BIOS:7600 Homework 2

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1. Derive the relationship between FDR and local FDR.

   $\mathbb{E}(\text{fdr}(z)|z \in \mathcal{Z}) = \mathbb{E}(\mathbb{P}(H_0|z)|z \in \mathcal{Z}) = \mathbb{P}(H_0|z \in \mathcal{Z}) = \mathbb{E}(A_Z/R_Z) = \text{Fdr}(\mathcal{Z})$.

2. Show that if $cX \sim \chi^2_\nu$, then $X \sim \text{Gamma}(\frac{\nu}{2}, \frac{c}{2})$.
   By CDF of the χ^2 distribution,

   $$\mathbb{P}(cX < x) = \frac{\gamma(\frac{\nu}{2}, \frac{x}{2})}{\Gamma(\frac{\nu}{2})}.$$

   Hence

   $$F_X(x) = \mathbb{P}(X < x) = \frac{\gamma(\frac{\nu}{2}, \frac{cx}{2})}{\Gamma(\frac{\nu}{2})} \sim \text{Gamma}(\frac{\nu}{2}, \frac{c}{2}).$$

3. State the answer.

   (a) In BH, the q-value with $z = 0$.
   $q = 0.5$.

   (b) What is the q-value with $z = 0$ if estimate π_0?

   (c) For GMM, what's the range of fdr for $z = 0$?
   $(0, \frac{1}{2})$.

   (d) For GMM, what's the range of fsr for $z \to 0^+$?