PYTHAGOREAN THEOREM

INTRODUCTION

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TOPIC OUTLINE

Pythagorean Theorem

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Statement I

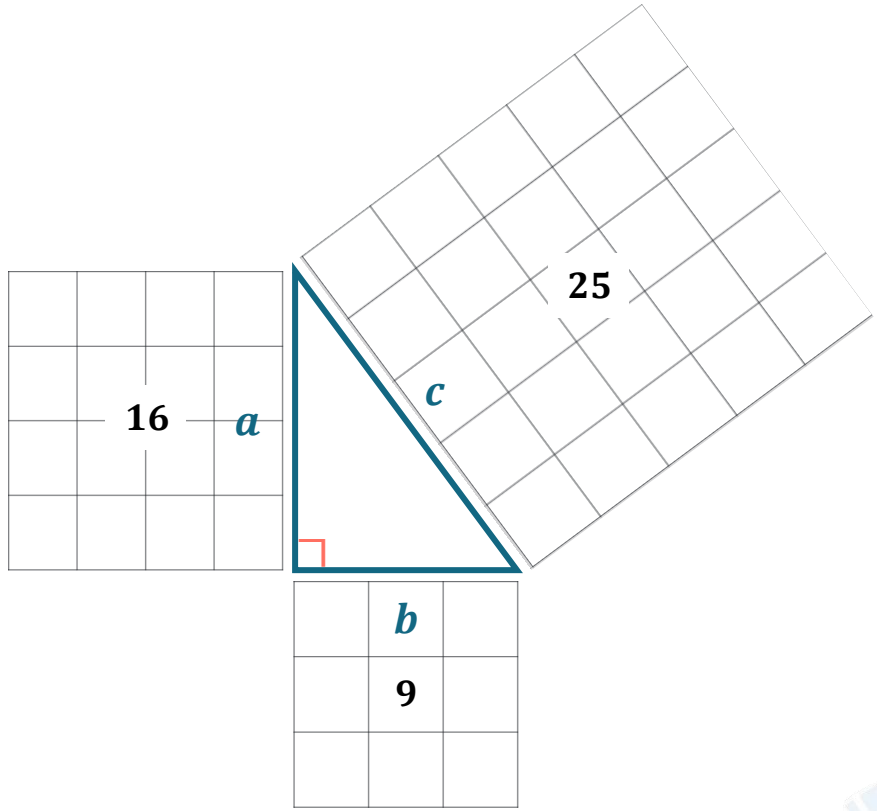
If a and b are the lengths of the legs of a right triangle, and c is the length of its hypotenuse, then

$$a^2 + b^2 = c^2$$

Statement II

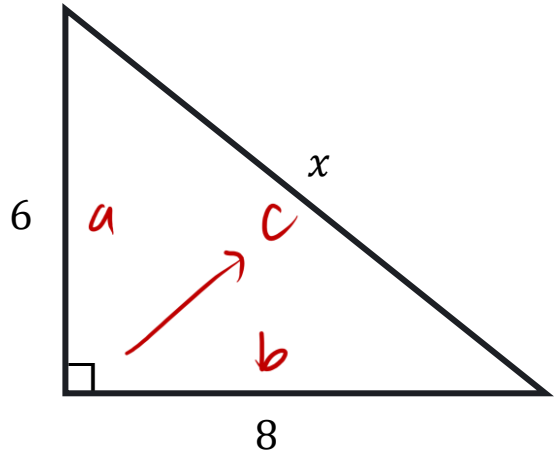
If the positive numbers a, b, and c satisfy $a^2 + b^2 = c^2$, then a triangle with these side lengths has a **right angle** opposite the side with length $\boldsymbol{c}$.

Right Triangle



EXERCISE

Find the value of x in the given right triangle.



Solution

$$c^2 = a^2 + b^2$$

$$x^2 = 6^2 + 8^2$$

$$x^2 = 36 + 64$$

$$x^2 = 100$$

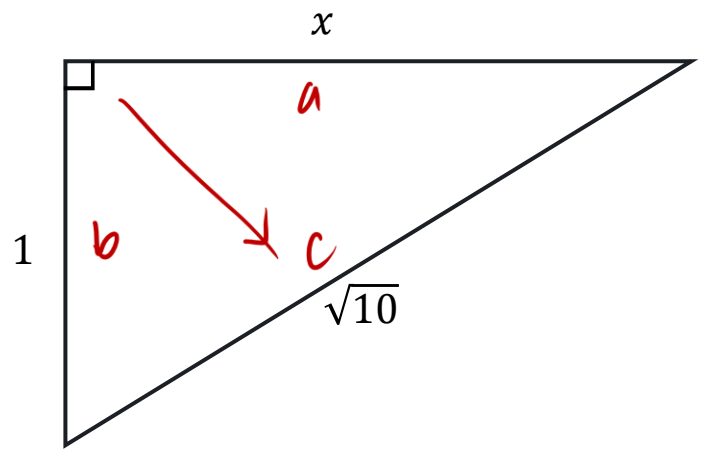
$$x = \sqrt{100}$$

$$\boxed{x = 10}$$

ans

EXERCISE

Compute the length of the missing side of the given right triangle.



Solution

$c^2 = a^2 + b^2$

$(\sqrt{10})^2 = x^2 + 1^2$

$10 = x^2 + 1$ (b)

$-1 \qquad -1$

$9 = x^2$

$x = \sqrt{9}$

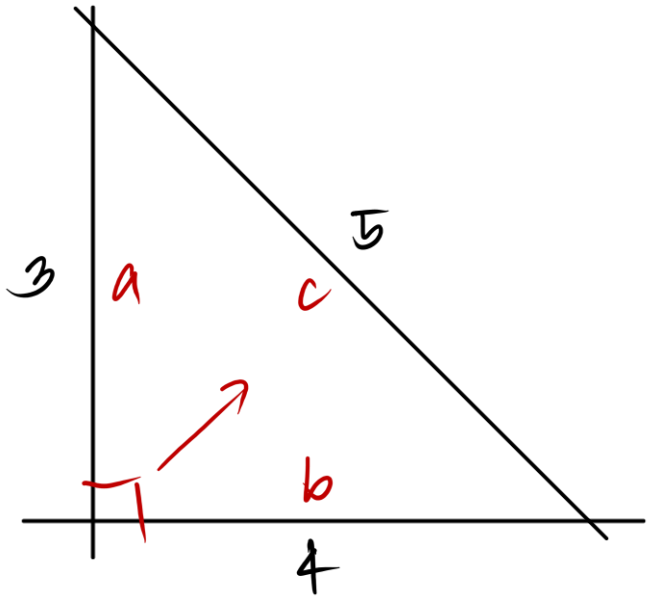
$x = 3$

ans

EXERCISE

Show that a triangle with sides 3, 4, and 5 is a right triangle.

longest side, c



Solution

$c^2 = a^2 + b^2$

$5^2 = 3^2 + 4^2$

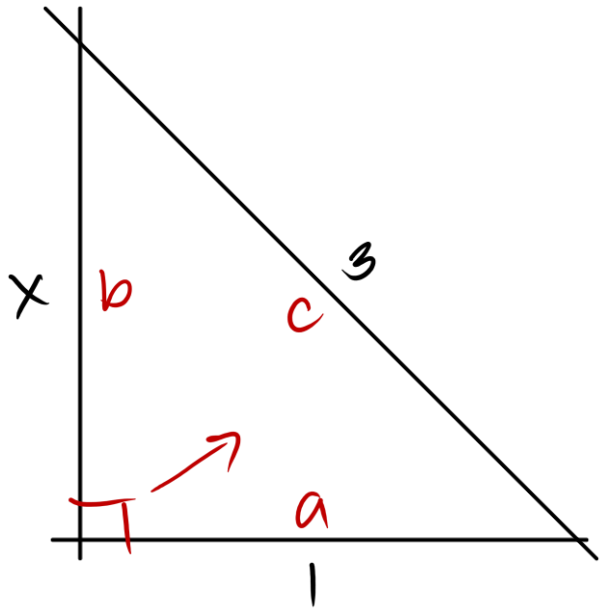
$25 = 9 + 16$

$25 = 25$ ✓

ans

EXERCISE

In a right triangle, one leg measures 1 unit, and the hypotenuse measures 3 units. Determine the length of the other leg.



Solution

$$c^2 = a^2 + b^2$$

$$3^2 = 1^2 + x^2$$

$$9 = 1 + x^2$$

$$-1 \quad -1$$

$$8 = x^2$$

$$x = \sqrt{8}$$

$$x = \sqrt{4 \cdot 2}$$

$$x = \sqrt{4} \cdot \sqrt{2}$$

$$\boxed{x = 2\sqrt{2}}$$

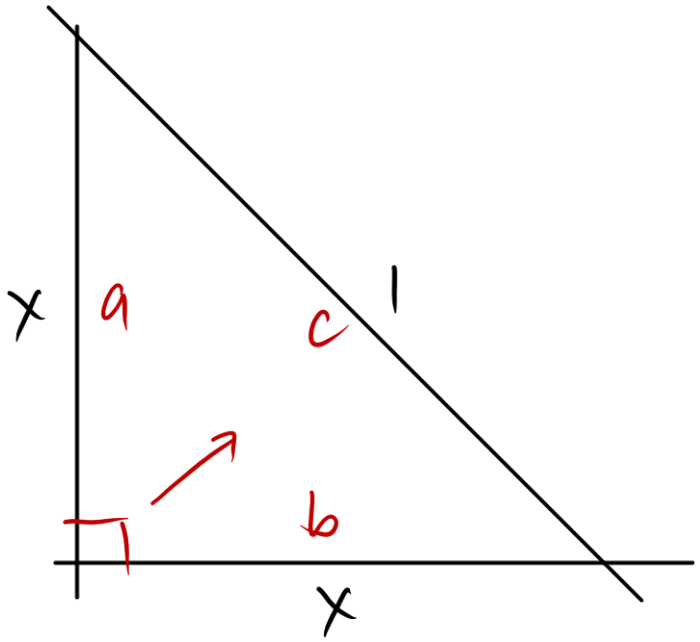
ans

EXERCISE

two sides are equal

In an isosceles right triangle, the hypotenuse has a length of 1 unit. Determine the length of one of the legs of the triangle.

c = 1



Solution

$$c^2 = a^2 + b^2$$

$$1^2 = x^2 + x^2$$

$$\frac{1}{2} = \frac{2x^2}{2}$$

$$\frac{1}{2} = x^2$$

$$x = \sqrt{\frac{1}{2}}$$

$$x = \frac{\sqrt{1}}{\sqrt{2}}$$

$$x = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

rationalize the denominator

$$x = \frac{\sqrt{2}}{(\sqrt{2})^2}$$

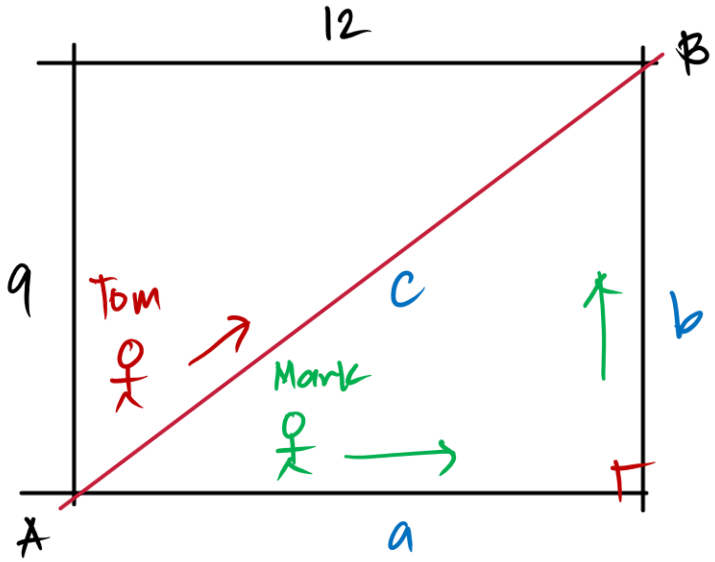
$$x = \frac{\sqrt{2}}{2}$$

ans

EXERCISE

A rectangular lot measures 12 meters in length and 9 meters in width. Tom walks diagonally across the lot from point A (one corner) to point B (the opposite corner), while Mark walks along the perimeter, following the two sides of the rectangle from point A to point B. How much farther does Mark walk than Tom?

Δ = Mark − Tom



Solution

$c^2 = a^2 + b^2$

$c^2 = 12^2 + 9^2$

$c^2 = 144 + 81$

$c^2 = 225$

$c = \sqrt{225}$

$c = 15$

Tom = 15 m

Mark = a + b

= 12 + 9

Mark = 21 m

Δ_{Mark} = Mark − Tom

= 21 − 15

Δ_{Mark} = 6 m

ans

Mark walks 6m more than Tom.

$\Delta_{Mark} = \frac{Mark - Tom}{Tom} \times 100$

$= \frac{21 - 15}{15}$ → 6

Δ_{Mark} = 40%

40% more than Tom

SEATWORK