Equations

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1 Introduction

This is a document that sets out what equations that may be expected within the Physics 2024 exam.

2 Waves

$$f = \frac{1}{T} \tag{1}$$

$$T = \frac{1}{f} \tag{2}$$

f is frequency, and T is the period. In periodic motion **frequency and period** are reciprocals of each other.

$$\omega = 2\pi f = \frac{2\pi}{T} \tag{3}$$

The Angular Frequency/Angular Velocity are related to the period and the frequency.

$$F_x = -kx \tag{4}$$

This is called **Hooke's Law**, the restoring force F_x exerted by an ideal spring is related to the displacemnt x and the force constant of spring k.

$$a_x = \frac{d^2x}{dt^2} = -\frac{k}{m}x \tag{5}$$

This is the equation for **Simple Harmonic Motion**, the acceleration in the x direction is related to the force constant, the displacement and the mass of the object.

$$\omega = \sqrt{\frac{k}{m}} \tag{6}$$

The **angular frequency** for **Simple Harmonic Motion** is the square root of the force constant divided by the mass of the object.

$$f = \frac{\omega}{2\pi} = 2\pi\sqrt{\frac{k}{m}} \tag{7}$$

The frequency can be represented as the angular frequency divided by two pi. Similarly we get the period as

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \tag{8}$$

$$x = Acos(\omega t + \phi) \tag{9}$$

x is the **Displacement in simple harmonic motion** as a function of time, where ϕ is the phase angle.

$$E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 = constant \tag{10}$$

E is the **total mechanical energy** in simple harmonic motion, where v_x is the velocity in the x direction and A is the amplitude of the wave.

$$v = \lambda f \tag{11}$$

For a **periodic wave**, the wave speed v is the product of the wavelength λ and the frequency of the wave f.

$$y(x,t) = Acos[2\pi(\frac{x}{\lambda} - \frac{t}{T}] \tag{12}$$

This is the **Wave function for a Sinusoidal wave** propagating in the +x direction.

The wave number is $k = \frac{2\pi}{\lambda}$. Using this and $\omega = vk$ we get the following.

$$y(x,t) = Acos(kx - \omega t) \tag{13}$$

which is a more general form of the wave function.

$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 (x,t)}{\partial t^2} \tag{14}$$

The **Wave Equation** involves the second partial derivatives of the wave function. where the first part is the second partial derivative with respect to x, and the second part is the second partial derivative with respect to t multiplied by 1/velocity squared.

$$v = \sqrt{\frac{F}{\mu}} \tag{15}$$

This is the speed of a transverse wave on a **string**, where F is the Tension in the string and μ is the mass per unit length.

$$P_{\mathbf{av}} = \frac{1}{2}\sqrt{\mu F}\omega^2 A^2 \tag{16}$$

This is the average power, of a sinusoidal wave on a **string**.

$$y(x,t) = y_1(x,t) + y_2(x,t) \tag{17}$$

This is the **Principle of superposition**, where y(x,t) is the wave function of combined wave.

$$y(x,t) = (A_{\mathbf{SW}} sinkx) sin\omega t \tag{18}$$

This is the wave function for a standing wave on a string, on a fixed end at x=0. $A_{\mathbf{SW}}$ is the standing-wave amplitude, and k is the wave number.

$$n = \frac{c}{v} \tag{19}$$

This is the **Index of refraction** of an optical material, where c is the speed of light in a *vacuum* and v is the speed of light in the *material*

$$\theta_r = \theta_a \tag{20}$$

This is the **Law of Reflection**. The angle of reflection is the same as angle of incidence, measured from the normal.

$$n_a sin\theta_a = n_b sin\theta_b \tag{21}$$

This is the **Law of Refraction**. where n_a is the index of refraction for material with incident light and n_b is for refracted light.

$$\lambda = \frac{\lambda_0}{n} \tag{22}$$

This is the **Wavelength of light in a material**, where λ_0 is the wavelength of light in a vacuum and n is the index of refraction.

$$sin\theta_{crit} = \frac{n_b}{n_a} \tag{23}$$

Theta is the **critical angle** for **total internal reflection** where n_a is the index of refraction of the first material and n_b is for the second.

$$E_p = 2E \mid cos\frac{\phi}{2} \mid \tag{24}$$

This is the **electric-field** amplitude in two-source interference. E is the amplitude of the wave from one source and ϕ is the phase difference.

$$I = I_{max} cos^2\phi \tag{25}$$

This is **Malus's Law**. I is the intensity of polarized light passed through an analyzer, ϕ is the angle between polarization axis of light and polarizing axis of analyzer.

$$dsin\theta = m\lambda \tag{26}$$

This is **Constructive interference for two slits** where $(m = 0, \pm 1, \pm 2, ...)$. This is where bright spots appear on a screen when there is TWO slits.

$$dsin\theta = (m + \frac{1}{2})\lambda \tag{27}$$

This is **Destructive interference for two slits** where $(m = 0, \pm 1, \pm 2, ...)$. This is where dark spots appear on a screen when there is TWO slits.

$$y_m = R\frac{m\lambda}{d} \tag{28}$$

This is **constructive interference, for Young's experiment**. This only works for small angles only. R is the distance from slits to screen. d is the distance between the slits.

$$I = I_0 cos^2\frac{\phi}{2} \tag{29}$$

This is the **Intensity in two-source interference**, where ϕ is the phase difference between waves.

$$\phi = \frac{2\pi}{\lambda}(r_2 - r_1) = k(r_2 - r_1) \tag{30}$$

This is the phase difference, $(r_2 - r_1)$ is the path difference, and k is the wave number.

$$2t = m\lambda \tag{31}$$

This is **constructive reflection** from thin film, with NO relative phase shift. If there is a relative phase shift you use the destructive reflection equation. $(m = 0, \pm 1, \pm 2, ...)$

$$2t = (m + \frac{1}{2})\lambda \tag{32}$$

This is **destructive reflection** from thin film,This is **constructive reflection** from thin film, with NO relative phase shift. If there is a relative phase shift you use the constructive reflection equation. $(m = 0, \pm 1, \pm 2, ...)$

$$sin\theta = \frac{m\lambda}{a} \tag{33}$$

This is where **dark fringes** appear for **single-slit diffraction**. $(m = 0, \pm 1, \pm 2, ...)$

$$I = I_0\{\frac{sin[\pi a(sin\theta)/\lambda]}{\pi a(sin\theta)/\lambda}\}^2 \tag{34}$$

This is **Intensity in single-slit diffraction**. I_0 is the intesnity at $\theta = 0$, a is the slit width. θ is the angle of line from center of slit to position on screen.

$$sin\theta_1 = 1.22\frac{\lambda}{D} \tag{35}$$

This is **diffraction by a circular aperture**, where θ_1 is the angular radius of the first dark ring = angular radius of Airy disk. D is the aperture diameter.

$$P_{max} = BkA \tag{36}$$

This is the **Pressure amplitude** of a sinusoidal **sound wave** .B is the bulk modulus of a medium, k is the wave number and A is the displacement amplitude.

$$\beta = (10\texttt{dB}) log \frac{I}{I_0} \tag{37}$$

This is the **sound intensity level**, where I_0 is the reference intensity ($= 10^{-12} W/m^2$), I is the intensity of sound.log is the logarithm to base 10.

$$f_{\texttt{beat}} = f_a - f_b \tag{38}$$

This is the **beat frequency** for waves a and b.

3 Special Relativity

3.1 Galilean Transformations

$$\begin{array}{cc} x = x' + vt' & v_x = v_{x'} + v \\ y = y' & v_y = v_{y'} \\ z = z' & v_z = v_{z'} \\ \multicolumn{2}{c}{t = t'} \end{array}$$

3.2 Lorentz Transformations

$$\begin{array}{cc} x = \gamma(x' + vt') & x' = \gamma(x - vt) \\ t = \gamma(t' + \frac{vx'}{c^2}) & t' = \gamma(t - \frac{vx}{c^2} \\ \multicolumn{2}{c}{y = y',\ z = z'} \end{array}$$

where

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

3.3 Length Contraction

$$l_0 = \gamma l$$

3.4 Time Dilation

$$\Delta t = \gamma \Delta t_0$$

3.5 Composition of Velocities

$$u_x = \frac{u'_x + v}{1 + \frac{v u'_x}{c^2}} \qquad u'_x = \frac{u_x - v}{1 - \frac{v u_x}{c^2}}$$

$$u_y = \frac{\frac{u'_y}{\gamma}}{1 + \frac{v u'_x}{c^2}} \qquad u'_y = \frac{\frac{u_y}{\gamma}}{1 - \frac{v u_x}{c^2}}$$

3.6 Relativistic Doppler Effect

For observer moving away:

$$\lambda' = \lambda\sqrt{\frac{1+v/c}{1-v/c}} \qquad f' = f\sqrt{\frac{1-v/c}{1+v/c}}$$

For when observer is moving towards:

$$\lambda' = \lambda\sqrt{\frac{1-v/c}{1+v/c}} \qquad f' = f\sqrt{\frac{1+v/c}{1-v/c}}$$

3.7 Mass, Energy, Kinetic Energy, Momentum

$$m(v) = m_0\gamma$$
$$E = mc^2$$
$$K.E. = m_0c^2(\gamma - 1)$$
$$P = m_0\gamma v$$

3.8 An Energy-Momentum Invariant

$$E^2 - P^2c^2 = m_0^2c^4$$

3.9 Momentum of a photon

$$P = \frac{E}{c} = \frac{hf}{c}$$