ELEMENT COMBINATION RULE

BASIC CIRCUIT ANALYSIS METHOD

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TOPIC OUTLINE

Ohm's Law

Series Network

Parallel Network

Series-Parallel Network

OHM'S LAW

OHM'S LAW

Ohm's Law states that the ratio of voltage ($\boldsymbol{V}$) to current ($\boldsymbol{I}$) is **constant** ($\boldsymbol{R}$).

Mathematical representation

$$\boldsymbol{R = \frac{V}{I}}$$

Basic Electrical Quantities

1. **Voltage** (**V**)

The measure of electrical potential energy per unit charge. It is the "**push**" or "force" that drives electric current through a circuit.

Formula

$$\boldsymbol{V = IR}$$

unit: Volt (V)

OHM'S LAW

Ohm's Law states that the ratio of voltage $(\boldsymbol{V})$ to current $(\boldsymbol{I})$ is **constant** $(\boldsymbol{R})$.

Mathematical representation

$$R = \frac{V}{I}$$

Basic Electrical Quantities

2. Current (I)

The **flow of electric charge**, typically carried by electrons in a conductor. It represents the rate at which charge flows through a point in a circuit.

Formula

$$I = \frac{V}{R}$$

unit: Ampere (A)

OHM'S LAW

Ohm's Law states that the ratio of voltage ($\boldsymbol{V}$) to current ($\boldsymbol{I}$) is **constant** ($\boldsymbol{R}$).

Mathematical representation

$$\boldsymbol{R} = \frac{\boldsymbol{V}}{\boldsymbol{I}}$$

Basic Electrical Quantities

3. **Resistance** (**R**)

The **opposition** to the flow of electric current in a material or component. It determines how much current will flow for a given voltage.

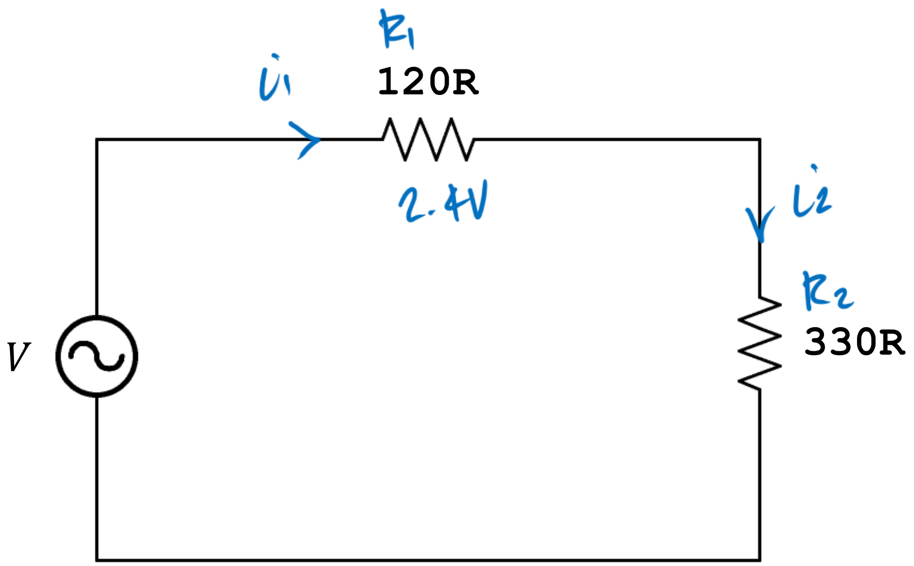
Formula

$$\boldsymbol{R} = \frac{\boldsymbol{V}}{\boldsymbol{I}}$$

unit: Ohm (Ω)

EXERCISE

In the given circuit, the voltage drop across a 120Ω resistor is measured as 2.4V. Determine the current flowing through the resistors.



Solution

$$i_1 = \frac{v_1}{R_1}$$

$$= \frac{2.4}{120}$$

$$\boxed{i_1 = 20mA}$$

ans

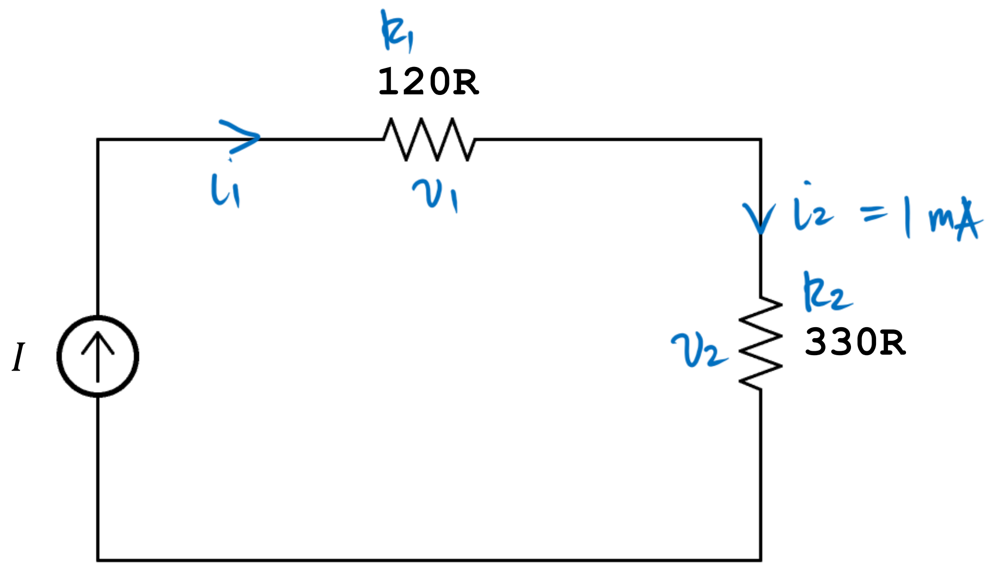
$$\boxed{i_2 = 20mA}$$

ans

Series network : $i_1 = i_2$

EXERCISE

A 330 Ω resistor in the given circuit carries a current of $1mA$. Calculate the voltage drop across the resistors.



Solution

$v_2 = i_2 R_2$

$v_2 = (1m)(330)$

$\boxed{v_2 = 330\,mV}$ ans

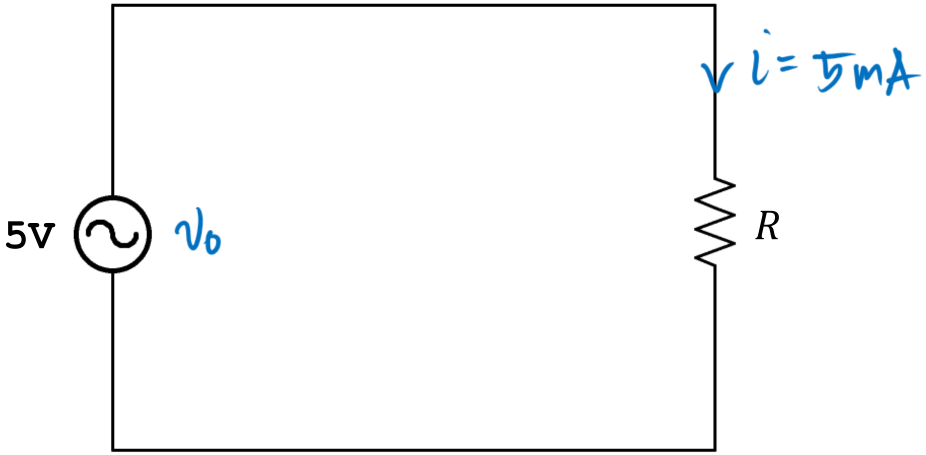
$v_1 = i_1 R_1$

$v_1 = (1m)(120)$

$\boxed{v_1 = 120\,mV}$ ans

EXERCISE

The given circuit has an applied voltage of 5V, resulting in a current flow of 5mA. Determine the resistance of the circuit.



Solution

$$R = \frac{v_o}{i}$$

$$= \frac{5}{5m}$$

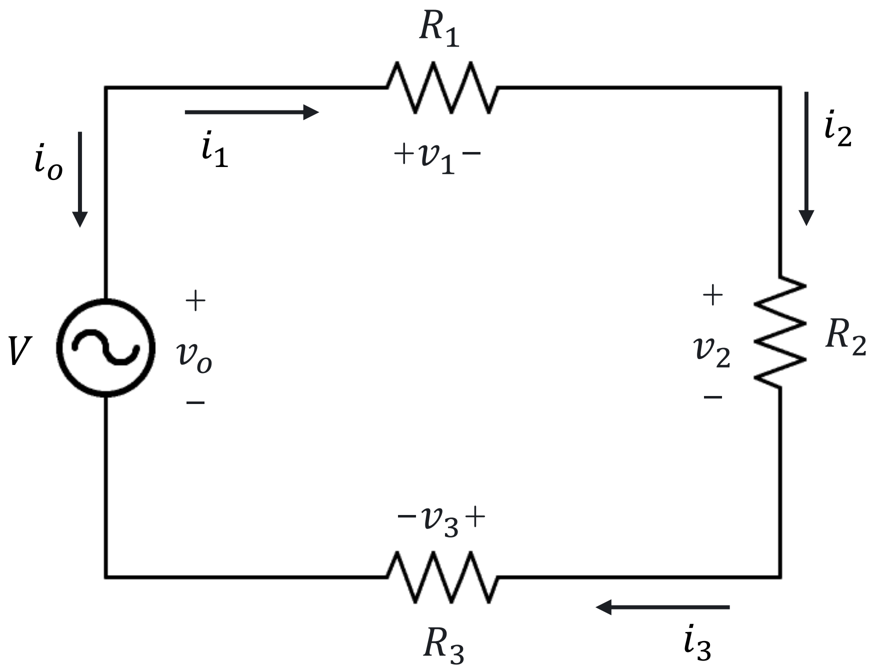
$$\boxed{R = 1k\Omega}$$

ans

SERIES NETWORK

SERIES NETWORK

Series Network

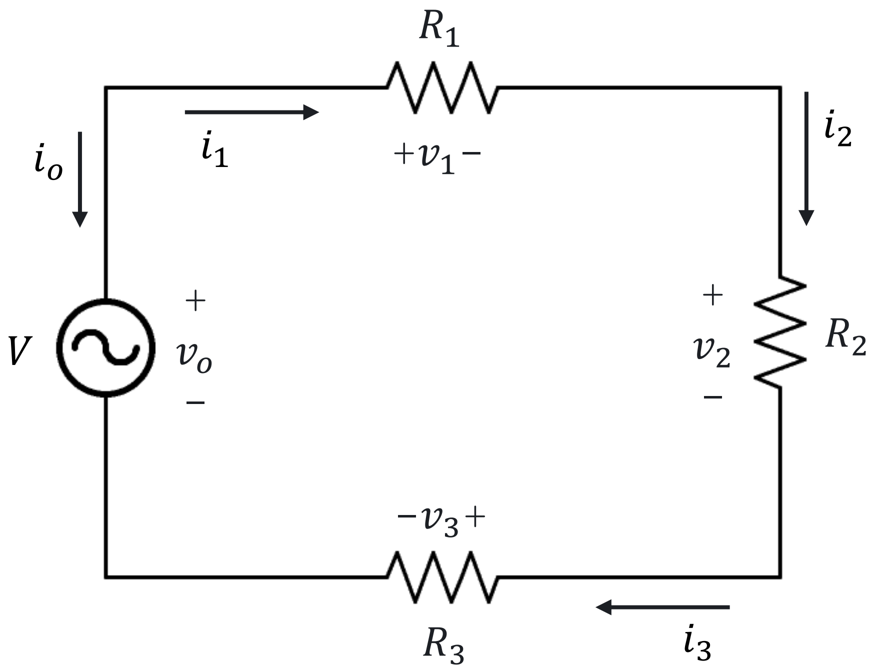


A **series network** refers to a configuration where components are connected end-to-end, forming a **single path** for current to flow.

CURRENT

Series Network

In a series network, the **same current** flows through all components.

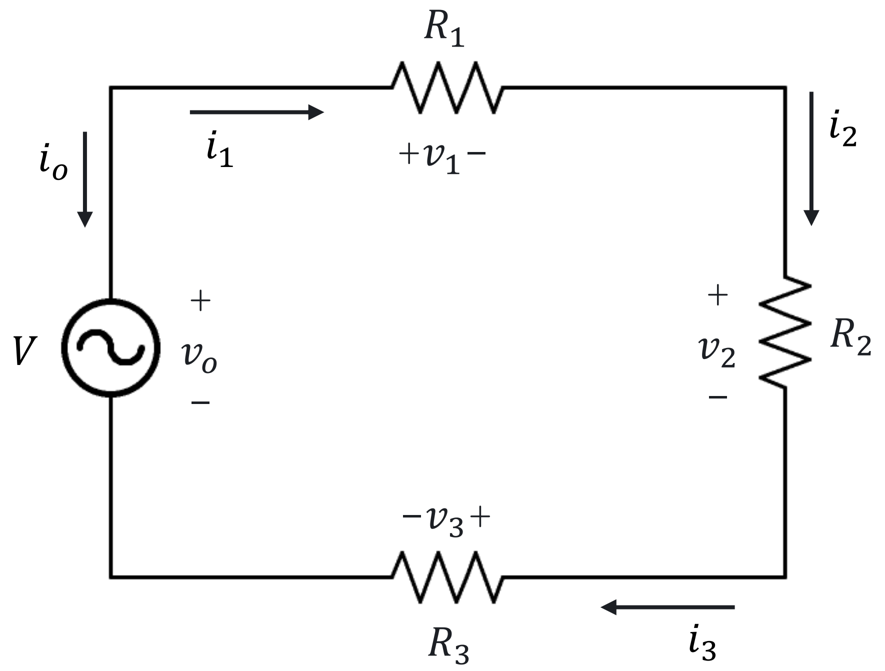


Mathematical representation

$$\boldsymbol{i_o = i_1 = i_2 = i_3 = \cdots i_n}$$

RESISTANCE

Series Network



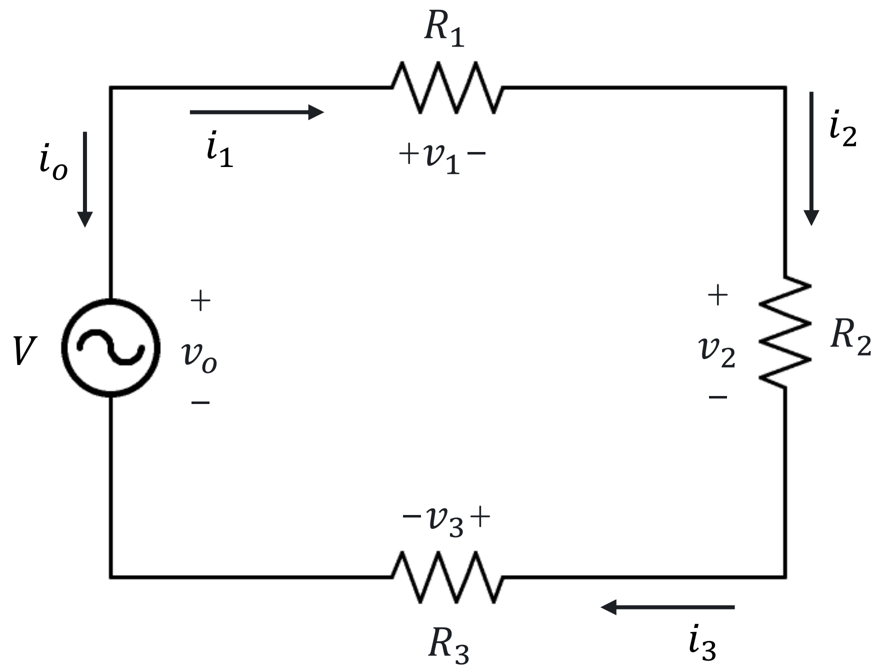
In a series network, the **total resistance** is the **sum** of the individual resistances.

Mathematical representation

$$R_o = R_1 + R_2 + R_3 + \cdots R_n$$

VOLTAGE

Series Network



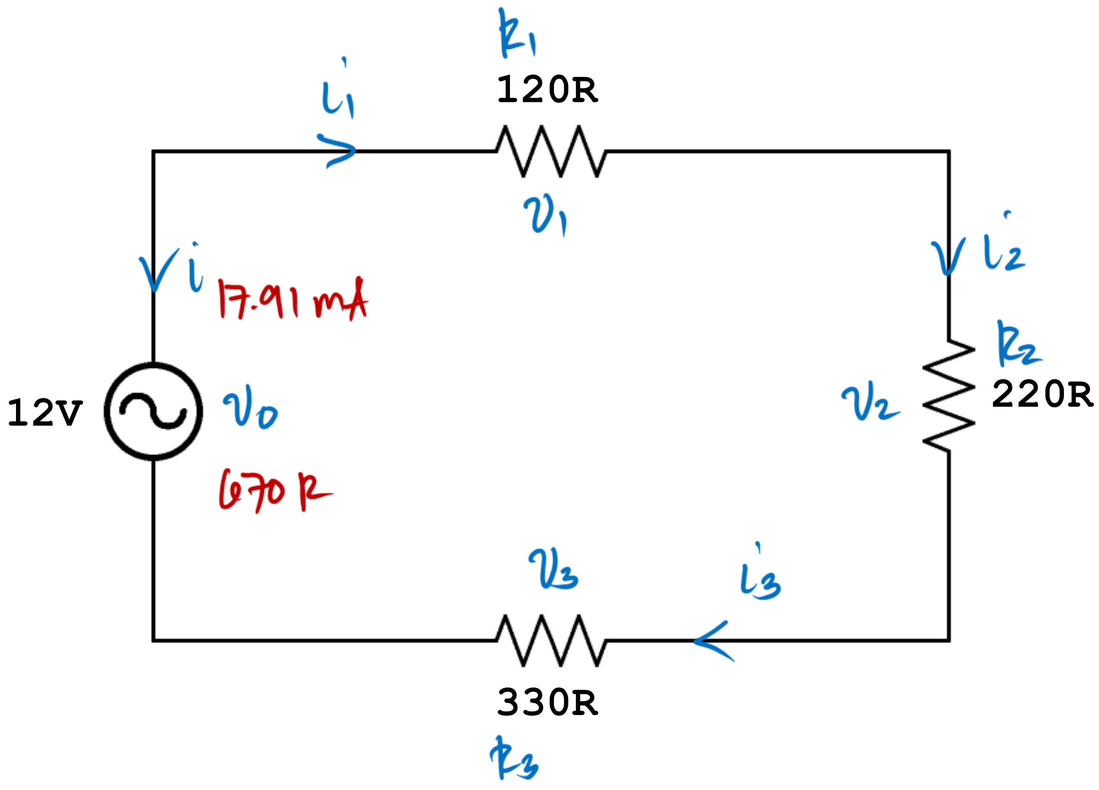
In a series network, the **total voltage** is the **sum** of the voltages across each individual component.

Mathematical representation

$$\boldsymbol{v_o = v_1 + v_2 + v_3 + \cdots v_n}$$

EXERCISE

For the given series circuit, determine the voltage drops across each individual resistor.



Solution

Total Resistance

$R_0 = R_1 + R_2 + R_3$

$R_0 = 120 + 220 + 330$

$R_0 = 670\Omega$

Total Current

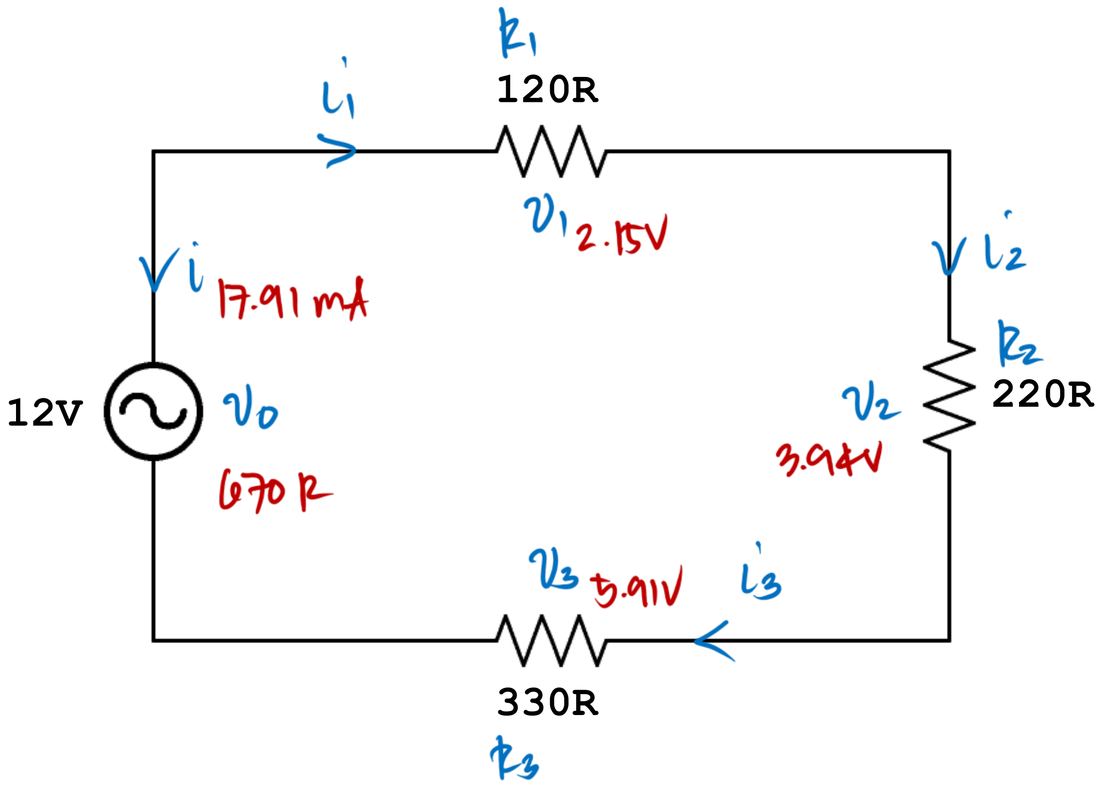
$i_0 = \frac{V_0}{R_0}$

$i_0 = \frac{12}{670}$

$i_0 = 17.91\ mA$

EXERCISE

For the given series circuit, determine the voltage drops across each individual resistor.



Solution

Series network

$i_o = i_1 = i_2 = i_3$

$v_1 = i_1 R_1$

$v_1 = 17.91m(120)$

$\boxed{v_1 = 2.15V}$ ans

$v_2 = i_1 R_2$

$v_2 = 17.91m(220)$

$\boxed{v_2 = 3.94V}$ ans

$v_3 = i_3 R_3$

$v_3 = 17.91m(330)$

$\boxed{v_3 = 5.91\ V}$ ans

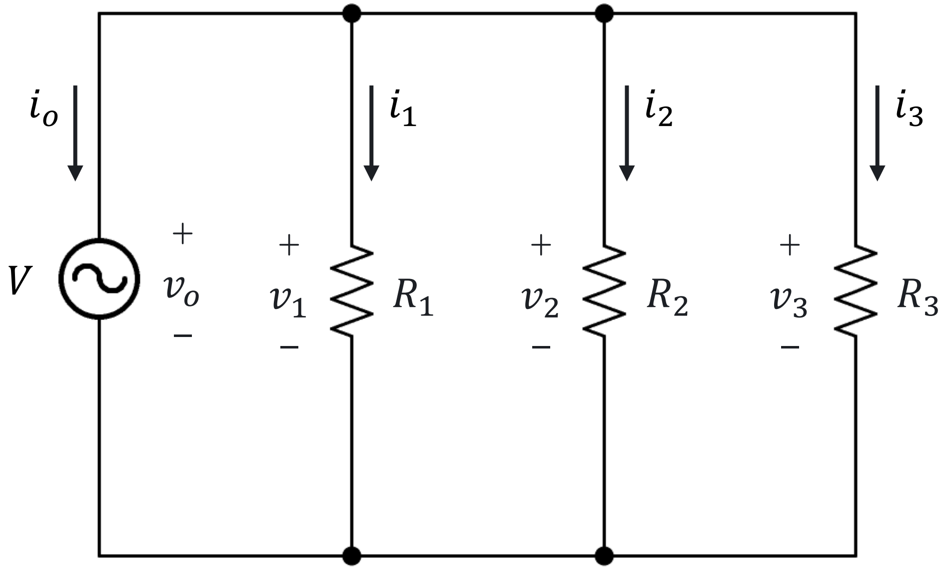
$v_o = v_1 + v_2 + v_3$

$v_o = 2.15 + 3.94 + 5.91$

$v_o = 12V$ ✓

PARALLEL NETWORK

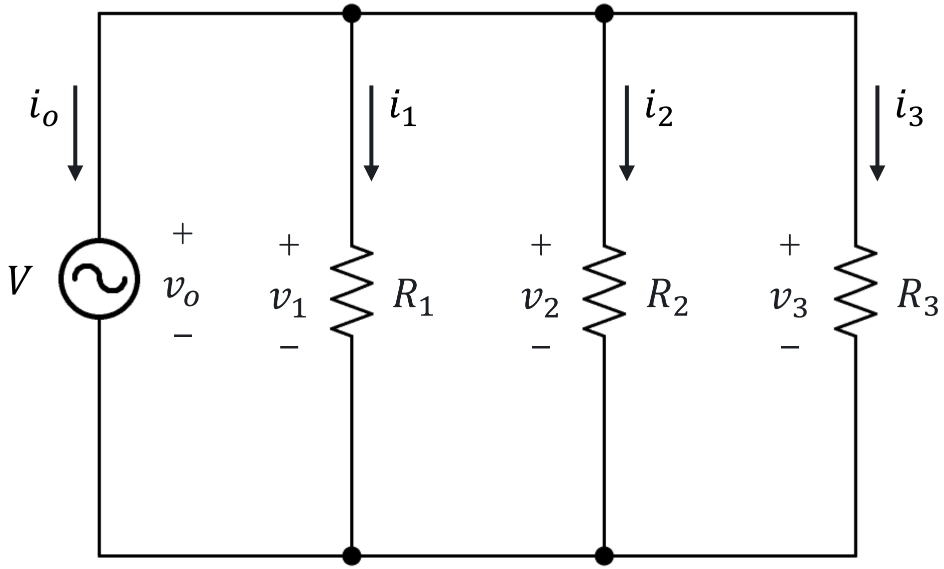
Parallel Network



A **parallel network** is a configuration where components are connected across the same two points, providing **multiple paths** for current to flow.

VOLTAGE

Parallel Network



In a parallel network, the **voltage** is the **same** across all components.

Mathematical representation

$$\boldsymbol{v_o = v_1 = v_2 = v_3 = \cdots v_n}$$

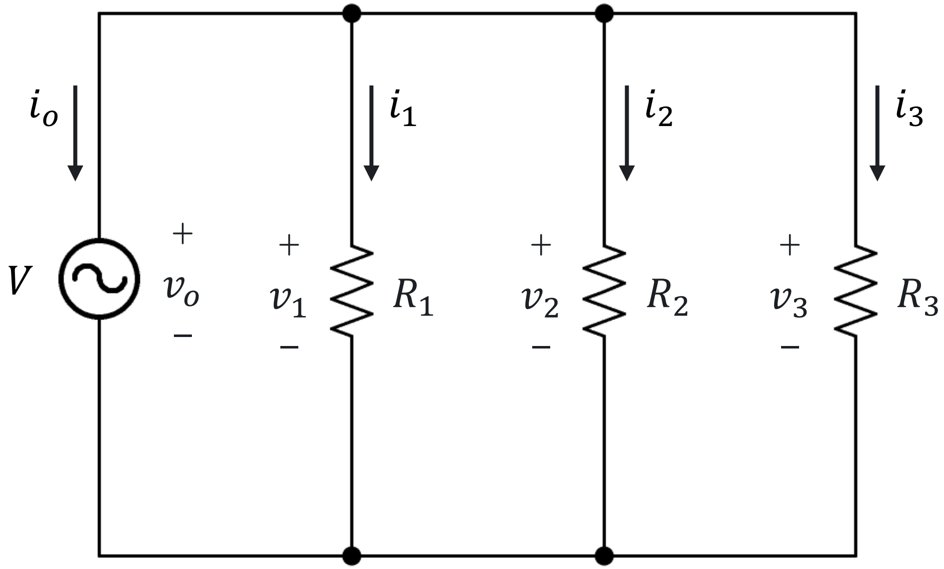
CONDUCTANCE

Conductance refers to the ability of the network to allow the flow of electric current. It is the **reciprocal of resistance** and is measured in siemens ($\boldsymbol{S}$).

Mathematical representation

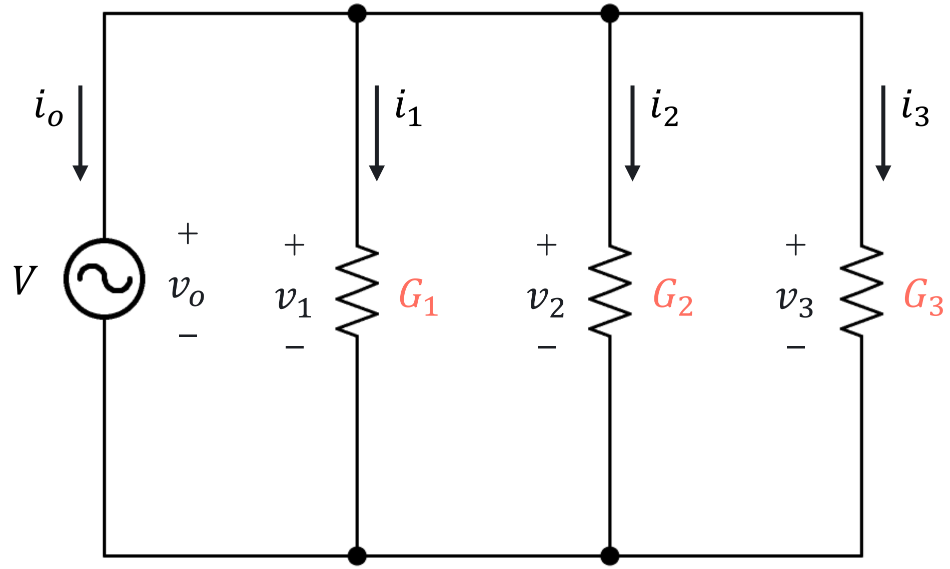
$$G = \frac{1}{R}$$

Parallel Network



CONDUCTANCE

Parallel Network



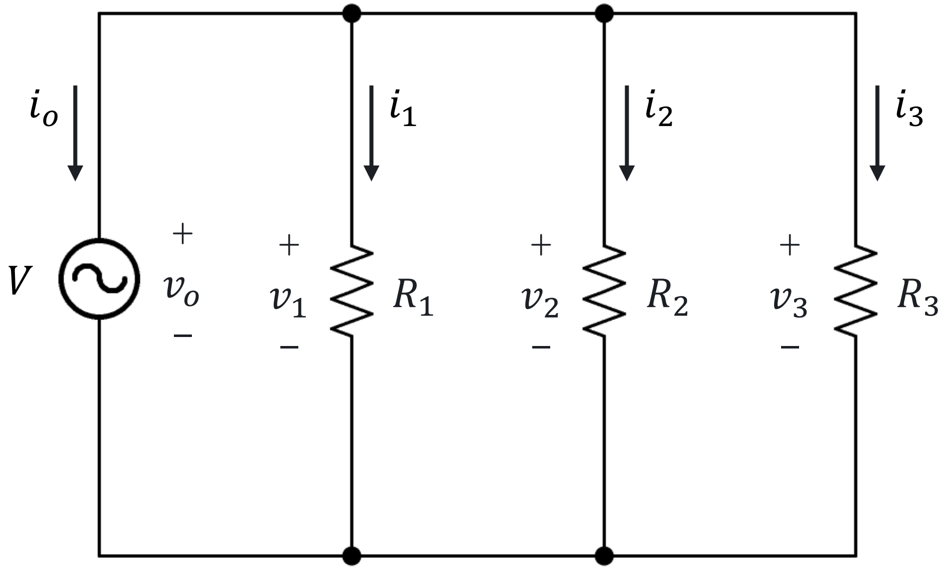
In a parallel network, the **total conductance** is the **sum** of the individual conductance of each resistor.

Mathematical representation

$$\boldsymbol{G_o = G_1 + G_2 + G_3 + \cdots G_n}$$

CURRENT

Parallel Network



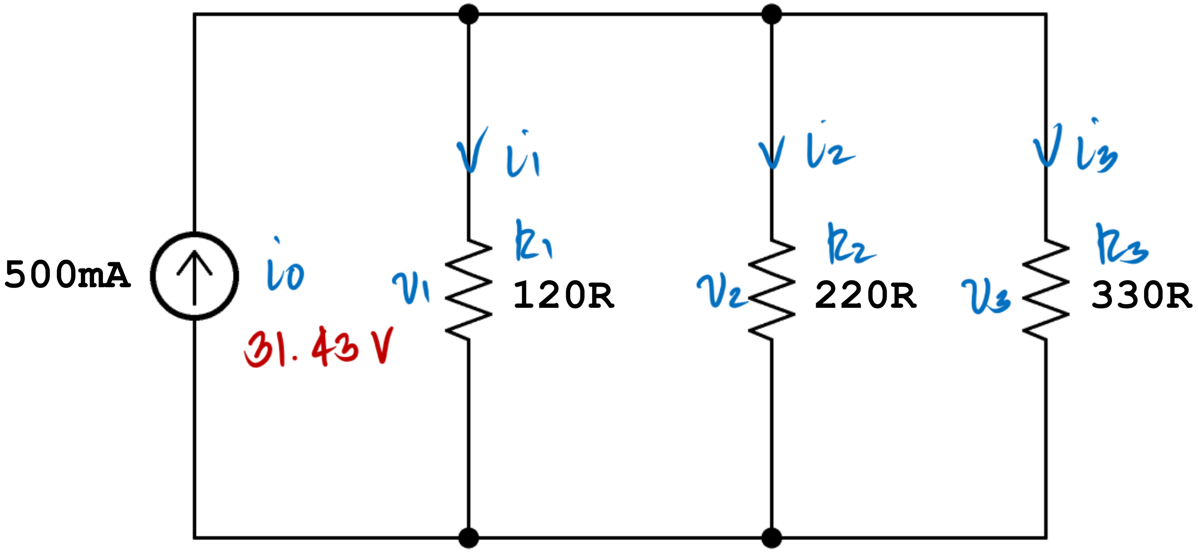
In a parallel network, the **total current** is the **sum** of the current flowing through each individual component.

Mathematical representation

$$\boldsymbol{i_o = i_1 + i_2 + i_3 + \cdots i_n}$$

EXERCISE

For the given parallel circuit, determine the current flowing through each individual resistor.



Solution

Total Conductance

$$G_0 = G_1 + G_2 + G_3$$

$$G_0 = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$G_0 = \frac{1}{120} + \frac{1}{220} + \frac{1}{330}$$

$$G_0 = \frac{7}{440} \, S$$

Total Resistance

$$R_0 = \frac{1}{G_0}$$

$$R_0 = \frac{440}{7} \, \Omega$$

Total Voltage

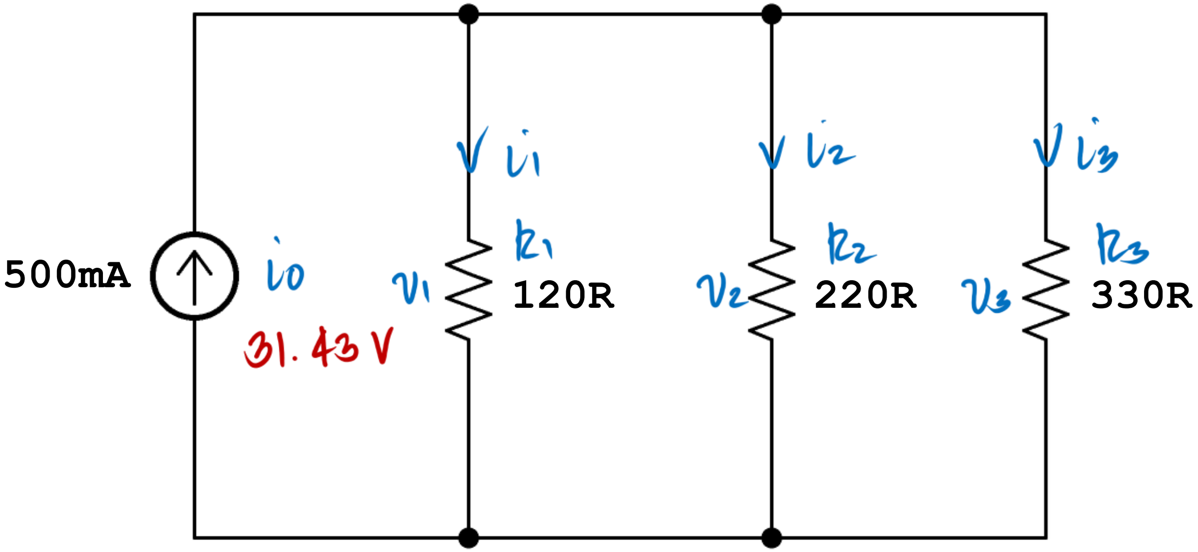
$$v_0 = i_0 R_0$$

$$v_0 = 500\,m\left(\frac{440}{7}\right)$$

$$v_0 = 31.43 \, V$$

EXERCISE

For the given parallel circuit, determine the current flowing through each individual resistor.



$i_o = 261.92 + 142.86 + 95.24 = \underline{500.02\ mA}$ ✓

Solution

Parallel Network

$v_o = v_1 = v_2 = v_3$

$i_1 = \dfrac{v_1}{R_1}$

$i_1 = \dfrac{31.43}{120}$

$\boxed{i_1 = 261.92\ mA}$ ans

$i_2 = \dfrac{v_2}{R_2}$

$= \dfrac{31.43}{220}$

$\boxed{i_2 = 142.86\ mA}$ ans

$i_3 = \dfrac{v_3}{R_3}$

$= \dfrac{31.43}{330}$

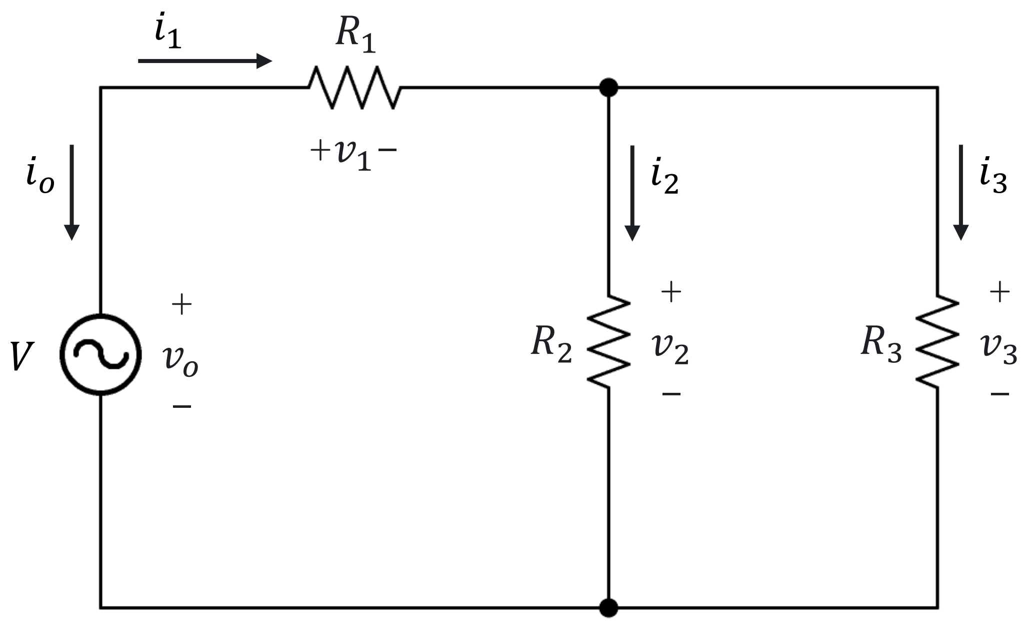
$\boxed{i_3 = 95.24\ mA}$ ans

SERIES PARALLEL NETWORK

SERIEL-PARALLEL NETWORK

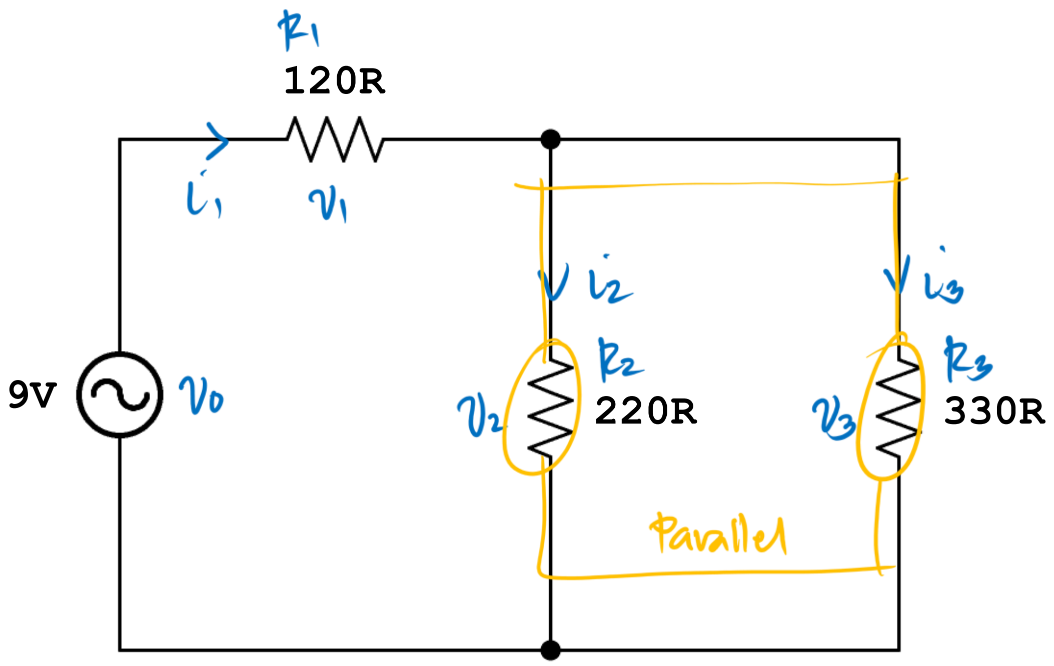
A **series-parallel** network is a type of electrical network that combines elements of both **series** and **parallel** circuits. These networks are commonly used in electrical and electronic systems to achieve desired voltage, current, and resistance characteristics.

Series-Parallel Network

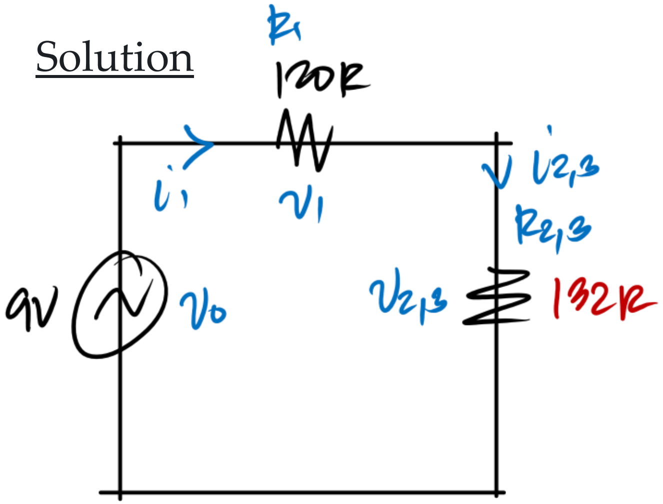


EXERCISE

Analyze the given circuit to determine both the current through and the voltage drop across each resistor.



Solution



Equivalent resistance of $R_2 \parallel R_3$

$$G_{2,3} = G_2 + G_3$$

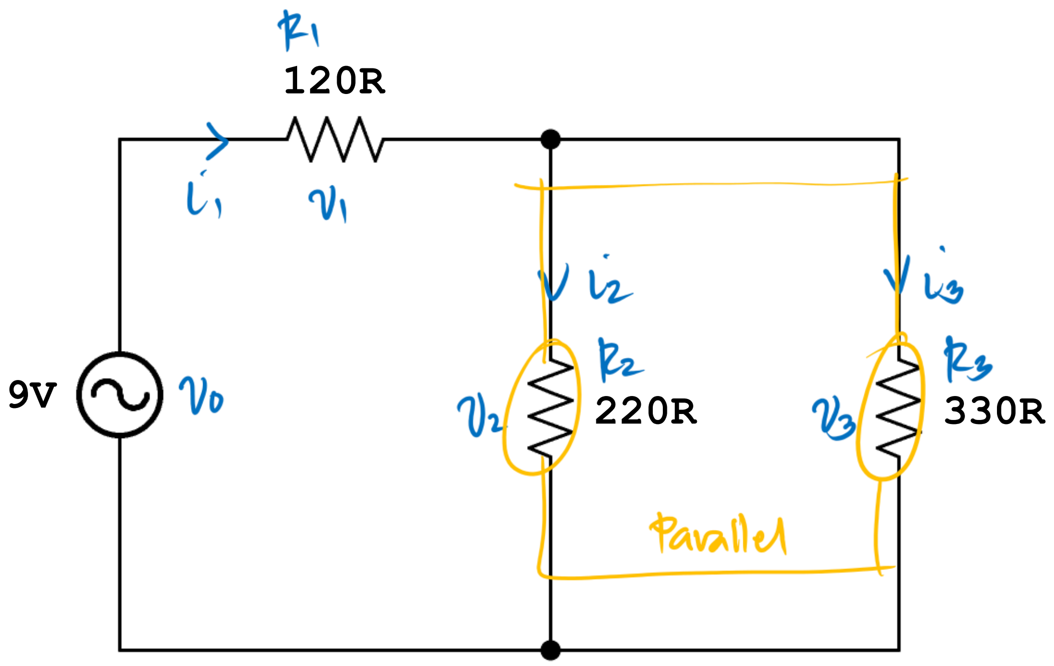
$$\frac{1}{R_{2,3}} = \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_{2,3}} = \frac{1}{220} + \frac{1}{330}$$

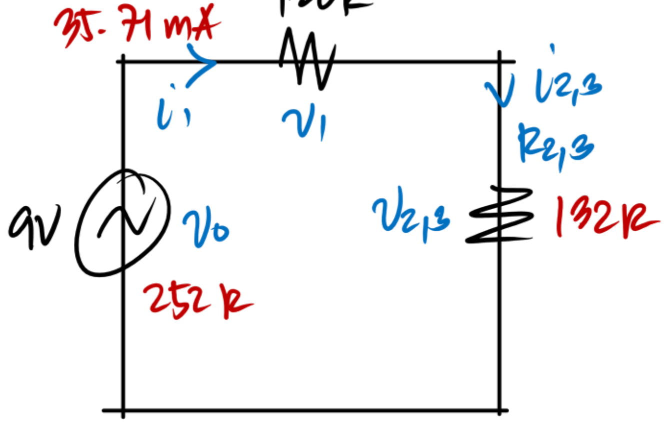
$$\underline{R_{2,3} = 132\,\Omega}$$

EXERCISE

Analyze the given circuit to determine both the current through and the voltage drop across each resistor.



Solution



Total Resistance

$R_o = R_1 + R_{2,3}$

$R_o = 120 + 132$

$R_o = 252\,\Omega$

Total Current

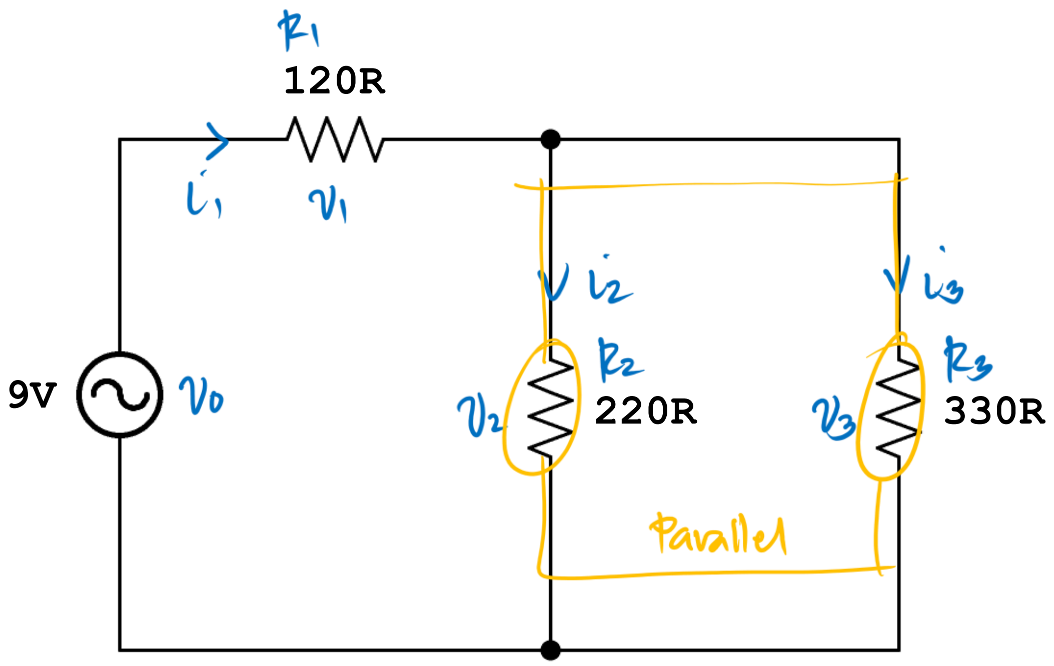
$i_o = \dfrac{V_o}{R_o}$

$= \dfrac{9}{252}$

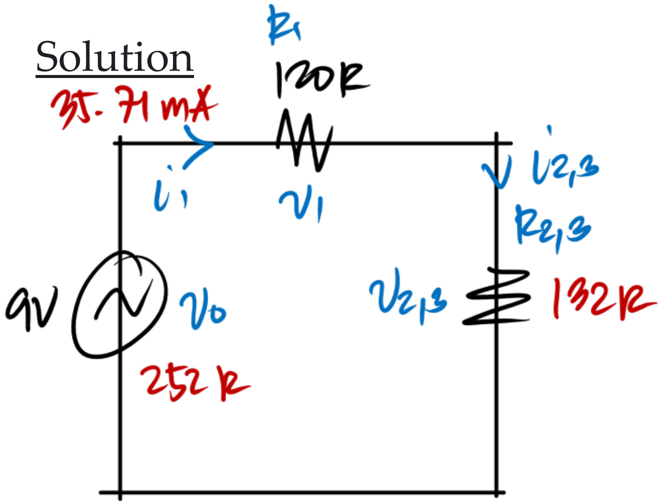
$i_o = 35.71\,mA$

EXERCISE

Analyze the given circuit to determine both the current through and the voltage drop across each resistor.



Solution



$v_1 = i_1 R_1$

$v_1 = 35.71m(120)$

$\boxed{v_1 = 4.28\ V}$ ans

$v_{2,3} = i_{2,3} R_{2,3}$

$v_{2,3} = 35.71m(132)$

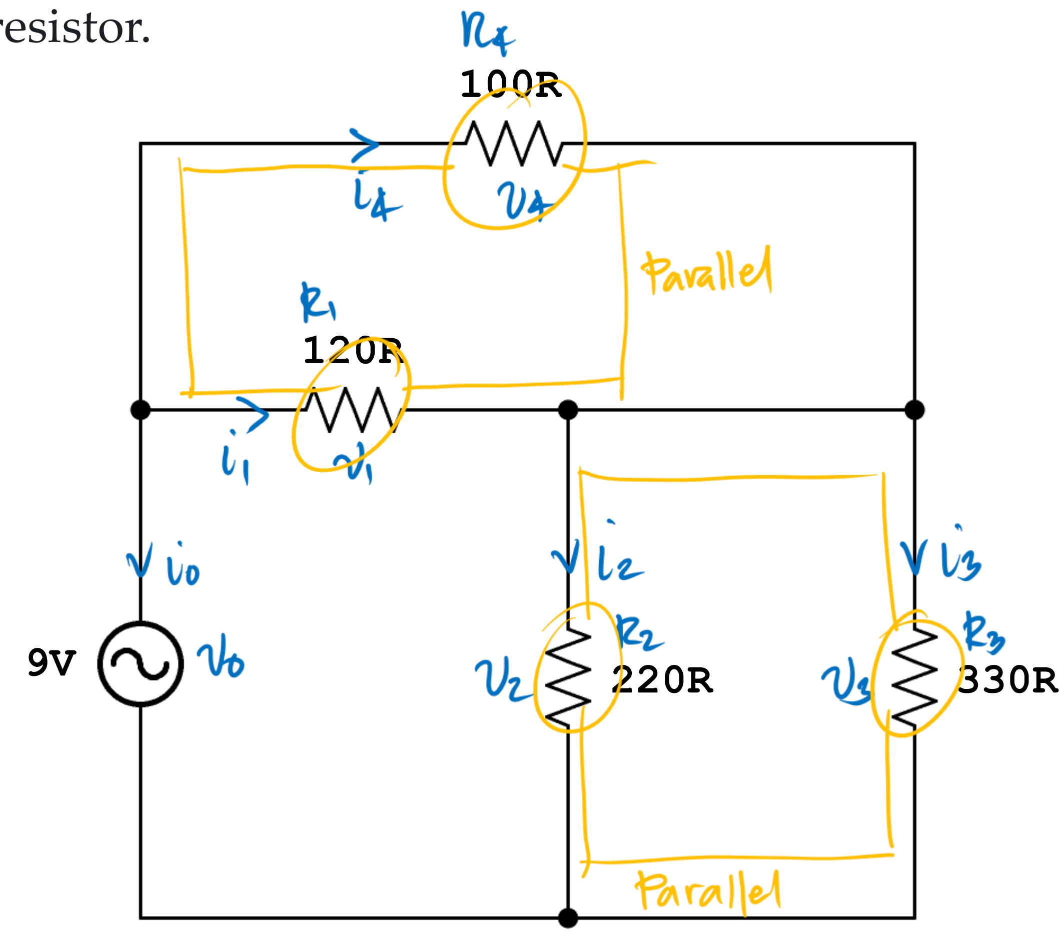
$v_{2,3} = 4.71\ V$

$\boxed{v_2 = 4.71\ V}$ ans

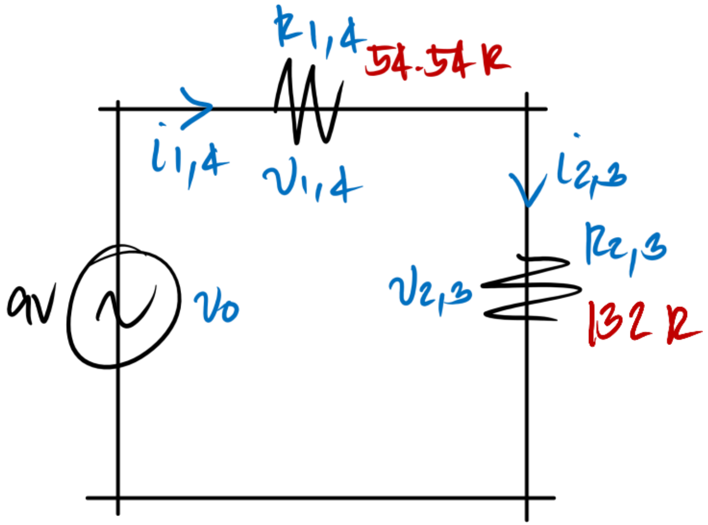
$\boxed{v_3 = 4.71\ V}$ ans

EXERCISE

Analyze the given circuit to determine both the current through and the voltage drop across each resistor.



Solution



$$G_{1,4} = G_1 + G_4$$

$$\frac{1}{R_{1,4}} = \frac{1}{R_1} + \frac{1}{R_4}$$

$$\frac{1}{R_{1,4}} = \frac{1}{120} + \frac{1}{100}$$

$$\underline{R_{1,4} = 54.54\,\Omega}$$

$$G_{2,3} = G_2 + G_3$$

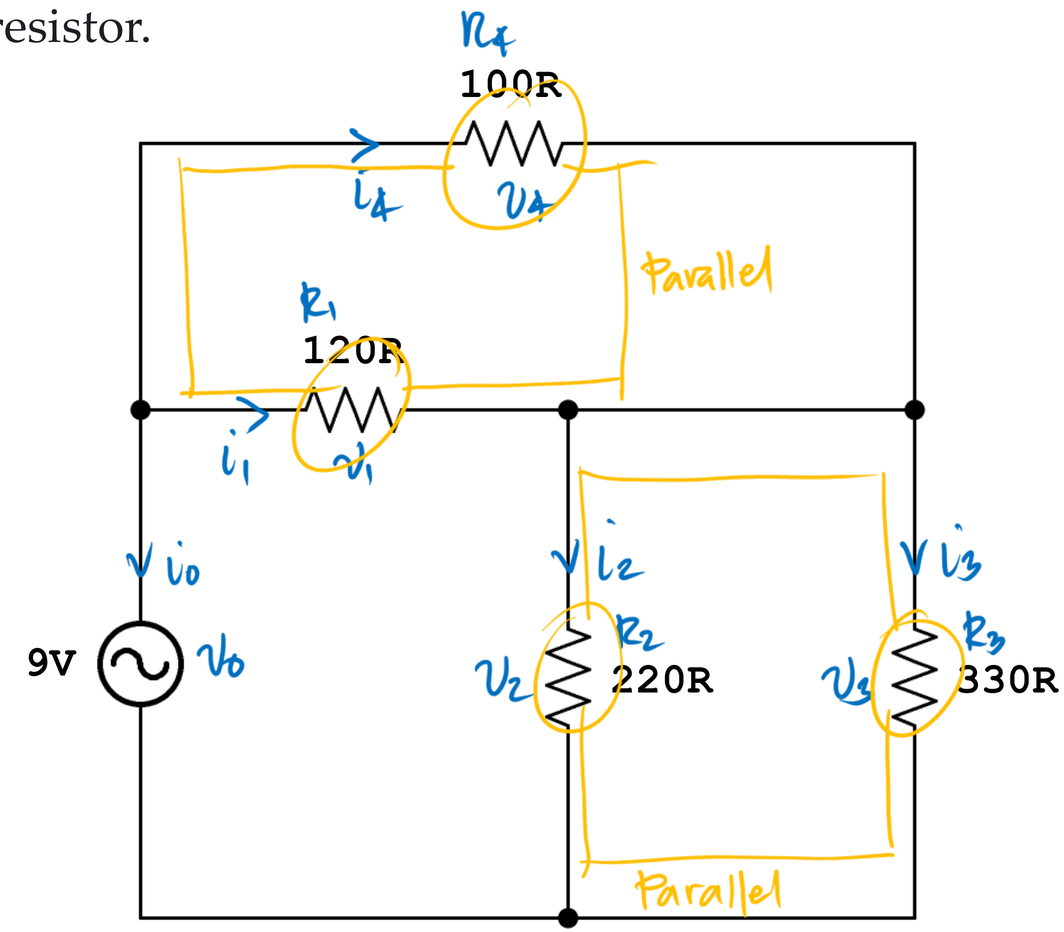
$$\frac{1}{R_{2,3}} = \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_{2,3}} = \frac{1}{220} + \frac{1}{330}$$

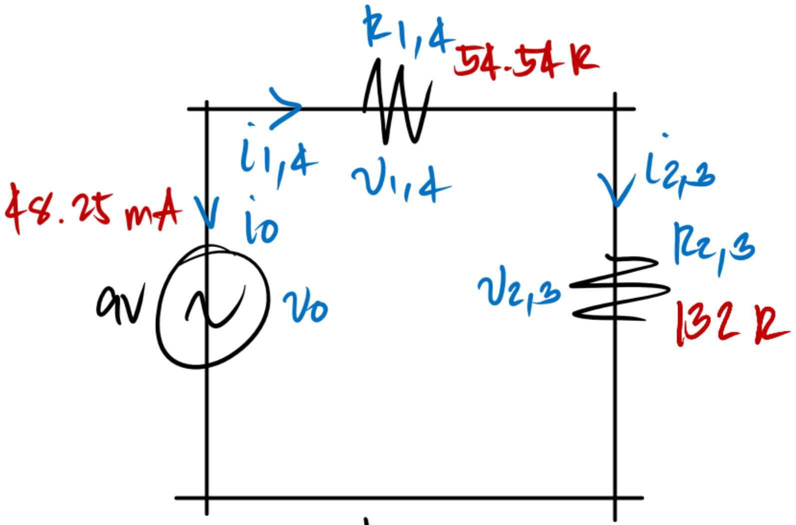
$$\underline{R_{2,3} = 132\,\Omega}$$

EXERCISE

Analyze the given circuit to determine both the current through and the voltage drop across each resistor.



Solution



$$i_o = \frac{v_o}{R_o}$$

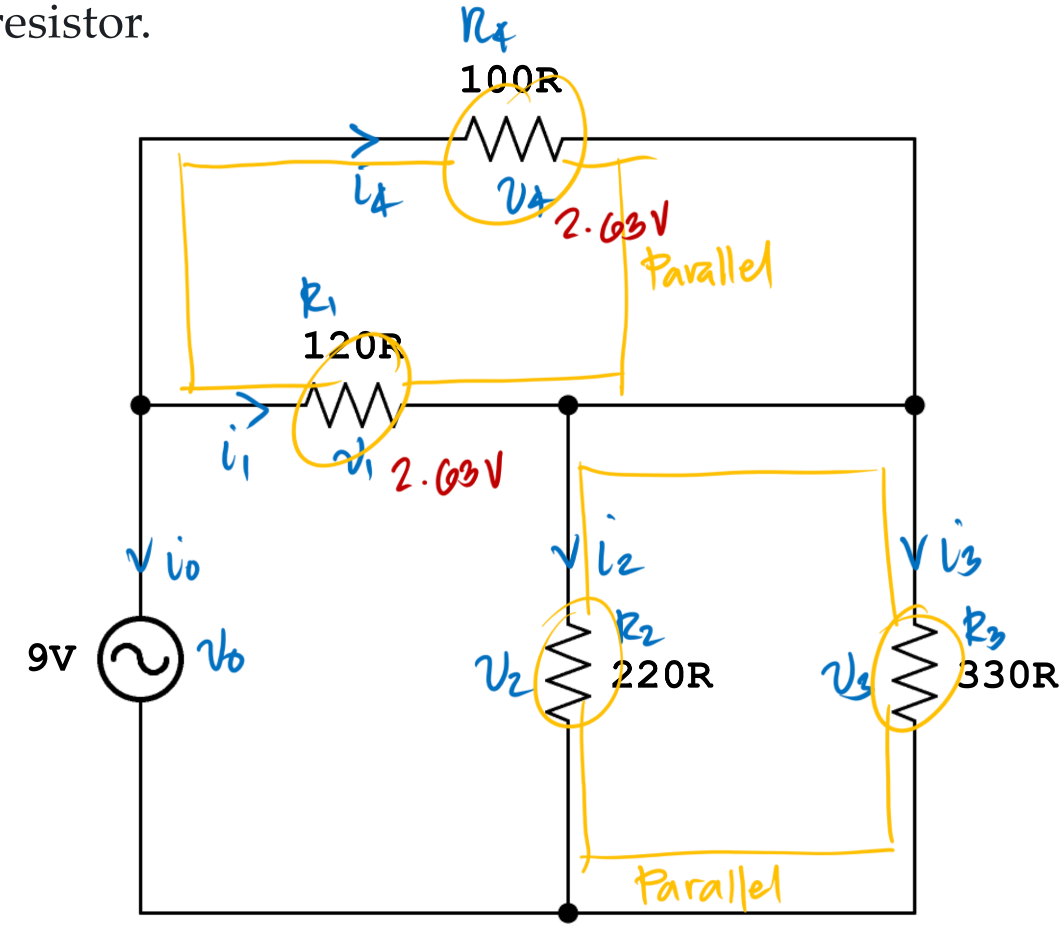
$$i_o = \frac{v_o}{R_{1,4} + R_{2,3}}$$

$$i_o = \frac{9}{54.54 + 132}$$

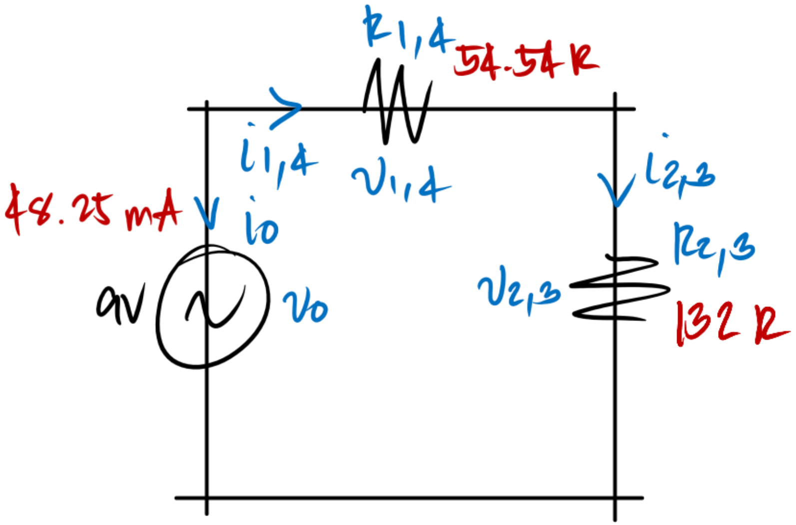
$$i_o = \underline{48.25\ mA}$$

EXERCISE

Analyze the given circuit to determine both the current through and the voltage drop across each resistor.



Solution



$$v_{1,4} = i_{1,4} R_{1,4}$$

$$= 48.25\,m\,(54.54)$$

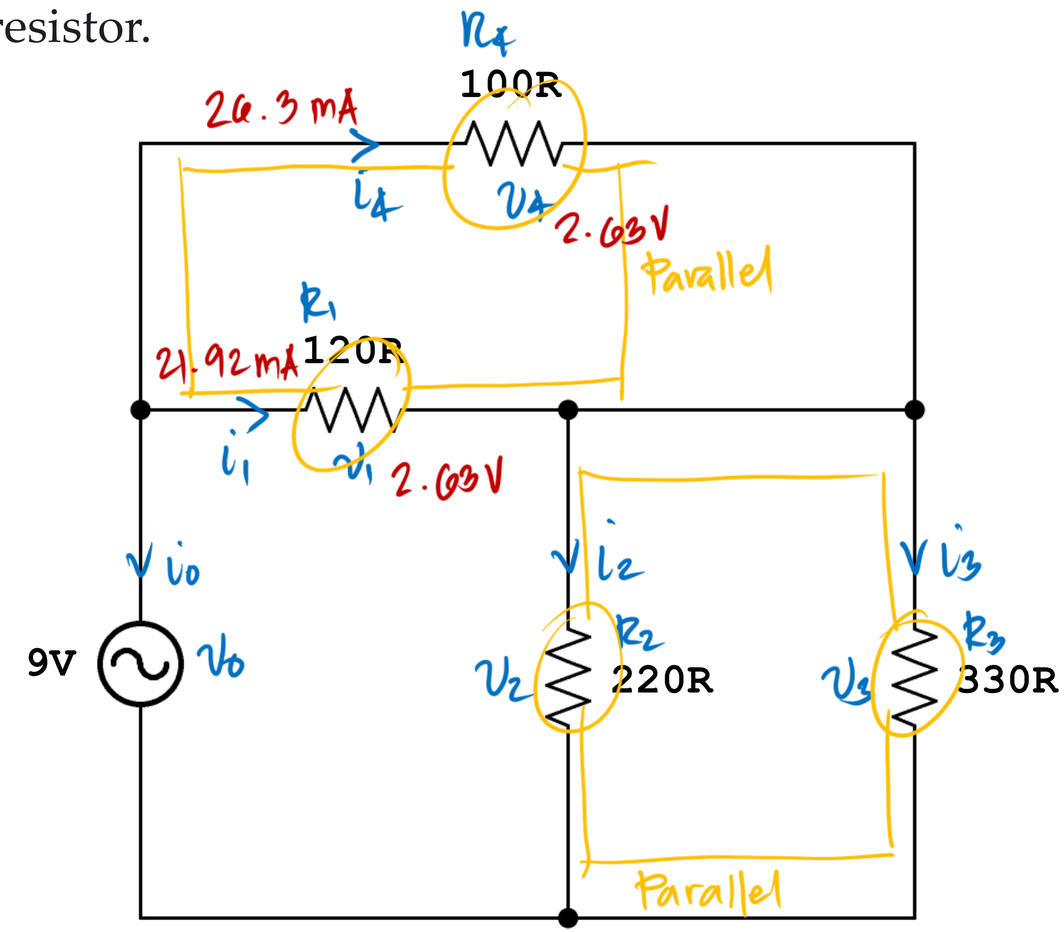
$$v_{1,4} = \underline{2.63\ V}$$

$$\boxed{v_1 = 2.63\ V}$$ ans

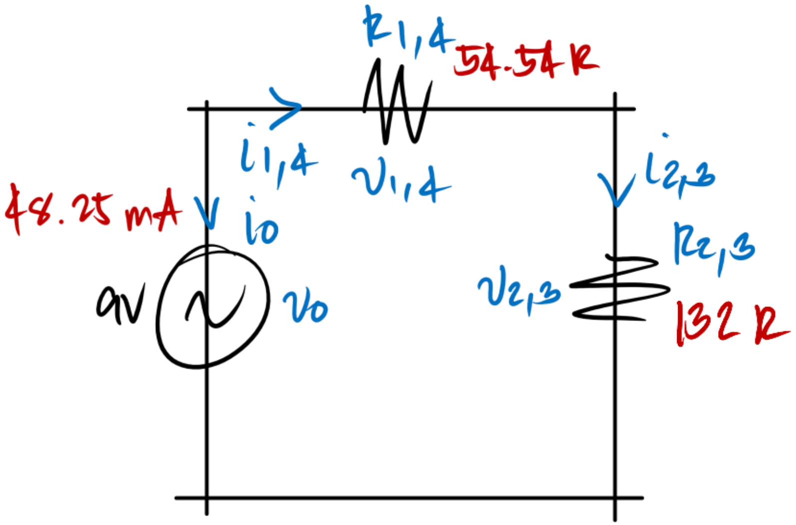
$$\boxed{v_4 = 2.63\ V}$$ ans

EXERCISE

Analyze the given circuit to determine both the current through and the voltage drop across each resistor.



Solution



$$i_1 = \frac{v_1}{R_1}$$

$$i_1 = \frac{2.63}{120}$$

$$\boxed{i_1 = 21.92\,mA}$$

ans

$$i_2 = \frac{v_2}{R_2}$$

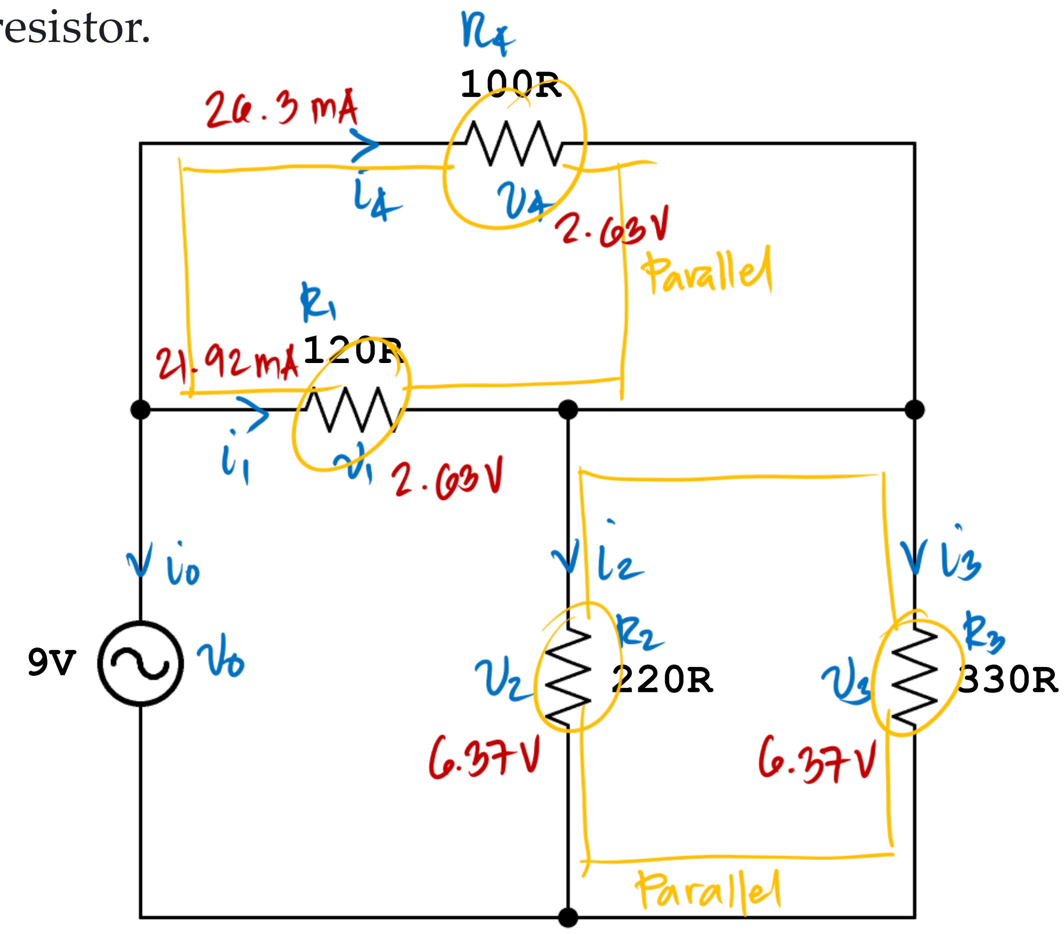
$$i_2 = \frac{2.63}{100}$$

$$\boxed{i_2 = 26.3\,mA}$$

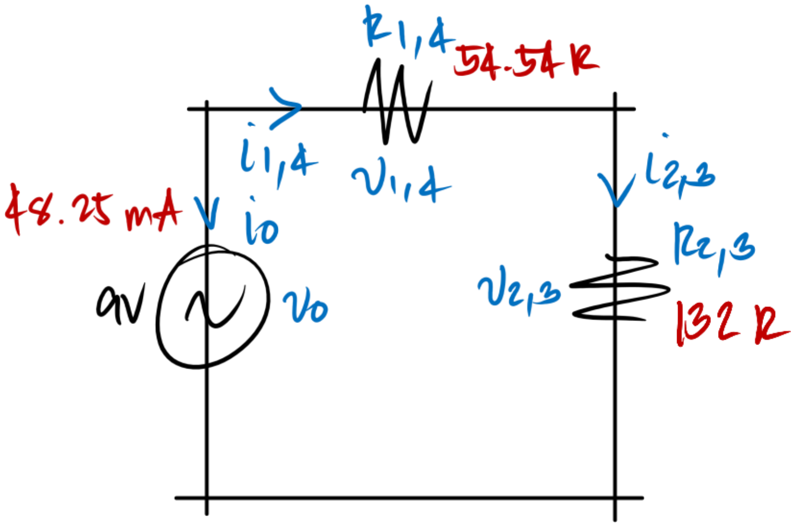
ans

EXERCISE

Analyze the given circuit to determine both the current through and the voltage drop across each resistor.



Solution



$$v_{2,3} = i_{2,3} R_{2,3}$$

$$v_{2,3} = 48.25m(132)$$

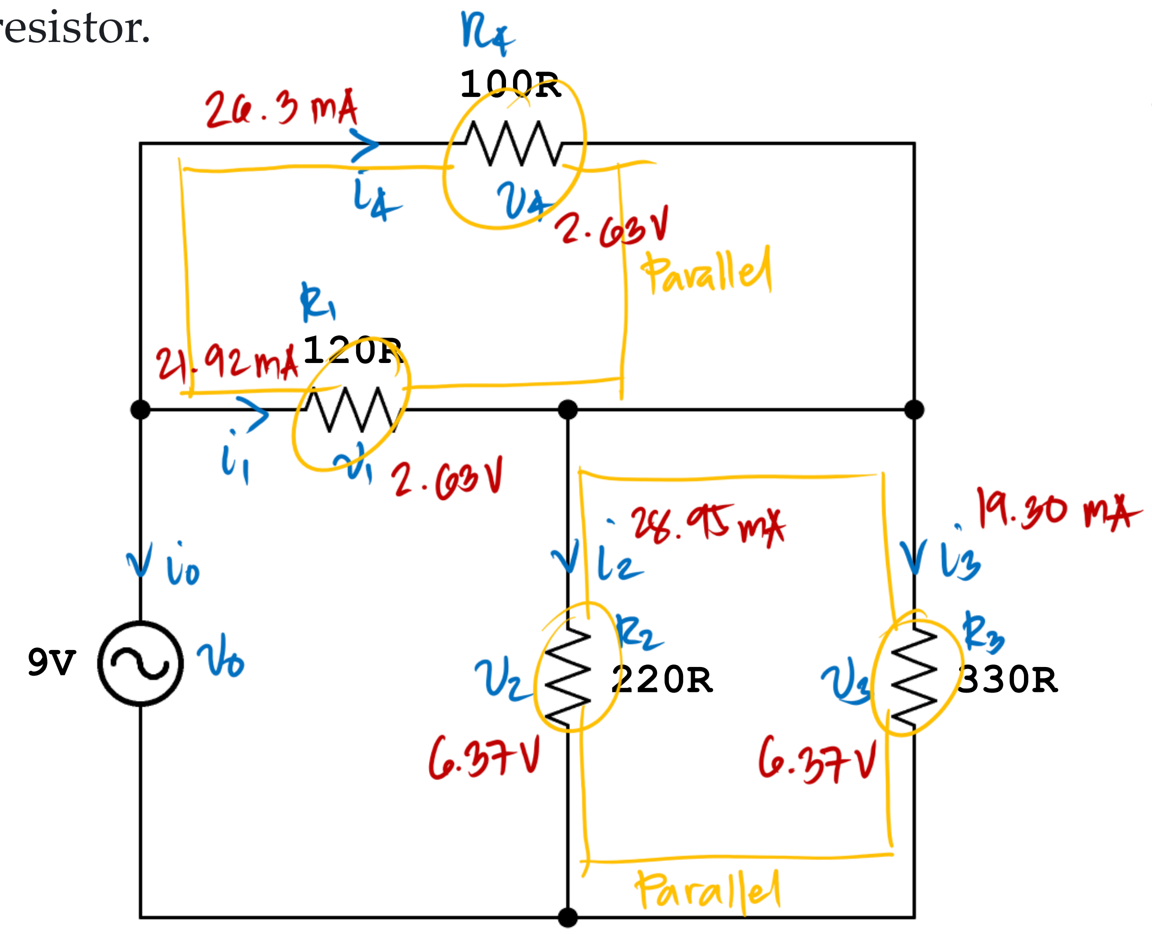
$$v_{2,3} = 6.37\,V$$

$$\boxed{v_2 = 6.37\,V}$$ ans

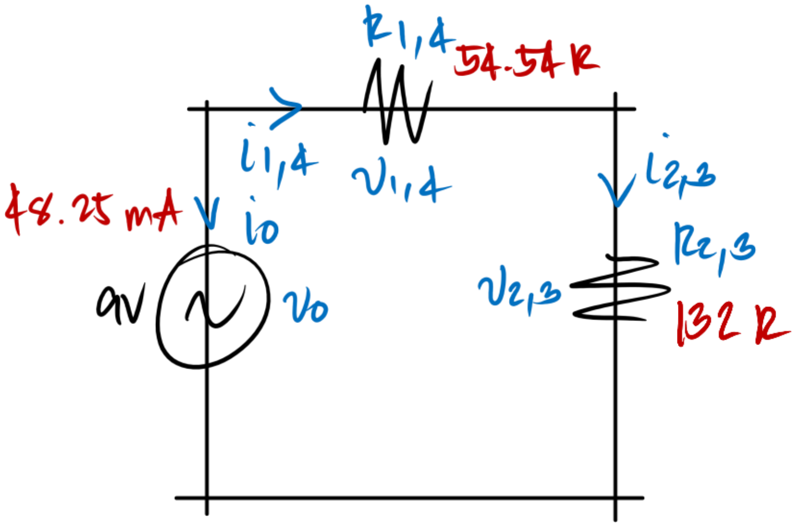
$$\boxed{v_3 = 6.37\,V}$$ ans

EXERCISE

Analyze the given circuit to determine both the current through and the voltage drop across each resistor.



Solution



$$i_2 = \frac{V_2}{R_2} \qquad i_3 = \frac{V_3}{R_3}$$

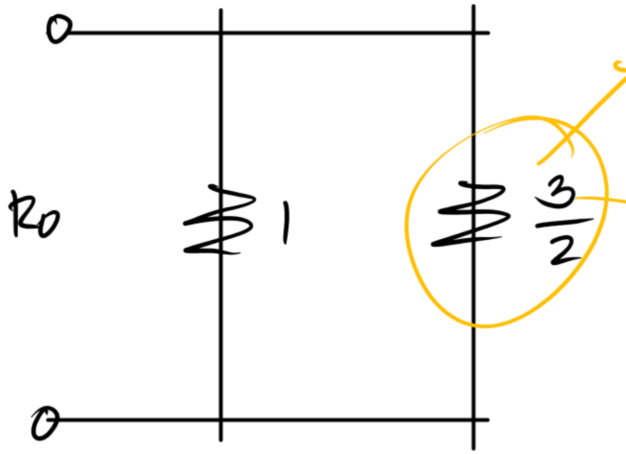
$$i_2 = \frac{6.37}{220} \qquad i_3 = \frac{6.37}{330}$$

$$\boxed{i_2 = 28.95\,mA} \qquad \boxed{i_3 = 19.30\,mA}$$

ans ans

EXERCISE

Using only $1K$ resistors, synthesize a resistor of $3/5K$ and $5/3K$. Use no more than four 1K resistors.



three 1k resistors

Solution

$R_o = \frac{3}{5}$

$G_o = \frac{5}{3}$

$G_o = \cancel{\frac{3}{3}}\,1 + \frac{2}{3}$

$G_o = \underbrace{1 + \frac{2}{3}}_{\text{parallel}}$

conductance

↓

resistance $= \frac{3}{2}$

Equivalent resistance

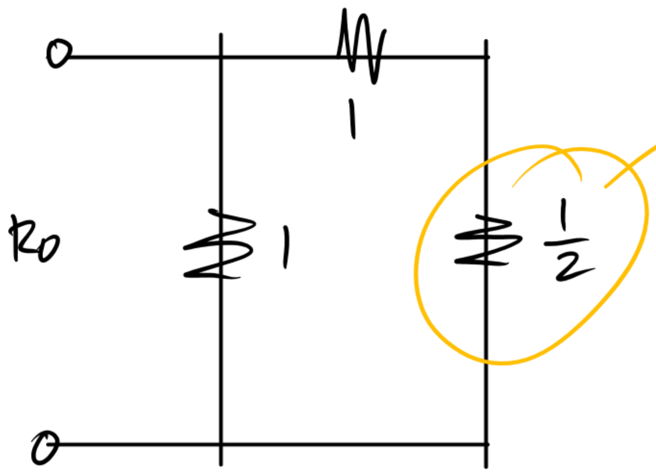
$R_{3T} = \frac{3}{2}$

$R_{3T} = \cancel{\frac{2}{2}}\,1 + \frac{1}{2}$

$R_{3T} = \underbrace{1 + \frac{1}{2}}_{\text{series}}$

EXERCISE

Using only $1K$ resistors, synthesize a resistor of $3/5K$ and $5/3K$. Use no more than four 1K resistors.



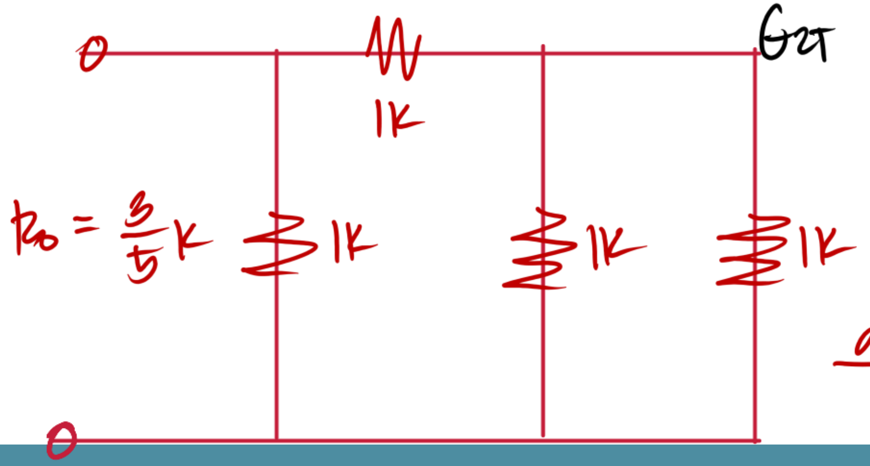
two 1K resistors

$R_{2T} = \frac{1}{2}$

$G_{2T} = 2$

$G_{2T} = 1 + 1$

parallel



ans

Solution

$R_o = \frac{3}{5}$

$G_o = \frac{5}{3}$

$G_o = \cancel{\frac{3}{3}}^{\,1} + \frac{2}{3}$

$G_o = 1 + \frac{2}{3}$

parallel

conductance

$\downarrow$

resistance $= \frac{3}{2}$

Equivalent resistance

$R_{3T} = \frac{3}{2}$

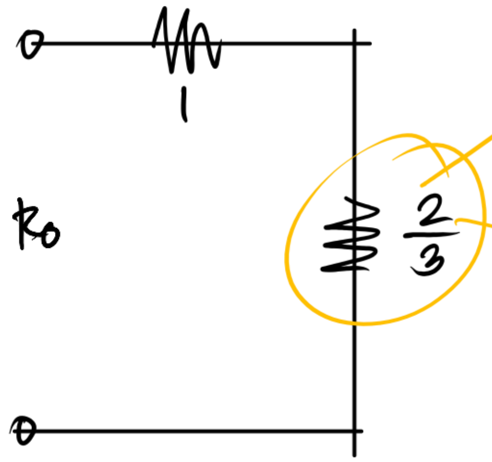
$R_{3T} = \cancel{\frac{2}{2}}^{\,1} + \frac{1}{2}$

$R_{3T} = 1 + \frac{1}{2}$

series

EXERCISE

Using only $1K$ resistors, synthesize a resistor of $3/5K$ and $5/3K$. Use no more than four 1K resistors.



Three 1K resistors

Solution

$R_o = \frac{5}{3}$

$R_o = \cancelto{1}{\frac{3}{3}} + \frac{2}{3}$

$R_o = 1 + \frac{2}{3}$

series

Equivalent Resistors

$R_{3T} = \frac{2}{3}$

$G_{3T} = \frac{3}{2}$

$G_{3T} = \cancelto{1}{\frac{2}{2}} + \frac{1}{2}$

$G_{3T} = 1 + \frac{1}{2}$

parallel

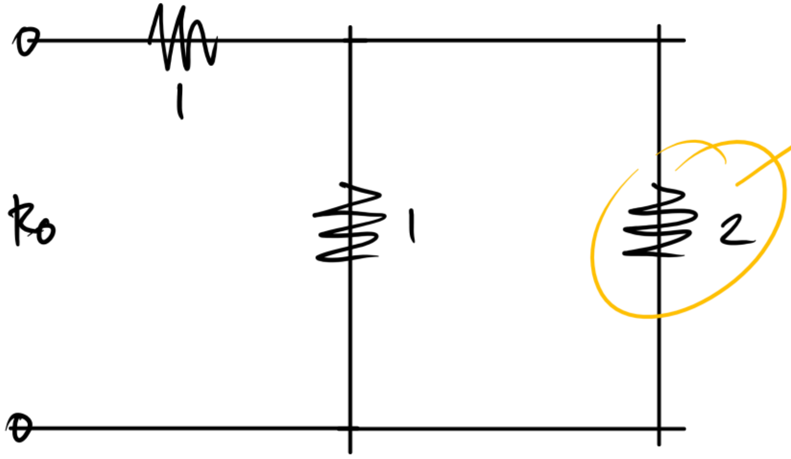
conductance

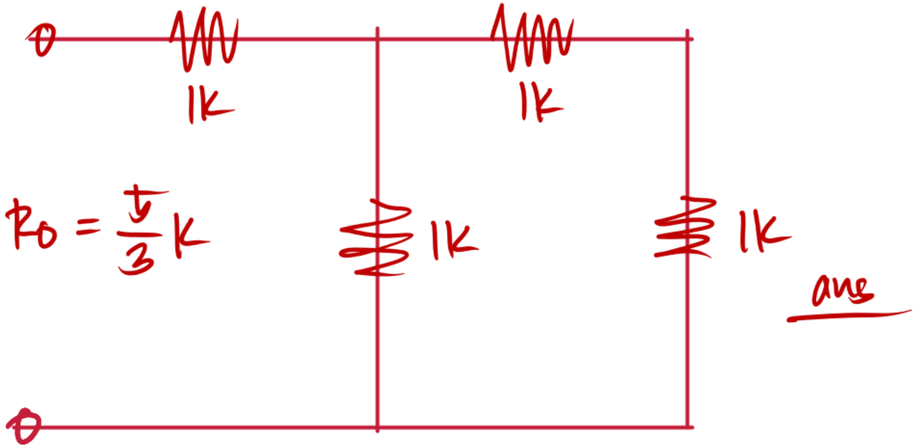
$\downarrow$

Resistance = 2

EXERCISE

Using only $1K$ resistors, synthesize a resistor of $3/5K$ and $5/3K$. Use no more than four 1K resistors.





Solution

$R_o = \frac{5}{3}$

$R_o = \cancel{\frac{3}{3}}^{1} + \frac{2}{3}$

$R_o = 1 + \frac{2}{3}$

series

Two 1K resistors

$R_{2T} = 2$

$R_{2T} = 1 + 1$

series

Equivalent Resistors

$R_{3T} = \frac{2}{3}$

$G_{3T} = \frac{3}{2}$

$G_{3T} = \cancel{\frac{2}{2}}^{1} + \frac{1}{2}$

$G_{3T} = 1 + \frac{1}{2}$

parallel

conductance

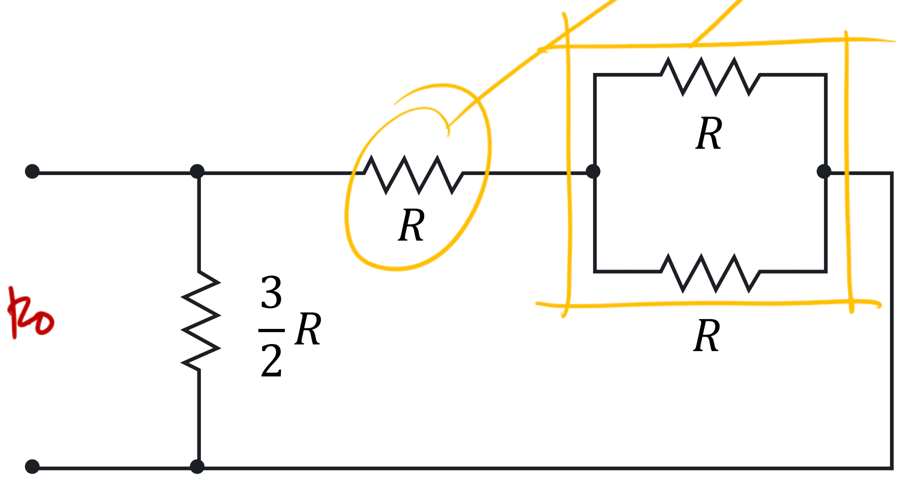
↓

Resistance = 2

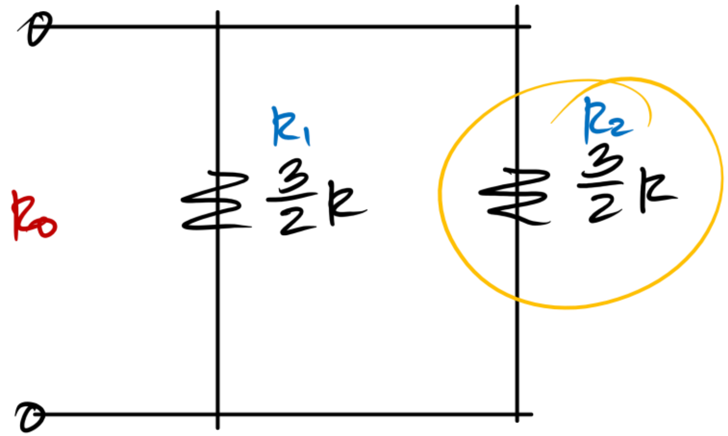
EXERCISE

Find the equivalent resistance of the given network as viewed from its port.

$R + \frac{1}{2}R = \frac{3}{2}R$



Solution



$G_o = G_1 + G_2$

$G_o = \frac{2}{3} + \frac{2}{3}$

$G_o = \frac{4}{3}$

$$\boxed{R_o = \frac{3}{4}R}$$

ans

EXERCISE

You are given a black box with three terminals, as shown in Fig.1. The box is known to contain five 1-ohm resistors.

Using an ohm-meter, you measure the resistance between terminals to be the following:

A–B: 1.5 ohms

B–C: 3 ohms

A–C: 2.5 ohms

Determine the configuration of the five resistors inside the box.

Solution

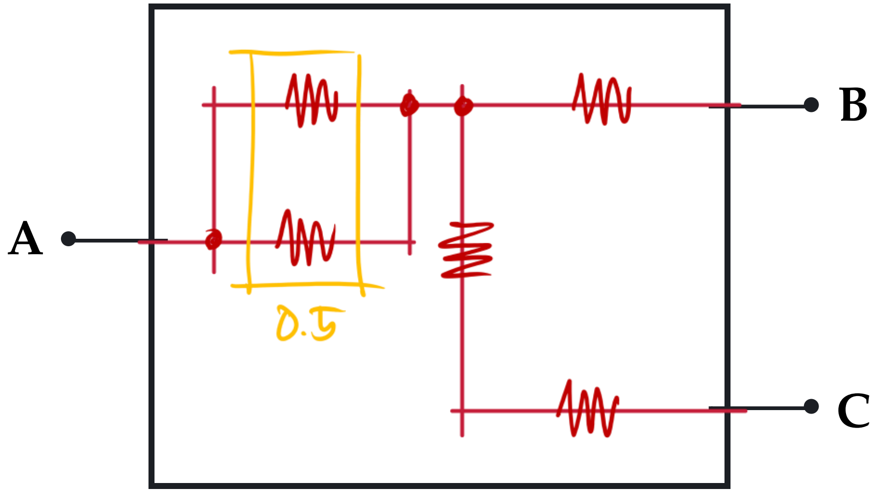


Fig.1. Black box

LABORATORY