DEGREE AND RADIAN

MEASURES OF ANGLES

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TOPIC OUTLINE

Degree

Radian

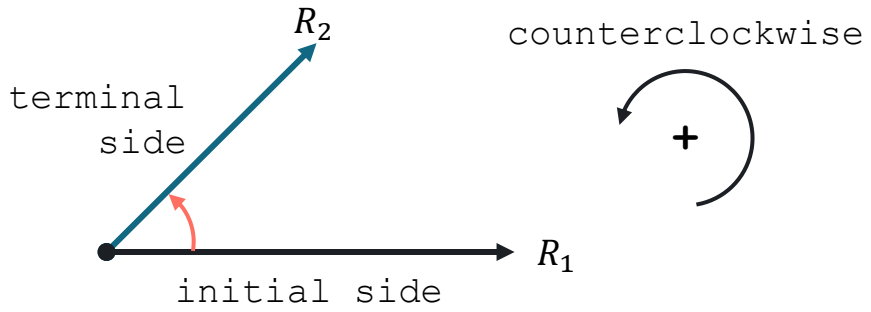
DEGREE

ANGLE

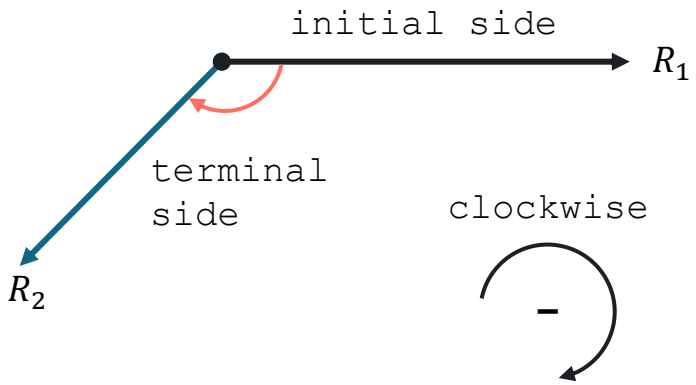
An **angle** is formed by two rays (R_1 and R_2) that share a common endpoint, called the vertex (O).

It can be interpreted as the **amount of rotation** from one ray (initial side) to another (terminal side) around the vertex.

Positive angle



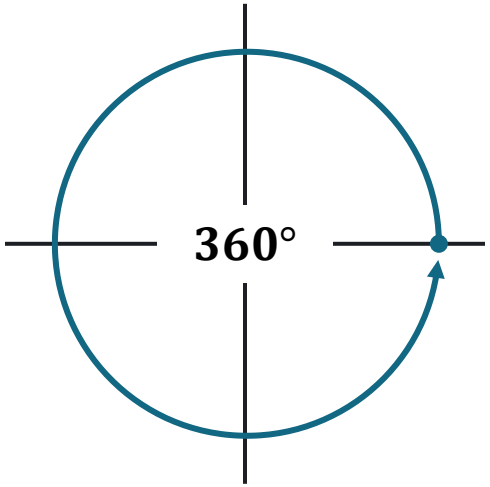
Negative angle



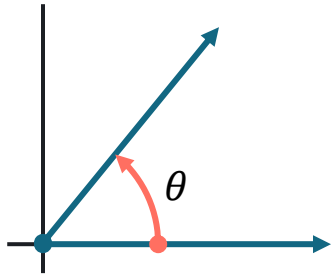
DEGREE

The **degree** (°) is the most commonly used unit for measuring angles.

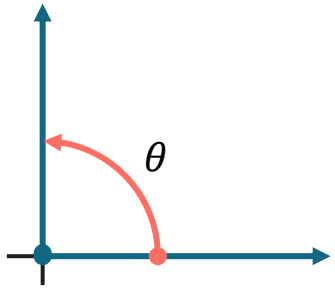
A full rotation around a circle corresponds to 360°.



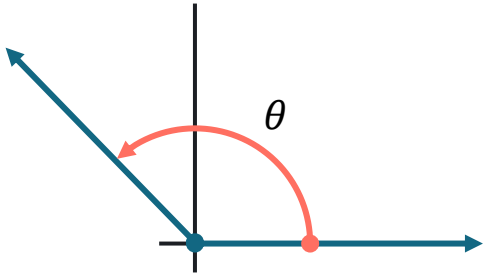
Classification of Angles



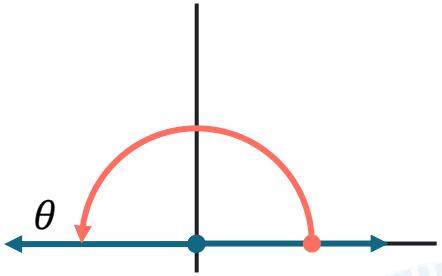
Acute angle
$0° < \theta < 90°$



Right angle
$\theta = 90°$



Obtuse angle
$90° < \theta < 180°$



Straight angle
$\theta = 180°$

COMPLEMENTARY ANGLES

If the sum of the measures of two positive angles is **90°**, the angles are **complementary** and the angles are complements of each other.

Formula

$$\theta_A + \theta_B = 90°$$

Example

Find the complement of an angle measuring 40°.

Let $\theta_A = 40°$

$\theta_A + \theta_B = 90°$

$40° + \theta_B = 90°$

$-40°$ $\quad$ $-40°$

$\theta_B = 50°$

ans

50° is a complement of 40°

50° and 40° are complementary angles

SUPPLEMENTARY ANGLES

If the sum of the measures of two positive angles is **180°**, the angles are **supplementary** and the angles are supplements of each other.

Formula

$$\theta_A + \theta_B = 180°$$

Example

Find the supplement of an angle measuring 40°.

Let $\theta_A = 40°$

$\theta_A + \theta_B = 180°$

$40° + \theta_B = 180°$

$-40° \qquad -40°$

$\theta_B = 140°$

ans

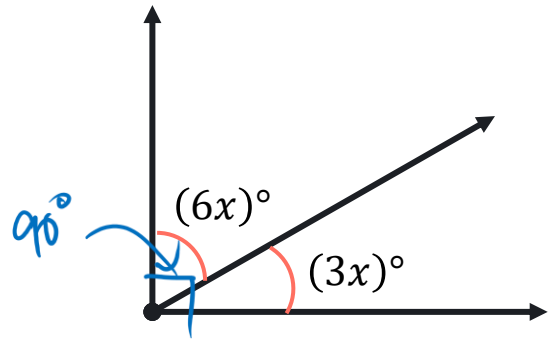
140° is a supplement of 40°

140° and 40° are supplementary angles

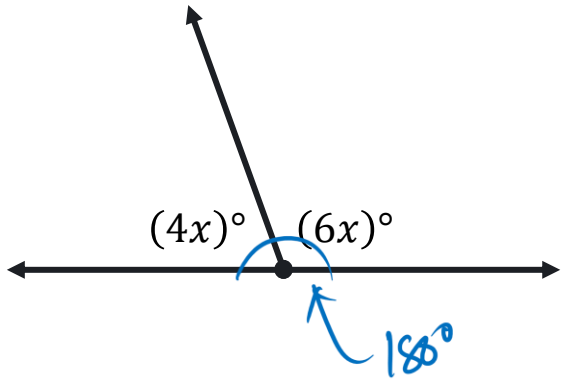
EXERCISE

Find the measure of each marked angle.

A.



B.



Solution

A. Complementary Angles

$6x + 3x = 90°$

$\frac{9x}{9} = \frac{90°}{9}$

$x = 10°$

$6x = 60°$ ans

$3x = 30°$ ans

B. Supplementary Angles

$4x + 6x = 180°$

$\frac{10x}{10} = \frac{180°}{10}$

$x = 18°$

$4x = 72°$ ans

$6x = 108°$ ans

DEGREES, MINUTES, SECONDS

One **minute** ($1'$) is $\frac{1}{60}$ of a degree.

$$1' = \frac{1^\circ}{60}$$

$$60' = 1^\circ$$

One **second** ($1''$) is $\frac{1}{60}$ of a minute.

$$1'' = \frac{1'}{60}$$

$$60'' = 1'$$

$$3600'' = 1^\circ$$

Example

Convert $74°08'14''$ to decimal degrees to the nearest thousandth.

$$74^\circ + 8' \cdot \frac{1^\circ}{60'} + 14'' \cdot \frac{1'}{60''} \cdot \frac{1^\circ}{60'}$$

min → deg sec → min → deg

$$\boxed{74.137^\circ}$$

ans

EXERCISE

Perform each calculation and express the result in degrees, rounded to the nearest thousandth.

a. $51°29' + 32°46'$

b. $90° - 73°12'$

b.

$$\begin{array}{r} 89\ 5 \\ \cancel{90}^{\circ}\ \cancel{00}' \\ -\ 73^{\circ}\ 12' \\ \hline 16^{\circ}\ 48' \end{array} \longrightarrow 90^{\circ} = 89^{\circ}\,60'$$

$16°\,48'$

ans

Solution

a.

$$\begin{array}{r} {\scriptstyle 1\ 1} \\ 51^{\circ}\ 29' \\ +\ 32^{\circ}\ 46' \\ \hline 84^{\circ}\ 15' \end{array}$$

75

-60 ← 60 min (MAX)

15

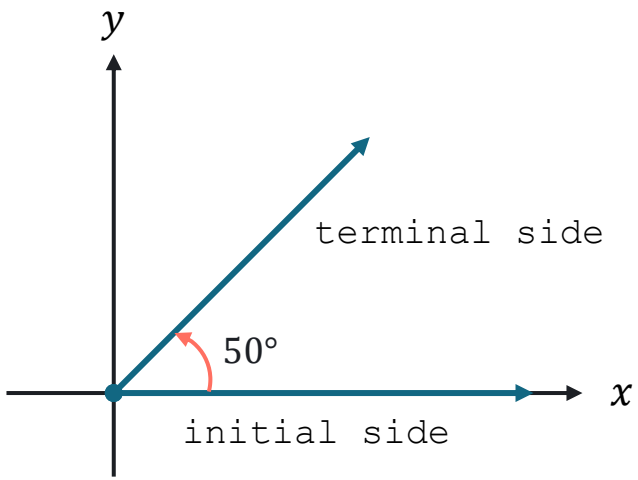
$84°\,15'$

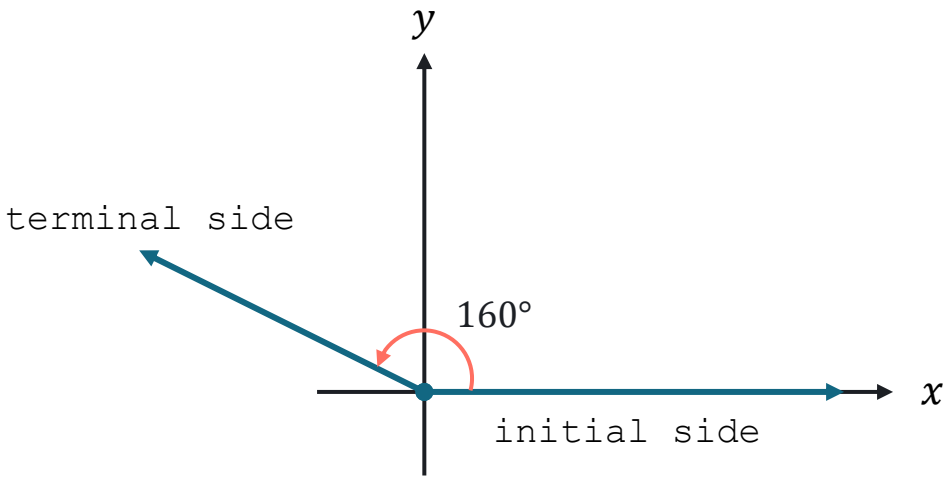
ans

STANDARD POSITION

An angle is in **standard position** if its vertex is at the **origin** and its initial side lies on the **positive x-axis**.

Angles in Standard Position

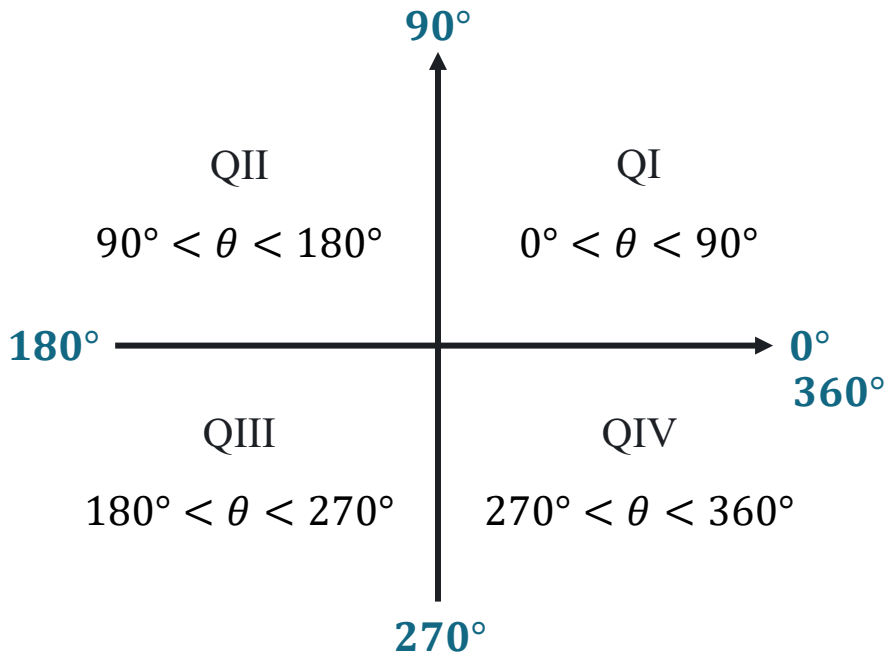




QUADRANTAL ANGLES

Angles in standard position whose terminal sides lie on the x-axis or y-axis, such as angles with measures **90°**, **180°**, **270°**, and so on, are **quadrantal angles**.

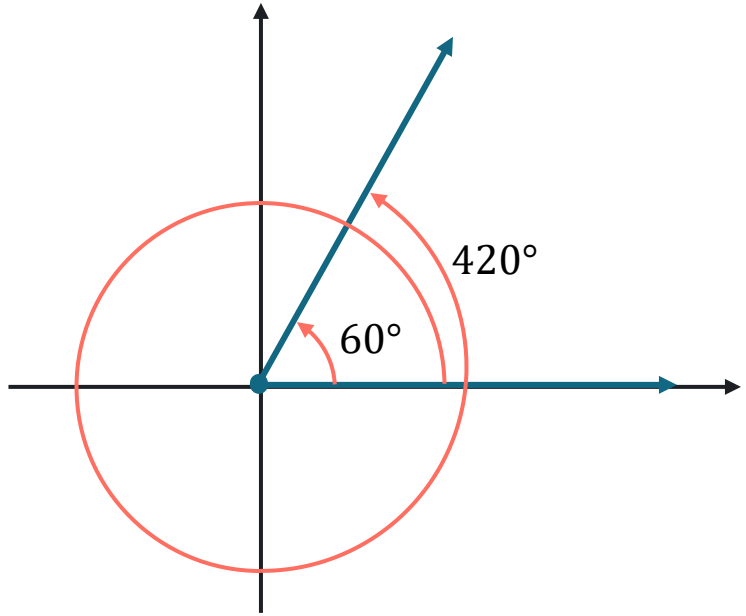
Quadrantal Angles



COTERMINAL ANGLES

Coterminal angles are angles that share the same terminal side when drawn in standard position. Their measures differ by a **multiple of 360°**, meaning they can be found by adding or subtracting 360° repeatedly.

Coterminal Angles



EXERCISE

Find the angle of least positive measure that is coterminal with each angle.

a. 908°

b. −75°

c. −800°

Solution

a. $\theta = 908^\circ - 2(360^\circ)$

$\theta = 188^\circ$

ans

b. $\theta = -75^\circ + 360^\circ$

$\theta = 285^\circ$

ans

c. $\theta = -800 + 3(360^\circ)$

$\theta = 280^\circ$

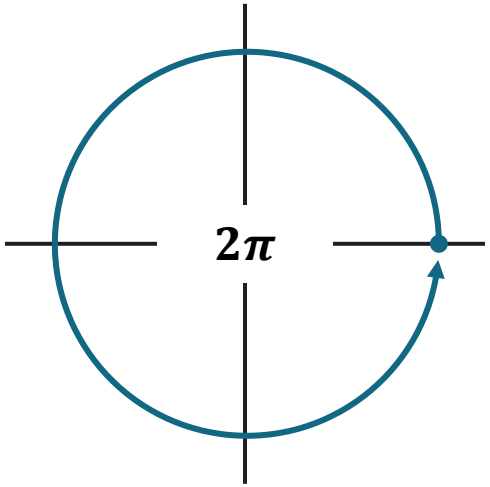
ans

RADIAN

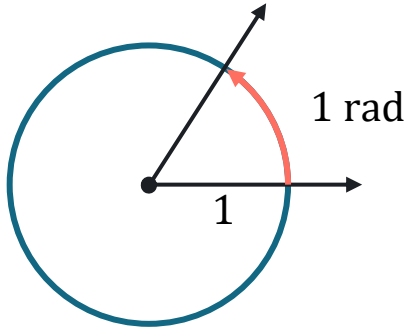
DEGREE

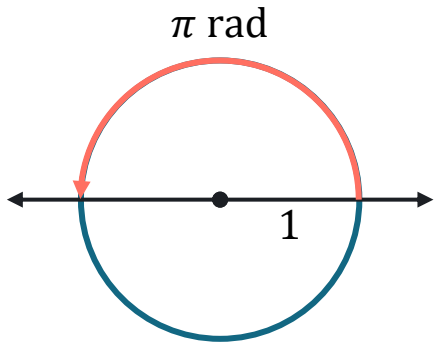
The **radian** (**rad**) is the angle subtended at the center of a circle by an arc whose length is equal to the radius of a circle.

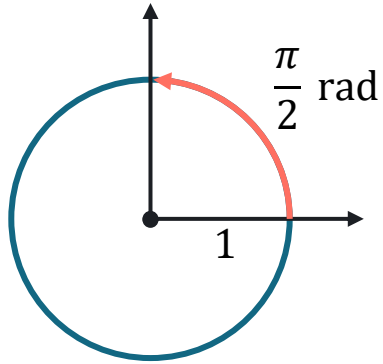
A full rotation around a circle corresponds to 2π radians.

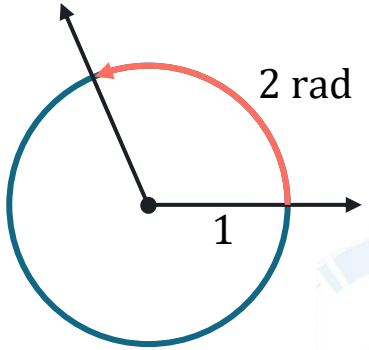


Unit Circle









EXERCISE

Find the radian measure of the angle with the given degree measure.

a. 72°

b. −60°

Solution

a. $\theta = 72^{\circ} \dfrac{\pi \text{ rad}}{180^{\circ}}$

$$\theta = \frac{2\pi}{5} \text{ rad}$$

ans

b. $\theta = -60^{\circ} \dfrac{\pi \text{ rad}}{180^{\circ}}$

coterminal angles

$\theta = 300^{\circ} \dfrac{\pi \text{ rad}}{180^{\circ}}$

$$\theta = \frac{5\pi}{3} \text{ rad}$$

ans

EXERCISE

Find the degree measure of the angle with the given radian measure.

$a.\ \dfrac{7\pi}{6}$

$b.\ -\dfrac{5\pi}{4}$

Solution

a. $\theta = \dfrac{7\pi \text{ rad}}{6} \dfrac{180^\circ}{\pi \text{ rad}}$

$\theta = 210^\circ$

ans

b. $\theta = -\dfrac{5\pi \text{ rad}}{4} \dfrac{180^\circ}{\pi \text{ rad}}$

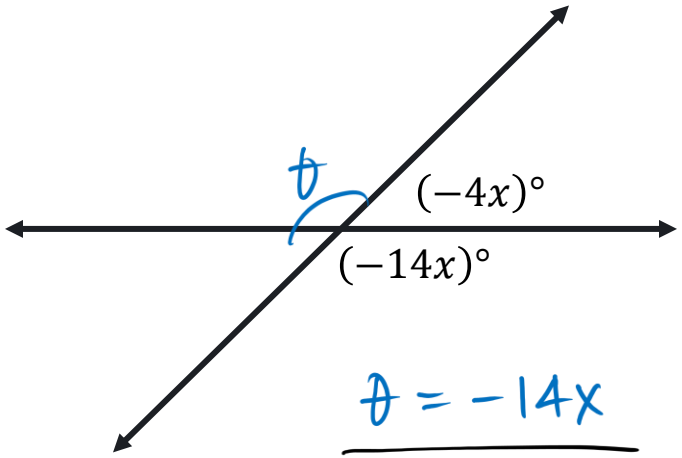
$\theta = -225^\circ$

$\theta = 135^\circ$

ans

EXERCISE

Find the measure of the marked angle.



$\theta = -14x$

Vertical angles

Solution

$-14x - 4x = 180^\circ$

$\frac{-18x}{-18} = \frac{180^\circ}{-18}$

$x = -10^\circ$

$-4x = 40^\circ$ ans

$-14x = 140^\circ$ ans

EXERCISE

A constant angular velocity disk drive spins a disk at a constant speed. Suppose a disk makes 480 revolutions per min. Through how many degrees will a point on the edge of the disk move in 2 sec?

1 rev = 360°

$\frac{\text{rev}}{\text{min}} \longrightarrow \frac{\text{deg}}{\text{sec}}$

Solution

Angular distance

$$W = \frac{480 \text{ rev}}{\text{min}} \cdot \frac{360^\circ}{1 \text{ rev}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \cdot 2 \text{ sec}$$

$$W = 5760^\circ$$

ans

SEATWORK