# SUM-OF-PRODUCTS FORM

## LOGIC MINIMIZATION

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# TOPIC OUTLINE

## Sum-of-Products (SOP) Form

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When two or more **product terms** are summed by Boolean addition, the resulting expression is a **sum-of-products** (**SOP**).

example

$f = AB + ABC$

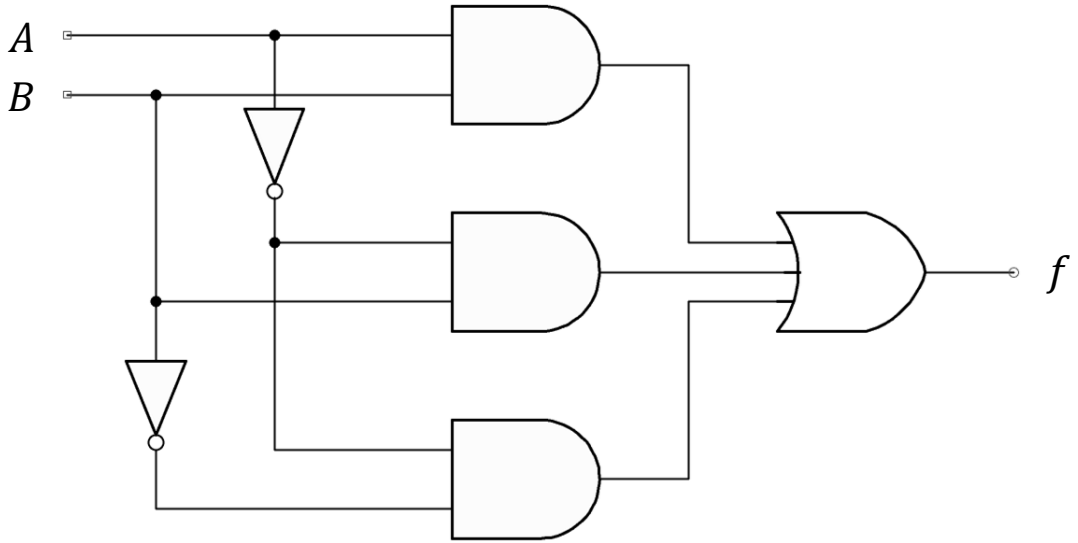
$f = A + \bar{A}\bar{B}C + BC\bar{D}$

$f = \bar{A}B + \bar{A}B\bar{C} + AC$

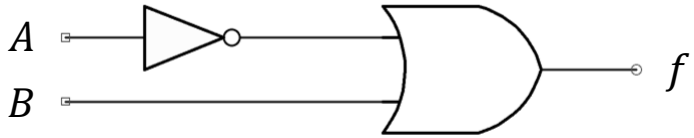
note

SOP expression can have the term $\bar{A}\bar{B}\bar{C}$ but not $\overline{ABC}$.

Canonical sum-of-products




Minimal-cost realization



# EXERCISE

Convert the given Boolean expression to SOP form.

$f = AB + B(CD + EF)$

$f = AB + BCD + BEF$

ans

Solution

# EXERCISE

Convert the given Boolean expression to SOP form.

$f = \overline{(\overline{A+B})+C}$

Solution

$f = (\overline{\overline{A+B}}) \cdot \overline{C}$

$f = (A+B)\overline{C}$

$f = A\overline{C} + B\overline{C}$

ans

# STANDARD SOP FORM

A **standard SOP** form ensures that each product term is a **minterm**.

**Minterm** is a **product term** that evaluates to **1** for exactly one unique combination of input values.

Minterms for the three-variable table

| Decimal | A | B | C | Minterm |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | $m_0 = \bar{A}\bar{B}\bar{C}$ |
| 1 | 0 | 0 | 1 | $m_1 = \bar{A}\bar{B}C$ |
| 2 | 0 | 1 | 0 | $m_2 = \bar{A}B\bar{C}$ |
| 3 | 0 | 1 | 1 | $m_3 = \bar{A}BC$ |
| 4 | 1 | 0 | 0 | $m_4 = A\bar{B}\bar{C}$ |
| 5 | 1 | 0 | 1 | $m_5 = A\bar{B}C$ |
| 6 | 1 | 1 | 0 | $m_6 = AB\bar{C}$ |
| 7 | 1 | 1 | 1 | $m_7 = ABC$ |

# EXERCISE

Convert the given Boolean expression to standard SOP form.

$$f = \overline{\overline{(\overline{A} + B)} + C}$$

and then represent the result using a truth table format.

note

A nonstandard SOP expression is converted into standard form using Boolean algebra rule:

$$A + \bar{A} = 1$$

Solution

$f = \overline{\overline{(\overline{A}+B)} \cdot \overline{C}}$

$f = (\overline{A}+B)\overline{C}$

$f = \overline{A}\overline{C} + B\overline{C}$

$\overline{A}\overline{C}(B+\overline{B}) \rightarrow A\overline{B}\overline{C} + \overline{A}\overline{B}\overline{C}$

$B\overline{C}(A+\overline{A}) \rightarrow A\overline{B}\overline{C} + \overline{A}B\overline{C}$

$f = A\overline{B}\overline{C} + \overline{A}\overline{B}\overline{C} + \overline{A}B\overline{C}$

ans

# EXERCISE

Convert the given Boolean expression to standard SOP form.

$$f = \overline{(\overline{A+B})+C}$$

and then represent the result using a truth table format.

$f = A\bar{B}\bar{C} + A\bar{B}\bar{C} + \bar{A}B\bar{C}$ (6, 4, 2)

ans

$f = \sum m(2,4,6)$

Solution

| | A | B | C | f |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 |
| 2 | 0 | 1 | 0 | 1 |
| 3 | 0 | 1 | 1 | 0 |
| 4 | 1 | 0 | 0 | 1 |
| 5 | 1 | 0 | 1 | 0 |
| 6 | 1 | 1 | 0 | 1 |
| 7 | 1 | 1 | 1 | 0 |

# EXERCISE

Convert the given Boolean expression into standard SOP form.

$$f = ABC + AB(C + D)$$

and then represent the result using a truth table format.

Solution

$f = ABC + \cancel{ABC} + ABD$

$f = ABC + ABD$

$ABC(D+\bar{D}) \longrightarrow ABCD + AB\bar{C}\bar{D}$

$ABD(C+\bar{C}) \longrightarrow ABCD + AB\bar{C}D$

$$f = ABCD + AB\bar{C}\bar{D} + AB\bar{C}D$$

ans

# EXERCISE

Convert the given Boolean expression into standard SOP form.

$$f = ABC + AB(C + D)$$

and then represent the result using a truth table format.

15 14 13

$f = ABCD + AB\bar{C}\bar{D} + AB\bar{C}D$

ans

$f = \sum m(13, 14, 15)$

Solution

| | A | B | C | D | f |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 |
| 2 | 0 | 0 | 1 | 0 | 0 |
| 3 | 0 | 0 | 1 | 1 | 0 |
| 4 | 0 | 1 | 0 | 0 | 0 |
| 5 | 0 | 1 | 0 | 1 | 0 |
| 6 | 0 | 1 | 1 | 0 | 0 |
| 7 | 0 | 1 | 1 | 1 | 0 |
| 8 | 1 | 0 | 0 | 0 | 0 |
| 9 | 1 | 0 | 0 | 1 | 0 |
| 10 | 1 | 0 | 1 | 0 | 0 |

| | A | B | C | D | f |
|---|---|---|---|---|---|
| 11 | 1 | 0 | 1 | 1 | 0 |
| 12 | 1 | 1 | 0 | 0 | 0 |
| 13 | 1 | 1 | 0 | 1 | 1 |
| 14 | 1 | 1 | 1 | 0 | 1 |
| 15 | 1 | 1 | 1 | 1 | 1 |

# EXERCISE

Convert the given Boolean expression into standard SOP form.

$$f = AB + B(C + D)$$

and then represent the result using a truth table format.

Solution

$f = AB + BC + BD$

$AB(C+\bar{C})(D+\bar{D}) \rightarrow ABCD + ABC\bar{D} + AB\bar{C}D + AB\bar{C}\bar{D}$

$BC(A+\bar{A})(D+\bar{D}) \rightarrow ABCD + ABC\bar{D} + \bar{A}BCD + \bar{A}BC\bar{D}$

$BD(A+\bar{A})(C+\bar{C}) \rightarrow ABCD + AB\bar{C}D + \bar{A}BCD + \bar{A}B\bar{C}D$

$f = ABCD + ABC\bar{D} + AB\bar{C}D + AB\bar{C}\bar{D} + \bar{A}BCD + \bar{A}BC\bar{D} + \bar{A}B\bar{C}D$

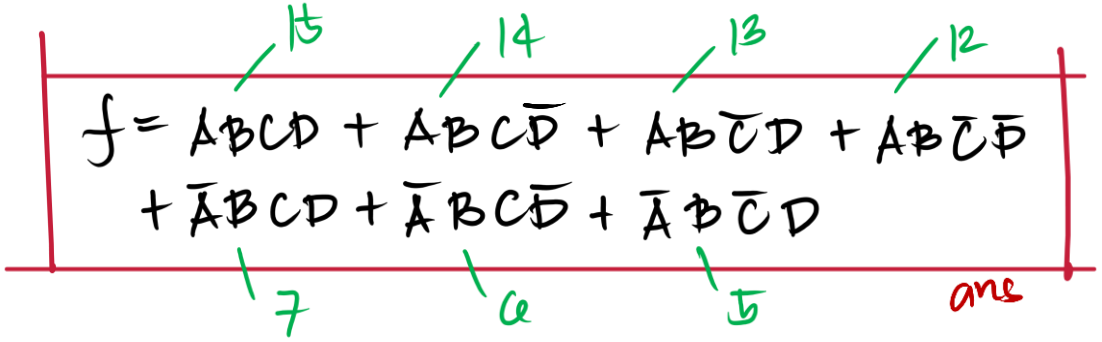
ans

# EXERCISE

Convert the given Boolean expression into standard SOP form.

$$f = AB + B(C + D)$$

and then represent the result using a truth table format.




$$f = \sum m(5, 6, 7, 12, 13, 14, 15)$$

Solution

| | A | B | C | D | f |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 |
| 2 | 0 | 0 | 1 | 0 | 0 |
| 3 | 0 | 0 | 1 | 1 | 0 |
| 4 | 0 | 1 | 0 | 0 | 0 |
| 5 | 0 | 1 | 0 | 1 | 1 |
| 6 | 0 | 1 | 1 | 0 | 1 |
| 7 | 0 | 1 | 1 | 1 | 1 |
| 8 | 1 | 0 | 0 | 0 | 0 |
| 9 | 1 | 0 | 0 | 1 | 0 |
| 10 | 1 | 0 | 1 | 0 | 0 |

| | A | B | C | D | f |
|---|---|---|---|---|---|
| 11 | 1 | 0 | 1 | 1 | 0 |
| 12 | 1 | 1 | 0 | 0 | 1 |
| 13 | 1 | 1 | 0 | 1 | 1 |
| 14 | 1 | 1 | 1 | 0 | 1 |
| 15 | 1 | 1 | 1 | 1 | 1 |

# LABORATORY