Electromagnetism

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Notes taken in Professor Lunghi's class, Hilary Term 2025

"If you can't explain it simply enough you don't understand it well enough"

\- Albert Einstein

Contents

1 Lecture 1

1.1 Principle of charge conservation:

The total charge of a closed system is conserved.
If charge cannot be created or destroyed, then charging a material requires a transfer.

1.2 Motion of electric charges across materials:

Conductors: Substances within which charge is (almost) completely free to move around.
Insulators: Substances within which charge cannot move around.
Semiconductors: Control the mobility of charge by changing temperature and by ”doping”.

1.3 The electric charge: SI Units:

Definition of Coulomb (C): The Coulomb is the quantity of electric charge carried in 1 second by a current of 1 ampere. **(for reference the charge of 1 electron is** $1.602 \cdot 10^{-19}$**C)**.
Definition of Ampere (A): The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed one meter apart in vacuum, would produce between conductors a force equal to $2 \cdot 10^{-7}$newtons per meter of length.

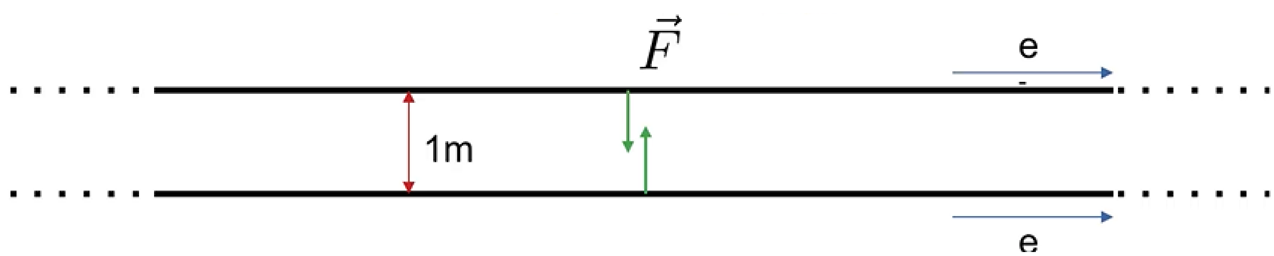


Figure 1: conductor of infinite length, of negligible circular cross-section, placed one meter apart in vacuum, producing a force.

2 Lecture 2

2.1 The Coulomb's Equation

The electric force between two point charges:
- The magnitude of the force between two point charges (F) is inversely proportional to the square of the distance between them (r).

$$|F| \propto \frac{1}{r^2} \tag{2.1}$$

- It is also proportional to the product of the individual charges of the particles (q)

$$|F| \propto |q_1 q_2| \tag{2.2}$$

$$\Rightarrow |F| \propto \frac{|q_1 q_2|}{r^2} \tag{2.3}$$

- The force direction lies along the line joining the two charges

$$\vec{F_{12}} \propto \frac{\hat{r}_{12}}{r^2} \tag{2.4}$$

"The force acting on 1 due to the presence of 2"

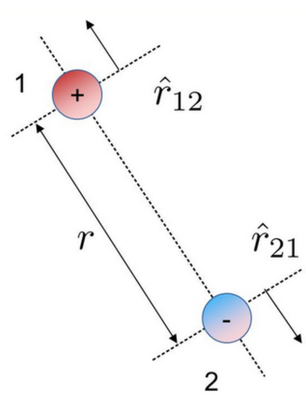


Figure 2: Two point charges acting on one another at a distance r

- Charges of equal sign repel each other and charges of different sign attract

$$\vec{F}_{12} = k\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12} \tag{2.5}$$

Units

- Force: newton (N) or (kg m $/s^2$)
- Distance: meter (m)
- Charge: coulomb (C)

Units of k must be Nm^2C^{-2} or $kg\ m^3\ C^{-2}\ s^{-2}$
$k \approx 8.987551 \cdot 10^9\ kg\ m^3\ s^{-2}\ C^{-2}$

Alternative definitions of k:
$k = \frac{1}{4\pi\epsilon_0}$, where $\epsilon_0 = 8.854 \cdot 10^{-12}C^2N^{-1}m^{-2}$ is known as the Vacuum dielectric permittivity.
$k = \frac{\mu_0}{4\pi}c^2$, where $\mu_0 = 1.257 \cdot 10^{-6}NC^{-2}s^2$ is known as magnetic permeability of vacuum.

2.2 Superposition Principle

The effect of each charge adds up according to vectors' algebra

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} \tag{2.6}$$

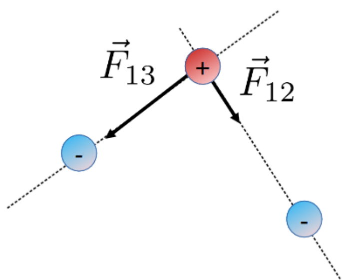


Figure 3: The superposition principle highlighting how a charge adds up according to vectors' algebra

2.3 Exploiting Symmetry

The symmetry of the forces must be coherent with the symmetry of the physical system.

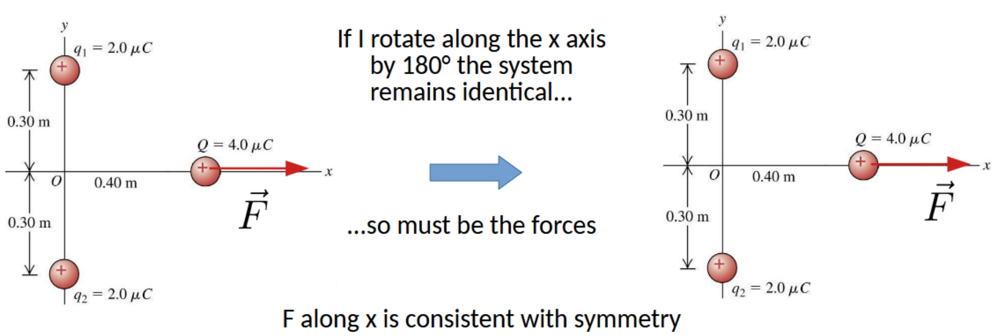


Figure 4: The system is coherent

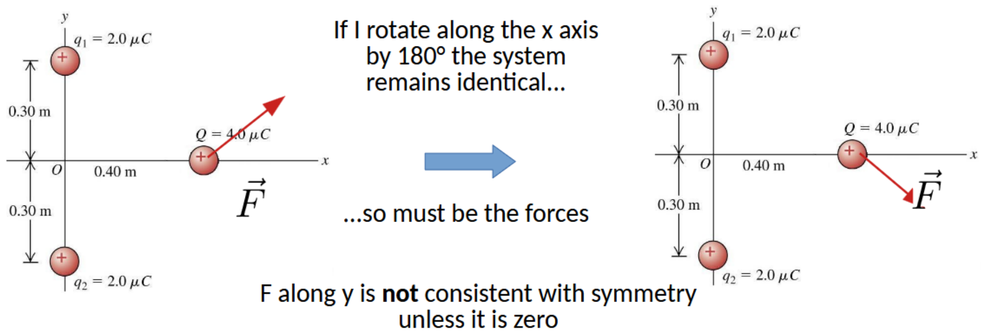
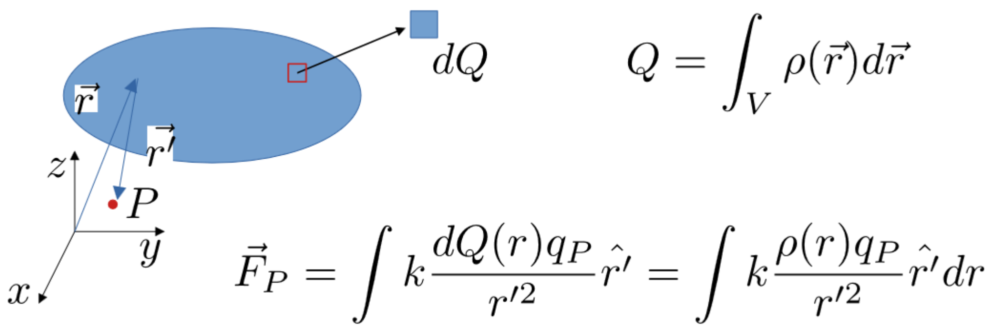


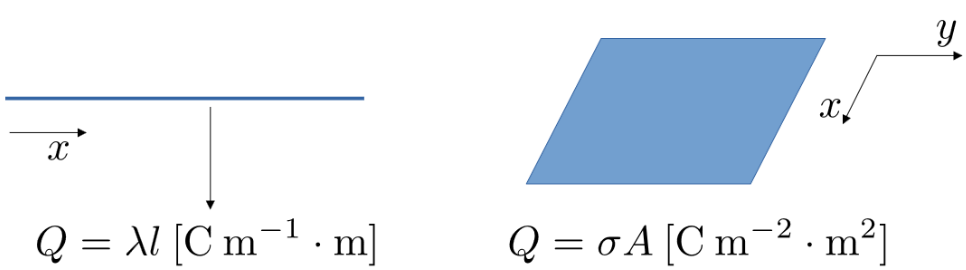
Figure 5: The system is not coherent

2.4 Complex Geometries

When we are dealing with a complex structure we can always reduce it to a sum(integral) of infinitesimal amounts of point charges.
For regular geometries and constant charge densities the problem can be simplified to



$$Q = \int_V \rho(\vec{r})d\vec{r} = \rho\int_V d\vec{r} = \rho V \tag{2.7}$$



3 Lecture 3

3.1 Fields as mathematical objects

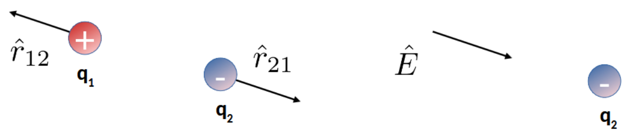
For our purposes a field is a function $f : R^d \to R^n$

Example 1: A temperature map

- d=2: surface of the globe.
- n=1: for each point $\vec{p} = (x, y)$ of the surface we have a scalar number associated, i.e. the temperature $(T = f(x, y))$

3.2 The Electric field

The electric field $\vec{E}$ is a property of space due to the presence of q_2. I still need a charge q_1 to sense its presence. The **Electric Field** created by A at point B is given by: $\vec{E} = \frac{\vec{F_0}}{q_0}$.



$$\vec{F_1} = k\frac{q_1 q_2}{r^2} r_{12} = \left(-k\frac{|q_2|}{r^2}\hat{r}_{12}\right) q_1 = \vec{E} q_1$$

It has units Newtons per Coulomb (N/C) and it its a **vector** quantity.

3.3 Electric Field Lines

- These are imaginary lines such that the direction of the electric field is a tangent to the electric field line.
- Field lines orientation is given by the electric field direction.
- The electric field always has only one direction so field lines never cross.
- Field lines always end up at a charge of at infinity.
- Field lines are drawn closer together where the field is stronger.
- Field lines are NOT trajectories of (charged) particles.

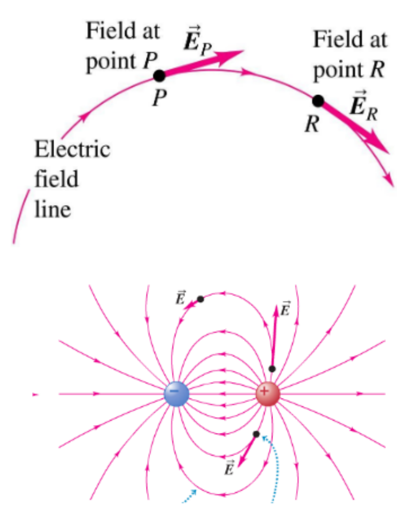
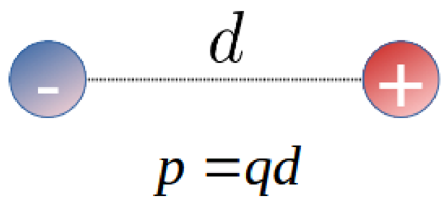


Figure 6: Demonstration of Electric Field Lines.

4 Lecture 4

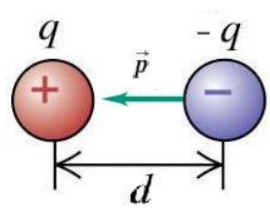
4.1 The Electric Dipole

An electric dipole **p** is defined as two charges of opposite sign **q** separated by distance **d**.



Assume we have an electric dipole with charges q and $-q$ and separated by a distance d.
The electric dipole moment p is defined: $p = qd$
The electric dipole moment vector is defined to have a magnitude p and direction from the negative charge to the positive charge.

$$\vec{E} = \frac{1}{4\pi\epsilon_0}(\frac{-\vec{p}}{r^3} + \frac{3(\vec{p}\cdot\vec{r})\vec{r}}{r^5} \tag{4.1}$$



4.2 Dipolar vs exact field for two charges

The dipolar field has been obtained by making the approximation r¿¿d, hence it is only a good description of the electric field at long distances from the charged object.

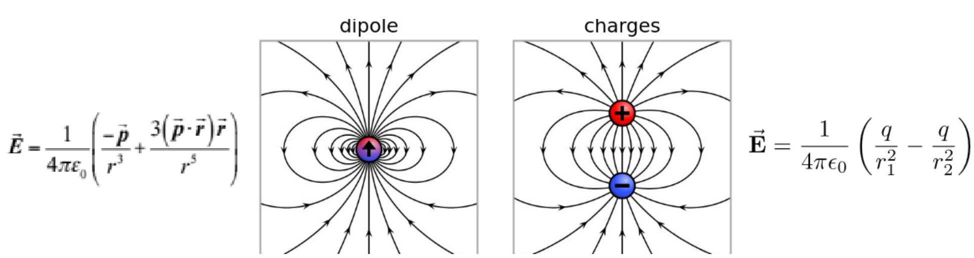
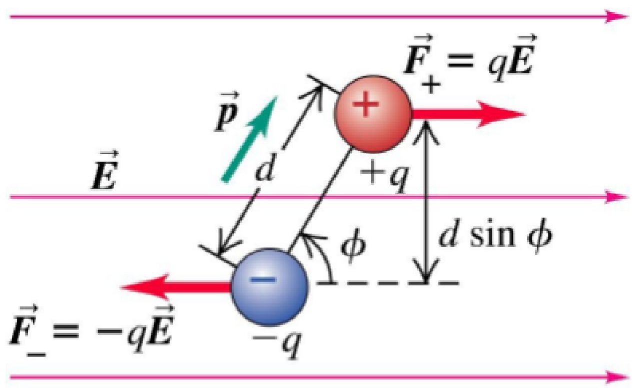


Figure 7: Comparison of a dipole electric field vs an exact field of two charges.

The introduction of the dipole approximation is however extremely important because virtually all charge distributions behave this way at long distances.
A dipole moment in a uniform electric field experiences a torque.
Each particle experiences a torque wrt to the middle-point of the dipole. $\vec{\tau} = \vec{r} \times \vec{F}$. which gives $|\vec{\tau}| = qE\frac{d}{2}\sin\varphi = \frac{pE}{2}\sin\varphi$.

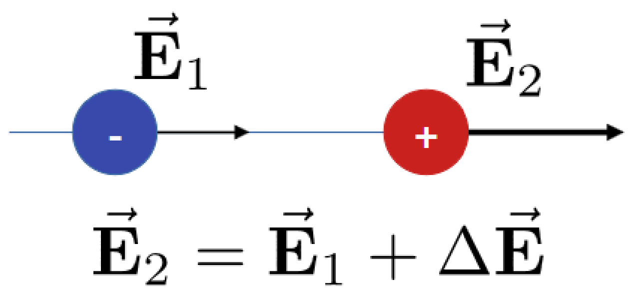
The total toque therefore is:

$$\vec{\tau}_{tot} = -qE\frac{d}{2}\sin(\phi)\hat{z} - qE\frac{d}{2}\sin(\phi)\hat{z} = -|p||E|\sin(\phi)\hat{z} \tag{4.2}$$



A dipole moment in a non-uniform electric field experience a net force
$\vec{F} = q_1\vec{E}_1 + q_2\vec{E}_2$
$\vec{F} = q(\vec{E}_1 + \Delta\vec{E}) - q\vec{E}_1 = q\Delta\vec{E}$
$\Delta\vec{E} \approx d\frac{d\vec{E}}{dx}$, which gives $\vec{F} = qd\frac{d\vec{E}}{dx}$.

$$\vec{F} = p\frac{d\vec{E}}{dx} \tag{4.3}$$



5 Lecture 5

5.1 The Gauss' Law

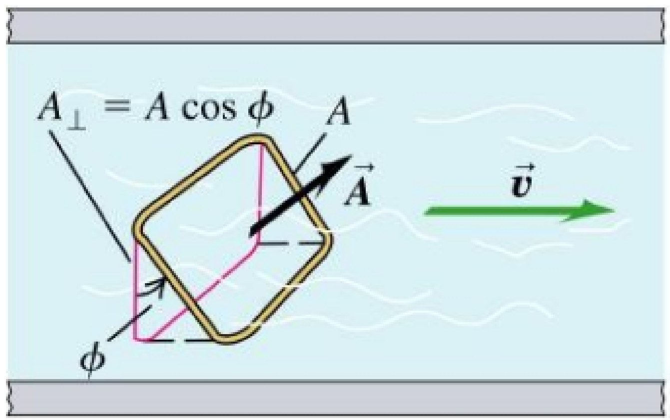
$$\Phi_E(A) = \oint_A \vec{E} \cdot \hat{n} dA = 4\pi k_e Q_{incl} \tag{5.1}$$

The Gauss' Law can be used either to determine the field given by the charge distribution or vice-verse. The surface A can be chosen arbitrarily, meaning that we can fully exploit the system's symmetry.

5.2 Flux

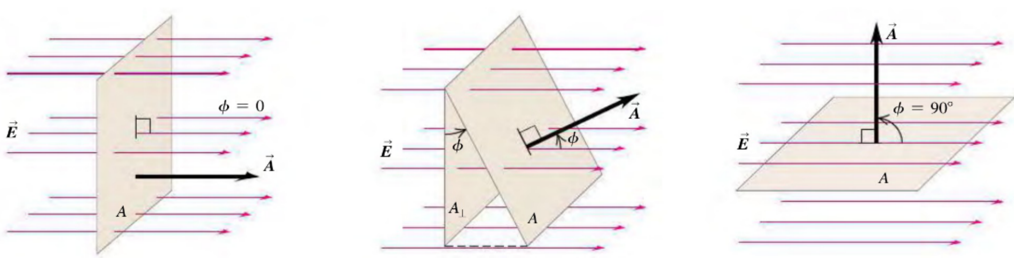
The flux of a moving liquid through a surface A is the amount of liquid that flows through it at a given time.
If a fluid flows through a surface (area A) with a volume *Vol* per unit time, the resulting velocity flux is:

$$\Phi_v = \frac{d(\mathrm{Vol})}{dt} = \frac{d(\mathrm{Ax})}{dt} \tag{5.2}$$
$$= vA_\perp = \vec{v} \cdot \vec{A} \tag{5.3}$$



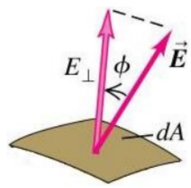
5.3 Flux of a uniform field

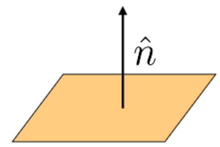
$$\Phi(\vec{E}) = \vec{E} \cdot \vec{A} = |E|\mathbf{cos}(\phi)A = E_\perp A \tag{5.4}$$



5.4 Flux

More generally, the flux of a vectorial field through a surface A is defined as the sum of the scalar product of the vector and a vector normal to A at any point of A.



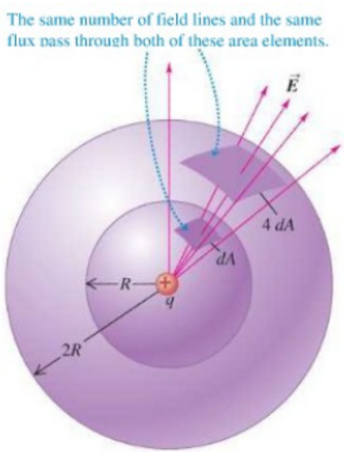


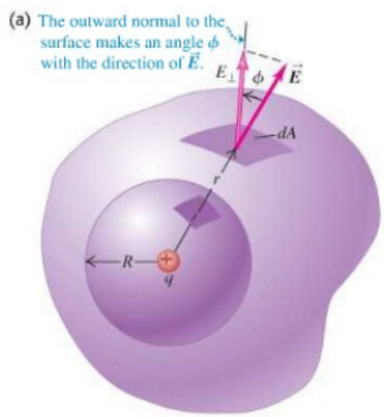
$$d\Phi_E = \vec{E} \cdot d\vec{A} = \vec{E} \cdot \hat{n} dA$$

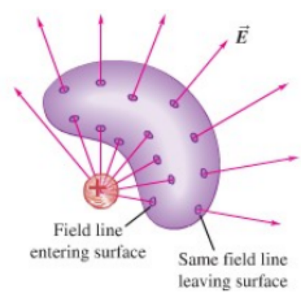
$$\Phi_E = \int_A \vec{E} \cdot \vec{A} = \int_A \vec{E} \cdot \hat{n} dA \tag{5.5}$$

5.5 Electric Field Flux

The flux is independent on the shape o the surface if the charge contained is the same.







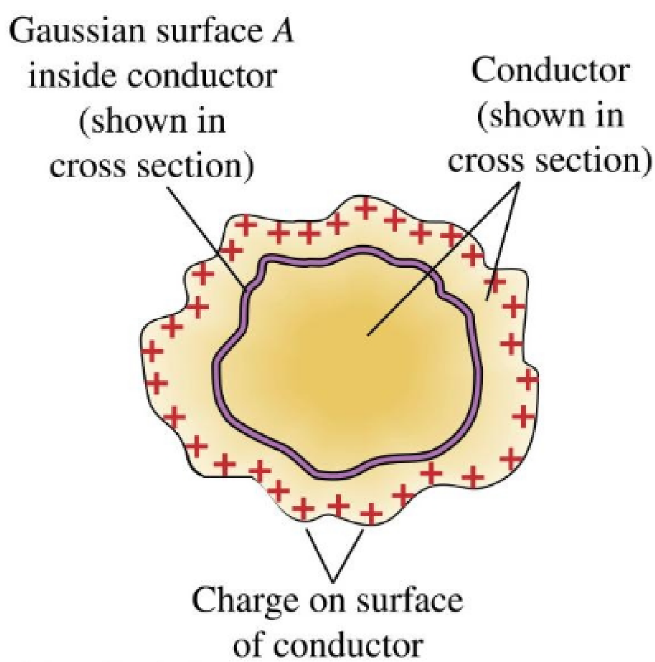
Zero flux

6 Lecture 6

6.1 Field of a charged conductor

Fact 1: in a conductor, charges are free to move (no friction), but are confined in the material.
Fact 2: in a electrostatic conditions charges must be at rest by definition.
Fact 3: if charges are at rest, $\vec{F} = 0$ on each of them.
Conclusion 1: Charges must be resting at the surface.

$$\Phi_E(A) = \oint_A \vec{E} \cdot \hat{n} dA = 4\pi k_e Q_{incl} \tag{6.1}$$

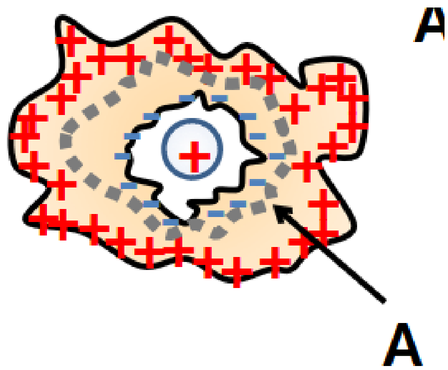


6.2 Field of a spherical charged conductor with a cavity

Consider a (positively) charged conducting object with a cavity. Construct a Gaussian surface A inside the conductor, surrounding the cavity.
Field within the conductor is zero so flux through A is zero:

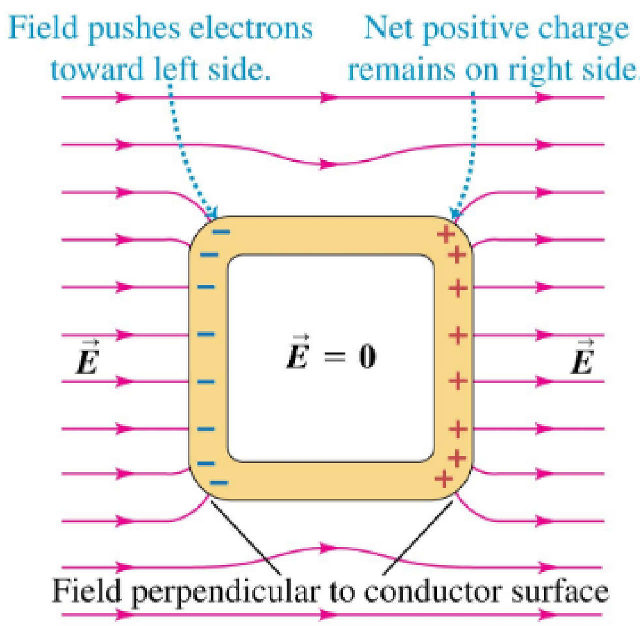
- there is no charge inside A
- and the positive charge must exist on **outer** surface.

If you put a point charge in the cavity you get the following:
Conclusion 2: Flux through surface A must still be zero as there is no electric field, so there must be a negative charge on interior surface to balance the total charge.



6.3 Electromagnetic Shield-Faraday cage

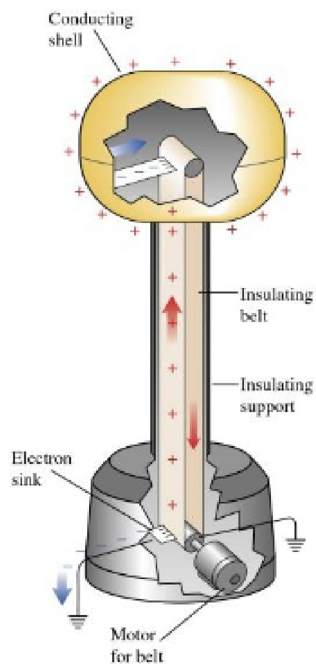
A conducting box is immersed in a uniform electric field.
The field of the induced charges on the box combines with the uniform field to give zero total field inside the box.



6.4 Van de Graff generator

Generation of large electrostatic charges

- Friction extract electrons from insulating belt
- Electrons are removed by grounding the contact
- Insulating belt transports positive charges to the interior belt with the conducting shell
- According to the Gauss' law, charges will move towards the surface.


Conducting
shell
Insulating
belt
Insulating
support
Electron
sink
Motor
for belt