MEASURES OF VARIABILITY

DESCRIPTIVE STATISTICS

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TOPIC OUTLINE

Measures of Variability

Range and Interquartile Range

Variance and Standard Deviation

Coefficient of Variation

MEASURES OF VARIABILITY

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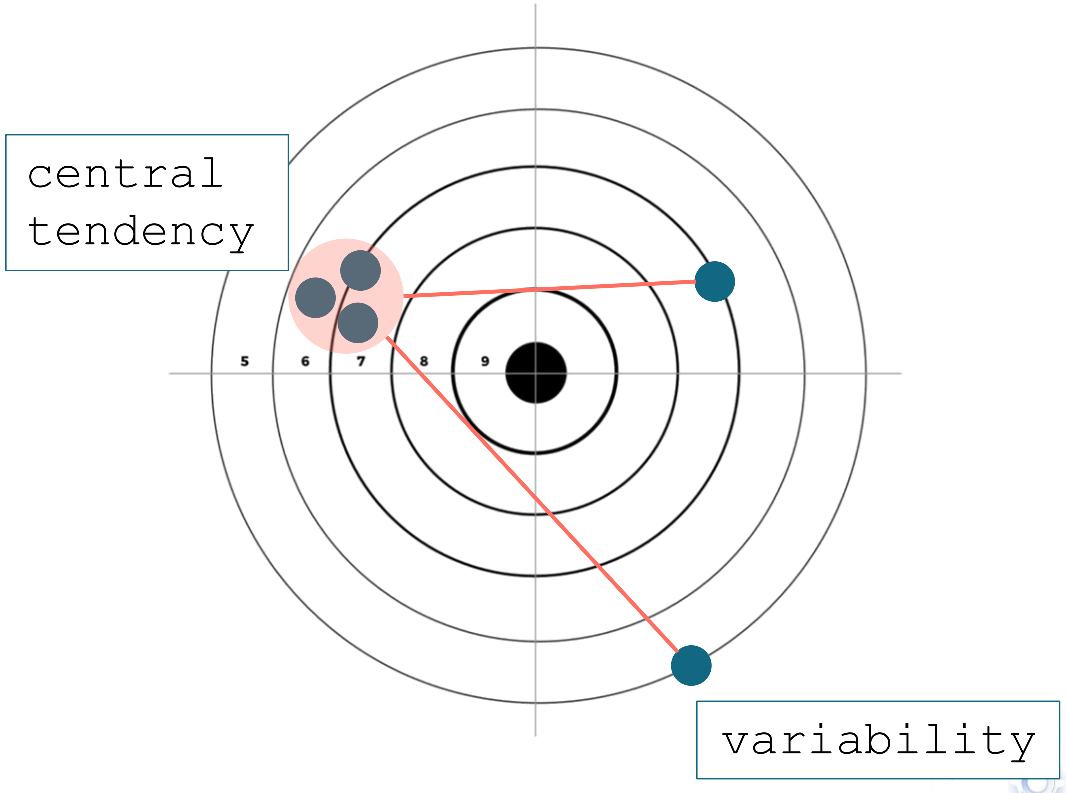
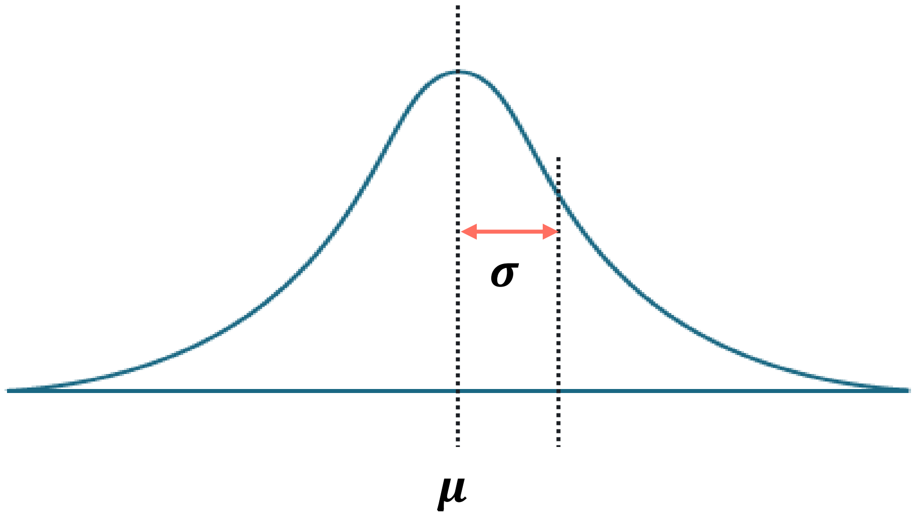
Dartboard Analogy



Measures of variability (or dispersion) describe how **spread out** or scattered the data points relative to the central tendency (mean, median, or mode).

MEASURES OF VARIABILITY

Normal Distribution



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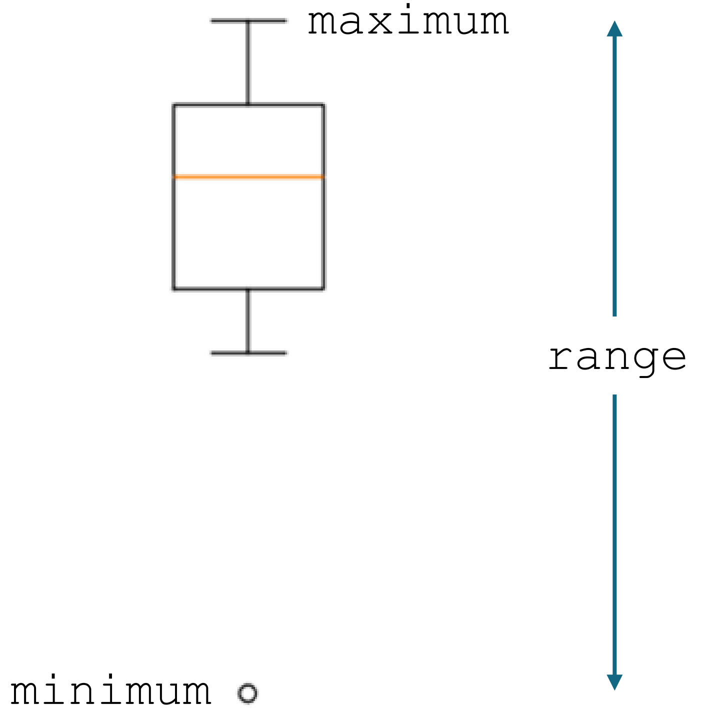
RANGE AND INTERQUARTILE RANGE

RANGE

Boxplot



Range is the simplest measure of variability and is calculated as the difference between the **maximum** and **minimum** values in a dataset.

Formula

$$\text{range} = \text{maximum value} - \text{minimum value}$$

INTERQUARTILE RANGE

The **interquartile range (IQR)** measures the spread of the middle 50% of the data, reducing the influence of outliers.

Formula

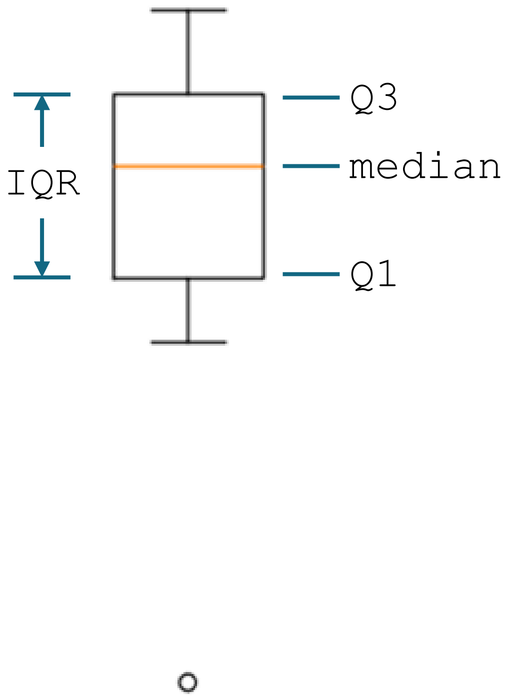
$$IQR = Q_3 - Q_1$$

where

Q_1 (first quartile) is the median of the lower half of the data (25%).

Q_3 (third quartile) is the median of the upper half of the data (75%).

Boxplot



OUTLIERS

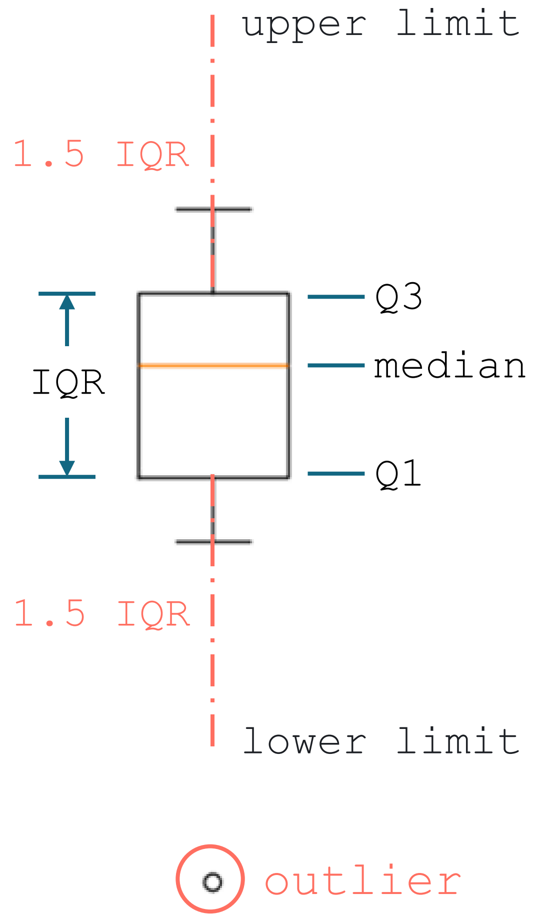
Outliers are data points that lie significantly **outside** the typical range of the rest of the dataset.

Formula

$$\textbf{Lower limit} = Q_1 - 1.5\ IQR$$

$$\textbf{Upper limit} = Q_3 + 1.5\ IQR$$

Boxplot



EXERCISE

The dataset provided contains the exam grades of 12 students. Calculate the **range** and **interquartile range** (IQR) to analyze the spread and variability of the grades.

solution

Exam Performance

Student	Grade
1	3.5
2	6.7
3	7
4	7.4
5	7.8
6	8.2
7	8.5
8	8.8
9	9
10	9.1
11	9.4
12	9.8

VARIANCE AND STANDARD DEVIATION

VARIANCE

Population Variance

$$\sigma^2 = \frac{\sum_{i=1}^{N}(x_i - \mu)^2}{N}$$

Variance measures the **average squared deviation** of each data point from the mean.

Sample Variance

$$s^2 = \frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n - 1}$$

STANDARD DEVIATION

The **standard deviation** is the **square root** of variance.

Population Standard Deviation

$$\sigma = \sqrt{\sigma^2}$$

Sample Standard Deviation

$$s = \sqrt{s^2}$$

EXERCISE

The dataset provided contains the sugar content (in grams) per serving for 10 popular breakfast cereals. Calculate the **variance** and **standard deviation** to measure the spread or variability in the sugar content across these cereals.

solution

Breakfast Cereal

Brand	Sugar
A	12
B	9
C	15
D	8
E	10
F	11
G	13
H	7
I	14
J	6

POOLED STANDARD DEVIATION

Variances add

$$\sigma_{total}^2 = \sigma_1^2 + \sigma_2^2 + \cdots \sigma_k^2$$

Pooled standard deviation is a weighted **average** of the standard deviations from two or more groups.

Standard deviations do not

$$\sigma_{total} \neq \sigma_1 + \sigma_2 + \cdots \sigma_k$$

Formula

$$\bar{\sigma}_{pooled} = \sqrt{\frac{\sigma_1^2 + \sigma_2^2 + \cdots \sigma_k^2}{k}}$$

where

k = number of groups

EXERCISE

The dataset provided contains the battery life (in hours) for smartphones from different models. Calculate the **pooled standard deviation** to measure the combined variability in battery life across these models.

solution

Battery Life

Model	Hours
A	12.5
A	12.8
A	12.7
A	13.3
A	12.6
B	13.5
B	14.1
B	13.9
B	14.3
B	13.7
C	11.8
C	11.9
C	12.1
C	12.2
C	11.6

COEFFICIENT OF VARIATION

COEFFICIENT OF VARIATION

Coefficient of variation (c_v) is a relative measure of variability, expressed as the **ratio** of the standard deviation to the mean.

Population Coefficient of Variation

$$c_v = \frac{\sigma}{\mu}$$

Sample Coefficient of Variation

$$\hat{c}_v = \frac{s}{\bar{x}}$$

EXERCISE

The provided dataset includes ice cream prices listed in both USD and PHP. Calculate the **standard deviation** and **coefficient of variation** for each currency to analyze the variability in prices.

solution

Ice Cream Price List

Brand	Price (USD)	Price (PHP)
Brand A	3.5	203
Brand B	4	232
Brand C	3.75	217.5
Brand D	4.25	246.5
Brand E	3.9	226.2
Brand F	4.1	237.8
Brand G	3.6	208.8
Brand H	4.5	261
Brand I	3.8	220.4
Brand J	4.15	240.7

LABORATORY