Review

Angular Velocity Vector

$$\vec{\omega} = \frac{d\theta_x}{dt}\hat{i} + \frac{d\theta_y}{dt}\hat{j} + \frac{d\theta_z}{dt}\hat{k} \quad \text{or} \quad \vec{\omega} = \omega_x\hat{i} + \omega_y\hat{j} + \omega_z\hat{k}$$

$$\frac{d\vec{r}}{dt} = \vec{v} = \vec{\omega} \times \vec{r}$$

The Gyroscope

$$\left|\frac{d\vec{L}_r}{dt}\right| = \Omega L_r, \quad \vec{\tau} = lW, \quad \Omega = \frac{lW}{I_0\omega_s}$$

where Ω is gyroscope's axis swing velocity

Tensor of Inertia

$$\tilde{I} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix}$$

note:

$I_{yx} = I_{xy}$,
$I_{zx} = I_{xz}$,
$I_{yz} = I_{zy}$.

Moment of Inertia + Products of Inertia

$$I_{xx} = \sum m_j (y_j^2 + z_j^2)$$
$$I_{xy} = -\sum m_j x_j y_j$$
$$I_{xz} = -\sum m_j x_j z_j$$

Angular Momentum

$$\vec{L} = \tilde{I}\vec{\omega}$$

$$L_x = I_{xx}\omega_x + I_{xy}\omega_y + I_{xz}\omega_z$$
$$L_y = I_{yx}\omega_x + I_{yy}\omega_y + I_{yz}\omega_z$$
$$L_z = I_{zx}\omega_x + I_{zy}\omega_y + I_{zz}\omega_z$$

or

$$\begin{bmatrix} L_x \\ L_y \\ L_z \end{bmatrix} = \begin{bmatrix} L_{xx} & L_{xy} & L_{xz} \\ L_{yx} & L_{yy} & L_{yz} \\ L_{zx} & L_{zy} & L_{zz} \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

rotational kinetic energy

$$K_{rot} = \frac{1}{2}\vec{\omega}\cdot\vec{L} = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2 + \frac{1}{2}I_3\omega_3^2$$

or

$$K_{rot} = \frac{L_1^2}{2I_1} + \frac{L_2^2}{2I_2} + \frac{L_3^2}{2I_3}$$

Rotation about a fixed point

$$I_{xx} = (I_0)_{xx} + M(Y^2 + Z^2)$$
$$I_{xy} = (I_0)_{xy} - MXY$$
$$I_{xz} = (I_0)_{xz} - MXZ$$