EE 236A HW7
Zhiyuan Cao
304397496
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Problem 1

(1)

We first introduce slack variables u and v and form the following tableau:

	x	y	1
u	-1	1	2
v	-1	-1	6
z	1	-1	0

The tableau is feasible. We follow the pivot and perform Jordan exchange on (y, v) and get the following tableau:

	x	v	1
u	-2	1	8
y	-1	-1	6
z	2	1	-6

Now, since both elements of row z is positive, we have reached optimality with $x, v = 0$, $y = 6$ and optimal value $z^\star = -6$.

(2)

See attached for the graphical representation

(3)

Again we form the following tableau:

	x	y	1
u	-1	1	2
v	-1	-1	6
z	-1	1	0

The tableau is feasible. We follow the pivot and perform Jordan exchange on (x, u) and get the following tableau:

	u	y	1
x	-1	1	2
v	1	-2	4
z	1	0	-2

Again, we have reached optimality with $u, y = 0$, $x = 2$ and optimal value $z^\star = -2$. Since the entry of y in the last row is 0, there exist multiple solutions. If we have $y = \lambda \geq 0$, from the first row we have $x = \lambda + 2$ and the second row $v = -2\lambda + 4 \geq 0$. Combine the two we acquire the optimal set $\{(x, y) | x - y = 2,\ y \leq 2\}$.

(4)

Again we form the following tableau:

	x	y	1
u	2	-1	1
v	-1	1	1
z	-1	-1	0

The tableau is feasible. We follow the pivot and perform Jordan exchange on (x, v) and get the following tableau:

	v	y	1
u	-2	1	3
x	-1	1	1
z	1	-2	-1

Since column 2 of row z is less than 0, we attempt another Jordan exchange. However, there is no pivot row in this case and therefore the problem is unbounded. The path is attached at the end of the document.

Problem 2

(1)

First, we notice that for the second constraint $b_2 > 0$. Therefore we introduce slack variables $z_0 = x_0$, x_3 and x_4 and form the following tableau:

	x_1	x_2	x_0	1
x_3	-1	-1	0	2
x_4	2	2	1	-10
z	-3	1	0	0
z_0	0	0	1	0

We perform an special pivot change between $(x_0\ x_4)$ and acquired the following tableau:

	x_1	x_2	x_4	1
x_3	-1	-1	0	2
x_0	-2	-2	1	10
z	-3	1	0	0
z_0	-2	-2	1	10

The tableau is feasible. We follow the pivot and perform Jordan exchange on (x_1, x_3) and get the following tableau:

	x_3	x_2	x_4	1
x_1	-1	-1	0	2
x_0	2	2	1	6
z	3	4	0	-6
z_0	2	0	1	6

This tableau satisfies optimality condition with optimal value $z_0 = 6$. Therefore the original problem is infeasible.

(2)

Again, we introduce slack variables $z_0 = x_0$, x_3 and x_4 and form the following tableau:

	x_1	x_2	x_0	1
x_3	2	-1	1	-1
x_4	1	2	1	-2
z	-1	1	0	0
z_0	0	0	1	0

We perform an special pivot change between (x_0 x_4) and acquired the following tableau:

	x_1	x_2	x_4	1
x_3	1	-3	1	1
x_0	-1	-2	1	2
z	-1	1	0	0
z_0	-1	-2	1	2

The tableau is feasible. We follow the pivot and perform Jordan exchange on (x_2, x_3) and get the following tableau:

	x_1	x_3	x_4	1
x_2	1/3	-1/3	1/3	1/3
x_0	-5/3	2/3	1/3	4/3
z	-2/3	-1/3	1/3	1/3
z_0	-5/3	2/3	1/3	4/3

Again, we perform another Jordan exchange on x_1 x_0:

	x_0	x_3	x_4	1
x_2	1/5	-1/5	2/5	3/5
x_1	-3/5	2/5	1/5	4/5
z	2/5	-3/5	1/5	-1/5
z_0	1	0	0	0

The tableau satisfies the optimality condition. Since $z_0^\star = 0$, the original problem is feasible. We remove the first column and last row and acquired a feasible tableau of the orignal problem:

	x_3	x_4	1
x_2	-1/5	2/5	3/5
x_1	2/5	1/5	4/5
z	-3/5	1/5	-1/5

From the pivot rule, we perform Jordan exchange on (x_2, x_3):

	x_2	x_4	1
x_3	-5	2	3
x_1	-2	1	2
z	3	-1	-2

We cannot find a pivot row corresponding to pivot column 2. Therefore the original problem is unbounded.

Problem 3

(1)

We can obviously have $x = 0$ and $t = max\{b_i,\ 0\}$ that is feasible.

(2)

We can write the duals of problem (1) and (2) as:

$$\begin{aligned} &maximize && b^T y \\ &subject\ to && A^T y \le c \\ & && y \ge 0 \end{aligned}$$

and:

$$\begin{aligned} &maximize && b^T y \\ &subject\ to && A^T y \le c \\ & && \mathbf{1}^T y \le M \\ & && y \ge 0 \end{aligned}$$

Obviously we have $d_1^\star \ge d_2^\star$, since d_2 has one extra constraint and therefore a smaller feasible set. For M that is big enough, we should have $y_1^\star$ satisfying $\mathbf{1}^T y \le M$. Under such condition, we have $d_1^\star = d_2^\star$ and from the strong duality $p_1^\star = p_2^\star$. Therefore if we have $x = x^\star$ for (1) and $x = x^\star$, $t = 0$ for (2), the objective value of (1) and (2) will be the same.

(3)

The "big M" LP can be written as follow:

$$\begin{aligned} \textit{minimize} \quad & z = x_1 - x_2 + Mt \\ \textit{subject to} \quad & -x_1 + x_2 + t \geq 2 \\ & -x_1 - x_2 + t \geq 6 \\ & x_1,\ x_2,\ t \geq 0 \end{aligned}$$

The tableau can be formed as followed:

	x_1	x_2	t	1
x_3	-1	1	1	-2
x_4	-1	-1	1	6
z	1	-1	M	0

From the feasible point $x = 0$, $t = 2$ we perform a Jordan exchange between (t, x_3).

	x_1	x_2	x_3	1
t	1	-1	1	2
x_4	0	-2	1	8
z	$M+1$	$-(M+1)$	M	$2M$

From the pivot rule, we perform a Jordan exchange between (t, x_2).

	x_1	t	x_3	1
x_2	1	-1	1	2
x_4	-2	2	-1	4
z	0	$(M+1)$	-1	-2

Again, perform a Jordan exchange between (x_3, x_4).

	x_1	t	x_4	1
x_2	-1	1	-1	6
x_3	-2	2	-1	4
z	2	$(M-1)$	1	-6

This tableau satisfies the optimality condition with $x_1 = 0$, $x_2 = 6$, and $p^\star = -6$.

Problem 4

From an active set of $\{3, 4, 5\}$ we can form a tableau as followed:

	x_4	x_5	x_3	1
x_1	-1	0	0	2
x_2	0	-1	0	2
x_6	0	0	-1	2
x_7	1	1	-1	0
z	-1	-1	-1	4

Based on Blands rule, we pick the smallest subscript with a negative entry at the last row. Therefore we perform a Jordan exchange between (x_3, x_6).

	x_4	x_5	x_6	1
x_1	-1	0	0	2
x_2	0	-1	0	2
x_3	0	0	-1	2
x_7	1	1	1	-2
z	-1	-1	1	2

Again we perform an Jordan exchange between (x_1, x_4).

	x_1	x_5	x_6	1
x_4	-1	0	0	2
x_2	0	-1	0	2
x_3	0	0	-1	2
x_7	-1	1	1	0
z	1	-1	1	0

Finally we perform an Jordan exchange betweern (x_5, x_2).

	x_1	x_2	x_6	1
x_4	-1	0	0	2
x_5	0	-1	0	2
x_3	0	0	-1	2
x_7	-1	-1	1	2
z	1	1	1	-2

The optimality condition is satisfied and the solution is $x = (0, 0, 2)$ and the optimal value is $p = -2$.

Problem 5

Monitoring is important in power system network. Suppose in a n-bus power system, we need to reach full observability of the system while minimizing the cost of implementation with a metering device called Phasor Measurement Unit (PMU) that is capable of monitoring all buses incident to the bus it has been installed. We need to determine where those PMUs should be installed. We introduce a indicator variable x_i:

$$x_1 = \begin{cases} 1 & \text{with PMU installed} \\ 0 & \text{otherwise.} \end{cases}$$

We construct an adjacent matrix A with element a_{ij}:

$$a_{ij} = \begin{cases} 1 & \text{bus } i \text{ is incident to bus } j \text{ or } i = j \\ 0 & \text{otherwise.} \end{cases}$$

An ILP can be formed as followed:

$$\begin{aligned}
&minimize && \sum_{i=1}^{n} c_i x_i \\
&subject\ to && \sum_{j=1}^{n} a_{ij} x_i \geq 1,\ \ i = 1, ..., n \\
& && x_i \in \{0, 1\}\ i = 1, ..., n
\end{aligned}$$

We can relax this ILP to LP as followed:

$$\begin{aligned}
&minimize && \sum_{i=1}^{n} c_i x_i \\
&subject\ to && \sum_{j=1}^{n} a_{ij} x_i \geq 1,\ \ i = 1, ..., n \\
& && x_i \geq 0\ i = 1, ..., n
\end{aligned}$$