Review

- ## Elastic collision theorem

$K = K_c + \frac{1}{2}(m_1 + m_2)|\vec{V}|^2$ where $K_c = \frac{1}{2}m_1|\vec{v}_{1c}|^2 + \frac{1}{2}m_2|\vec{v}_{2c}|^2$

$K = \frac{1}{2}\mu|\vec{v}_{rel}|^2 + \frac{1}{2}(m_1+m_2)|\vec{V}|^2$

$Q = \frac{1}{2}\mu\left(|\vec{v}_{rel}|^2 - |\vec{v}_{rel}'|^2\right), \quad 0 \leq Q \leq \frac{1}{2}\mu|\vec{v}_{rel}|^2$

→ For elast. collisions in COM frame, particle speeds don't change but directions can.

Scattering angle $\tan\theta_1 = \frac{|\vec{v}_{1c}'|\sin\theta}{|\vec{V}| + |\vec{v}_{1c}'|\cos\theta}$, if collision elastic: $|\vec{v}_{1c}| = |\vec{v}_{1c}'|$

$\frac{|\vec{V}|}{|\vec{v}_{1c}|} = \frac{m_1}{m_2}$

Fixed axis Rotation

$\vec{L} = \vec{r}\times\vec{p}, \Rightarrow \vec{L} = I\omega\hat{k}$

$\vec{\tau} = \vec{r}\times\vec{F} \Rightarrow |\vec{\tau}| = |\vec{r}||\vec{F}|\sin\theta \qquad \tau = \frac{dL}{dt}$

$I = \sum_{i=1}^{N} m_i \rho_i^2$, where ρ_i = perpendicular distance to axis of rotation

$I = \int_m \rho^2\, dm \qquad \rho = \sqrt{x^2+y^2}$

1d: $dm = \lambda(s)\, ds$ ← length

2d: $dm = \lambda(A)\, dA$ ← Area

3d: $dm = \lambda(V)\, dV$ ← Volume

Parallel axis Theorem : $I_0 = I_A + ML^2$

$\vec{L}_z = I_0\omega + (\vec{R}\times M\vec{V})_z$

$\vec{\tau}_z = (\tau_z)_0 + (\vec{R}\times\vec{F})_z$

$K = \frac{1}{2}I_0\omega^2 + \frac{1}{2}M|\vec{V}|^2$

} motion involving translation and rotation.

$\vec{R} = \frac{1}{M}\sum_{i=1}^{N} m_i\vec{r}_i, \quad M = \sum_{i=1}^{N} m_i$

Dynamics of fixed axis rotation

$\vec{L}_z = I_0\omega$

$\vec{\tau}_z = I_0\alpha$

$\vec{K} = \frac{1}{2}I\omega^2$

Collisions extra

$\vec{V} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2}, \quad \vec{u} = (\vec{v}_1 - \vec{v}_2), \quad \vec{v}_{1c} = \vec{v}_1 - \vec{V}, \quad \vec{v}_{2c} = \vec{v}_2 - \vec{V}$

$\vec{p}_{1c} = m_1\vec{v}_{1c}, \quad \vec{p}_{2c} = m_2\vec{v}_{2c}$

$\mu = \frac{m_1 m_2}{m_1+m_2}$

$m_1\vec{v}_{1c} - m_2\vec{v}_{2c} = 0$

$m_1\vec{v}_{1c}' - m_2\vec{v}_{2c}' = 0$