THE KARNAUGH MAP

LOGIC MINIMIZATION

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TOPIC OUTLINE

SOP Minimization

POS Minimization

SOP MINIMIZATION

KARNAUGH MAP

A **Karnaugh map** is an array of cells in which each cell represents a binary value of the input variables. It provides a systematic method for simplifying Boolean expressions.

3-variable K-map

AB \ C	0	1
00	$\bar{A}\bar{B}\bar{C}$	$\bar{A}\bar{B}C$
01	$\bar{A}B\bar{C}$	$\bar{A}BC$
11	$AB\bar{C}$	ABC
10	$A\bar{B}\bar{C}$	$A\bar{B}C$

KARNAUGH MAP

A **Karnaugh map** is an array of cells in which each cell represents a binary value of the input variables. It provides a systematic method for simplifying Boolean expressions.

4-variable K-map

AB \ CD	00	01	11	10
00	$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}\bar{B}\bar{C}D$	$\bar{A}\bar{B}CD$	$\bar{A}\bar{B}C\bar{D}$
01	$\bar{A}B\bar{C}\bar{D}$	$\bar{A}B\bar{C}D$	$\bar{A}BCD$	$\bar{A}BC\bar{D}$
11	$AB\bar{C}\bar{D}$	$AB\bar{C}D$	$ABCD$	$ABC\bar{D}$
10	$A\bar{B}\bar{C}\bar{D}$	$A\bar{B}\bar{C}D$	$A\bar{B}CD$	$A\bar{B}C\bar{D}$

MAPPING A STANDARD SOP EXPRESSION

A **1** is placed on the K-map cell that corresponds to the value of a **product term** in the expression.

3-variable K-map

$$f = \bar{A}\bar{B}\bar{C} + \bar{A}BC + AB\bar{C} + ABC$$

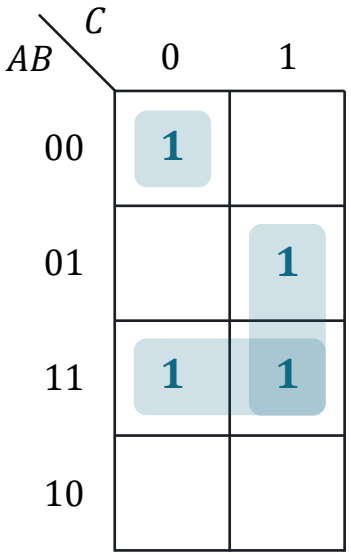
000 011 110 111

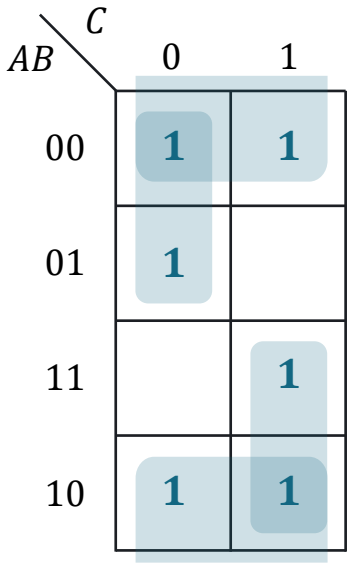
AB \ C	0	1
00	**1**	
01		**1**
11	**1**	**1**
10		

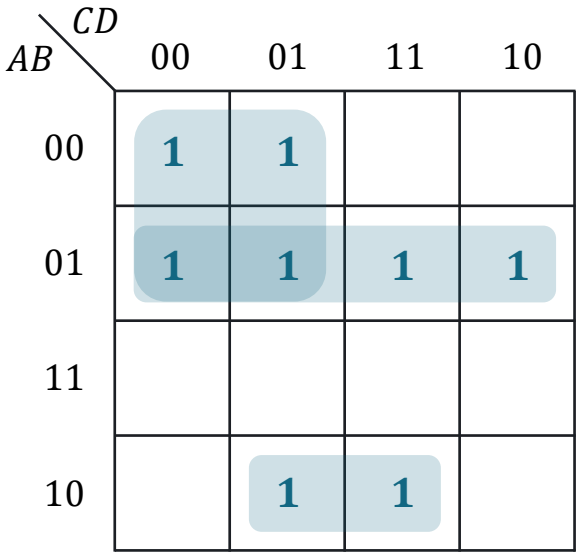
GROUPING 1s

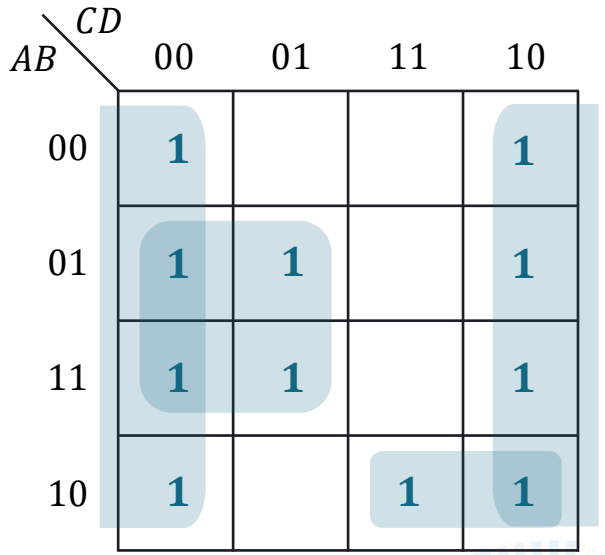
A group must contain either 1, 2, 4, 8, or 16 cells,

which are all **powers of two**.



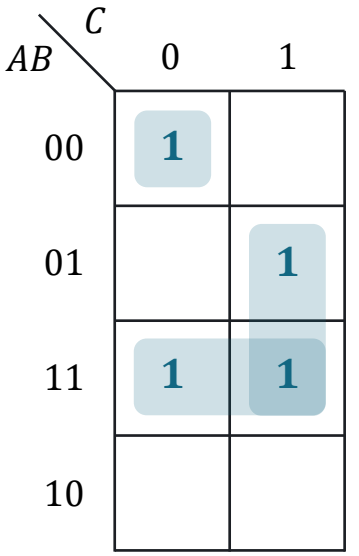




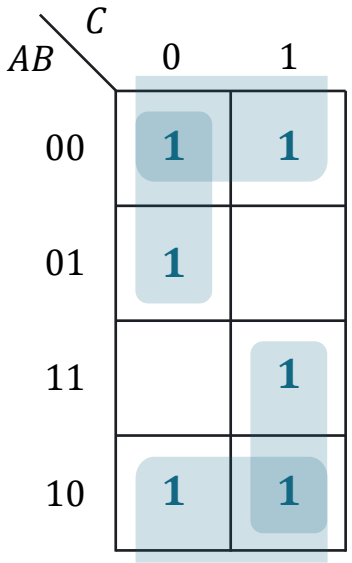


SOP MINIMIZATION

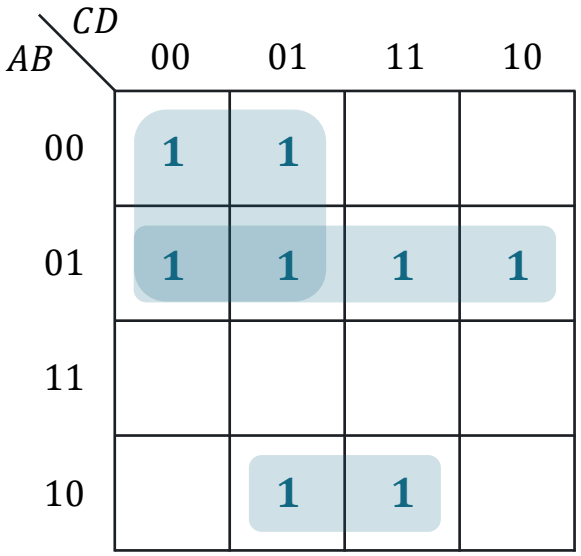
The process that results in an expression containing the fewest possible terms with the fewest possible variables is called **minimization**.



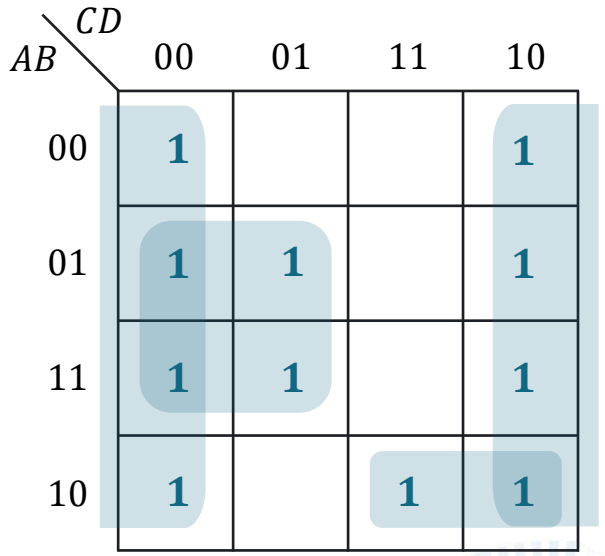
$f = \bar{A}\bar{B}\bar{C} + AB + BC$



$f = \bar{B} + \bar{A}\bar{C} + AC$



$f = \bar{A}\bar{C} + \bar{A}B + A\bar{B}D$



$f = \bar{D} + B\bar{C} + A\bar{B}C$

Variables that occur both **complemented** and **uncomplemented** within the group are eliminated. These are called **contradictory variables**.

EXERCISE

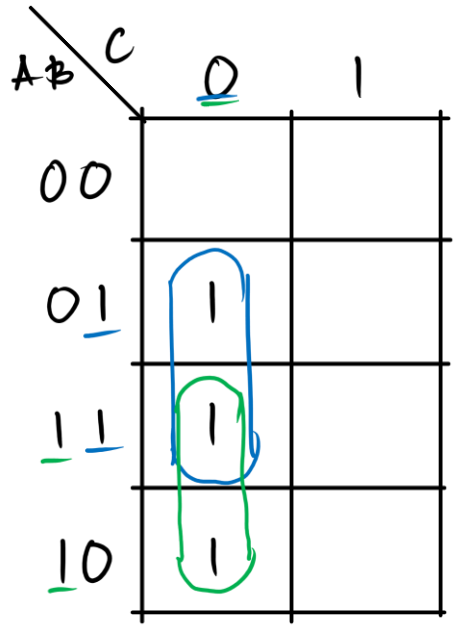
Use a Karnaugh map to minimize the given standard SOP expression.

$$f = \sum m(2, 4, 6)$$

$$f = \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C}$$

$$010 \quad 100 \quad 110$$

Solution



$$f = B\bar{C} + A\bar{C}$$

$$f = \bar{C}(A + B)$$

ans

EXERCISE

Use a Karnaugh map to minimize the given standard SOP expression.

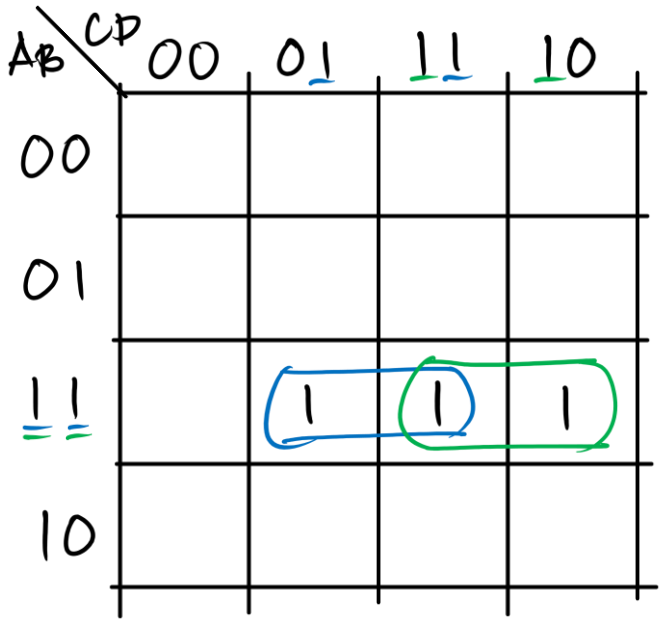
$$f = \sum m(13, 14, 15)$$

$f = A\bar{B}CD + A\bar{B}C\bar{D} + ABCD$

13 14 15

1101 1110 1111

Solution



$f = AB\bar{D} + ABC$

$f = AB(C + D)$

ans

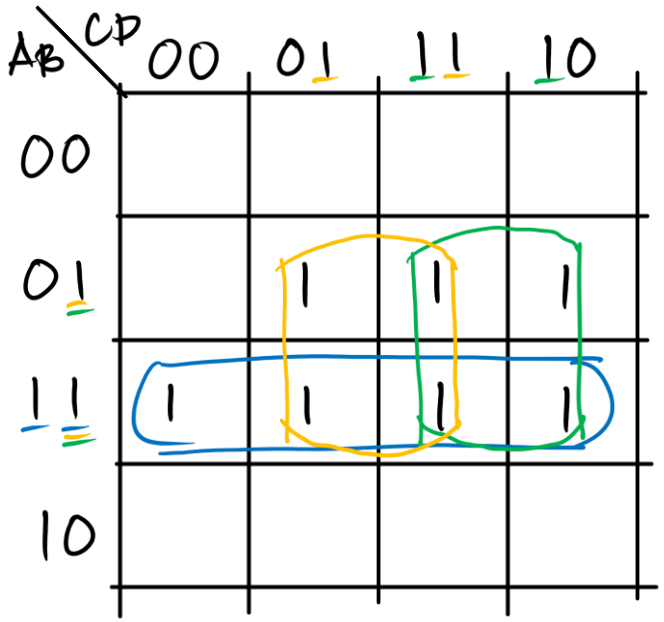
EXERCISE

Use a Karnaugh map to minimize the given standard SOP expression.

$$f = \sum m(5, 6, 7, 12, 13, 14, 15)$$

$$f = \bar{A}B\bar{C}D + \bar{A}BC\bar{D} + \bar{A}BCD + AB\bar{C}\bar{D} +$$
$$AB\bar{C}D + ABC\bar{D} + ABCD$$

Solution



$$f = AB + BD + BC$$

$$f = B(A + C + D)$$

ans

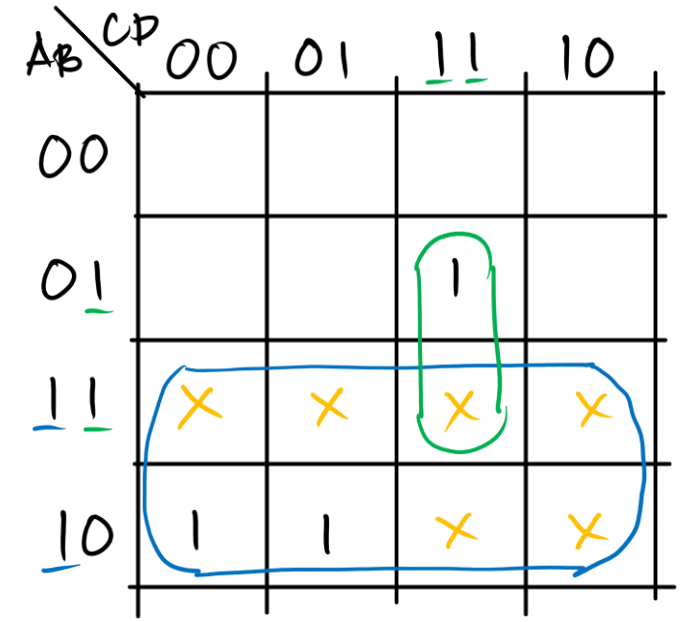
EXERCISE

Map the given table to a Karnaugh map and generate the minimized expression.

A	B	C	D	f
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1

A	B	C	D	f
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

Solution



note

X represents "don't care" term – it can be treated as either 1 or 0.

POS MINIMIZATION

KARNAUGH MAP

3-variable K-map

AB \ C	0	1
00	$A+B+C$	$A+B+\bar{C}$
01	$A+\bar{B}+C$	$A+\bar{B}+\bar{C}$
11	$\bar{A}+\bar{B}+C$	$\bar{A}+\bar{B}+\bar{C}$
10	$\bar{A}+B+C$	$\bar{A}+B+\bar{C}$

4-variable K-map

AB \ CD	00	01	11	10
00	$A+B+C+D$	$A+B+C+\bar{D}$	$A+B+\bar{C}+\bar{D}$	$A+B+\bar{C}+D$
01	$A+\bar{B}+C+D$	$A+\bar{B}+C+\bar{D}$	$A+\bar{B}+\bar{C}+\bar{D}$	$A+\bar{B}+\bar{C}+D$
11	$\bar{A}+\bar{B}+C+D$	$\bar{A}+\bar{B}+C+\bar{D}$	$\bar{A}+\bar{B}+\bar{C}+\bar{D}$	$\bar{A}+\bar{B}+\bar{C}+D$
10	$\bar{A}+B+C+D$	$\bar{A}+B+C+\bar{D}$	$\bar{A}+B+\bar{C}+\bar{D}$	$\bar{A}+B+\bar{C}+D$

MAPPING A STANDARD POS EXPRESSION

A **0** is placed on the K-map cell that corresponds to the value of a **sum term** in the expression.

3-variable K-map

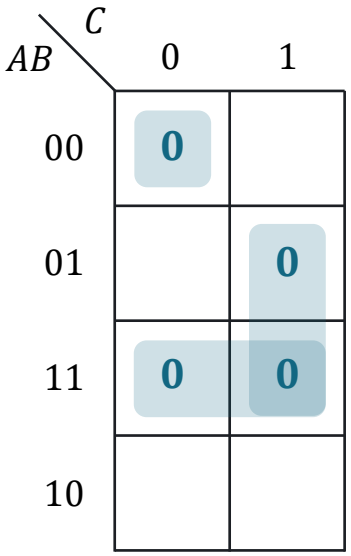
$$f = (A + B + C)(A + \bar{B} + \bar{C})(\bar{A} + \bar{B} + C)(\bar{A} + \bar{B} + \bar{C})$$
$$000 \qquad 011 \qquad 110 \qquad 111$$

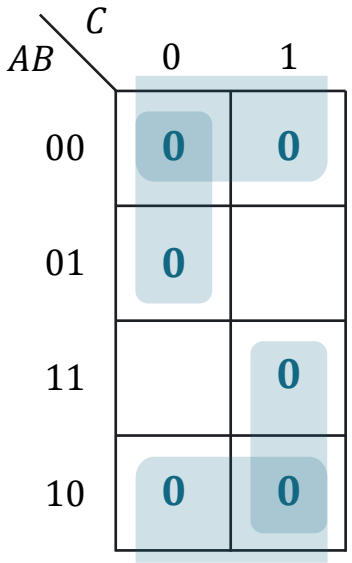
AB \ C	0	1
00	**0**	
01		**0**
11	**0**	**0**
10		

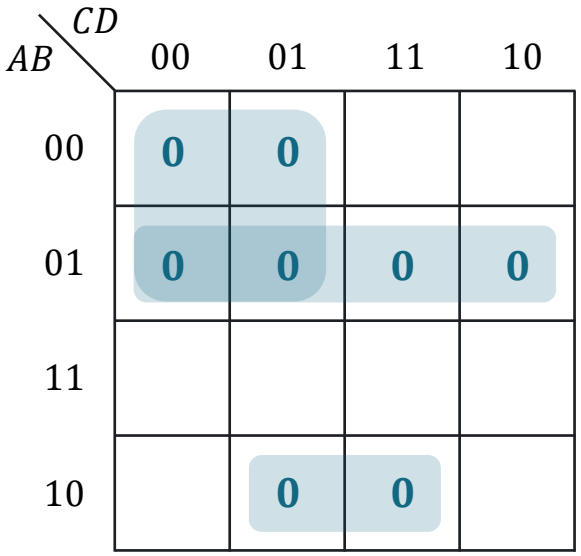
GROUPING 0s

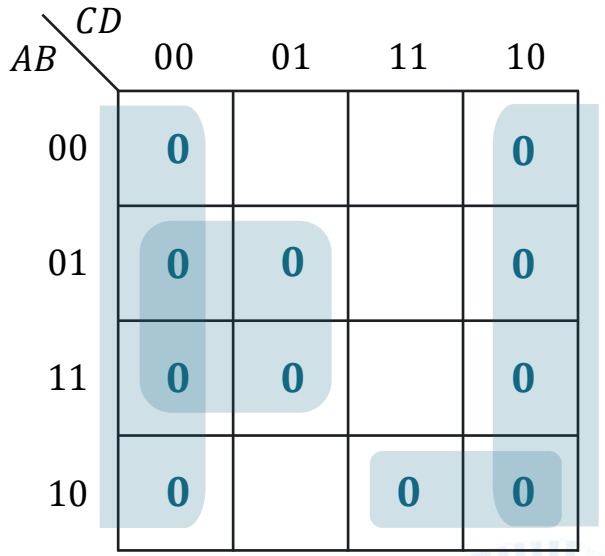
A group must contain either 1, 2, 4, 8, or 16 cells,

which are all **powers of two**.



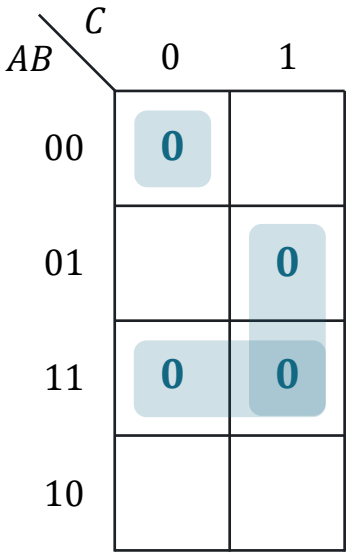




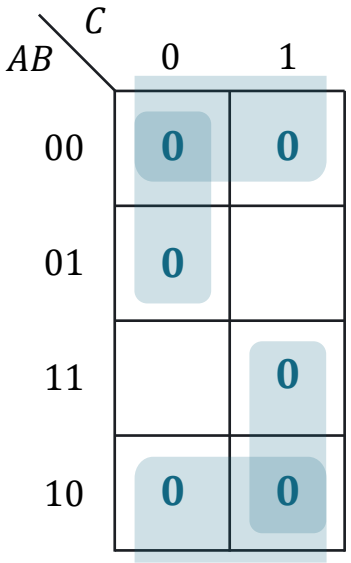


POS MINIMIZATION

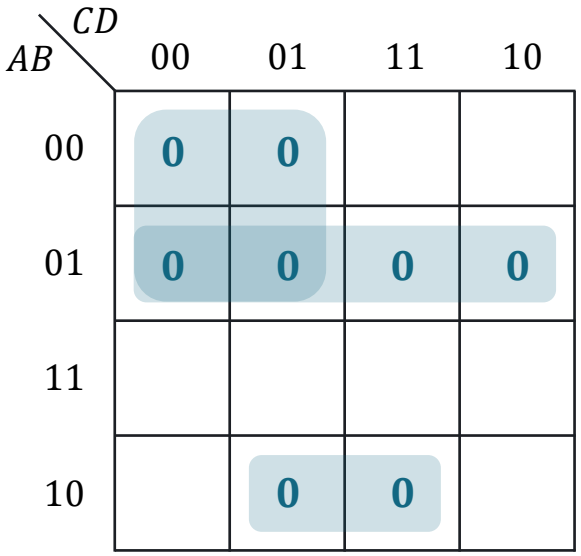
The process that results in an expression containing the fewest possible terms with the fewest possible variables is called **minimization**.



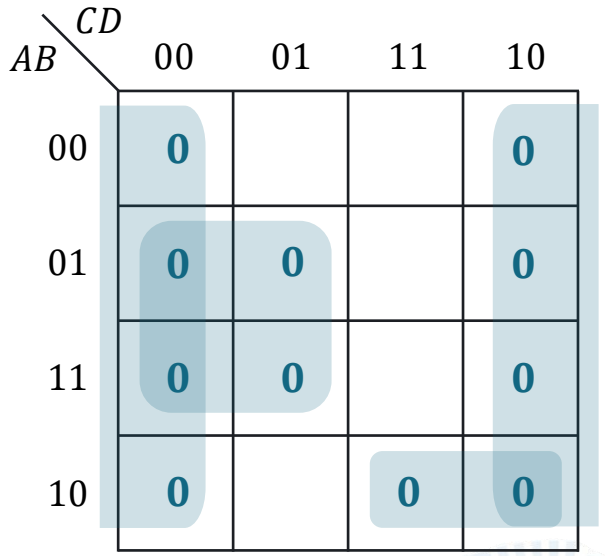
$f = (A + B + C)(\bar{A} + \bar{B})(\bar{B} + \bar{C})$



$f = (B)(A + C)(\bar{A} + \bar{C})$



$f = (A + C)(A + \bar{B})(\bar{A} + B + \bar{D})$



$f = (D)(\bar{B} + C)(\bar{A} + B + \bar{C})$

Variables that occur both **complemented** and **uncomplemented** within the group are eliminated. These are called **contradictory variables**.

EXERCISE

Use a Karnaugh map to minimize the given standard POS expression.

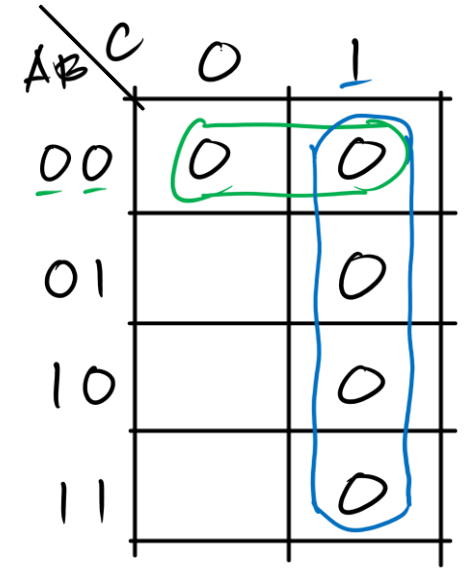
$$f = \prod M(0, 1, 3, 5, 7)$$

$$f = (A+B+C)(A+B+\bar{C})(A+\bar{B}+\bar{C})(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+\bar{C})$$

0 1 3 5 7

000 001 011 101 111

Solution



$$f = \bar{C}(A+B)$$

ans

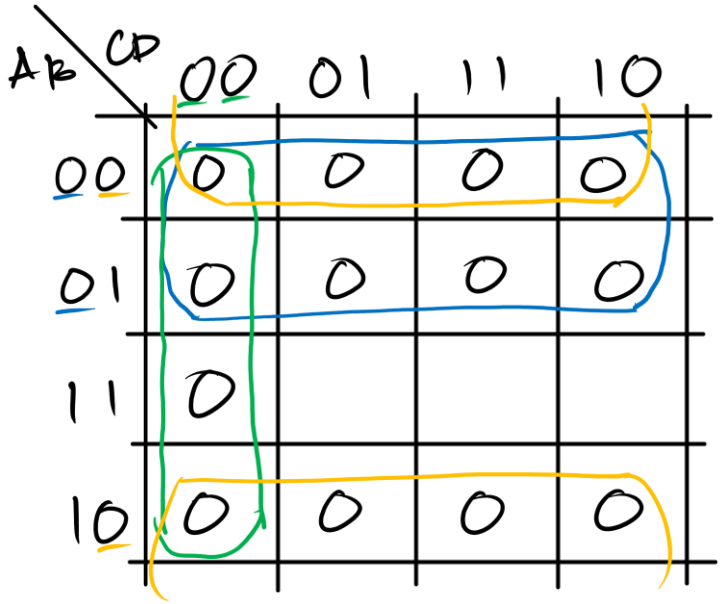
EXERCISE

Use a Karnaugh map to minimize the given standard POS expression.

$$f = \prod M(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12)$$

$$f = (A+B+C+D)(A+B+C+\bar{D})(A+B+\bar{C}+D)$$
$$(A+B+\bar{C}+\bar{D})(A+\bar{B}+C+D)(A+\bar{B}+C+\bar{D})$$
$$(A+\bar{B}+\bar{C}+D)(A+\bar{B}+\bar{C}+\bar{D})(\bar{A}+B+C+D)$$
$$(\bar{A}+B+C+\bar{D})(\bar{A}+B+\bar{C}+D)(\bar{A}+B+\bar{C}+\bar{D})$$
$$(\bar{A}+\bar{B}+C+D)$$

Solution



$$f = \bar{A}(C+D)B$$

$$\boxed{f = \bar{A}B(C+D)}$$

ans

EXERCISE

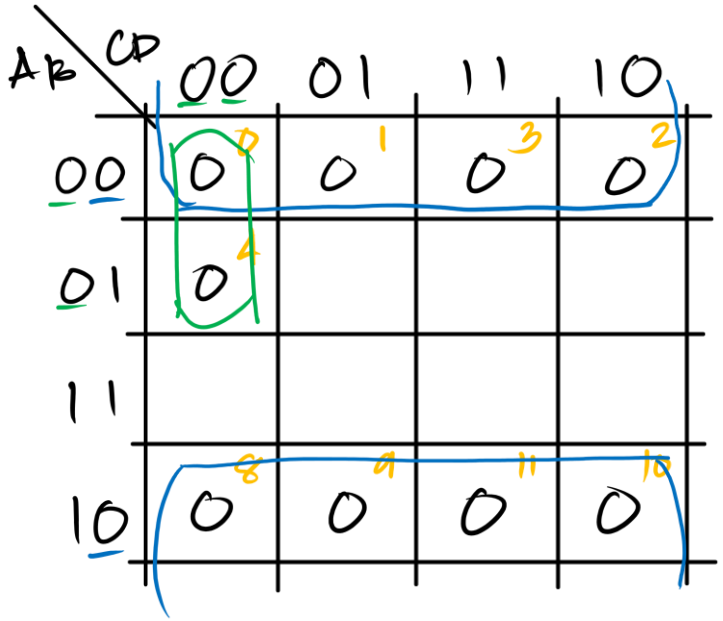
Use a Karnaugh map to minimize the given standard SOP expression.

$$f = \prod M(0, 1, 2, 3, 4, 8, 9, 10, 11)$$

$f = \bar{B}(A + C + D)$

ans

Solution



EXERCISE

Map the given table to a Karnaugh map and generate the minimized expression.

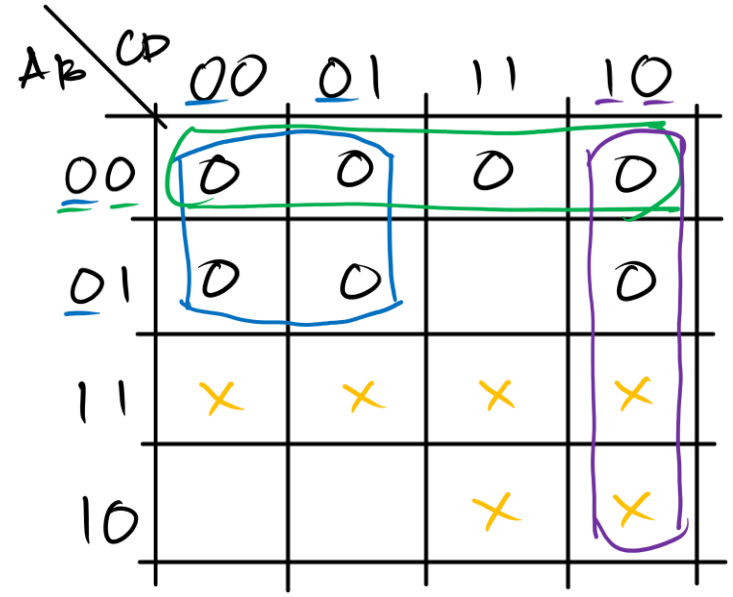
A	B	C	D	f
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1

A	B	C	D	f
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

note

X represents "don't care" term – it can be treated as either 1 or 0.

Solution



LABORATORY