



BOOLEAN ALGEBRA

LOGIC MINIMIZATION

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TOPIC OUTLINE

Laws of Boolean Algebra

Rules of Boolean Algebra

DeMorgan's Theorem



LAWS OF BOOLEAN **ALGEBRA**



BOOLEAN ALGEBRA

Boolean algebra is the mathematics of digital logic. It was formulated in 1874 by George Boole.



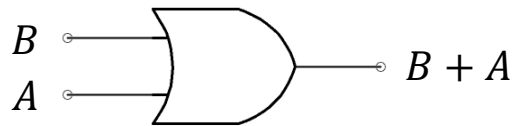
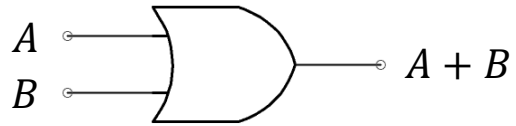
George Boole



COMMUTATIVE LAWS

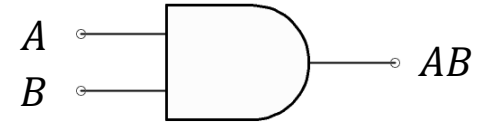
Commutative law of addition

$$A + B = B + A$$



Commutative law of multiplication

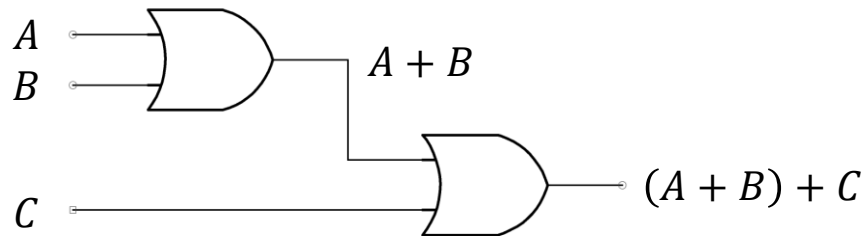
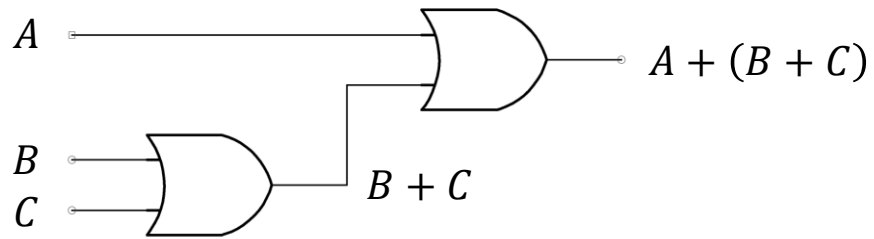
$$AB = BA$$



ASSOCIATIVE LAWS

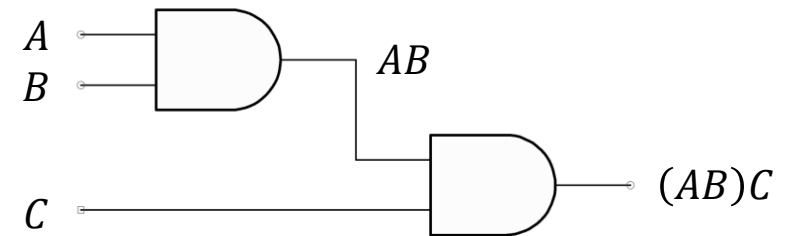
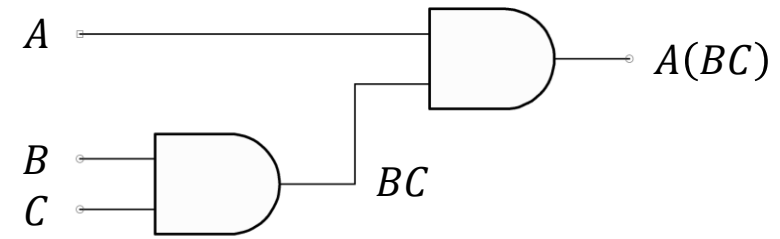
Associative law of addition

$$A + (B + C) = (A + B) + C$$



Associative law of multiplication

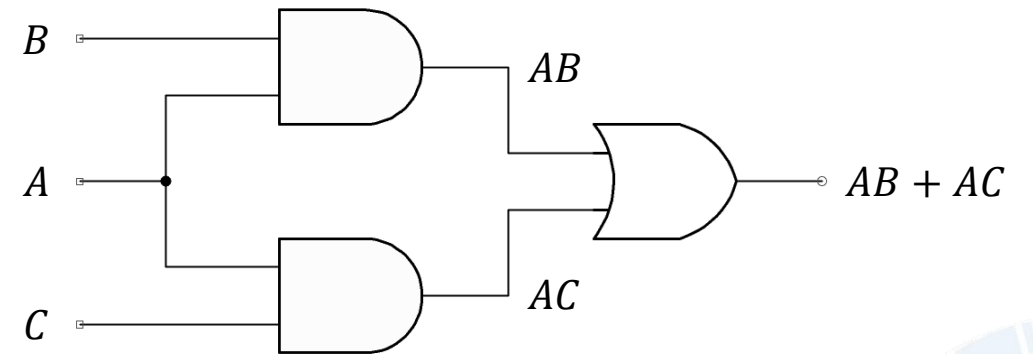
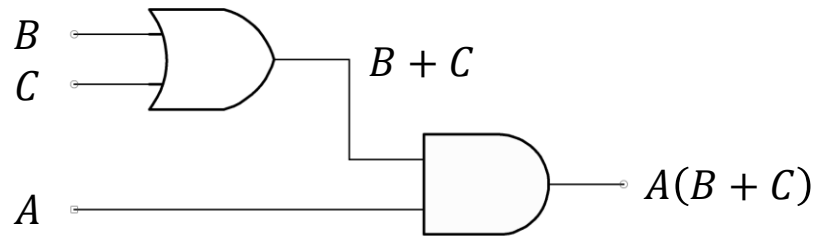
$$A(BC) = (AB)C$$



DISTRIBUTIVE LAW

Distributive law

$$A(B + C) = AB + AC$$



RULES OF BOOLEAN ALGEBRA



BASIC RULES OF BOOLEAN ALGEBRA

Basic rules of Boolean algebra are useful in manipulating and simplifying Boolean expressions.

Basic rules of Boolean algebra

1. $A + 0 = A$

2. $A + 1 = 1$

3. $A \cdot 0 = 0$

4. $A \cdot 1 = A$

5. $A + A = A$

6. $A + \bar{A} = 1$

7. $A \cdot A = A$

8. $A \cdot \bar{A} = 0$

9. $\bar{\bar{A}} = A$

10. $A + AB = A$

11. $A + \bar{A}B = A + B$

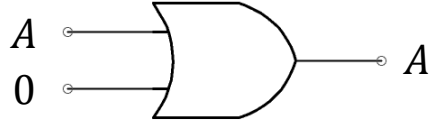
12. $(A + B)(A + C) = A + BC$



RULE 1 AND 2

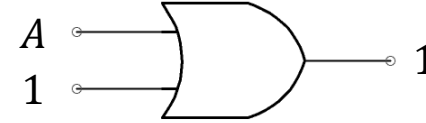
Rule 1: $A + 0 = A$

A variable Ored with 0 is always equal to the variable.



Rule 2: $A + 1 = 1$

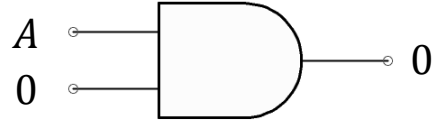
A variable Ored with 1 is always equal to 1.



RULE 3 AND 4

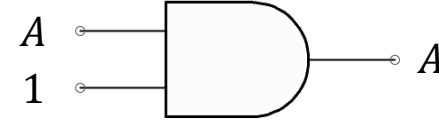
Rule 3: $A \cdot 0 = 0$

A variable ANDed with 0 is always equal to 0



Rule 4: $A \cdot 1 = A$

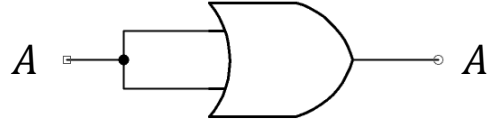
A variable ANDed with 1 is always equal to the variable.



RULE 5 AND 6

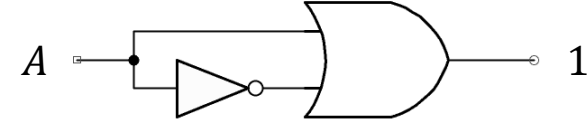
Rule 5: $A + A = A$

A variable ORed with itself is always equal to the variable.



Rule 6: $A + \bar{A} = 1$

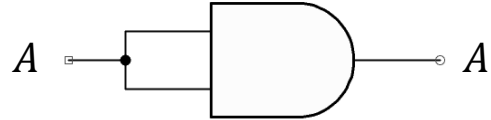
A variable ORed with its complement is always equal to 1.



RULE 7 AND 8

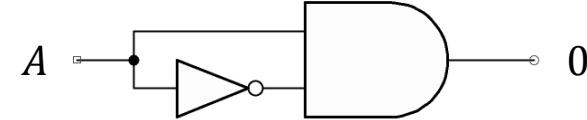
Rule 7: $A \cdot A = A$

A variable ANDed with itself is always equal to the variable.



Rule 8: $A \cdot \bar{A} = 0$

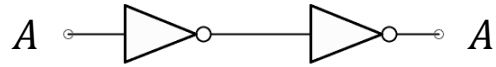
A variable ANDed with its complement is always equal to 0.



RULE 9 AND 10

Rule 9: $\overline{\overline{A}} = A$

A double complement of a variable is always equal to the variable.



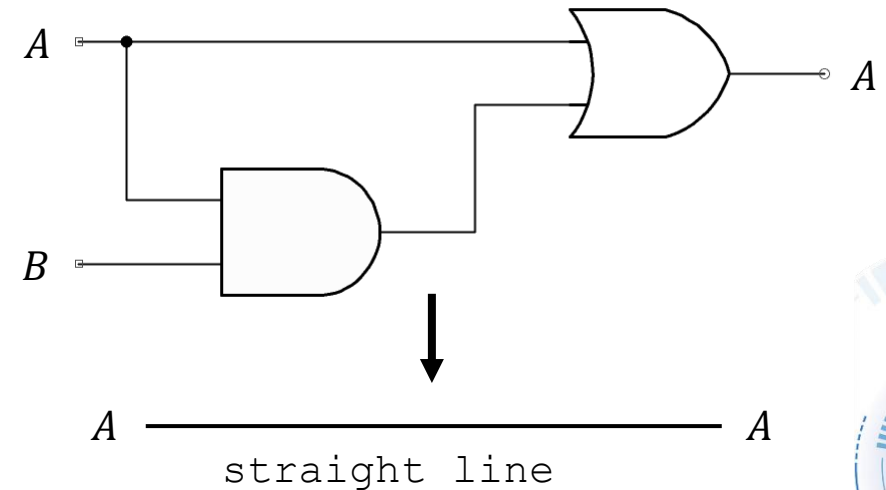
Rule 10: $A + AB = A$

$$A + AB = A \cdot 1 + AB$$

$$= A(1 + B)$$

$$= A(1)$$

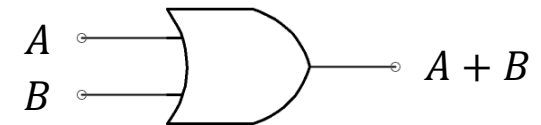
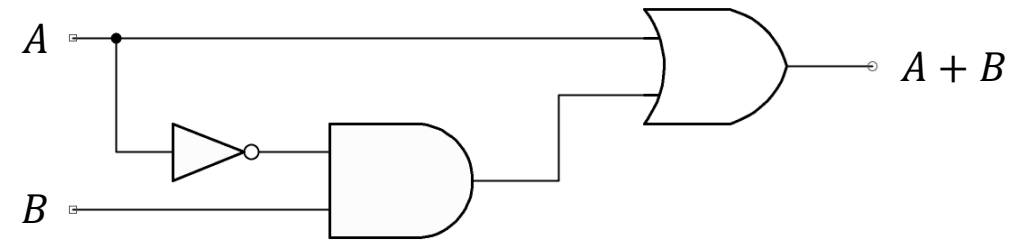
$$= A$$



RULE 11

Rule 11: $A + \bar{A}B = A + B$

$$\begin{aligned} A + \bar{A}B &= (A + AB) + \bar{A}B \\ &= (AA + AB) + \bar{A}B \\ &= AA + AB + A\bar{A} + \bar{A}B \\ &= (A + \bar{A})(A + B) \\ &= 1 \cdot (A + B) \\ &= A + B \end{aligned}$$



RULE 12

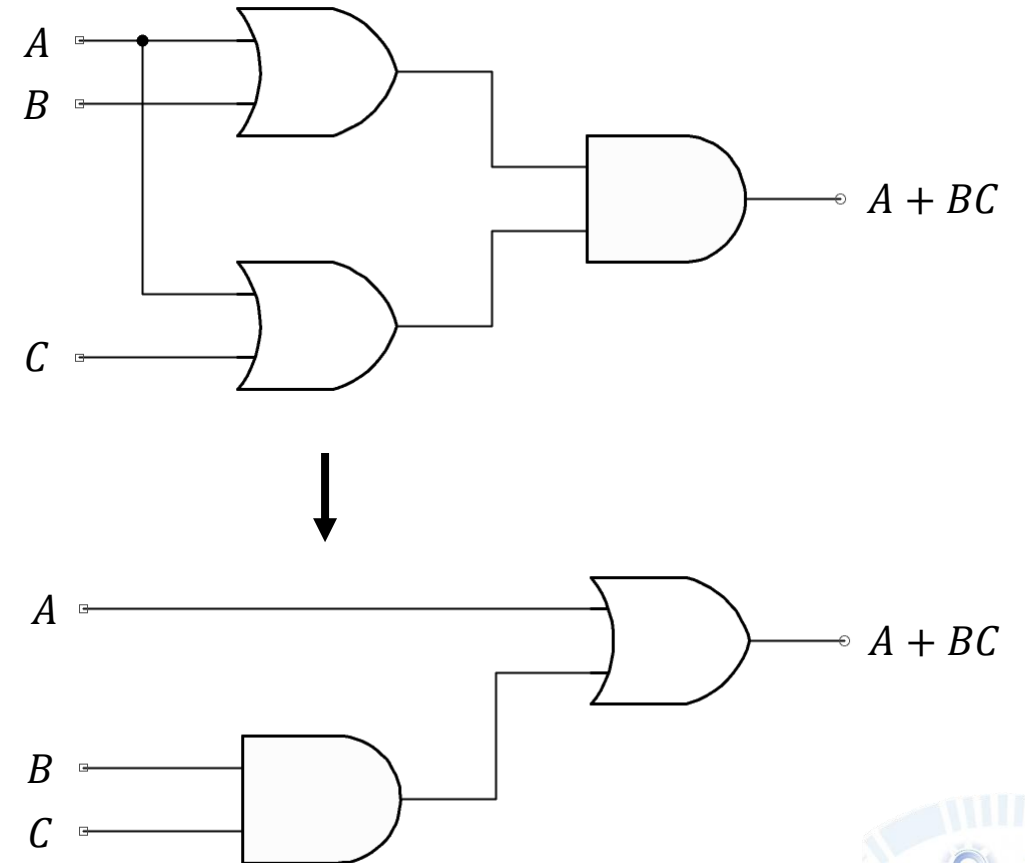
Rule 12: $(A + B)(A + C) = A + BC$

$$(A + B)(A + C) = AA + AC + AB + BC$$

$$= A + AC + AB + BC$$

$$= A + AB + BC$$

$$= A + BC$$



DEMORGAN'S THEOREMS



FIRST THEOREM

DeMorgan's first theorem states that the complement of a product of variables is equal to the sum of the complements of the variables.

Logic Expression

$$\overline{XY} = \bar{X} + \bar{Y}$$

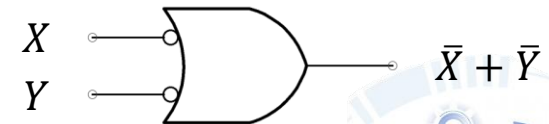
Truth Table

X	Y	\overline{XY}	$\bar{X} + \bar{Y}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

NAND



Negative-OR



SECOND THEOREM

DeMorgan's second theorem states that the complement of a sum of variables is equal to the product of the complements of the variables.

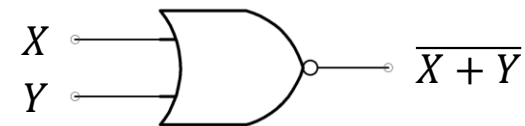
Logic expression

$$\overline{X + Y} = \bar{X} \cdot \bar{Y}$$

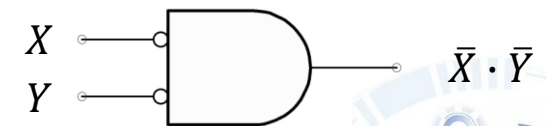
Truth Table

X	Y	$\overline{X + Y}$	$\bar{X} \cdot \bar{Y}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

NOR



Negative-AND



EXERCISE

Apply DeMorgan's theorems to the expression:

$$f = \overline{(A + B)C}$$

Solution



EXERCISE

Simplify the Boolean expression:

$$f = AB + A(B + C) + B(B + C)$$

Solution



EXERCISE

Simplify the Boolean expression:

$$f = [A\bar{B}(C + BD) + \bar{A}\bar{B}]C$$

Solution



EXERCISE

Apply DeMorgan's theorems to the expression:

$$f = \overline{(\bar{A} + B) + CD}$$

Solution



EXERCISE

Simplify the Boolean expression:

$$f = A\bar{B} + A(\overline{B + C}) + B(\overline{B + C})$$

Solution



LABORATORY

