PHYS:5905 Homework 2

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1. Larmor Motion in constant, uniform magnetic field with zero electric field.
   (a) Figure 1 shows the plots of $x(t)$, where the numerical solution is computed with $N = 2000$ timesteps:
   (b) Figure 2 is the error plot with respect to the number of timesteps. The slope is $k = -1.00393$.
2. $E \times B$ drift in a constant, uniform magnetic and perpendicular electric field.
   (a) Figure 3 shows the plots of $x(t)$, where the numerical solution is computed with $N = 2000$ timesteps:
   (b) Figure 4 is the error plot with respect to the number of timesteps. The slope is $k = -1.00389$.
   We notice that the slopes in both problems are just the same when the number of timesteps N is large enough. This shows that the forward difference method is asymptotically linear.
3. Second-order timestepping for $E \times B$ drift.
   (a) Figure 5 shows the plots of $x(t)$, where the numerical solution is computed with $N = 1000$ timesteps and with leapfrog method:
   (b) Figure 6 is the error plot with respect to the number of timesteps. The slope is $k = -2$. (I wonder if the abnormal points are caused by the instability since I first use Euler forward scheme to get x_1 and v_1 with given x_0 and v_0.)
4. Use Taylor expansion to demonstrate the order of Leapfrog method
   In fact, consider the equation
   $$\frac{dx}{dt} = v. \tag{1}$$
   The Taylor expansion at x_j leads to
   $$x_{j+1} = x_j + v_j \Delta t + \frac{1}{2} v'(t_j)(\Delta t)^2 + O((\Delta t)^3), \tag{2}$$

and

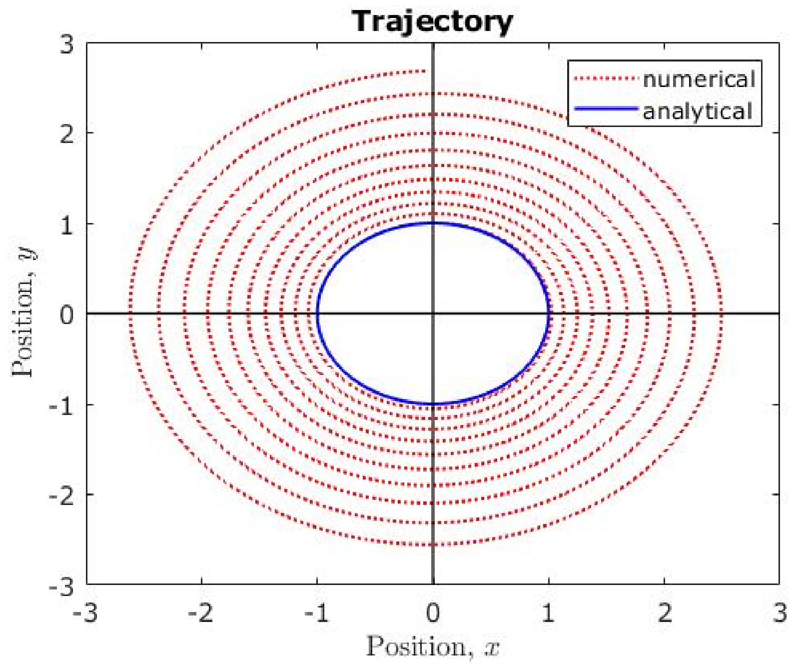
$$x_{j-1} = x_j - v_j \Delta t + \frac{1}{2} v'(t_j)(\Delta t)^2 + O((\Delta t)^3). \tag{3}$$

Hence

$$x_{j+1} - x_{j-1} = 2 v_j \Delta t + O((\Delta t)^3), \tag{4}$$

and we get

$$v_j = \left. \frac{dx}{dt} \right|_{t=t_j} = \frac{x_{j+1} - x_{j-1}}{2\Delta t} + O((\Delta t)^2). \tag{5}$$



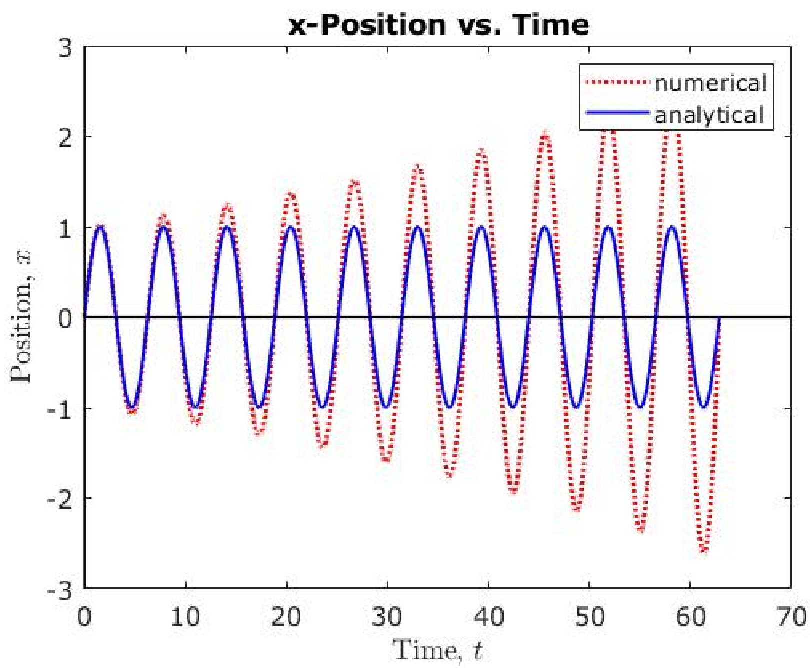


Figure 1: The trajectory in the (x, y) plane (top) and the position x as function of time t (bottom). The dot lines are numerical solutions solved with $N = 2000$ timesteps and the solid lines are the analytical solutions.

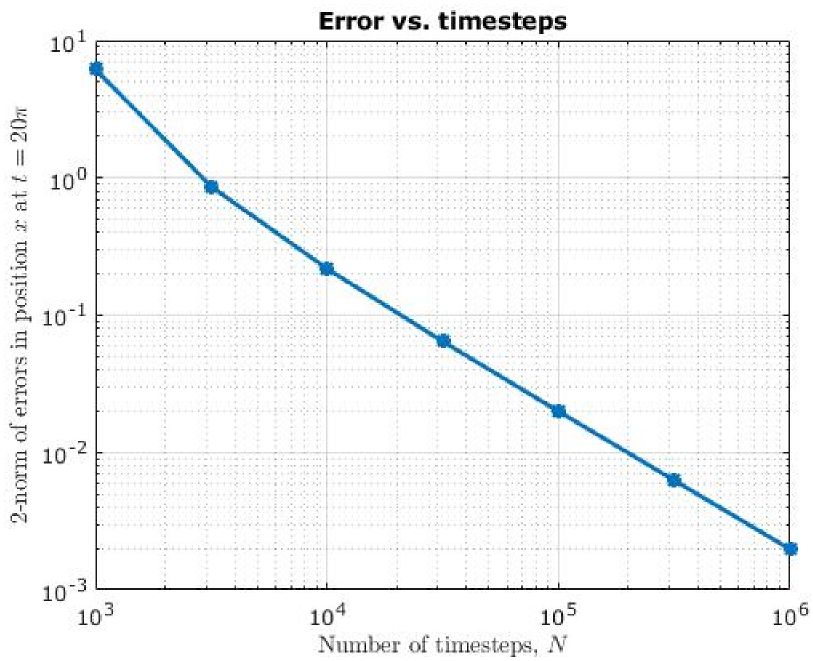


Figure 2: The error at $t = 20\pi$ with respect to the number of timesteps N.

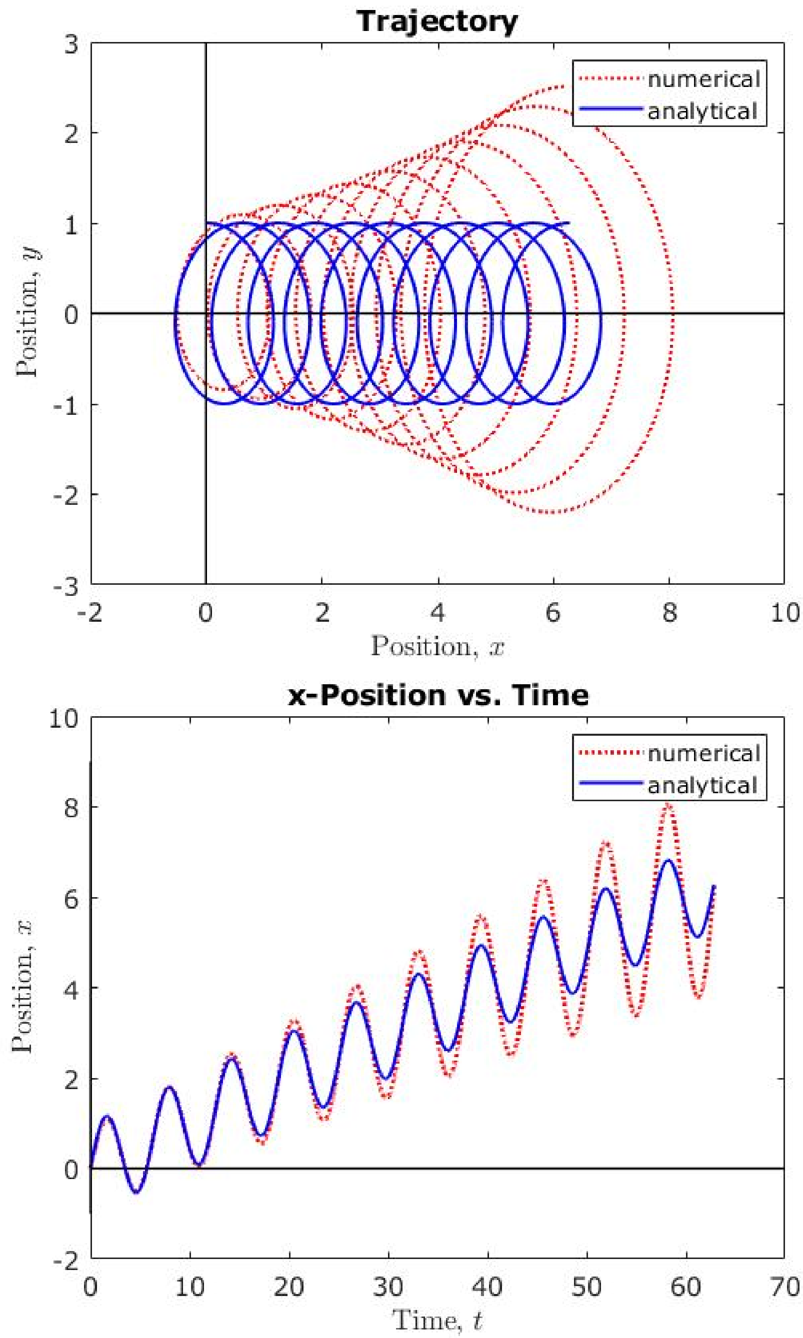


Figure 3: The trajectory in the (x, y) plane (top) and the position x as function of time t (bottom). The dot lines are numerical solutions solved with $N = 2000$ timesteps and the solid lines are the analytical solutions.

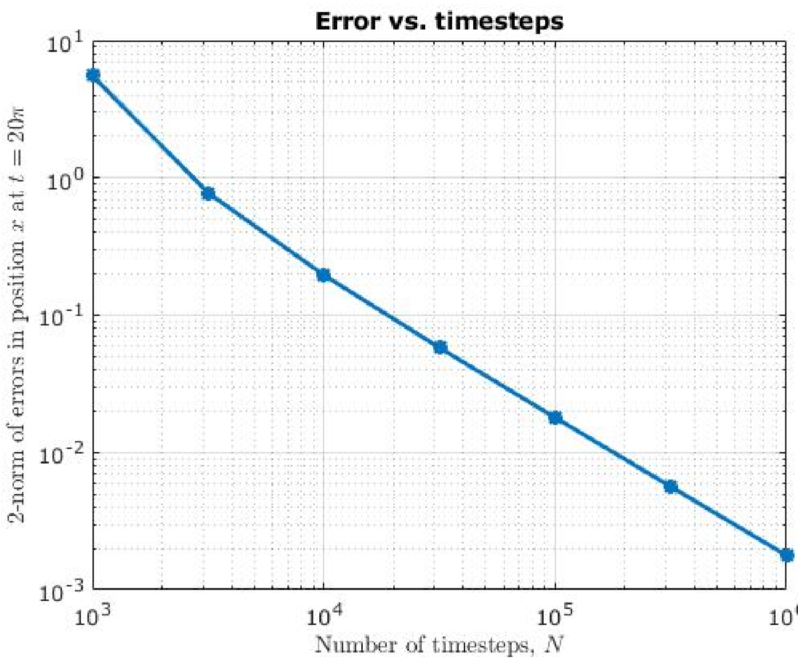


Figure 4: The error at $t = 20\pi$ with respect to the number of timesteps N.

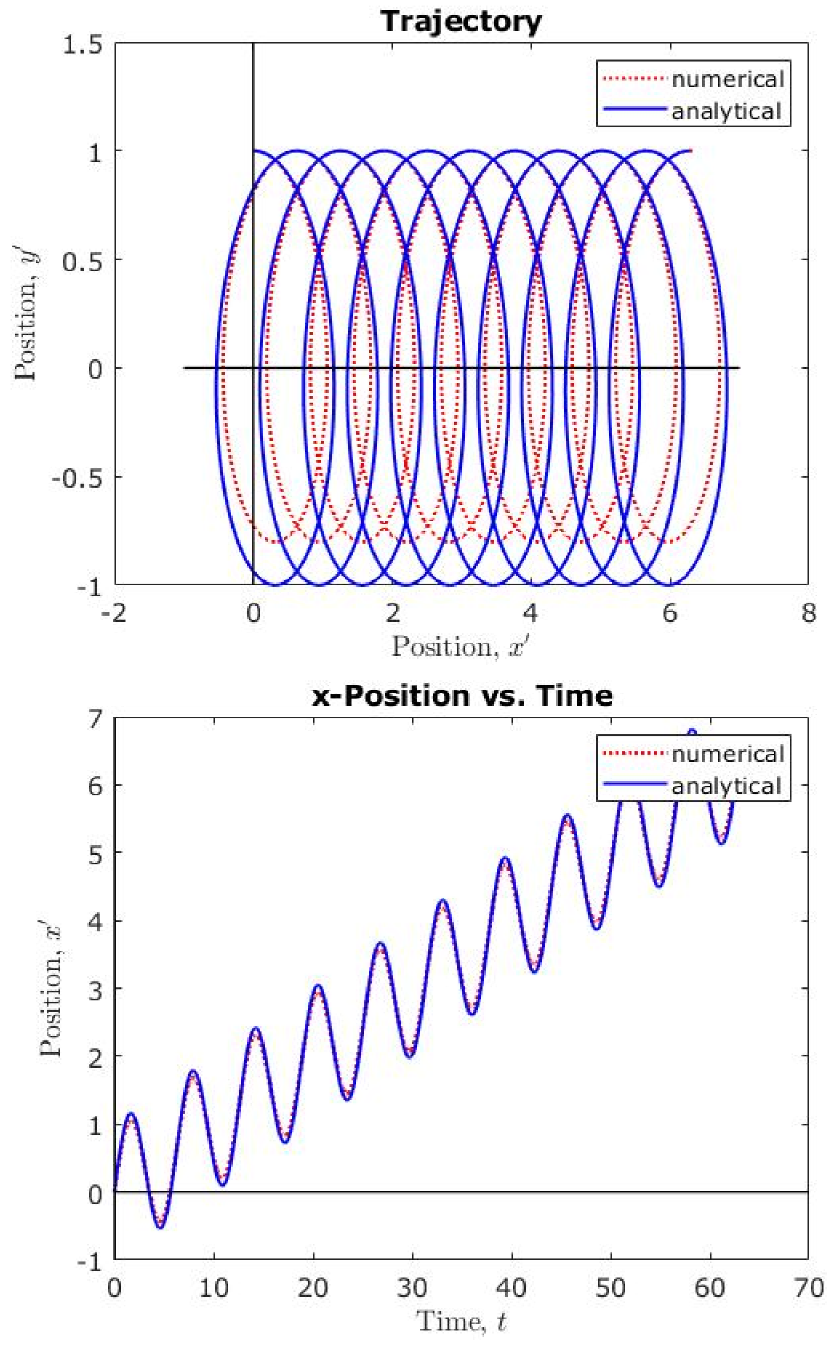


Figure 5: The trajectory in the (x, y) plane (top) and the position x as function of time t (bottom). The dot lines are numerical solutions solved with $N = 1000$ timesteps and the solid lines are the analytical solutions.

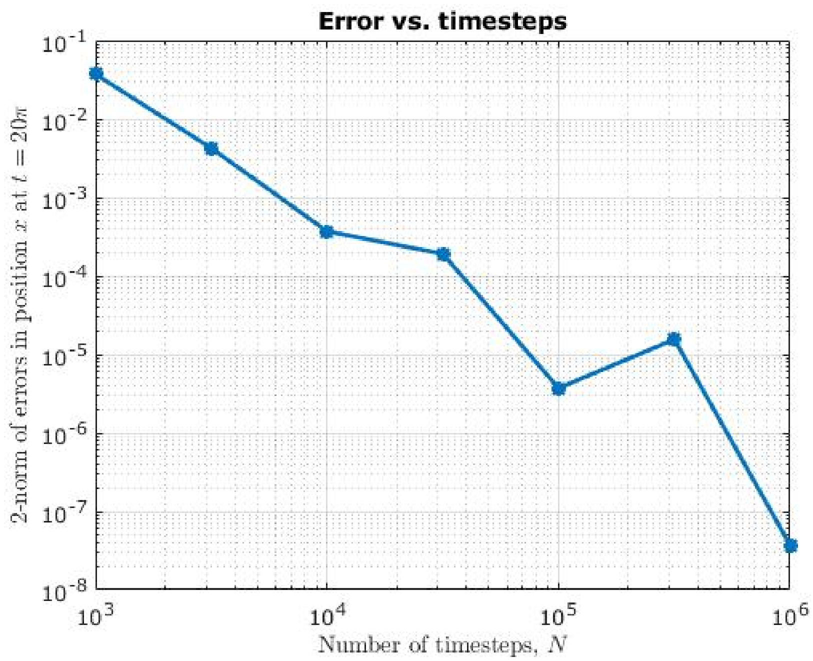


Figure 6: The error at $t = 20\pi$ with respect to the number of timesteps N.