GRAPH OF TRIGONOMETRIC FUNCTIONS

TRIGONOMETRIC FUNCTION OF ANGLES

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TOPIC OUTLINE

Graph of Trigonometric Functions

Amplitude and Period of Sine Function

GRAPH OF TRIGONOMETRIC FUNCTIONS

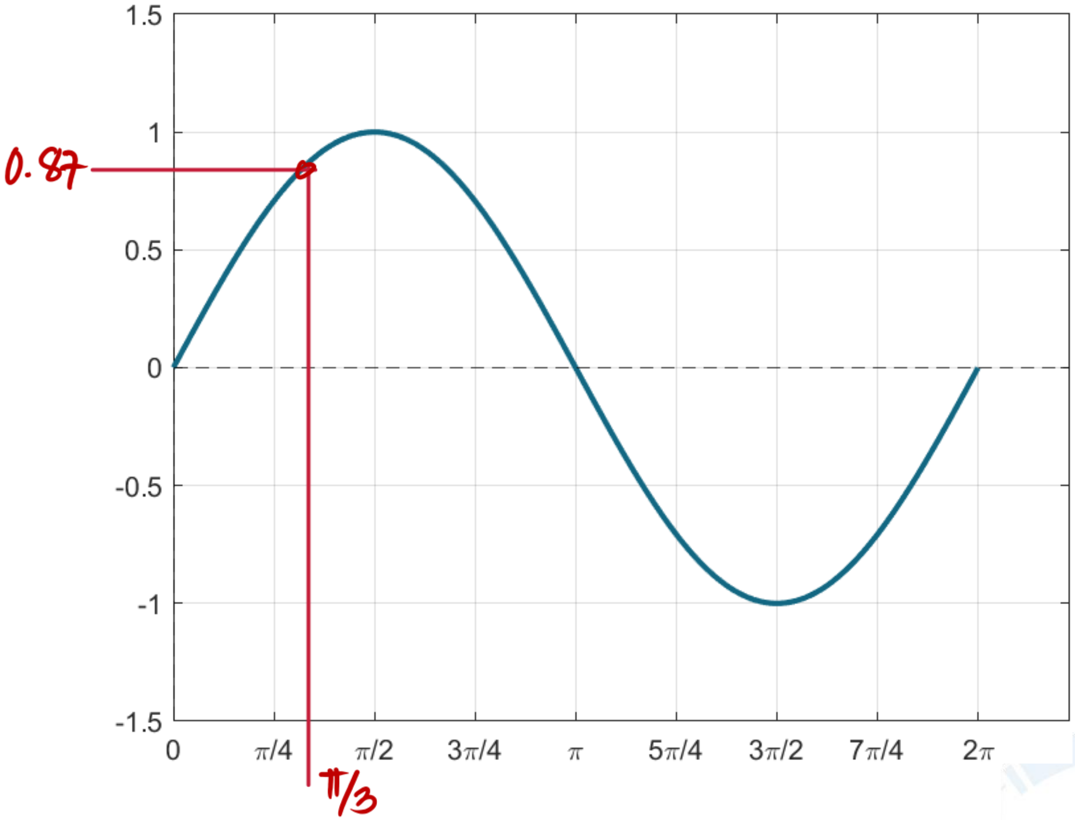
SINE FUNCTION

$f(x) = \sin x$

x	$f(x)$
0	0
$\pi/6$	0.5
$\pi/4$	0.71
$\pi/3$	0.87
$\boldsymbol{\pi/2}$	1
$2\pi/3$	0.87
$3\pi/4$	0.71
$5\pi/6$	0.5
$\boldsymbol{\pi}$	0

x	$f(x)$
$7\pi/6$	−0.5
$5\pi/4$	−0.71
$4\pi/3$	−0.87
$\boldsymbol{3\pi/2}$	−1
$5\pi/3$	−0.87
$7\pi/4$	−0.71
$11\pi/6$	−0.5
$\boldsymbol{2\pi}$	0

Graph of $\sin x$



Domain: All real numbers $(-\infty, \infty)$

Range: $[-1, 1]$

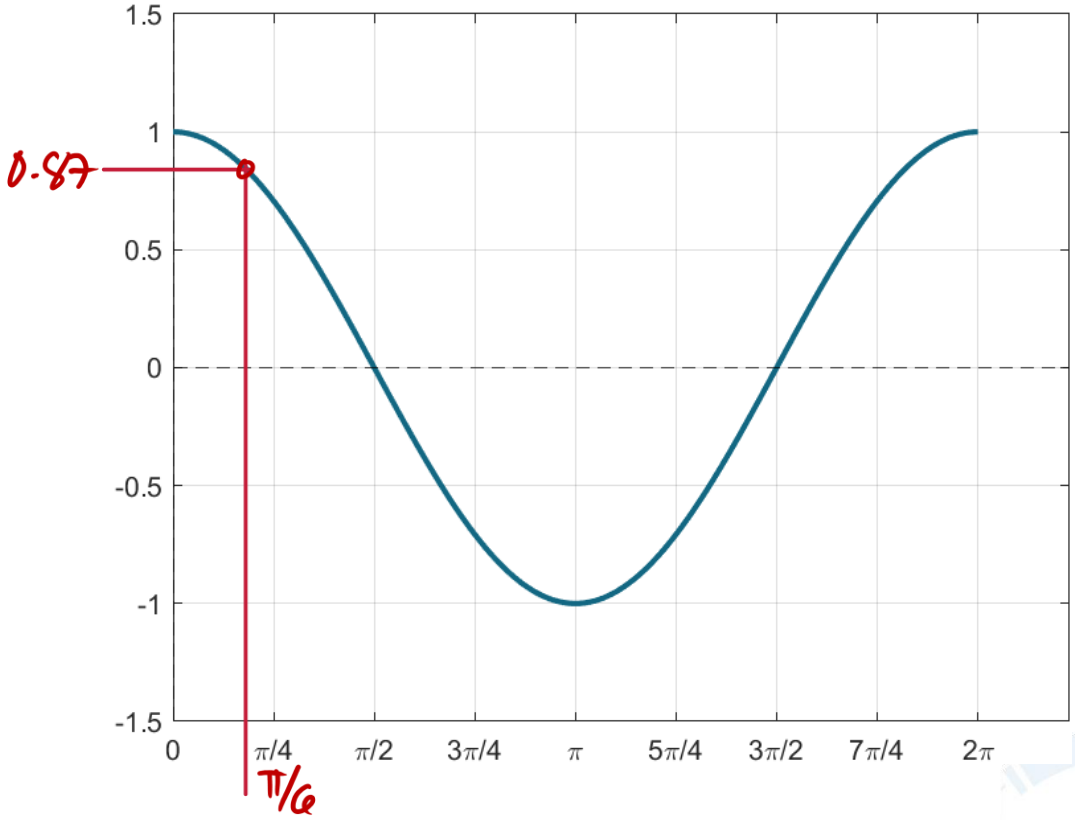
COSINE FUNCTION

$f(x) = \cos x$

x	$f(x)$
0	1
$\pi/6$	0.87
$\pi/4$	0.71
$\pi/3$	0.5
$\boldsymbol{\pi/2}$	0
$2\pi/3$	-0.5
$3\pi/4$	-0.71
$5\pi/6$	-0.87
$\boldsymbol{\pi}$	-1

x	$f(x)$
$7\pi/6$	-0.87
$5\pi/4$	-0.71
$4\pi/3$	-0.5
$\boldsymbol{3\pi/2}$	0
$5\pi/3$	0.5
$7\pi/4$	0.71
$11\pi/6$	0.87
$\boldsymbol{2\pi}$	1

Graph of $\cos x$



Domain: All real numbers $(-\infty, \infty)$

Range: $[-1, 1]$

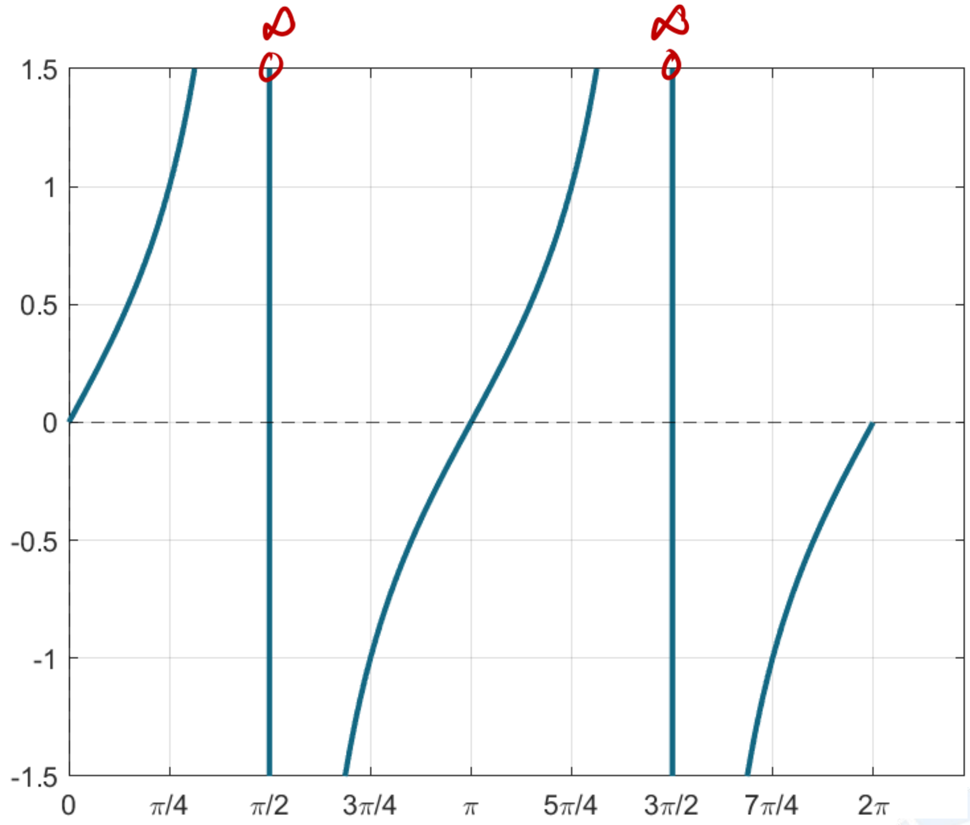
TANGENT FUNCTION

$f(x) = \tan x$

x	$f(x)$
0	0
$\pi/6$	0.58
$\pi/4$	1
$\pi/3$	1.73
$\boldsymbol{\pi/2}$	∞
$2\pi/3$	-1.73
$3\pi/4$	-1
$5\pi/6$	-0.58
$\boldsymbol{\pi}$	0

x	$f(x)$
$7\pi/6$	0.58
$5\pi/4$	1
$4\pi/3$	1.73
$\boldsymbol{3\pi/2}$	∞
$5\pi/3$	-1.73
$7\pi/4$	-1
$11\pi/6$	-0.58
$\boldsymbol{2\pi}$	0

Graph of $\tan x$



Domain: All real numbers except odd multiples of $\pi/2$

Range: $[-\infty, \infty]$

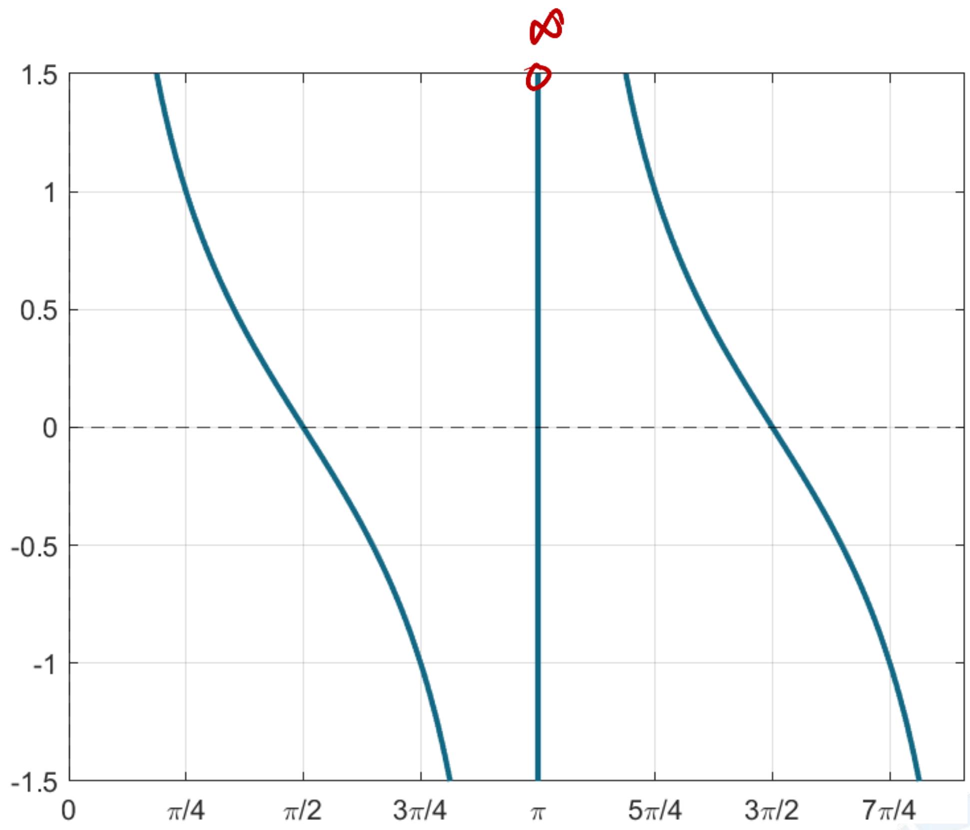
COTANGENT FUNCTION

$f(x) = \cot x$

x	$f(x)$
0	∞
$\pi/6$	1.73
$\pi/4$	1
$\pi/3$	0.58
$\boldsymbol{\pi/2}$	0
$2\pi/3$	−0.58
$3\pi/4$	−1
$5\pi/6$	−1.73
$\boldsymbol{\pi}$	∞

x	$f(x)$
$7\pi/6$	1.73
$5\pi/4$	1
$4\pi/3$	0.58
$\boldsymbol{3\pi/2}$	0
$5\pi/3$	−0.58
$7\pi/4$	−1
$11\pi/6$	−1.73
$\boldsymbol{2\pi}$	∞

Graph of $\cot x$



Domain: All real numbers except integer multiples of π

Range: $[-\infty, \infty]$

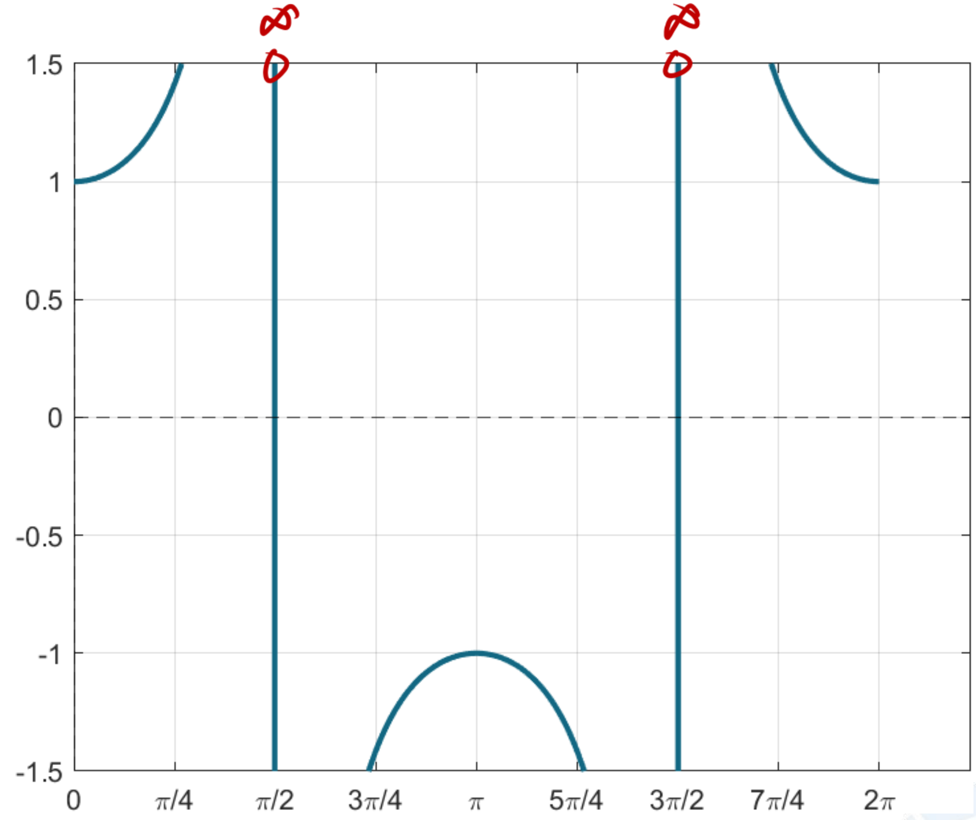
SECANT FUNCTION

$f(x) = \sec x$

x	$f(x)$
0	1
$\pi/6$	1.15
$\pi/4$	1.41
$\pi/3$	2
$\pi/2$	∞
$2\pi/3$	-2
$3\pi/4$	-1.41
$5\pi/6$	-1.15
π	-1

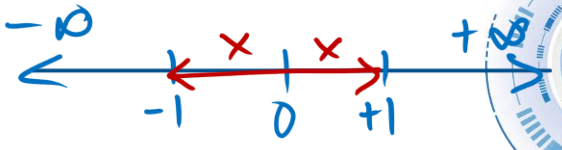
x	$f(x)$
$7\pi/6$	-1.15
$5\pi/4$	-1.41
$4\pi/3$	-2
$3\pi/2$	∞
$5\pi/3$	2
$7\pi/4$	1.41
$11\pi/6$	1.15
2π	1

Graph of $\sec x$



Domain: All real numbers except odd multiples of $\pi/2$

Range: $(-\infty, -1] \cup [1, \infty)$



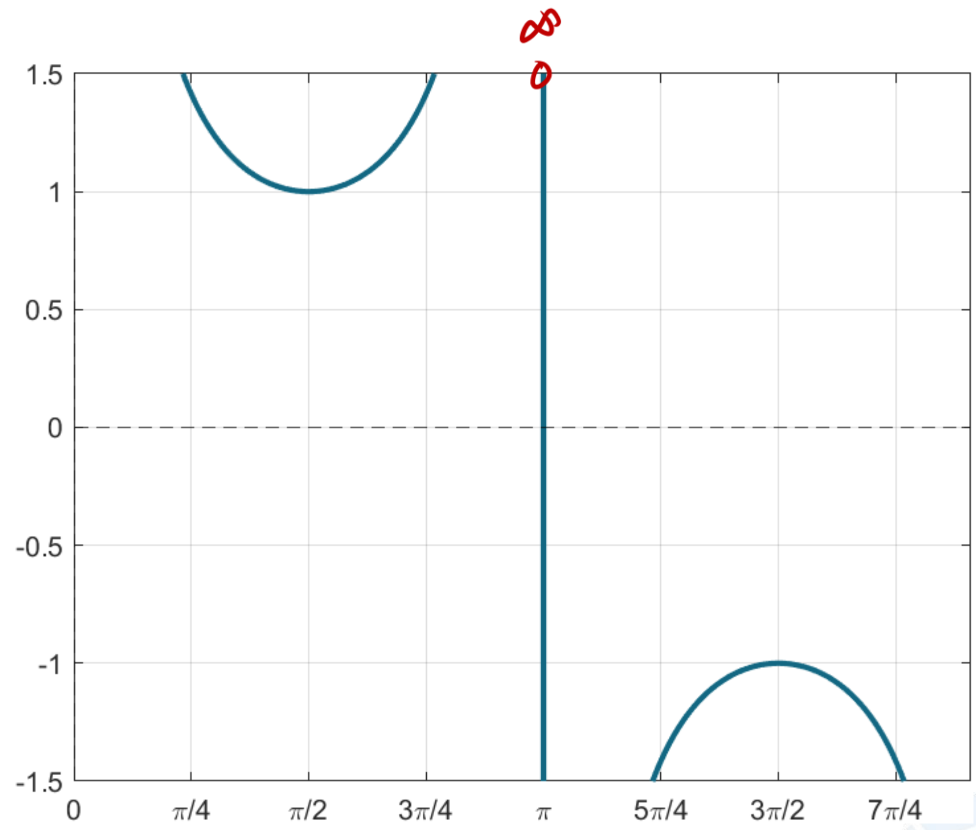
COSECANT FUNCTION

$f(x) = \csc x$

x	$f(x)$
0	∞
$\pi/6$	2
$\pi/4$	1.41
$\pi/3$	1.15
$\boldsymbol{\pi/2}$	1
$2\pi/3$	1.15
$3\pi/4$	1.41
$5\pi/6$	2
$\boldsymbol{\pi}$	∞

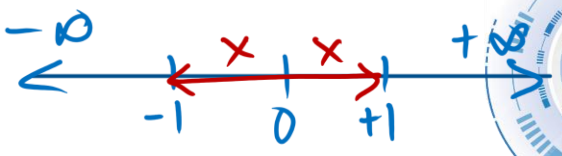
x	$f(x)$
$7\pi/6$	−2
$5\pi/4$	−1.41
$4\pi/3$	−1.15
$\boldsymbol{3\pi/2}$	−1
$5\pi/3$	−1.15
$7\pi/4$	−1.41
$11\pi/6$	−2
$\boldsymbol{2\pi}$	∞

Graph of $\csc x$



Domain: All real numbers except odd multiples of π

Range: $(-\infty, -1] \cup [1, \infty)$



AMPLITUDE AND PERIOD OF SINE FUNCTION

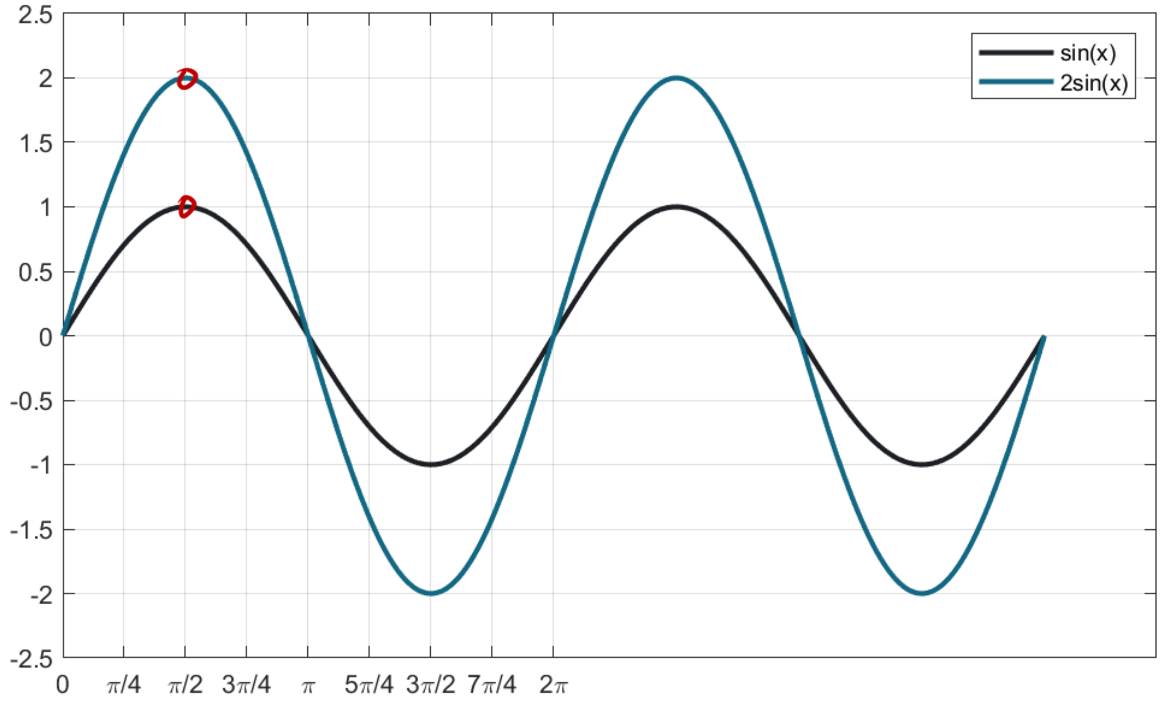
AMPLITUDE

$f(x) = a \sin x$

Let the amplitude $a = 2$

x	$\sin x$	$2 \sin x$
0	0	0
$\pi/4$	0.71	1.42
$\pi/2$	1	2
$3\pi/4$	0.71	1.42
π	0	0
$5\pi/4$	-0.71	-1.42
$3\pi/2$	-1	-2
$7\pi/4$	-0.71	-1.42
2π	0	0

Graph of $2 \sin x$



What would be the graph of $3 \sin x$?

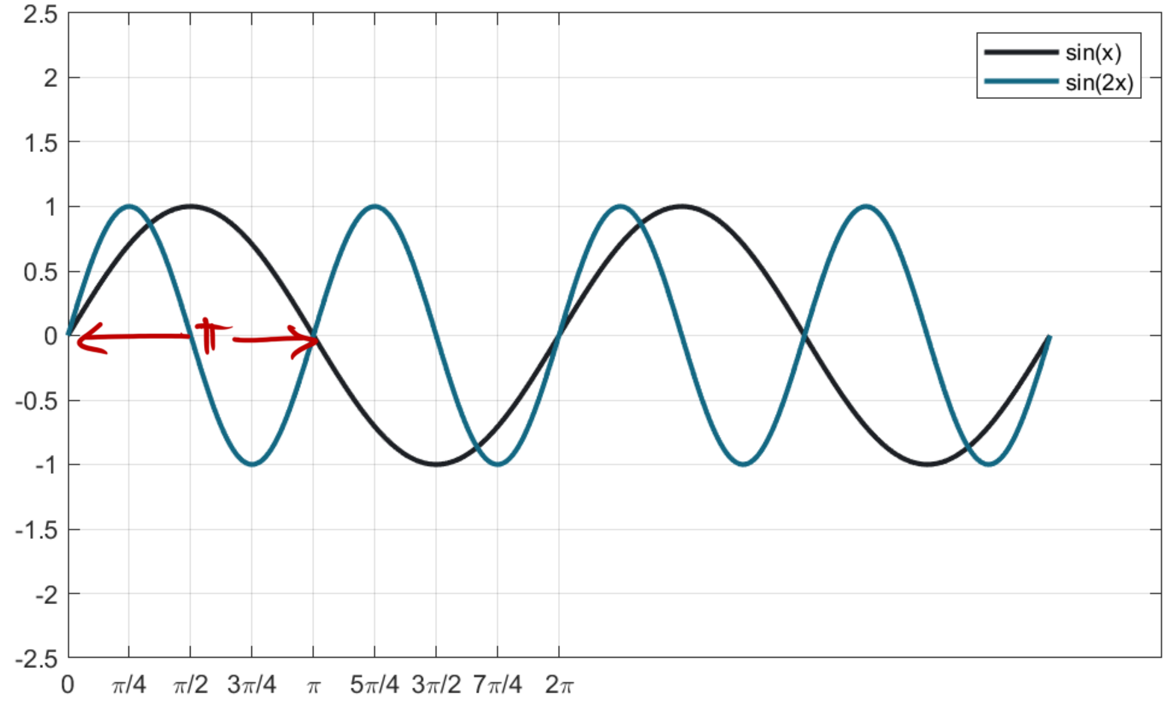
PERIOD

$f(x) = \sin bx$

Let $b = 2$

x	$\sin x$	$\sin 2x$
0	0	0
$\pi/4$	0.71	1
$\pi/2$	1	0
$3\pi/4$	0.71	-1
π	0	0
$5\pi/4$	-0.71	1
$3\pi/2$	-1	0
$7\pi/4$	-0.71	-1
2π	0	0

Graph of $\sin 2x$



$$\text{period} = \frac{2\pi}{b} = \frac{\overset{1}{\cancel{2}}\pi}{\cancel{2}} = \underline{\pi}$$

1 cycle

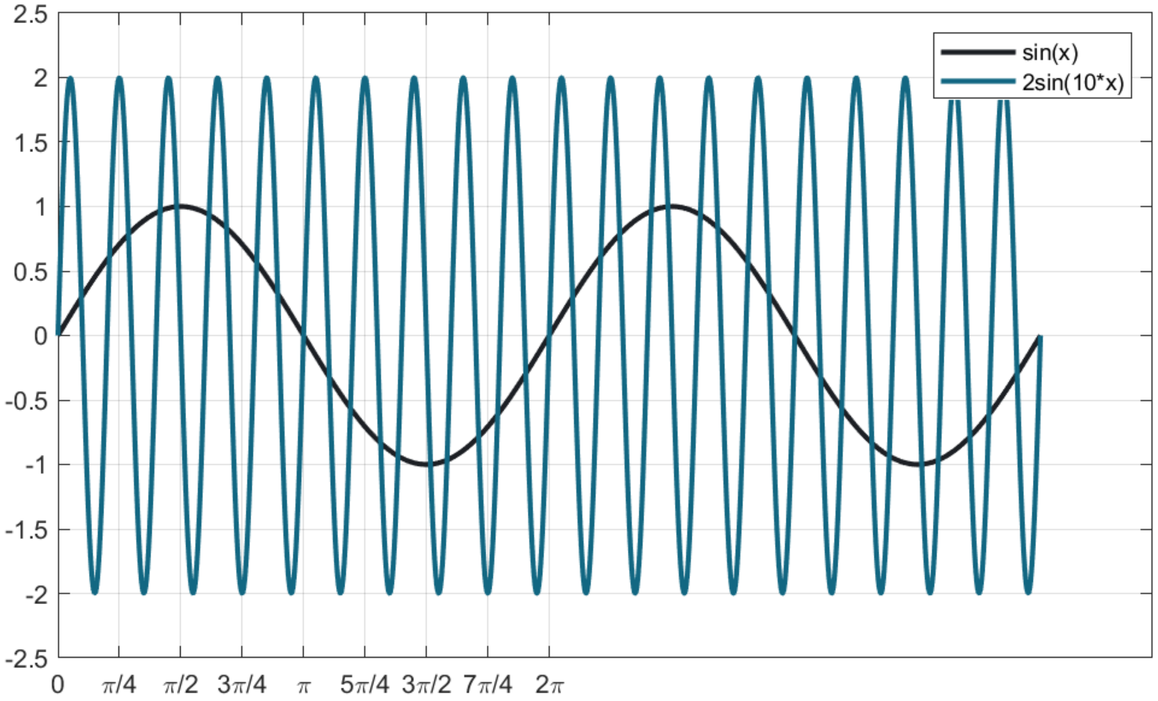
AMPLITUDE AND PERIOD

$f(x) = a \sin bx$

Let a $= 2$, $b = 10$

x	$\sin x$	$2\sin 10x$
0	0	0
$\pi/4$	0.71	2
$\pi/2$	1	0
$3\pi/4$	0.71	-2
π	0	0
$5\pi/4$	-0.71	2
$3\pi/2$	-1	0
$7\pi/4$	-0.71	-2
2π	0	0

Graph of $\sin 10x$



1 cycle

$$\text{period} = \frac{2\pi}{b} = \frac{2\pi}{10} = \frac{\pi}{5}$$

EXERCISE

The average temperature (in °F) at Mould Bay, Canada, can be approximated by the function

$$f(x) = 34 \sin\left[\frac{\pi}{6}(x - 4.3)\right]$$

where x is the month and $x = 1$ corresponds to January, $x = 2$ to February, and so on.

Using this model:

a. What is the maximum temperature predicted?

a

b. What is the period of the temperature cycle?

need "b"

c. What is the average temperature in May?

$x = 5$

Solution

$$f(x) = \underbrace{34}_{a} \sin\left[\frac{\pi}{6}x - \frac{4.3}{x}\right]$$

a. $\boxed{a = 34\ {}^\circ F}$

ans

EXERCISE

The average temperature (in °F) at Mould Bay, Canada, can be approximated by the function

$$f(x) = 34\sin\left[\frac{\pi}{6}(x - 4.3)\right]$$

where x is the month and $x = 1$ corresponds to January, $x = 2$ to February, and so on.

Using this model:

a

a. What is the maximum temperature predicted?

need "b"

b. What is the period of the temperature cycle?

c. What is the average temperature in May?

$x=5$

Solution

$$f(x) = 34\sin\left[\underbrace{\frac{\pi}{6}x}_{bx} - \frac{4.3}{x}\right]$$

b. $b\cancel{x} = \frac{\pi}{6}\cancel{x}$

$b = \frac{\pi}{6}$

$\text{period} = \frac{2\pi}{b}$

$\text{period} = \frac{2\pi}{\pi/6} \longrightarrow 2\cancel{\pi} \cdot \frac{6}{\cancel{\pi}}$

$\boxed{\text{period} = 12 \text{ months}}$

ans

EXERCISE

The average temperature (in °F) at Mould Bay, Canada, can be approximated by the function

$$f(x) = 34 \sin\left[\frac{\pi}{6}(x - 4.3)\right]$$

where x is the month and $x = 1$ corresponds to January, $x = 2$ to February, and so on.

Using this model:

a. What is the maximum temperature predicted? a

b. What is the period of the temperature cycle? need "b"

c. What is the average temperature in May? $x = 5$

Solution

c. $f(5) = 34 \sin\left[\frac{\pi}{6}(5 - 4.3)\right]$

$$\boxed{f(5) = 12.18\ °F}$$

ans

EXERCISE

The light from the moon, in lux, on the night of the day t^{th} of 2016, is

$$L(t) = 0.25 - \sin\left(\frac{2\pi(t-2)}{28.5}\right)$$

What is the period of the light from the moon?

need "b"

Solution

$$L(t) = 0.25 - \sin\left[\underbrace{\frac{2\pi}{28.5}t}_{bt} - \frac{4\pi}{28.5}\right]$$

$$\text{period} = \frac{2\pi}{b}$$

$$\text{period} = \frac{2\pi}{2\pi/28.5} \longrightarrow \cancel{2\pi} \cdot \frac{28.5}{\cancel{2\pi}}$$

$$\boxed{\text{period} = 28.5 \text{ days}}$$

ans

EXERCISE

The solar constant S is the amount of energy per unit area that reaches Earth's atmosphere from the sun. It is equal to 1367 watts per m^2 but varies slightly throughout the seasons. This fluctuation ΔS in S can be calculated using the formula

$$\boldsymbol{\Delta S = 0.034S \sin\left[\frac{2\pi(82.5 - N)}{365.25}\right]}$$

In this formula, N is the day number covering a four-year period, where $N = 1$ corresponds to January 1 of a leap year and $N = 1461$ corresponds to December 31 of the fourth year.

a. Calculate ΔS for $N = 80$, which is the spring equinox in the first year.

b. Calculate ΔS for $N = 1268$, which is the summer solstice in the fourth year.

c. What is the maximum value of ΔS?

d. Find a value for N where ΔS is equal to 0.

EXERCISE

The solar constant S is the amount of energy per unit area that reaches Earth's atmosphere from the sun. It is equal to 1367 watts per m^2 but varies slightly throughout the seasons. This fluctuation ΔS in S can be calculated using the formula

S

$$\Delta S = 0.034S \sin\left[\frac{2\pi(82.5 - N)}{365.25}\right]$$

In this formula, N is the day number covering a four-year period, where $N = 1$ corresponds to January 1 of a leap year and $N = 1461$ corresponds to December 31 of the fourth year.

a. Calculate ΔS for $N = 80$, which is the spring equinox in the first year.

$$\Delta S = 0.034(1367)\sin\left[\frac{2\pi(82.5-80)}{365.25}\right]$$

$$\boxed{\Delta S = 1.998\ W/m^2}$$

ans

EXERCISE

The solar constant S is the amount of energy per unit area that reaches Earth's atmosphere from the sun. It is equal to 1367 watts per m^2 but varies slightly throughout the seasons. This fluctuation ΔS in S can be calculated using the formula

S

$$\Delta S = 0.034S \sin\left[\frac{2\pi(82.5 - N)}{365.25}\right]$$

In this formula, N is the day number covering a four-year period, where $N = 1$ corresponds to January 1 of a leap year and $N = 1461$ corresponds to December 31 of the fourth year.

b. Calculate ΔS for $N = 1268$, which is the summer solstice in the fourth year.

$$\Delta S = 0.034(1367)\sin\left[\frac{2\pi(82.5 - 1268)}{365.25}\right]$$

$$\boxed{\Delta S = -46.461 \ W/m^2}$$

ans

EXERCISE

The solar constant S is the amount of energy per unit area that reaches Earth's atmosphere from the sun. It is equal to $\underset{S}{\underline{1367}}$ watts per m^2 but varies slightly throughout the seasons. This fluctuation ΔS in S can be calculated using the formula

$$\Delta S = 0.034S \sin\left[\frac{2\pi(82.5 - N)}{365.25}\right]$$

In this formula, N is the day number covering a four-year period, where $N = 1$ corresponds to January 1 of a leap year and $N = 1461$ corresponds to December 31 of the fourth year.

c. What is the maximum value of ΔS?

$a = 0.034(1367)$

$\boxed{a = 46.478 \ W/m^2}$

ans

EXERCISE

The solar constant S is the amount of energy per unit area that reaches Earth's atmosphere from the sun. It is equal to 1367 watts per m^2 but varies slightly throughout the seasons. This fluctuation ΔS in S can be calculated using the formula

$$\Delta S = 0.034S \sin\left[\frac{2\pi(82.5 - N)}{365.25}\right]$$

In this formula, N is the day number covering a four-year period, where $N = 1$ corresponds to January 1 of a leap year and $N = 1461$ corresponds to December 31 of the fourth year.

d. Find a value for N where ΔS is equal to 0.

$$\frac{0 = 0.034S \sin\left[\frac{2\pi(82.5-N)}{365.2\pi}\right]}{0.034S}$$

$$\sin^{-1}(0) = \frac{2\pi(82.5-N)}{365.2\pi}$$

$$0 = 82.5 - N$$
$$+N \qquad +N$$

$$\boxed{N = 82.5 \text{ days}}$$

ans

SEATWORK