BIOS:7600 Homework 2

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February 9, 2019

1. Derive the relationship between FDR and local FDR.

   $\mathbb{E}(\text{fdr}(z)|z \in \mathcal{Z}) = \mathbb{E}(\mathbb{P}(H_0|z)|z \in \mathcal{Z}) = \mathbb{P}(H_0|z \in \mathcal{Z}) = \mathbb{E}(A_Z/R_Z) = \text{Fdr}(\mathcal{Z})$.

2. Show that if $cX \sim \chi^2_\nu$, then $X \sim \text{Gamma}(\frac{\nu}{2}, \frac{c}{2})$.
   By CDF of the χ^2 distribution,

   $$\mathbb{P}(cX < x) = \frac{\gamma(\frac{\nu}{2}, \frac{x}{2})}{\Gamma(\frac{\nu}{2})}.$$

   Hence

   $$F_X(x) = \mathbb{P}(X < x) = \frac{\gamma(\frac{\nu}{2}, \frac{cx}{2})}{\Gamma(\frac{\nu}{2})} \sim \text{Gamma}(\frac{\nu}{2}, \frac{c}{2}).$$

3. State the answer.

   (a) In BH, the q-value with $z = 0$.
   $q = 0.5$.

   (b) What is the q-value with $z = 0$ if estimate π_0?

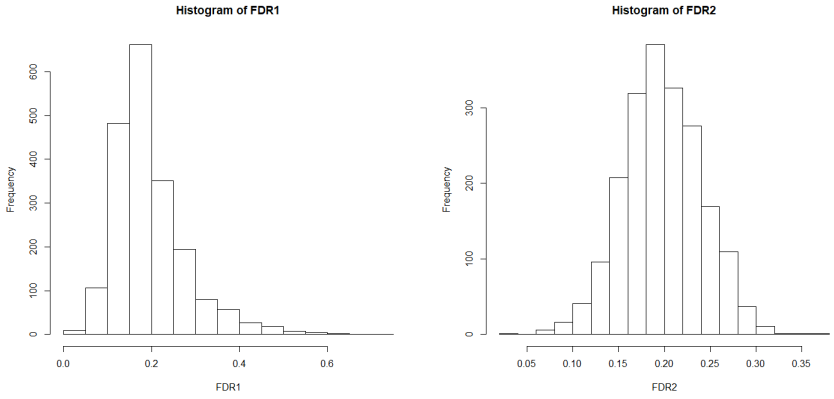
   (c) For GMM, what's the range of fdr for $z = 0$?
   $(0, \frac{1}{2})$.

   (d) For GMM, what's the range of fsr for $z \to 0^+$?

4. (I wonder how to get the p-values by z-values. I suppose here to use two-tail test, but I don't know if it's correct.)

   The histograms of the FDRs are shown as follows. The left one is for the first set of Z-values, and the right one is for the second set. We can see that correlations do not have a large influence on the mean of FDRs, but result in a larger standard deviation.

group	mean	std
1	0.1971197	0.08333976
2	0.198178	0.04258965


Histogram of FDR1
Frequency
0
100
200
300
400
500
600
0.0
0.2
0.4
0.6
FDR1
Histogram of FDR2
Frequency
0
100
200
300
0.05
0.10
0.15
0.20
0.25
0.30
0.35
FDR2