PRODUCT-OF-SUMS FORM

LOGIC MINIMIZATION

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TOPIC OUTLINE

Product-of-Sums (POS) Form

PRODUCT-OF-SUMS FORM

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When two or more **sum terms** are multiplied, the resulting expression is a **product-of-sums** (**POS**).

<u>example</u>

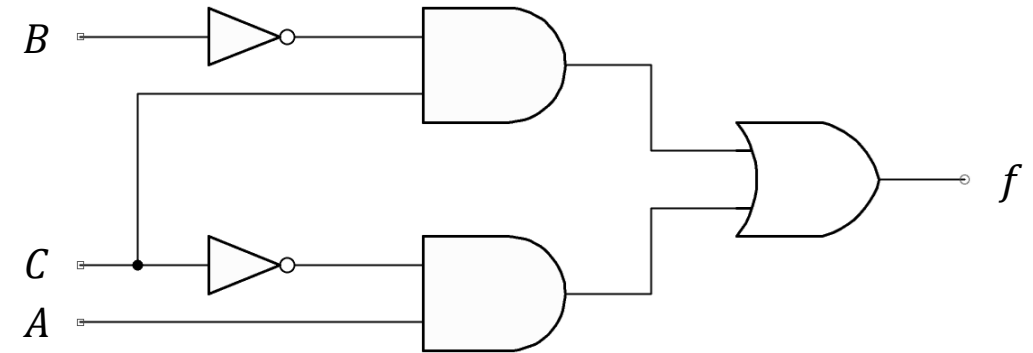$$f = (\bar{A} + B)(A + \bar{B} + C)$$

$$f = (\bar{A} + \bar{B} + \bar{C})(C + \bar{D} + E)(\bar{B} + C + D)$$

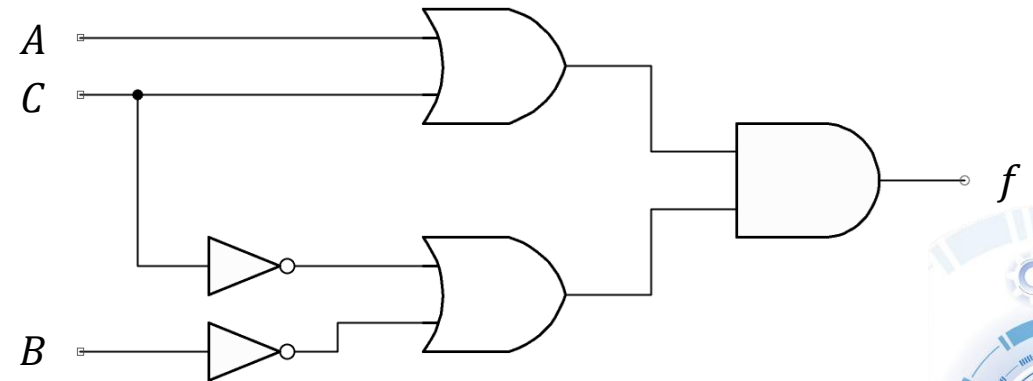$$f = (A + B)(A + \bar{B} + C)(\bar{A} + C)$$

<u>note</u>

POS expression can have the term $\bar{A} + \bar{B} + \bar{C}$ but not $\overline{A + B + C}$.

Minimal SOP realization

Minimal POS realization

EXERCISE

Convert the given Boolean expressions to POS form.

$$f = \overline{(A + B) + C}$$

Solution

$$f = \overline{(A + B)} \cdot \tilde{C}$$

$$\boxed{f = (A + B)\tilde{C}}$$

ans

STANDARD POS FORM

Maxterms for the three-variable table

Decimal	A	B	C	Maxterm
0	0	0	0	$M_0 = A + B + C$
1	0	0	1	$M_1 = A + B + \bar{C}$
2	0	1	0	$M_2 = A + \bar{B} + C$
3	0	1	1	$M_3 = A + \bar{B} + \bar{C}$
4	1	0	0	$M_4 = \bar{A} + B + C$
5	1	0	1	$M_5 = \bar{A} + B + \bar{C}$
6	1	1	0	$M_6 = \bar{A} + \bar{B} + C$
7	1	1	1	$M_7 = \bar{A} + \bar{B} + \bar{C}$

A **standard POS form** ensures that each product term is a **maxterm**.

Maxterm is a **sum term** that evaluates to **0** for exactly one unique combination of input values.

EXERCISE

Convert the given Boolean expressions to standard POS form.

$$f = \overline{(A + B) + C}$$

and then represent the result using a truth table format.

note

A nonstandard POS expression is converted into standard form using Boolean algebra rule:

$$A \cdot \bar{A} = 0$$

$$f = \overline{(A + \cancel{B})} \cdot \bar{C}$$

$$f = (A + \cancel{B})\bar{C}$$

$$(\cancel{A} + \cancel{B}) + C\bar{C} \longrightarrow (A + B + C)(A + B + \bar{C}) \ \ast$$

$$\textcolor{red}{x + yz = (x + y)(x + z) \ ; \ \text{Rule } 12}$$

$$\bar{C} + A\bar{A} \longrightarrow (\bar{C} + A)(\bar{C} + \bar{A})$$

$$(A + \bar{C}) + B\bar{B} \longrightarrow (A + \cancel{B} + \bar{C})(A + \bar{B} + \bar{C}) \ \ast$$

$$(\bar{C} + \bar{A}) + B\bar{B} \longrightarrow (\bar{A} + B + \bar{C})(\bar{A} + \bar{B} + \bar{C}) \ \ast$$

$$f = \frac{(A + B + C)(A + B + \bar{C})(A + \bar{B} + \bar{C})}{(\bar{A} + B + \bar{C})(\bar{A} + \bar{B} + \bar{C})}$$

ans

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Convert the given Boolean expressions to standard POS form.

$$f = \overline{(A + B) + C}$$

and then represent the result using a truth table format.

$$f = \frac{(A + B + C)(A + B + \overline{C})(A + \overline{B} + \overline{C})}{(\overline{A} + B + \overline{C})(\overline{A} + \overline{B} + \overline{C})}$$

ans

$$f = \prod M(0, 1, 3, 5, 7)$$

Solution

	A	B	C	f
0	0	0	0	0
1	0	0	1	0
2	0	1	0	1
3	0	1	1	0
4	1	0	0	1
5	1	0	1	0
6	1	1	0	1
7	1	1	1	0

EXERCISE

Convert the given Boolean expressions to standard POS form.

$$f = ABC + AB(C + D)$$

and then represent the result using a truth table format.

$$0, 1, 2, 3, 4, 5, 6, 7$$

Solution

$$f = AB\left[\cancel{C} + (C+D)\right]$$

$$f = AB(C+D)$$

$$A + B\bar{B} \rightarrow (A+B)(A+\bar{B})$$

$$(A+B) + C\bar{C} \rightarrow (A+B+C)(A+B+\bar{C})$$

$$(A+B+C) + D\bar{D} \rightarrow (A+B+C+D)^{0}(A+B+C+\bar{D})^{1} \text{ *}$$

$$(A+B+\bar{C}) + D\bar{D} \rightarrow (A+B+\bar{C}+D)^{2}(A+B+\bar{C}+\bar{D})^{3} \text{ *}$$

$$(A+\bar{B}) + C\bar{C} \rightarrow (A+\bar{B}+C)(A+\bar{B}+\bar{C})$$

$$(A+\bar{B}+C) + D\bar{D} \rightarrow (A+\bar{B}+C+D)^{4}(A+\bar{B}+C+\bar{D})^{5} \text{ *}$$

$$(A+\bar{B}+\bar{C}) + D\bar{D} \rightarrow (A+\bar{B}+\bar{C}+D)^{6}(A+\bar{B}+\bar{C}+\bar{D})^{7} \text{ *}$$

EXERCISE

Convert the given Boolean expressions to standard POS form.

$$f = ABC + AB(C + D)$$

and then represent the result using a truth table format.

$0, 1, 2, 3, 4, 5, 6, 7$

$8, 9, 10, 11, 12$

$$f = \Pi M \begin{pmatrix} 0, 1, 2, 3, 4, 5, 6, \\ 7, 8, 9, 10, 11, 12 \end{pmatrix}$$

ans

$$f = AB\left[\cancel{C} + (C+D)\right]$$

$$f = AB(C+D)$$

OK

$\overline{B} + A\overline{A} \longrightarrow (A + B)(\overline{A} + B)$

$(\overline{A} + B) + C\overline{C} \longrightarrow (\overline{A} + B + C)(\overline{A} + B + \overline{C})$

$(\overline{A} + B + C) + D\overline{D} \longrightarrow (\overline{A} + B + C + D)(\overline{A} + B + C + \overline{D})$ ✳ 8, 9

$(\overline{A} + B + \overline{C}) + D\overline{D} \longrightarrow (\overline{A} + B + \overline{C} + D)(\overline{A} + B + \overline{C} + \overline{D})$ ✳ 10, 11

$(C + D) + B\overline{B} \longrightarrow (B + C + D)(\overline{B} + C + D)$

$(B + C + D) + A\overline{A} \longrightarrow (A + B + C + D)(\overline{A} + B + C + D)$ ✳ 0, 8

$(\overline{B} + C + D) + A\overline{A} \longrightarrow (A + \overline{B} + C + D)(\overline{A} + \overline{B} + C + D)$ ✳ 4, 12

EXERCISE

Convert the given Boolean expressions to standard POS form.

$$f = ABC + AB(C + D)$$

and then represent the result using a truth table format.

0, 1, 2, 3, 4, 5, 6, 7

8, 9, 10, 11, 12

$$f = \Pi M \begin{pmatrix} 0, 1, 2, 3, 4, 5, 6, \\ 7, 8, 9, 10, 11, 12 \end{pmatrix}$$

ans

	A	B	C	D	f
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	0
4	0	1	0	0	0
5	0	1	0	1	0
6	0	1	1	0	0
7	0	1	1	1	0
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	0

	A	B	C	D	f
11	1	0	1	1	0
12	1	1	0	0	0
13	1	1	0	1	1
14	1	1	1	0	1
15	1	1	1	1	1

EXERCISE

Convert the given Boolean expressions to standard POS form.

$$f = A\bar{B} + \bar{B}(C + D)$$

and then represent the result using a truth table format.

$$f = \prod M(0,1,2,3,4,8,9,10,11)$$

ans

Solution

$$f = B\left[A + (C+D)\right]$$

$$B + A\bar{A} \rightarrow (A+B)(\bar{A}+B)$$

$$(A+B) + C\bar{C} \rightarrow (A+B+C)(A+B+\bar{C})$$

$$(A+B+C) + D\bar{D} \rightarrow (A+B+C+D)(A+B+C+\bar{D}) \quad \overset{0}{*} \quad \overset{1}{*}$$

$$(A+B+\bar{C}) + D\bar{D} \rightarrow (A+B+\bar{C}+D)(A+B+\bar{C}+\bar{D}) \quad \overset{2}{*} \quad \overset{3}{*}$$

$$(\bar{A}+B) + C\bar{C} \rightarrow (\bar{A}+B+C)(\bar{A}+B+\bar{C})$$

$$(\bar{A}+B+C) + D\bar{D} \rightarrow (\bar{A}+B+C+D)(\bar{A}+B+C+\bar{D}) \quad \overset{8}{*} \quad \overset{9}{*}$$

$$(\bar{A}+B+\bar{C}) + D\bar{D} \rightarrow (\bar{A}+B+\bar{C}+D)(\bar{A}+B+\bar{C}+\bar{D}) \quad \overset{10}{*} \quad \overset{11}{*}$$

$$(A+C+D) + B\bar{B} \rightarrow (A+B+C+D)(A+\bar{B}+C+D) \quad \overset{0}{*} \quad \overset{4}{*}$$

EXERCISE

Convert the given Boolean expressions to standard POS form.

$$f = AB + B(C + D)$$

and then represent the result using a truth table format.

$$f = \Pi M(0,1,2,3,4,8,9,10,11)$$

ans

	A	B	C	D	f
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	0
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	1
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	0

	A	B	C	D	f
11	1	0	1	1	0
12	1	1	0	0	1
13	1	1	0	1	1
14	1	1	1	0	1
15	1	1	1	1	1

LABORATORY