KIRCHHOFF'S CURRENT AND VOLTAGE LAW

BASIC CIRCUIT ANALYSIS METHOD

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TOPIC OUTLINE

Circuit Convention

Kirchhoff’s Current Law (KCL)

Kirchhoff’s Voltage Law (KVL)

CIRCUIT CONVENTION

CONVENTION

A **convention** is a widely accepted practice, method, or behavior that is followed by common **agreement** or tradition, rather than by formal rules.

example

Color coding in Offices

red – urgent documents

blue – general files

green – financial records

This is a common practice but not formally regulated.

STANDARD

A **standard** is a formal, established guideline, rule, or specification that is often **mandatory** and enforced by an authoritative body or organization.

example

IEC 60062 Resistor Color Code

black – 0

brown – 1

red – 2

.

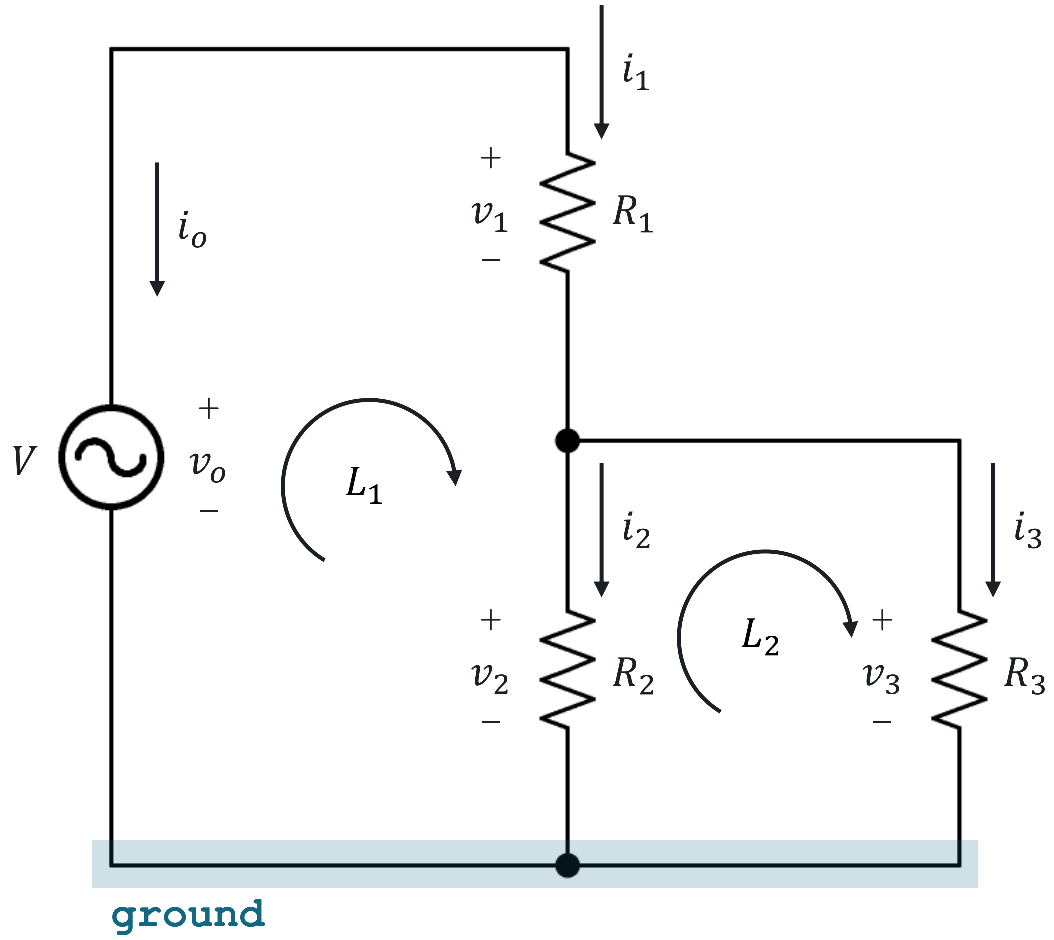
.

.

white – 9

Resistors have colored bands that represent specific digits, multipliers, and tolerance values.

LABELING VARIABLES



Steps in Labeling Variables

1. **Label the Reference Node** `(ground)`

Select a reference node with the most connections or the negative `(-)` terminal of a voltage source.

2. **Label Node Voltages**

Mark higher potentials as positive `(+)` relative to the reference node.

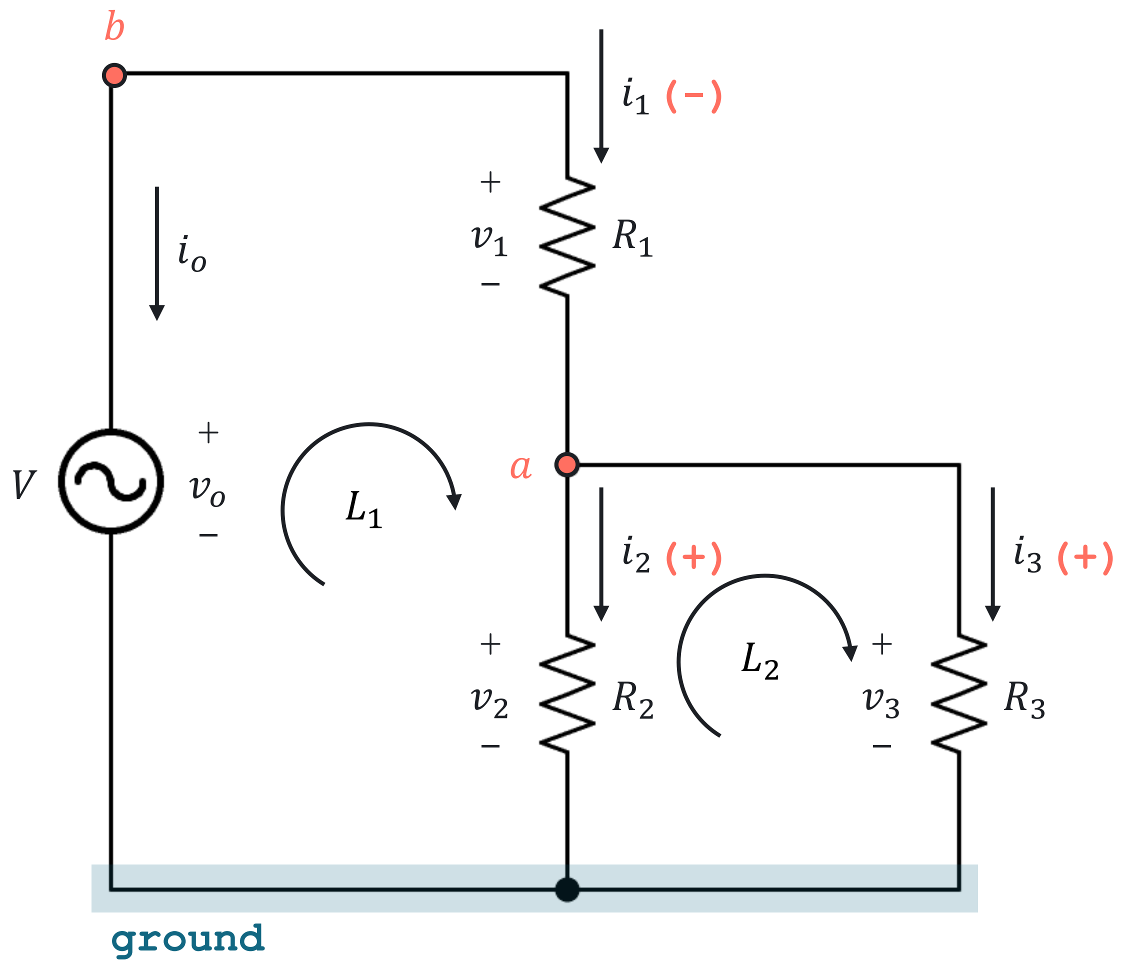
3. **Label Currents**

Entering the positive `(+)` terminal of a component.

4. **Create a voltage loop**

Follow the defined current directions.

CIRCUIT CONVENTION

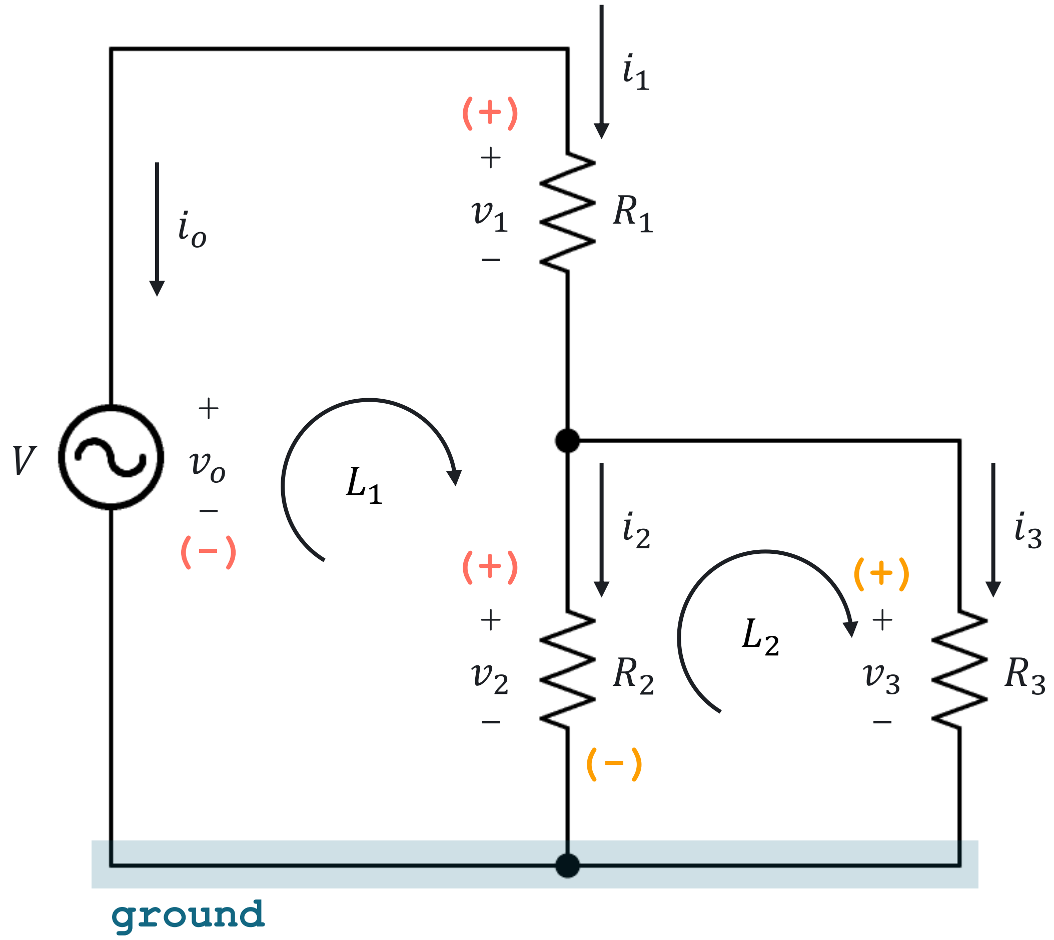


Current Flow Convention

- Current **entering** a node is negative **(−)**
- Current **leaving** a node is positive **(+)**

@a	**@b**
$-i_1$	$+i_o$
$+i_2$	$+i_1$
$+i_3$	

CIRCUIT CONVENTION



Voltage Loop Convention

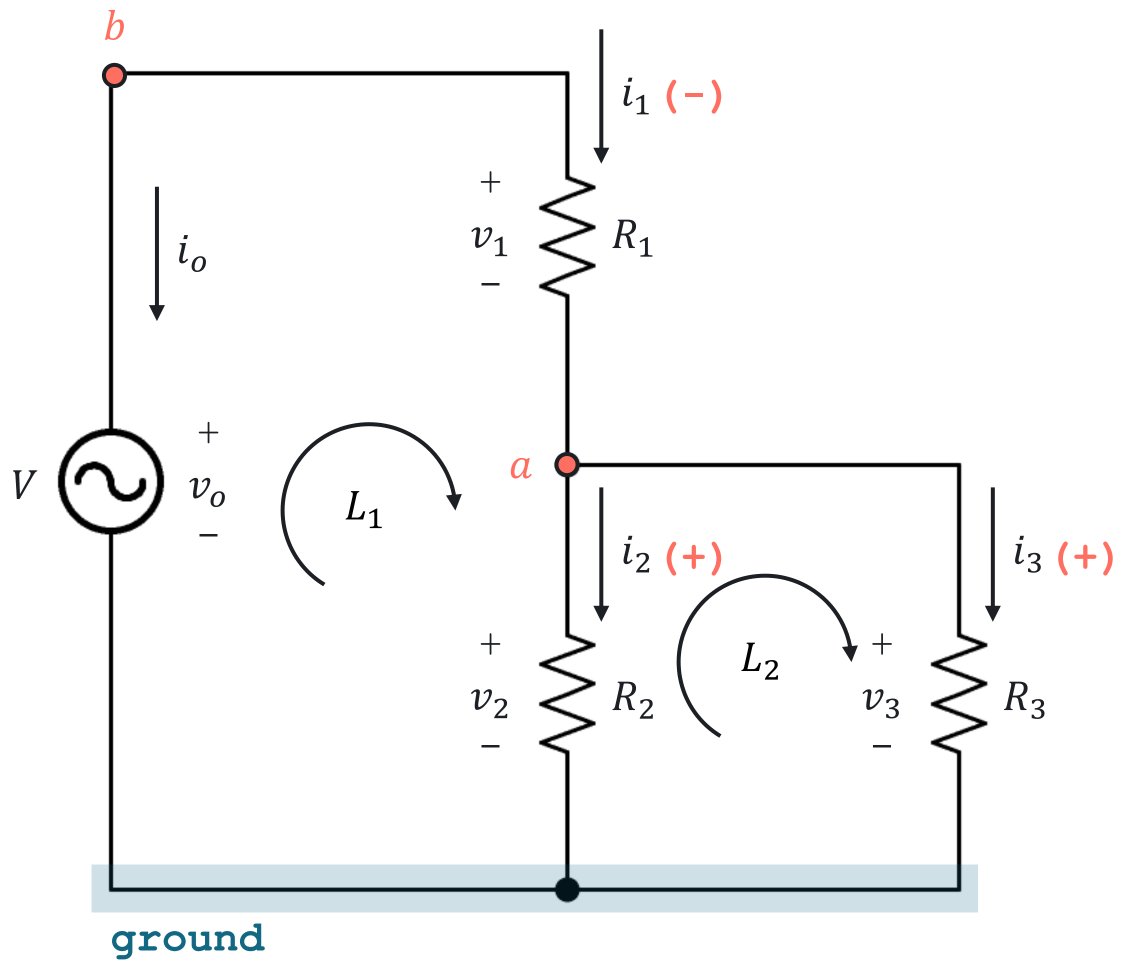
The **"sign"** of voltage of the element is the **first sign** the loop encounters.

@L_1	**@L_2**
$-v_o$	$-v_2$
$+v_1$	$+v_3$
$+v_2$	

KIRCHHOFF'S

CURRENT AND VOLTAGE LAW

KIRCHHOFF'S CURRENT LAW



Kirchhoff's current law states that summation of currents going-in and going-out a node is zero.

$$\sum i_j = 0$$

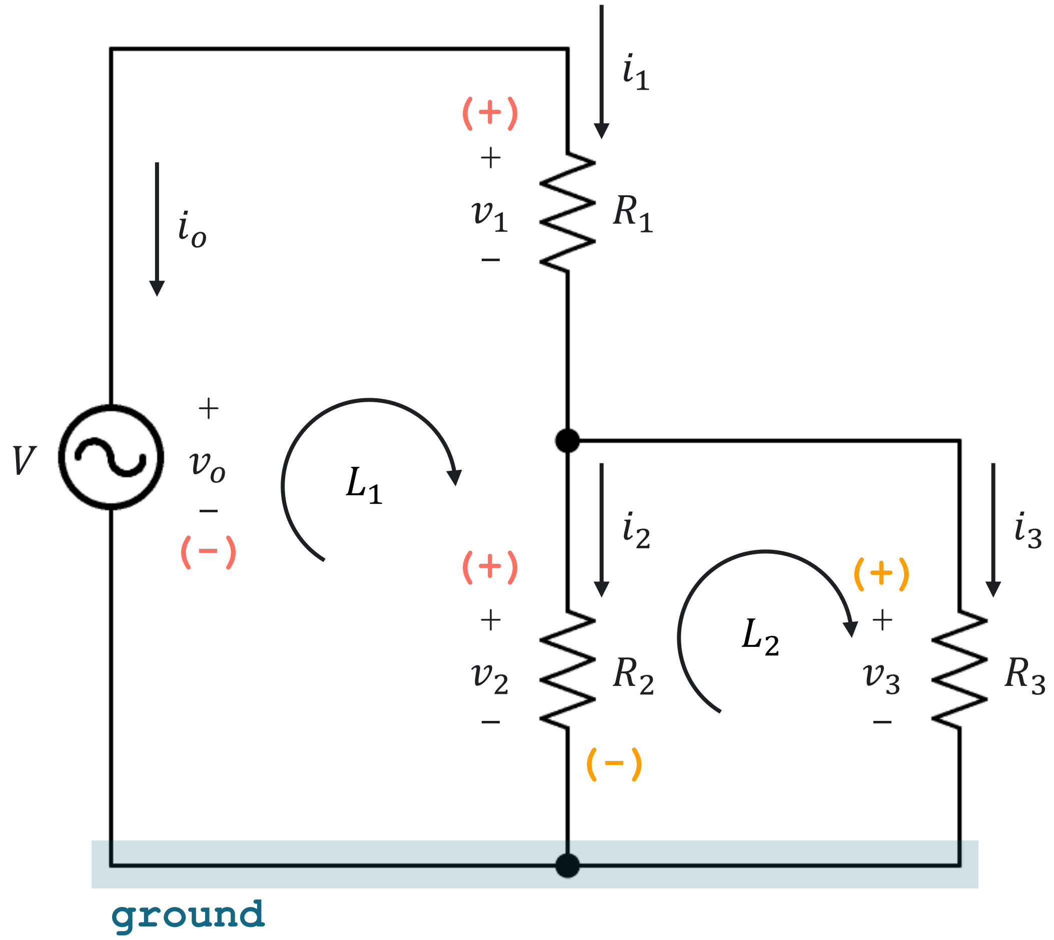
KCL @a

$$-i_1 + i_2 + i_3 = 0$$

KCL @b

$$i_o + i_1 = 0$$

KIRCHHOFF'S VOLTAGE LAW



Kirchhoff's voltage law states that the summation of voltages in a closed-loop is zero.

$$\sum v_j = 0$$

KVL @L_1

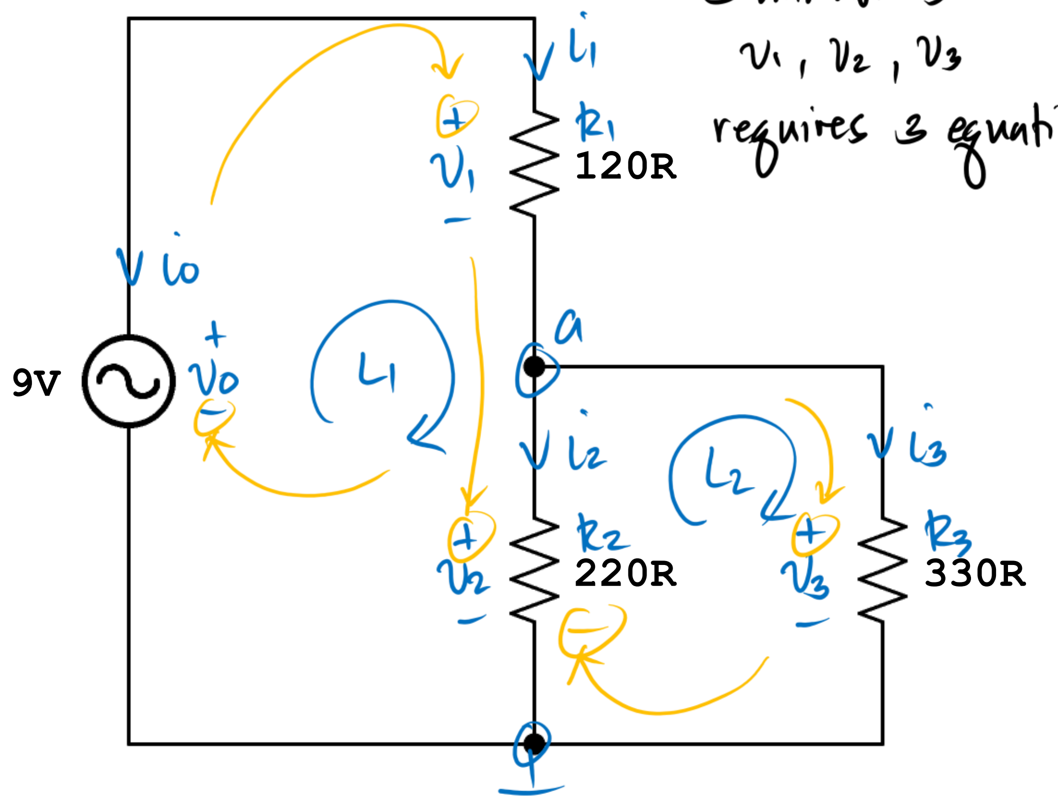
$$-v_o + v_1 + v_2 = 0$$

KVL @L_2

$$-v_2 + v_3 = 0$$

EXERCISE

Analyze the given circuit to determine both the current through and the voltage drop across each resistor.



3 unknowns

v_1, v_2, v_3

requires 3 equations

Solution

KCL @ a

$$-i_1 + i_2 + i_3 = 0$$

$$-\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3} = 0$$

$$-v_1 G_1 + v_2 G_2 + v_3 G_3 = 0 \quad \text{eq (1)}$$

KVL @ L_1

$$-v_0 + v_1 + v_2 = 0$$

$$v_1 + v_2 = v_0 \quad \text{eq (2)}$$

KVL @ L_2

$$-v_2 + v_3 = 0 \quad \text{eq (3)}$$

EXERCISE

Gaussian elimination method

	v_1	v_2	v_3		
	$-\frac{1}{120}$	$\frac{1}{220}$	$\frac{1}{330}$	0	r_1
	1	1	0	9	r_2
	0	-1	1	0	r_3

System of linear equations

$$-v_1 G_1 + v_2 G_2 + v_3 G_3 = 0 \quad eq ①$$

$$v_1 + v_2 = v_0 \quad eq ②$$

$$-v_2 + v_3 = 0 \quad eq ③$$

$$-v_1 \frac{1}{120} + v_2 \frac{1}{220} + \frac{1}{330} = 0 \quad ①$$

$$v_1 + v_2 = 9 \quad ②$$

$$-v_2 + v_3 = 0 \quad ③$$

EXERCISE

Gaussian elimination method

	v_1	v_2	v_3		
	$-\frac{1}{120}$	$\frac{1}{220}$	$\frac{1}{330}$	0	r_1
	1	1	0	9	$r_2 \leftarrow 120 r_1 + r_2$
	0	-1	1	0	r_3

System of linear equations

	v_1	v_2	v_3		
	$-\frac{1}{120}$	$\frac{1}{220}$	$\frac{1}{330}$	0	r_1
	0	$\frac{17}{11}$	$\frac{4}{11}$	9	r_2
	0	-1	1	0	r_3

EXERCISE

Gaussian elimination method

	v_1	v_2	v_3		
	$-\frac{1}{120}$	$\frac{1}{220}$	$\frac{1}{330}$	0	r_1
	0	$\frac{17}{11}$	$\frac{4}{11}$	9	r_2
	0	0	$\frac{21}{11}$	9	r_3

System of linear equations

	v_1	v_2	v_3		
	$-\frac{1}{120}$	$\frac{1}{220}$	$\frac{1}{330}$	0	r_1
	0	$\frac{17}{11}$	$\frac{4}{11}$	9	r_2
	0	-1	1	0	$r_3 \leftarrow \frac{17}{11} r_3 + r_2$

EXERCISE

Gaussian elimination method

$$
\begin{array}{ccc|c|c}
v_1 & v_2 & v_3 & & \\
\hline
-\frac{1}{120} & \frac{1}{220} & \frac{1}{330} & 0 & r_1 \\
0 & \frac{17}{11} & \frac{4}{11} & 9 & r_2 \\
0 & 0 & \frac{21}{11} & 9 & r_3
\end{array}
$$

System of linear equations

from r_3

$$\frac{21}{11} v_3 = 9$$

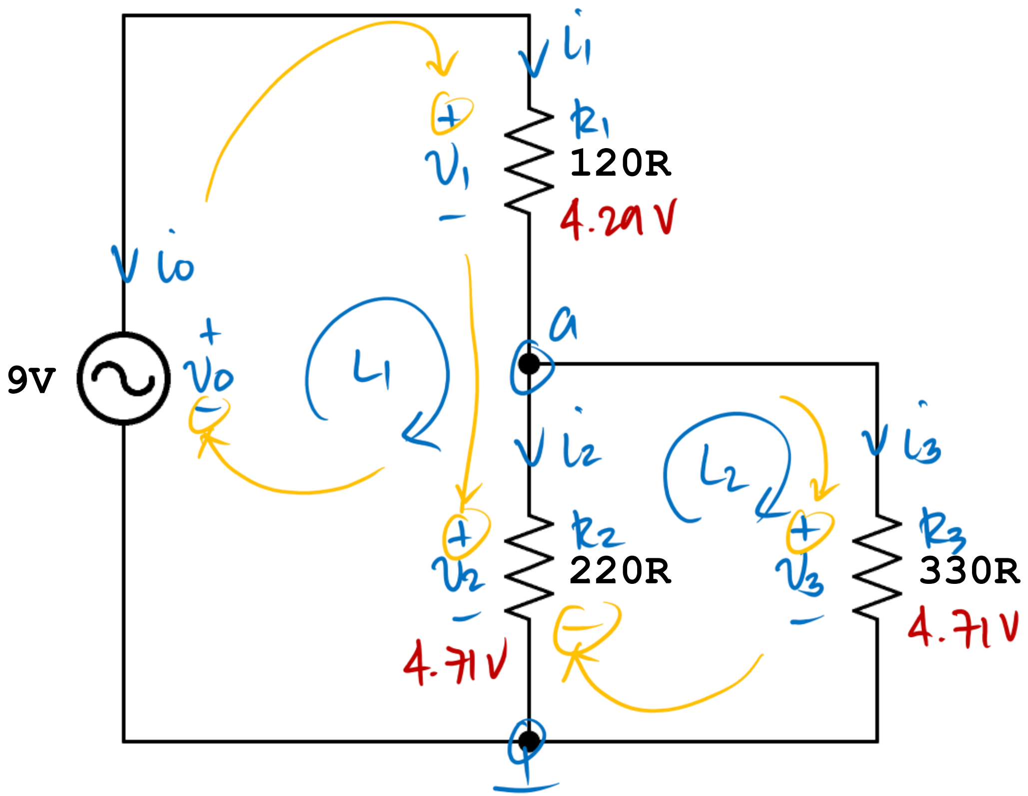
$$v_3 = \frac{9(11)}{21}$$

$$\boxed{v_3 = 4.71\ V}$$

ans

EXERCISE

Analyze the given circuit to determine both the current through and the voltage drop across each resistor.



Solution

KVL @ L_2

$$-v_2 + v_3 = 0$$

$$v_2 = v_3$$

$$\boxed{v_2 = 4.71\,V}$$

ans

KVL @ L_1

$$-v_0 + v_1 + v_2 = 0$$

$$v_1 = v_0 - v_2$$

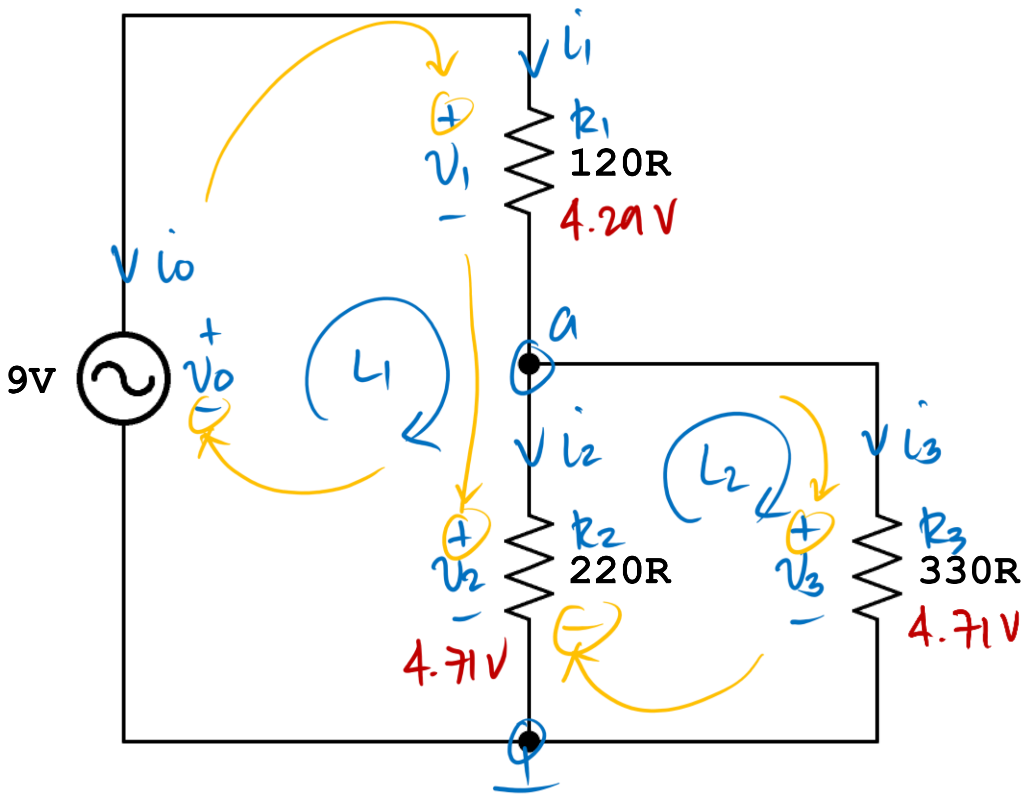
$$= 9 - 4.71$$

$$\boxed{v_1 = 4.29\,V}$$

ans

EXERCISE

Analyze the given circuit to determine both the current through and the voltage drop across each resistor.



Solution

$$i_1 = \frac{v_1}{R_1} = \frac{4.29}{120}$$

$$\boxed{i_1 = 35.75\,mA}$$

ans

$$i_2 = \frac{v_2}{R_2} = \frac{4.71}{220}$$

$$\boxed{i_2 = 21.41\,mA}$$

ans

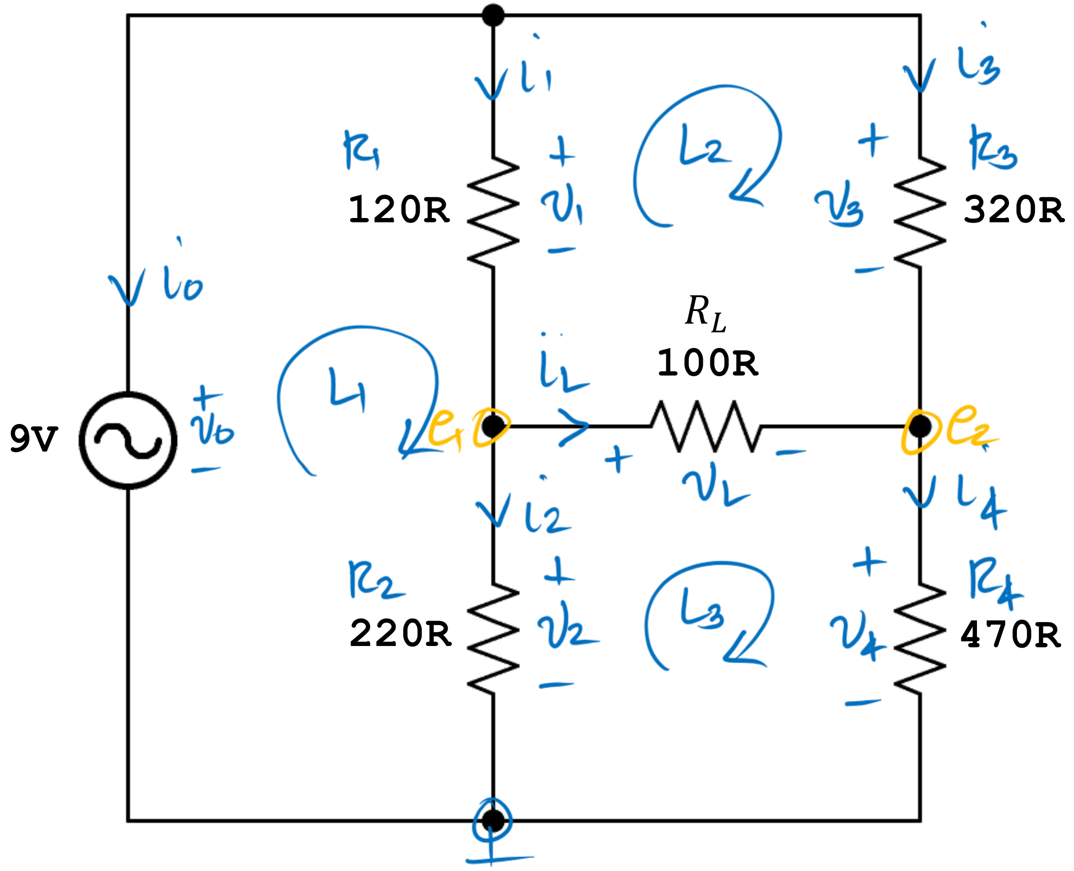
$$i_3 = \frac{v_3}{R_3} = \frac{4.71}{330}$$

$$\boxed{i_3 = 14.27\,mA}$$

ans

EXERCISE

Determine the voltage drop across the load resistor and the current flowing through it.



Solution

KCL @ e_1

$$-i_1 + i_2 + i_L = 0$$

$$-\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_L}{R_L} = 0$$

$$-v_1G_1 + v_2G_2 + v_LG_L = 0 \quad (\text{eq.}1)$$

KCL @ e_2

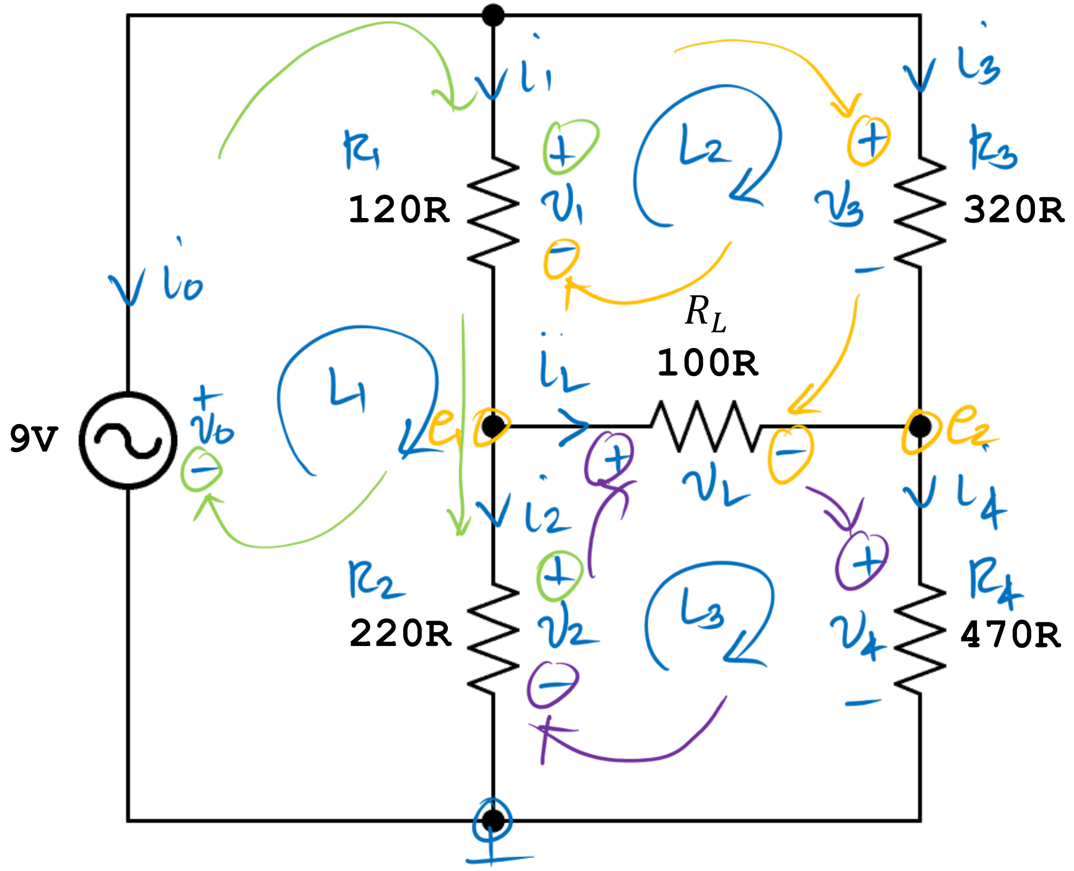
$$-i_3 + i_4 - i_L = 0$$

$$-\frac{v_3}{R_3} + \frac{v_4}{R_4} - \frac{v_L}{R_L} = 0$$

$$-v_3G_3 + v_4G_4 - v_LG_L = 0 \quad (\text{eq.}2)$$

EXERCISE

Determine the voltage drop across the load resistor and the current flowing through it.



Solution

KVL@ L_1

$$-v_0 + v_1 + v_2 = 0$$

$$v_1 + v_2 = v_0 \quad (eq.3)$$

KVL@ L_2

$$-v_1 + v_3 - v_L = 0 \quad (eq.4)$$

KVL@ L_3

$$-v_2 + v_4 + v_L = 0 \quad (eq.5)$$

EXERCISE

Gaussian Elimination Method

$$
\begin{array}{c|ccccc|c|l}
 & v_1 & v_2 & v_3 & v_4 & v_L & & \\
\hline
 & -\frac{1}{120} & \frac{1}{220} & 0 & 0 & \frac{1}{100} & 0 & r_1 \\
 & 0 & 0 & -\frac{1}{320} & \frac{1}{470} & -\frac{1}{100} & 0 & r_2 \\
 & 1 & 1 & 0 & 0 & 0 & 9 & r_3 \\
 & -1 & 0 & 1 & 0 & -1 & 0 & r_4 \\
 & 0 & -1 & 0 & 1 & 1 & 0 & r_5
\end{array}
$$

System of Linear Equations

$$-v_1 \overset{\frac{1}{120}}{\cancel{G_1}} + v_2 \overset{\frac{1}{220}}{\cancel{G_2}} + v_L \overset{\frac{1}{100}}{\cancel{G_L}} = 0 \quad (eq.1)$$

$$-v_3 \overset{\frac{1}{320}}{\cancel{G_3}} + v_4 \overset{\frac{1}{470}}{\cancel{G_4}} - v_L \overset{\frac{1}{100}}{\cancel{G_L}} = 0 \quad (eq.2)$$

$$v_1 + v_2 = \overset{9}{\cancel{10}} \quad (eq.3)$$

$$-v_1 + v_3 - v_L = 0 \quad (eq.4)$$

$$-v_2 + v_4 + v_L = 0 \quad (eq.5)$$

EXERCISE

Gaussian Elimination Method

swap

$$
\begin{array}{c|ccccc|c|c}
 & v_1 & v_2 & v_3 & v_4 & v_L & & \\
\hline
 & -\frac{1}{120} & \frac{1}{220} & 0 & 0 & \frac{1}{100} & 0 & r_1 \\
 & 0 & 0 & -\frac{1}{320} & \frac{1}{470} & -\frac{1}{100} & 0 & r_2 \\
 & 1 & 1 & 0 & 0 & 0 & 9 & r_3 \\
 & -1 & 0 & 1 & 0 & -1 & 0 & r_4 \\
 & 0 & -1 & 0 & 1 & 1 & 0 & r_5
\end{array}
$$

System of Linear Equations

$$
\begin{array}{c|ccccc|c|c}
 & v_1 & v_2 & v_3 & v_4 & v_L & & \\
\hline
 & -\frac{1}{120} & \frac{1}{220} & 0 & 0 & \frac{1}{100} & 0 & r_1 \\
 & 0 & -1 & 0 & 1 & 1 & 0 & r_2 \\
 & 0 & 0 & -\frac{1}{320} & \frac{1}{470} & -\frac{1}{100} & 0 & r_3 \\
 & -1 & 0 & 1 & 0 & -1 & 0 & r_4 \\
 & 1 & 1 & 0 & 0 & 0 & 9 & r_5
\end{array}
$$

EXERCISE

Gaussian Elimination Method

v_1	v_2	v_3	v_4	v_L		
$-\frac{1}{120}$	$\frac{1}{220}$	0	0	$\frac{1}{100}$	0	r_1
0	-1	0	1	1	0	r_2
0	0	$-\frac{1}{320}$	$\frac{1}{470}$	$-\frac{1}{100}$	0	r_3
0	$\frac{6}{11}$	-1	0	$\frac{11}{5}$	0	r_4
1	1	0	0	0	9	r_5

System of Linear Equations

v_1	v_2	v_3	v_4	v_L		
$-\frac{1}{120}$	$\frac{1}{220}$	0	0	$\frac{1}{100}$	0	r_1
0	-1	0	1	1	0	r_2
0	0	$-\frac{1}{320}$	$\frac{1}{470}$	$-\frac{1}{100}$	0	r_3
-1	0	1	0	-1	0	$r_4 \leftarrow 120\,r_1 - r_4$
1	1	0	0	0	9	r_5

EXERCISE

Gaussian Elimination Method

$$
\begin{array}{ccccc|c|l}
v_1 & v_2 & v_3 & v_4 & v_L & & \\
\hline
-\frac{1}{120} & \frac{1}{220} & 0 & 0 & \frac{1}{100} & 0 & r_1 \\
0 & -1 & 0 & 1 & 1 & 0 & r_2 \\
0 & 0 & -\frac{1}{320} & \frac{1}{470} & -\frac{1}{100} & 0 & r_3 \\
0 & \frac{6}{11} & -1 & 0 & \frac{11}{5} & 0 & r_4 \leftarrow \frac{6}{11} r_2 + r_4 \\
1 & 1 & 0 & 0 & 0 & 9 & r_5
\end{array}
\longrightarrow
$$

System of Linear Equations

$$
\begin{array}{ccccc|c|l}
v_1 & v_2 & v_3 & v_4 & v_L & & \\
\hline
-\frac{1}{120} & \frac{1}{220} & 0 & 0 & \frac{1}{100} & 0 & r_1 \\
0 & -1 & 0 & 1 & 1 & 0 & r_2 \\
0 & 0 & -\frac{1}{320} & \frac{1}{470} & -\frac{1}{100} & 0 & r_3 \\
0 & 0 & -1 & \frac{6}{11} & \frac{151}{55} & 0 & r_4 \\
1 & 1 & 0 & 0 & 0 & 9 & r_5
\end{array}
$$

EXERCISE

Gaussian Elimination Method

$$
\begin{array}{ccccc|c|l}
v_1 & v_2 & v_3 & v_4 & v_L & & \\
\hline
-\frac{1}{120} & \frac{1}{220} & 0 & 0 & \frac{1}{100} & 0 & r_1 \\
0 & -1 & 0 & 1 & 1 & 0 & r_2 \\
0 & 0 & -\frac{1}{320} & \frac{1}{470} & -\frac{1}{100} & 0 & r_3 \\
0 & 0 & 0 & \frac{70}{517} & -\frac{327}{55} & 0 & r_4 \\
1 & 1 & 0 & 0 & 0 & 9 & r_5
\end{array}
$$

System of Linear Equations

$$
\begin{array}{ccccc|c|l}
v_1 & v_2 & v_3 & v_4 & v_L & & \\
\hline
-\frac{1}{120} & \frac{1}{220} & 0 & 0 & \frac{1}{100} & 0 & r_1 \\
0 & -1 & 0 & 1 & 1 & 0 & r_2 \\
0 & 0 & -\frac{1}{320} & \frac{1}{470} & -\frac{1}{100} & 0 & r_3 \\
0 & 0 & -1 & \frac{6}{11} & \frac{151}{55} & 0 & r_4 \leftarrow 320\, r_3 - r_4 \\
1 & 1 & 0 & 0 & 0 & 9 & r_5
\end{array}
$$

EXERCISE

Gaussian Elimination Method

v_1	v_2	v_3	v_4	v_L		
$-\frac{1}{120}$	$\frac{1}{220}$	0	0	$\frac{1}{100}$	0	r_1
0	-1	0	1	1	0	r_2
0	0	$-\frac{1}{320}$	$\frac{1}{470}$	$-\frac{1}{100}$	0	r_3
0	0	0	$\frac{70}{517}$	$-\frac{327}{55}$	0	r_4
1	1	0	0	0	9	r_5

$r_5 \leftarrow 120 r_1 + r_5 \longrightarrow$

System of Linear Equations

v_1	v_2	v_3	v_4	v_L		
$-\frac{1}{120}$	$\frac{1}{220}$	0	0	$\frac{1}{100}$	0	r_1
0	-1	0	1	1	0	r_2
0	0	$-\frac{1}{320}$	$\frac{1}{470}$	$-\frac{1}{100}$	0	r_3
0	0	0	$\frac{70}{517}$	$-\frac{327}{55}$	0	r_4
0	$\frac{17}{11}$	0	0	$\frac{6}{5}$	9	r_5

EXERCISE

Gaussian Elimination Method

$$
\begin{array}{ccccc|c|l}
v_1 & v_2 & v_3 & v_4 & v_L & & \\
\hline
-\frac{1}{120} & \frac{1}{220} & 0 & 0 & \frac{1}{100} & 0 & r_1 \\
0 & -1 & 0 & 1 & 1 & 0 & r_2 \\
0 & 0 & -\frac{1}{320} & \frac{1}{470} & -\frac{1}{100} & 0 & r_3 \\
0 & 0 & 0 & \frac{70}{517} & -\frac{327}{55} & 0 & r_4 \\
0 & 0 & 0 & \frac{17}{11} & \frac{151}{55} & 9 & r_5
\end{array}
$$

System of Linear Equations

$$
\begin{array}{ccccc|c|l}
v_1 & v_2 & v_3 & v_4 & v_L & & \\
\hline
-\frac{1}{120} & \frac{1}{220} & 0 & 0 & \frac{1}{100} & 0 & r_1 \\
0 & -1 & 0 & 1 & 1 & 0 & r_2 \\
0 & 0 & -\frac{1}{320} & \frac{1}{470} & -\frac{1}{100} & 0 & r_3 \\
0 & 0 & 0 & \frac{70}{517} & -\frac{327}{55} & 0 & r_4 \\
0 & \frac{17}{11} & 0 & 0 & \frac{6}{5} & 9 & r_5 \leftarrow \frac{17}{11} r_2 + r_5
\end{array}
$$

EXERCISE

Gaussian Elimination Method

$$
\begin{array}{c}
\left[\begin{array}{ccccc|c}
v_1 & v_2 & v_3 & v_4 & v_L & \\
\hline
-\frac{1}{120} & \frac{1}{220} & 0 & 0 & \frac{1}{100} & 0 \\
0 & -1 & 0 & 1 & 1 & 0 \\
0 & 0 & -\frac{1}{320} & \frac{1}{470} & -\frac{1}{100} & 0 \\
0 & 0 & 0 & \frac{70}{517} & -\frac{327}{55} & 0 \\
0 & 0 & 0 & \frac{17}{11} & \frac{151}{55} & 9
\end{array}\right]
\end{array}
\begin{array}{l}
\\
r_1 \\
r_2 \\
r_3 \\
r_4 \\
r_5 \leftarrow \frac{17}{11} r_4 - \frac{70}{517} r_5
\end{array}
$$

$\rightarrow$

System of Linear Equations

$$
\begin{array}{c}
\left[\begin{array}{ccccc|c}
v_1 & v_2 & v_3 & v_4 & v_L & \\
\hline
-\frac{1}{120} & \frac{1}{220} & 0 & 0 & \frac{1}{100} & 0 \\
0 & -1 & 0 & 1 & 1 & 0 \\
0 & 0 & -\frac{1}{320} & \frac{1}{470} & -\frac{1}{100} & 0 \\
0 & 0 & 0 & \frac{70}{517} & -\frac{327}{55} & 0 \\
0 & 0 & 0 & 0 & -9.56 & -1.22
\end{array}\right]
\end{array}
\begin{array}{l}
\\
r_1 \\
r_2 \\
r_3 \\
r_4 \\
r_5
\end{array}
$$

EXERCISE

Gaussian Elimination Method

from r_5

$$\frac{\cancel{-9.56}\, v_L}{\cancel{-9.56}} = \frac{-1.22}{-9.56}$$

$$\boxed{v_L = 127.62\,\text{mV}}$$

ans

System of Linear Equations

	v_1	v_2	v_3	v_4	v_L		
	$-\frac{1}{120}$	$\frac{1}{220}$	0	0	$\frac{1}{100}$	0	r_1
	0	-1	0	1	1	0	r_2
	0	0	$-\frac{1}{320}$	$\frac{1}{470}$	$-\frac{1}{100}$	0	r_3
	0	0	0	$\frac{70}{517}$	$-\frac{327}{55}$	0	r_4
	0	0	0	0	-9.56	-1.22	r_5

LABORATORY