

Algorithms in pseudocode for implementing natural and clamped cubic splines follow follows.

## Natural Cubic Spline

To construct the cubic spline interpolant  $S$  for the function  $f$ , defined at the numbers  $x_0 < x_1 < \dots < x_n$ , satisfying  $S''(x_0) = S''(x_n) = 0$ :

**INPUT**  $n; x_0, x_1, \dots, x_n; a_0 = f(x_0), a_1 = f(x_1), \dots, a_n = f(x_n)$ .

**OUTPUT**  $a_j, b_j, c_j, d_j$  for  $j = 0, 1, \dots, n - 1$ .

(Note:  $S(x) = S_j(x) = a_j + b_j(x - x_j) + c_j(x - x_j)^2 + d_j(x - x_j)^3$  for  $x_j \leq x \leq x_{j+1}$ .)

**Step 1** For  $i = 0, 1, \dots, n - 1$  set  $h_i = x_{i+1} - x_i$ .

**Step 2** For  $i = 1, 2, \dots, n - 1$  set

$$\alpha_i = \frac{3}{h_i}(a_{i+1} - a_i) - \frac{3}{h_{i-1}}(a_i - a_{i-1}).$$

**Step 3** Set  $l_0 = 1$ ; (Steps 3, 4, 5, and part of Step 6 solve a tridiagonal linear system using a method described in Algorithm 6.7.)

$$\mu_0 = 0;$$

$$z_0 = 0.$$

**Step 4** For  $i = 1, 2, \dots, n - 1$

$$\text{set } l_i = 2(x_{i+1} - x_{i-1}) - h_{i-1}\mu_{i-1};$$

$$\mu_i = h_i/l_i;$$

$$z_i = (\alpha_i - h_{i-1}z_{i-1})/l_i.$$

**Step 5** Set  $l_n = 1$ ;

$$z_n = 0;$$

$$c_n = 0.$$

**Step 6** For  $j = n - 1, n - 2, \dots, 0$

$$\text{set } c_j = z_j - \mu_j c_{j+1};$$

$$b_j = (a_{j+1} - a_j)/h_j - h_j(c_{j+1} + 2c_j)/3;$$

$$d_j = (c_{j+1} - c_j)/(3h_j).$$

**Step 7** OUTPUT ( $a_j, b_j, c_j, d_j$  for  $j = 0, 1, \dots, n - 1$ );

STOP.

