HYPOTHESIS TESTING

INFERENTIAL STATISTICS

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TOPIC OUTLINE

Hypothesis Test

Significance Level

Rejection Region

Critical Value and Z-score

HYPOTHESIS TEST

HYPOTHESIS

A **hypothesis** is an initial **assumption** formed before collecting data, and it serves as a statement about a **population** parameter rather than about the sample data.

Steps in Data-Driven Decision Making

1. Formulate a hypothesis
2. Find the right test (e.g., z-test, t-test)
3. Execute the test
4. Make a decision

HYPOTHESIS TEST

A **hypothesis test** is simply comparing reality to an assumption and asking, "**Did things change?**"

Null Hypothesis (H_o)

Represents **no change**, no effect, or the status quo.

Alternative Hypothesis (H_a)

Represents the possibility that things did change or that there is a **significant difference**.

EXERCISE

A fitness tracker company claims their device measures heart rate with **95%** accuracy compared to medical-grade monitors. An independent lab wants to verify this claim.

Null Hypothesis

$$\boldsymbol{H_o : \mu_o = 95}$$

The average accuracy is 95%.

Alternative Hypothesis

$$\boldsymbol{H_a : \mu_o \neq 95}$$

The average accuracy differs from 95%.

EXERCISE

A manufacturer claims that their new energy-efficient LED bulbs have an average lifespan of **at least 25,000 hours**. A consumer group suspects that the actual lifespan is shorter and decides to test this claim.

Null Hypothesis

$$H_o\colon \mu_o = 25,000$$

The average lifespan of the LED is 25,000 hours.

Alternative Hypothesis

$$H_a\colon \mu_o < 25,000$$

The average lifespan of the LED is less than 25,000 hours.

EXERCISE

A study suggests that storing apples in a controlled atmosphere **extends** their shelf life beyond **30 days**. A food scientist wants to verify if this method truly increases shelf life compared to conventional storage.

Null Hypothesis

$$\boldsymbol{H_o: \mu_o = 30}$$

Controlled-atmosphere storage shelf life is 30 days.

Alternative Hypothesis

$$\boldsymbol{H_a: \mu_o > 30}$$

Controlled-atmosphere storage increases shelf life beyond 30 days.

SIGNIFICANCE LEVEL

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The **significance level** ($\boldsymbol{\alpha}$) determines the threshold for deciding whether to **reject** the null hypothesis ($\boldsymbol{H_o}$).

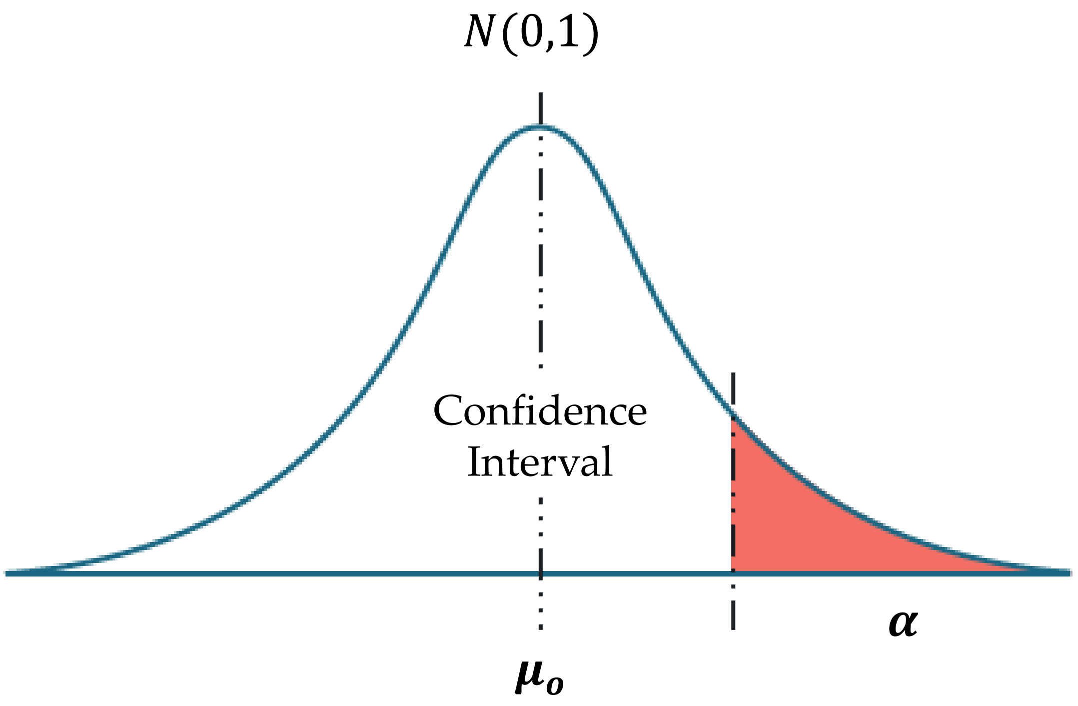
Typical values for $\boldsymbol{\alpha}$:

0.01

0.05

0.1

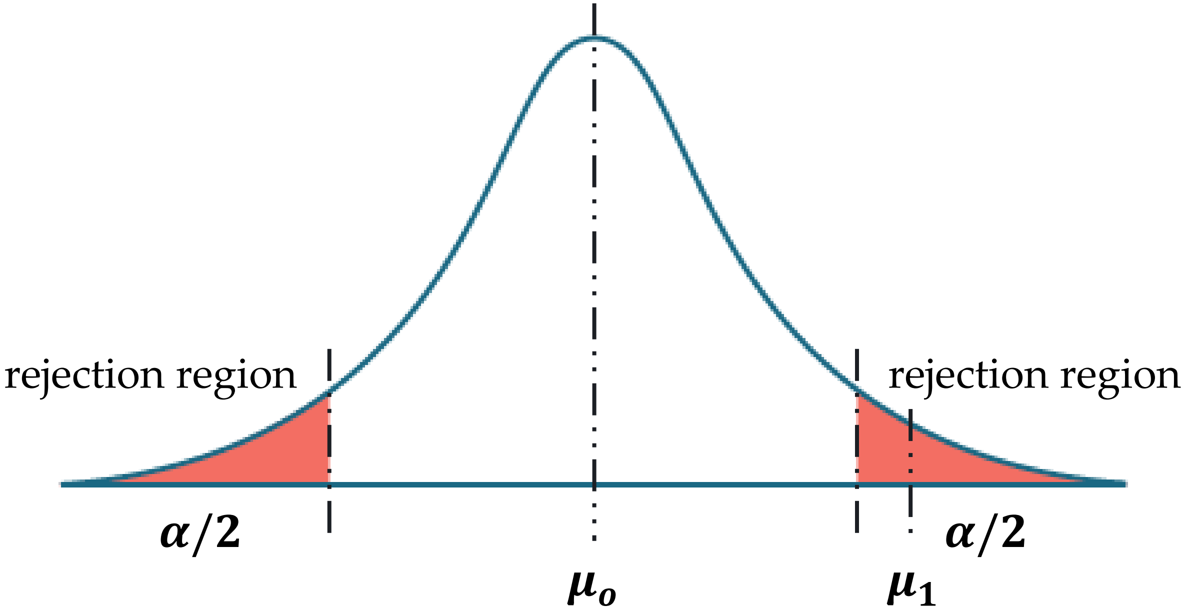
Standard Normal Distribution



REJECTION REGION

REJECTION REGION

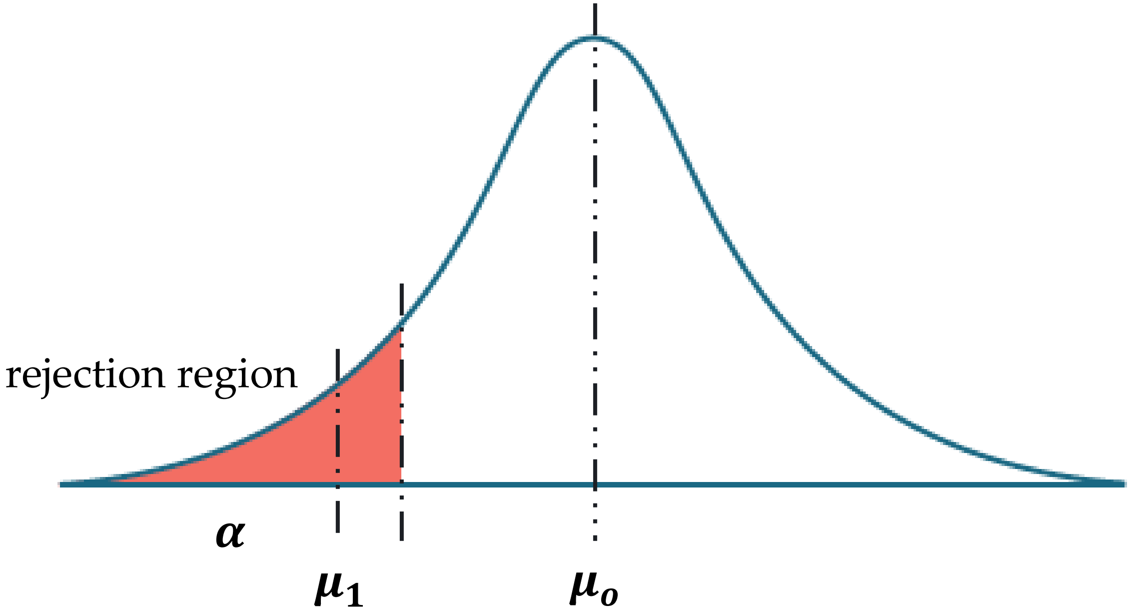
Two-Tailed Test



H_o: $\mu_1 = \mu_o$

H_a: $\mu_1 \neq \mu_o$

One-Tailed Test

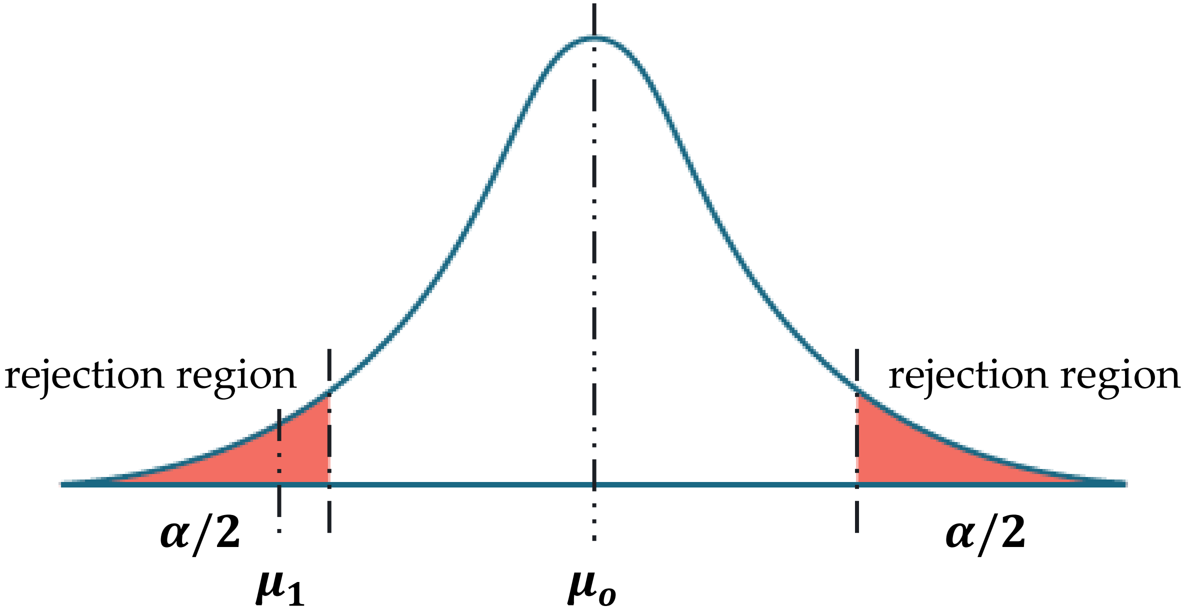


H_o: $\mu_1 = \mu_o$

H_a: $\mu_1 < \mu_o$

REJECTION REGION

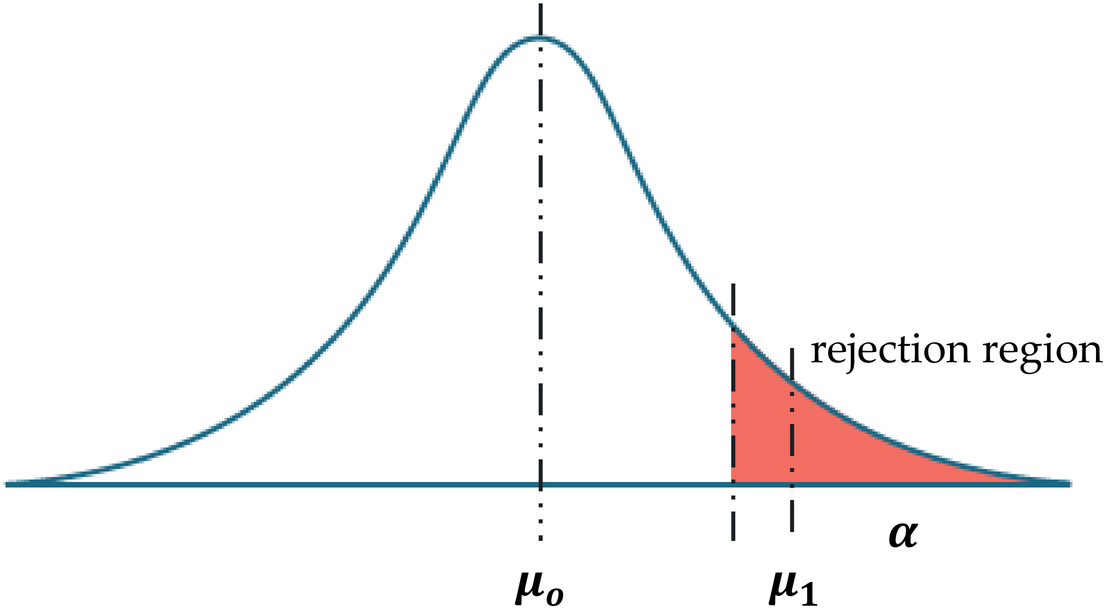
Two-Tailed Test



H_o: $\mu_1 = \mu_o$

H_a: $\mu_1 \neq \mu_o$

One-Tailed Test



H_o: $\mu_1 = \mu_o$

H_a: $\mu_1 > \mu_o$

CRITICAL VALUE AND Z-SCORE

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lowercase z

z refers to the **critical value** obtained from the standard normal distribution table (z-table).

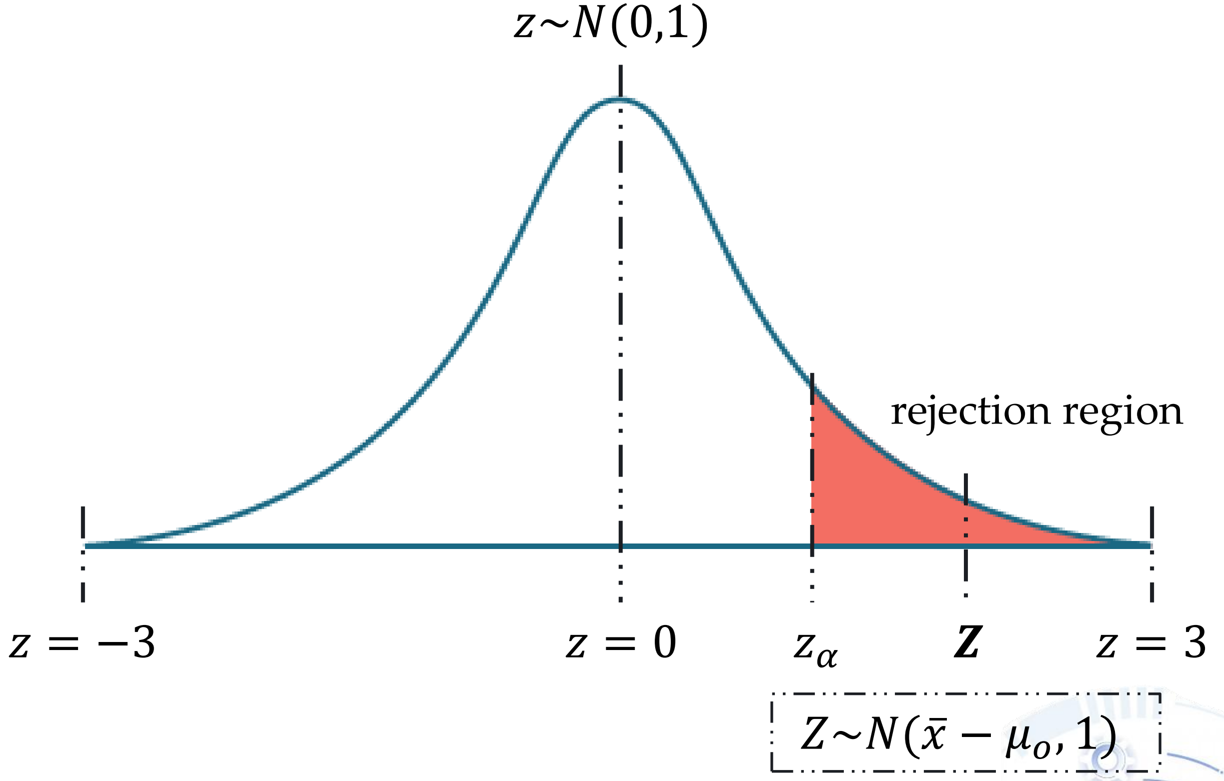
uppercase Z

Z is a standardized variable associated with the test called the **Z-score**.

Formula:

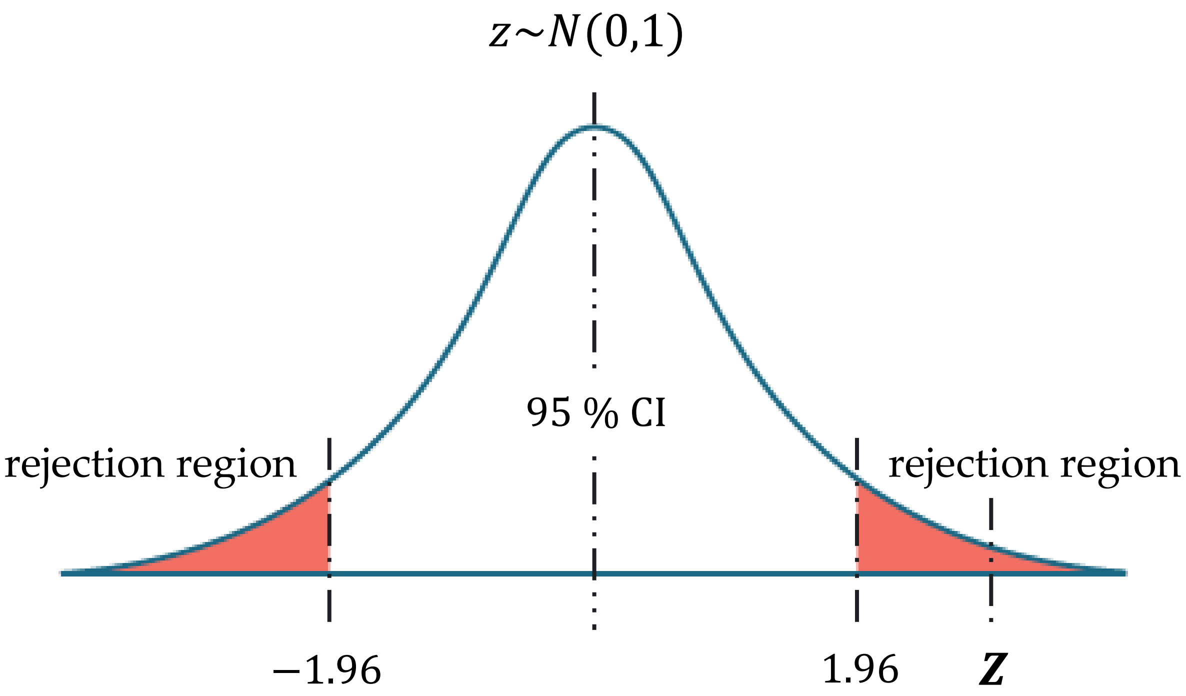
$$Z = \frac{\bar{x} - \mu_o}{\sigma/\sqrt{n}}$$

One-Tailed Test



EXERCISE

Two-Tailed Test



$\alpha = 0.05$

$z_{0.025} = 1.96$

Null Hypothesis

$\boldsymbol{H_o: \mu_o = 95}$

The average accuracy is 95%.

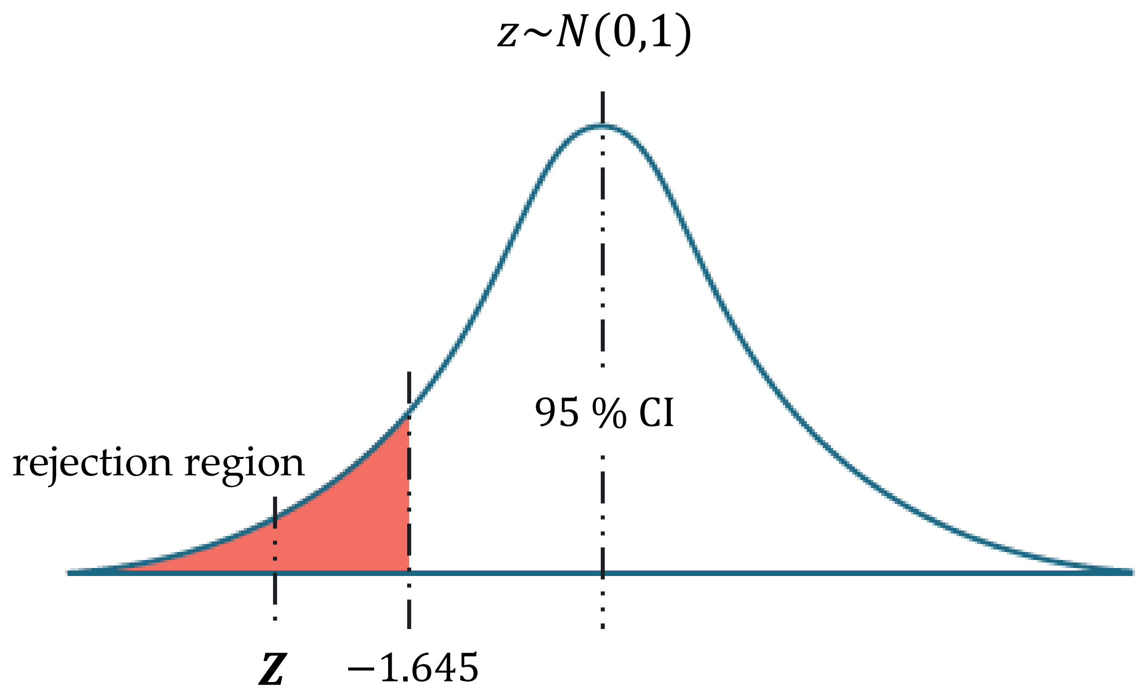
Alternative Hypothesis

$\boldsymbol{H_a: \mu_o \neq 95}$

The average accuracy differs from 95%.

EXERCISE

One-Tailed Test



$\alpha = 0.05$

$z_{0.05} = 1.645$

Null Hypothesis

$$H_o: \mu_o = 25,000$$

The average lifespan of the LED is 25,000 hours.

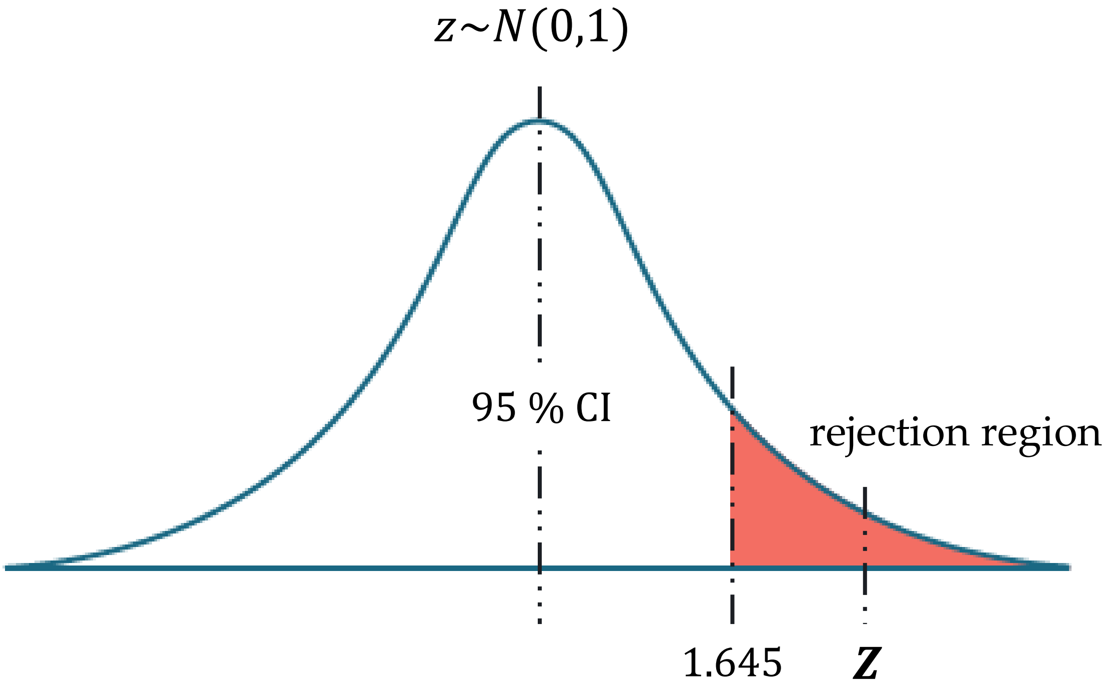
Alternative Hypothesis

$$H_a: \mu_o < 25,000$$

The average lifespan of the LED is less than 25,000 hours.

EXERCISE

One-Tailed Test



$\alpha = 0.05$

$z_{0.05} = 1.645$

Null Hypothesis

$\boldsymbol{H_o}$: $\boldsymbol{\mu_o = 30}$

Controlled-atmosphere storage shelf life is 30 days.

Alternative Hypothesis

$\boldsymbol{H_a}$: $\boldsymbol{\mu_o > 30}$

Controlled-atmosphere storage increases shelf life beyond 30 days.

EXERCISE

A manufacturing process is claimed to have an average defect rate of **10.32** units, with a known standard deviation of **3.17** units. The Statistical Process Control (SPC) department suspects this claim may no longer be valid and collects a **random sample** of **30** production units to test whether the true average **defect rate differs** significantly from **10.32**.

Dataset:

defects-data-30-samples.csv

Solution

LABORATORY