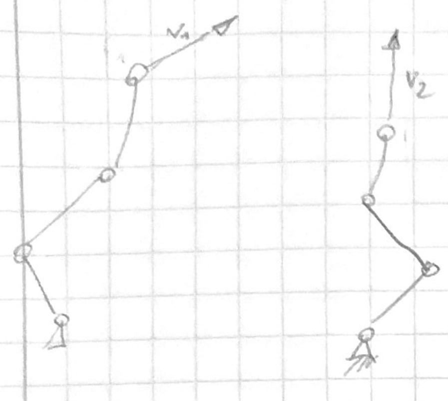

$$v = v_1 \times v_2$$

$$e = \| v \|_2$$

$$\dot{e} = \frac{v^T \dot{v}}{\| v \|_2}$$

$$\dot{v} = \dot{v}_1 \times v_2 + v_1 \times \dot{v}_2$$

$$= {}^{v}J^{v_1} \cdot \dot{q} \times v_2 + v_1 \times {}^{v}J^{v_2} \dot{q} \, , \quad {}^{v}J^{v_j} = [k_1 \times v_j, \ldots, k_n \times v_j]$$

$$= \underbrace{\left(\tilde{S}^{v_1} \, {}^{v}J^{v_2} - \tilde{S}^{v_2} \, {}^{v}J^{v_1} \right)}_{\bar{J}_T} \dot{q} \qquad \tilde{S}^{v_j} = \begin{bmatrix} 0 & -v_{jz} & v_{jy} \\ v_{jz} & 0 & -v_{jx} \\ -v_{jy} & v_{jx} & 0 \end{bmatrix}$$

$$\Rightarrow \boxed{J_T = \frac{v^T}{\| v \|} \bar{J}_T}$$

$$\boxed{\dot{J}_T = \frac{v^T}{\| v \|_2} \dot{\bar{J}}_T + \left[\frac{\dot{v}^T}{\| v \|} - \frac{(v^T \dot{v}) v^T}{(v^T v)^{3/2}} \right] \bar{J}_T}$$

$$\dot{\bar{J}}_T = \tilde{S}^{\dot{v}_1} \, {}^{v}J^{v_2} - \tilde{S}^{\dot{v}_2} \, {}^{v}J^{v_1} + \tilde{S}^{v_1} \, {}^{v}\dot{J}^{v_2} - \tilde{S}^{v_2} \, {}^{v}\dot{J}^{v_1}$$

NOTE: could derive w. rotational part of the Jacobian, but that results in an affine system

i.e.: $$\dot{v} = ({}^{R}J^{v_1} q \times v_1) \times v_2 + v_1 \times ({}^{R}J^{v_2} \dot{q} \times v_2)$$

$$= \left(\tilde{S}^{v_1} \, {}^{R}J^{v_2} - \tilde{S}^{v_2} \, {}^{R}J^{v_1} \right) \dot{q} + 2v$$