EE 236B HW1
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T2.7

$||x-a||_2 \leq ||x-b||_2$

$\Rightarrow (x-a)^T(x-a) \leq (x-b)^T(x-b)$

$\Rightarrow x^Tx - 2a^Tx + a^Ta \leq x^Tx - 2b^Tx + b^Tb$

$\Rightarrow (b-a)^Tx \leq (b^Tb - a^Ta)/2$

Therefore, $\{x \mid ||x-a||_2 \leq ||x-b||_2\}$ is a halfspace.

T2.12

(d)

$||x-x_0||_2 \leq ||x-y||_2$

$\Rightarrow (x-x_0)^T(x-x_0) \leq (x-y)^T(x-y)$

$\Rightarrow x^Tx - 2x_0^Tx + x_0^Tx_0 \leq x^Tx - 2y^Tx + y^Ty$

$\Rightarrow (y-x_0)^Tx \leq (y^Ty - x_0^Tx)/2$

For a given $y \in S$, $\{x \mid ||x-x_0||_2 \leq ||x-y||_2\}$ represents a halfspace in R^n, and is a convex set. Therefore, $\{x \mid ||x-x_0||_2 \leq ||x-y||_2$ for any y $\in S\}$ can be written as $\cap_{y\in S}\{x \mid ||x-x_0||_2 \leq ||x-y||_2\}$ and is a convex set.

(e)

We could apply the same method and conclude that for each x that satisfies $\mathrm{dist}(x,S) \leq \mathrm{dist}(x,T)$, there is always a $y \in S$ and $z \in T$ that makes it equivalent to $||x-y||_2 \leq ||x-z||_2$, which is a halfspace. However, this does not mean all points that satisfies $\{x \mid \mathrm{dist}(x,S) \leq \mathrm{dist}(x,T)\}$ are also in that particular halfspace. To see this, consider $S = \{(x,y) \in R^2 \mid x^2 + y^2 = 1, y \leq 0\}$ and $T = \{(x,y) \in R^2 \mid x^2+y^2 = 2, y \leq 0\}$ and $A = \{(x,y) \in R^2 \mid \mathrm{dist}(x,S) \leq \mathrm{dist}(x,T)\}$. Obviously A is not a convex set.

(f)

Assume $S = \{x \mid ||x-a||_2 \leq \theta ||x-b||_2\}$. Again, we could rewrite the inequality as follow:

$$\underbrace{(1-\theta^2)}_{a}x^Tx + \underbrace{2(\theta^2 b - a)^T}_{b}x + \underbrace{(a^Ta - \theta^2 b^T b)}_{c} \le 0$$

$\Rightarrow a||x-h||_2 + k \le 0$, where $h = -b/2a$ and $k = c - b^2/4a$.

Now we could apply the definition and assume $x_1, x_2 \in S$ and $0 \le \theta \le 1$. We have

$$a||\theta x_1 + (1-\theta)x_2 + h||_2 + k \le a\theta||x_1 + h||_2 + a(1-\theta)||x_2 + h|| + k \le 0$$

Therefore, S is a convex set.

T2.16

Consider two points $(x', y_1' + y_2')$, $(x'', y_1'' + y_2'') \in S$ with $(x', y_1'), (x'', y_1'') \in S_1$ and $(x', y_2'), (x'', y_2'') \in S_2$, and $0 \le \theta \le 1$. By applying the definition, we have

$$\begin{aligned}(x, y) = \theta(x', y_1' + y_2') + (1-\theta)(x'', y_1'' + y_2'') \\ = (\theta x', \theta(y_1' + y_2')) + ((1-\theta)x'', (1-\theta)(y_1'' + y_2''))\end{aligned}$$

Since both S_1 and S_2 are convex, $(\theta x', \theta y_1') + ((1-\theta)x'', (1-\theta)y_1'') \in S_1$ and $(\theta x', \theta y_2') + ((1-\theta)x'', (1-\theta)y_2'') \in S_2$. Therefore $(x, y) \in S$ since it is a partial sum of two points from S_1 and S_2.

Add.1

(a)

We could simplify the original fitting problem as follow:

$$\begin{aligned}&\min \sum_{i=1}^{m}((u_i - u_c)^2 + (v_i - v_c)^2 - R^2)^2 \\ &\Rightarrow \min \sum_{i=1}^{m}(u_i^2 + v_i^2 - 2u_iu_c - 2v_iv_c + (v_c^2 + u_c^2 - R^2))^2\end{aligned}$$

To write it into a linear least-square problem, we could have $a_i^T = [-2u_i \ -2v_i \ 1]^T$ and $b_i = -u_i^2 - v_i^2$, which are the row vectors of A and b, and variable $x = [x_1 \ x_2 \ x_3]^T$.

(b)

If we expand the normal equation with A and b we selected from (a), we get the following equation from the third row of expansion:

$$
\begin{aligned}
&2x_1 \sum^n u_i + 2x_2 \sum^n v_i - nx_3 = \sum^n (u_i^2 + v_i^2) \\
&\Rightarrow nx_1^2 + nx_2^2 - nx_3 = nx_1^2 + nx_2^2 - 2x_1 \sum^n u_i - 2x_2 \sum^n v_i + \sum^n (u_i^2 + v_i^2) \\
&\Rightarrow n(x_1^2 + x_2^2 - x_3) = \sum^n (x_1 - u_i)^2 + \sum^n (x_2 - v_i)^2 \geq 0
\end{aligned}
$$

i.e. $\hat{x}_1^2 + \hat{x}_2^2 - \hat{x}_3 \geq 0$.

(c)

Figure 1 presents the least-square fit of a circle to points.

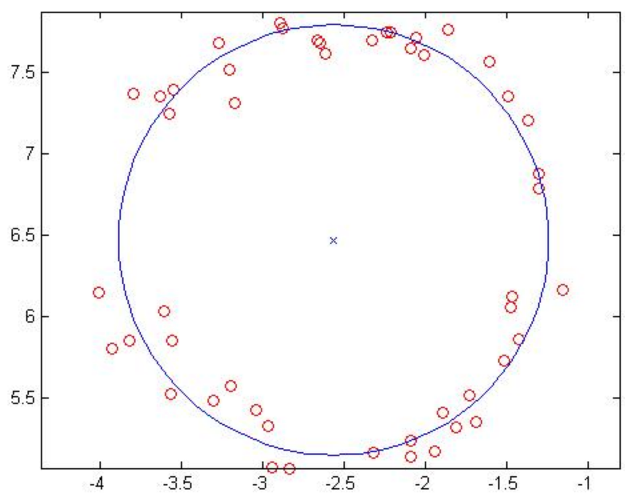

Figure 1: Least Square Fit

Add.2

(a)

From Figure 2. The optimal γ is 0.35 and the corresponding objective value is 0.4809.

(b)

$x = [1\ 0\ 1\ 0\ 0\ 1\ 0\ 1\ 0\ 1]^T$ and $p = 0.8628$.

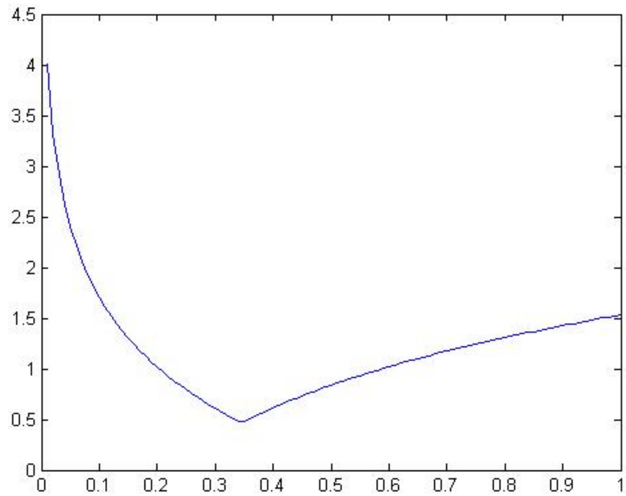

Figure 2: Equal Lamp Power

(c)

$\rho = 0.2200$, $x = [0.5\ 0.4773\ 0.0832\ 0\ 0.4559\ 0.4353\ 0.4595\ 0.4306\ 0.4035\ 0.4525]^T$ and $p = 0.4440$.

(d)

With cvx we acquired a solution $x = [1\ 0.1165\ 0\ 0\ 1\ 0\ 1\ 0.0249\ 0\ 1]^T$ and an objective value $p = 0.4198$.

(e)

With cvx we acquired a solution $x = [1\ 0.1896\ 0\ 0\ 1\ 0\ 1\ 0.1640\ 0\ 1]^T$ and an objective value $p = 0.3664$.

(f)

With cvx we acquired the exact solution as $x = [1\ 0.2023\ 0\ 0\ 1\ 0\ 1\ 0.1882\ 0\ 1]^T$ and objective value $p = 0.3575$.