SPECIAL RIGHT TRIANGLES

RIGHT TRIANGLES

prepared by:

Gyro A. Madrona

Electronics Engineer

TOPIC OUTLINE

The Unit Circle

45-45-90 Triangle
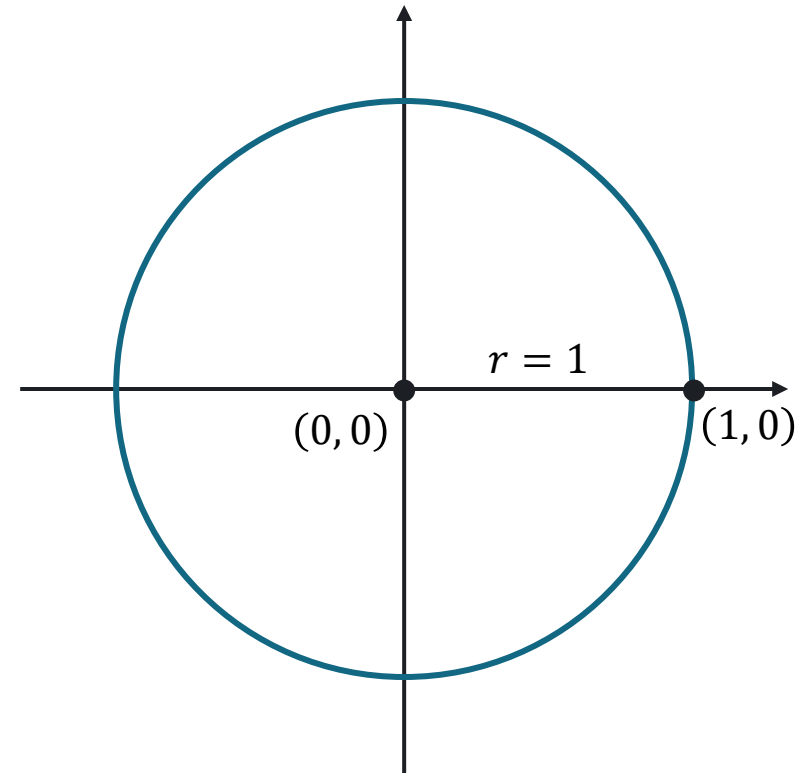
30-60-90 Triangle

UNIT CIRCLE

UNIT CIRCLE

Unit Circle

A **unit circle** is a circle with radius **1 unit**, centered at the origin of the xy-plane.
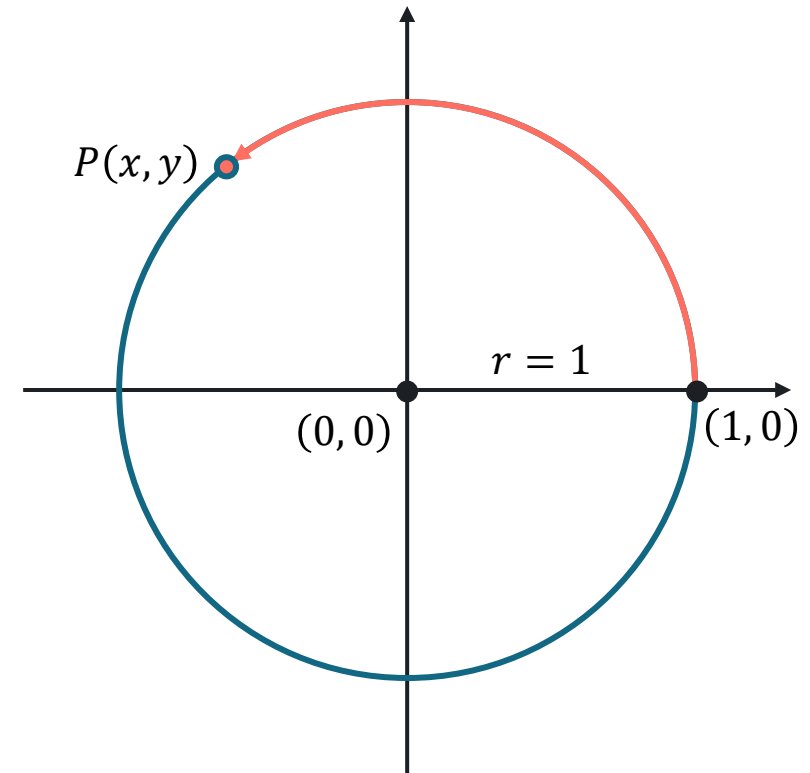
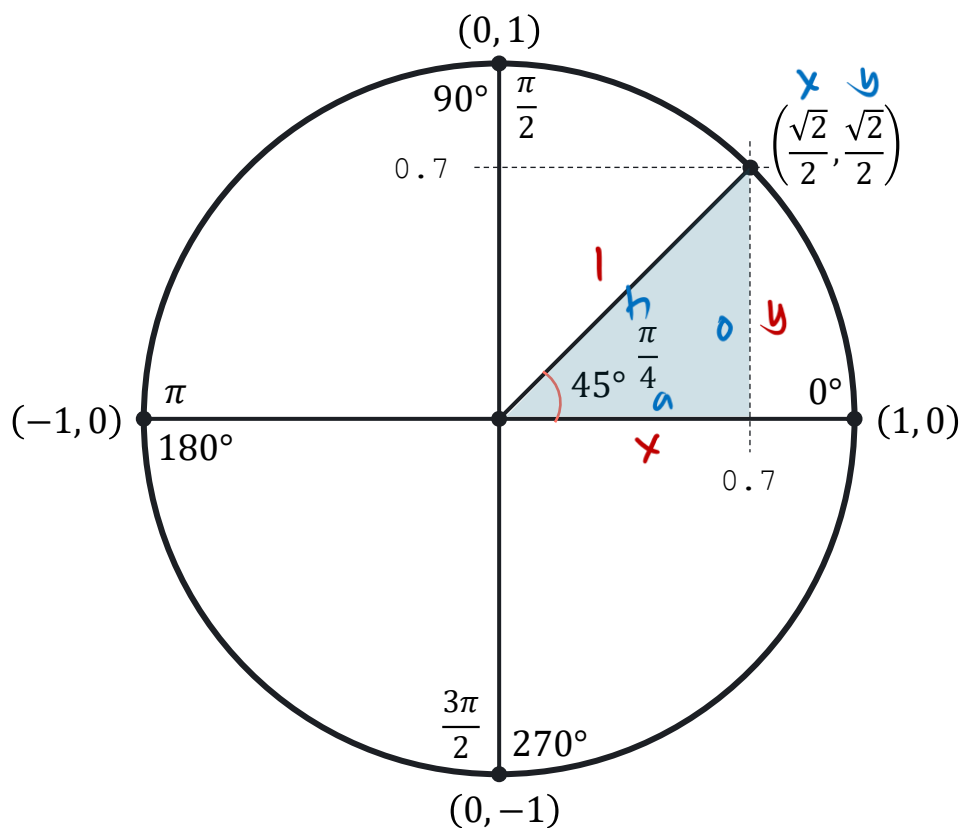Equation

$$x^2 + y^2 = 1$$

TERMINAL POINTS

Terminal Point on the Unit Circle

The **terminal point** is a point on the unit circle that corresponds to a given **angle** measured from the positive x-axis.

$P(x, y)$

$r = 1$

$(0, 0)$

$(1, 0)$

45° ANGLE

$$\sin \theta = \frac{o}{h} \qquad \cos \theta = \frac{a}{h} \qquad \tan \theta = \frac{o}{a}$$

$$\sin \theta = \frac{y}{1} \qquad \cos \theta = \frac{x}{1} \qquad \tan \theta = \frac{y}{x}$$

$$\underline{\sin \theta = y} \qquad \underline{\cos \theta = x} \qquad \tan \theta = \frac{\sin \theta}{\cos \theta}$$
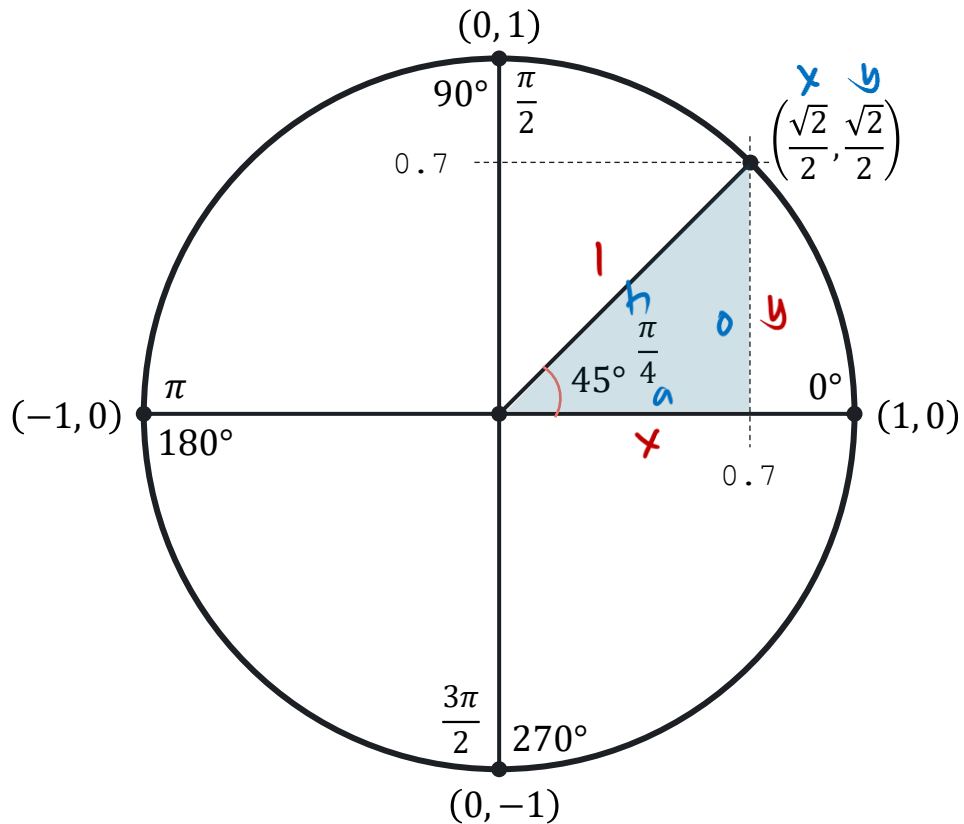
$$\boxed{\sin 45^0 = \frac{\sqrt{2}}{2}}$$
ans

$$\csc 45^0 = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$\boxed{\csc 45^0 = \frac{\sqrt{2}}{2}}$$
ans

45° ANGLE

Trigonometric function values for 45°

(0, 1)

90° $\frac{\pi}{2}$

0.7

$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

x y

1 h

o y

$\frac{\pi}{4}$

45° a

x

0.7

π
180°
(−1, 0)

0°
(1, 0)

$\frac{3\pi}{2}$ 270°

(0, −1)

$\sin \theta = \frac{o}{h}$ $\qquad \cos \theta = \frac{a}{h}$ $\qquad \tan \theta = \frac{o}{a}$

$\sin \theta = \frac{y}{1}$ $\qquad \cos \theta = \frac{x}{1}$ $\qquad \tan \theta = \frac{y}{x}$

$\underline{\sin \theta = y}$ $\qquad \underline{\cos \theta = x}$ $\qquad \tan \theta = \frac{\sin \theta}{\cos \theta}$

$\cos 45° = \frac{\sqrt{2}}{2}$

ans

$\sec 45° = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\sec 45° = \frac{\sqrt{2}}{2}$

ans

45° ANGLE

Trigonometric function values for 45°

$$\sin \theta = \frac{o}{h}$$

$$\cos \theta = \frac{a}{h}$$

$$\tan \theta = \frac{o}{a}$$

$$\sin \theta = \frac{y}{1}$$

$$\cos \theta = \frac{x}{1}$$

$$\tan \theta = \frac{y}{x}$$

$$\underline{\sin \theta = y}$$
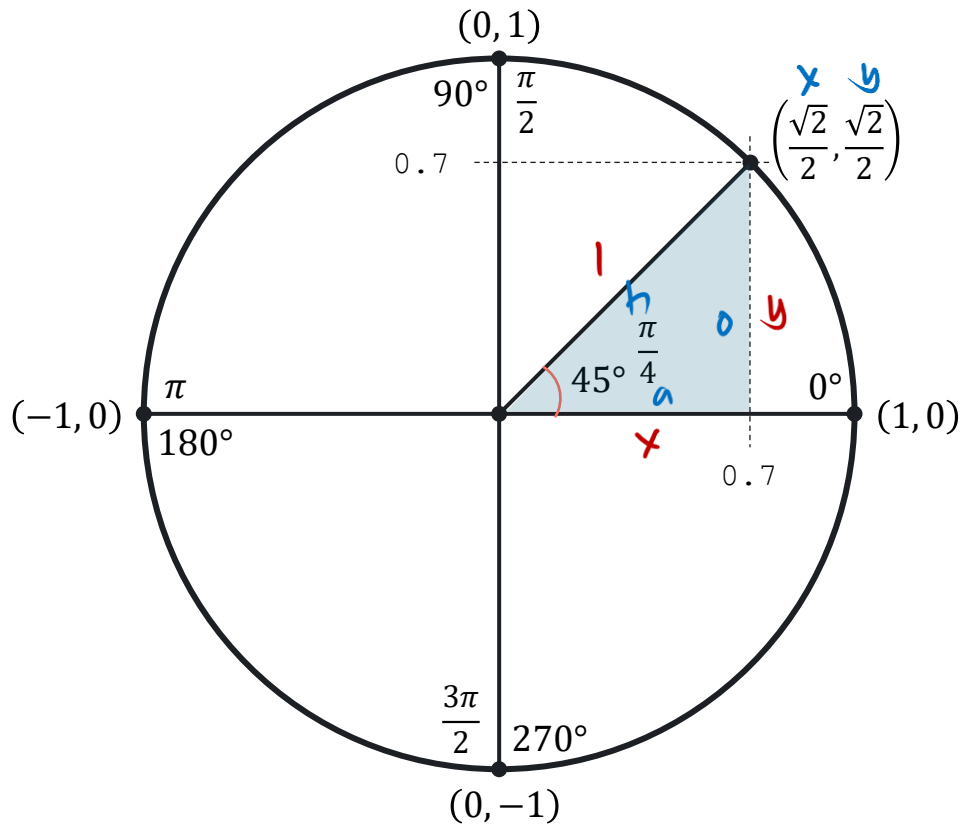
$$\underline{\cos \theta = x}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$
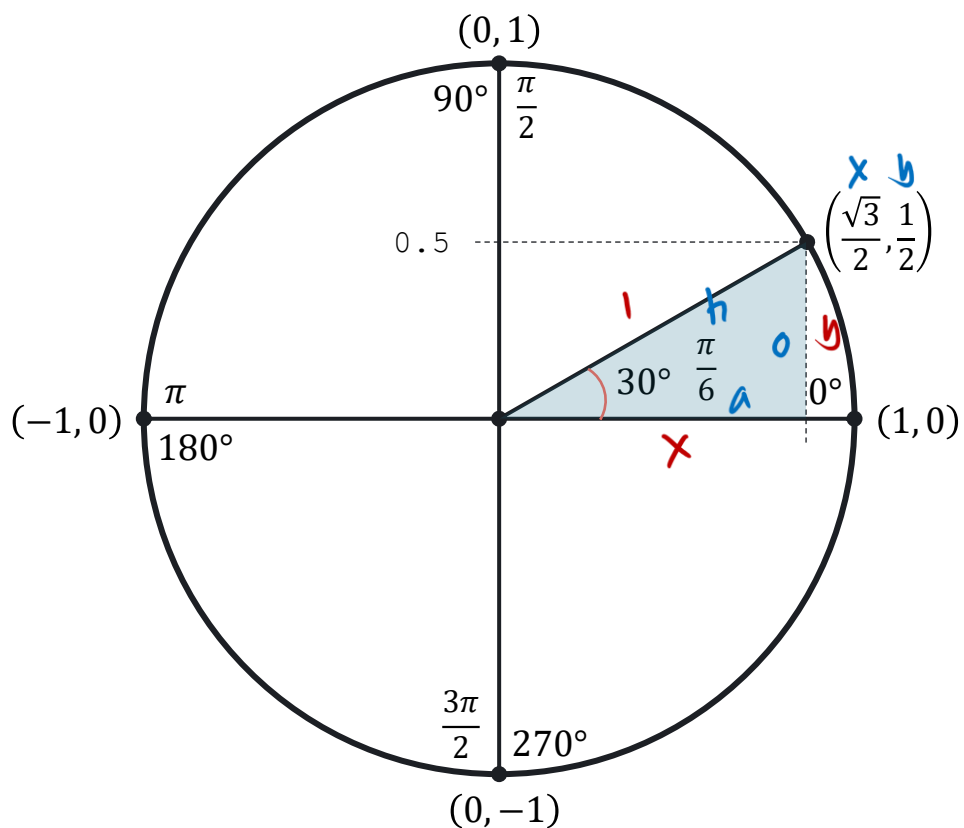
$$\tan 45° = \frac{\sqrt{2}/2}{\sqrt{2}/2}$$

$$\boxed{\tan 45° = 1}$$ ans

$$\boxed{\cot 45° = 1}$$ ans

30° ANGLE

Trigonometric function values for 30°

$$\sin \theta = \frac{o}{h} \qquad \cos \theta = \frac{a}{h} \qquad \tan \theta = \frac{o}{a}$$

$$\sin \theta = \frac{y}{1} \qquad \cos \theta = \frac{x}{1} \qquad \tan \theta = \frac{y}{x}$$

$$\underline{\sin \theta = y} \qquad \underline{\cos \theta = x} \qquad \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\boxed{\sin 30° = \frac{1}{2}}$$
ans

$$\boxed{\csc 30° = 2}$$
ans

30° ANGLE

Trigonometric function values for 30°

$$\sin \theta = \frac{o}{h} \qquad \cos \theta = \frac{a}{h} \qquad \tan \theta = \frac{o}{a}$$

$$\sin \theta = \frac{y}{1} \qquad \cos \theta = \frac{x}{1} \qquad \tan \theta = \frac{y}{x}$$

$$\sin \theta = y \qquad \cos \theta = x \qquad \tan \theta = \frac{\sin \theta}{\cos \theta}$$
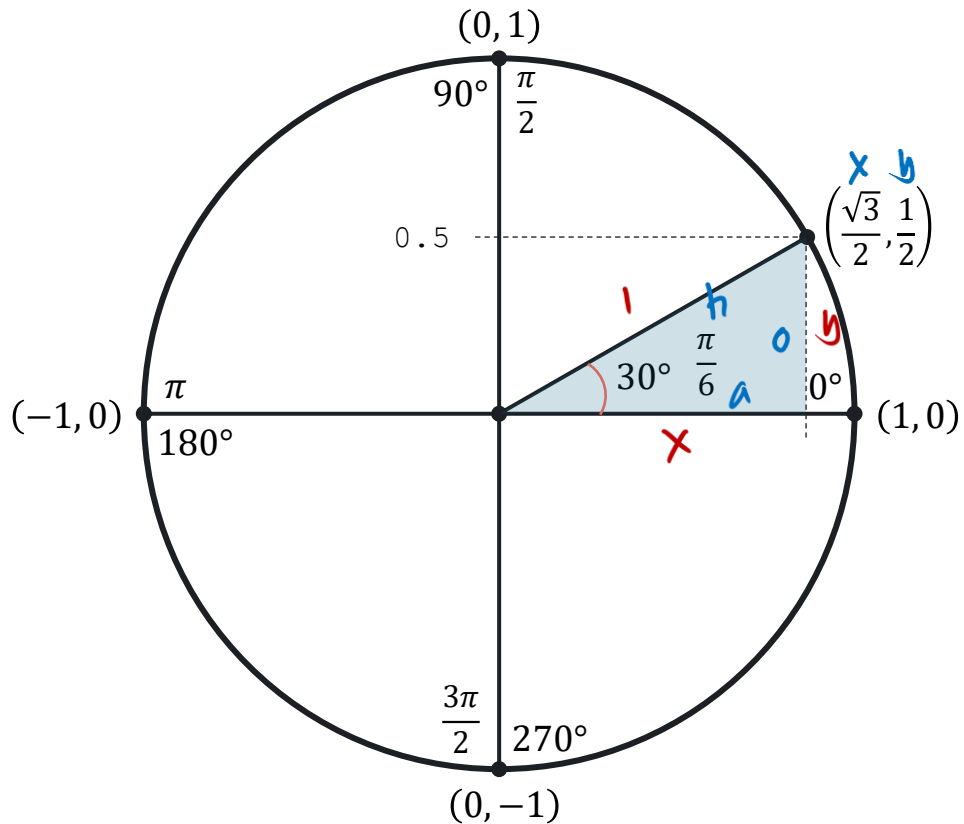
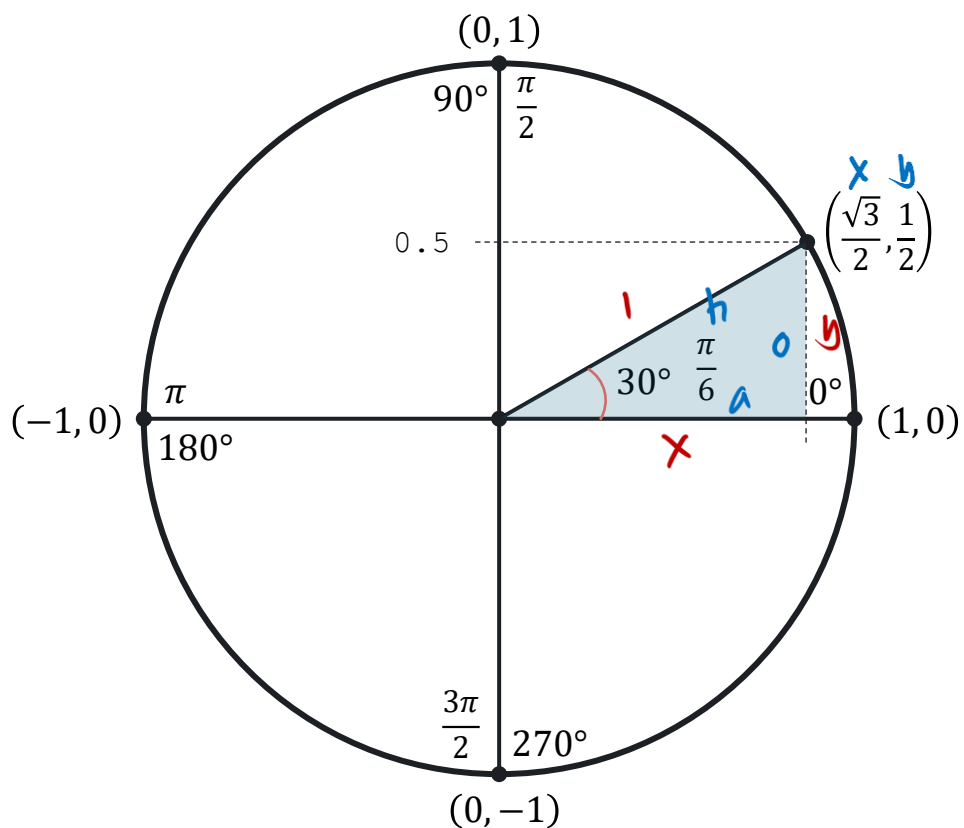$$\boxed{\cos 30° = \frac{\sqrt{3}}{2}}$$
ans

$$\sec 30° = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\boxed{\sec 30° = \frac{2\sqrt{3}}{3}}$$
ans

30° ANGLE

Trigonometric function values for 30°

$$\sin \theta = \frac{o}{h} \qquad \cos \theta = \frac{a}{h} \qquad \tan \theta = \frac{o}{a}$$

$$\sin \theta = \frac{y}{1} \qquad \cos \theta = \frac{x}{1} \qquad \tan \theta = \frac{y}{x}$$

$$\underline{\sin \theta = y} \qquad \underline{\cos \theta = x} \qquad \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\tan 30° = \frac{1/2}{\sqrt{3}/2}$$

$$\tan 30° = \frac{1}{2} \cdot \frac{2}{\sqrt{3}}$$

$$\tan 30° = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \longrightarrow \boxed{\cot 30° = \sqrt{3}}$$
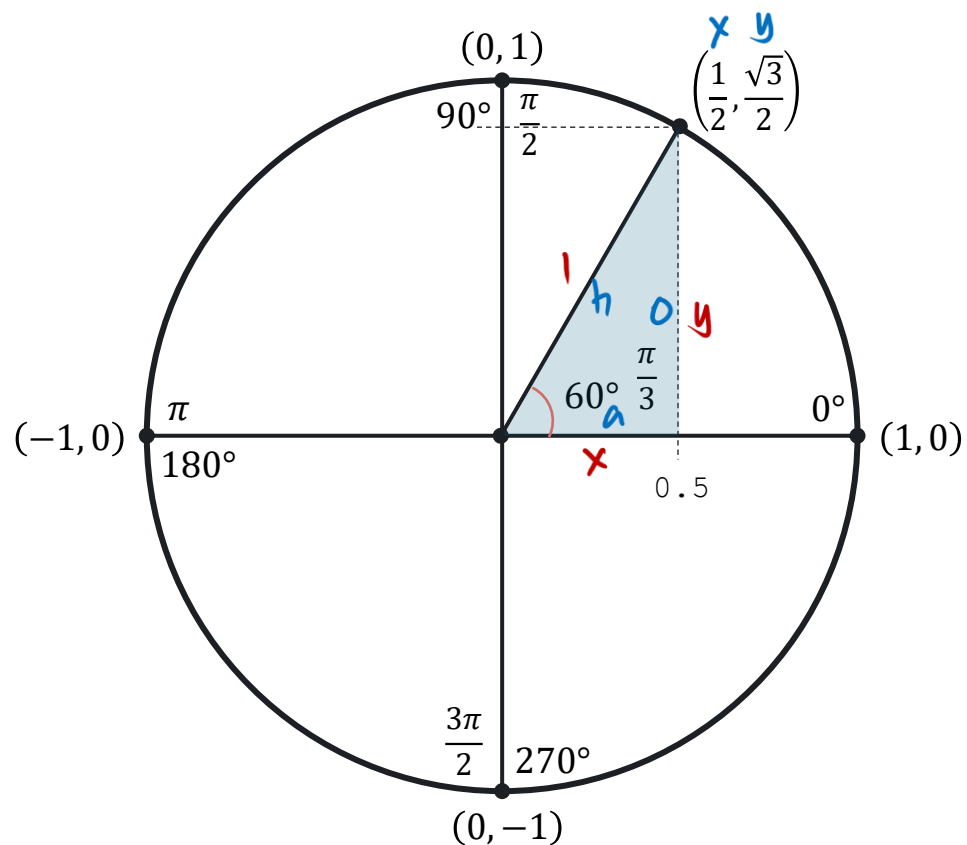
ans

$$\boxed{\tan 30° = \frac{\sqrt{3}}{3}}$$

ans

60° ANGLE

$$\sin\theta = \frac{o}{h}$$

$$\cos\theta = \frac{a}{h}$$

$$\tan\theta = \frac{o}{a}$$

$$\sin\theta = \frac{y}{1}$$

$$\cos\theta = \frac{x}{1}$$

$$\tan\theta = \frac{y}{x}$$

$$\underline{\sin\theta = y}$$

$$\underline{\cos\theta = x}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\boxed{\sin 60^0 = \frac{\sqrt{3}}{2}}$$
ans

$$\csc 60^0 = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\boxed{\csc 60^0 = \frac{2\sqrt{3}}{3}}$$
ans

60° ANGLE

(0, 1)

$\left(\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}\right)$

90° $\dfrac{\pi}{2}$

x y

1 h o y

60° $\dfrac{\pi}{3}$ a

π 180° (−1, 0)

x

0.5

0° (1, 0)

$\dfrac{3\pi}{2}$ 270°

(0, −1)

Trigonometric function values for 60°

$$\sin \theta = \frac{o}{h} \qquad \cos \theta = \frac{a}{h} \qquad \tan \theta = \frac{o}{a}$$

$$\sin \theta = \frac{y}{1} \qquad \cos \theta = \frac{x}{1} \qquad \tan \theta = \frac{y}{x}$$

$$\underline{\sin \theta = y} \qquad \underline{\cos \theta = x} \qquad \tan \theta = \frac{\sin \theta}{\cos \theta}$$
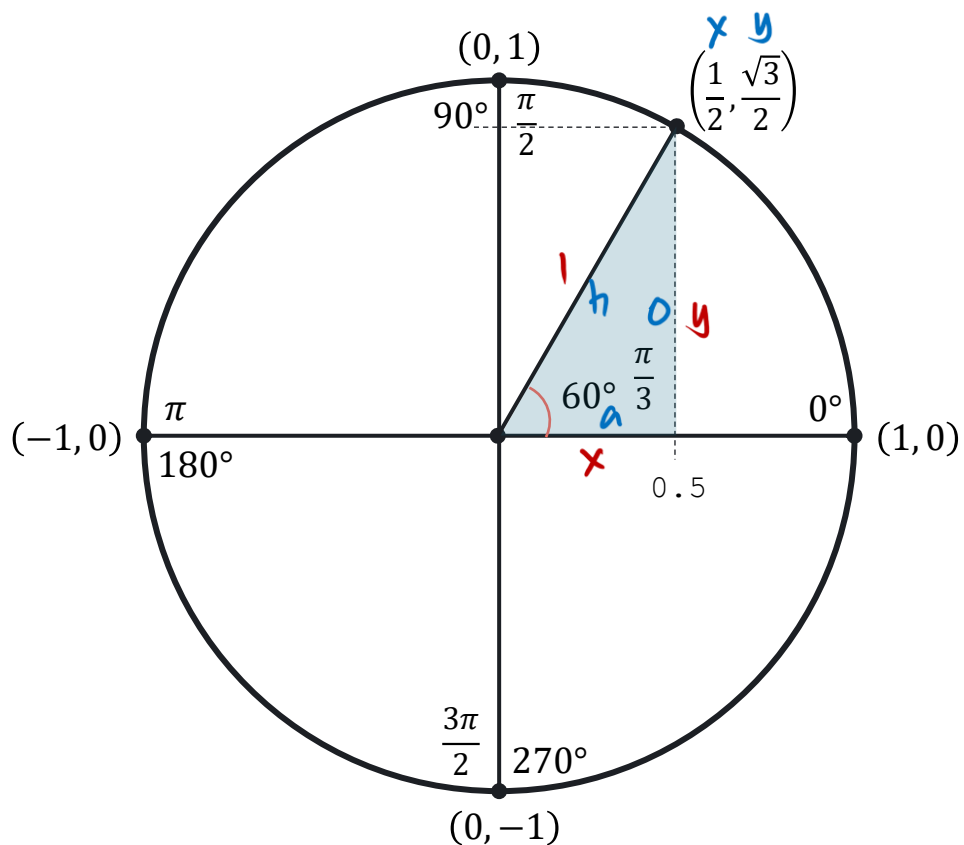
$$\boxed{\cos 60° = \frac{1}{2}}$$
ans

$$\boxed{\sec 60° = 2}$$
ans

60° ANGLE

Trigonometric function values for 60°

$$\sin \theta = \frac{o}{h} \qquad \cos\theta = \frac{a}{h} \qquad \tan\theta = \frac{o}{a}$$

$$\sin \theta = \frac{y}{1} \qquad \cos\theta = \frac{x}{1} \qquad \tan\theta = \frac{y}{x}$$

$$\underline{\sin \theta = y} \qquad \underline{\cos\theta = x} \qquad \tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\tan 60° = \frac{\sqrt{3}/2}{1/2} \qquad \cot 60° = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\tan 60° = \frac{\sqrt{3}}{\cancel{2}} \cdot \frac{\cancel{2}}{1} \qquad \boxed{\cot 60° = \frac{\sqrt{3}}{3}}$$
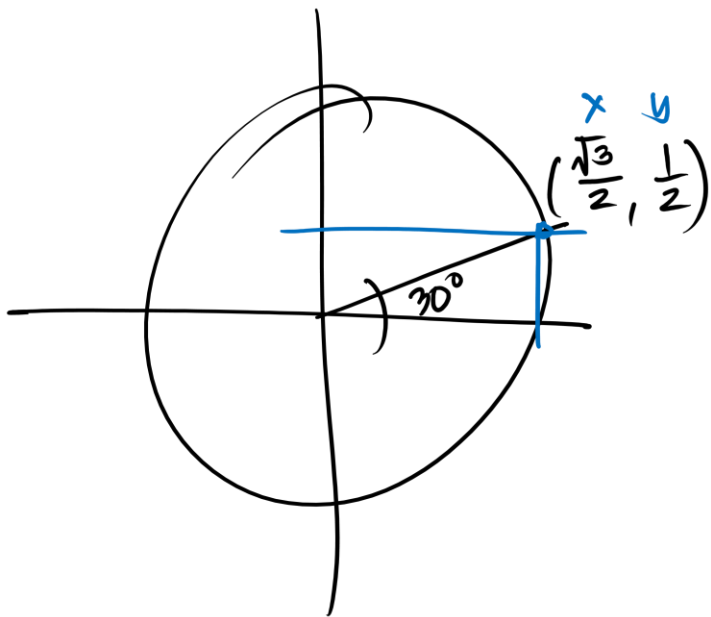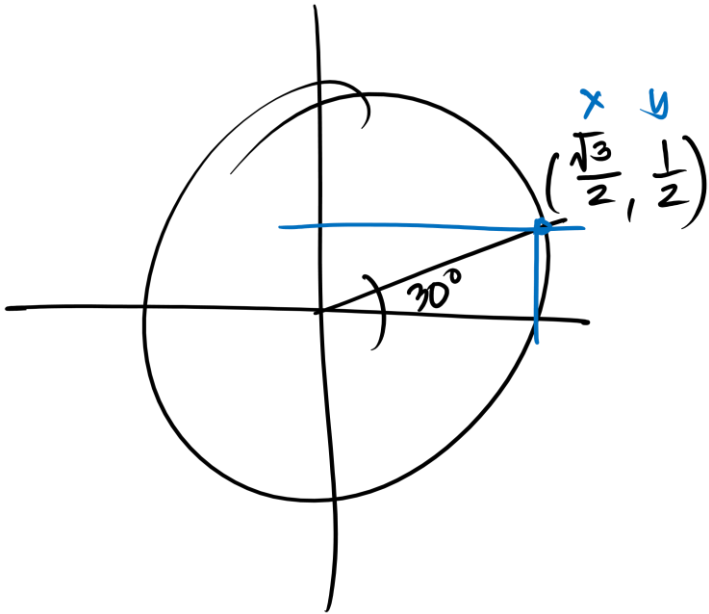
$$\boxed{\tan 60° = \sqrt{3}}$$

ans

ans

EXERCISE

Evaluate the expression without using a calculator.

a. $\sin 30° \csc 30°$

b. $\sin \dfrac{\pi}{6} + \cos \dfrac{\pi}{6}$

c. $\sin 30° \cos 60° + \sin 60° \cos 30°$

Solution

a. $\sin 30° \csc 30°$

$= \sin 30° \cdot \dfrac{1}{\sin 30°}$

$= \dfrac{1}{2} \cdot \dfrac{1}{\frac{1}{2}}$

$= \dfrac{1}{2} \cdot 2$

$= \boxed{1}$

ans

EXERCISE

Evaluate the expression without using a calculator.

a. $\sin 30° \csc 30°$

b. $\sin \dfrac{\pi}{6} + \cos \dfrac{\pi}{6}$ ⟶ $\dfrac{\pi}{6} rad = 30°$

c. $\sin 30° \cos 60° + \sin 60° \cos 30°$

<u>Solution</u>

b. $\sin \dfrac{\pi}{6} + \cos \dfrac{\pi}{6}$

$= \dfrac{1}{2} + \dfrac{\sqrt{3}}{2}$
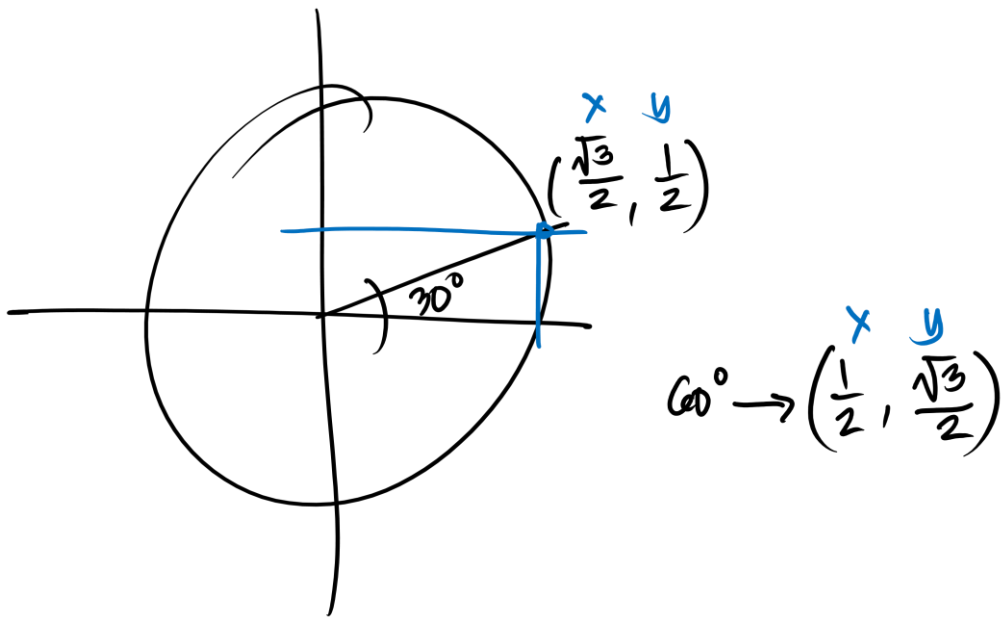
$= \boxed{\dfrac{1 + \sqrt{3}}{2}}$ ans

EXERCISE

Evaluate the expression without using a calculator.

a. $\sin 30° \csc 30°$

b. $\sin \dfrac{\pi}{6} + \cos \dfrac{\pi}{6}$

c. $\sin 30° \cos 60° + \sin 60° \cos 30°$

c. $\sin 30° \cos 60° + \sin 60° \cos 30°$

$= \dfrac{1}{2} \cdot \dfrac{1}{2} + \dfrac{\sqrt{3}}{2} \cdot \dfrac{\sqrt{3}}{2}$

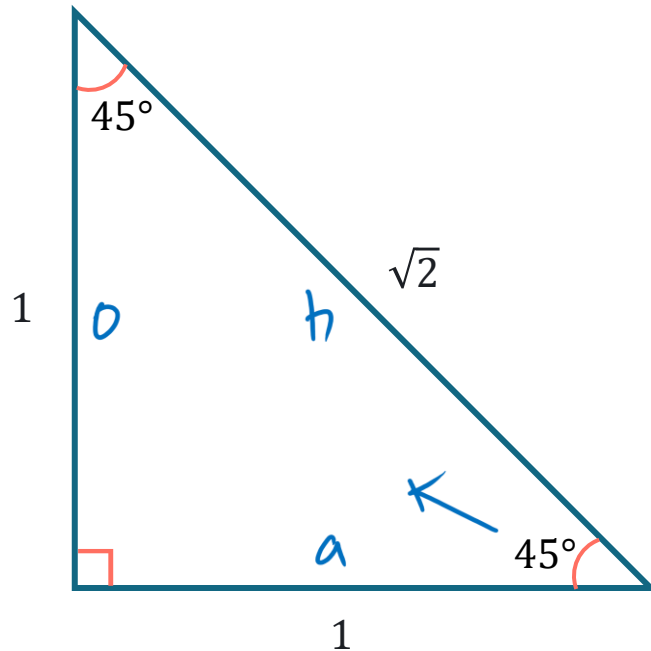$= \dfrac{1}{4} + \dfrac{3}{4}$

$= \dfrac{4}{4}$

$= \boxed{1}$ ans

SPECIAL RIGHT TRIANGLES

45-45-90 TRIANGLE

$$\sin 45^\circ = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$\boxed{\sin 45^\circ = \frac{\sqrt{2}}{2}}$$

ans

$$\cos 45^\circ = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$
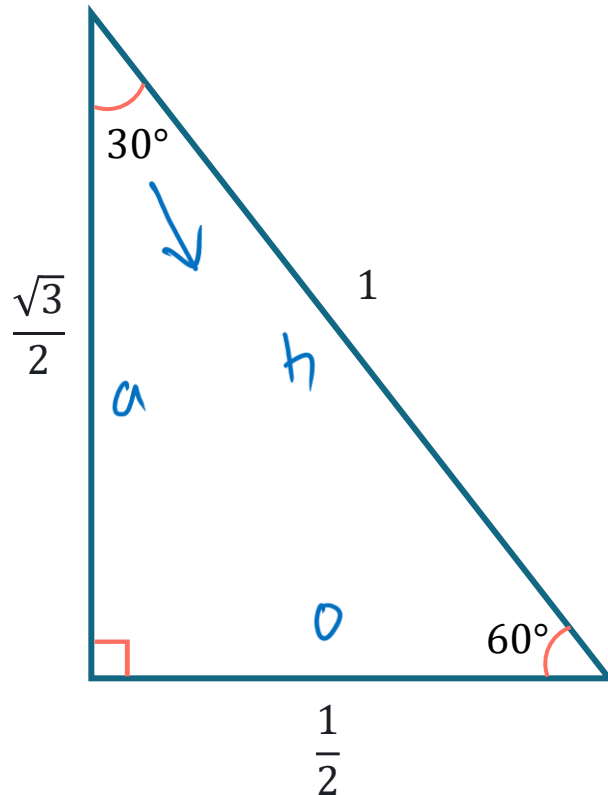
$$\boxed{\cos 45^\circ = \frac{\sqrt{2}}{2}}$$

ans

$$\tan 45^\circ = \frac{1}{1}$$

$$\boxed{\tan 45^\circ = 1}$$

ans

Triangle diagram: 45° at top, right angle at bottom left, 45° at bottom right. Left side labeled 1 (o), hypotenuse labeled $\sqrt{2}$ (h), bottom side labeled 1 (a).

30-60-90 TRIANGLE

Triangle with angles 30°, 60°, 90°. Sides labeled: $\frac{\sqrt{3}}{2}$ (left side, a), 1 (hypotenuse, h), $\frac{1}{2}$ (bottom, o).

Trigonometric function values for 30°

$$\sin 30° = \frac{1/2}{1}$$

$$\boxed{\sin 30° = \frac{1}{2}} \quad ans$$

$$\cos 30° = \frac{\sqrt{3}/2}{1}$$

$$\boxed{\cos 30° = \frac{\sqrt{3}}{2}} \quad ans$$
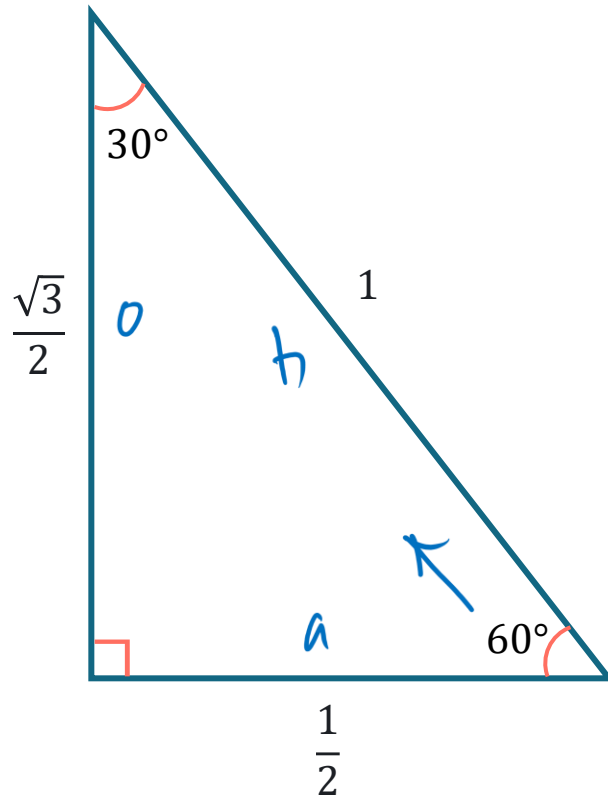
$$\tan 30° = \frac{1/2}{\sqrt{3}/2}$$

$$\tan 30° = \frac{1}{2} \cdot \frac{2}{\sqrt{3}}$$

$$\tan 30° = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\boxed{\tan 30° = \frac{\sqrt{3}}{3}} \quad ans$$

30-60-90 TRIANGLE

$$\sin 60° = \frac{\sqrt{3}/2}{1}$$

$$\boxed{\sin 60° = \frac{\sqrt{3}}{2}} \quad ans$$

$$\cos 60° = \frac{1/2}{1}$$

$$\boxed{\cos 60° = \frac{1}{2}} \quad ans$$
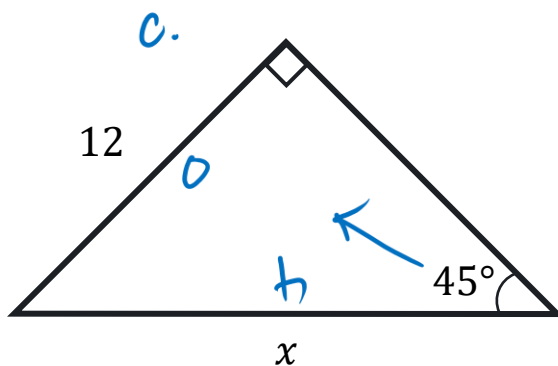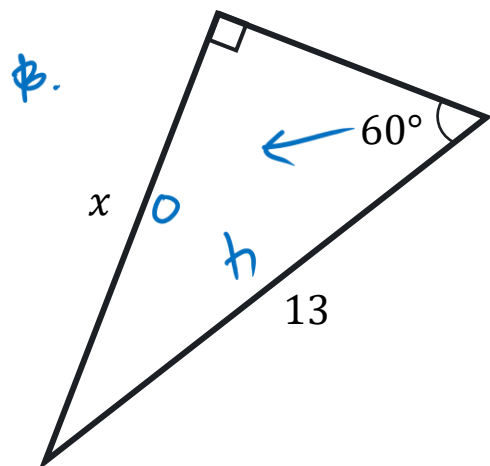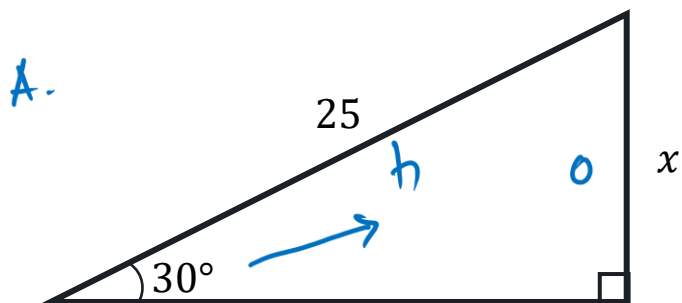
$$\tan 60° = \frac{\sqrt{3}/2}{1/2}$$

$$\tan 60° = \frac{\sqrt{3}}{2} \cdot \frac{2}{1}$$

$$\boxed{\tan 60° = \frac{\sqrt{3}}{2}} \quad ans$$

Triangle diagram:
- 30° (top angle)
- $\frac{\sqrt{3}}{2}$ (left side, labeled o)
- 1 (hypotenuse, labeled h)
- $\frac{1}{2}$ (bottom side, labeled a)
- 60° (bottom right angle)

EXERCISE

Find the side labeled x.

A.

25 · h · o · x · 30°

B.

x · o · h · 60° · 13

C.

12 · o · h · 45° · x

A. $\sin 30° = \dfrac{x}{25}$

$x = 25 \sin 30°$

$\boxed{x = 12.5}$ ans

B. $\sin 60° = \dfrac{x}{13}$

$x = 13 \sin 60°$

$\boxed{x = 11.26}$ ans

C. $\sin 45° = \dfrac{12}{x}$

$x = \dfrac{12}{\sin 45°}$

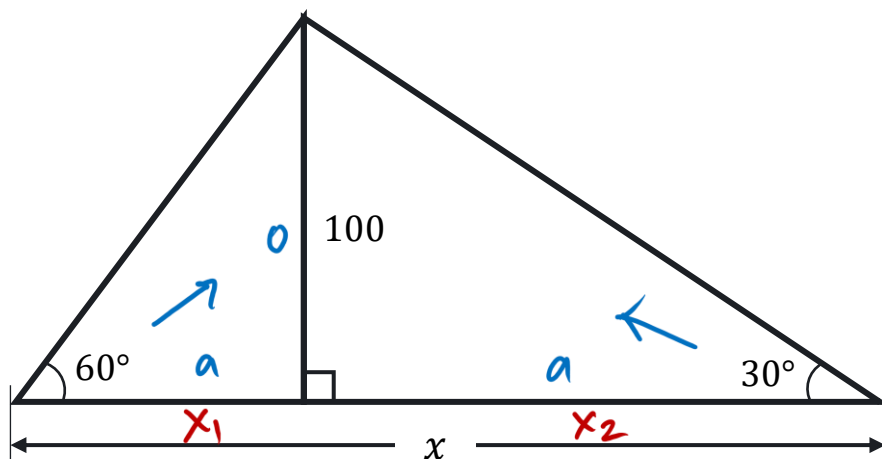$\boxed{x = 16.97}$ ans

EXERCISE

Find x correct to one decimal place.

$$X = X_1 + X_2$$

Solution

$$\sin 60° = \frac{100}{X_1}$$

$$X_1 = \frac{100}{\sin 60°}$$

$$\underline{X_1 = 115.47}$$

$$\sin 30° = \frac{100}{X_2}$$

$$X_2 = \frac{100}{\sin 30°}$$

$$\underline{X_2 = 200}$$

$$X = X_1 + X_2$$

$$X = 115.47 + 200$$

$$\boxed{X = 315.47}$$
$$ans$$

EXERCISE

Find x correct to one decimal place.

SEATWORK