



$$d = P_1 - P_2$$

$$e = \|d\|_2 - r$$

$$\dot{e} = \frac{d^T \dot{d}}{\|d\|_2}$$

$$P_1 = k + v^T (P_2 - k)v$$

$$\dot{d} = \dot{P}_1 - \dot{P}_2 = \dot{k} + \dot{v}^T (P_2 - k)v - \dot{P}_2 + v^T [(P_2 - k)\dot{v} + (\dot{P}_2 - \dot{k})v], \quad \dot{v} = J_v \cdot \dot{q}, \quad J_v^T = k_i \times v_i$$

$$= J_k \dot{q} + \underbrace{\dot{q}^T J_v^T (P_2 - k)v}_{v (P_2 - k)^T J_v \dot{q}} - J_{P_2} \dot{q} + v^T (P_2 - k) J_v \dot{q} + \underbrace{v^T (\dot{P}_2 \dot{q} - \dot{P}_k \dot{q})v}_{v \cdot v^T (J_{P_2} - J_k) \dot{q}}$$

$$= \underbrace{\left[ J_k + v (P_2 - k)^T J_v - J_{P_2} + v^T (P_2 - k) J_v + v \cdot v^T (J_{P_2} - J_k) \right]}_{\bar{J}_T} \dot{q}$$

$$\boxed{J_T = \frac{d^T}{\|d\|_2} \cdot \bar{J}_T}$$

$$\boxed{\dot{J}_T = \frac{d^T}{\|d\|_2} \dot{\bar{J}}_T + \left[ \frac{\dot{d}^T}{\|d\|_2} - \frac{(d^T \dot{d}) d^T}{(\|d\|_2)^3} \right] \bar{J}_T}$$

$$\begin{aligned} \dot{\bar{J}}_T = & \left[ \dot{J}_k + v (P_2 - k)^T \dot{J}_v + [\dot{v} (P_2 - k)^T + v (\dot{P}_2 - \dot{k})^T] J_v - \dot{J}_{P_2} + v^T (P_2 - k) \dot{J}_v + [\dot{v}^T (P_2 - k) + v^T (\dot{P}_2 - \dot{k})] J_v + \right. \\ & \left. + v \cdot v^T (\dot{J}_{P_2} - \dot{J}_k) + \underbrace{[\dot{v} v^T + v \dot{v}^T]}_{\text{symmetric}} (J_{P_2} - J_k) \right] \end{aligned}$$