

# PHYS:5905 Homework 4

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## 1. Magnetic Mirror Confinement

- (a) Verify that  $\nabla \cdot B = 0$  for this magnetic mirror field.

*Proof.* In fact, we only need to prove this for the normalized magnetic field  $B'$ .

$$\begin{aligned}\nabla \cdot B' &= \frac{\partial}{\partial x'} B'_x + \frac{\partial}{\partial y'} B'_y + \frac{\partial}{\partial z'} B'_z \\ &= -\frac{\pi}{2L'} \delta B'_z \sin\left(\frac{2\pi z'}{L}\right) - \frac{\pi}{2L'} \delta B'_z \sin\left(\frac{2\pi z'}{L}\right) + \frac{2\pi}{2L'} \delta B'_z \sin\left(\frac{2\pi z'}{L'}\right) \\ &= 0.\end{aligned}$$

□

- (b) For an initial position  $x_0/r_L = (0.1, 0, 8)$ , and initial velocity  $v_0/v_\perp = (0, 1, 0)$ , plot the trajectory of the particle on the  $(z, x)$  plane over a simulation time  $\Omega T = 10\pi$ .

The trajectory is shown in Figure 1.

- (c) Plot the 3D trajectory of the particle of the same simulation over plotted region.

The trajectory is shown in Figure 2.

- (d) Plot the evolution of normalized magnetic moment.

The plot is shown in Figure 3, where we observe that with the increase of timesteps  $N$ , the conservation gets better.

- (e) Plot the evolution of normalized kinetic energy.

The plot is shown in Figure 4, where we observe that with the increase of timesteps  $N$ , the conservation gets better.

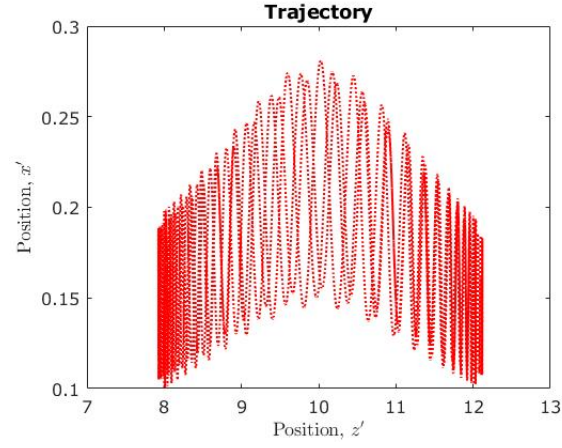


Figure 1: The trajectory in the  $(z, x)$  plane with AB3 scheme and  $N = 5000$  timesteps, and final time  $T' = 10\pi$ .

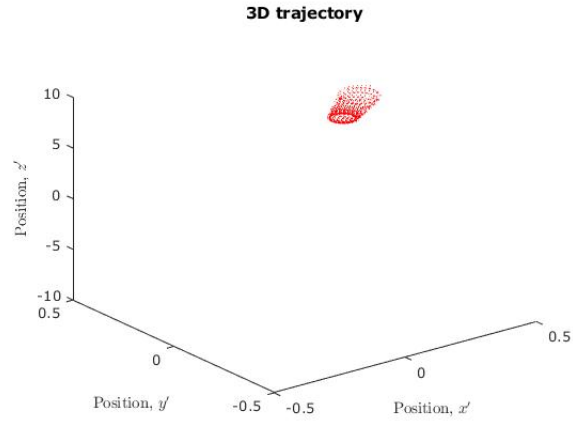


Figure 2: The trajectory in the  $(x, y, z)$  plane with AB3 scheme and  $N = 5000$  timesteps, and final time  $T' = 10\pi$ .

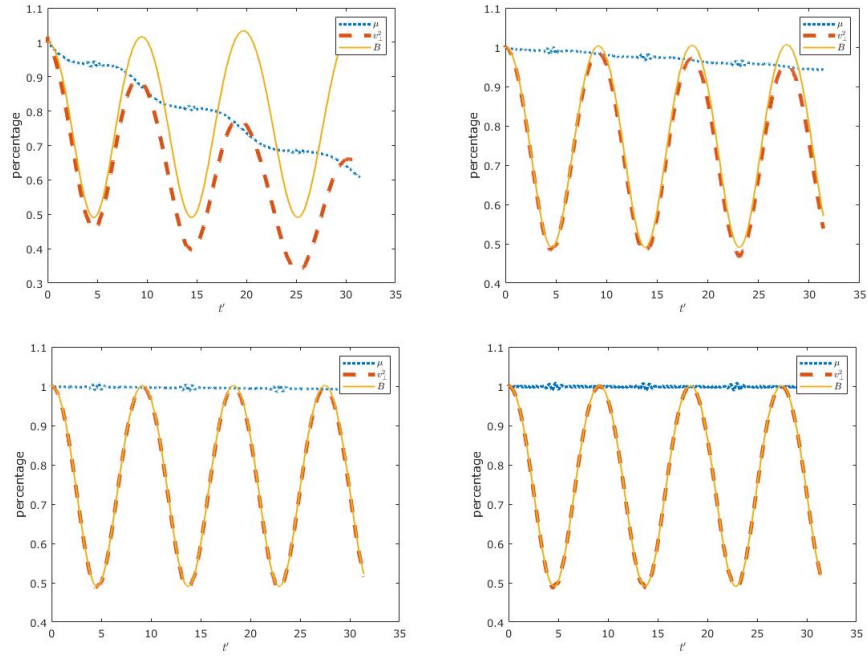


Figure 3: The normalized magnetic moment  $\mu$ , normalized perpendicular velocity  $v_{\perp}^2$  and magnetic field magnitude  $B$ . Top: number of time steps  $N = 5000, 10000$ . Bottom:  $N = 20000, 50000$ .

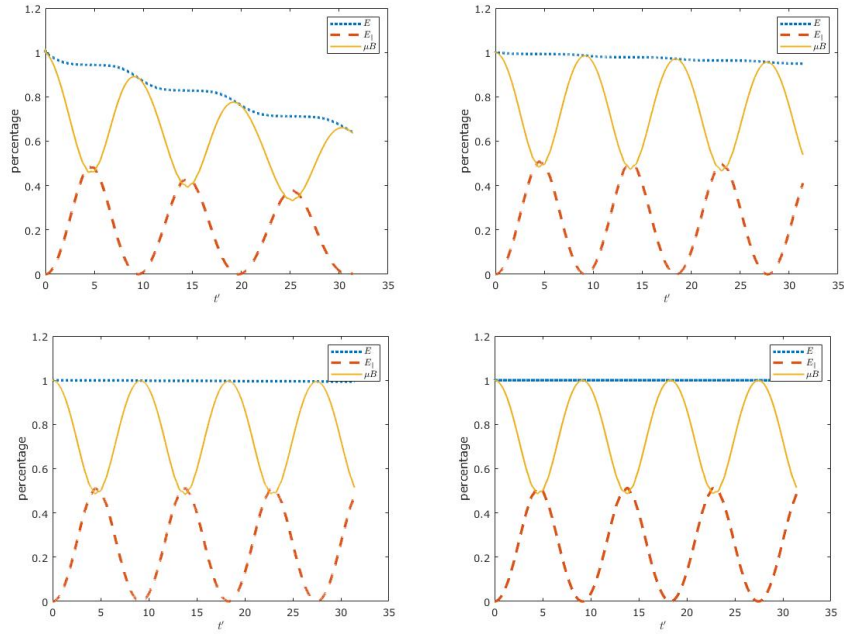


Figure 4: The normalized total kinetic energy  $E$ , normalized parallel kinetic energy  $E_{\parallel}$  and  $\mu B$ . Top: number of time steps  $N = 5000, 10000$ . Bottom:  $N = 20000, 50000$ .