



$$v = v_1 \times v_2$$

$$e = \|v\|_2$$

$$\dot{e} = \frac{v^T \dot{v}}{\|v\|_2}$$



$$\dot{v} = \dot{v}_1 \times v_2 + v_1 \times \dot{v}_2$$

$$= v_1^T \dot{q} \times v_2 + v_1 \times v_2^T \dot{q}, \quad \tilde{J}^v = [k_1 \times v_1, \dots, k_n \times v_n]$$

$$= \underbrace{(\tilde{S}^{v_1} v_1^T \tilde{J}^{v_2} - \tilde{S}^{v_2} v_2^T \tilde{J}^{v_1})}_{\tilde{J}_T} \dot{q}$$

$$\tilde{S}^v = \begin{bmatrix} 0 & -v_z & v_y \\ v_z & 0 & -v_x \\ -v_y & v_x & 0 \end{bmatrix}$$

$$\Rightarrow \tilde{J}_T = \frac{v^T}{\|v\|} \tilde{J}_T$$

$$\dot{\tilde{J}}_T = \frac{v^T}{\|v\|_2} \dot{\tilde{J}}_T + \left[\frac{\dot{v}^T}{\|v\|} - \frac{(v^T \dot{v}) v^T}{(v^T v)^{3/2}} \right] \tilde{J}_T$$

$$\dot{\tilde{J}}_T = \tilde{S}^{v_1} v_1^T \dot{\tilde{J}}^{v_2} - \tilde{S}^{v_2} v_2^T \dot{\tilde{J}}^{v_1} + \tilde{S}^{v_1} v_1^T \dot{\tilde{J}}^{v_2} - \tilde{S}^{v_2} v_2^T \dot{\tilde{J}}^{v_1}$$