DEGREE AND RADIAN

MEASURES OF ANGLES

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TOPIC OUTLINE

Degree

Radian

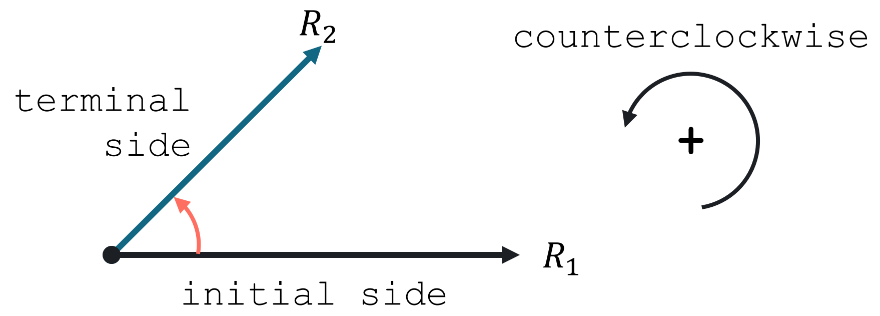
DEGREE

ANGLE

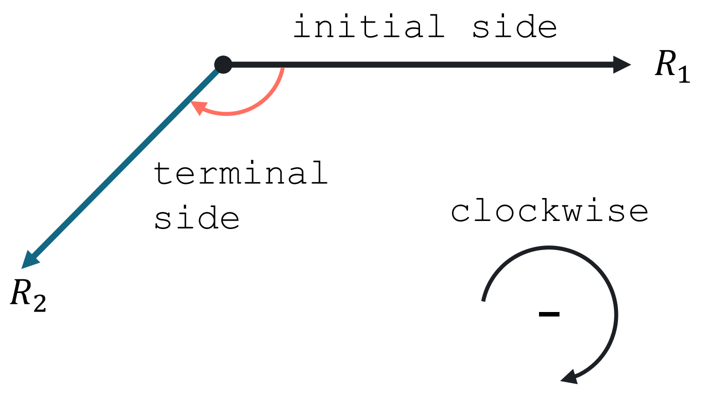
An **angle** is formed by two rays (R_1 and R_2) that share a common endpoint, called the vertex (O).

It can be interpreted as the **amount of rotation** from one ray (initial side) to another (terminal side) around the vertex.

Positive angle



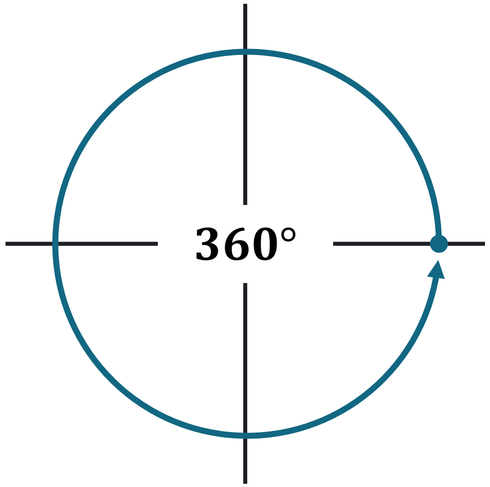
Negative angle



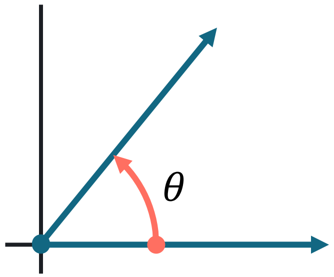
DEGREE

The **degree** (°) is the most commonly used unit for measuring angles.

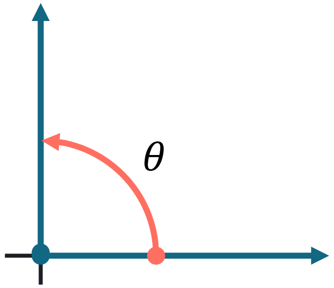
A full rotation around a circle corresponds to 360°.



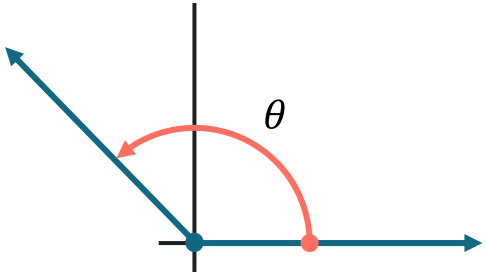
Classification of Angles



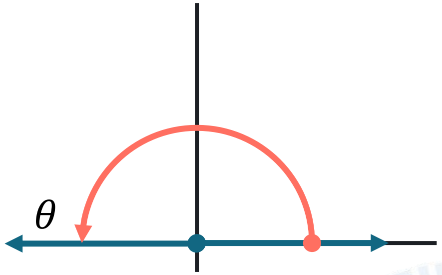
Acute angle
$0° < \theta < 90°$



Right angle
$\theta = 90°$



Obtuse angle
$90° < \theta < 180°$



Straight angle
$\theta = 180°$

COMPLEMENTARY ANGLES

If the sum of the measures of two positive angles is **90°**, the angles are **complementary** and the angles are complements of each other.

Formula

$$\boldsymbol{\theta_A + \theta_B = 90^\circ}$$

Example

Find the complement of an angle measuring 40°.

Let $\theta_A = 40^\circ$

$\theta_A + \theta_B = 90^\circ$

$40^\circ + \theta_B = 90^\circ$

$-40^\circ \qquad -40^\circ$

$\boxed{\theta_B = 50^\circ}$

ans

50° is a complement of 40°

50° and 40° are complementary angles

SUPPLEMENTARY ANGLES

If the sum of the measures of two positive angles is **180°**, the angles are **supplementary** and the angles are supplements of each other.

Formula

$$\boldsymbol{\theta_A + \theta_B = 180°}$$

Example

Find the supplement of an angle measuring 40°.

Let $\theta_A = 40°$

$\theta_A + \theta_B = 180°$

$40° + \theta_B = 180°$

$-40° \qquad\qquad -40°$

$\boxed{\theta_B = 140°}$

ans

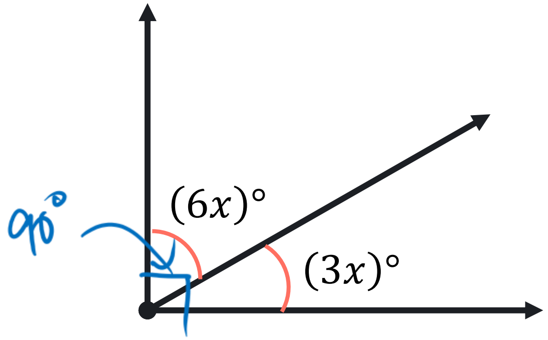
140° is a supplement of 40°

140° and 40° are supplementary angles

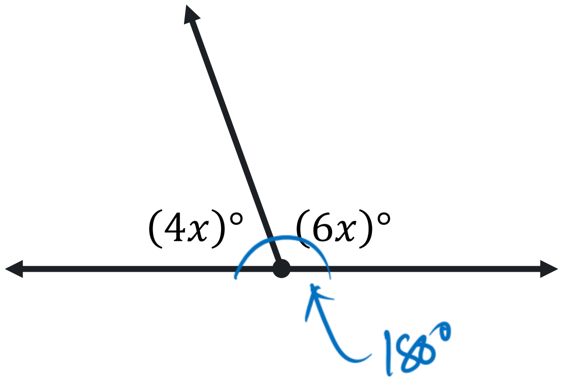
EXERCISE

Find the measure of each marked angle.

A.



B.



Solution

A. Complementary Angles

$$6x + 3x = 90^\circ$$

$$\frac{\cancel{9}x}{\cancel{9}} = \frac{90^\circ}{9}$$

$$x = 10^\circ$$

$$\boxed{6x = 60^\circ}$$ ans

$$\boxed{3x = 30^\circ}$$ ans

B. Supplementary Angles

$$4x + 6x = 180^\circ$$

$$\frac{\cancel{10}x}{\cancel{10}} = \frac{180^\circ}{10}$$

$$x = 18^\circ$$

$$\boxed{4x = 72^\circ}$$ ans

$$\boxed{6x = 108^\circ}$$ ans

DEGREES, MINUTES, SECONDS

One **minute** ($1'$) is $\frac{1}{60}$ of a degree.

$$1' = \frac{1^\circ}{60}$$

$$60' = 1^\circ$$

One **second** ($1''$) is $\frac{1}{60}$ of a minute.

$$1'' = \frac{1'}{60}$$

$$60'' = 1'$$

$$3600'' = 1^\circ$$

Example

Convert $74^\circ 08' 14''$ to decimal degrees to the nearest thousandth.

$$74^\circ + \cancel{8'}\frac{1^\circ}{60\cancel{'}} + 14\cancel{''}\frac{1\cancel{'}}{60\cancel{''}} \cdot \frac{1^\circ}{60\cancel{'}}$$

min → deg　　sec → min → deg

$$\boxed{74.137^\circ}$$

ans

EXERCISE

Perform each calculation and express the result in degrees, rounded to the nearest thousandth.

$a.\ 51°29' + 32°46'$

$b.\ 90° - 73°12'$

b.

$$\begin{array}{r} \overset{89}{\cancel{90}}°\ \overset{5}{\cancel{6}}0' \\ -\ 73°\ 12' \\ \hline 16°\ 48' \end{array}$$

$90° = 89°60'$

$$\boxed{16°48'}$$

ans

Solution

a.

$$\begin{array}{r} 51°\ 29' \\ +\ 32°\ 46' \\ \hline 84°\ 15' \end{array}$$

$$\begin{array}{r} 75 \\ -60 \\ \hline 15 \end{array}$$

60 min (MAX)

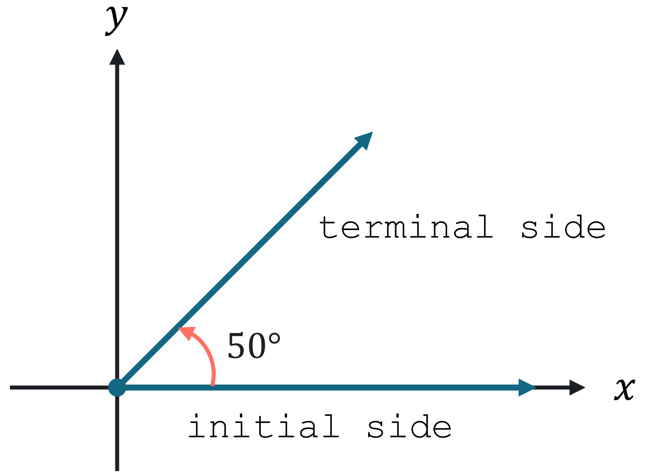
$$\boxed{84°15'}$$

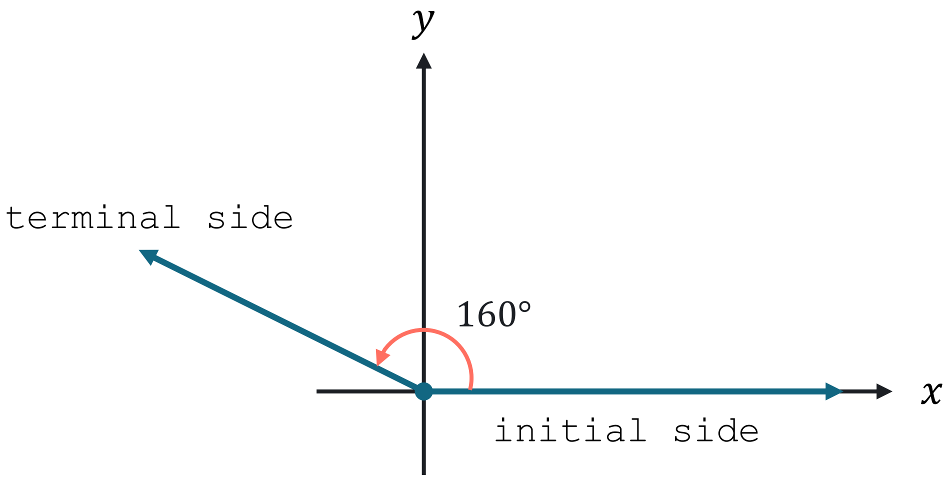
ans

STANDARD POSITION

An angle is in **standard position** if its vertex is at the **origin** and its initial side lies on the **positive x-axis**.

Angles in Standard Position

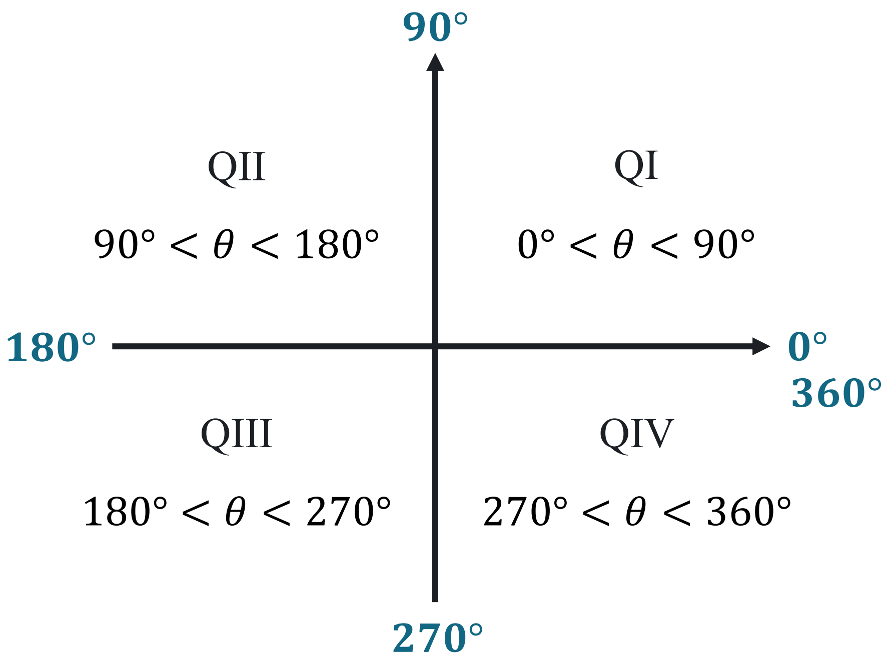




QUADRANTAL ANGLES

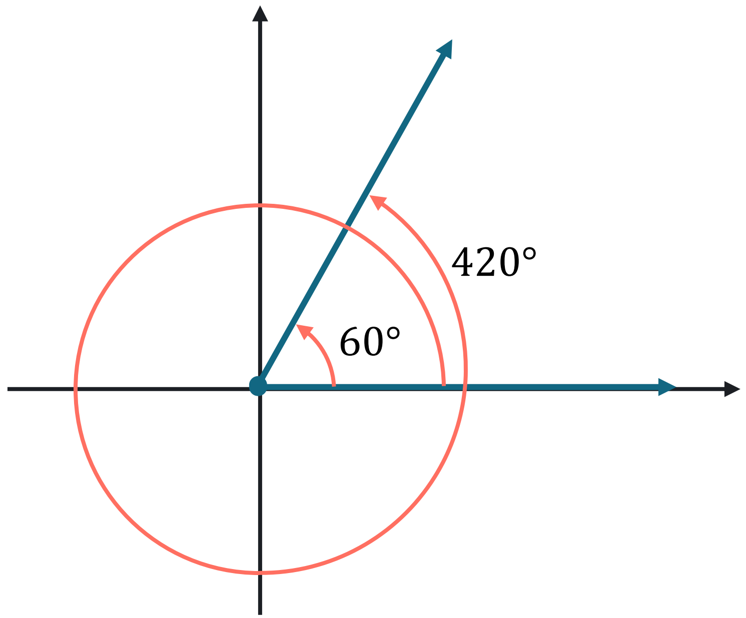
Angles in standard position whose terminal sides lie on the x-axis or y-axis, such as angles with measures $\mathbf{90°}$, $\mathbf{180°}$, $\mathbf{270°}$, and so on, are **quadrantal angles**.

Quadrantal Angles



COTERMINAL ANGLES

Coterminal Angles



Coterminal angles are angles that share the same terminal side when drawn in standard position. Their measures differ by a **multiple of 360°**, meaning they can be found by adding or subtracting 360° repeatedly.

EXERCISE

Find the angle of least positive measure that is coterminal with each angle.

$a.\ 908°$

$b.\ -75°$

$c.\ -800°$

Solution

a. $\theta = 908° - 2(360°)$

$$\boxed{\theta = 188°}$$

ans

b. $\theta = -75° + 360°$

$$\boxed{\theta = 285°}$$

ans

c. $\theta = -800 + 3(360°)$

$$\boxed{\theta = 280°}$$

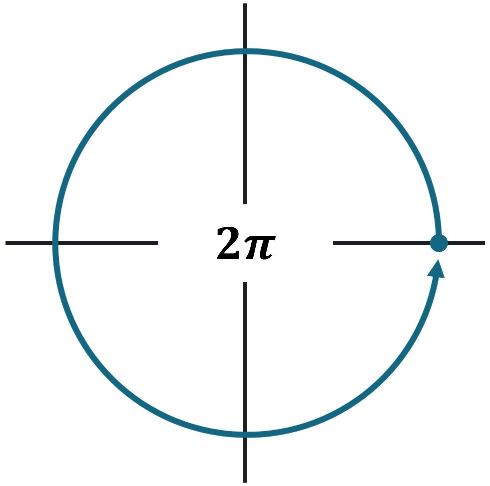
ans

RADIAN

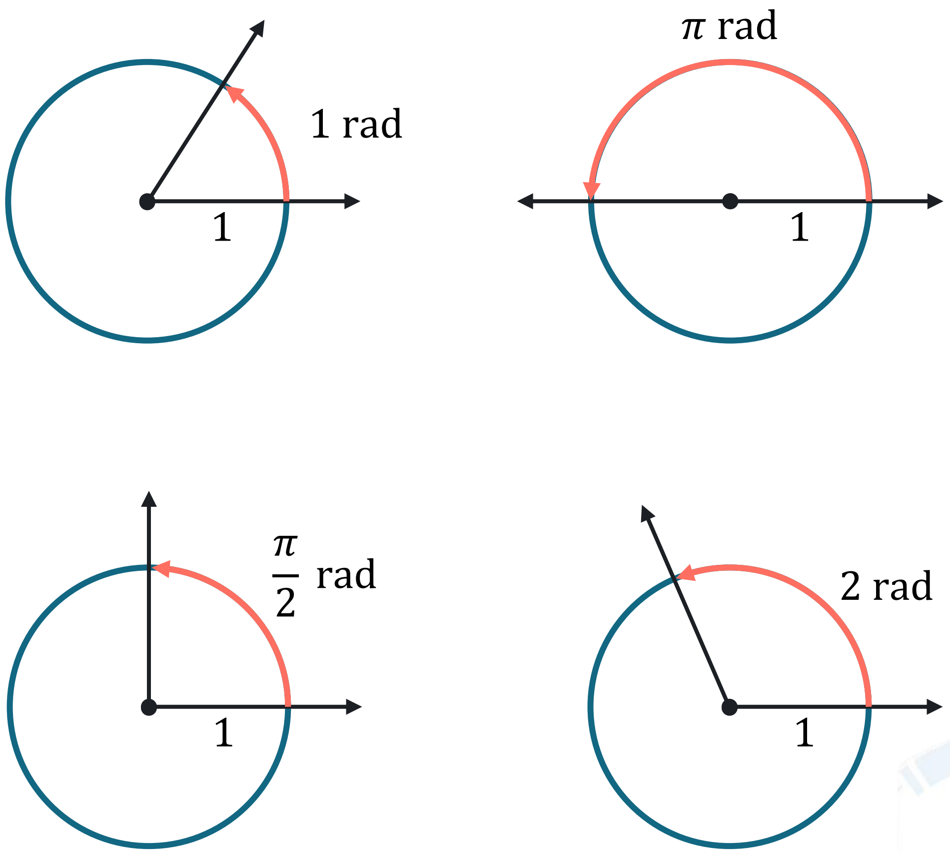
DEGREE

The **radian** (**rad**) is the angle subtended at the center of a circle by an arc whose length is equal to the radius of a circle.

A full rotation around a circle corresponds to 2π radians.



Unit Circle



EXERCISE

Find the radian measure of the angle with the given degree measure.

$a.\ 72°$

$b.\ -60°$

Solution

a. $\theta = 72\cancel{°} \dfrac{\pi \text{ rad}}{180\cancel{°}}$

$$\boxed{\theta = \frac{2\pi}{5} \text{ rad}}$$

ans

b. $\theta = -60° \dfrac{\pi \text{ rad}}{180°}$

coterminal angles

$\theta = 300\cancel{°} \dfrac{\pi \text{ rad}}{180\cancel{°}}$

$$\boxed{\theta = \frac{5\pi}{3} \text{ rad}}$$

ans

EXERCISE

Find the degree measure of the angle with the given radian measure.

$a.\ \dfrac{7\pi}{6}$

$b.\ -\dfrac{5\pi}{4}$

Solution

a. $\theta = \dfrac{7\pi \cancel{\text{rad}}}{6} \cdot \dfrac{180^\circ}{\pi \cancel{\text{rad}}}$

$\boxed{\theta = 210^\circ}$

ans

b. $\theta = -\dfrac{5\pi \cancel{\text{rad}}}{4} \cdot \dfrac{180^\circ}{\pi \cancel{\text{rad}}}$

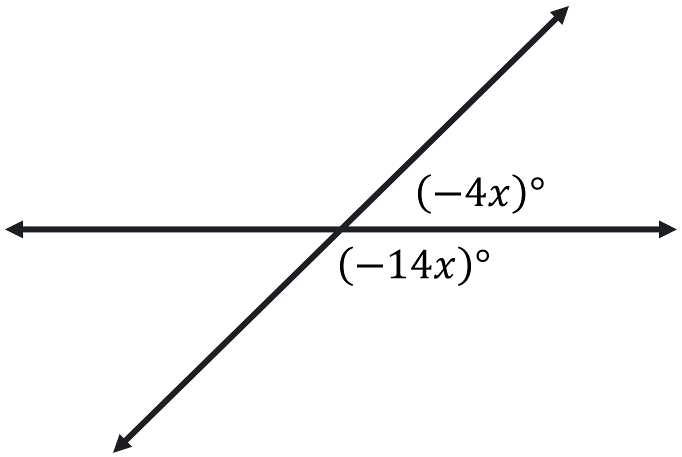
$\theta = -225^\circ$

$\boxed{\theta = 135^\circ}$

ans

EXERCISE

Find the measure of the marked angle.



Solution

EXERCISE

A constant angular velocity disk drive spins a disk at a constant speed. Suppose a disk makes 480

1 rev = 360°

revolutions per min. Through how many degrees will a point on the edge of the disk move in 2 sec?

Solution

$$\frac{\text{rev}}{\text{min}} \longrightarrow \frac{\text{deg}}{\text{sec}}$$

1 rev

360°

SEATWORK