

Lessons from Radar: New Approaches in Autonomous Navigation & Tracking

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Radar-Based SLAM & MTT Lessons from Radar - RFSs RFS-SLAM Future Work in RFS Based SLAM

Presentation Outline

1. Radar-Based SLAM & Multi-Target Tracking.
2. Lessons from Radar – Random Finite Set (RFS) Formulations.
 - Motivation for new Mapping/SLAM concepts.
3. Simultaneous Localisation & Mapping (SLAM) with RFSs.
 - Relation between RFS and RV-SLAM
 - RFS versus Random Vector (RV)-SLAM Results.
4. Future Work in RFS based Mapping/SLAM.

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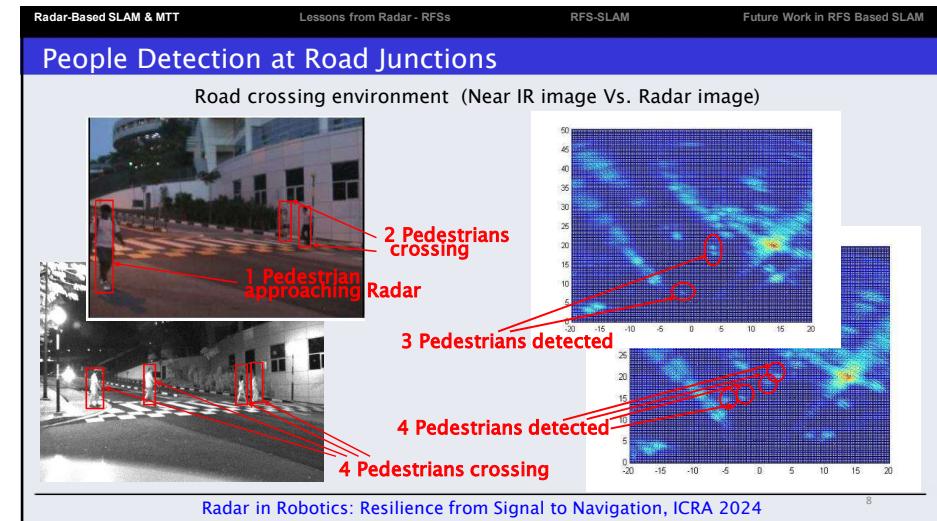
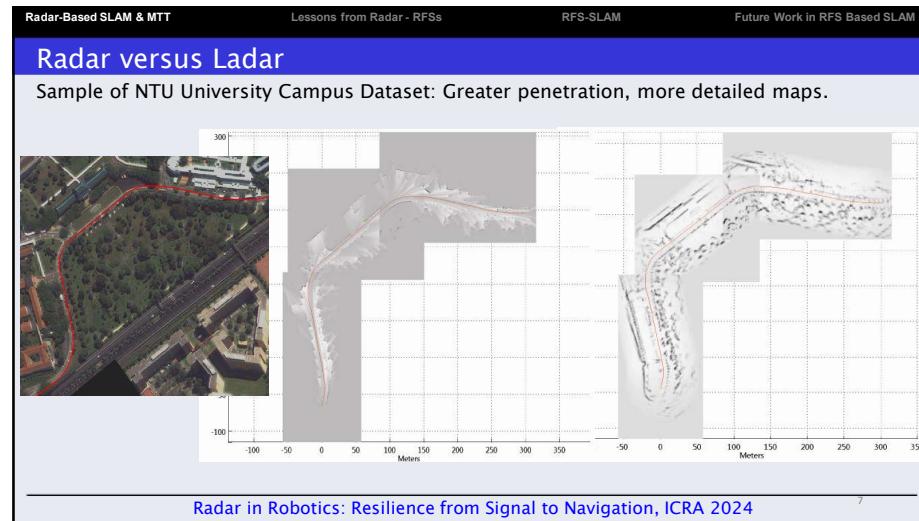
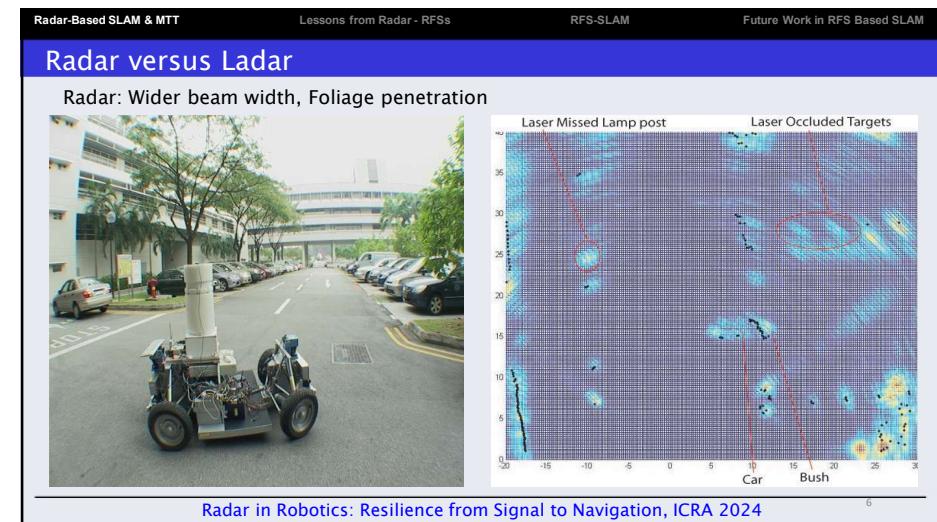
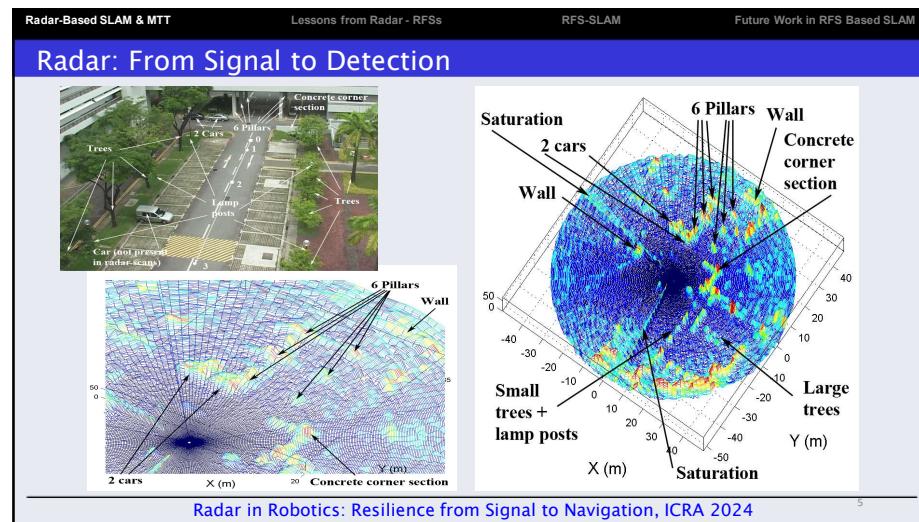
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Radar: From Signal to Detection

Millimetre Wave Radar

- 360 deg. scanning unit, 77GHz FMCW
- 0.25m range res., 200m max range
- Per Bearing angle -> Multiple Targets
- 1.8 deg. beam width

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SLAM with MMW Radar

Sample data registered from radar.

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SLAM with MMW Radar

SLAM input: Odometry path + radar data

Extracted point feature measurements registered to odometry.

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SLAM with MMW Radar

NN-EKF FastSLAM PHD-SLAM

meters meters meters

EKF, FastSLAM and PHD-SLAM with Radar data.

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SLAM with Marine Radar: Singapore – MIT Alliance: CENSAM Project

Autonomous Kayak Surface Vehicle with Radar

Environmental landmarks
m-band radar
ASC

- Environmental monitoring of coastal waters.
- Navigation and map info. necessary above/below water surface.
- Fusion of sea surface radar, sub-sea sonar data for combined surface/sub-sea mapping.

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SLAM with Marine Radar: Singapore – MIT Alliance: CENSAM Project

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RFS Versus Vector Based SLAM

GPS Trajectory (Green Line), GPS point feature coordinates (Green Points), Point feature measurement history (Black dots).

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RFS Versus Vector Based SLAM

Top: Posterior MH-FastSLAM estimate (red).
Bottom: Posterior RB-PHD SLAM estimate (blue).
Ground truth (Green).

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RFS Versus Vector Based SLAM

Feature Count

(Red) MH-FastSLAM Feature Number estimate.
(Blue) PRB-PHD SLAM Feature Number estimate.
(Green) Actual Number to enter FoV at each time index.

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Space Situational Awareness – Space Debris Tracking




Chilbolton LEO CAMRa

Radar detections from south U.K: Defunct ADEOS II pass.

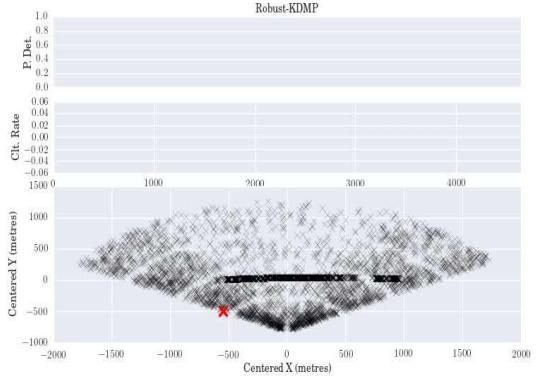
[3] A. Pak, J. Correa, M. Adams, "Robust Joint Target Detection and Tracking for Space Situational Awareness", Journal of Guidance, Control and Dynamics, Vol. 41, No. 1, Jan. 2018.

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Preliminary Results: Space Debris tracking

Robust-KDMP

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Sensing the Environment

Clearpath Robotic Skid Steer Platform



- Acumine Radar 360 deg. scanning unit, 94GHz FMCW
- Sick LD-LRS1000 Scanning LRF
- Microsoft Kinect camera system

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Sensing the Environment: Detection Errors

The random nature of detections

(a) Radar point detections based on an OS-CFAR detector.

(b) Laser range data with a single RANSAC line detector.

(c) Visual SURF features.

Radar LRF + Line RANSAC Visual SURF Features

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Back to Basics: Why Random Finite Sets? Robotic Mapping Error

(a)

Meters

$M = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$

$X_{0:k}$

$\hat{M} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

(b)

Meters

$M = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$

$X_{0:k}$

$\hat{M} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Map error: $\|M - \hat{M}\| = 2$

$\|M - \hat{M}\| = ???$

Set based error metrics exist: Hausdorff, OMAT, OSPA, COLA ...

[6] D. Schumacher, B.-T. Vo, B.-N. Vo, "A Consistent Metric for Performance Evaluation of Multi-Object Filters", IEEE Trans. Signal Proc. Vol 56, No. 8, pages 3447 – 3457, 2008.
[7] P. Barrios, M. Adams, et al, "Metrics for Evaluating Feature-Based Mapping Performance" IEEE Transactions on Robotics, Vol. 33, No. 1, pages 198 to 213, February 2017.

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Why Random Finite Sets? Robotic Mapping Error

- Ground truth feature/target map.

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Why Random Finite Sets? Robotic Mapping Error

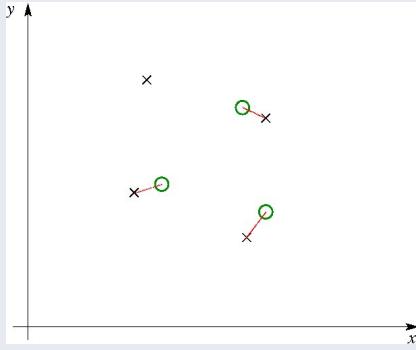
- Ground truth feature/target map.
- Map estimate.

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Why Random Finite Sets? Robotic Mapping Error

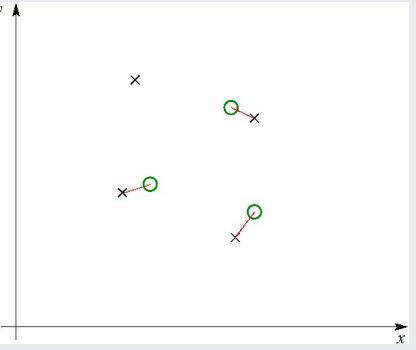


- Ground truth feature/target map.
- Map estimate.
- Data association & map management

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Why Random Finite Sets? Robotic Mapping Error

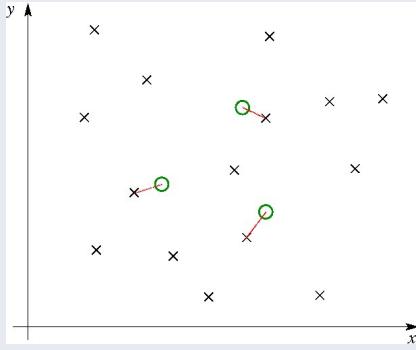


- Ground truth feature/target map.
- Map estimate.
- Data association & map management
- Result: Can now specify map error (e.g. average Euclidean distance between associated features).

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Why Random Finite Sets? Robotic Mapping Error

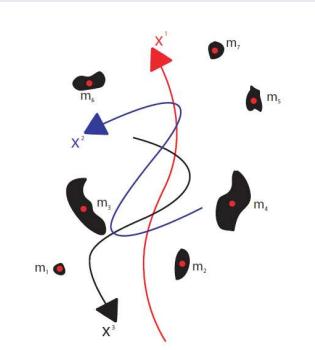


- Ground truth feature/target map.
- Map estimate.
- Data association & map management
- Result: Can now specify map error (e.g. average Euclidean distance between associated features).
- However, this map estimate would give **the same error**, despite many more false positives.

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What's Wrong with Random Vectors?



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What's Wrong with Random Vectors?

Given \mathbf{X}^1 :

$$\mathbf{M} = [\mathbf{m}_1, \mathbf{m}_2, \mathbf{m}_3, \mathbf{m}_4, \mathbf{m}_5, \mathbf{m}_6, \mathbf{m}_7]$$

Given \mathbf{X}^2 :

$$\mathbf{M} = [\mathbf{m}_4, \mathbf{m}_3, \mathbf{m}_2, \mathbf{m}_1, \mathbf{m}_5, \mathbf{m}_7, \mathbf{m}_6]$$

Given \mathbf{X}^3 :

$$\mathbf{M} = [\mathbf{m}_6, \mathbf{m}_7, \mathbf{m}_5, \mathbf{m}_4, \mathbf{m}_3, \mathbf{m}_2, \mathbf{m}_1]$$

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What's Wrong with Random Vectors?

Given \mathbf{X}^1 :

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Given \mathbf{X}^3 :

$$\mathbf{M} = [\mathbf{m}_6, \mathbf{m}_7, \mathbf{m}_5, \mathbf{m}_4, \mathbf{m}_3, \mathbf{m}_2, \mathbf{m}_1]$$

- Estimated map vector depends on vehicle trajectory ???

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What's Wrong with Random Vectors?

$$\begin{aligned}\widehat{\mathbf{M}} &= [m_1 \ m_2 \ m_3 \ m_4 \ m_5 \ m_6 \ m_7]^T \\ \widehat{\mathbf{M}} &= [m_4 \ m_2 \ m_3 \ m_1 \ m_5 \ m_7 \ m_6]^T \\ &\vdots \\ \widehat{\mathbf{M}} &= [m_6 \ m_7 \ m_5 \ m_4 \ m_3 \ m_2 \ m_1]^T \\ \widehat{\mathbf{M}} &= \{m_1 \ m_2 \ m_3 \ m_4 \ m_5 \ m_6 \ m_7\}\end{aligned}$$

- RFS makes more sense as order of features *cannot/should not* be significant.

[8] J. Mullane, B.-N. Vo, M. Adams, B.-T. Vo, A Random-Finite-Set Approach to Bayesian SLAM, IEEE Transactions on Robotics, Vol. 27, No. 2, pages 268 – 282, April 2011.

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What's Wrong with Random Vectors?

Untangle:

$$\mathbf{Z} = [\mathbf{z}_1, \mathbf{z}_2, \mathbf{z}_3, \mathbf{z}_4, \mathbf{z}_5, \mathbf{z}_6, \mathbf{z}_7]$$

?

$$\mathbf{M} = [\mathbf{m}_1, \mathbf{m}_2, \mathbf{m}_3, \mathbf{m}_4, \mathbf{m}_5, \mathbf{m}_6, \mathbf{m}_7]$$

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Random Finite Sets Circumvent the Need for Data Association

Untangle:

$$\mathbf{Z} = [\mathbf{z}_1, \mathbf{z}_2, \mathbf{z}_3, \mathbf{z}_4, \mathbf{z}_5, \mathbf{z}_6, \mathbf{z}_7]$$

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$$\mathbf{M} = [\mathbf{m}_1, \mathbf{m}_2, \mathbf{m}_3, \mathbf{m}_4, \mathbf{m}_5, \mathbf{m}_6, \mathbf{m}_7]$$

Current vector formulations *require* data association (DA) prior to Bayesian update:

Why? Features & measurements rigidly ordered in vector-valued map state.

RFS approach *circumvents* external DA decisions.

Why? Features & measurements are finite valued sets. No distinct order assumed.

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The Mathematics of “Map Management”

How to integrate new map features:

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The Mathematics of “Map Management”

How to integrate new map features:

...with random vectors:

$$\widehat{\mathbf{M}}_{k-1} = [m_1 \ m_2 \ m_3]^T$$

$$\widehat{\mathbf{M}}_k \stackrel{?}{=} [\mathbf{m}_1 \ \mathbf{m}_2 \ \mathbf{m}_3]^T \cup "[\mathbf{m}_4]$$

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The Mathematics of "Map Management"

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$$\widehat{M}_{k-1} = [m_1 \ m_2 \ m_3]^T$$

$$\widehat{M}_k \stackrel{?}{=} [m_1 \ m_2 \ m_3]^T + [m_4]$$

...with random sets:

$$\widehat{\mathcal{M}}_{k-1} = \{m_1 \ m_2 \ m_3\}$$

$$\widehat{\mathcal{M}}_k = \{m_1 \ m_2 \ m_3\} \cup \{m_4\}$$

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The Mathematics of Missing or Extra Measurements

How to write measurement/observation model:

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The Mathematics of Missing or Extra Measurements

How to write measurement/observation model:

...with random vectors:

$$Z_k = h([m_1 \ m_2 \ m_3 \ m_4], X_k) + \text{noise}$$

i.e. : $[z_1 \ z_2 \ z_3 \ z_4 \ z_5]^T \stackrel{?}{=} h([m_1 \ m_2 \ m_3 \ m_4], X_k) + \text{noise}$

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The Mathematics of Missing or Extra Measurements

How to write measurement/observation model:

...with random vectors:

$$Z_k = h([m_1 \ m_2 \ m_3 \ m_4], X_k) + \text{noise}$$

i.e. : $[z_1 \ z_2 \ z_3 \ z_4 \ z_5]^T \stackrel{?}{=} h([m_1 \ m_2 \ m_3 \ m_4], X_k) + \text{noise}$

...with random sets:

$$\mathcal{Z}_k = \bigcup_{m \in \mathcal{M}_k} \mathcal{D}_k(m, X_k) \cup \mathcal{C}_k(X_k)$$

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State-of-art SLAM: Modelling Measurements & Map

$\mathcal{Z}_k \equiv \{\mathbf{z}_k^1, \mathbf{z}_k^2, \dots, \mathbf{z}_k^{n_k}\}$

Collected measurements/detections random in number, and have no specific order.

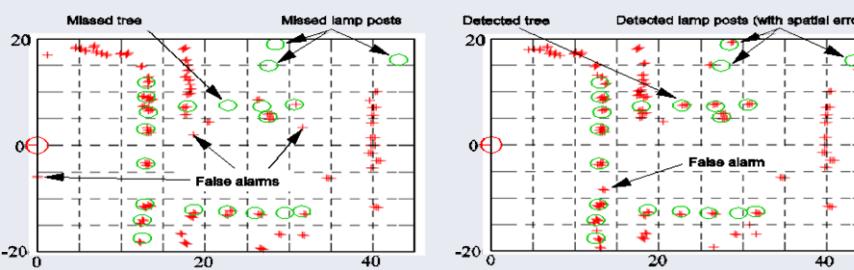
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State-of-art SLAM: Modelling Measurements & Map

$\mathcal{Z}_k \equiv \{\mathbf{z}_k^1, \mathbf{z}_k^2, \dots, \mathbf{z}_k^{n_k}\}$

Collected measurements/detections random in number, and have no specific order.



(a) Radar point detections based on an OS-CFAR detector.

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State-of-art SLAM: Modelling Measurements & Map

$\mathcal{Z}_k \equiv \{\mathbf{z}_k^1, \mathbf{z}_k^2, \dots, \mathbf{z}_k^{n_k}\}$

Collected measurements/detections random in number, and have no specific order.

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State-of-art SLAM: Modelling Measurements & Map

$\mathcal{Z}_k \equiv \{\mathbf{z}_k^1, \mathbf{z}_k^2, \dots, \mathbf{z}_k^{n_k}\}$ Collected measurements/detections random in number, and have no specific order.

$$\mathcal{Z} = \{z^1, \dots, z^3\} = \{[r^1 \theta^1]^T, \dots, [r^3 \theta^3]^T\} \quad (1)$$

Hence, at any instant, a sensor can be considered to collect a finite set $\mathcal{Z} = \{z^1, \dots, z^3\}$ of measurements z^1, \dots, z^3 from a measurement space \mathcal{Z}_0 as follows:

$\mathcal{Z} =$	\emptyset	(no features detected)
$\mathcal{Z} =$	$\{z^1\}$	(one feature z^1 detected)
$\mathcal{Z} =$	$\{z^1, z^2\}$	(two features z^1, z^2 detected)
\vdots	\vdots	\vdots
$\mathcal{Z} =$	$\{z^1, \dots, z^3\}$	(3 features z^1, \dots, z^3 detected)

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RFS SLAM: General Measurement Likelihood

Introduce SLAM state probability of detection: $P_D(\mathbf{x}_k, \mathbf{m})$

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RFS SLAM: General Measurement Likelihood

Introduce SLAM state probability of detection: $P_D(\mathbf{x}_k, \mathbf{m})$

...and likelihood of measurements being clutter: $p(\mathcal{Z}_k)$

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RFS SLAM: General Measurement Likelihood

Introduce SLAM state probability of detection: $P_D(\mathbf{x}_k, \mathbf{m})$

...and likelihood of measurements being clutter: $p(\mathcal{Z}_k)$

...and assuming multi-Bernoulli distribution:

$$p(\mathcal{Z}_k | \mathcal{M}_k, \mathbf{x}_{0:k}) = \sum_{\theta} \left(\underbrace{p(\mathcal{Z}_k^\theta | \mathcal{M}_k^\theta, \mathbf{x}_{0:k}, \theta)}_{\text{Spatial measurement likelihood for state-measurement pairs under association } \theta} \underbrace{\left[\prod_{\mathbf{m} \in \mathcal{M}_k^\theta} P_D(\mathbf{x}_k, \mathbf{m}) \right]}_{\text{For all associated state elements: Product of their detection probabilities}} \underbrace{\left[\prod_{\mathbf{m} \in \bar{\mathcal{M}}_k^\theta} (1 - P_D(\mathbf{x}_k, \mathbf{m})) \right]}_{\text{For all unassociated state elements: Product of their misdetection probabilities}} \underbrace{p_\kappa(\mathcal{Z}_k^\theta)}_{\text{Likelihood of non-associated measurements being clutter}} \right)$$

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SLAM Ideal Detection Conditions

- The RFS formulation is equivalent to the vector-based formulation when:
 - Map size is fixed / deterministic (with one ordering)
 - Data association is assumed
 - Probability of detection equals 1 for associated landmarks
 - Probability of non-associated measurements being clutter equals 1
- RFS-SLAM is a generalization of random-vector SLAM

[10] K. Leung, F. Inostroza, M. Adams, *Relating Random Vector and Random Finite Set Estimation in Navigation, Mapping and Tracking*, IEEE Trans. Signal Processing, Vol. 65, No. 17, Sept. 2017.

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Difference Between RFS and Vector State Models

Bayesian Estimation:

Model state as random Vector \mathbf{X} :

$$p(\mathbf{X} | \mathcal{Z}) = \frac{p(\mathcal{Z} | \mathbf{X})p(\mathbf{X})}{p(\mathcal{Z})}$$

Change order (permute) elements of \mathbf{X} - different Bayesian estimate.

Model state as random Set \mathcal{X} :

$$p(\mathcal{X} | \mathcal{Z}) = \frac{p(\mathcal{Z} | \mathcal{X})p(\mathcal{X})}{p(\mathcal{Z})}$$

Change order (permute) elements of \mathcal{X} - same Bayesian estimate.

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SLAM with Random Finite Sets (RFS)

Why use Random Finite Sets (RFS)?

- Set algebra
- Implicit data association
- Allows mathematically consistent definition of error metrics

RFS Solutions:

- Probability Hypothesis Density (PHD) Filter
- Labelled Multi-Bernoulli (LMB) Filter
- Joint Vector-Set (JVS)-SLAM solver

[8] J. Mullane, B.-N. Vo, M. Adams, B.-T. Vo, *A Random Finite Set Approach to Bayesian SLAM*, IEEE Trans. Robotics, Vol. 27, No. 2, April 2011.
 [11] L. Gao, G. Battistelli, and L. Chisci, *"PHD-SLAM 2.0: Efficient SLAM in the presence of mis detections and clutter"*, IEEE Trans. Robotics, 2021.
 [12] H. Deusch, S. Reuter, K. Dietmayer, *"The Labeled Multi-Bernoulli SLAM Filter"*, IEEE Signal Proc. Letters, Vol 22, No. 10, Oct. 2015.
 [13] D. Moratuwage, M. Adams, F. Inostroza, *δ -Generalized Labeled Multi-Bernoulli Simultaneous Localization and Mapping with an Optimal Kernel-based Particle Filtering Approach*, MDPI Sensors, Vol. 19, Issue 10, May 2019.

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RFS – GraphSLAM Solutions with Non-Linear Least Squares Solvers

Joint Vector-Set (JVS)-SLAM Solver:

- Seek the posterior: $p(\mathbf{x}_{0:k}, \mathcal{M}_k | \mathcal{Z}_{1:k}, \mathbf{u}_{0:k})$
- Observation: the map set \mathcal{M}_k and the vector trajectory $\mathbf{x}_{0:k}$ are correlated.
- Need a mixed vector-set (**hybrid**) distribution.

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A Joint Vector-Set (JVS)-SLAM Distribution

- Start with a single element set and a vector
- Define the Joint Vector-Set (JVS) Object: $\chi_k = (\mathbf{x}_{0:k}, \mathcal{M})$

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Radar-Based SLAM & MTT Lessons from Radar - RFSs RFS-SLAM Future Work in RFS Based SLAM

A Joint Vector-Set (JVS)-SLAM Distribution

- Start with a single element set and a vector
- Define the Joint Vector-Set (JVS) Object: $\chi_k = (\mathbf{x}_{0:k}, \mathcal{M})$
- The following distribution results:

$$\pi(\chi_K) = \begin{cases} (1-r)\pi_0(\mathbf{x}_{0:K}) & \chi_K = (\mathbf{x}_{0:K}, \emptyset), \\ r\pi_1(\mathbf{x}_{0:K}, m) & \chi_K = (\mathbf{x}_{0:K}, \{m\}), \\ 0 & \text{else,} \end{cases}$$

$\pi_0(\mathbf{x}_{0:k})$: Vector distribution on vehicle trajectory, given empty map.
 $\pi_1(\mathbf{x}_{0:K}, m)$: Joint dist. on vehicle traj. & map element m tuple, given that map is singleton.
 r : existence probability of the map element m .

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Apply Multi-Bernoulli Filter: *Each Bernoulli component has distinct map size & data assoc.*
Weights from JVS-SLAM solver, spatial distribution from g2o.

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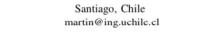
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JVS-SLAM (HGLMB-SLAM) vs g2o Back-End + ML Data Assoc.

Unifying the SLAM Back and Front Ends:
A Bayesian Approach with Random Finite Sets.



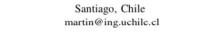
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HGLMB SLAM

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JVS-SLAM vs g2o Back-End + NN & ML Data Assoc. Front-End

Video – stacked.

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JVS-SLAM vs ORBSLAM 3 – Challenging EuRoC Dataset

APE w.r.t. translation part (m) (with origin alignment)

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JVS-SLAM vs ORBSLAM 3 – Challenging EuRoC Dataset

APE w.r.t. translation part (m) (with SE(3) Umeyama alignment)

Euroc Micro Aerial Vehicle, Visual-Inertial dataset.

“EASY”

“MEDIUM”

“DIFFICULT”

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JVS-SLAM vs ORBSLAM 3 – Challenging EuRoC Dataset

APE(m)

Euroc Micro Aerial Vehicle, Visual-Inertial dataset.

“EASY”

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“DIFFICULT”

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Radar-Based SLAM & MTT	Lessons from Radar - RFSs	RFS-SLAM	Future Work in RFS Based SLAM
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Presentation Outline

1. Radar-Based SLAM & Multi-Target Tracking.
2. Lessons from Radar – Random Finite Set (RFS) Formulations.
 - Motivation for new Mapping/SLAM concepts.
3. Simultaneous Localisation & Mapping (SLAM) with RFSs.
 - Relation between RFS and RV-SLAM
 - RFS versus Random Vector (RV)-SLAM Results.
4. Future Work in RFS based Mapping/SLAM.

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Radar-Based SLAM & MTT	Lessons from Radar - RFSs	RFS-SLAM	Future Work in RFS Based SLAM
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Conclusions

Radar taught us how to reformulate SLAM/MTT!

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Radar-Based SLAM & MTT	Lessons from Radar - RFSs	RFS-SLAM	Future Work in RFS Based SLAM
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Conclusions

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State of art RV SLAM methods:

- Deal with detection uncertainty via heuristic methods.
- Apply external (to Bayes/ML estimator) measurement to feature/track association methods. **Requires computational approximations.**

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Radar-Based SLAM & MTT	Lessons from Radar - RFSs	RFS-SLAM	Future Work in RFS Based SLAM
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Conclusions

Radar taught us how to reformulate SLAM/MTT!

State of art RV SLAM methods:

- Deal with detection uncertainty via heuristic methods.
- Apply external (to Bayes/ML estimator) measurement to feature/track association methods. **Requires computational approximations.**

State of art RFS SLAM methods:

- Intrinsically account for detection & spatial uncertainty.
- Apply Bayes/ML estimation to **all** measurements/estimated features/targets. **Requires computational approximations.**

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Radar-Based SLAM & MTT Lessons from Radar - RFSs RFS-SLAM Future Work in RFS Based SLAM

Future Work

1. Probabilistic sensor modelling – improve detection statistics, e.g. occlusions.
2. Incorporate hypothesis tree structure (eg: Hsiao & Kaess).
3. Extended target filtering.

[14] Hsiao M and Kaess M, MH-ISAM2: Multi-hypothesis iSAM using Bayes tree and hypo-tree. In: 2019 International Conference on Robotics and Automation (ICRA). pp. 1274-1280.

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