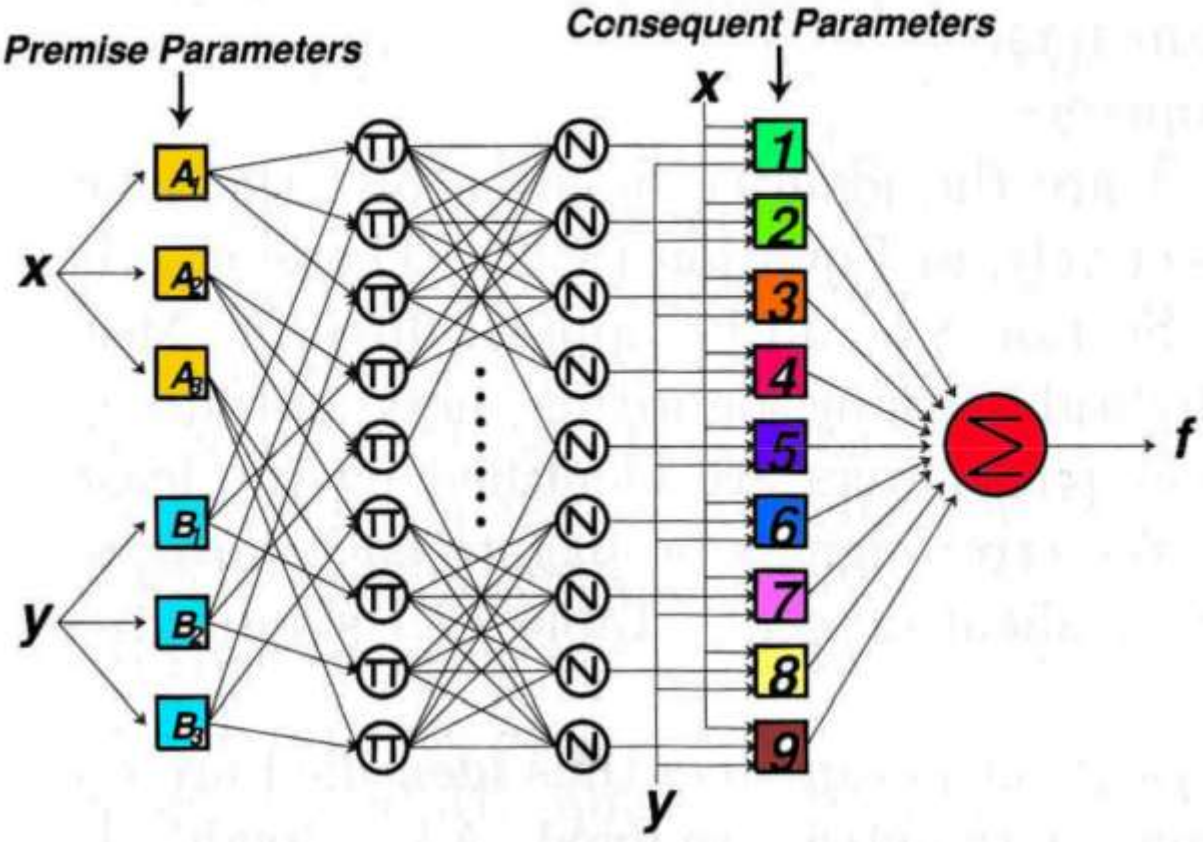
ANFIS Architecture: two inputs Sugeno with nine rules



Let's see our membership functions

$$\mu_{A_i}(x) = \frac{1}{1 + \left(\frac{x - c_{A_i}}{a_{A_i}}\right)^{2b_{A_i}}} \qquad i = 1, 2, \ldots, n_A$$

$$\mu_{B_i}(y) = \frac{1}{1 + \left(\frac{y - c_{B_i}}{a_{B_i}}\right)^{2b_{B_i}}} \qquad i = 1, 2, \ldots, n_B$$

The summation of all incoming signals is

$$f_p = \frac{\sum w_i f_i}{\sum w_i} = \frac{u_p}{v_p} \qquad (1)$$

Here,

$$u_p = \sum_{ia}\left(\mu_{A_{ia}}(x)\cdot\sum_{ib}\mu_{B_{ib}}(y)\cdot f_{ia,ib}(x,y)\right) = \sum_{ib}\left(\mu_{B_{ib}}(y)\cdot\sum_{ia}\mu_{A_{ia}}(x)\cdot f_{ia,ib}(x,y)\right) \quad (2)$$

$$v_p = \sum_{ia,ib}\mu_{A_{ia}}(x)\cdot\mu_{B_{ib}}(y) \quad (3)$$

And

$$f_i = p_i\cdot x + q_i\cdot y + r_i \quad (4)$$

Derivative of u_p and v_p by μ

$$\frac{\partial u_p}{\partial \mu_{A_{ia}}} = \sum_{ib}\mu_{B_{ib}}(y)\cdot f_{ia,ib}(x,y) \quad (5.1)$$

$$\frac{\partial u_p}{\partial \mu_{B_{ib}}} = \sum_{ia}\mu_{A_{ia}}(x)\cdot f_{ia,ib}(x,y) \quad (5.2)$$

$$\frac{\partial v_p}{\partial \mu_{A_{ia}}} = \sum_{ib}\mu_{B_{ib}}(y) \quad (6.1)$$

$$\frac{\partial v_p}{\partial \mu_{B_{ib}}} = \sum_{ia}\mu_{A_{ia}}(x) \quad (6.2)$$

Derivative of μ by a and c

$$\frac{\partial \mu_i(t)}{\partial a_i} = 2\mu_i^2(t-c_i)\cdot\frac{1}{a_i^2} \quad (7.1)$$

$$\frac{\partial \mu_i(t)}{\partial c_i} = 2\mu_i^2\cdot\frac{1}{a_i} \quad (7.2)$$

Backward pass with gradient descents (η – learning rate)

$$T(p+1) = T(p) - \eta\cdot\frac{\partial E}{\partial T} \quad (8)$$

The error, which is equal to

$$E = \sum_{p} \left(d_p - f_p\right)^2 \tag{9}$$

where d_p – target value.