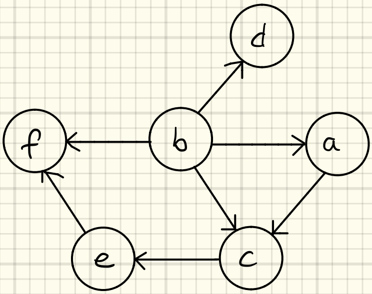
ECE521 - Assignment 4

Jianwei Sun (1000009821)
Raymond Ly (999959497)

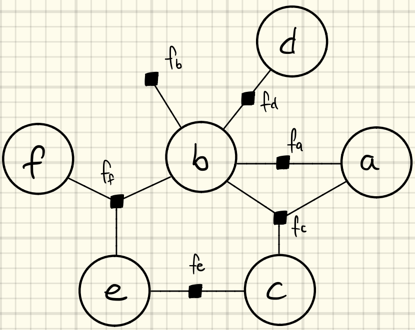
1. Graphical Models

1.1 Graphical models from factorization

1. $P(a,b,c,d,e,f) = P(a|b)\,P(b)\,P(c|a,b)\,P(d|b)\,P(e|c)\,P(f|b,e)$

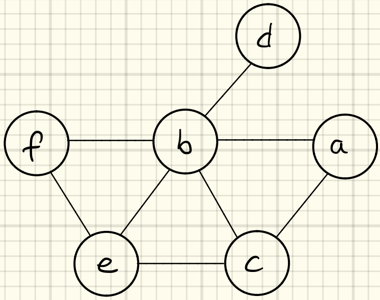


2.

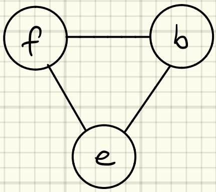


f_a : $p(a|b)$
f_b : $p(b)$
f_c : $p(c|a,b)$
f_d : $p(d|b)$
f_e : $p(e|c)$
f_f : $p(f|b,e)$

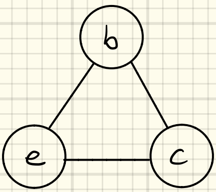
3.



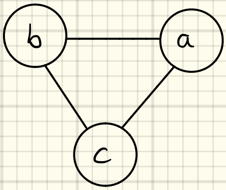
maximum cliques:



$\psi(b,e,f) = p(f|b,e)$



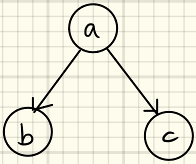
$\psi(b,e,c) = p(e|c)p(b)$



$\psi(a,b,c) = p(a|b)\,p(c|a,b)$

1.2 Conversion between graphical models

1.1 a.



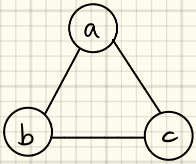
$$p(b|a)p(c|a)p(a)$$

$$f_1(a,b,c) = p(b|a)p(c|a)$$

$$f_2(a) = p(a)$$

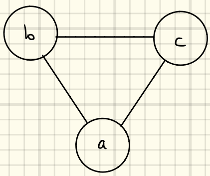
b. b and c are independent because they each have a "dongle" of a factor node. The factor f5 must be either p(b|c) or p(c|b) because it only connects nodes b and c, which would imply that b and c are not independent. These two statements contradict; hence, an equivalent BN does not exist. In the BN, b and c both have to be parent nodes. There is no placement of the directed edge f5 that would allow both b and c to be parent nodes.

1.2 a.



$$\Psi(a,b,c) = f_1(a,b,c)\, f_2(a)$$

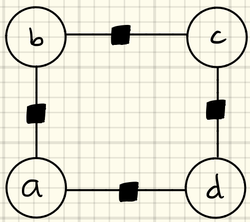
b.



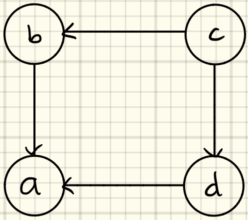
$$\Psi(a,b,c) = f_1(b)\, f_2(c)\, f_3(a,b)\, f_4(a,c)\, f_5(b,c)$$

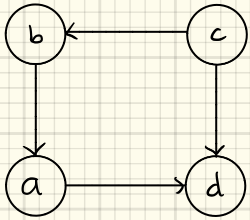
2.

1.



2. It is not possible to sketch the equivalent BN because the diagonal elements are conditionally independent (using the Markov Blanket), but there is no way to draw the conditional independence with a BN. There would always be at least one dependency. For instance, consider the possible cases (graph must be acyclic):





$c \perp\!\!\!\perp a \mid b,d$ but $b \not\perp\!\!\!\perp d \mid a,c$ $\quad$ $b \perp\!\!\!\perp d \mid a,c$ but $a \not\perp\!\!\!\perp c \mid b,d$

1.3 Conditional independence in Bayesian Networks

1. $P(a,b,c,d,e,f) = p(d \mid b,c)\, p(f \mid b)\, p(c \mid b,e)\, p(e)\, p(b \mid a)\, p(a)$

2. $a \perp\!\!\!\perp c$ F – cascade, no condition

$a \perp\!\!\!\perp c \mid b$ T – Cascade, conditional

$e \perp\!\!\!\perp b$ T – V-structure, no condition

$e \perp\!\!\!\perp b \mid c$ F – V-structure, conditional

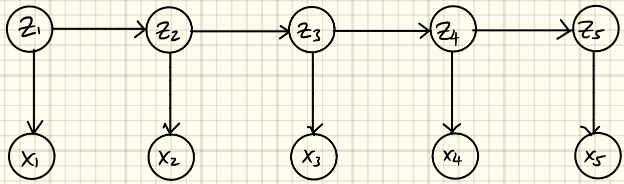
$a \perp\!\!\!\perp e$ T – e and b are independent based on V-structure. a is only connected to b, so e and a must be independent

$a \perp\!\!\!\perp e \mid c$ F – e and b are dependent given c, based on the V-structure. b is dependent on a, so e and a are not conditionally independent.

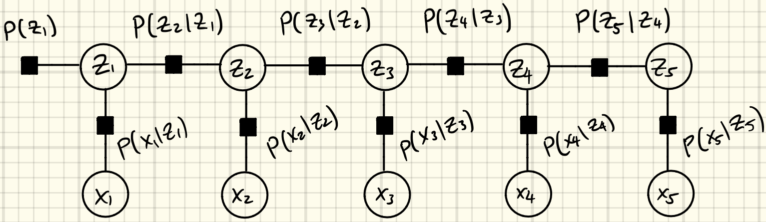
3. Hidden Markov Models

3.1 Factor graph representation

1.



2.



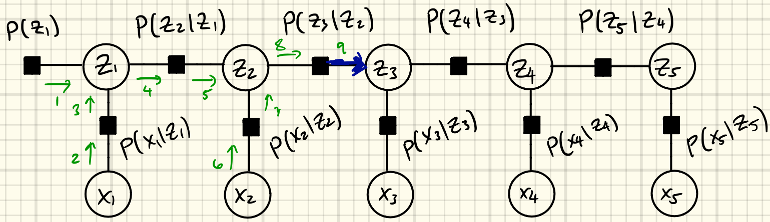
3.2 Inference by passing messages

1. variable to factor

$$\mu_{z_4 \to f_{z_3 z_4}}(z_4) = \mu_{f_{x_4 z_4} \to z_4}(z_4)\, \mu_{f_{z_5 z_4} \to z_4}(z_4)$$

3.3 Message-passing as bi-direction RNNs

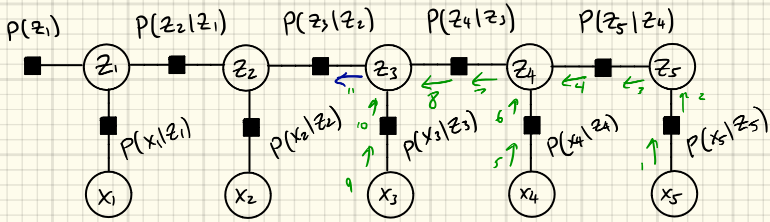
1.



Let $\otimes$ denote element-wise multiplication.

$$\mu_{f_{z_2,z_3}\to z_3}(z_3) = \underset{9}{T}\left(\left[\underset{5}{T}\left[\underset{1}{\pi} \underset{4}{\otimes} \underset{3}{W}^T \underset{2}{x_1}\right]\right] \underset{8}{\otimes} \left[\underset{7}{W}^T \underset{6}{x_2}\right]\right)$$

2.



$$\mu_{z_3\to f_{z_2,z_3}}(z_3) = \left[\underset{8}{T}\left(\left[T \underset{4}{\otimes} (\underset{2,3}{W}^T \underset{1}{x_5}\right] \underset{7}{\otimes} \left[\underset{6}{W}^T \underset{5}{x_4}\right]\right)\right] \underset{11}{\otimes} (\underset{10}{W}^T \underset{9}{x_3})$$