



LIMIT OF A FUNCTION

INTRODUCTION

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TOPIC OUTLINE

Limit of a Function

Limit Laws

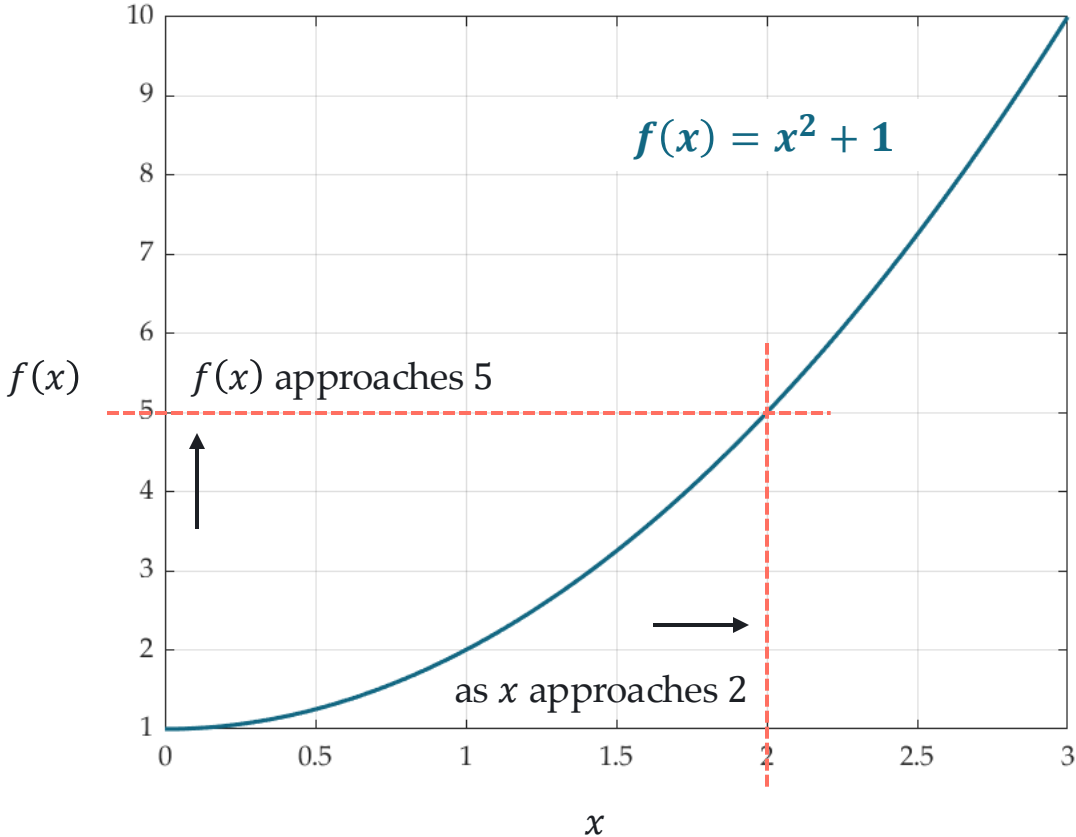
- Sum, Difference, Constant Multiple
- Product, Quotient, Power, Root



LIMIT OF A FUNCTION



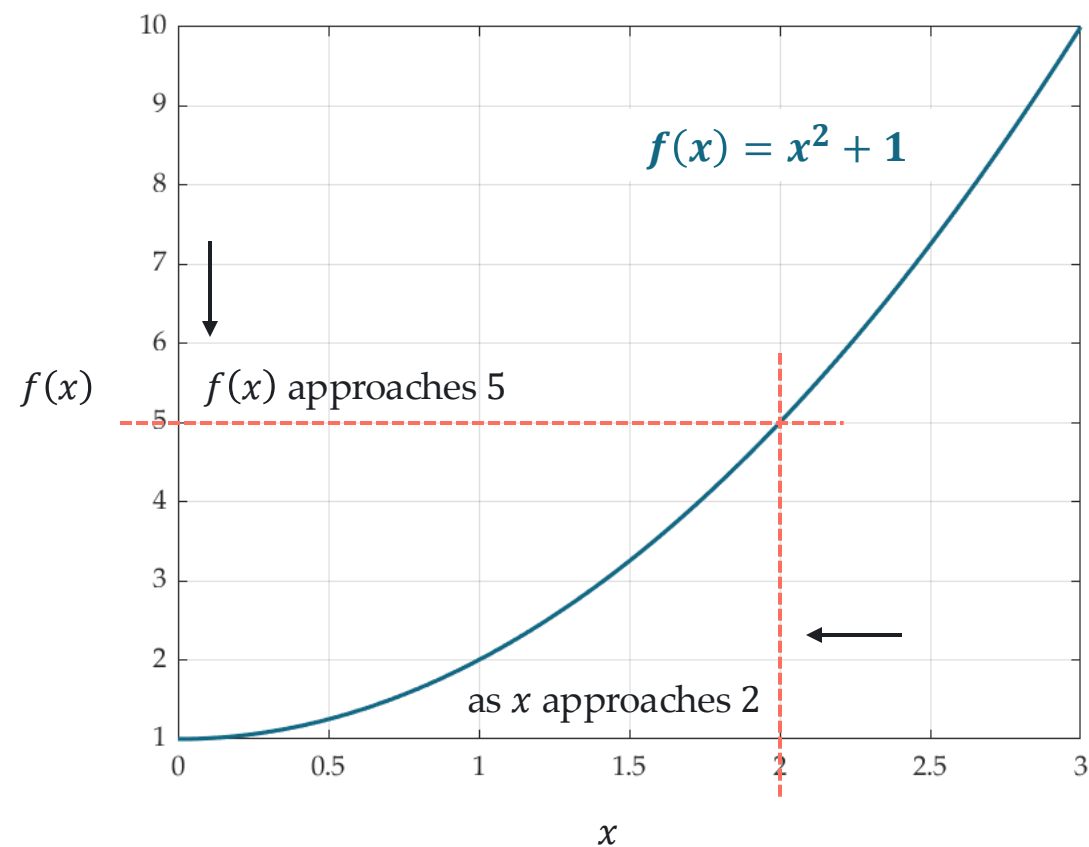
PARABOLA



x	$f(x)$
0	1
1	2
1.5	3.25
1.8	4.24
1.9	4.61
1.91	4.6481
1.92	4.6864
1.93	4.7249
1.95	4.8025
1.96	4.8416
1.97	4.8809
1.98	4.9204
1.99	4.9601
2	5



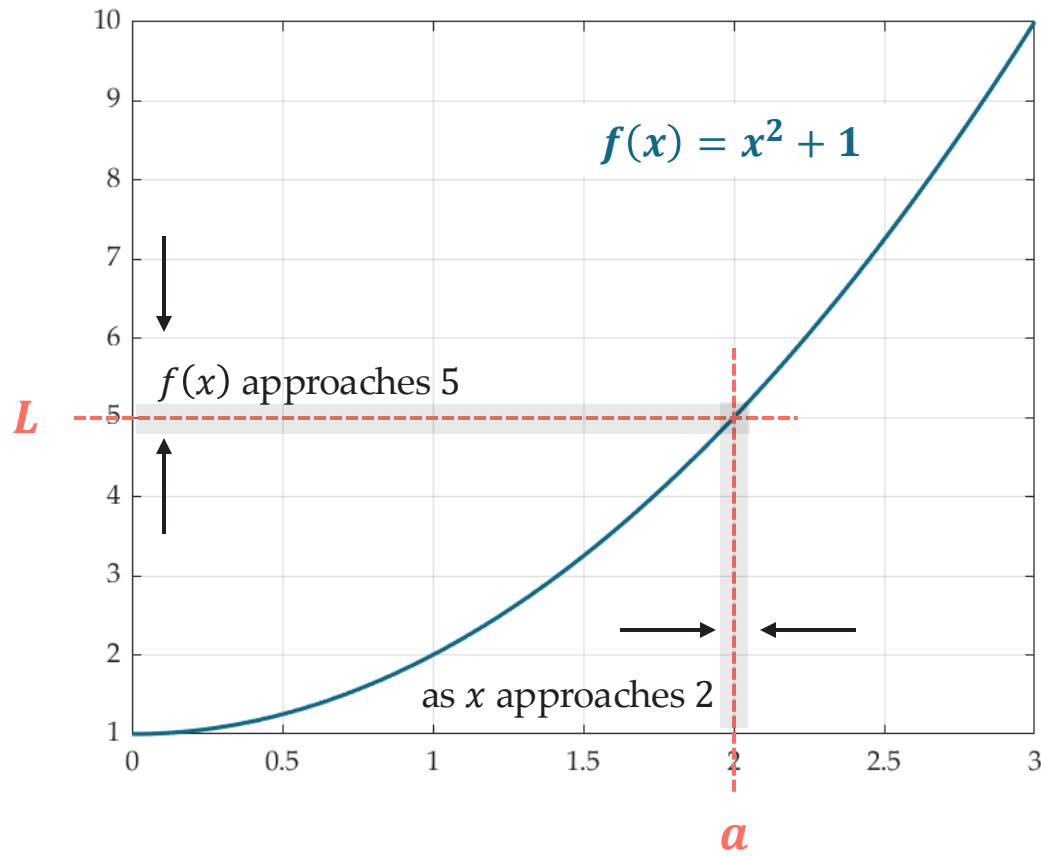
PARABOLA



x	$f(x)$
3	10
2.5	7.25
2.3	6.29
2.1	5.41
2.09	5.3681
2.08	5.3264
2.07	5.2849
2.06	5.2436
2.05	5.2052
2.04	5.1616
2.03	5.1209
2.02	5.0804
2.01	5.0401
2	5



DEFINITION OF LIMIT



Suppose $f(x)$ is **defined** when x is near the number a , then the limit of $f(x)$, as x approaches a , equals L .

notation

$$\lim_{x \rightarrow a} f(x) = L$$

The limit of $(x^2 + 1)$ as x approaches 2 is 5.

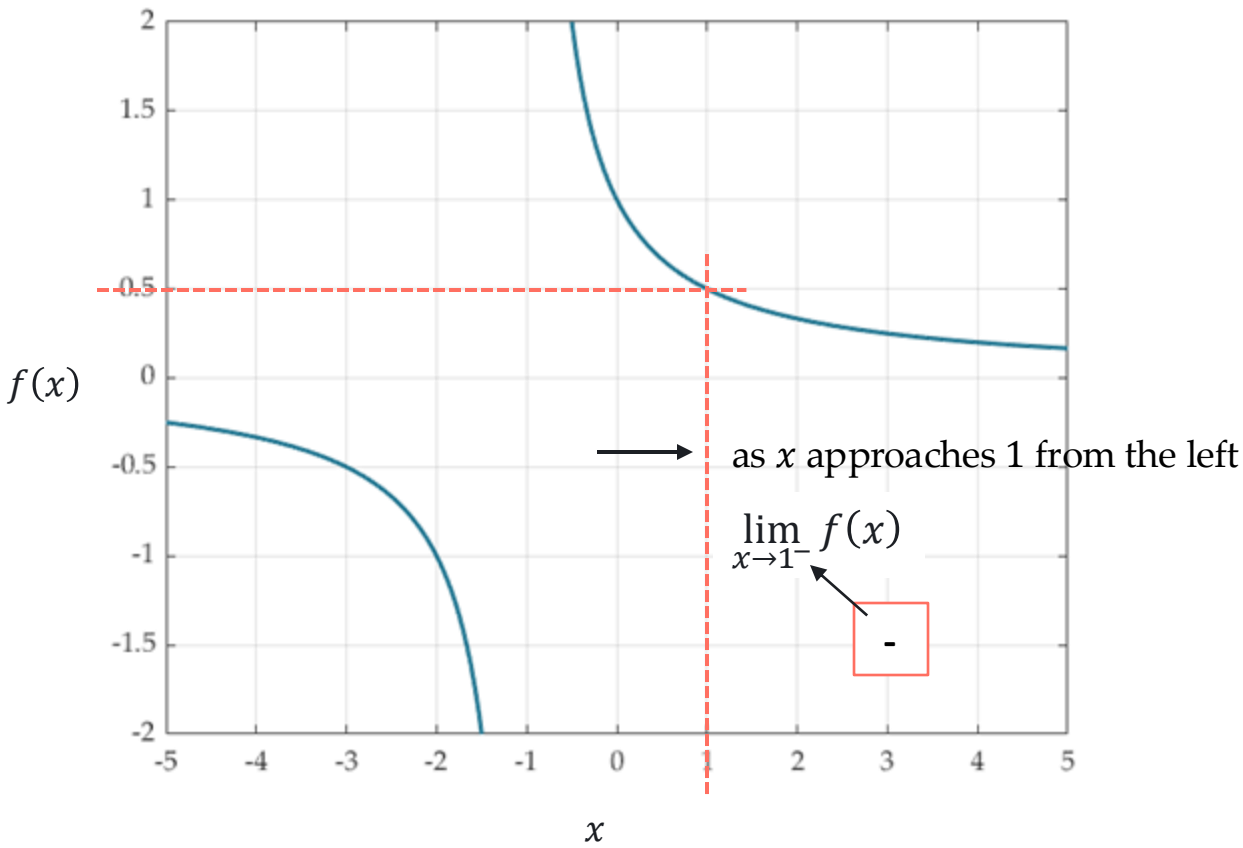
$$\lim_{x \rightarrow 2} (x^2 + 1) = 5$$

$$\lim_{x \rightarrow 3} (x^2 + 1) = 10$$



EXERCISE

Determine $\lim_{x \rightarrow 1} \frac{x-1}{x^2-1}$

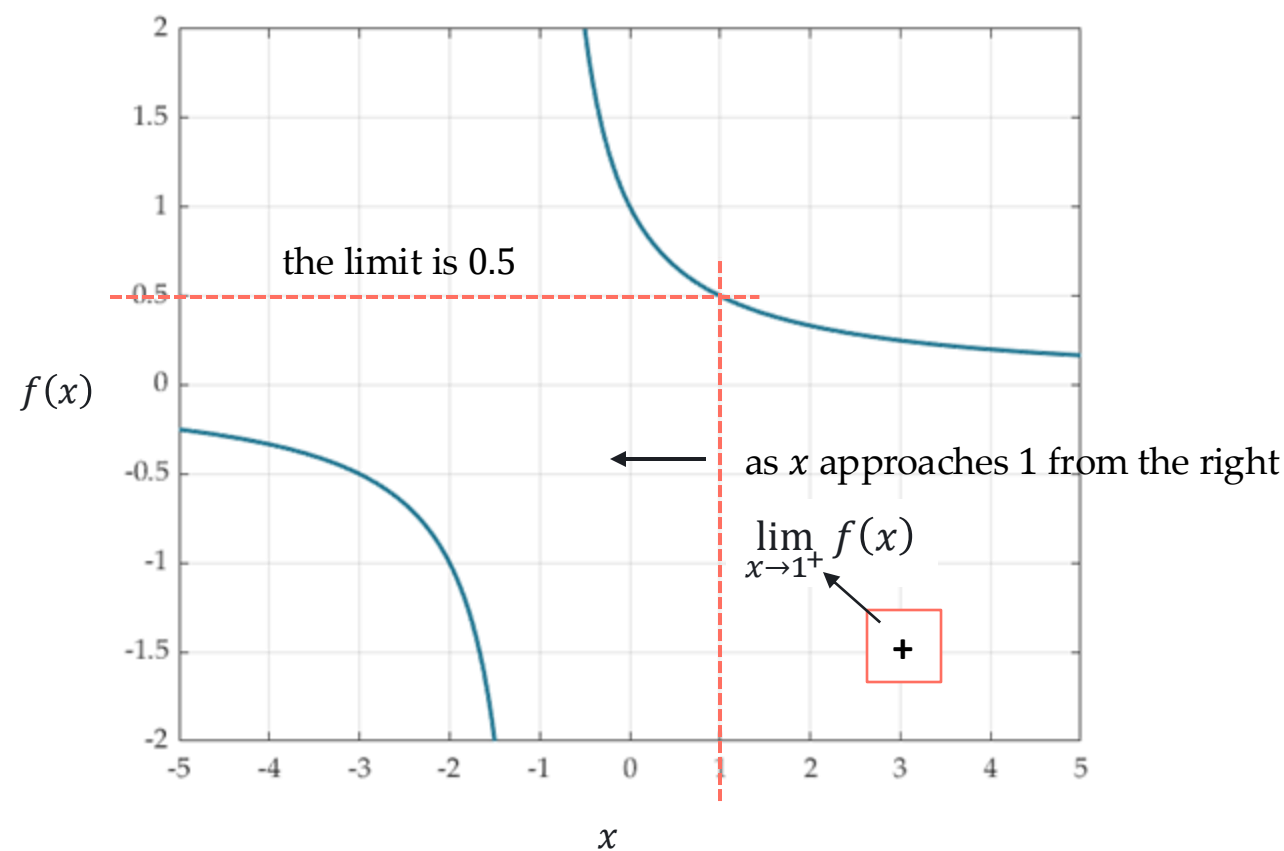


x	$f(x)$
0	1
0.5	0.66667
0.6	0.62500
0.7	0.58824
0.8	0.55556
0.9	0.52632
0.95	0.51282
0.96	0.51020
0.97	0.50761
0.98	0.50505
0.99	0.50251
1	∞



EXERCISE

Determine $\lim_{x \rightarrow 1} \frac{x-1}{x^2-1}$

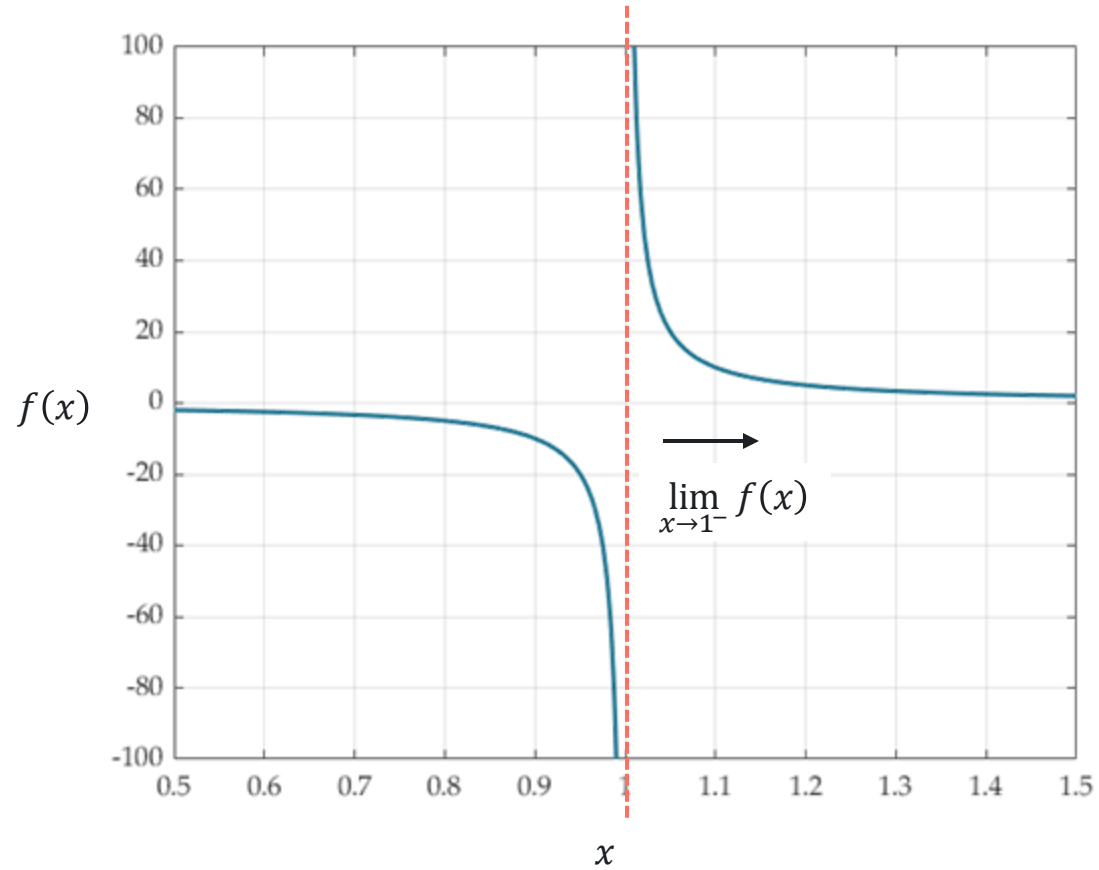


x	$f(x)$
2.0	0.33333
1.5	0.4
1.4	0.41667
1.3	0.43478
1.2	0.45455
1.1	0.47619
1.05	0.48780
1.04	0.49020
1.03	0.49261
1.02	0.49505
1.01	0.49751
1	∞



EXERCISE

Determine $\lim_{x \rightarrow 1} \frac{1}{x-1}$

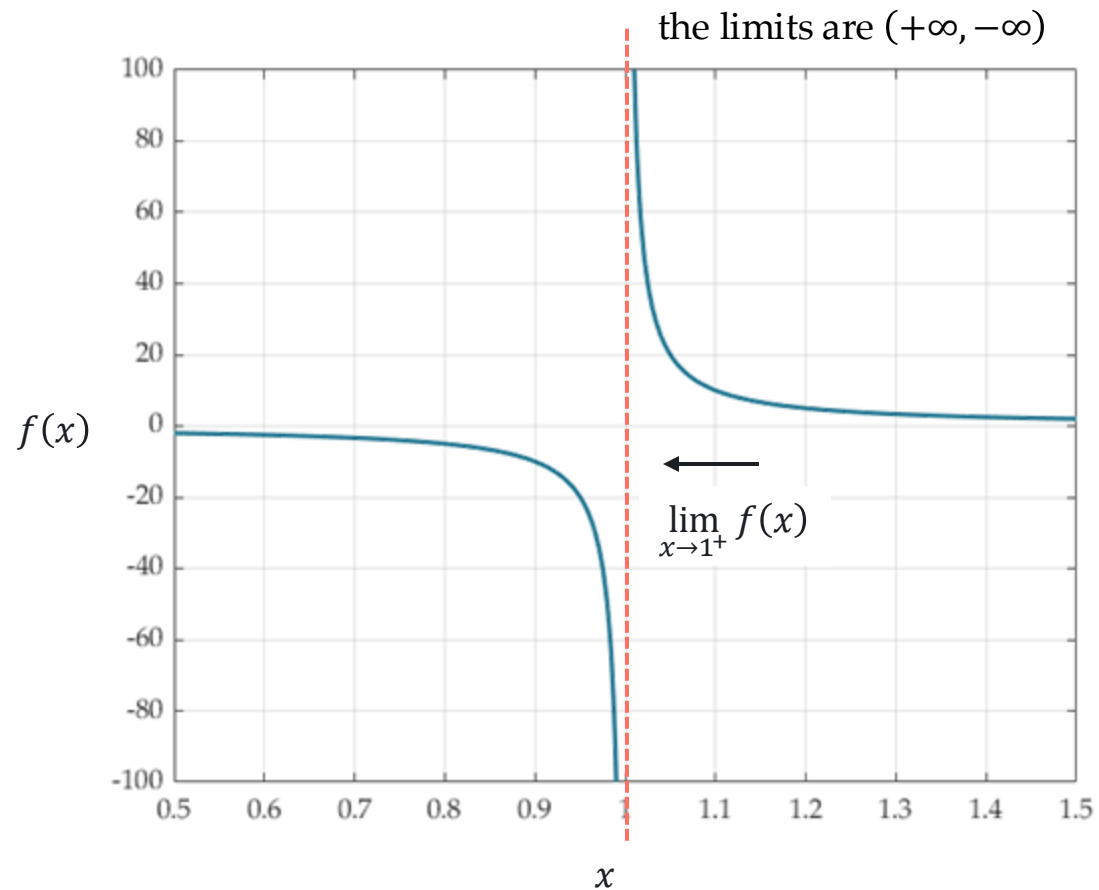


x	$f(x)$
0.9	-10
0.95	-20
0.99	-100
0.999	-1,000
0.9999	-10,000
0.99999	-100,000
0.999999	-1,000,000
$\rightarrow 1^-$	$-\infty$



EXERCISE

Determine $\lim_{x \rightarrow 1} \frac{1}{x-1}$



x	$f(x)$
1.1	10
1.05	20
1.01	100
1.001	1,000
1.0001	10,000
1.00001	100,000
1.000001	1,000,000
$1^+ \leftarrow$	$+\infty$

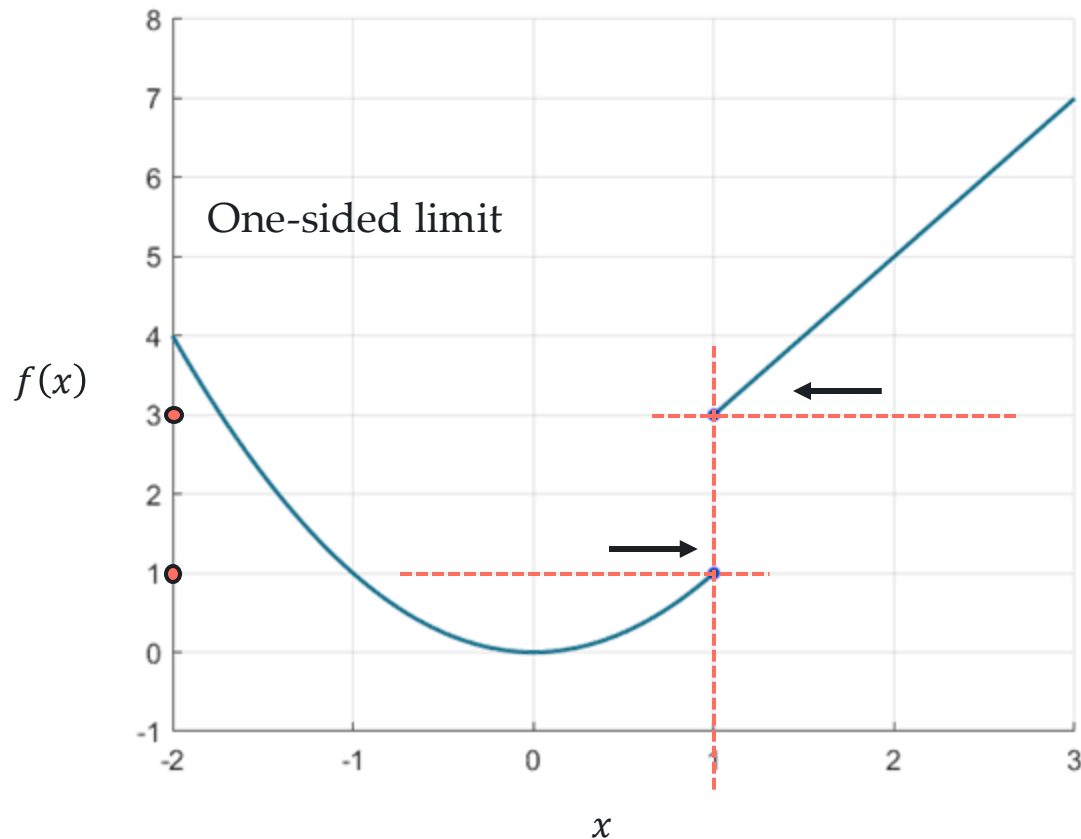
The **limit does not exist** (DNE), since the left-hand limit and the right-hand limit are not equal.



EXERCISE

Determine $\lim_{x \rightarrow 1} \begin{cases} x^2, & \text{if } x < 1 \\ 2x + 1, & \text{if } x \geq 1 \end{cases}$

piecewise function



Left-hand limit

$$\lim_{x \rightarrow 1^-} f(x) = 1$$

Right-hand limit

$$\lim_{x \rightarrow 1^+} f(x) = 3$$

The **limit does not exist** (DNE), since the left-hand limit and the right-hand limit are not equal.



LIMIT LAWS



SUM LAW

$$\lim_{x \rightarrow a} [f(x) + g(x)] = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$$

Evaluate $\lim_{x \rightarrow 2} (x^2 + 1)$

$$= \lim_{x \rightarrow 2} (x^2) + \lim_{x \rightarrow 2} (1)$$

$$= 2^2 + 1$$

$$= \boxed{5}$$

ans



EXERCISE

Evaluate $\lim_{x \rightarrow -1} (2x^2 + 5x + 3)$

$$= \lim_{x \rightarrow -1} (2x^2) + \lim_{x \rightarrow -1} (5x) + \lim_{x \rightarrow -1} (3)$$

$$= 2(-1)^2 + 5(-1) + 3$$

$$= 2 - 5 + 3$$

$$= \boxed{0}$$

ans

Evaluate $\lim_{x \rightarrow 0} (x^3 + 2x + 7)$

$$= \lim_{x \rightarrow 0} (x^3) + \lim_{x \rightarrow 0} (2x) + \lim_{x \rightarrow 0} (7)$$

$$= \cancel{0^3} + 2\cancel{0} + 7$$

$$= \boxed{7}$$

ans



DIFFERENCE LAW

$$\lim_{x \rightarrow a} [f(x) - g(x)] = \lim_{x \rightarrow a} f(x) - \lim_{x \rightarrow a} g(x)$$

Evaluate $\lim_{x \rightarrow 4} (x^2 - 3x)$

$$= \lim_{x \rightarrow 4} (x^2) - \lim_{x \rightarrow 4} (3x)$$

$$= 4^2 - 3(4)$$

$$= 16 - 12$$

$$= \boxed{4}$$

ans



EXERCISE

Evaluate $\lim_{x \rightarrow -3} (2x^2 - 5x + 3)$

$$= \lim_{x \rightarrow -3} (2x^2) - \lim_{x \rightarrow -3} (5x) + \lim_{x \rightarrow -3} (3)$$

$$= 2(-3)^2 - 5(-3) + 3$$

$$= 18 + 15 + 3$$

$$= \boxed{36}$$

ans

Evaluate $\lim_{x \rightarrow 0} (x^3 - 2x - 7)$

$$= \lim_{x \rightarrow 0} (x^3) - \lim_{x \rightarrow 0} (2x) - \lim_{x \rightarrow 0} (7)$$

$$= \cancel{(0)}^0 - 2\cancel{(0)}^0 - 7$$

$$= \boxed{-7}$$

ans



CONTANT MULTIPLE LAW

$$\lim_{x \rightarrow a} [cf(x)] = c \lim_{x \rightarrow a} f(x)$$

Evaluate $\lim_{x \rightarrow 2} (5x^3)$

$$= 5 \lim_{x \rightarrow 2} (x^3)$$

$$= 5(2^3)$$

$$= 5(8)$$

$$= \boxed{40}$$

Ans



EXERCISE

Evaluate $\lim_{x \rightarrow 3} (4x^3)$

$$= 4 \lim_{x \rightarrow 3} (x^3)$$

$$= 4(3^3)$$

$$= 4(27)$$

$$= \boxed{108}$$

ans

Evaluate $\lim_{x \rightarrow -2} (6x^3)$

$$= 6 \lim_{x \rightarrow -2} (x^3)$$

$$= 6(-2)^3$$

$$= \boxed{-48}$$

ans



PRODUCT LAW

$$\lim_{x \rightarrow a} [f(x)g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$$

Evaluate $\lim_{x \rightarrow 3} [x^2(x - 1)]$

$$= \lim_{x \rightarrow 3} (x^2) \cdot \lim_{x \rightarrow 3} (x - 1)$$

$$= (3^2) \cdot (3 - 1)$$

$$= (9) \cdot (2)$$

$$= \boxed{18}$$

ans



EXERCISE

Evaluate $\lim_{x \rightarrow -1} (x^2(x - 5))$

$$= \lim_{x \rightarrow -1} (x^2) \cdot \lim_{x \rightarrow -1} (x - 5)$$

$$= (-1)^2 \cdot (-1 - 5)$$

$$= (1) \cdot (-6)$$

$$= \boxed{-6}$$

ans

Evaluate $\lim_{x \rightarrow -2} (2x^3(x + 5))$

$$= \lim_{x \rightarrow -2} (2x^3) \cdot \lim_{x \rightarrow -2} (x + 5)$$

$$= 2(-2)^3 \cdot (-2 + 5)$$

$$= 2(-8) \cdot (3)$$

$$= (-16) \cdot (3)$$

$$= \boxed{-48}$$

ans



QUOTIENT LAW

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$$

if $\lim_{x \rightarrow a} g(x) \neq 0$

Evaluate $\lim_{x \rightarrow 1} \left(\frac{1}{x-1} \right)$

$$= \frac{\lim_{x \rightarrow 1} (1)}{\lim_{x \rightarrow 1} (x-1)}$$

$$= \frac{1}{(1-1)} \rightarrow 0$$

$$= \boxed{\infty, \text{ DNE}}$$

ans



EXERCISE

Evaluate $\lim_{x \rightarrow 2} \left(\frac{x-1}{x^2-1} \right)$

$$= \frac{\lim_{x \rightarrow 2} (x-1)}{\lim_{x \rightarrow 2} (x^2-1)}$$

$$= \frac{(2-1)}{(2^2-1)}$$

$$= \boxed{\frac{1}{3}}$$

ans

Evaluate $\lim_{x \rightarrow 1} \left(\frac{x-1}{x^2-1} \right)$

$$= \lim_{x \rightarrow 1} \frac{(x-1)}{(x+1)(x-1)}$$

$$= \frac{\lim_{x \rightarrow 1} (1)}{\lim_{x \rightarrow 1} (x+1)}$$

$$= \frac{1}{(1+1)}$$

$$= \boxed{\frac{1}{2}}$$

ans



POWER LAW

$$\lim_{x \rightarrow a} [f(x)]^n = \left[\lim_{x \rightarrow a} f(x) \right]^n$$

Evaluate $\lim_{x \rightarrow 2} (x^3 - 2x)^2$

$$= \left[\lim_{x \rightarrow 2} (x^3 - 2x) \right]^2$$

$$= \left[\lim_{x \rightarrow 2} (x^3) - \lim_{x \rightarrow 2} (2x) \right]^2$$

$$= \left[(2^3) - 2(2) \right]^2$$

$$= (8 - 4)^2$$

$$= \boxed{16}$$

ans



EXERCISE

Evaluate $\lim_{x \rightarrow -2} [(x^3 + 2x)^2]$

$$= \left[\lim_{x \rightarrow -2} (x^3 + 2x) \right]^2$$

$$= [(-2)^3 + 2(-2)]^2$$

$$= (-8 - 4)^2$$

$$= \boxed{144}$$

ans

Evaluate $\lim_{x \rightarrow 1} [(x^3 - 5x)^2]$

$$= \left[\lim_{x \rightarrow 1} (x^3 - 5x) \right]^2$$

$$= [1^3 - 5(1)]^2$$

$$= (1 - 5)^2$$

$$= \boxed{16}$$

ans



ROOT LAW

$$\lim_{x \rightarrow a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \rightarrow a} f(x)}$$

where n is a positive integer

Evaluate $\lim_{x \rightarrow 4} \sqrt{x+2}$

$$= \sqrt{\lim_{x \rightarrow 4} (x+2)}$$

$$= \sqrt{(4+2)}$$

$$= \boxed{\sqrt{6}}$$

ans



EXERCISE

Evaluate $\lim_{x \rightarrow 6} \sqrt{x - 2}$

$$= \sqrt{\lim_{x \rightarrow 6} (x - 2)}$$

$$= \sqrt{(6 - 2)}$$

$$= \boxed{2}$$

ans

Evaluate $\lim_{x \rightarrow -4} \sqrt{x^2 + 2x + 1}$

$$= \sqrt{\lim_{x \rightarrow -4} (x^2 + 2x + 1)}$$

$$= \sqrt{(-4)^2 + 2(-4) + 1}$$

$$= \sqrt{16 - 8 + 1}$$

$$= \sqrt{9}$$

$$= \boxed{3}$$

ans



LABORATORY

