FUNDAMENTAL IDENTITIES

TRIGONOMETRIC IDENTITIES

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TOPIC OUTLINE

Reciprocal Identities

Quotient Identities

Pythagorean Identities

Even-Odd Identities

FUNDAMENTAL IDENTITIES

RECIPROCAL AND QUOTIENT IDENTITIES

Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta}$$

$$\cos\theta = \frac{1}{\sec\theta}$$

$$\tan\theta = \frac{1}{\cot\theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

PYTHAGOREAN IDENTITIES

	Alternative Forms	
$\sin^2\theta + \cos^2\theta = 1$	$\sin^2\theta = 1 - \cos^2\theta$	$\cos^2\theta = 1 - \sin^2\theta$
$\tan^2\theta + 1 = \sec^2\theta$	$\tan^2\theta = \sec^2\theta - 1$	$1 = \sec^2\theta - \tan^2\theta$
$1 + \cot^2\theta = \csc^2\theta$	$\cot^2\theta = \csc^2\theta - 1$	$1 = \csc^2\theta - \cot^2\theta$

EVEN-ODD IDENTITIES

$\sin(-\theta) = -\sin\theta$ $\quad\quad$ $\csc(-\theta) = -\csc\theta$

$\cos(-\theta) = \cos\theta$ $\quad\quad$ $\sec(-\theta) = \sec\theta$

$\tan(-\theta) = -\tan\theta$ $\quad\quad$ $\cot(-\theta) = -\cot\theta$

EXERCISE

For each expression in Column I, choose the expression from Column II that completes an identity.

Solution

I	II
1. $\frac{\cos x}{\sin x} =$ ____	A. $\sin^2 x + \cos^2 x$
2. $\tan x =$ ____	B. $\cot x$
3. $\cos(-x) =$ ____	C. $\sec^2 x$
4. $\tan^2 x + 1 =$ ____	D. $\frac{\sin x}{\cos x}$
5. $1 =$ ____	E. $\cos x$

EXERCISE

Simplify the expression

$a.\ -\tan x \cos x$

$b.\ \dfrac{\sec x}{\csc x}$

Solution

EXERCISE

Simplify the expression

$$\frac{1+\cot^2\theta}{1-\csc^2\theta}$$

Solution

EXERCISE

Write $\cos x$ in terms of $\tan x$.

Solution

EXERCISE

Write the given expression in terms of sine and cosine, and then simplify the expression so that no quotients appear and all functions are of θ only.

$$\frac{1-\cos^2(-\theta)}{1+\tan^2(-\theta)}$$

Solution

EXERCISE

Write each expression in terms of sine and cosine, and then simplify the expression so that no quotients appear and all functions are of θ only.

$a.\ \sin^2\theta\,(\csc^2\theta - 1)$

$b.\ \sec\theta\cot\theta\sin\theta$

$c.\ (1-\cos\theta)(1+\sec\theta)$

$d.\ \dfrac{1+\tan(-\theta)}{\tan(-\theta)}$

$e.\ \dfrac{\sec^2\theta - 1}{\csc^2\theta - 1}$

Solution

SEATWORK