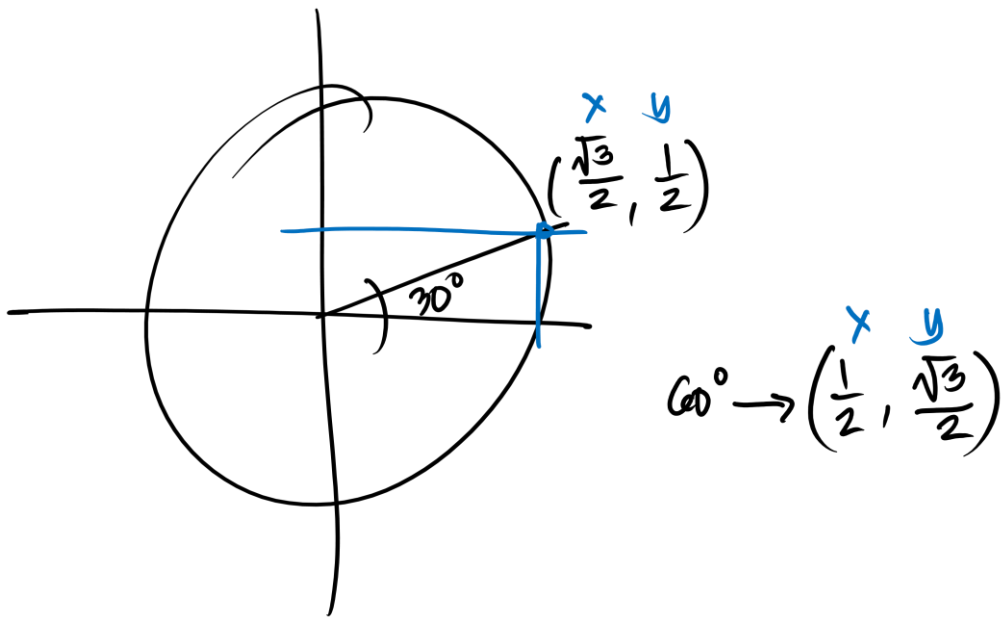
EXERCISE

Evaluate the expression without using a calculator.

a. $\sin 30^\circ \csc 30^\circ$

b. $\sin \frac{\pi}{6} + \cos \frac{\pi}{6}$

c. $\sin 30^\circ \cos 60^\circ + \sin 60^\circ \cos 30^\circ$



Solution

c. $\sin 30^\circ \cos 60^\circ + \sin 60^\circ \cos 30^\circ$

$$= \frac{1}{2} \cdot \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2}$$

$$= \frac{1}{4} + \frac{3}{4}$$

$$= \frac{4}{4}$$

$$= \boxed{1}$$

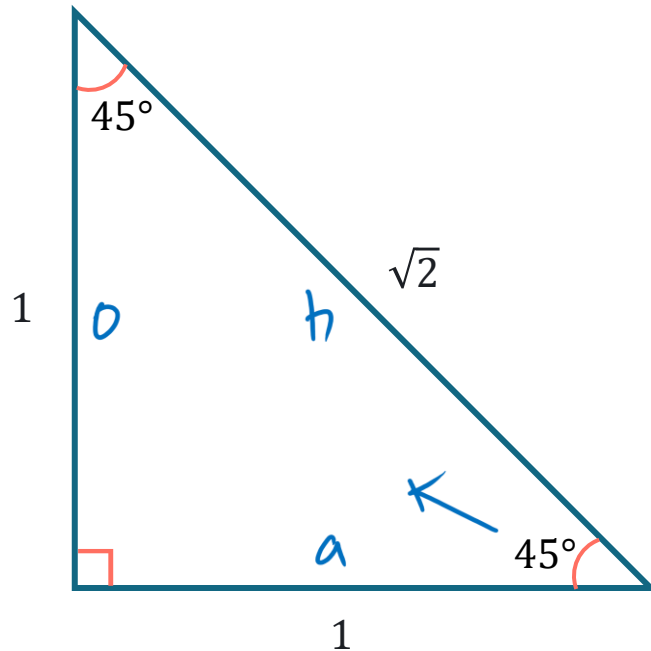
ans



SPECIAL RIGHT TRIANGLES



45-45-90 TRIANGLE



Trigonometric function values for 45°

$$\sin 45^\circ = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$\sin 45^\circ = \frac{\sqrt{2}}{2}$$

ans

$$\cos 45^\circ = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$\cos 45^\circ = \frac{\sqrt{2}}{2}$$

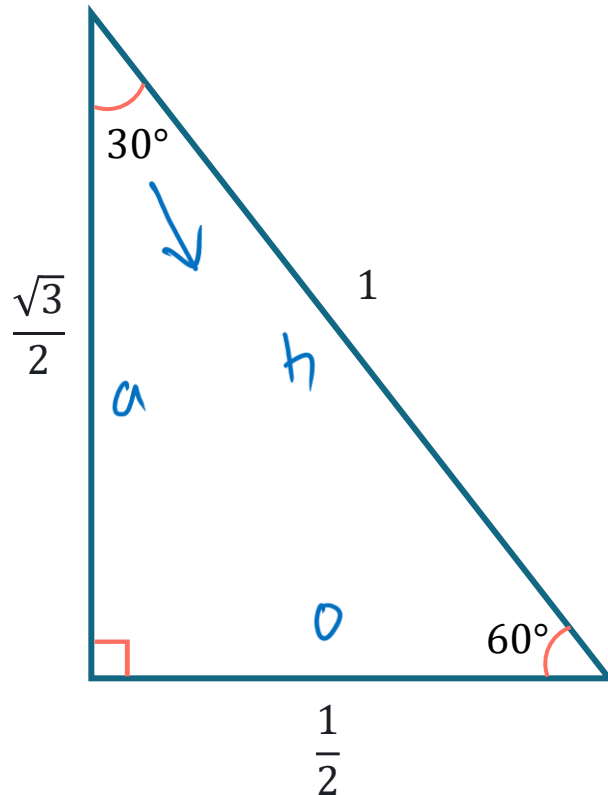
ans

$$\tan 45^\circ = \frac{1}{1}$$

$$\tan 45^\circ = 1$$

ans

30-60-90 TRIANGLE



Trigonometric function values for 30°

$$\sin 30^\circ = \frac{1/2}{1}$$

$$\boxed{\sin 30^\circ = \frac{1}{2}} \quad \text{ans}$$

$$\cos 30^\circ = \frac{\sqrt{3}/2}{1}$$

$$\boxed{\cos 30^\circ = \frac{\sqrt{3}}{2}} \quad \text{ans}$$

$$\tan 30^\circ = \frac{1/2}{\sqrt{3}/2}$$

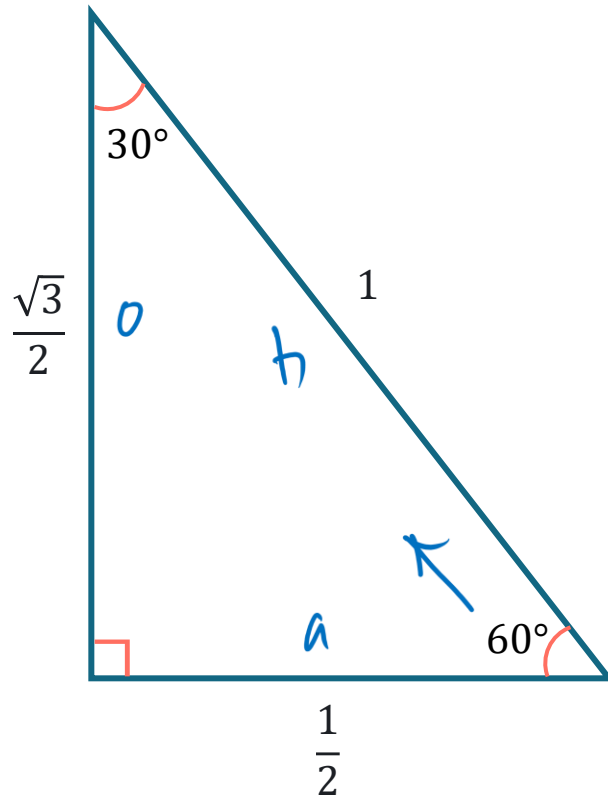
$$\tan 30^\circ = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{3}}$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\boxed{\tan 30^\circ = \frac{\sqrt{3}}{3}} \quad \text{ans}$$



30-60-90 TRIANGLE



Trigonometric function values for 60°

$$\sin 60^\circ = \frac{\sqrt{3}/2}{1}$$

$$\boxed{\sin 60^\circ = \frac{\sqrt{3}}{2}} \quad \text{ans}$$

$$\cos 60^\circ = \frac{1/2}{1}$$

$$\boxed{\cos 60^\circ = \frac{1}{2}} \quad \text{ans}$$

$$\tan 60^\circ = \frac{\sqrt{3}/2}{1/2}$$

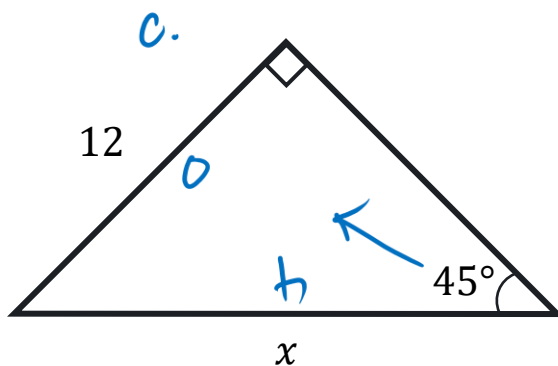
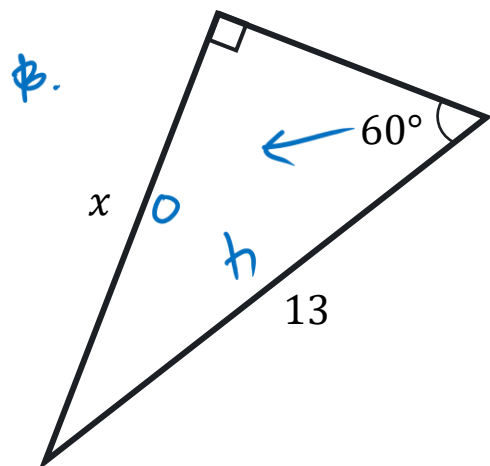
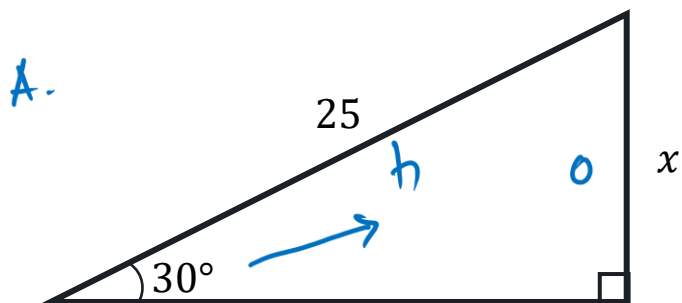
$$\tan 60^\circ = \frac{\sqrt{3}}{2} \cdot \frac{2}{1}$$

$$\boxed{\tan 60^\circ = \frac{\sqrt{3}}{2}} \quad \text{ans}$$



EXERCISE

Find the side labeled x .



Solution

$$A. \sin 30^\circ = \frac{x}{25}$$

$$x = 25 \sin 30^\circ$$

$$x = 12.5$$

ans

$$C. \sin 45^\circ = \frac{12}{x}$$

$$x = \frac{12}{\sin 45^\circ}$$

$$x = 16.97$$

ans

$$B. \sin 60^\circ = \frac{x}{13}$$

$$x = 13 \sin 60^\circ$$

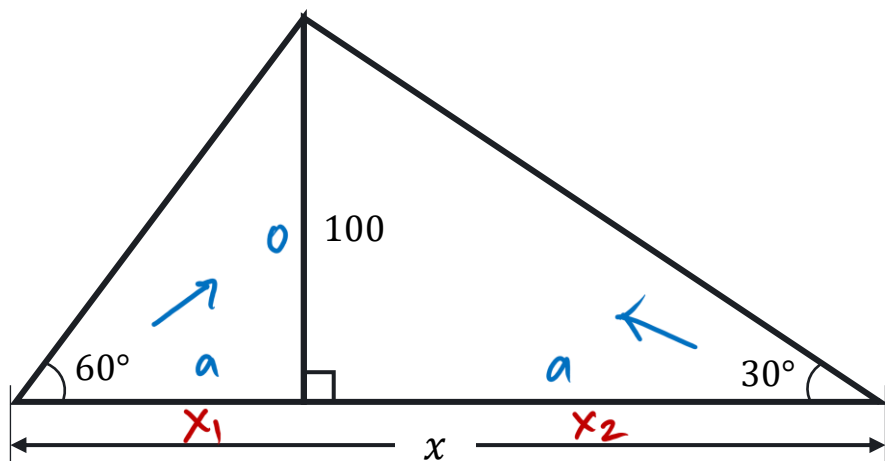
$$x = 11.26$$

ans



EXERCISE

Find x correct to one decimal place.



$$\underline{x = x_1 + x_2}$$

Solution

$$\tan 60^\circ = \frac{100}{x_1}$$

$$x_1 = \frac{100}{\tan 60^\circ}$$

$$\underline{x_1 = 57.73}$$

$$\tan 30^\circ = \frac{100}{x_2}$$

$$x_2 = \frac{100}{\tan 30^\circ}$$

$$\underline{x_2 = 173.21}$$

$$x = x_1 + x_2$$

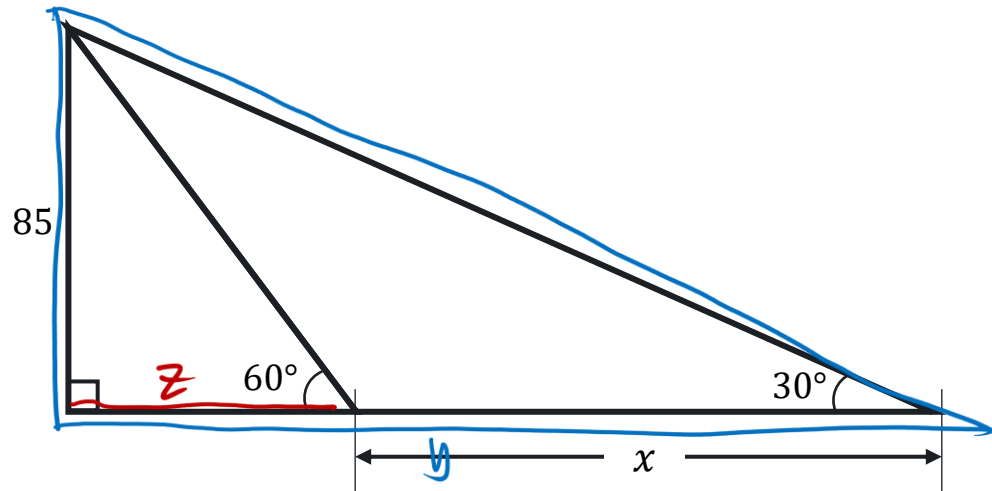
$$x = 57.73 + 173.21$$

$$\boxed{x = 230.9}$$

ans

EXERCISE

Find x correct to one decimal place.



$$\underline{x = y - z}$$

Solution

$$\tan 30^\circ = \frac{85}{y}$$

$$y = \frac{85}{\tan 30^\circ}$$

$$\underline{y = 147.22}$$

$$\tan 60^\circ = \frac{85}{z}$$

$$z = \frac{85}{\tan 60^\circ}$$

$$\underline{z = 49.07}$$

$$x = y - z$$

$$x = 147.22 - 49.07$$

$$\boxed{x = 98.2}$$

ans

SEATWORK

