CAPACITOR

TRANSIENT RESPONSE

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TOPIC OUTLINE

RC Circuit

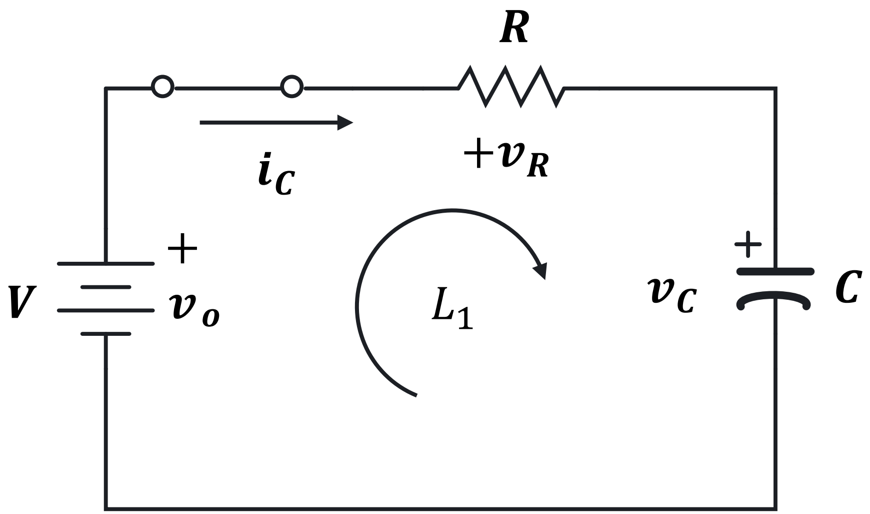
Charging a Capacitor

Discharging a Capacitor

Transient Response

CHARGING A CAPACITOR

RC CIRCUIT



KVL @ L_1:

$$-v_o + v_R + v_C = 0$$

$$v_R + v_C = v_o$$

$$i_C R + v_C = v_o \quad ; i_C = C\frac{d}{dt}v_C$$

$$RC\frac{d}{dt}v_C + v_C = v_o$$

$$\frac{d}{dt}v_C + \frac{1}{RC}v_C = \frac{v_o}{RC}$$

... first-order ODE

$$v_C(t) = v_o\left(1 - e^{-\frac{t}{RC}}\right)$$

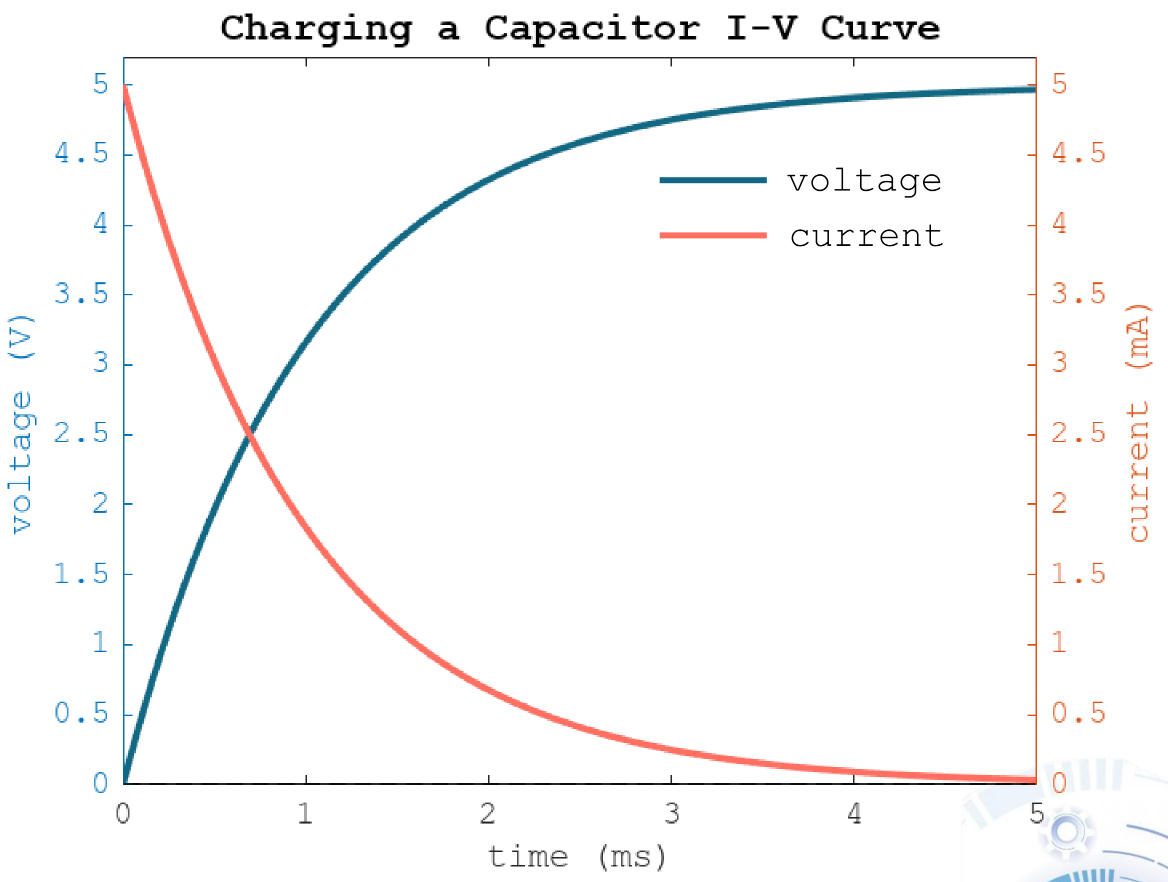
CAPACITOR VOLTAGE

Charging equation:

$$v_c(t) = v_o\left(1 - e^{-\frac{t}{\tau}}\right)$$

where: $\tau = RC$

The **voltage** across the capacitor **starts at zero** and exponentially increases to $\boldsymbol{v_o}$ volts (source voltage).



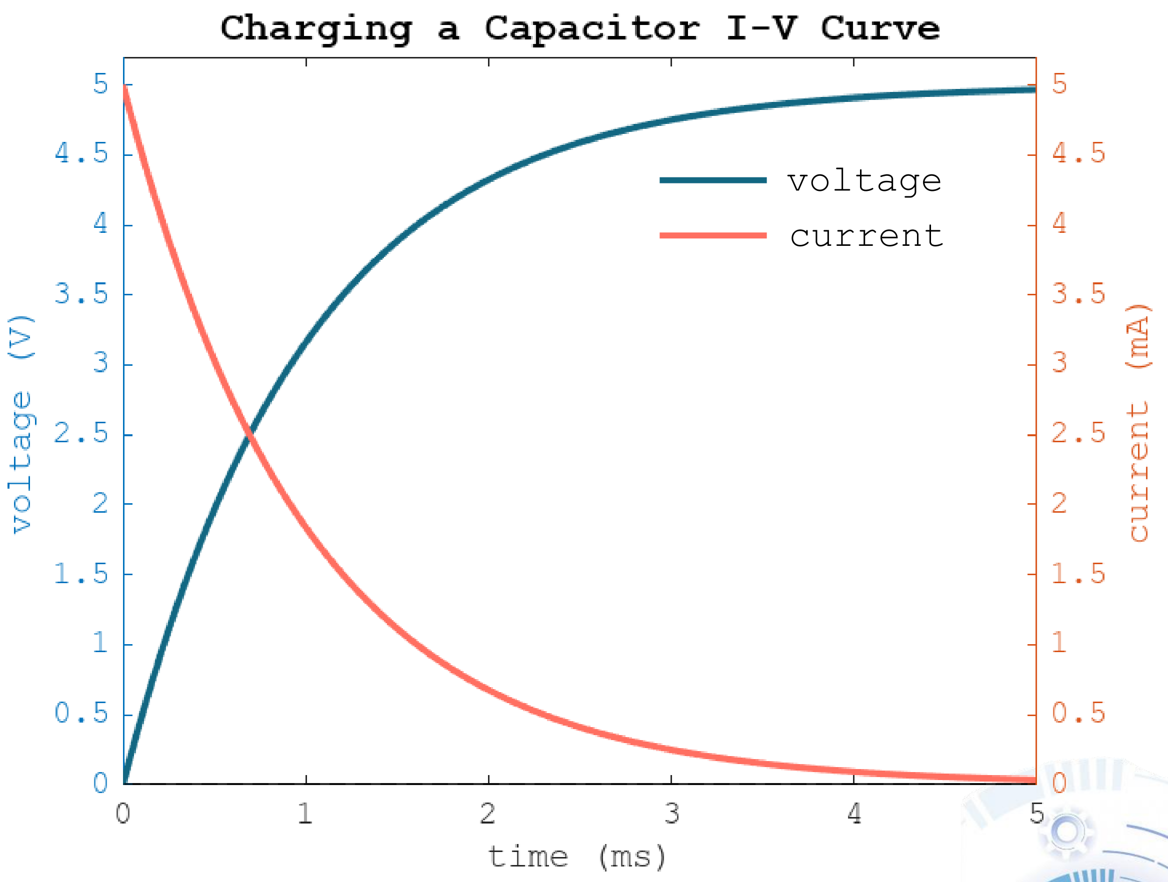
CAPACITOR CURRENT

Charging equation:

$$i_c(t) = \frac{v_o}{R} e^{-\frac{t}{\tau}}$$

where: $\tau = RC$

The **current** through the capacitor instantly jumps to its **maximum value** of $\frac{v_o}{R}$ amperes then decays exponentially to zero.



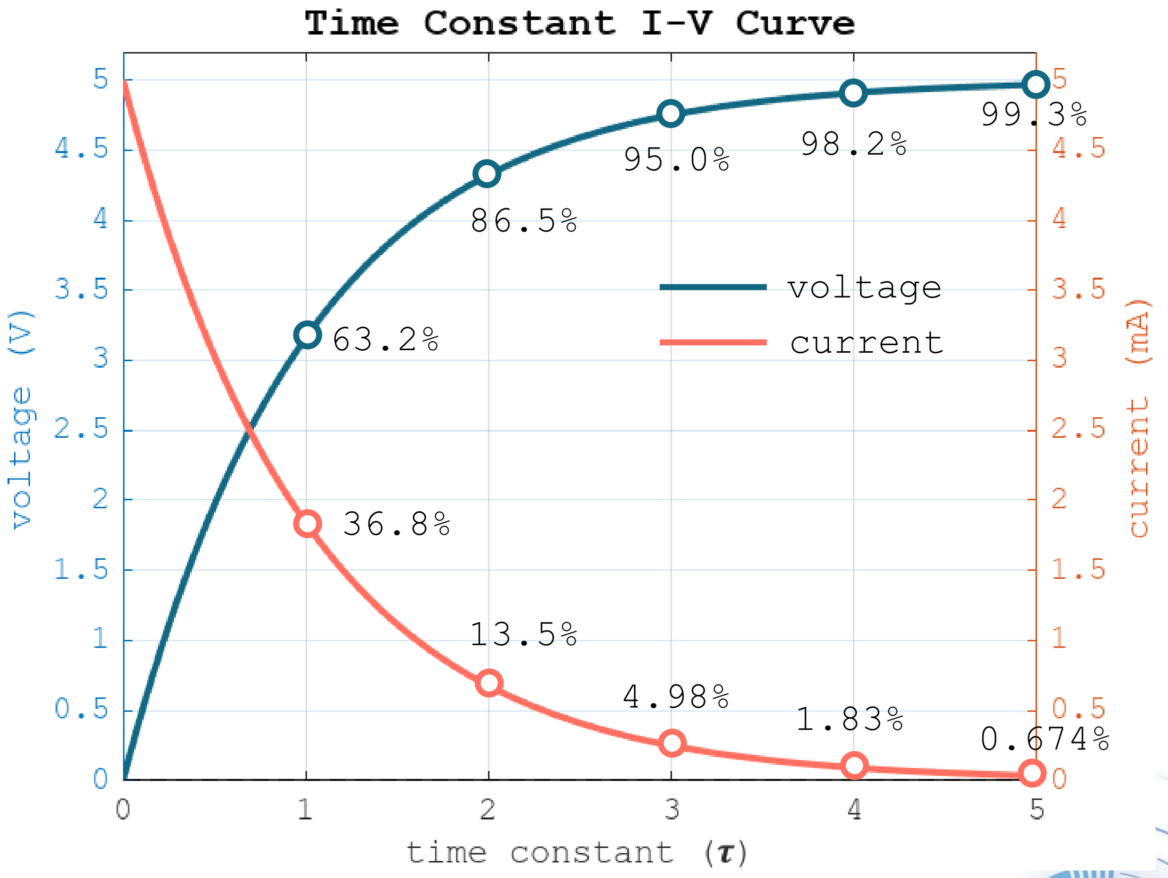
TIME CONSTANT

The time constant τ is a measure of how quickly a capacitor charges or discharges in an RC circuit.

Formula:

$$\tau = RC$$

unit: second



EXERCISE

A $\mathbf{100}\ \boldsymbol{\mu F}$ capacitor is connected to a $\mathbf{12}\ \boldsymbol{V}$ DC power supply through a resistor of $\mathbf{1}\ \boldsymbol{K\Omega}$. Determine the **time** it takes for the capacitor to charge to $\mathbf{86.5}\%$ of its maximum voltage.

Solution:

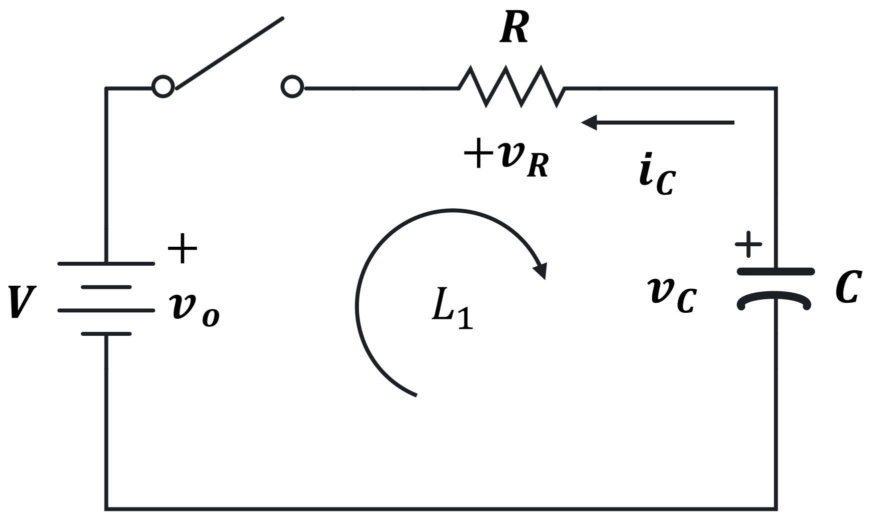
EXERCISE

A $\mathbf{100}\ \boldsymbol{\mu F}$ capacitor is connected to a $\mathbf{12}\ \boldsymbol{V}$ DC power supply through a resistor of $\mathbf{1}\ \boldsymbol{K\Omega}$. Determine the **voltage** across the capacitor after $\mathbf{200}\ \boldsymbol{ms}$ of charging.

Solution:

DISCHARGING A CAPACITOR

RC CIRCUIT



KVL @ L_1:

$$v_R + v_C = 0$$

$$i_C R + v_C = 0 \qquad ; i_C = C\frac{d}{dt}v_C$$

$$RC\frac{d}{dt}v_C + v_C = 0$$

$$\frac{d}{dt}v_C + \frac{1}{RC}v_C = 0$$

... first-order ODE

$$v_C(t) = v_o e^{-\frac{t}{RC}}$$

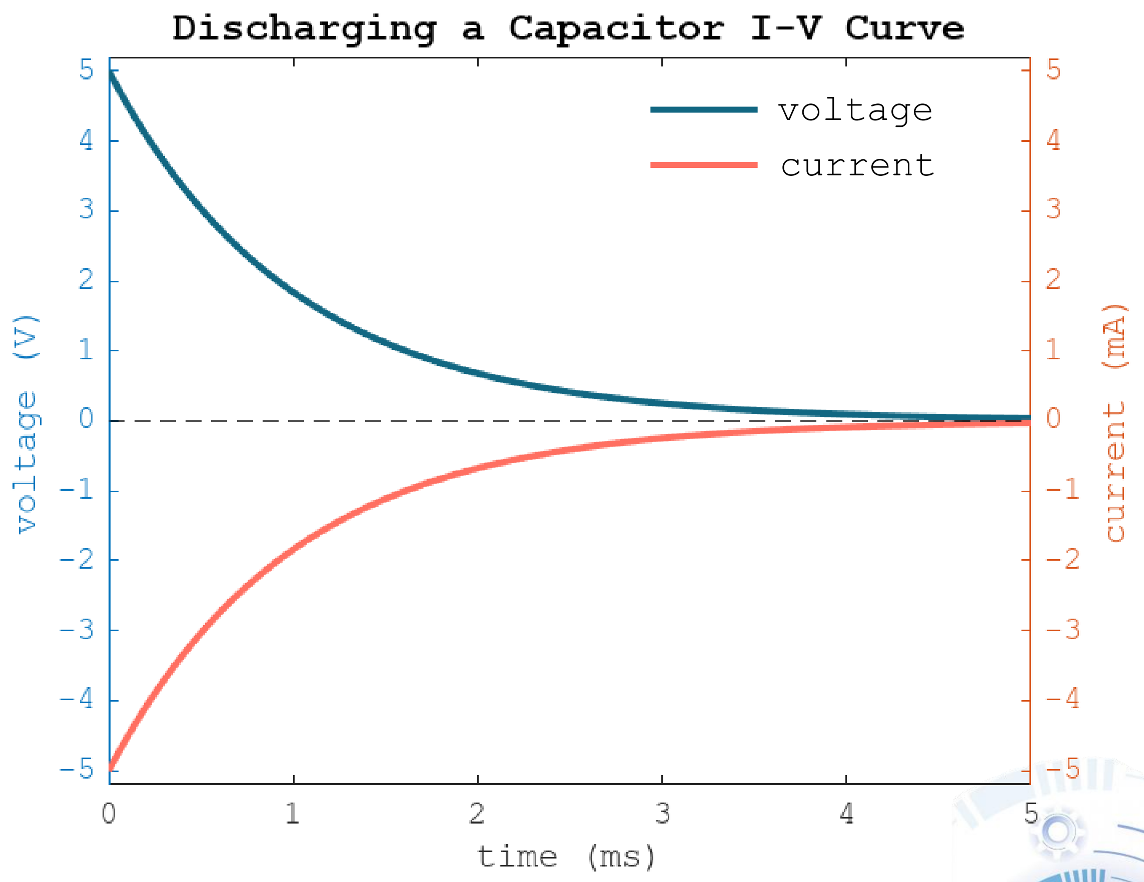
CAPACITOR VOLTAGE

Discharging equation:

$$v_c(t) = v_o e^{-\frac{t}{\tau}}$$

where: $\tau = RC$

The **voltage** across the capacitor starts at its maximum voltage $\boldsymbol{v_o}$ then decays exponentially to **zero**.



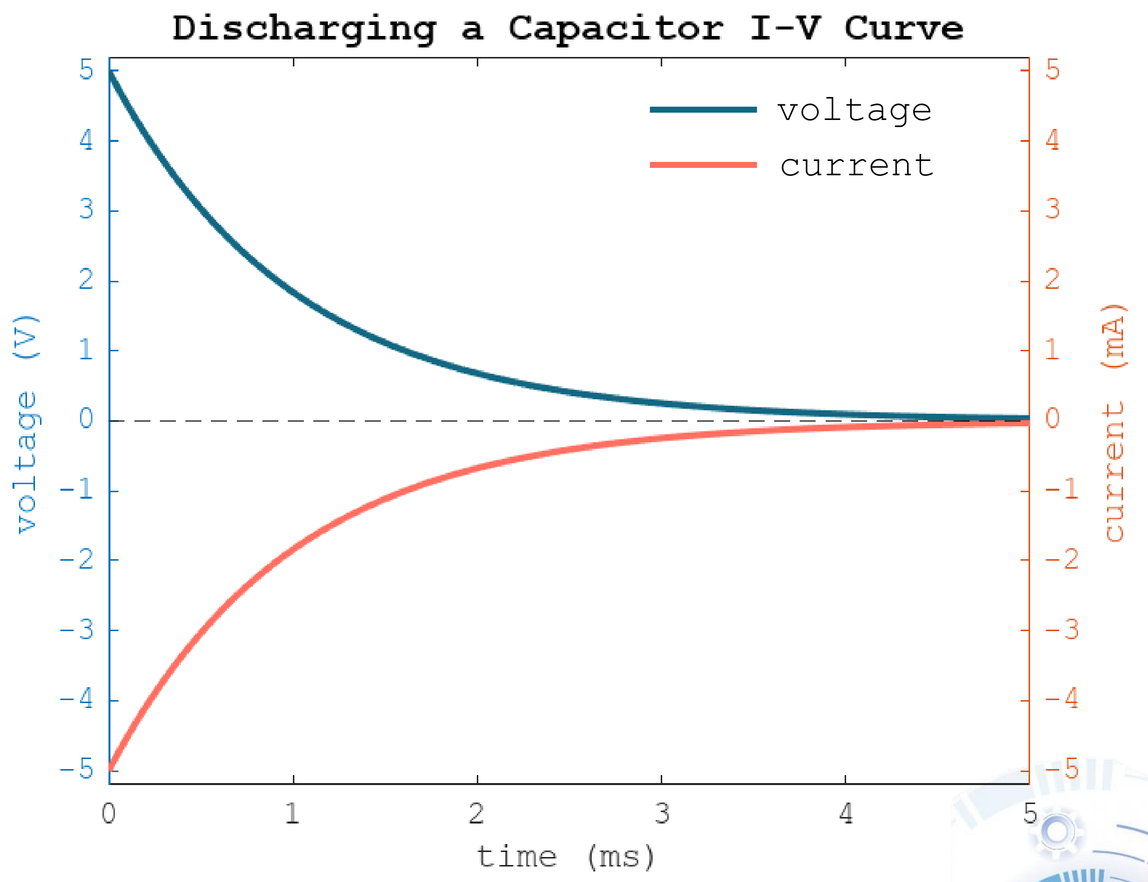
CAPACITOR CURRENT

Discharging equation:

$$i_c(t) = -\frac{v_o}{R}\left(e^{-\frac{t}{\tau}}\right)$$

where: $\tau = RC$

The **current** through the capacitor instantly jumps to its maximum value, but in opposite direction of $-\frac{v_o}{R}$ then decays exponentially to **zero**.

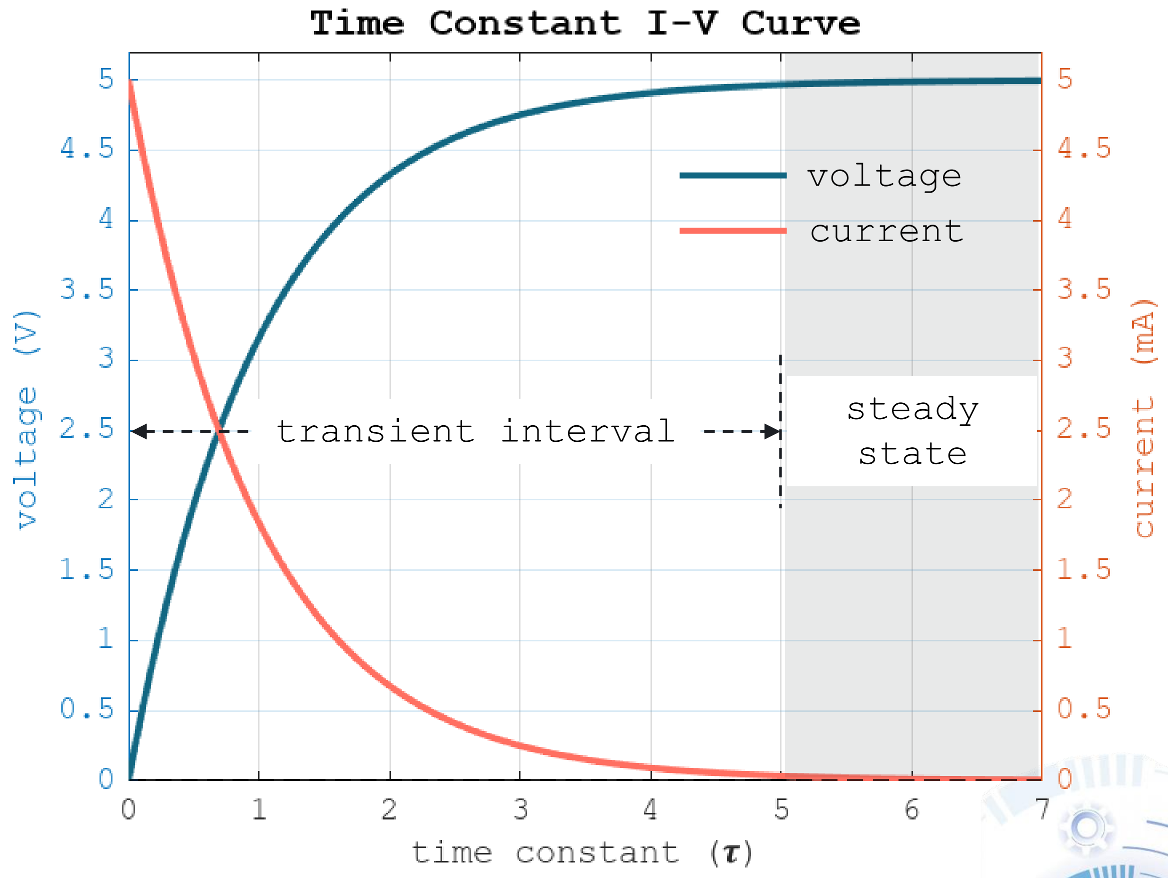


EXERCISE

A $\mathbf{200}\ \boldsymbol{\mu F}$ capacitor is initially charged to $\mathbf{12}\ \boldsymbol{V}$. It is then disconnected from the power supply and discharged a resistor of $\mathbf{1.5}\ \boldsymbol{K\Omega}$. Determine the **voltage** across the capacitor after $\mathbf{0.1}\ \boldsymbol{s}$ of discharging.

TRANSIENT RESPONSE

The **transient response** of a capacitor describes the time-dependent changes in voltage across the capacitor and the current through it. The transient phase is typically considered to last for approximately **five time constants** 5τ after which the system is assumed to have reached steady-state conditions.



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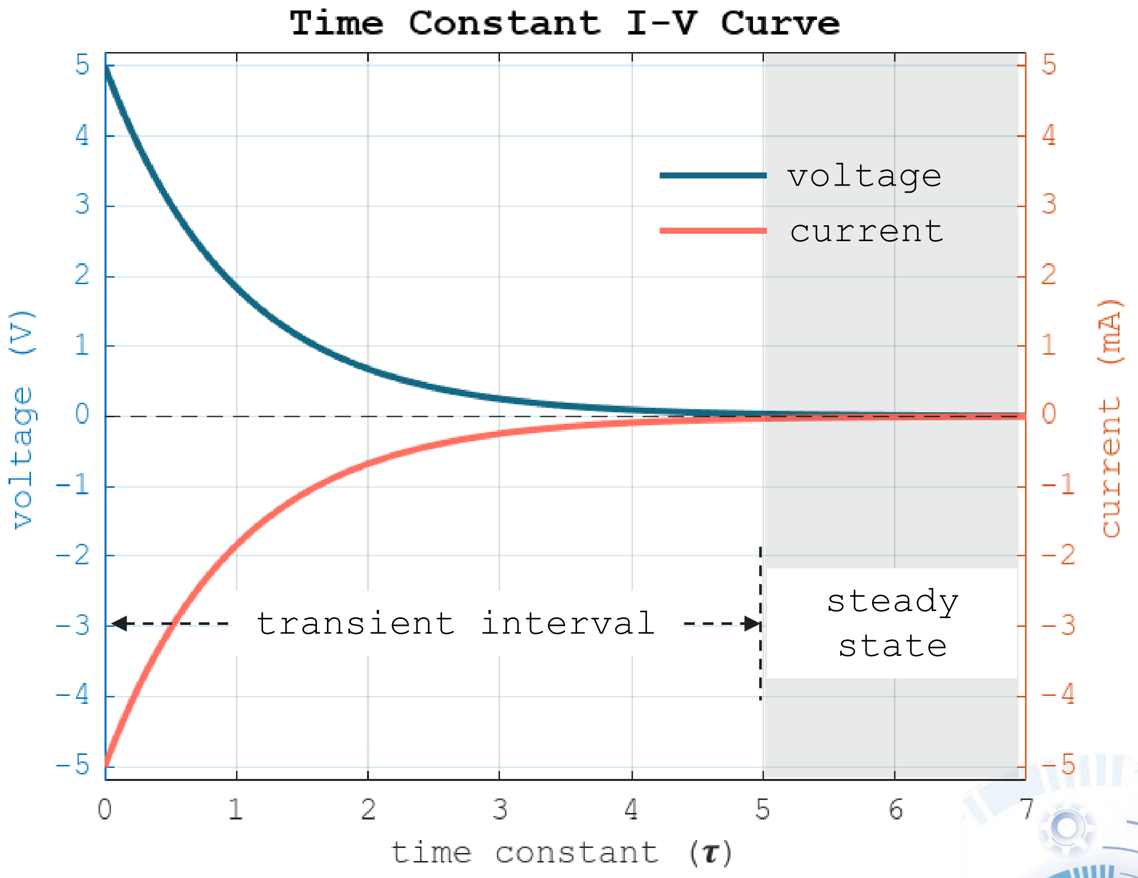

LABORATORY