TRANSIENT RESPONSE OF CAPACITOR

RC CIRCUITS

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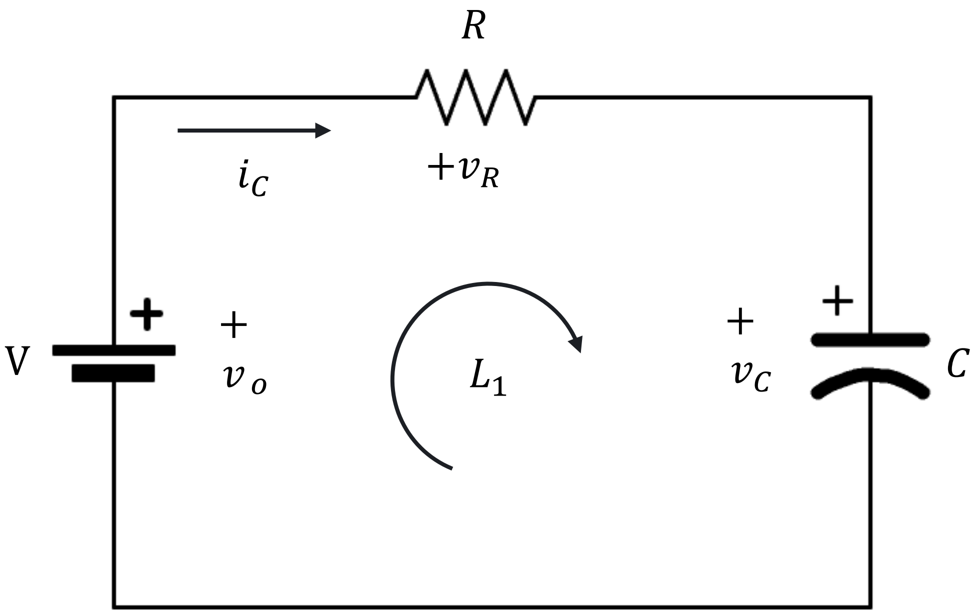
TOPIC OUTLINE

Charging a Capacitor

Discharging a Capacitor

CHARGING A CAPACITOR

RC CIRCUIT



KVL $@L_1$

$$-v_o + v_R + v_C = 0$$

$$v_R + v_C = v_o$$

$$i_C R + v_C = v_o \qquad ; i_C = C\frac{d}{dt}v_C$$

$$RC\frac{d}{dt}v_C + v_C = v_o$$

$$\frac{d}{dt}v_C + \frac{1}{RC}v_C = \frac{v_o}{RC}$$

... first-order ODE

$$v_C(t) = v_o\left(1 - e^{-\frac{t}{RC}}\right)$$

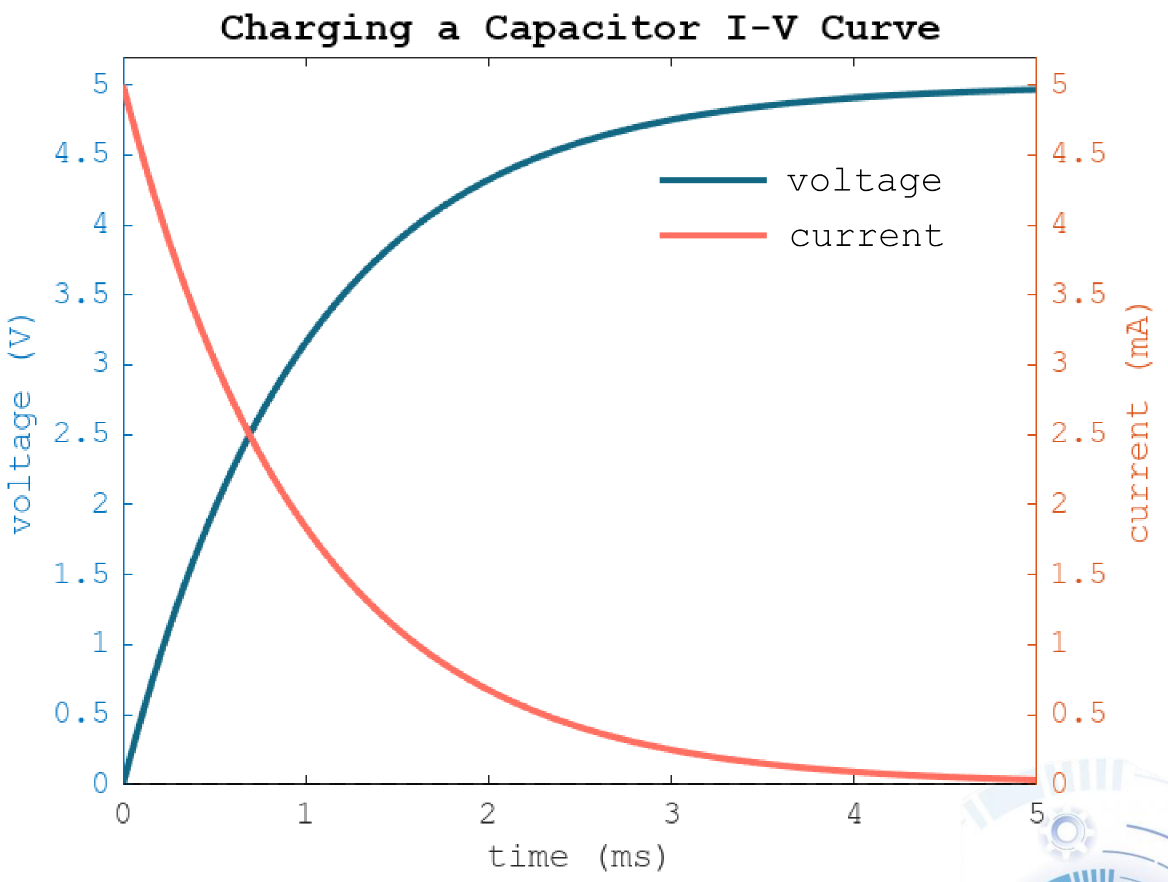
CAPACITOR VOLTAGE

Charging Equation

$$v_c(t) = v_o\left(1 - e^{-\frac{t}{\tau}}\right)$$

where: $\tau = RC$

The **voltage** across the capacitor **starts at zero** and exponentially increases to its maximum voltage (v_o).



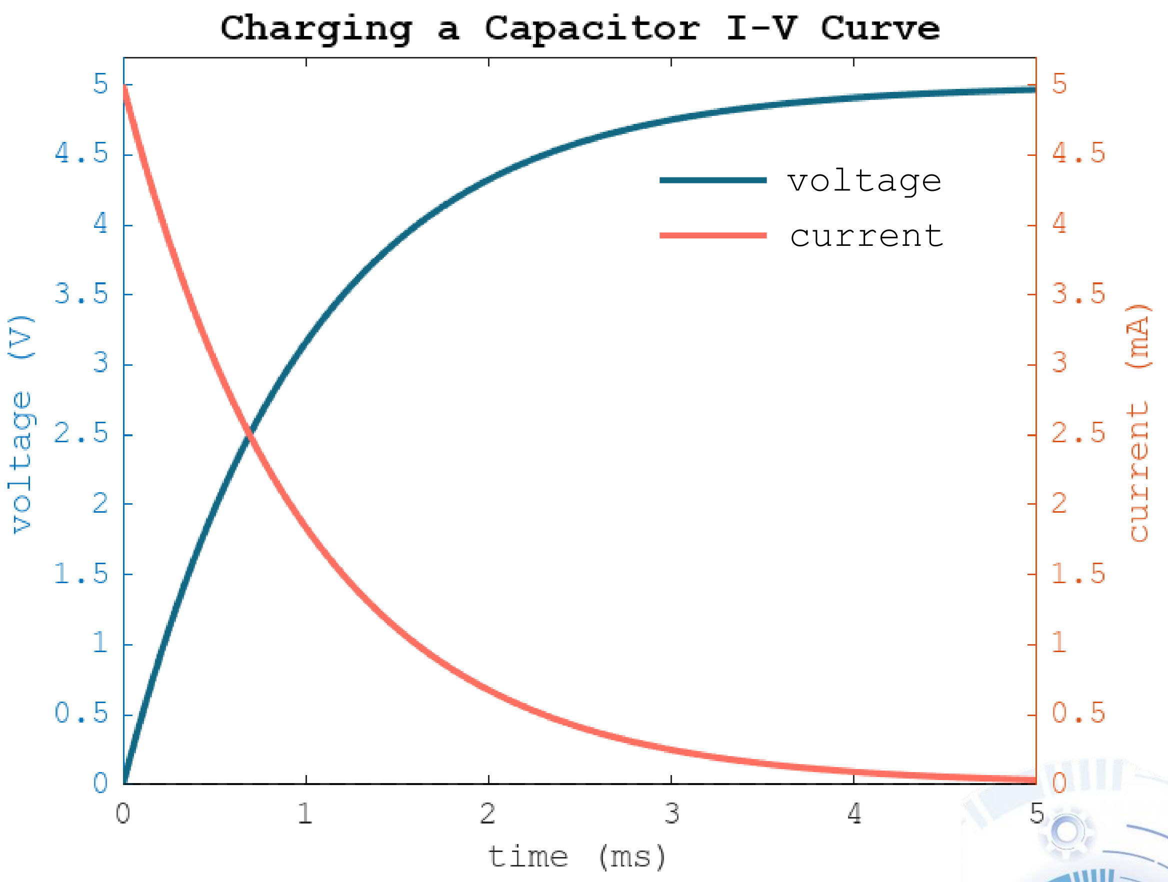
CAPACITOR CURRENT

Charging Equation

$$i_c(t) = \frac{v_o}{R} e^{-\frac{t}{\tau}}$$

where: $\tau = RC$

The **current** through the capacitor instantly jumps to its **maximum value** $\left({}^{v_o}/_{R}\right)$ amperes then decays exponentially to zero.



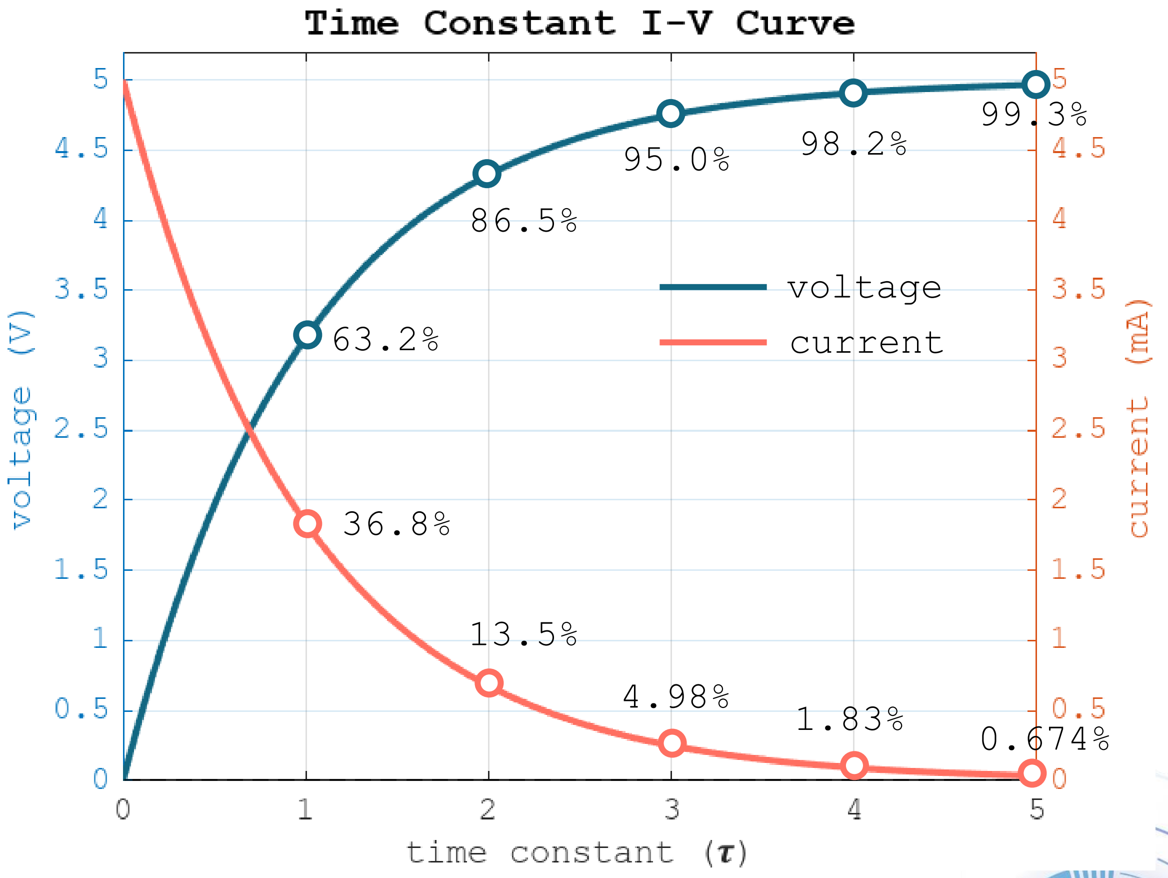
TIME CONSTANT

The **time constant** (τ) is a measure of how quickly a capacitor charges or discharges in an RC circuit.

Formula

$$\tau = RC$$

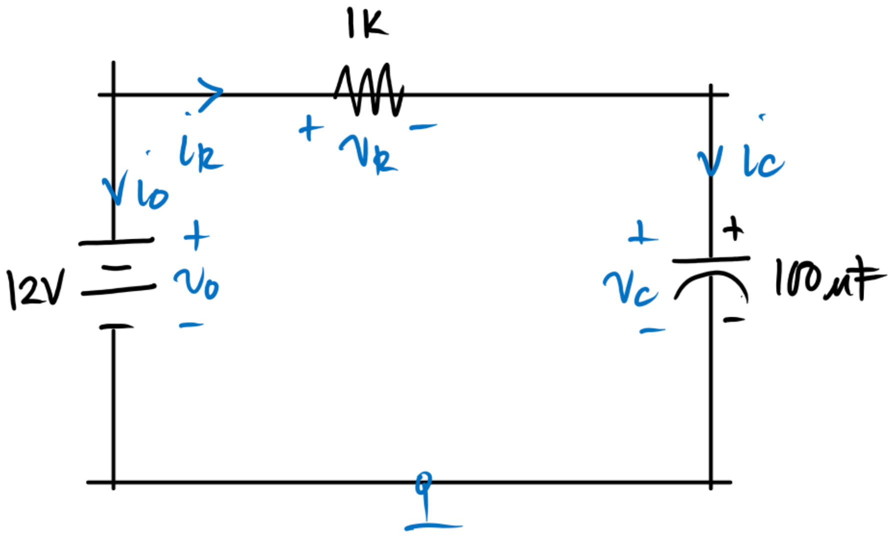
unit: second



EXERCISE

A 100μF capacitor is connected to a 12V DC power supply through a resistor of 1$K\Omega$. Determine the time it takes for the capacitor to charge to 86.5% of its maximum voltage.

2τ



Solution

$\tau = RC$

$\tau = 1K(100\mu)$

$\tau = 100\ ms$

2τ for 86.5%

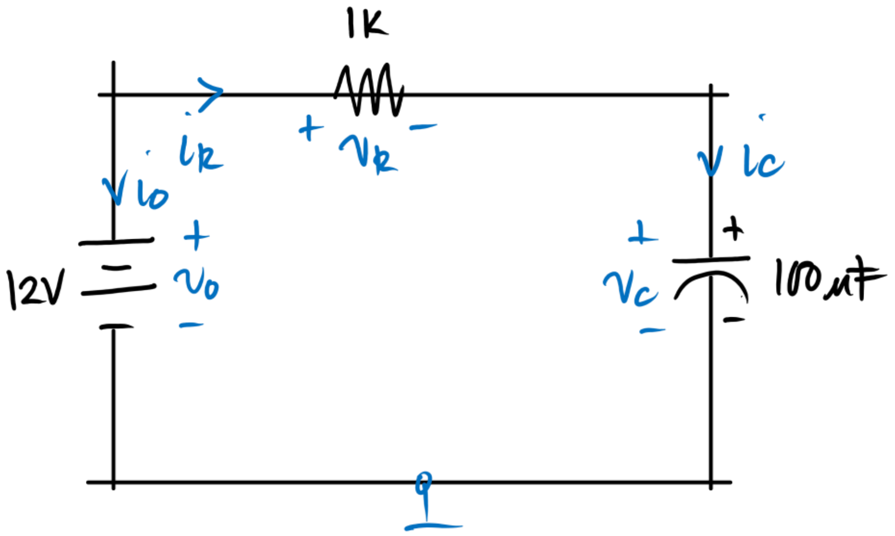
$2\tau = 2(100m)$

$\boxed{2\tau = 200\ ms}$

ans

EXERCISE

A 100μF capacitor is connected to a 12V DC power supply through a resistor of 1$K\Omega$. Determine the voltage across the capacitor after $\underbrace{200\ ms}_{t}$ of charging.



Solution

$\tau = RC$

$\tau = 1k(100\mu)$

$\tau = 100\ ms$

$v(t) = V_o\left(1 - e^{-\frac{t}{\tau}}\right)$

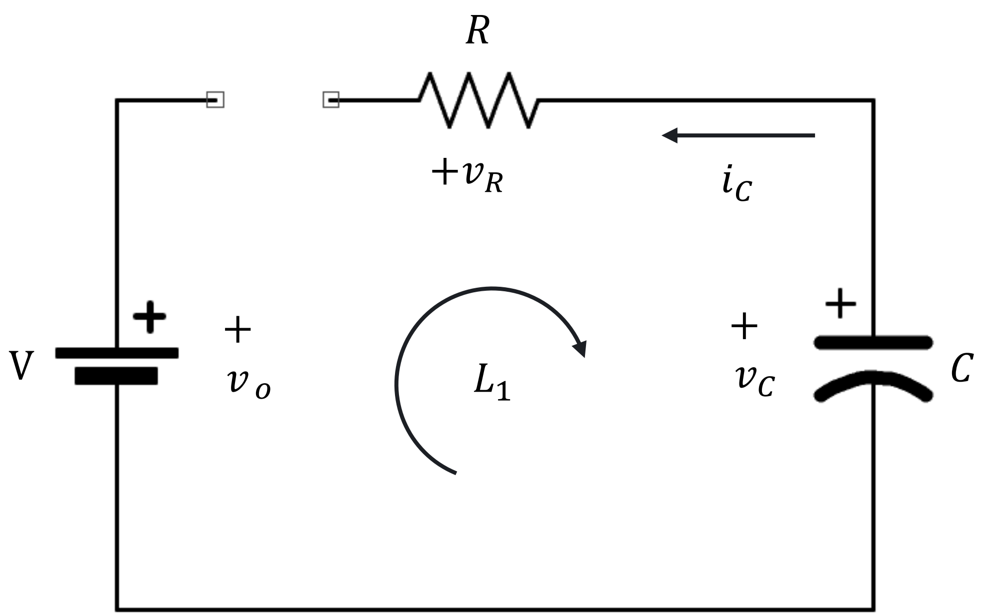
$v(200ms) = 12\left(1 - e^{-\frac{200}{100}}\right)$

$\boxed{v(200ms) = 10.38\ V}$

ans

DISCHARGING A CAPACITOR

RC CIRCUIT



KVL @L_1

$$v_R + v_C = 0$$

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$$RC\frac{d}{dt}v_C + v_C = 0$$

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... first-order ODE

$$v_C(t) = v_o e^{-\frac{t}{RC}}$$

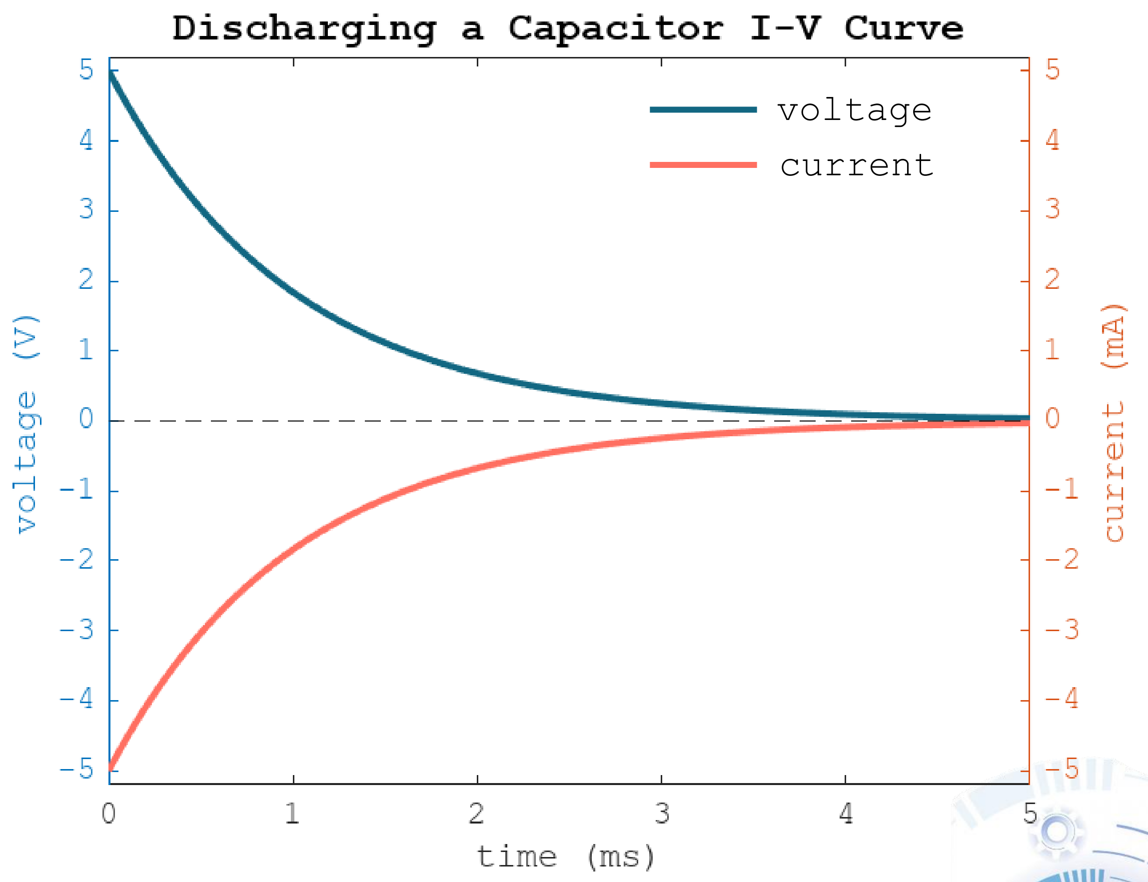
CAPACITOR VOLTAGE

Discharging Equation

$$v_c(t) = v_o e^{-\frac{t}{\tau}}$$

where: $\tau = RC$

The **voltage** across the capacitor starts at its maximum voltage (v_o) then decays exponentially to **zero**.



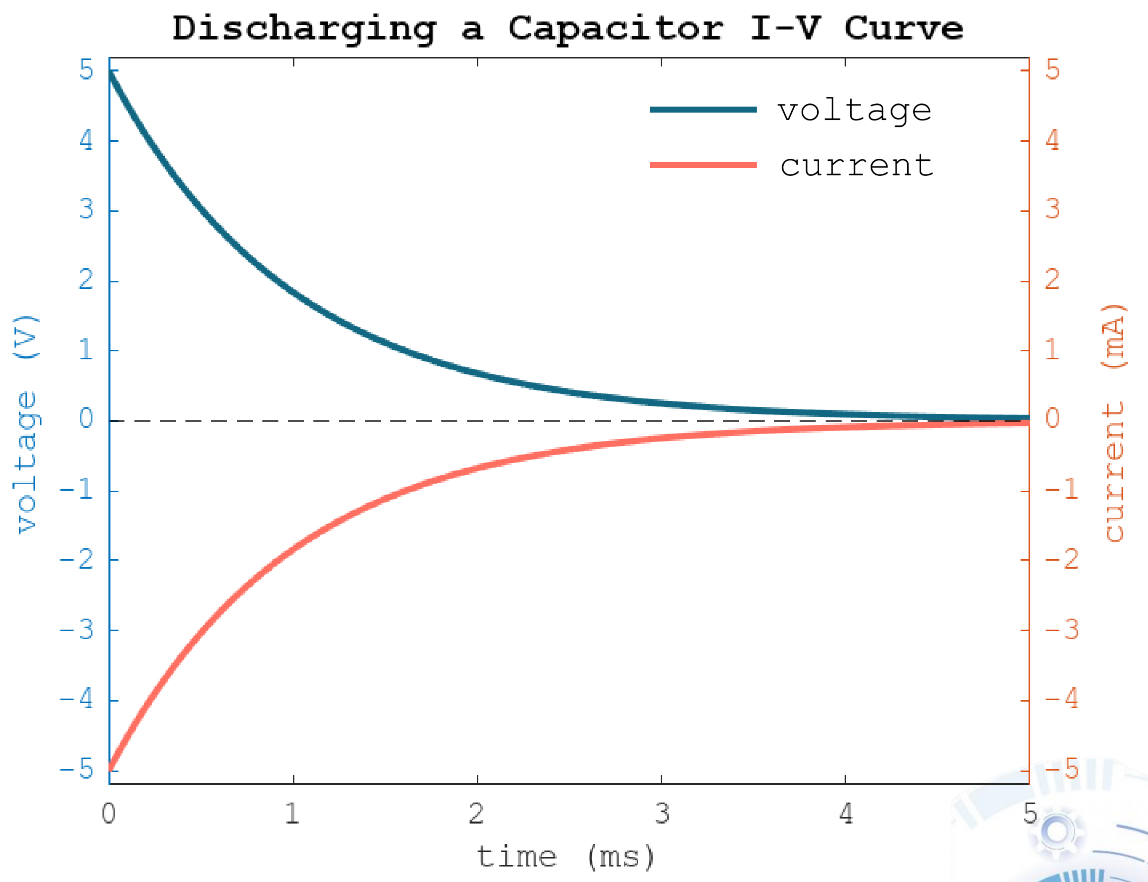
CAPACITOR CURRENT

Discharging Equation

$$i_c(t) = -\frac{v_o}{R}\left(e^{-\frac{t}{\tau}}\right)$$

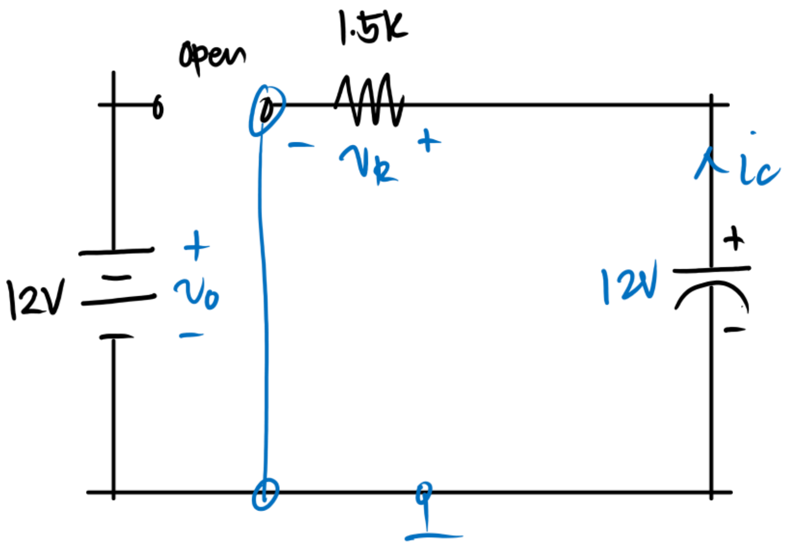
where: $\tau = RC$

The **current** through the capacitor instantly jumps to its maximum value, but in opposite direction $\left(-{}^{v_o}/_{R}\right)$ then decays exponentially to **zero**.



EXERCISE

A 200μF capacitor is initially charged to 12V. It is then disconnected from the power supply and discharged a resistor of 1.5$K\Omega$. Determine the voltage across the capacitor after $\underbrace{0.1s}_{t}$ of discharging.



Solution

$\tau = RC$

$\tau = 1.5k(200\mu)$

$\tau = 300\text{ ms}$

$v(t) = V_0 e^{-\frac{t}{\tau}}$

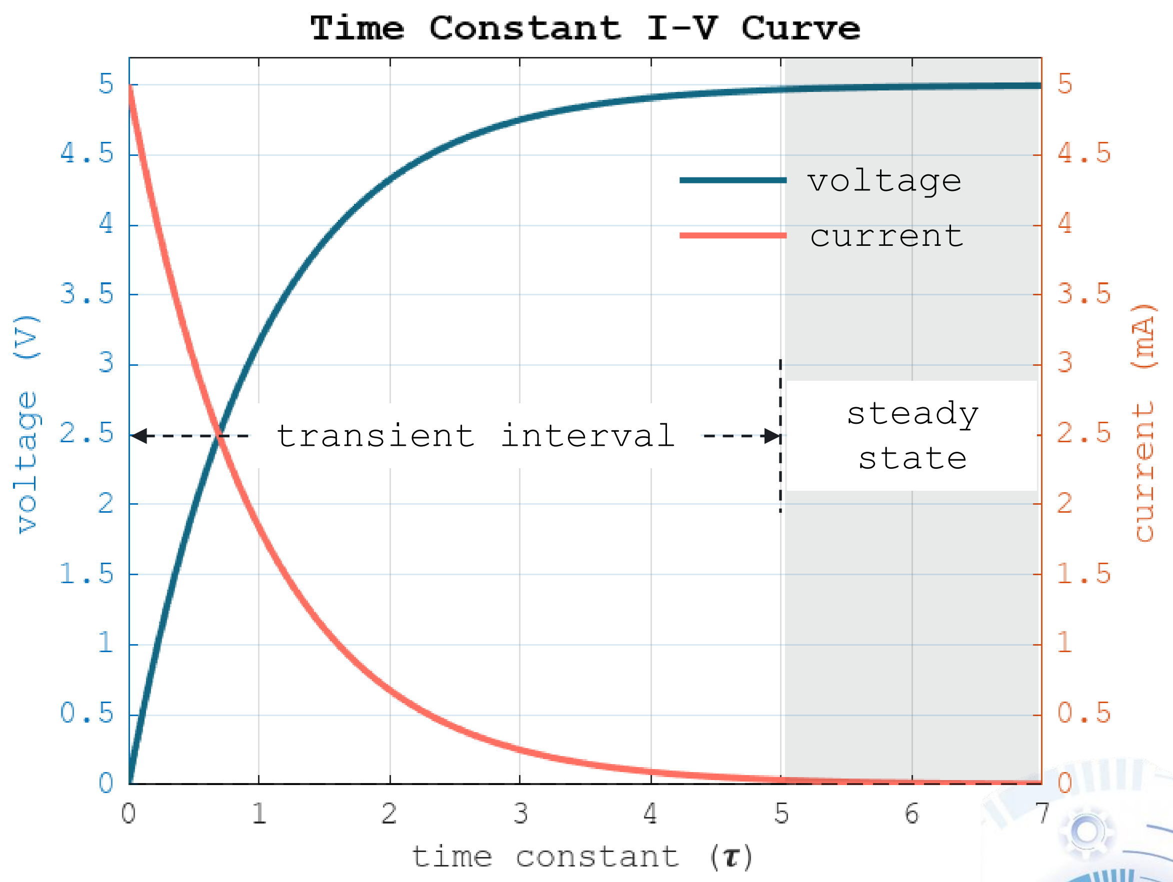
$v(300\text{ms}) = 12 e^{-\frac{0.1}{0.3}}$

$\boxed{v(300\text{ms}) = 8.60V}$

ans

TRANSIENT RESPONSE

The **transient response** of a capacitor describes the time-dependent changes in voltage across the capacitor and the current through it. The transient phase is typically considered to last for approximately **five time constants** (5τ) after which the system is assumed to have reached steady-state conditions.



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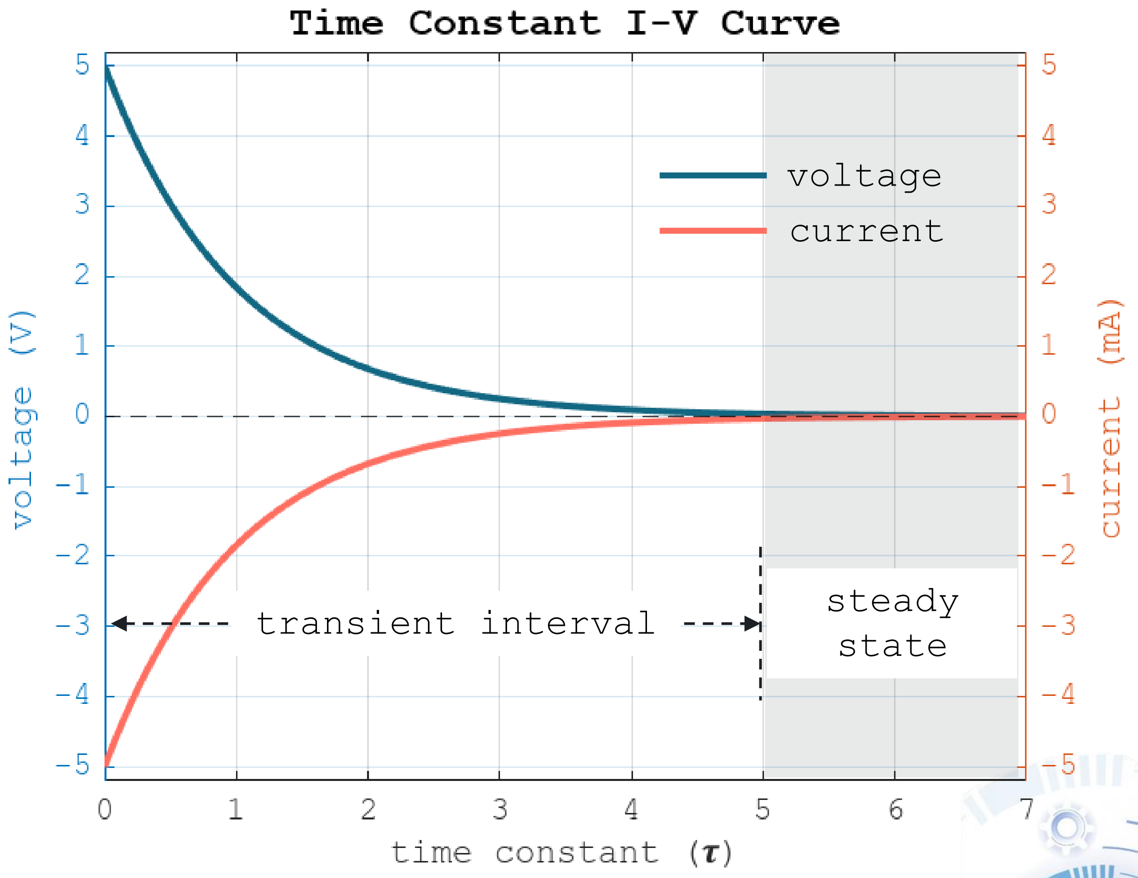

LABORATORY