# DUA BEFORE STUDYING

## Dua Before Studying

Oh Allah! Make useful for me what you have taught me and teach me knowledge that will be useful to me. Oh Allah! I ask you for the understanding of the prophets and the memory of the messengers, and those nearest to you.Oh Allah! Make my tongue full of your remembrance and my heart with awe of you. Oh Allah! You do whatever you wish, and you are my availer and protector and best of aid.

للّٰهُمَّ انْفَعْنِي بِمَا عَلِّمْتَنِي وَ عَلِّمْنِي مَا يَنْفَعُنِي

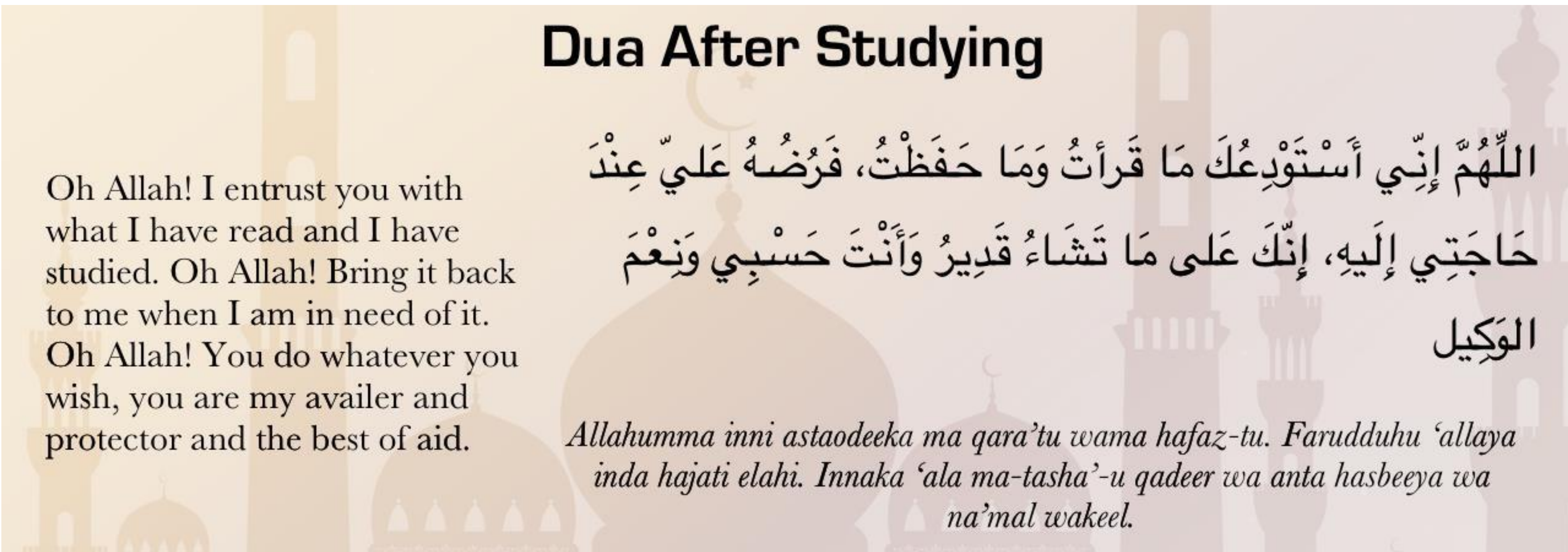
اللّٰهُمَّ إِنِّي أَسْأَلُكَ فَهْمَ النَّ بِيِّينَ وَ حِفْظَ الْمُرْسَلِينَ الْمُقَرَّبِينَ.

اللّٰهُمَّ اجْعَلْ لِسَانِي عَامِرًا بِذِكْرِكَ وَ قَلْبِي بِخَشْيَتِكَ. .

إِنَّكَ عَلَى مَا تَشَاءُ قَدِيرٌ وَ أَنْتَ حَسْبُنَا اللّٰهُ وَ نِعْمَ الْوَكِيلُ.

*Allahumma infa'nii bimaa 'allamtanii wa'allimnii maa yanfa'uunii. Allahumma inii as'aluka fahmal-nabiyyen wa hifzal mursaleen al-muqarrabeen. Allahumma ijal leesanee 'aiman bi dhikrika wa qalbi bi khashyatika. Innaka 'ala ma-tasha'u qadeer wa anta hasbun-allahu wa na'mal wakeel.*

# DUA AFTER STUDYING



# ~CHAPTER 5~

# DISCRETE PROBABILITY DISTRIBUTIONS

# OUTLINE

| | |
|---|---|
| 5-1 | Probability Distributions |
| 5-2 | Mean, Variance, Standard Deviation, and Expectation |
| 5-3 | The Binomial Distribution |
| 5-4 | Other Types of Distributions |

# LEARNING OBJECTIVES

1. Construct a probability distribution for a random variable.
2. Find the mean, variance, standard deviation, and expected value for a discrete random variable.
3. Find the exact probability for $X$ successes in $n$ trials of a binomial experiment.
4. Find the mean, variance, and standard deviation for the variable of a binomial distribution.
5. Find probabilities for outcomes of variables, using the Poisson, hypergeometric, and multinomial distributions.

# 5.1 Probability Distributions

# INTRODUCTION (1)

1. Many decisions in business, insurance, and any other real-life situations are made by assigning probabilities to all possible outcomes pertaining to the situation and then evaluating the results.
2. For example, a saleswoman can compute the probability that she will make 0, 1, 2, or 3 or more sales in a single day.
3. Once these probabilities are assigned, statistics such as the mean, variance, and standard deviation can be computed.

# INTRODUCTION (2)

1. The saleswoman will be able to compute the average number of sales she makes per week, and if she is working on commission, she will be able to approximate her weekly income over a period of time, say, monthly.
2. This chapter explains the concepts and applications of what is called a probability distribution.

# PROBABILITY DISTRIBUTIONS (1)

**Definition**

**Classification: Discrete**

**Classification: Continuous**

**Important note**: Need to recall Chapter 1, on variable matters.

# PROBABILITY DISTRIBUTIONS (2)

1. A random variable is a variable whose values are determined by chance.
2. In other words, random variable is one that is associated with probability.
3. This chapter only use discrete random variables.
4. A discrete probability distribution consists of the values a random variable can assume and the corresponding probabilities of the values.

# PROBABILITY DISTRIBUTIONS (3)

The procedure shown here for constructing a probability distribution for a discrete random variable uses the probability experiment of tossing three coins. Recall that when three coins are tossed, the sample space is represented as TTT, TTH, THT, HTT, HHT, HTH, THH, HHH; and if $X$ is the random variable for the number of heads, then $X$ assumes the value 0, 1, 2, or 3.

Probabilities for the values of $X$ can be determined as follows:

| **No heads** | **One head** | | | **Two heads** | | | **Three heads** |
|---|---|---|---|---|---|---|---|
| TTT | TTH | THT | HTT | HHT | HTH | THH | HHH |
| $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ |
| $\frac{1}{8}$ | $\frac{3}{8}$ | | | $\frac{3}{8}$ | | | $\frac{1}{8}$ |

Hence, the probability of getting no heads is $\frac{1}{8}$, one head is $\frac{3}{8}$, two heads is $\frac{3}{8}$, and three heads is $\frac{1}{8}$. From these values, a probability distribution can be constructed by listing the outcomes and assigning the probability of each outcome, as shown here.

| **Number of heads $X$** | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| **Probability $P(X)$** | $\frac{1}{8}$ | $\frac{3}{8}$ | $\frac{3}{8}$ | $\frac{1}{8}$ |

# EXAMPLE 5-1: ROLLING A DIE

- Construct a probability distribution for rolling a single die.

# EXAMPLE 5-1: ROLLING A DIE

- Construct a probability distribution for rolling a single die.

**Solution**

Since the sample space is 1, 2, 3, 4, 5, 6 and each outcome has a probability of $\frac{1}{6}$, the distribution is as shown.

| **Outcome $X$** | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| **Probability $P(X)$** | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ |

Probability distributions can be shown graphically by representing the values of $X$ on the $x$ axis and the probabilities $P(X)$ on the $y$ axis.

# EXAMPLE 5-2: TOSSING COINS

- Represent graphically the probability distribution for the sample space for tossing three coins.

| Number of heads $X$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| Probability $P(X)$ | $\frac{1}{8}$ | $\frac{3}{8}$ | $\frac{3}{8}$ | $\frac{1}{8}$ |

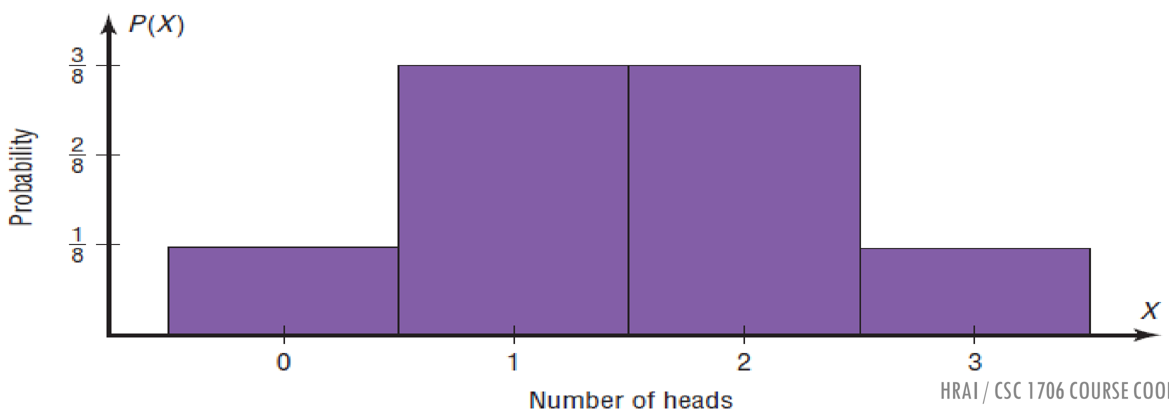
# EXAMPLE 5-2: TOSSING COINS

- Represent graphically the probability distribution for the sample space for tossing three coins.

| Number of heads $X$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| Probability $P(X)$ | $\frac{1}{8}$ | $\frac{3}{8}$ | $\frac{3}{8}$ | $\frac{1}{8}$ |

**Solution**

The values that $X$ assumes are located on the $x$ axis, and the values for $P(X)$ are located on the $y$ axis. The graph is shown in Figure 5–1.



# SUMMARY FOR EXAMPLES 5-1 & 5-2

1. Note that for visual appearances, it is not necessary to start with 0 at the origin.
2. Illustrations used in both examples are of theoretical probability distributions.
3. It is not a need to actually perform the experiments to compute the probabilities.
4. In contrast, to construct actual probability distributions, the observation of the variable over a period of time must be used.
5. It is called empirical probability distributions.

# EXAMPLE 5-3: BASEBALL WORLD SERIES

## Baseball World Series

The baseball World Series is played by the winner of the National League and the American League. The first team to win four games wins the World Series. In other words, the series will consist of four to seven games, depending on the individual victories. The data shown consist of 40 World Series events. The number of games played in each series is represented by the variable $X$. Find the probability $P(X)$ for each $X$, construct a probability distribution, and draw a graph for the data.

| $X$ | Number of games played |
|---|---|
| 4 | 8 |
| 5 | 7 |
| 6 | 9 |
| 7 | 16 |
| | 40 |

# EXAMPLE 5-3: BASEBALL WORLD SERIES

**Solution**

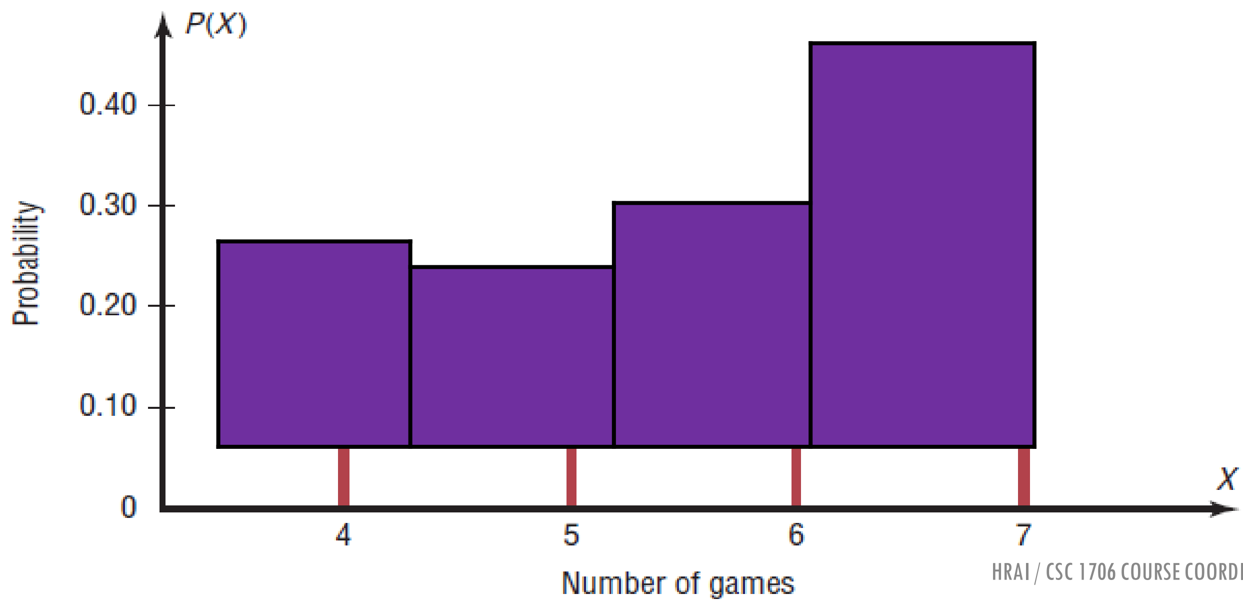
The probability $P(X)$ can be computed for each $X$ by dividing the number of games $X$ by the total.

For 4 games, $\frac{8}{40} = 0.200$ For 6 games, $\frac{9}{40} = 0.225$

For 5 games, $\frac{7}{40} = 0.175$ For 7 games, $\frac{16}{40} = 0.400$

The probability distribution is

| **Number of games $X$** | 4 | 5 | 6 | 7 |
|---|---|---|---|---|
| **Probability $P(X)$** | 0.200 | 0.175 | 0.225 | 0.400 |



# TWO REQUIREMENTS FOR A PROBABILITY DISTRIBUTION (1)

1. The sum of the probabilities of all the events in the sample space must equal 1; that is, $\Sigma P(X) = 1$.

1. This sum cannot be less than 1 or greater than 1 since the sample space includes all possible outcomes of the probability experiment.

# TWO REQUIREMENTS FOR A PROBABILITY DISTRIBUTION (2)

2. The probability of each event in the sample space must be between or equal to 0 and 1. That is, $0 \leq P(X) \leq 1$.

1. The reason (as stated in Chapter 4) is that the range of the probability of any individual value can be 0, 1, or any value between 0 and 1.
2. A probability cannot be a negative number or greater than 1.

# EXAMPLE 5-4: PROBABILITY DISTRIBUTION

- Determine whether each distribution is a probability distribution:

a.

| $X$ | 4 | 6 | 8 | 10 |
|---|---|---|---|---|
| $P(X)$ | $-0.6$ | 0.2 | 0.7 | 1.5 |

c.

| $X$ | 8 | 9 | 12 |
|---|---|---|---|
| $P(X)$ | $\frac{2}{3}$ | $\frac{1}{6}$ | $\frac{1}{6}$ |

b.

| $X$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $P(X)$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ |

d.

| $X$ | 1 | 3 | 5 | 7 | 9 |
|---|---|---|---|---|---|
| $P(X)$ | 0.3 | 0.1 | 0.2 | 0.4 | $-0.7$ |

# 5-1 EXERCISES

*For Exercises 13 through 18, state whether the variable is discrete or continuous.*

**13.** The number of cheeseburgers a fast-food restaurant serves each day

**14.** The number of people who play the state lottery each day

**15.** The weight of an automobile

**16.** The time it takes to have a medical physical exam

**17.** The number of mathematics majors in your school

**18.** The blood pressures of all patients admitted to a hospital on a specific day

# 5-1 EXERCISES (CONT'D.)

*For Exercises 19 through 26, construct a probability distribution for the data and draw a graph for the distribution.*

**19. Medical Tests** The probabilities that a patient will have 0, 1, 2, or 3 medical tests performed on entering a hospital are $\frac{6}{15}$, $\frac{5}{15}$, $\frac{3}{15}$, and $\frac{1}{15}$, respectively.

# 5-1 EXERCISES (CONT'D.)

*For Exercises 19 through 26, construct a probability distribution for the data and draw a graph for the distribution.*

**21. Birthday Cake Sales** The probabilities that a bakery has a demand for 2, 3, 5, or 7 birthday cakes on any given day are 0.35, 0.41, 0.15, and 0.09, respectively.

# 5-1 EXERCISES (CONT'D.)

**27. Triangular Numbers** The first six triangular numbers (1, 3, 6, 10, 15, 21) are printed one each on one side of a card. The cards are placed face down and mixed. Choose two cards at random, and let $x$ be the sum of the two numbers. Construct the probability distribution for this random variable $x$.

# 5-1 EXERCISES (CONT'D.)

**30. Mathematics Tutoring Center** At a drop-in mathematics tutoring center, each teacher sees 4 to 8 students per hour. The probability that a tutor sees 4 students in an hour is 0.117; 5 students, 0.123; 6 students, 0.295; and 7 students, 0.328. Find the probability that a tutor sees 8 students in an hour, construct the probability distribution, and draw the graph.

# 5.2 | Mean, Variance, Standard Deviation, and Expectation

# MEAN, VARIANCE, STANDARD DEVIATION, AND EXPECTATION (1)

MEAN: $\mu = \sum X \cdot P(X)$

The mean of a random variable with a discrete probability distribution is

$$\mu = X_1 \cdot P(X_1) + X_2 \cdot P(X_2) + X_3 \cdot P(X_3) + \cdots + X_n \cdot P(X_n)$$
$$= \Sigma X \cdot P(X)$$

where $X_1, X_2, X_3, \ldots, X_n$ are the outcomes and $P(X_1), P(X_2), P(X_3), \ldots, P(X_n)$ are the corresponding probabilities.

*Note:* $\Sigma X \cdot P(X)$ means to sum the products.

# MEAN, VARIANCE, STANDARD DEVIATION, AND EXPECTATION (2)

VARIANCE:

$$\sigma^2 = \sum \left\lfloor X^2 \cdot P(X) \right\rfloor - \mu^2$$

Find the variance of a probability distribution by multiplying the square of each outcome by its corresponding probability, summing those products, and subtracting the square of the mean. The formula for the variance of a probability distribution is

$$\sigma^2 = \Sigma[X^2 \cdot P(X)] - \mu^2$$

The standard deviation of a probability distribution is

$$\sigma = \sqrt{\sigma^2} \qquad \text{or} \qquad \sqrt{\Sigma[X^2 \cdot P(X)] - \mu^2}$$

# MEAN, VARIANCE, STANDARD DEVIATION, AND EXPECTATION (3)

Rounding Rule:

1. The mean, variance, and standard deviation should be rounded to one more decimal place than the outcome X.
2. When fractions are used, they should be reduced to lowest terms.

# EXAMPLE 5-6: CHILDREN IN A FAMILY *(MEAN)*

- In families with five children, find the mean number of children who will be girls.

# EXAMPLE 5-6: CHILDREN IN A FAMILY *(MEAN)*

**Solution:**

1. First, it is necessary to find the probability distribution for the number of females.

2. There are 32 outcomes, and there is one way for a family to have no girls.

3. There are five ways to have one girl, that is, FMMMM, MFMMM, MMFMM, MMMFM, MMMMF.

4. Continue with two females and three males, three females and two males, four females and one male, and five females.

5. The probability distribution is:

| Number of girls $X$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Probability $P(X)$ | $\frac{1}{32}$ | $\frac{5}{32}$ | $\frac{10}{32}$ | $\frac{10}{32}$ | $\frac{5}{32}$ | $\frac{1}{32}$ |

# EXAMPLE 5-6: CHILDREN IN A FAMILY *(MEAN)*

| Number of girls $X$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Probability $P(X)$ | $\frac{1}{32}$ | $\frac{5}{32}$ | $\frac{10}{32}$ | $\frac{10}{32}$ | $\frac{5}{32}$ | $\frac{1}{32}$ |

The mean is

$$\mu = \Sigma X \cdot P(X) = 0 \cdot \tfrac{1}{32} + 1 \cdot \tfrac{5}{32} + 2 \cdot \tfrac{10}{32} + 3 \cdot \tfrac{10}{32} + 4 \cdot \tfrac{5}{32} + 5 \cdot \tfrac{1}{32} = 2\tfrac{1}{2} = 2.5$$

**The mean number of who will be girls in a family of five children is 2.5.**

# EXAMPLE 5-8: TRIPS OF 5 NIGHTS OR MORE *(MEAN)*

- The probability distribution shown represents the number of trips of five nights or more that American adults take per year.
- (That is, 6% do not take any trips lasting five nights or more, 70% take one trip lasting five nights or more per year, etc.)
- Find the mean.

| Number of trips $X$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Probability $P(X)$ | 0.06 | 0.70 | 0.20 | 0.03 | 0.01 |

# EXAMPLE 5-8: TRIPS OF 5 NIGHTS OR MORE *(MEAN)*

| Number of trips $X$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Probability $P(X)$ | 0.06 | 0.70 | 0.20 | 0.03 | 0.01 |

$$\begin{aligned}\mu &= \sum X \cdot P(X) \\ &= 0(0.06) + 1(0.70) + 2(0.20) \\ &\quad + 3(0.03) + 4(0.01) \\ &= 1.23\end{aligned}$$

**The mean of the number of trips lasting 5 nights or more per year taken by American adults is 1.23.**

# EXAMPLE 5-9: ROLLING A DIE *(VARIANCE AND STANDARD DEVIATION)*

- Compute the variance and standard deviation for the following probability distribution.
- The mean μ = 3.5.

| **Outcome $X$** | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| **Probability $P(X)$** | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ |

# EXAMPLE 5-9: ROLLING A DIE *(VARIANCE AND STANDARD DEVIATION)*

| **Outcome** $X$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| **Probability** $P(X)$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ |

**Solution**

Recall that the mean is $\mu = 3.5$ . Square each outcome and multiply by the corresponding probability, sum those products, and then subtract the square of the mean.

$$\sigma^2 = (1^2 \cdot \tfrac{1}{6} + 2^2 \cdot \tfrac{1}{6} + 3^2 \cdot \tfrac{1}{6} + 4^2 \cdot \tfrac{1}{6} + 5^2 \cdot \tfrac{1}{6} + 6^2 \cdot \tfrac{1}{6}) - (3.5)^2 = 2.9$$

To get the standard deviation, find the square root of the variance.

$$\sigma = \sqrt{2.9} = 1.7$$

# EXPECTATION (1)

1. The expected value, or expectation, of a discrete random variable of a probability distribution is the theoretical average of the variable.
2. Expected value is used in various types of games of chance, in insurance, and in other areas, such as decision theory.
3. The expected value is, by definition, the mean of the probability distribution.

# EXPECTATION (2)

The **expected value** of a discrete random variable of a probability distribution is the theoretical average of the variable. The formula is

$$\mu = E(X) = \Sigma X \cdot P(X)$$

The symbol $E(X)$ is used for the expected value.

The formula for the expected value is the same as the formula for the theoretical mean. The expected value, then, is the theoretical mean of the probability distribution. That is, $E(X) = \mu$.

When expected value problems involve money, it is customary to round the answer to the nearest cent.

# EXAMPLE 5-13: SELECTING BALLS *(EXPECTATION)*

- You have six balls numbered 1, 2, 3, 5, 8 and 13 are placed in a box. A ball is selected at random , and its number is recorded and it is replaced.
- Find the expected value of the number that will occur.

# EXAMPLE 5-13: SELECTING BALLS *(EXPECTATION)*

**Solution:**

Since the balls are replaced, the probability for each number is $\frac{1}{6}$, so the probability distribution is:

| Number ($X$) | 1 | 2 | 3 | 5 | 8 | 13 |
|---|---|---|---|---|---|---|
| Probability $P(X)$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ |

$$E(X) = 1 \cdot \tfrac{1}{6} + 2 \cdot \tfrac{1}{6} + 3 \cdot \tfrac{1}{6} + 5 \cdot \tfrac{1}{6} + 8 \cdot \tfrac{1}{6} + 13 \cdot \tfrac{1}{6} = 5\tfrac{1}{3}$$

# 5-2 EXERCISES

1. **Defective DVDs** From past experience, a company found that in cartons of DVDs, 90% contain no defective DVDs, 5% contain one defective DVD, 3% contain two defective DVDs, and 2% contain three defective DVDs. Find the mean, variance, and standard deviation for the number of defective DVDs.

# 5-2 EXERCISES (CONTD.)

6. **Traffic Accidents** The county highway department recorded the following probabilities for the number of accidents per day on a certain freeway for one month. The number of accidents per day and their corresponding probabilities are shown. Find the mean, variance, and standard deviation.

| **Number of accidents $X$** | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| **Probability $P(X)$** | 0.4 | 0.2 | 0.2 | 0.1 | 0.1 |

# 5-2 EXERCISES (CONTD.)

**9. Students Using the Math Lab** The number of students using the Math Lab per day is found in the distribution below. Find the mean, variance, and standard deviation for this probability distribution.

| $X$ | 6 | 8 | 10 | 12 | 14 |
|---|---|---|---|---|---|
| $P(X)$ | 0.15 | 0.3 | 0.35 | 0.1 | 0.1 |

What is the probability that fewer than 8 or more than 12 use the lab in a given day?

# 5-2 EXERCISES (CONTD.)

11. **Grab Bags** A craft store has 25 assorted grab bags on sale for \$3.00 each. Fifteen of the bags contain \$3.00 worth of merchandise, six contain \$2.00 worth, two contain \$5.00 worth of merchandise, and there are one each containing \$10.00 and \$20.00 worth of merchandise. Suppose that you purchase one bag; what is your expected gain or loss?

# 5-2 EXERCISES (CONTD.)

**18. Life Insurance** A 35-year-old woman purchases a \$100,000 term life insurance policy for an annual payment of \$360. Based on a period life table for the U.S. government, the probability that she will survive the year is 0.999057. Find the expected value of the policy for the insurance company.

# 5.3 | The Binomial Distribution

# THE BINOMIAL DISTRIBUTION (1)

1. Many types of probability problems…
   i. have only two possible outcomes
   ii. they can be reduced to two outcomes.
2. Examples (1): when a coin is tossed it can land on heads or tails, when a baby is born it is either a boy or girl, etc.
3. Example (2): A multiple-choice question, even though there are four or five answer questions, can be reduced to two options: correct or incorrect.
4. Situations like these are called *binomial experiments*.

# THE BINOMIAL DISTRIBUTION (2)

5. The binomial experiment is a probability experiment that satisfies these requirements …
   i. There must be a fixed number of trials.
   ii. Each trial can have only two outcomes.
   iii. The outcomes of each trial must be independent of each other.
   iv. The probability of ******success* must remain the same for each trial.

# THE BINOMIAL DISTRIBUTION (3)

6. ***Success** – does not simple imply that something good or positive has occurred:
   - For example, in a probability experiment of selecting 10 people and let S represent the number of people who were in an automobile accident in the last 6 months.
   - In this case, a success would not be a positive or good thing.

# THE BINOMIAL DISTRIBUTION (4)

- Decide whether each experiment is a binomial experiment, If not, state the reason why.

1. Selecting 20 university students and recording their class rank.
2. Selecting 20 students from a university and recording their gender.
3. Selecting five students from a large school and asking them if they are on dean's list.
4. Recording the number of children in 50 randomly selected families.

# THE BINOMIAL DISTRIBUTION (5)

7. In binomial experiments, the outcomes are usually classified as successes or failures.
8. These outcomes and the their corresponding probabilities are called *binomial distribution.*

# THE BINOMIAL DISTRIBUTION (6)

| | |
|---|---|
| $P(S)$ | The symbol for the probability of success |
| $P(F)$ | The symbol for the probability of failure |
| $p$ | The numerical probability of success |
| $q$ | The numerical probability of failure |
| $P(S) = p$ | and $P(F) = 1 - p = q$ |
| $n$ | The number of trials |
| $X$ | The number of successes in *n trials* |

Note that $X = 0, 1, 2, 3, \ldots, n$

# THE BINOMIAL DISTRIBUTION (7)

9. In a binomial experiment, the probability of exactly $X$ successes in $n$ trials is:

$$P(X) = \frac{n!}{(n-X)!X!} \cdot p^X \cdot q^{n-X}$$

# EXAMPLE 5-16: TOSSING COINS

- A coin is tossed 3 times.
- Find the probability of getting exactly two heads.

# EXAMPLE 5-16: TOSSING COINS

- A coin is tossed 3 times.
- Find the probability of getting exactly two heads.

**Solution**

This problem can be solved by looking at the sample space. There are three ways to get two heads.

HHH, HHT, HTH, THH, TTH, THT, HTT, TTT

The answer is $\frac{3}{8}$, or 0.375.

# EXAMPLE 5-16: TOSSING COINS

- A coin is tossed 3 times.
- Find the probability of getting exactly two heads.

Looking at the problem in Example 5–15 from the standpoint of a binomial experiment, one can show that it meets the four requirements.

1. There are a fixed number of trials (three).
2. There are only two outcomes for each trial, heads or tails.
3. The outcomes are independent of one another (the outcome of one toss in no way affects the outcome of another toss).
4. The probability of a success (heads) is $\frac{1}{2}$ in each case.

In this case, $n = 3, X = 2, p = \frac{1}{2}$, and $q = \frac{1}{2}$. Hence, substituting in the formula gives

$$P(2 \text{ heads}) = \frac{3!}{(3-2)!2!} \cdot \left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^1 = \frac{3}{8} = 0.375$$

which is the same answer obtained by using the sample space.

# EXAMPLE 5-17: SURVEY ON DOCTOR VISITS

- A survey found that one out of five Americans say he or she has visited a doctor in any given month.
- If 10 people are selected at random, find the probability that exactly 3 will have visited a doctor last month.

# EXAMPLE 5-17: SURVEY ON DOCTOR VISITS

- A survey found that one out of five Americans say he or she has visited a doctor in any given month.
- If 10 people are selected at random, find the probability that exactly 3 will have visited a doctor last month.

$$P(X) = \frac{n!}{(n-X)!X!} \cdot p^X \cdot q^{n-X}$$

$$n = 10, \text{"one out of five"} \rightarrow p = \tfrac{1}{5}, X = 3$$

$$P(3) = \frac{10!}{7!3!} \cdot \left(\frac{1}{5}\right)^3 \cdot \left(\frac{4}{5}\right)^7 = 0.201$$

# EXAMPLE 5-18: SURVEY ON EMPLOYMENT

- A survey from Teenage Research Unlimited (Northbrook, Illinois) found that 30% of teenage consumers receive their spending money from part-time jobs.
- If 5 teenagers are selected at random, find the probability that at least 3 of them will have part-time jobs.

# EXAMPLE 5-18: SURVEY ON EMPLOYMENT

- A survey from Teenage Research Unlimited (Northbrook, Illinois) found that 30% of teenage consumers receive their spending money from part-time jobs.
- If 5 teenagers are selected at random, find the probability that at least 3 of them will have part-time jobs.

$$n = 5,\ p = 0.30, \text{"at least 3"} \rightarrow X = 3, 4, 5$$

$$P(3) = \frac{5!}{2!3!} \cdot (0.30)^3 \cdot (0.70)^2 = 0.132$$

$$P(4) = \frac{5!}{1!4!} \cdot (0.30)^4 \cdot (0.70)^1 = 0.028$$

$$P(5) = \frac{5!}{0!5!} \cdot (0.30)^5 \cdot (0.70)^0 = 0.002$$

$$P(X \geq 3) = 0.132 + 0.028 + 0.002 = 0.162$$

# GET READY!

**Please take out this distribution table:**

**Binomial or TABLE B**

# TABLE B

**Table B** The Binomial Distribution

| | | *p* | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ***n*** | ***x*** | **0.05** | **0.1** | **0.2** | **0.3** | **0.4** | **0.5** | **0.6** | **0.7** | **0.8** | **0.9** | **0.95** |
| 2 | 0 | 0.902 | 0.810 | 0.640 | 0.490 | 0.360 | 0.250 | 0.160 | 0.090 | 0.040 | 0.010 | 0.002 |
| | 1 | 0.095 | 0.180 | 0.320 | 0.420 | 0.480 | 0.500 | 0.480 | 0.420 | 0.320 | 0.180 | 0.095 |
| | 2 | 0.002 | 0.010 | 0.040 | 0.090 | 0.160 | 0.250 | 0.360 | 0.490 | 0.640 | 0.810 | 0.902 |
| 3 | 0 | 0.857 | 0.729 | 0.512 | 0.343 | 0.216 | 0.125 | 0.064 | 0.027 | 0.008 | 0.001 | |
| | 1 | 0.135 | 0.243 | 0.384 | 0.441 | 0.432 | 0.375 | 0.288 | 0.189 | 0.096 | 0.027 | 0.007 |
| | 2 | 0.007 | 0.027 | 0.096 | 0.189 | 0.288 | 0.375 | 0.432 | 0.441 | 0.384 | 0.243 | 0.135 |
| | 3 | | 0.001 | 0.008 | 0.027 | 0.064 | 0.125 | 0.216 | 0.343 | 0.512 | 0.729 | 0.857 |
| 4 | 0 | 0.815 | 0.656 | 0.410 | 0.240 | 0.130 | 0.062 | 0.026 | 0.008 | 0.002 | | |
| | 1 | 0.171 | 0.292 | 0.410 | 0.412 | 0.346 | 0.250 | 0.154 | 0.076 | 0.026 | 0.004 | |
| | 2 | 0.014 | 0.049 | 0.154 | 0.265 | 0.346 | 0.375 | 0.346 | 0.265 | 0.154 | 0.049 | 0.014 |
| | 3 | | 0.004 | 0.026 | 0.076 | 0.154 | 0.250 | 0.346 | 0.412 | 0.410 | 0.292 | 0.171 |
| | 4 | | | 0.002 | 0.008 | 0.026 | 0.062 | 0.130 | 0.240 | 0.410 | 0.656 | 0.815 |
| 5 | 0 | 0.774 | 0.590 | 0.328 | 0.168 | 0.078 | 0.031 | 0.010 | 0.002 | | | |
| | 1 | 0.204 | 0.328 | 0.410 | 0.360 | 0.259 | 0.156 | 0.077 | 0.028 | 0.006 | | |
| | 2 | 0.021 | 0.073 | 0.205 | 0.309 | 0.346 | 0.312 | 0.230 | 0.132 | 0.051 | 0.008 | 0.001 |
| | 3 | 0.001 | 0.008 | 0.051 | 0.132 | 0.230 | 0.312 | 0.346 | 0.309 | 0.205 | 0.073 | 0.021 |
| | 4 | | | 0.006 | 0.028 | 0.077 | 0.156 | 0.259 | 0.360 | 0.410 | 0.328 | 0.204 |

**6 pages**

# EXAMPLE 5-19: TOSSING COINS

- A coin is tossed 3 times.
- Find the probability of getting exactly two heads, using Table B.

# EXAMPLE 5-19: TOSSING COINS

- A coin is tossed 3 times.
- Find the probability of getting exactly two heads, using Table B.

$$n = 3, p = \tfrac{1}{2} = 0.5, X = 2 \quad \rightarrow P(2) = \boxed{0.375}$$

$p = 0.5$

| | | | | | | | $p$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $n$ | $X$ | 0.05 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 0.95 |
| 2 | 0 | | | | | | | | | | | |
| | 1 | | | | | | | | | | | |
| | 2 | | | | | | | | | | | |
| 3 | 0 | | | | | | 0.125 | | | | | |
| $n = 3$ | 1 | | | | | | 0.375 | | | | | |
| | 2 | | | | | | 0.375 | | | | | |
| $X = 2$ | 3 | | | | | | 0.125 | | | | | |

# THE BINOMIAL DISTRIBUTION (8)

10. The mean, variance, and standard deviation of a variable that has the *binomial distribution* can be found by using the following formulas:

$$\text{Mean: } \mu = np$$

$$\text{Variance: } \sigma^2 = npq$$

$$\text{Standard Deviation: } \sigma = \sqrt{npq}$$

# EXAMPLE 5-22: TOSSING A COIN

- A coin is tossed 4 times.
- Find the mean, variance, and standard deviation of the number of heads that will be obtained.

# EXAMPLE 5-22: TOSSING A COIN

- A coin is tossed 4 times.
- Find the mean, variance, and standard deviation of the number of heads that will be obtained.

**Solution** 1

With the formulas for the binomial distribution and $n = 4, p = \frac{1}{2}$, and $q = \frac{1}{2}$, the results are

$$\mu = n \cdot p = 4 \cdot \tfrac{1}{2} = 2$$
$$\sigma^2 = n \cdot p \cdot q = 4 \cdot \tfrac{1}{2} \cdot \tfrac{1}{2} = 1$$
$$\sigma = \sqrt{1} = 1$$

# EXAMPLE 5-22: TOSSING A COIN

- A coin is tossed 4 times.
- Find the mean, variance, and standard deviation of the number of heads that will be obtained.

**Solution 2 :**

Formulas for expected value can also be used. The distribution is shown below:

| No. of heads $X$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Probability $P(X)$ | $\frac{1}{16}$ | $\frac{4}{16}$ | $\frac{6}{16}$ | $\frac{4}{16}$ | $\frac{1}{16}$ |

$$\mu = E(X) = \Sigma X \cdot P(X) = 0 \cdot \tfrac{1}{16} + 1 \cdot \tfrac{4}{16} + 2 \cdot \tfrac{6}{16} + 3 \cdot \tfrac{4}{16} + 4 \cdot \tfrac{1}{16} = \tfrac{32}{16} = 2$$

$$\sigma^2 = \Sigma X^2 \cdot P(X) - \mu^2$$

$$= 0^2 \cdot \tfrac{1}{16} + 1^2 \cdot \tfrac{4}{16} + 2^2 \cdot \tfrac{6}{16} + 3^2 \cdot \tfrac{4}{16} + 4^2 \cdot \tfrac{1}{16} - 2^2 = \tfrac{80}{16} - 4 = 1$$

$$\sigma = \sqrt{1} = 1$$

Hence, the simplified binomial formulas give the same results.

# 5-3 EXERCISES

**2.** Which of the following are binomial experiments or can be reduced to binomial experiments?

*a.* Testing one brand of aspirin by using 10 people to determine whether it is effective

*b.* Asking 100 people if they smoke

*c.* Checking 1000 applicants to see whether they were admitted to White Oak College

*d.* Surveying 300 prisoners to see how many different crimes they were convicted of

*e.* Surveying 300 prisoners to see whether this is their first offense

# 5-3 EXERCISES (CONT'D.)

**4.** Compute the probability of $X$ successes, using Table B in Appendix A.

*a.* $n = 15, p = 0.80, X = 12$

*b.* $n = 17, p = 0.05, X = 0$

*c.* $n = 20, p = 0.50, X = 10$

*d.* $n = 16, p = 0.20, X = 3$

# 5-3 EXERCISES (CONT'D.)

**6.** Compute the probability of $X$ successes, using the binomial formula.

*a.* $n = 9, X = 0, p = 0.42$

*b.* $n = 10, X = 5, p = 0.37$

# 5-3 EXERCISES (CONT'D.)

*For Exercises 7 through 16, assume all variables are binomial. (*Note: *If values are not found in Table B of Appendix A, use the binomial formula.)*

**9. Driving to Work Alone** It is reported that 77% of workers aged 16 and over drive to work alone. Choose 8 workers at random. Find the probability that

*a.* All drive to work alone

*b.* More than one-half drive to work alone

*c.* Exactly 3 drive to work alone

Source: www.factfinder.census.gov

# 5-3 EXERCISES (CONT'D.)

*For Exercises 7 through 16, assume all variables are binomial. (*Note: *If values are not found in Table B of Appendix A, use the binomial formula.)*

**11. Survey on Concern for Criminals** In a survey, 3 of 4 students said the courts show "too much concern" for criminals. Find the probability that at most 3 out of 7 randomly selected students will agree with this statement.

Source: *Harper's Index.*

# 5-3 EXERCISES (CONT'D.)

**17.** Find the mean, variance, and standard deviation for each of the values of $n$ and $p$ when the conditions for the binomial distribution are met.

*a.* $n = 100, p = 0.75$
*b.* $n = 300, p = 0.3$
*c.* $n = 20, p = 0.5$
*d.* $n = 10, p = 0.8$

# 5-3 EXERCISES (CONT'D.)

**31. Survey of High School Seniors** Of graduating high school seniors, 14% said that their generation will be remembered for their social concerns. If 7 graduating seniors are selected at random, find the probability that either 2 or 3 will agree with that statement.

Source: *USA TODAY.*

# 5.4 Other Types of Distributions

# OTHER TYPES OF DISTRIBUTIONS

1. Multinomial distribution
2. Poisson distribution
3. Hypergeometric distribution
4. Geometric distribution

# MULTINOMIAL DISTRIBUTION (1)

1. The multinomial distribution is similar to the binomial distribution but…
   - has the advantage of allowing one to compute probabilities when there are more than two outcomes.

2. The binomial distribution is a special case of the multinomial distribution.

$$P(X) = \frac{n!}{X_1!\,X_2!\,X_3!\cdots X_k!} \cdot p_1^{X_1} \cdot p_2^{X_2} \cdot p_3^{X_3} \cdots p_k^{X_k}$$

# MULTINOMIAL DISTRIBUTION (2)

3. The multinomial distribution is a probability experiment that satisfies these requirements:
   i. There must be a fixed number of trials.
   ii. Each trial has a specific – but not necessarily the same – numbers of outcomes.
   iii. The trials are independent.
   iv. The probability of a particular outcome remains the same.

# EXAMPLE 5-25: LEISURE ACTIVITIES

- In a large city, 50% of the people choose a movie, 30% choose dinner and a play, and 20% choose shopping as a leisure activity.
- If a sample of 5 people is randomly selected, find the probability that 3 are planning to go to a movie, 1 to a play, and 1 to a shopping mall.

# EXAMPLE 5-25: LEISURE ACTIVITIES

- In a large city, 50% of the people choose a movie, 30% choose dinner and a play, and 20% choose shopping as a leisure activity.
- If a sample of 5 people is randomly selected, find the probability that 3 are planning to go to a movie, 1 to a play, and 1 to a shopping mall.

Solution: It will be good to use the following table.

$n = 5$

| **Event (*E*)** | **Movie** | **Dinner & Play** | **Shopping** |
|---|---|---|---|
| **Probability (*P*)** | **0.5** | **0.3** | **0.2** |
| **The no. of occurrences (*X*)** | **3** | **1** | **1** |

# EXAMPLE 5-25: LEISURE ACTIVITIES

Solution: It will be good to use the following table.

$n = 5$

| **Event ($E$)** | **Movie** | **Dinner & Play** | **Shopping** |
|---|---|---|---|
| **Probability ($P$)** | **0.5** | **0.3** | **0.2** |
| **The no. of occurrences ($X$)** | **3** | **1** | **1** |

Then put the values inside the formula.

$$P(X) = \frac{n!}{X_1!X_2!X_3!\cdots X_k!} \cdot p_1^{X_1} \cdot p_2^{X_2} \cdot p_3^{X_3} \cdots p_k^{X_k}$$

$$P(X) = \frac{5!}{3!1!1!} \cdot (0.50)^3 (0.30)^1 (0.20)^1 = \boxed{0.15}$$

# POISSON DISTRIBUTION (1)

1. The Poisson distribution is a distribution useful when…
   - *n* is large and *p* is small and
   - when the independent variables occur over a period of time.
2. The Poisson distribution can also be used when…
   - a density of items is distributed over a given area or volume,
   - such as the number of plants growing per acre or
   - the number of defects in a given length of videotape.

# POISSON DISTRIBUTION (2)

3. The Poisson distribution is a probability experiment that satisfies these requirements:
   i. There random variable X is the number of occurrence of an event over some interval (i.e., length, area, volume, period of time, etc.).
   ii. The occurrences occur randomly.
   iii. The occurrences are independent of one another.
   iv. The average number of occurrences over an interval is known.

# GET READY!

**Please take out this distribution table:**

**Poisson or TABLE C**

# TABLE C

**Table C** The Poisson Distribution

| | λ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $x$ | **0.1** | **0.2** | **0.3** | **0.4** | **0.5** | **0.6** | **0.7** | **0.8** | **0.9** | **1.0** |
| 0 | .9048 | .8187 | .7408 | .6703 | .6065 | .5488 | .4966 | .4493 | .4066 | .3679 |
| 1 | .0905 | .1637 | .2222 | .2681 | .3033 | .3293 | .3476 | .3595 | .3659 | .3679 |
| 2 | .0045 | .0164 | .0333 | .0536 | .0758 | .0988 | .1217 | .1438 | .1647 | .1839 |
| 3 | .0002 | .0011 | .0033 | .0072 | .0126 | .0198 | .0284 | .0383 | .0494 | .0613 |
| 4 | .0000 | .0001 | .0003 | .0007 | .0016 | .0030 | .0050 | .0077 | .0111 | .0153 |
| 5 | .0000 | .0000 | .0000 | .0001 | .0002 | .0004 | .0007 | .0012 | .0020 | .0031 |
| 6 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0001 | .0002 | .0003 | .0005 |
| 7 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0001 |
| | λ | | | | | | | | | |
| $x$ | **1.1** | **1.2** | **1.3** | **1.4** | **1.5** | **1.6** | **1.7** | **1.8** | **1.9** | **2.0** |
| 0 | .3329 | .3012 | .2725 | .2466 | .2231 | .2019 | .1827 | .1653 | .1496 | .1353 |
| 1 | .3662 | .3614 | .3543 | .3452 | .3347 | .3230 | .3106 | .2975 | .2842 | .2707 |
| 2 | .2014 | .2169 | .2303 | .2417 | .2510 | .2584 | .2640 | .2678 | .2700 | .2707 |
| 3 | .0738 | .0867 | .0998 | .1128 | .1255 | .1378 | .1496 | .1607 | .1710 | .1804 |
| 4 | .0203 | .0260 | .0324 | .0395 | .0471 | .0551 | .0636 | .0723 | .0812 | .0902 |
| 5 | .0045 | .0062 | .0084 | .0111 | .0141 | .0176 | .0216 | .0260 | .0309 | .0361 |

•
•
•

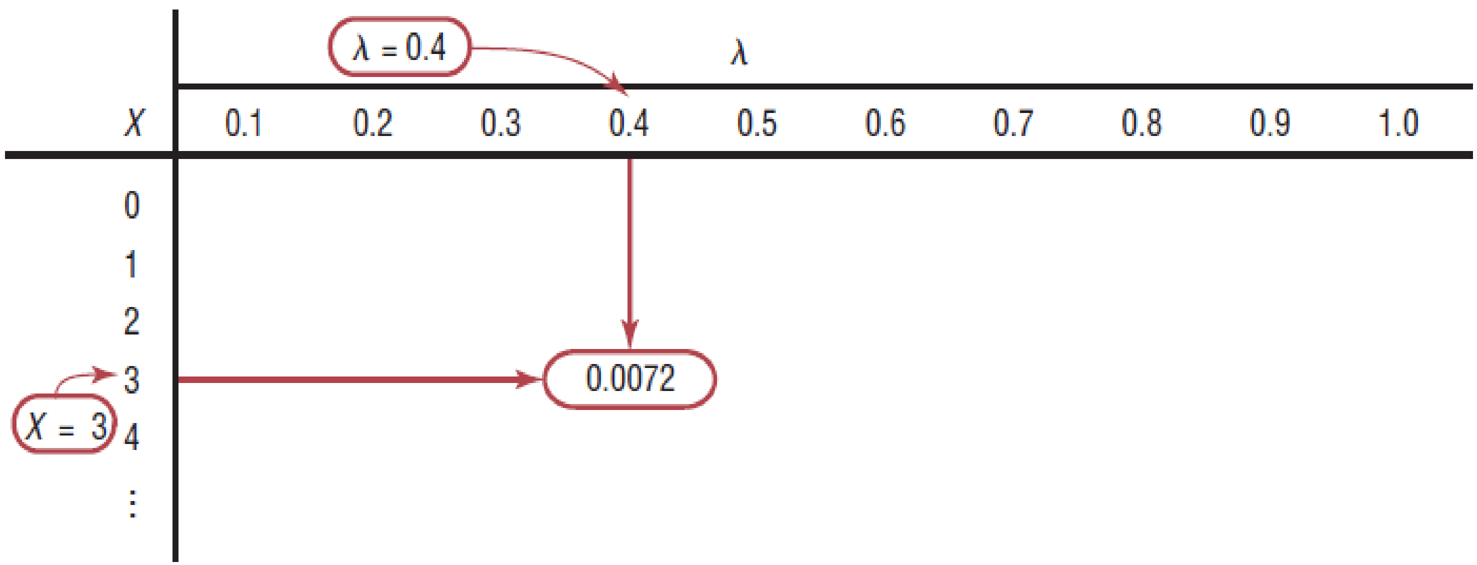
**7 pages**

# POISSON DISTRIBUTION (3)

4. Since the mathematics involved in computing Poisson probabilities is somewhat complicated, tables have been compiled for these probabilities.
5. Table C in Appendix A gives P for various values for λ and X.
6. In next example, where X is 3 and λ is 0.4, the table gives the value 0.0072 for the probability (P).

# POISSON DISTRIBUTION (4)

7. In the example of X is 3 and λ is 0.4, the table gives the value 0.0072 for the probability (P).



# EXAMPLE 5-27: TYPOGRAPHICAL ERRORS

- If there are 200 typographical errors randomly distributed in a 500-page manuscript,
- find the probability that a given page contains exactly 3 errors.

# EXAMPLE 5-27: TYPOGRAPHICAL ERRORS

- If there are 200 typographical errors randomly distributed in a 500-page manuscript,
- find the probability that a given page contains exactly 3 errors.

Solution:

First, find the mean number λ of errors. With 200 errors distributed over 500 pages, each page has an average of $\lambda = \frac{200}{500} = 0.4$ errors per page.

$$P(X;\lambda) = \frac{e^{-\lambda}\lambda^{X}}{X!} = \frac{e^{-0.4}(0.4)^{3}}{3!} = 0.0072$$

**Thus, there is less than 1% chance that any given page will contain exactly 3 errors.**

# EXAMPLE 5-30: LEFT-HANDED PEOPLE

- If approximately 2% of the people in a room of 200 people are left-handed,
- find the probability that exactly 5 people there are left-handed.

# EXAMPLE 5-30: LEFT-HANDED PEOPLE

- If approximately 2% of the people in a room of 200 people are left-handed,
- find the probability that exactly 5 people there are left-handed.

**Solution**

Since $\lambda = n \cdot p$, then $\lambda = (200)(0.02) = 4$. Hence,

$$P(X; \lambda) = \frac{(2.7183)^{-4}(4)^5}{5!} = 0.1563$$

which is verified by the formula ${}_{200}C_5(0.02)^5(0.98)^{195} \approx 0.1579$. The difference between the two answers is based on the fact that the Poisson distribution is an approximation and rounding has been used.

# HYPERGEOMETRIC DISTRIBUTION (1)

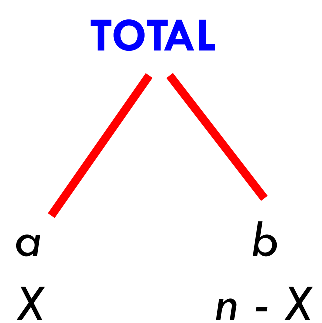
1. The hypergeometric distribution is a distribution of a variable that has…
   - two outcomes when…
   - sampling is done without replacement.

# HYPERGEOMETRIC DISTRIBUTION (2)

2. Given a population with only two types of objects (females and males, defective and nondefective, successes and failures, etc.),

3. such that there are:
   - ***a*** items of one kind
   - and ***b*** items of another kind
   - and ***a* + *b*** equals the total population,
   - the probability ***P(X)*** of selecting without replacement a sample of size ***n*** with ***X*** items of type ***a*** and ***n* – *X*** items of type ***b*** *is*

$$P(X) = \frac{{}_{a}C_{X} \cdot {}_{b}C_{n-X}}{{}_{a+b}C_{n}}$$



# HYPERGEOMETRIC DISTRIBUTION (3)

4. The Hypergeometric distribution is a probability experiment that satisfies these requirements:
   i. There are a fixed number of trial.
   ii. There are two outcomes, and they can be classified as success or failure.
   iii. The sample is selected without replacement.

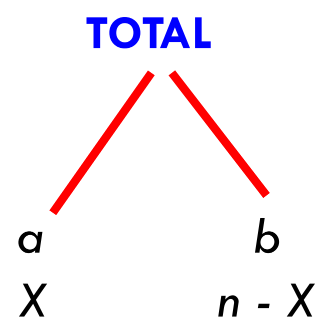
# EXAMPLE 5-32: HOUSE INSURANCE

- A recent study found that 2 out of every 10 houses in a neighborhood have no insurance.
- If 5 houses are selected from 10 houses, find the probability that exactly 1 will be uninsured.

# EXAMPLE 5-32: HOUSE INSURANCE

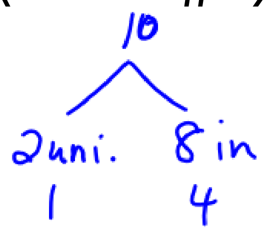
- A recent study found that 2 out of every 10 houses in a neighborhood have no insurance.
- If 5 houses are selected from 10 houses, find the probability that exactly 1 will be uninsured.

$$a = 2, a + b = 10 \rightarrow b = 8, \quad X = 1, n = 5 \rightarrow n - X = 4$$




$$P(X) = \frac{{}_{a}C_{X} \cdot {}_{b}C_{n-X}}{{}_{a+b}C_{n}}$$

$$P(X) = \frac{{}_{2}C_{1} \cdot {}_{8}C_{4}}{{}_{10}C_{5}} = \frac{2 \cdot 70}{252} = \frac{140}{252} = \boxed{\frac{5}{9}}$$



# EXAMPLE 5-33: DEFECTIVE COMPRESSOR TANKS

A lot of 12 compressor tanks is checked to see whether there are any defective tanks. Three tanks are checked for leaks. If 1 or more of the 3 is defective, the lot is rejected. Find the probability that the lot will be rejected if there are actually 3 defective tanks in the lot.

# EXAMPLE 5-33: DEFECTIVE COMPRESSOR TANKS

A lot of 12 compressor tanks is checked to see whether there are any defective tanks. Three tanks are checked for leaks. If 1 or more of the 3 is defective, the lot is rejected. Find the probability that the lot will be rejected if there are actually 3 defective tanks in the lot.

**Solution**

Since the lot is rejected if at least 1 tank is found to be defective, it is necessary to find the probability that none are defective and subtract this probability from 1.

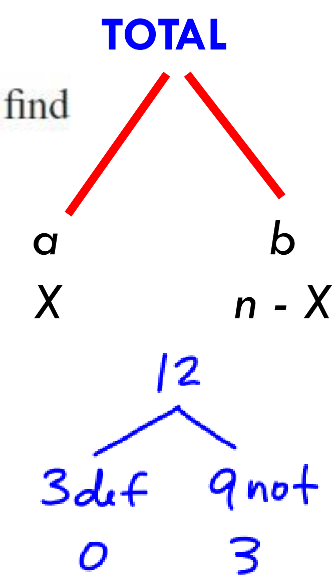
Here, $a = 3$, $b = 9$, $n = 3$, and $X = 0$; so

$$P(X) = \frac{{}_3C_0 \cdot {}_9C_3}{{}_{12}C_3} = \frac{1 \cdot 84}{220} = 0.38$$

Hence,

$$P(\text{at least 1 defective}) = 1 - P(\text{no defectives}) = 1 - 0.38 = 0.62$$

There is a 0.62, or 62%, probability that the lot will be rejected when 3 of the 12 tanks are defective.

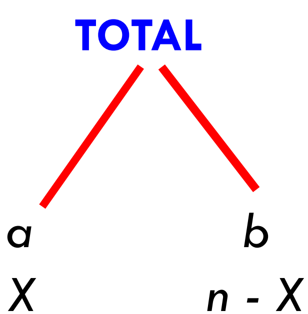


# ADDITIONAL EXAMPLE (1)

- Ten people apply for a job as assistant manager of a restaurant. Five have completed college and five have not.
- If the manager selects 3 applicants at random, find the probability that all 3 are college graduate.

# ADDITIONAL EXAMPLE (2)

- Ten people apply for a job as assistant manager of a restaurant. Five have completed college and five have not.
- If the manager selects 3 applicants at random, find the probability that all 3 are college graduate.




10

5g 5not

3 0

$$P(3\text{grads}) = \frac{{}_5C_3 \cdot {}_5C_0}{{}_{10}C_3}$$

$$= \frac{10}{120}$$

$$= 8.3\%$$

# GEOMETRIC DISTRIBUTIONS (1)

1. This distribution can be used when we have an experiment that has…
   - two outcomes and
   - is repeated until a successful outcome is obtained.

# GEOMETRIC DISTRIBUTIONS (2)

2. If p is the probability of a success on each trial of a binomial experiment
   - and n is the number of the trial at which the first success occurs,
   - then the probability of getting the first success on the nth trial is:

$$P(n) = p(1 - p)^{n-1}$$

Where n is 1, 2, 3…

# GEOMETRIC DISTRIBUTIONS (3)

3. The geometric distribution is a probability experiment that satisfies these requirements:
    i. Each trial has two outcomes that can be either success or failure.
    ii. The outcomes are independent or each other.
    iii. The probability of a success is the same for each trial.
    iv. The experiment continues until a success is obtained.

# EXAMPLE 5-34: TOSSING COINS

- A coin is tossed.
- Find the probability of getting the first head on the third toss.

# EXAMPLE 5-34: TOSSING COINS

- A coin is tossed.
- Find the probability of getting the first head on the third toss.

**Solution:** 1

The objective for tossing a coin and getting a head on the third toss is TTH. The probability for this outcome is:

$$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)=\frac{1}{8}$$

# EXAMPLE 5-34: TOSSING COINS

- A coin is tossed.
- Find the probability of getting the first head on the third toss.

**Solution:** 2

Now by using the formula, you can get the same results:

$$P(n) = p(1-p)^{n-1} = \frac{1}{2}\left(1 - \frac{1}{2}\right)^{3-1} = \frac{1}{2}\left(\frac{1}{2}\right)^{2} = \frac{1}{8}$$

**There is a 1 out of 8 chance or 0.125 probability of getting the first head on the third toss of a coin.**

# 5-4 EXERCISES

**3. M&M's Color Distribution** According to the manufacturer, M&M's are produced and distributed in the following proportions: 13% brown, 13% red, 14% yellow, 16% green, 20% orange, and 24% blue. In a random sample of 12 M&M's, what is the probability of having 2 of each color?

# 5-4 EXERCISES (CONT'D.)

**9. Study of Robberies** A recent study of robberies for a certain geographic region showed an average of 1 robbery per 20,000 people. In a city of 80,000 people, find the probability of the following.

*a.* 0 robberies

*b.* 1 robbery

*c.* 2 robberies

*d.* 3 or more robberies

# 5-4 EXERCISES (CONT'D.)

**10. Misprints on Manuscript Pages** In a 400-page manuscript, there are 200 randomly distributed misprints. If a page is selected, find the probability that it has 1 misprint.

# 5-4 EXERCISES (CONT'D.)

22. **Job Applications** Ten people apply for a job at Computer Warehouse. Five are college graduates and five are not. If the manager selects 3 applicants at random, find the probability that all 3 are college graduates.

# 5-4 EXERCISES (CONT'D.)

**25. Shooting an Arrow** Mark shoots arrows at a target and hits the bull's-eye about 40% of the time. Find the probability that he hits the bull's-eye on the third shot.

# IMPORTANT FORMULAS

Formula for the mean of a probability distribution:

$$\mu = \Sigma X \cdot P(X)$$

Formulas for the variance and standard deviation of a probability distribution:

$$\sigma^2 = \Sigma[X^2 \cdot P(X)] - \mu^2$$
$$\sigma = \sqrt{\Sigma[X^2 \cdot P(X)] - \mu^2}$$

Formula for expected value:

$$E(X) = \Sigma X \cdot P(X)$$

Formula for the multinomial distribution:

$$P(X) = \frac{n!}{X_1! \cdot X_2! \cdot X_3! \cdot \cdots \cdot X_k!} \cdot p_1^{X_1} \cdot p_2^{X_2} \cdot \cdots \cdot p_k^{X_k}$$

(The $X$s sum to $n$ and the $p$s sum to one)

Binomial probability formula:

$$P(X) = \frac{n!}{(n - X)!X!} \cdot p^X \cdot q^{n-X} \quad \text{where } X = 0, 1, 2, 3, \ldots n$$

Formula for the mean of the binomial distribution:

$$\mu = n \cdot p$$

Formulas for the variance and standard deviation of the binomial distribution:

$$\sigma^2 = n \cdot p \cdot q \qquad \sigma = \sqrt{n \cdot p \cdot q}$$

Formula for the Poisson distribution:

$$P(X; \lambda) = \frac{e^{-\lambda}\lambda^X}{X!} \qquad \text{where } X = 0, 1, 2, \ldots$$

Formula for the hypergeometric distribution:

$$P(X) = \frac{{}_aC_X \cdot {}_bC_{n-X}}{{}_{a+b}C_n}$$