

PHYS:5905 Homework 4

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February 12, 2019

1. Magnetic Mirror Confinement

- (a) Verify that $\nabla \cdot B = 0$ for this magnetic mirror field.

Proof. In fact, we only need to prove this for the normalized magnetic field B' .

$$\begin{aligned}\nabla \cdot B' &= \frac{\partial}{\partial x'} B'_x + \frac{\partial}{\partial y'} B'_y + \frac{\partial}{\partial z'} B'_z \\ &= -\frac{\pi}{2L'} \delta B'_z \sin\left(\frac{2\pi z'}{L}\right) - \frac{\pi}{2L'} \delta B'_z \sin\left(\frac{2\pi z'}{L}\right) + \frac{2\pi}{2L'} \delta B'_z \sin\left(\frac{2\pi z'}{L'}\right) \\ &= 0.\end{aligned}$$

□

- (b) For an initial position $x_0/r_L = (0.1, 0, 4)$, and initial velocity $v_0/v_\perp = (0, 1, 0)$, plot the trajectory of the particle on the (z, x) plane over a simulation time $\Omega T = 5\pi$.

The trajectory is shown in Figure 1.

- (c) Plot the 3D trajectory of the particle of the same simulation over plotted region.

The trajectory is shown in Figure 2.

- (d) Plot the evolution of normalized magnetic moment.

The plot is shown in Figure 3, where we observe that with the increase of timesteps N , the conservation gets better.

- (e) Plot the evolution of normalized kinetic energy.

The plot is shown in Figure 4, where we observe that with the increase of timesteps N , the conservation gets better.

2. Implementation of Adaptive Runge-Kutta (RK45).

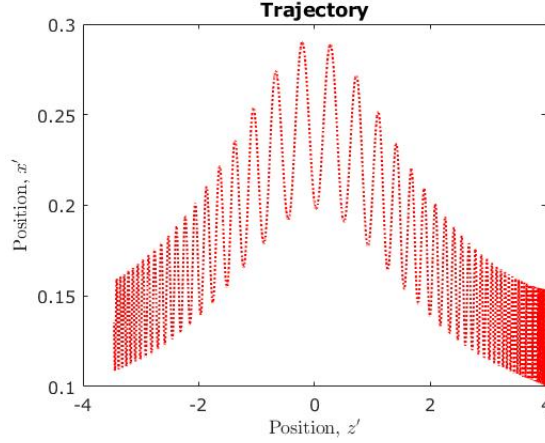


Figure 1: The trajectory in the (z, x) plane with AB3 scheme and $N = 5000$ timesteps, and final time $T' = 5\pi$.

- (a) Run AB3 with $N = 10000$ steps and compute the error at $\Omega t = 20\pi$. Set relative tolerance for RK45 to the same value, determine how many time steps it requires.

For AB3, $e = 5.3978 \times 10^{-6}$. For RK45, the number of time steps with $RelTol = e$ is $N = 993$.

- (b) Plot the trajectory for RK45 with $RelTol = 4 \times 10^{-3}$. How many steps does this require? Total relative error?

The plot is shown in Figure 5, where the number of time steps is $N = 257$, and relative total error with respect to analytical solution $e = 4.94 \times 10^{-3}$.

- (c) How many RK45 steps required if change $RelTol = 1 \times 10^{-6}$. Total relative error? Why total error larger than specified tolerance?

$N = 1153$ for $RelTol = 1 \times 10^{-6}$. The relative total error is $e = 1.266 \times 10^{-6}$.

It's larger than $RelTol$ since in `ode45`, the program uses the solution of 5^{th} order method to estimate the real solution for adaptive control of the step size of the 4^{th} order method. However, the solution of 5^{th} order method is not accurate so it still has some small error, which makes the final relative error slightly larger than $RelTol$.

3. Magnetic Mirror Integration with RK45.

- (a) Using conservation of energy as a measure of accuracy, how many AB3 steps need for energy loss less than 0.1%? Set RK45 $RelTol = 10^{-3}$, how many steps require, error in energy?

For AB3, $N = 2208$.

For RK45, $N = 341$, error in energy is 3.24×10^{-3} .

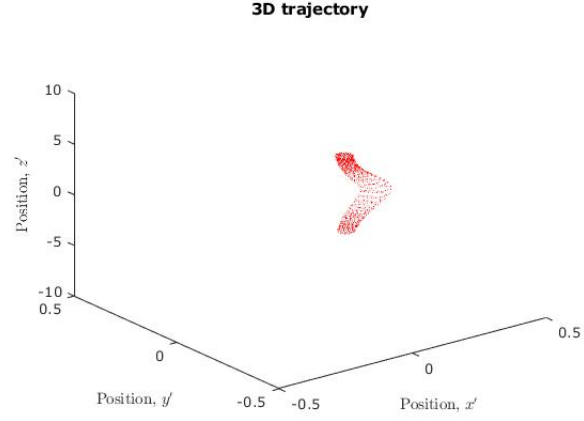


Figure 2: The trajectory in the (x, y, z) plane with AB3 scheme and $N = 5000$ timesteps, and final time $T' = 5\pi$.

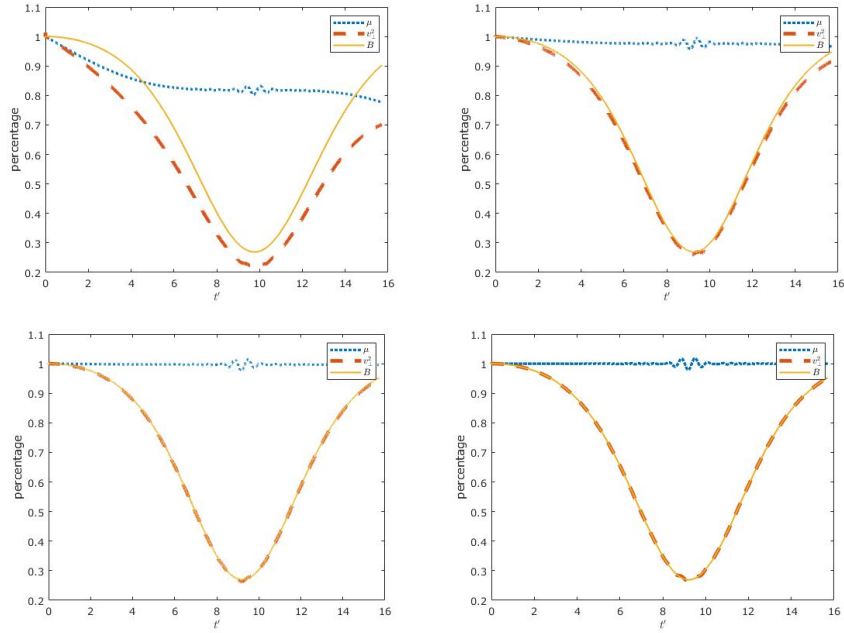


Figure 3: The normalized magnetic moment μ , normalized perpendicular velocity v_{\perp}^2 and magnetic field magnitude B . Top: number of time steps $N = 5000, 10000$. Bottom: $N = 20000, 50000$.

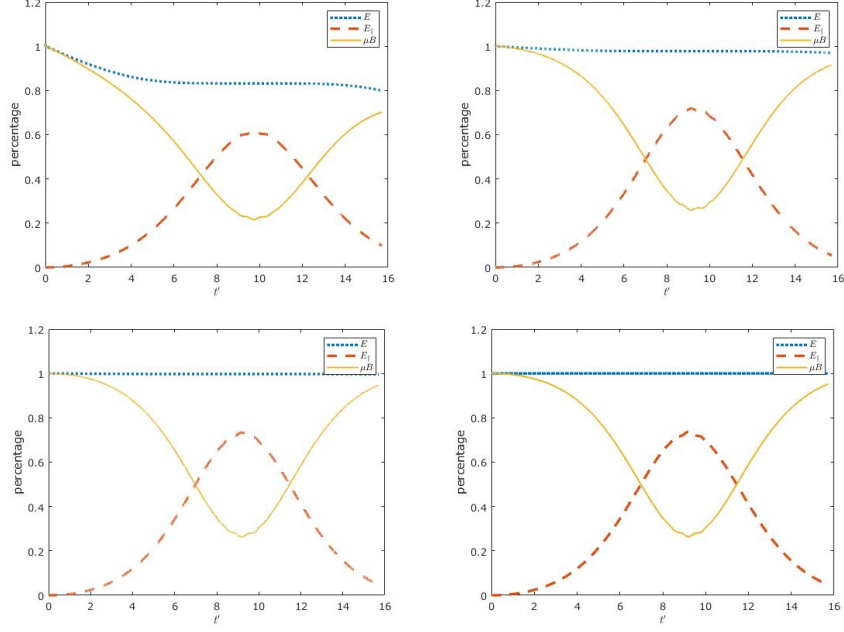


Figure 4: The normalized total kinetic energy E , normalized parallel kinetic energy E_{\parallel} and μB . Top: number of time steps $N = 5000, 10000$. Bottom: $N = 20000, 50000$.

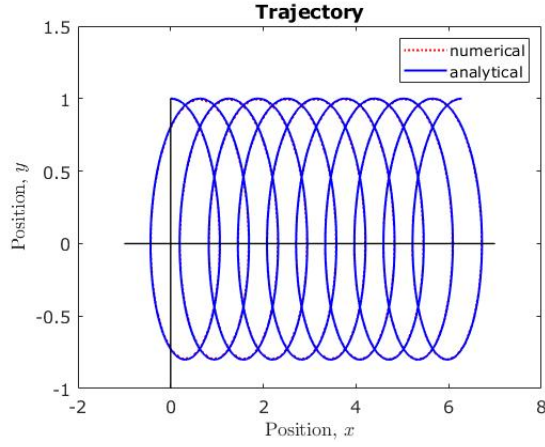


Figure 5: The trajectory in the (x, y) plane with RK45 scheme and $N = 257$ timesteps, and final time $T' = 20\pi$.