

# PHYS:5905 Homework 4

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## 1. Magnetic Mirror Confinement

- (a) Verify that  $\nabla \cdot B = 0$  for this magnetic mirror field.

*Proof.* In fact, we only need to prove this for the normalized magnetic field  $B'$ .

$$\begin{aligned}\nabla \cdot B' &= \frac{\partial}{\partial x'} B'_x + \frac{\partial}{\partial y'} B'_y + \frac{\partial}{\partial z'} B'_z \\ &= -\frac{\pi}{2L'} \delta B'_z \sin\left(\frac{2\pi z'}{L}\right) - \frac{\pi}{2L'} \delta B'_z \sin\left(\frac{2\pi z'}{L}\right) + \frac{2\pi}{2L'} \delta B'_z \sin\left(\frac{2\pi z'}{L'}\right) \\ &= 0.\end{aligned}$$

□

- (b) For an initial position  $x_0/r_L = (0.1, 0, 8)$ , and initial velocity  $v_0/v_\perp = (0, 1, 0)$ , plot the trajectory of the particle on the  $(z, x)$  plane over a simulation time  $\Omega T = 10\pi$ .

The trajectory is shown in Figure 1.

- (c) Plot the 3D trajectory of the particle of the same simulation over plotted region.

The trajectory is shown in Figure 2.

- (d) Plot the evolution of normalized magnetic moment.

The plot is shown in Figure 3, where we observe that with the increase of timesteps  $N$ , the conservation gets better.

- (e) Plot the evolution of normalized kinetic energy.

The plot is shown in Figure 4, where we observe that with the increase of timesteps  $N$ , the conservation gets better.

## 2. Implementation of Adaptive Runge-Kutta (RK45).

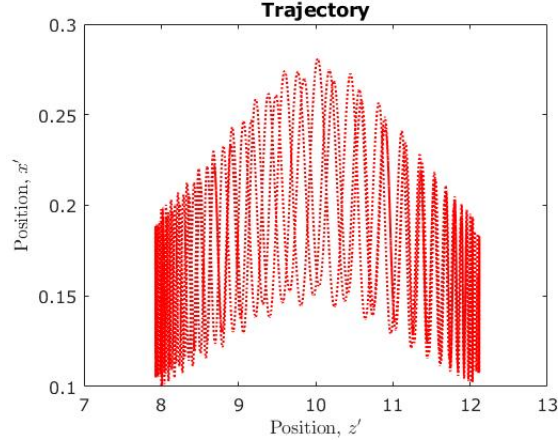


Figure 1: The trajectory in the  $(z, x)$  plane with AB3 scheme and  $N = 5000$  timesteps, and final time  $T' = 10\pi$ .

- (a) Run AB3 with  $N = 10000$  steps and compute the error at  $\Omega t = 20\pi$ . Set relative tolerance for RK45 to the same value, determine how many time steps it requires.

For AB3,  $e = 5.3978 \times 10^{-6}$ . For RK45, the number of time steps with  $RelTol = e$  is  $N = 993$ .

- (b) Plot the trajectory for RK45 with  $RelTol = 4 \times 10^{-3}$ . How many steps does this require? Total relative error?

The plot is shown in Figure 5, where the number of time steps is  $N = 257$ , and relative total error with respect to analytical solution  $e = 4.94 \times 10^{-3}$ .

- (c) How many RK45 steps required if change  $RelTol = 1 \times 10^{-6}$ . Total relative error? Why total error larger than specified tolerance?

$N = 1153$  for  $RelTol = 1 \times 10^{-6}$ . The relative total error is  $e = 1.266 \times 10^{-6}$ .

It's larger than  $RelTol$  since in `ode45`, the program uses the solution of  $5^{th}$  order method to estimate the real solution for adaptive control of the step size of the  $4^{th}$  order method. However, the solution of  $5^{th}$  order method is not accurate so it still has some small error, which makes the final relative error slightly larger than  $RelTol$ .

### 3. Magnetic Mirror Integration with RK45.

- (a) Using conservation of energy as a measure of accuracy, how many AB3 steps need for energy loss less than 0.1%? Set RK45  $RelTol = 10^{-3}$ , how many steps require, error in energy?

For AB3,  $N = 2208$ .

For RK45,  $N = 341$ , error in energy is  $3.24 \times 10^{-3}$ .

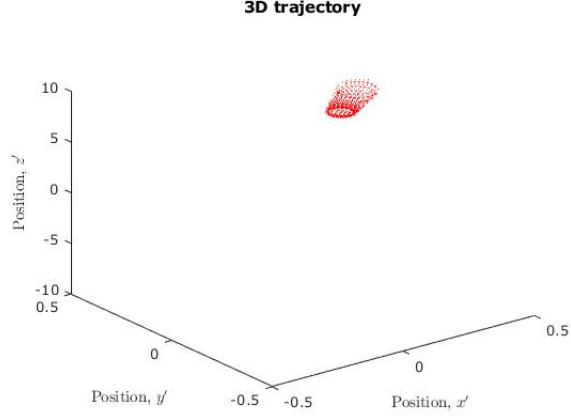


Figure 2: The trajectory in the  $(x, y, z)$  plane with AB3 scheme and  $N = 5000$  timesteps, and final time  $T' = 10\pi$ .

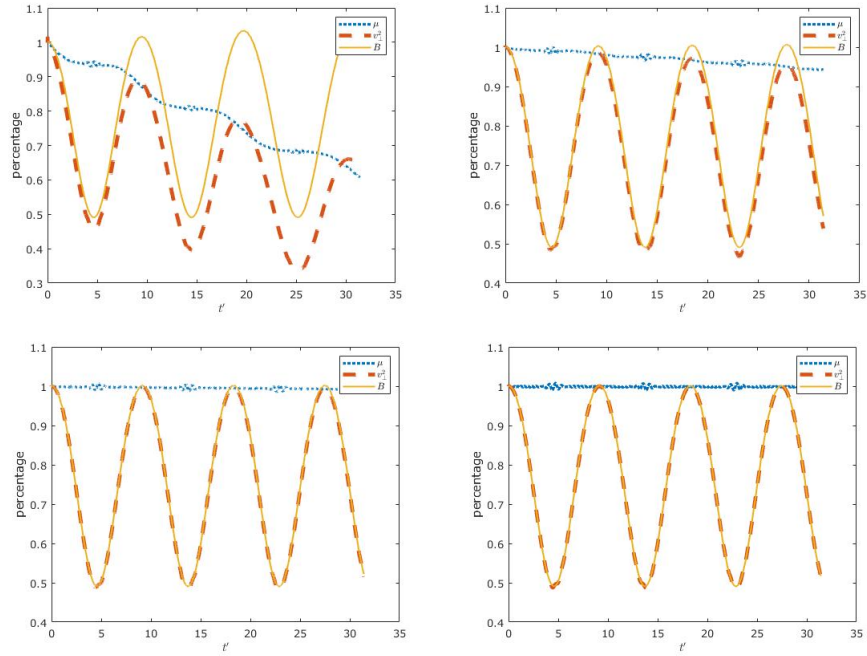


Figure 3: The normalized magnetic moment  $\mu$ , normalized perpendicular velocity  $v_{\perp}^2$  and magnetic field magnitude  $B$ . Top: number of time steps  $N = 5000, 10000$ . Bottom:  $N = 20000, 50000$ .

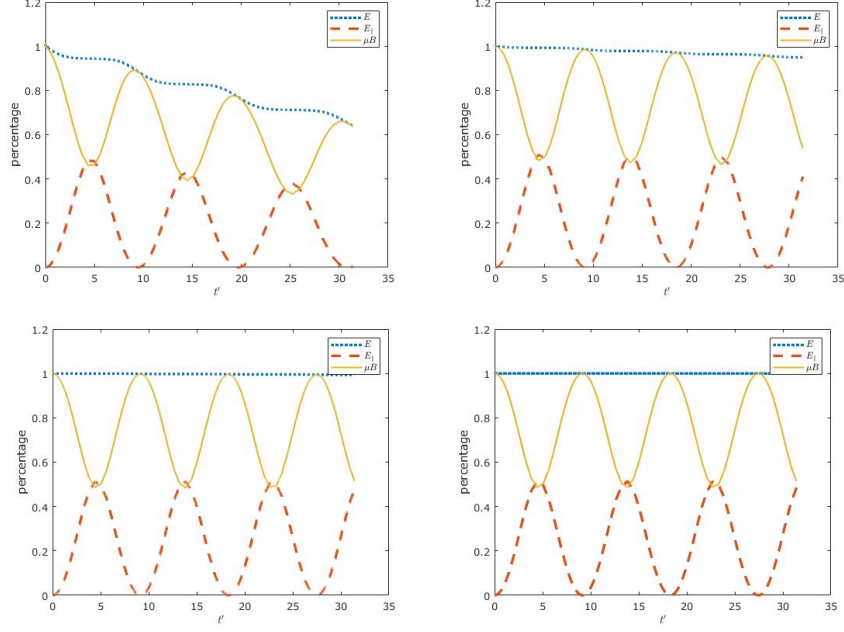


Figure 4: The normalized total kinetic energy  $E$ , normalized parallel kinetic energy  $E_{\parallel}$  and  $\mu B$ . Top: number of time steps  $N = 5000, 10000$ . Bottom:  $N = 20000, 50000$ .

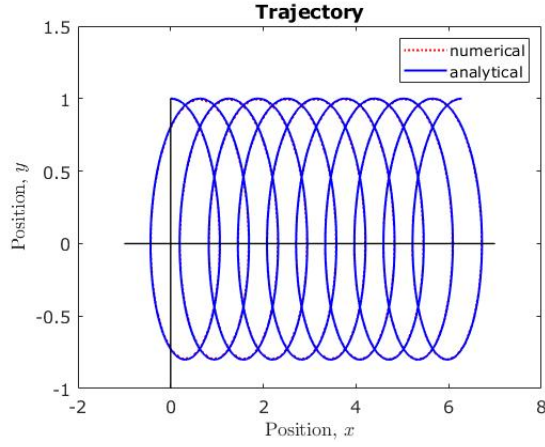


Figure 5: The trajectory in the  $(x, y)$  plane with RK45 scheme and  $N = 257$  timesteps, and final time  $T' = 20\pi$ .