TRIGONOMETRIC FUNCTIONS

DIFFERENTIAL CALCULUS

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TOPIC OUTLINE

Derivative of Trigonometric Functions

- sine
- cosine
- tangent
- cotangent
- secant
- cosecant

DERIVATIVE OF TRIGONOMETRIC FUNCTIONS

RECIPROCAL AND RATIO IDENTITIES

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

PYTHAGOREAN IDENTITIES

$$\cos^2\theta + \sin^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

SINE

$$\frac{d}{dx}(\sin x) = \cos x$$

Differentiate $y = \sin x + 5$.

Evaluate $\frac{d}{dx}(3\sin x)$.

COSINE

$$\frac{d}{dx}(\cos x) = -\sin x$$

Differentiate $y = \cos x + 5$.

if $u = -\cos x$, find $\frac{du}{dx}$.

TANGENT

Differentiate $y = \frac{\sin x}{\cos x}$.

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

COTANGENT

Differentiate $y = \frac{\cos x}{\sin x}$.

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

SECANT

Differentiate $y = \frac{1}{\cos x}$.

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

COSECANT

Differentiate $y = \frac{1}{\sin x}$.

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

EXERCISE

Differentiate $y = x^2 \sin x$.

Solution

EXERCISE

Differentiate $y = \sqrt{x}\sin x$.

Solution

EXERCISE

Differentiate $y = 3x^2 - 2\cos x$.

Solution

EXERCISE

If $u(x) = \frac{\cos x}{1-\sin x}$, find $\frac{du(x)}{dx}$.

Solution

EXERCISE

Evaluate $\frac{d}{d\theta}\left(\frac{\sec\theta}{1+\sec\theta}\right)$.

Solution

EXERCISE

Differentiate $g(t) = 4\sec t + \tan t$.

Solution

EXERCISE

Differentiate $f(\theta) = \dfrac{\sec\theta}{1+\tan\theta}$.

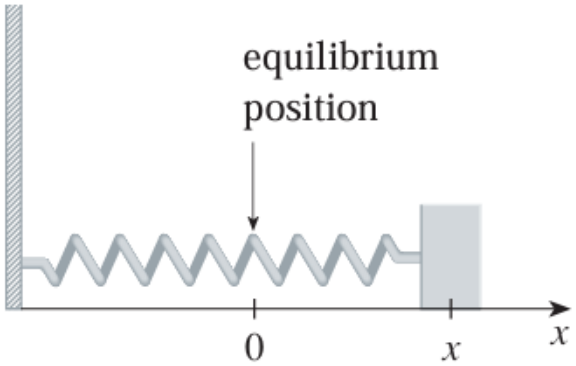
Solution

EXERCISE

A mass on a spring vibrates horizontally on a smooth level surface (see figure). Its equation of motion is $\boldsymbol{x(t) = 8 \sin t}$, where t is in seconds and x in centimeters.

a. Find the velocity at time t

b. Find the position of the mass at time $t = \frac{2\pi}{3}$



Solution

LABORATORY