



CHAIN RULE

DIFFERENTIAL CALCULUS

prepared by:

Gyro A. Madrona
Electronics Engineer

TOPIC OUTLINE

Chain Rule

- Leibniz Notation



CHAIN RULE



CHAIN RULE

Differentiate the function $y = \sin(2x)$.

If g is differentiable at x and f is differentiable at $g(x)$, then the composite function defined by

$$F(x) = f(g(x))$$

is differentiable at x and F' is given by product

$$F'(x) = f'(g(x)) \cdot g'(x)$$

Differentiate the function $y = \cos(\sqrt{x})$.



EXERCISE

Differentiate the function $y = \sqrt{x^2 + 1}$.

Differentiate the function $y = \tan(5x^3 + 2x)$.



LEIBNIZ NOTATION

Differentiate the function $y = \sin(2x)$.

If $y = f(u)$ and $u = g(x)$ are both differentiable functions, then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Differentiate the function $y = \cos(\sqrt{x})$.



EXERCISE

Differentiate $y = \sin x^2$.

Differentiate $y = x \sin \frac{1}{x}$.



EXERCISE

Differentiate $y = \frac{x^3}{\sqrt{1+x^2}}$.

Differentiate $g(t) = \left(\frac{t-2}{2t+1}\right)^9$.



EXERCISE

The displacement of a particle on a vibrating string is given by the equation

$$s(t) = 10 + \frac{1}{4}\sin(10\pi t)$$

where s is measured in centimeters and t in seconds.

Find the velocity of the particle in t seconds.

Solution



EXERCISE

Differentiate $y = \frac{\cos \pi x}{\sin \pi x + \cos \pi x}$.

Differentiate $y = [x + (x + \sin^2 x)^3]^4$.



EXERCISE

A Cepheid variable star is a star whose brightness alternately increases and decreases. The most easily visible such star is Delta Cephei, for which the interval between times of maximum brightness is 5.4 days. The average brightness of this star is 4.0 and its brightness changes by ± 0.35 . In view of these data, the brightness of Delta Cephei at time t , where t is measured in days, has been modeled by the function

$$B(t) = 4.0 + 0.35 \sin\left(\frac{2\pi t}{5.4}\right)$$

Find the rate of change of the brightness after t days.

Solution



LABORATORY

