EE 236B HW6
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A12.6

(a)

The problem can be formed as follow:

$$\begin{array}{ll} minimize & t \\ subject\ to & ||A\omega||_2 \leq t \\ & F\omega = 1, \end{array}$$

where

$$A = \begin{bmatrix} \cos\gamma_1(\theta) & ... & \cos\gamma_n(\theta) & -\sin\gamma_1(\theta) & ... & -\sin\gamma_n(\theta) \\ \sin\gamma_1(\theta) & ... & \sin\gamma_n(\theta) & cos\gamma_1(\theta) & ... & \cos\gamma_n(\theta) \end{bmatrix}$$

$$F = \begin{bmatrix} \cos\gamma_1(\theta^{tar}) & ... & \cos\gamma_n(\theta^{tar}) & -\sin\gamma(\theta^{tar}) & ... & -\sin\gamma_n(\theta^{tar}) \\ \sin\gamma(\theta^{tar}) & ... & \sin\gamma_n(\theta^{tar}) & cos\gamma(\theta^{tar}) & ... & \cos\gamma_n(\theta^{tar}) \end{bmatrix},$$

and $\omega = [\omega_{re}\ \omega_{im}]^T$.

(b)

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clc;clear;
rand('state',0);
n = 40;
X = 30*[rand(1,n); rand(1,n)];
delta = 15*pi/180;
t_tar=15*pi/180;
N = 400 ;
theta = linspace(t_tar + delta, 2*pi + t_tar-delta, N)';
A = exp(1i*[cos(theta), sin(theta)]*X);
F = exp(1i*[cos(t_tar), sin(t_tar)]*X);
cvx_begin
    variable omega(n) complex
    minimize  max(abs(A*omega))
    subject to
        F*omega == 1
cvx_end
t = linspace(-180, 180, N)';
A1 = exp(1i * [cosd(t), sind(t)]*X);
g = abs(A1*omega);
semilogy(t, g);
```

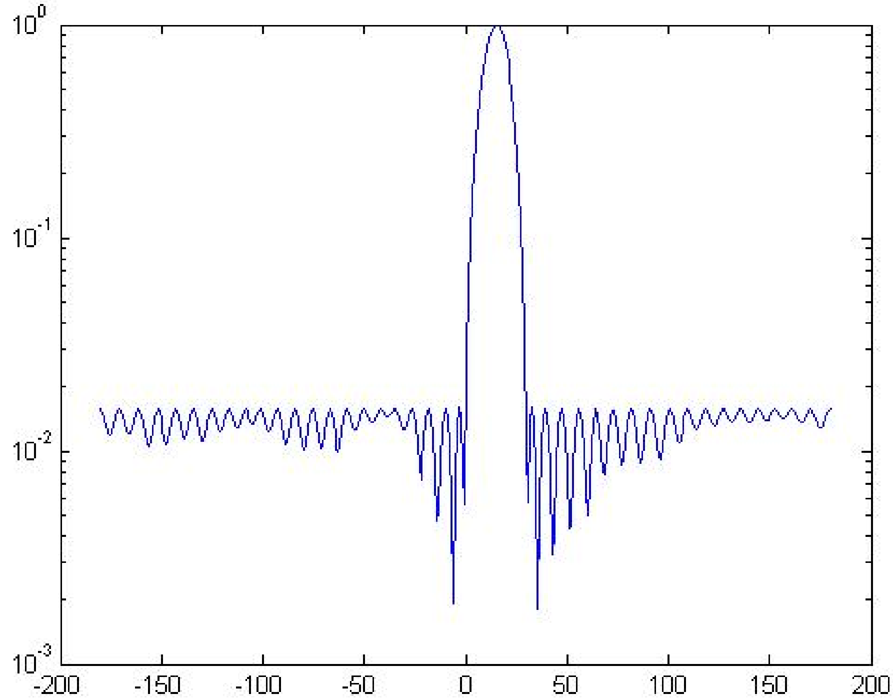

Figure 1: Gain

A4.4

We can rewrite the problem as

$$\begin{array}{ll} \textit{minimize} & ||Az-b||_2^2 \\ \textit{subject to} & z^TCz + 2f^Tz = 0, \end{array}$$

where

$$A = \begin{bmatrix} -2y_1^T & 1 \\ \dots & \dots \\ -2y_5^T & 1 \end{bmatrix} \qquad b = \begin{bmatrix} d_1^2 - ||y_1||_2^2 \\ \dots \\ d_5^2 - ||y_5||_2^2 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad f = \begin{bmatrix} 0 \\ 0 \\ -1/2 \end{bmatrix}$$

The Lagrangian is

$$\begin{aligned} L(z,\nu) &= z^TA^TAz - 2b^TAz + b^Tb + \nu^T(z^TCz + 2f^Tz) \\ &= z^T(A^TA + \nu C)z - 2(A^Tb - \nu f)^Tz + b^Tb \end{aligned}$$

which is bounded below if

$$A^TA + \nu C \succeq 0 \qquad A^Tb - \nu f \in \text{Range}(A^TA + \nu C)$$

and we have

$$\nabla L(z,v) = 2(A^TA+\nu C)z - 2(A^Tb-\nu f) = 0 \Rightarrow z = (A^TA+\nu C)^+(A^Tb-\nu f)$$

Therefore $g(\nu)$ can be written as

$$g(\nu) = -(A^b-\nu f)^T(A^TA+\nu C)^+(A^Tb-\nu f)+b^Tb$$

The dual can therefore be written as an SDP

$$\begin{array}{ll} \textit{maximize} & -t+b^Tb \\ \textit{subject to} & \begin{bmatrix} A^TA+\nu C & A^Tb-\nu f \\ (A^Tb-\nu f)^T & t \end{bmatrix} \succeq 0 \end{array}$$

This can be easily solved through CVX. This gives $\nu^\star = 0.5896$, which gives $z^\star = [1.33, 0.64, 2.18]^T$. Therefore $x^\star = [1.33, 0.64]^T$.

T4.43

(b)

From the property that $\lambda_1 \leq t$ if and only if $A(x) \preceq tI$. The problem can be formed as

$$\begin{array}{ll} \textit{minimize} & t-d \\ \textit{subject to} & dI \preceq A(x) \preceq tI \end{array}$$

(c)

The problem can be formed as

$$\begin{array}{ll} \textit{minimize} & t/d \\ \textit{subject to} & dI \preceq A(x) \preceq tI \end{array}$$

T5.19

(a)

The given LP is equivalent to the following ILP since its constraint matrix is total unimodular.

$$\begin{array}{ll} \textit{minimize} & x^Ty \\ \textit{subject to} & y \in \{0,\ \mathbf{1}\} \\ & \mathbf{1}^Ty = r \end{array}$$

This optimal value of this IPL is equal to $f(x)$.

(b)

$$\begin{aligned} L(y,u,v,t) &= -x^T y + u^T(y-\mathbf{1}) - v^T y + t(\mathbf{1}^T y - r) \\ &= (-x+u-v+t\mathbf{1})^T y - 1^T u - tr \end{aligned}$$

$$\begin{aligned} g(u,v,t) &= \inf_y L(y,u,v,t) \\ &= \begin{cases} -rt - \mathbf{1}^T u & u - v + t\mathbf{1} - x = 0,\ u, v \succeq 0 \\ -\infty & \text{otherwise} \end{cases} \end{aligned}$$

This can be written as

$$\begin{array}{ll} \textit{minimize} & rt + 1^T u \\ \textit{subject to} & t\mathbf{1} + 1 \succeq x \\ & u \succeq 0 \end{array}$$

(c)

from (b), $f(x) \le 0.8$ if and only if $0.1nt + 1^T u \le 0.8$, $t\mathbf{1} + u \ge x$, and $u \ge 0$. Therefore it can be written as a QP as

$$\begin{array}{ll} \textit{minimize} & x^T \Sigma x \\ \textit{subject to} & \bar{p}^T x \ge r_{min} \\ & \mathbf{1}^T x = 1, \qquad x \succeq 0 \\ & 0.1nt + 1^T u \le 0.8 \\ & t\mathbf{1} + u \ge x \\ & u \ge 0 \end{array}$$

where x, t, u are the variables.

T5.21

(a)

This is a convex problem since the objective function is a convex function and the constraint is also convex. The optimal value $p^\star = 1$ and $x^\star = 0$, $y^\star \in R_+$.

(b)

The dual can be written as

$$\begin{array}{ll} \textit{maximize} & 0 \\ \textit{subject to} & \lambda \ge 0 \end{array}$$

Therefore $d^\star = 0$. The duality gap $p^\star - d^\star = 1$.

(c)

The Slater's condition does not hold in this case.

A4.30

(a)

$$\begin{aligned} L(x,y,\lambda) &= c^T x + \frac{1}{\mu}\sum_{i=1}^n \log(1+e^{\mu y_i}) + \lambda^T(Ax-b-y) \\ &= (c-A^T\lambda)^T x + (\frac{1}{\mu}\sum_{i=1}^n \log(1+e^{\mu y_i}) - \lambda^T y) - b^T\lambda \end{aligned}$$

$$\begin{aligned} g(\lambda) &= \inf_{x,y} L(x,y,\lambda) \\ &= \inf_y (\frac{1}{\mu}\sum_{i=1}^n \log(1+e^{\mu y_i}) - \lambda^T y - b^T\lambda) \qquad \text{if } c - A^T\lambda = 0,\ \lambda \geq 0 \end{aligned}$$

Taking partial over y and set it to 0, we have

$$y_i = \frac{1}{u}\log\frac{\lambda_i}{1-\lambda_i}$$

Therefore we have

$$g(\lambda) = \frac{1}{u}\sum_i^n (\log\frac{1}{1-\lambda_i} - \lambda_i \log(\frac{\lambda_i}{1-\lambda_i})) - b^T\lambda \qquad \text{if } \lambda \preceq 1$$

The dual can we written as:

$$\begin{array}{ll} \textit{maximize} & \frac{1}{u}\sum_i^m (\log\frac{1}{1-\lambda_i} - \lambda_i\log(\frac{\lambda_i}{1-\lambda_i})) - b^T\lambda \\ \textit{subject to} & c - A^T\lambda = 0 \\ & 0 \preceq \lambda \preceq 1 \end{array}$$

(b)

Rewrite objective function of (25) as $f(z) - b^T z$. To see $p^\star \leq q^\star$, notice that $z^\star$ is dual feasible for (25) and $f(z) \geq 0$ for z feasible. Therefore we always have $f(z) - b^T z \geq f(z^\star) - b^T z^\star \geq b^T z^\star$. Since strong duality holds followed by Slater's condition, we have $p^\star \leq q^\star$.

To see $q^\star \le p^\star + m \log 2/\mu$. Take derivative of $f(z)$ and we have $f'(0.5) = 0$, i.e., $z = 0.5$ maximize $f(z)$ and $f(0.5) = m \log 2/\mu$. With $z^\star$ maximizing $-b^T z$, assume z' maximize dual of (25), we have $q^\star = d^\star = f(z') - b^T z' \le f(0.5) - b^T z^\star = p^\star + m \log 2/\mu$.