LIMIT OF A FUNCTION

INTRODUCTION

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TOPIC OUTLINE

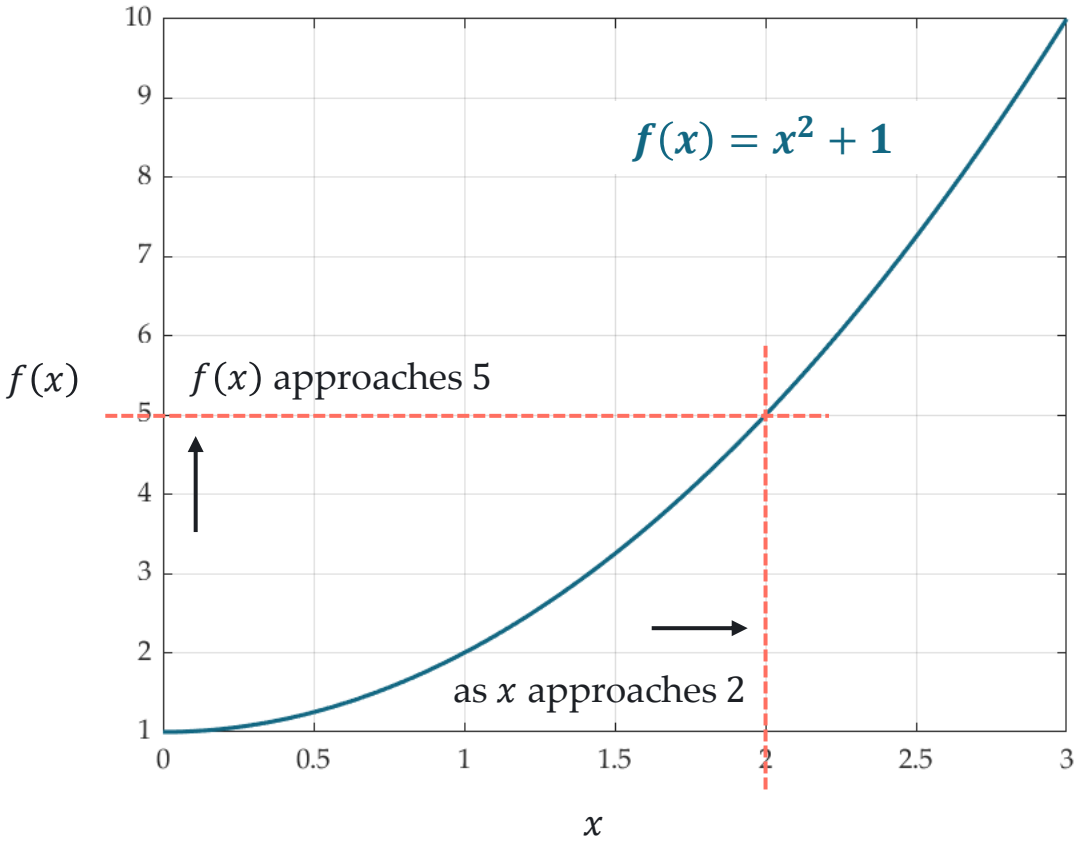
Limit of a Function

Limit Laws

- **Sum, Difference, Constant Multiple**
- **Product, Quotient, Power, Root**

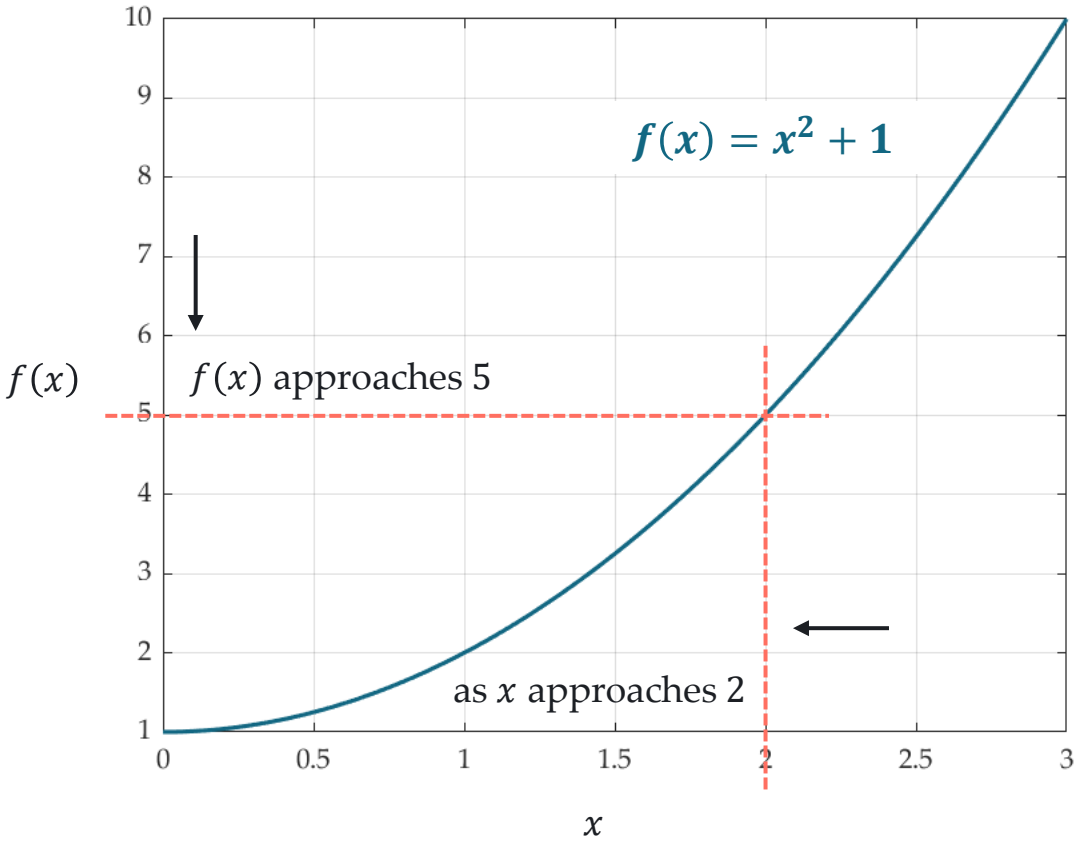
LIMIT OF A FUNCTION

PARABOLA



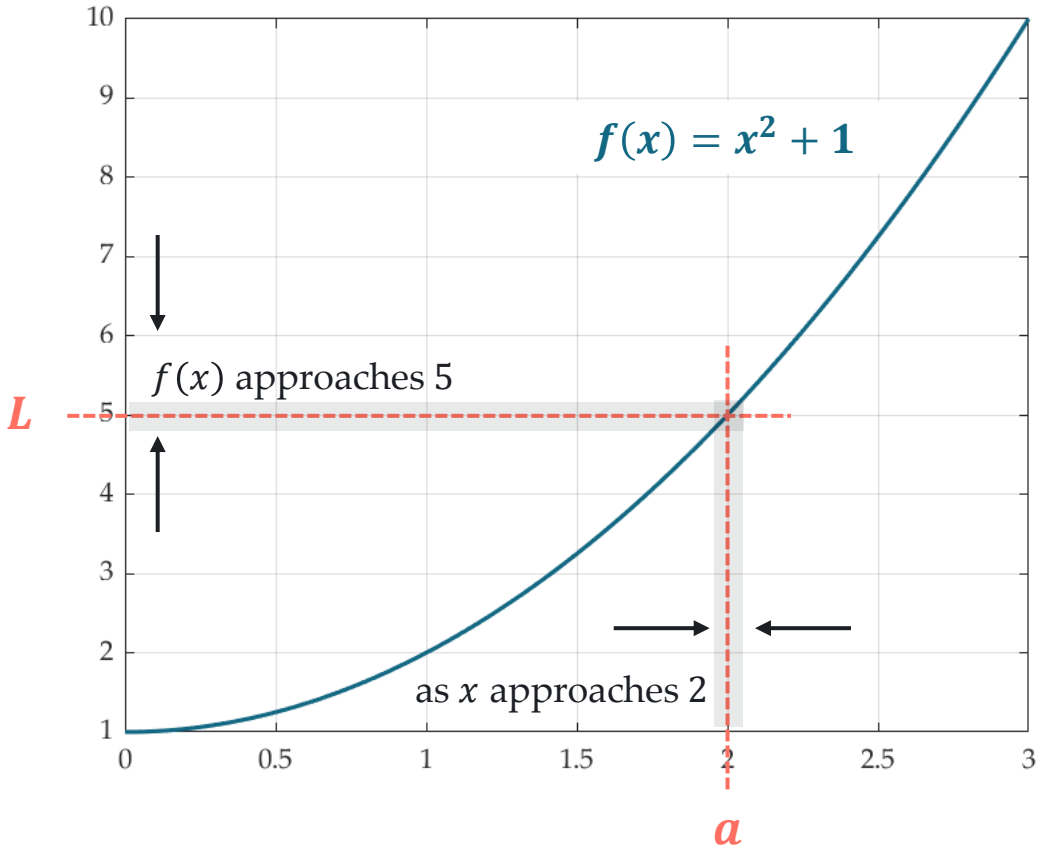
x	$f(x)$
0	1
1	2
1.5	3.25
1.8	4.24
1.9	4.61
1.91	4.6481
1.92	4.6864
1.93	4.7249
1.95	4.8025
1.96	4.8416
1.97	4.8809
1.98	4.9204
1.99	4.9601
2	5

PARABOLA



x	$f(x)$
3	10
2.5	7.25
2.3	6.29
2.1	5.41
2.09	5.3681
2.08	5.3264
2.07	5.2849
2.06	5.2436
2.05	5.2052
2.04	5.1616
2.03	5.1209
2.02	5.0804
2.01	5.0401
2	5

DEFINITION OF LIMIT



Suppose $f(x)$ is **defined** when x is near the number a, then the limit of $f(x)$, as x approaches a, equals L.

notation

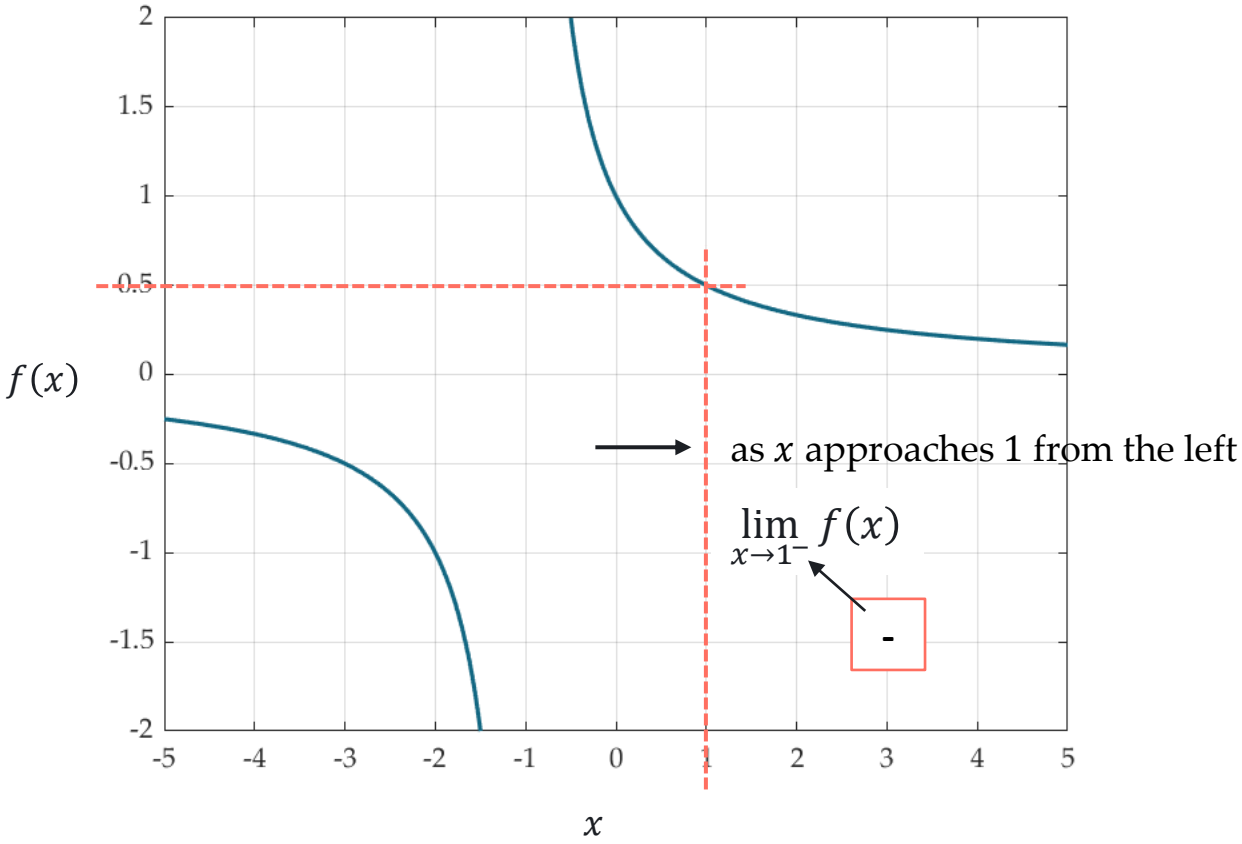
$$\lim_{x \to a} f(x) = L$$

The limit of $(x^2 + 1)$ as x approaches 2 is 5.

$$\lim_{x \to 2} (x^2 + 1) = 5$$

EXERCISE

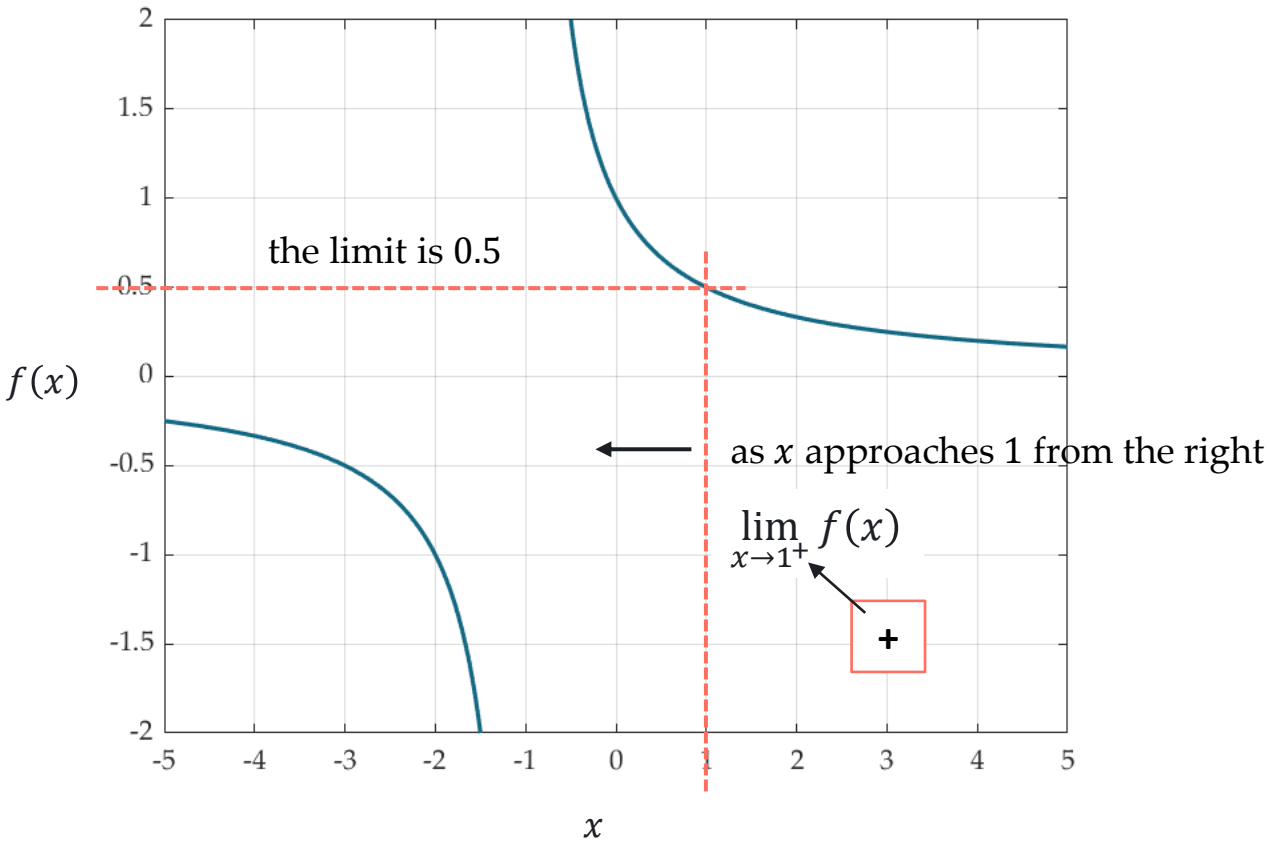
Determine $\lim_{x \to 1} \frac{x-1}{x^2-1}$



x	$f(x)$
0	1
0.5	0.66667
0.6	0.62500
0.7	0.58824
0.8	0.55556
0.9	0.52632
0.95	0.51282
0.96	0.51020
0.97	0.50761
0.98	0.50505
0.99	0.50251
1	∞

EXERCISE

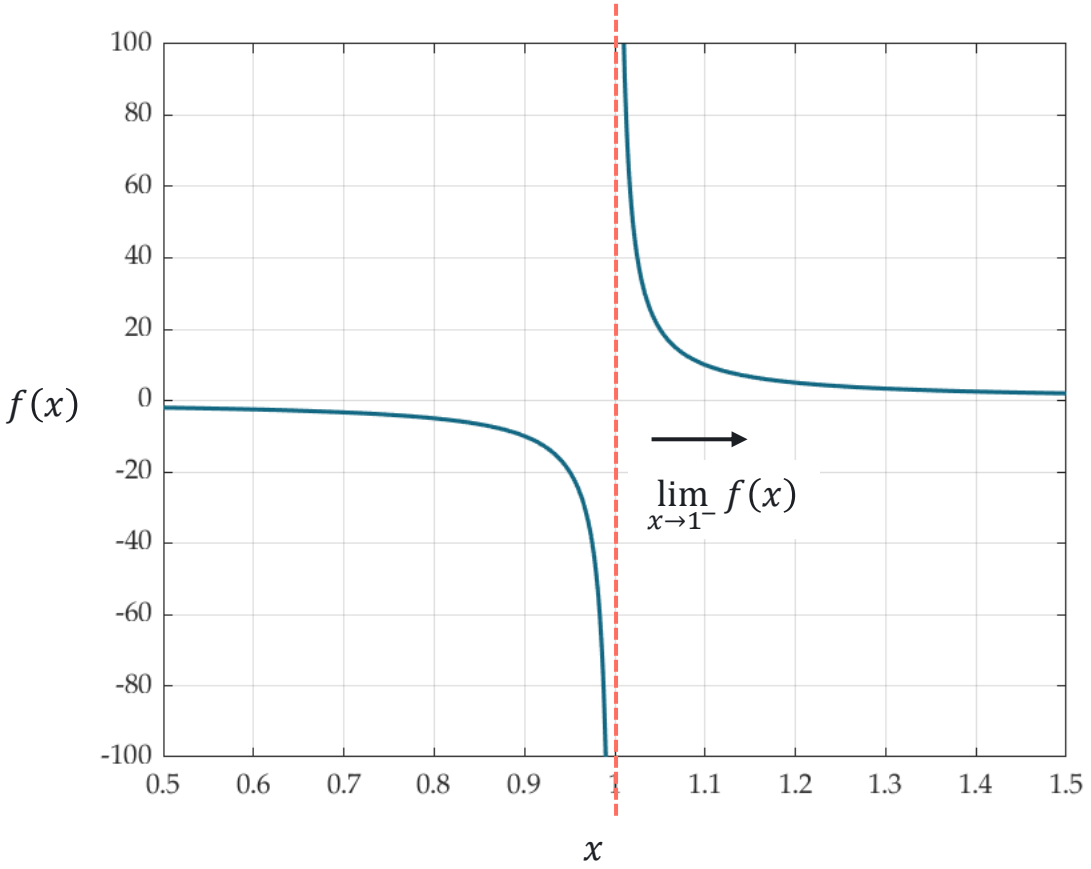
Determine $\lim_{x \to 1} \frac{x-1}{x^2-1}$



x	$f(x)$
2.0	0.33333
1.5	0.4
1.4	0.41667
1.3	0.43478
1.2	0.45455
1.1	0.47619
1.05	0.48780
1.04	0.49020
1.03	0.49261
1.02	0.49505
1.01	0.49751
1	∞

EXERCISE

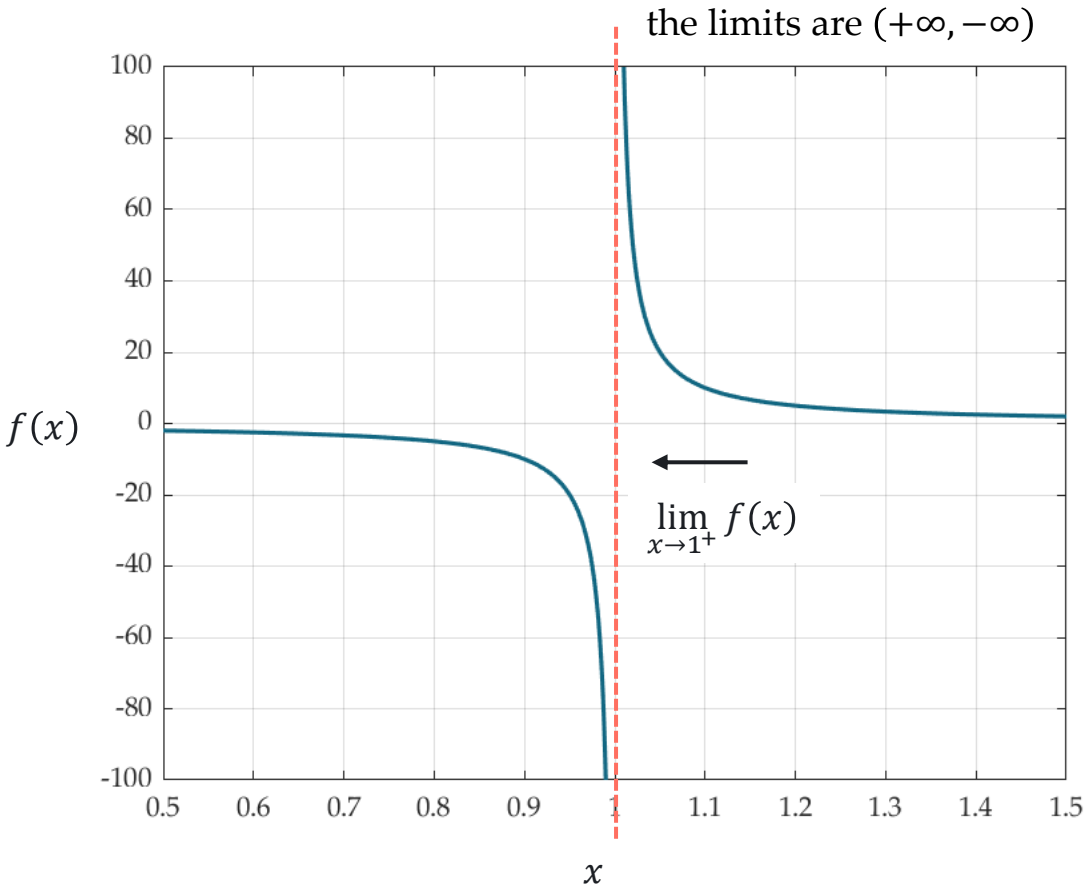
Determine $\lim_{x \to 1} \frac{1}{x-1}$



x	$f(x)$
0.9	-10
0.95	-20
0.99	-100
0.999	-1,000
0.9999	-10,000
0.99999	-100,000
0.999999	-1,000,000
$\to 1^-$	$-\infty$

EXERCISE

Determine $\lim_{x \to 1} \frac{1}{x-1}$



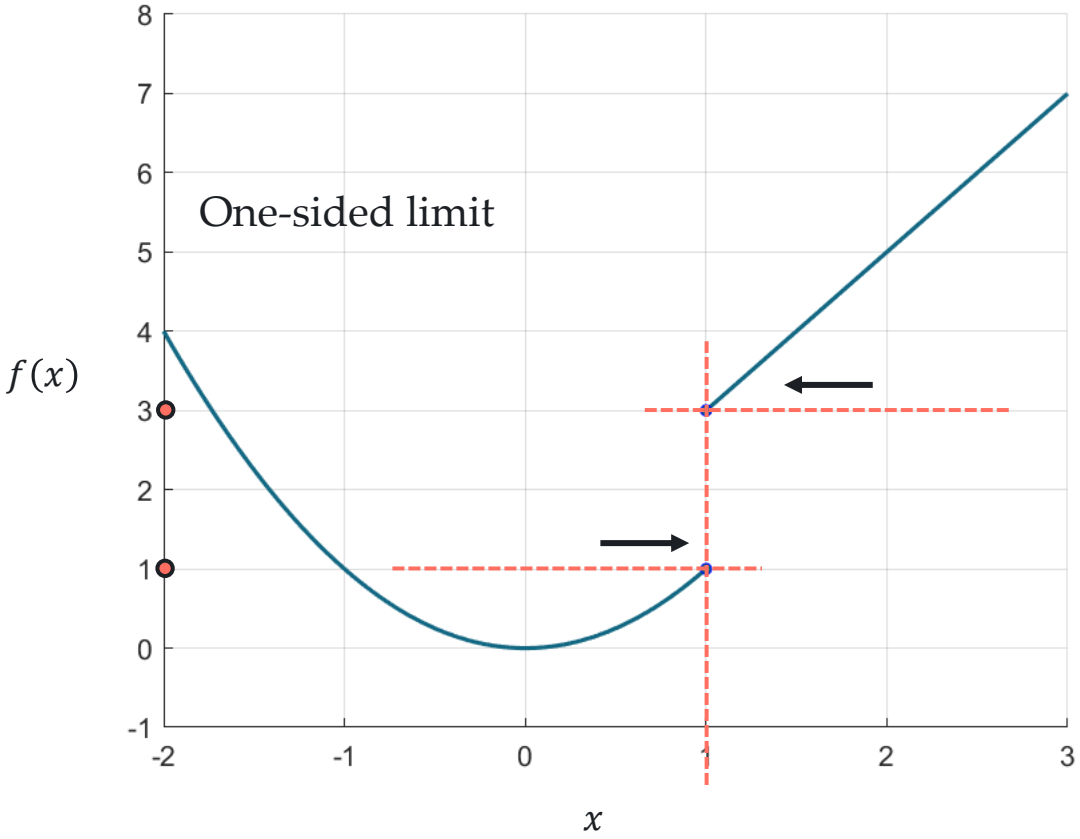
x	$f(x)$
1.1	10
1.05	20
1.01	100
1.001	1,000
1.0001	10,000
1.00001	100,000
1.000001	1,000,000
$1^+ \leftarrow$	$+\infty$

The **limit does not exist** (DNE), since the left-hand limit and the right-hand limit are not equal.

EXERCISE

Determine $\lim_{x \to 1} \begin{cases} x^2, & \text{if } x < 1 \\ 2x + 1, & \text{if } x \geq 1 \end{cases}$

piecewise function



Left-hand limit

$$\lim_{x \to 1^-} f(x) = 1$$

Right-hand limit

$$\lim_{x \to 1^+} f(x) = 3$$

The **limit does not exist** (DNE), since the left-hand limit and the right-hand limit are not equal.

LIMIT LAWS

SUM LAW

Evaluate $\lim_{x \to 2}(x^2 + 1)$

$$\lim_{x \to a}[f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$$

EXERCISE

Evaluate $\lim_{x \to -1} (2x^2 + 5x + 3)$

Evaluate $\lim_{x \to 0} (x^3 + 2x + 7)$

DIFFERENCE LAW

Evaluate $\lim_{x \to 4}(x^2 - 3x)$

$$\lim_{x \to a}[f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$$

EXERCISE

Evaluate $\lim_{x \to -3} (2x^2 - 5x + 3)$

Evaluate $\lim_{x \to 0} (x^3 - 2x - 7)$

CONTANT MULTIPLE LAW

Evaluate $\lim_{x \to 2}(5x^3)$

$$\lim_{x \to a}[cf(x)] = c\lim_{x \to a} f(x)$$

EXERCISE

Evaluate $\lim_{x \to 3}(4x^3)$

Evaluate $\lim_{x \to -2}(6x^3)$

PRODUCT LAW

Evaluate $\lim_{x \to 3}[x^2(x-1)]$

$$\lim_{x \to a}[f(x)g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$$

EXERCISE

Evaluate $\lim_{x \to -1} \left(x^2(x-5)\right)$

Evaluate $\lim_{x \to -2} \left(2x^3(x+5)\right)$

QUOTIENT LAW

Evaluate $\lim_{x \to 1} \left(\frac{1}{x-1} \right)$

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$$

if $\lim_{x \to a} g(x) \neq 0$

EXERCISE

Evaluate $\lim_{x \to 2} \left(\frac{x-1}{x^2-1} \right)$

Evaluate $\lim_{x \to 1} \left(\frac{x-1}{x^2-1} \right)$

POWER LAW

Evaluate $\lim_{x \to 2}(x^3 - 2x)^2$

$$\lim_{x \to a}[f(x)]^n = \left[\lim_{x \to a} f(x)\right]^n$$

EXERCISE

Evaluate $\lim_{x \to -2} [(x^3 + 2x)^2]$

Evaluate $\lim_{x \to 1} [(x^3 - 5x)^2]$

ROOT LAW

Evaluate $\lim_{x \to 4} \sqrt{x + 2}$

$$\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)}$$

where n is a positive integer

EXERCISE

Evaluate $\lim_{x \to 6} \sqrt{x - 2}$

Evaluate $\lim_{x \to -4} \sqrt{x^2 + 2x + 1}$

LABORATORY