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Secure Multiparty Computation Based on Linear Secret Sharing

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What is MPC

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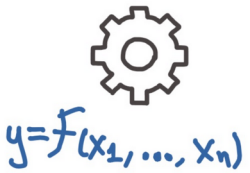
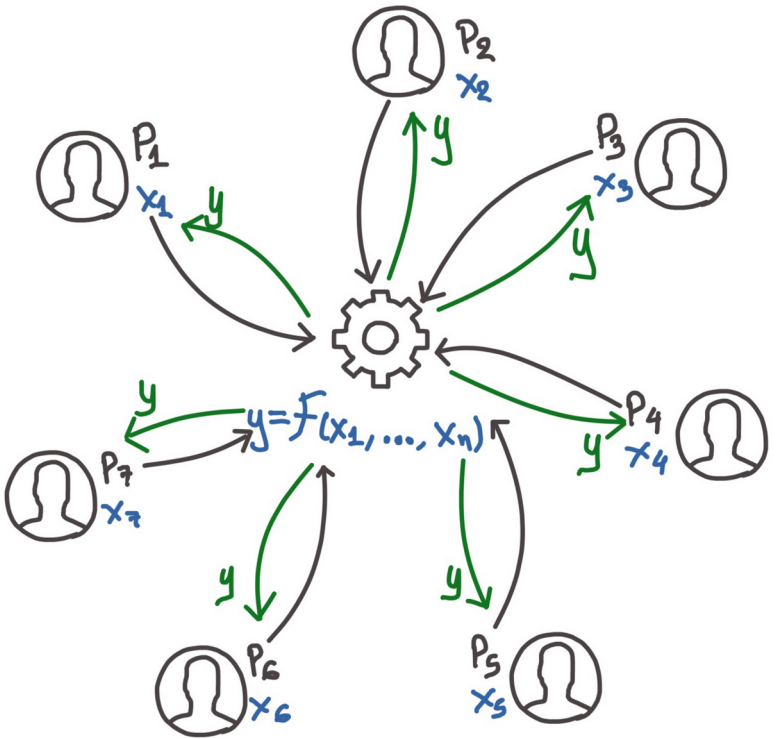


What is MPC



Properties

- Privacy: Any information learnt by P_i can be derived by x_i and y
  - Auction, output y is the highest bid
  - Nothing should be learnt for the other bids
  - But we do learn that they are lower than
- Correctness: The output received by each player is correct
  - Auction, party with highest bid will win
  - All parties will know it

Properties

- Independence of inputs: Corrupt parties must choose inputs independent of honest parties
- Fairness: Corrupt parties receive output if and only if honest parties do
- Guaranteed output delivery (Robustness): Corrupt parties cannot prevent honest parties from receiving the output
  – Stronger than fairness

Not exhaustive
Not all properties always guaranteed

Formal definition

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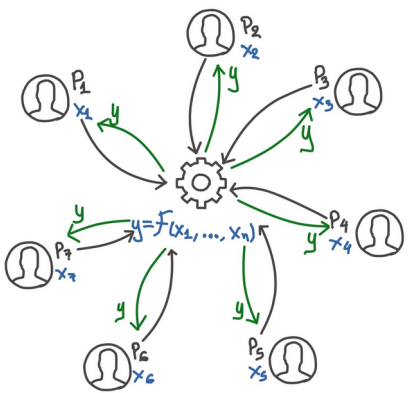
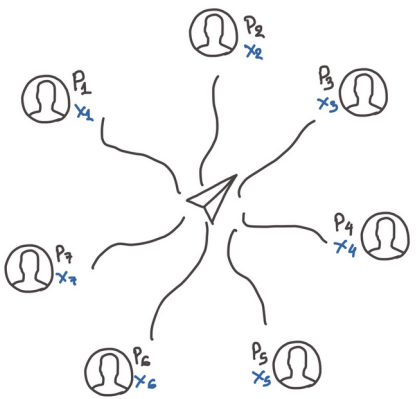
Ideal world

- An external trusted functionality does the computation
- Properties hold by definition



Real world

- No trusted party, parties run protocol
- Prove that the adversary cannot do any worse than in ideal world



Additional definition parameters

- Adversarial behavior
  - Passive (honest-but-curious, semi-honest)
  - Active (malicious)
  - Covert
- Corruption strategy
  - Static
  - Adaptive
  - Mobile (proactive security)
- Corruption thresholds
  - Honest supermajority ($t < n / 3$)
  - Honest majority ($t < n / 2$)
  - Dishonest majority ($t < n$)
- Type of security
  - Information theoretic
  - Computational
- Modular composition
  - Sequential (stand-alone setting)
  - Parallel (universal composability, UC)

Security with abort

MPC approaches

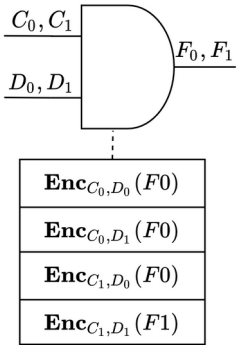
First step

- Write *F* as an arithmetic circuit *C* of *add* and *multiply* gates.
- Evaluate *C* gate by gate
- Addition and multiplication are universal over F_p

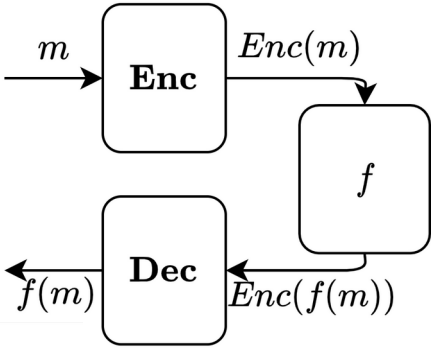
Whatever needs to be computed,
can be computed securely

Three approaches to evaluate the circuit

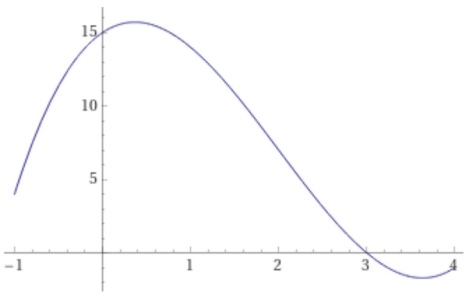
- Garbled circuits



- Homomorphic encryption

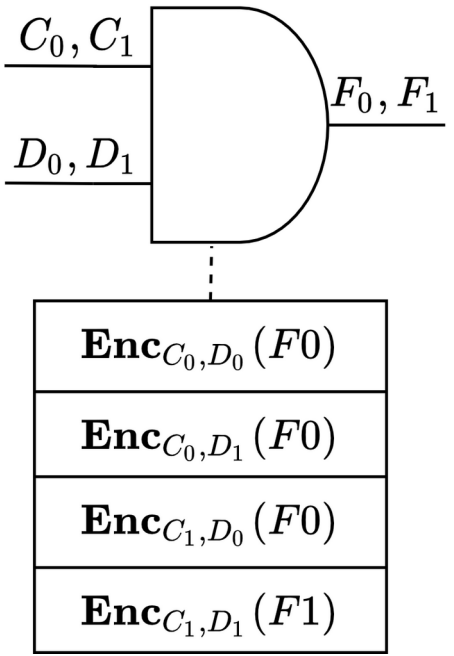


- Secret sharing

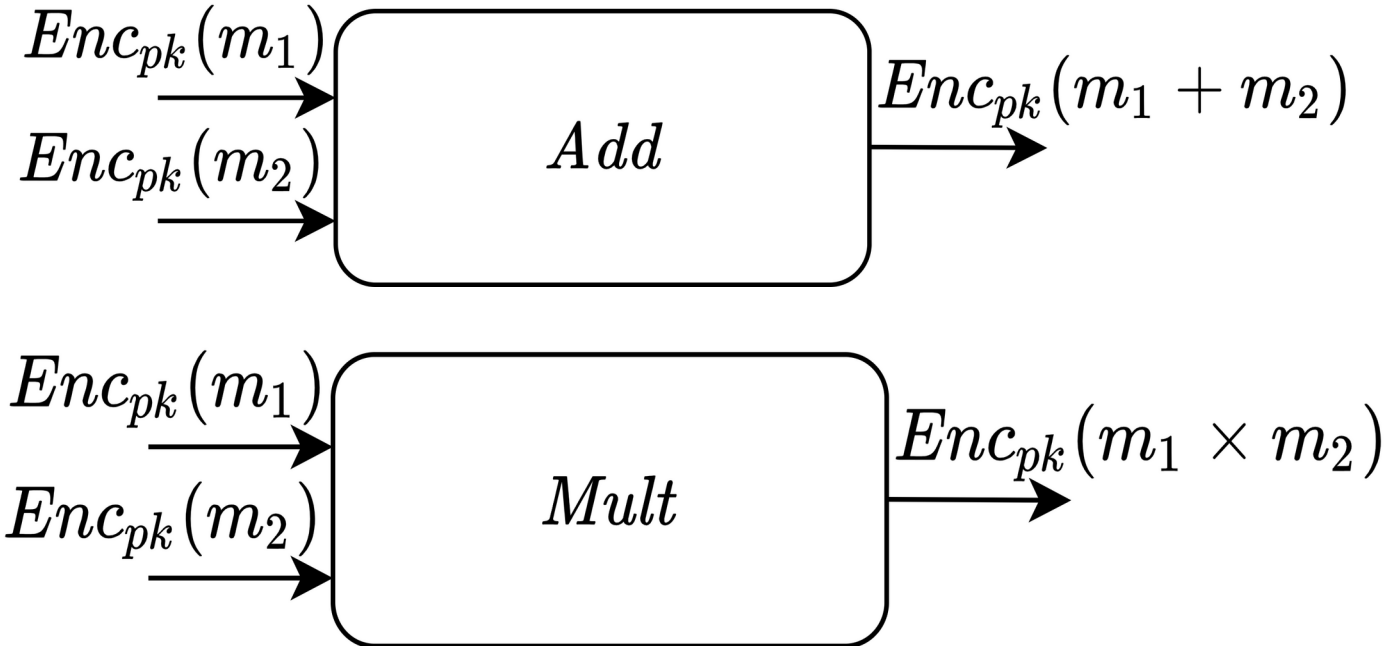


1. Garbled circuits

- Garbler and Evaluator [Yao82]
- Treat gate as matrix
  For example, AND gate has 4 rows, one for each possible input pair
- Encrypt each row, send only the keys that decrypt one input
- When output also encrypted, we can use it as input to the next gate



2. Fully homomorphic encryption

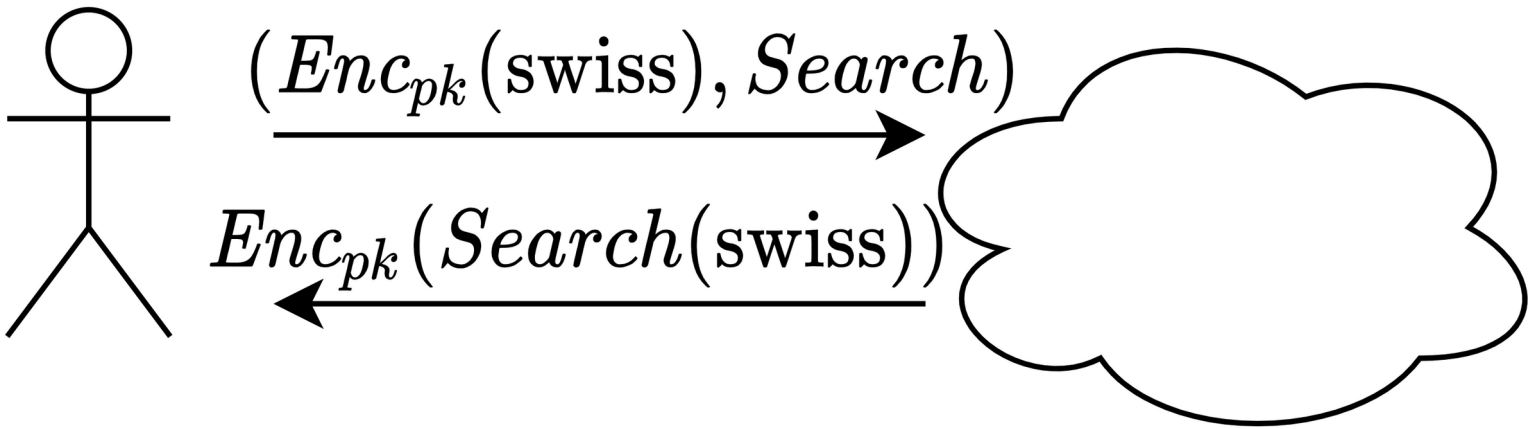


- *Add* and *Mult* are specific to the scheme

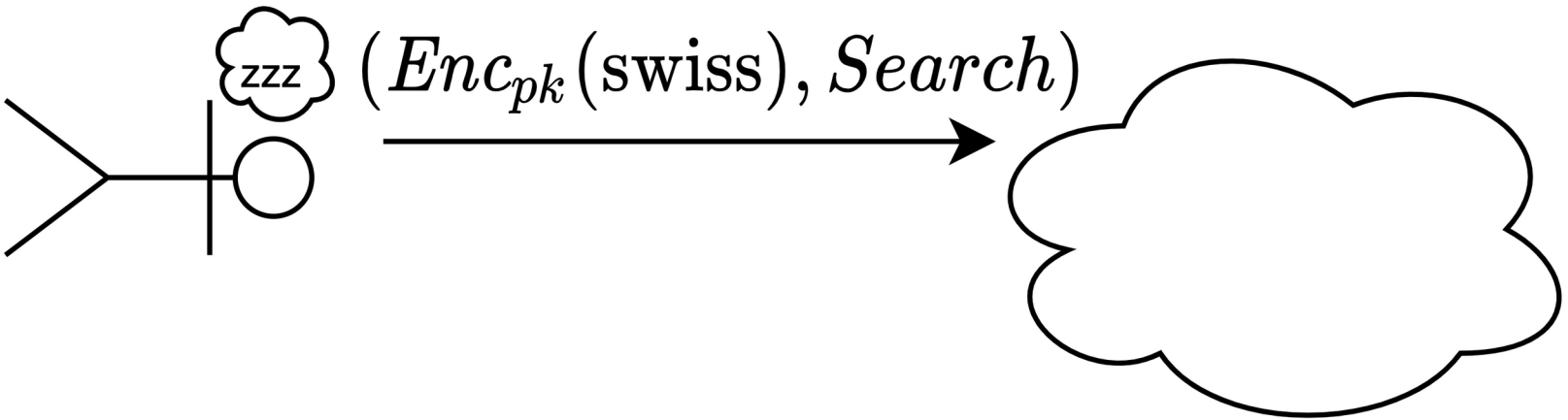
2. Fully homomorphic encryption

- For MPC, we also need partial decryption (*sk* is shared among parties)
- For passive, computational security with two rounds of communication:
- Each p_i encrypts its input and broadcasts
- Parties compute the circuit on ciphertexts
- Each p_i partially decrypts result and broadcasts
- Parties combine partial decryptions to obtain result

2. Fully homomorphic encryption is promising



2. Fully homomorphic encryption is slow



2. Fully homomorphic encryption vs (P/S) HE

- Partially homomorphic encryption
- Somewhat homomorphic encryption
- Examples:
  - ElGamal: $Enc_Y(m) = (g^r, mY^r)$
  - RSA: $Enc_e(m) = m^e$
  - Both partially homomorphic under multiplication

2. An example from SPDZ

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- Use SHE to obtain an additive sharing of number *m* [DPSZ12]

Protocol Reshare

Usage: Input is $e_{\mathbf{m}}$, where $e_{\mathbf{m}} = \mathsf{Enc}_{\mathsf{pk}}(\mathbf{m})$ is a public ciphertext and a parameter enc, where $enc =$ NewCiphertext or $enc =$ NoNewCiphertext. Output is a share $\mathbf{m}_i$ of $\mathbf{m}$ to each player P_i; and if $enc =$ NewCiphertext, a ciphertext $e'_{\mathbf{m}}$. The idea is that $e_{\mathbf{m}}$ could be a product of two ciphertexts, which Reshare converts to a "fresh" ciphertext $e'_{\mathbf{m}}$. Since Reshare uses distributed decryption (that may return an incorrect result), it is not guaranteed that $e_{\mathbf{m}}$ and $e'_{\mathbf{m}}$ contain the same value, but it *is* guaranteed that $\sum_i \mathbf{m}_i$ is the value contained in $e'_{\mathbf{m}}$.

Reshare$(e_{\mathbf{m}}, enc)$:

1. Each player P_i samples a uniform $\mathbf{f}_i \in (\mathbb{F}_{p^k})^s$. Define $\mathbf{f} := \sum_{i=1}^{n} \mathbf{f}_i$.
2. Each player P_i computes and broadcasts $e_{\mathbf{f}_i} \leftarrow \mathsf{Enc}_{\mathsf{pk}}(\mathbf{f}_i)$.
3. Each player P_i runs Π_{ZKPoPK} acting as a prover on $e_{\mathbf{f}_i}$. The protocol aborts if any proof fails.
4. The players compute $e_{\mathbf{f}} \leftarrow e_{\mathbf{f}_1} \boxplus \cdots \boxplus e_{\mathbf{f}_n}$, and $e_{\mathbf{m}+\mathbf{f}} \leftarrow e_{\mathbf{m}} \boxplus e_{\mathbf{f}}$.
5. The players invoke $\mathcal{F}_{\text{KeyGenDec}}$ to decrypt $e_{\mathbf{m}+\mathbf{f}}$ and thereby obtain $\mathbf{m} + \mathbf{f}$.
6. P_1 sets $\mathbf{m}_1 \leftarrow \mathbf{m} + \mathbf{f} - \mathbf{f}_1$, and each player P_i $(i \neq 1)$ sets $\mathbf{m}_i \leftarrow -\mathbf{f}_i$.
7. If $enc =$ NewCiphertext, all players set $e'_{\mathbf{m}} \leftarrow \mathsf{Enc}_{pk}(\mathbf{m} + \mathbf{f}) \boxminus e_{\mathbf{f}_1} \boxminus \cdots \boxminus e_{\mathbf{f}_n}$, where a default value for the randomness is used when computing $\mathsf{Enc}_{pk}(\mathbf{m} + \mathbf{f})$.

Fig. 4. The sub-protocol for additively secret sharing a plaintext $\mathbf{m} \in (\mathbb{F}_{p^k})^s$ on input a ciphertext $e_{\mathbf{m}} = \mathsf{Enc}_{\mathsf{pk}}(\mathbf{m})$.

3. Secret sharing

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- Share a value *x* among *n* participants, so that [Shamir79]
  - *t* + *1* can recover the secret
  - any *t* have no information about it

- Share
  - Degree-*t* random polynomial: $f(x) = k + a_1 x + \ldots + a_t x^t$
  - Give each party the share $s_i = f(i)$

- Reconstruct
  - *t* + 1 pairs (i, s_i) uniquely determine *f*
  - Lagrange interpolation

3. General secret sharing (LSSS)

- Share a value *x* among *n* parties, given access structure *A*, so that [CDM00]
  - An authorized set in *A* can recover the secret
  - Any other set has no information about it

- MSP (labeled 2D matrix *M*) is equivalent to LSSS

- Share
  - Random vector $\boldsymbol{r} = (k, a_1, \ldots, a_{d-1})$
  - Calculate shares as $\boldsymbol{s} = M\boldsymbol{r}$

- Reconstruct
  - For quorum *A* with shares $\boldsymbol{s}_A$ find recombination vector $\boldsymbol{\lambda}_A$ such that $\boldsymbol{\lambda}_A M_A = \boldsymbol{e}$
  - The value is $x = \boldsymbol{\lambda}_A \boldsymbol{s}_A$

3. Secret sharing - Add

- Players hold sharings
  - *[x]* of *x*, made with *deg-t* polynomial
  - *[y]* of y, made with *deg-t* polynomial

- Obtain sharing *[x + y]* of *x + y* by locally adding shares

- No interaction

3. Secret sharing - Multiply

- Players hold sharings
  - *[x]* of *x*, made with *deg-t* polynomial f_1
  - *[y]* of y, made with *deg-t* polynomial f_2

- Obtain sharing *[xy]* of *xy* by locally multiplying shares

- But polynomial $g = f_1 f_2$ has degree *2t*

3. Secret sharing - Multiply

- Degree reduction
- Luckily, we have *2t + 1* shares of *g* (we started with *t < n / 2*)
- These shares determine *g(0)* as $g(0) = \sum_{i=1}^{2t+1} \lambda_i g(i)$
- Each p_i shares *g(i)* with deg-t polynomial
- Parties now calculate $[g(0)] = \sum_{i=1}^{2t+1} \lambda_i [g(i)]$
- This is a sharing of *g(0)* with the correct degree

3. Secret sharing - Multiply

- Similar idea for LSSS (Maurer)
- Requires the exchange of n^2 elements (each party send n elements)

3. Secret sharing - Multiply with Beaver trick

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- Assume *[a], [b], [c]*, with *ab* = *c* and *a,b,c* unknown, are available [Beaver91]
- Parties open *[ε]* = *[x]* – *[a]*. Reconstruct *ε*
- Parties open *[δ]* = *[y]* – *[b].* Reconstruct *δ*
- Parties compute *[z]* = *[c]* + *ε[b]* + *δ[a]* + *εδ* locally
- Now *2n* elements are exchanged (each party send *2* elements)

Three approaches to evaluate the circuit - Summary

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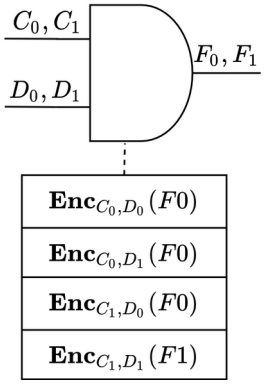
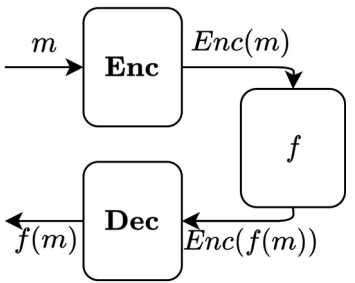
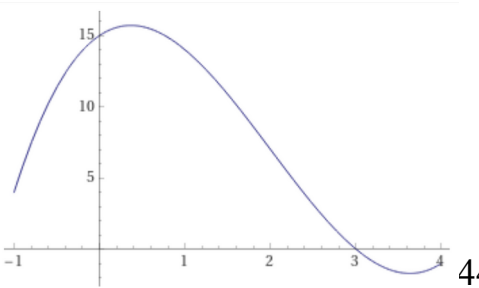
- Garbled circuits
  - 2PC
  - Low communication complexity
  - Practical and efficient for Boolean operations
  - Large circuit size for arithmetic operations



- Homomorphic encryption
  - Low communication complexity
  - Slow (computationally expensive operations)



- Secret sharing
  - No computationally expensive PK operations
  - High communication complexity
  - Number of rounds depends on multiplicative depth



Combine the three approaches: The preprocessing model

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[DPSZ12]

- Very fast online phase
  - Information theoretic primitives
  - No PK
  - Assume everything is given

- We saw how parties can add and multiply values, given sharings + Beaver triples

- Slow offline phase
  - Create everything for online phase
  - Heavy HE
  - Does not depend on circuit *C*
  - (it is not really offline)

- We saw how parties can create sharings (Beaver triples is similar)

From passive to active security

From passive to active security

- Adversary can send false shares
- We need a way to verify
- One solution: Verifiable secret sharing (Commitments)
  - Information-theoretic
  - Computational

Don’t slow me down!

From passive to active security

- Sacrifice security properties to gain efficiency
- Dishonest majority, security with abort
- We can detect cheating, not correct it

Message Authentication Codes (MACs)

- Key $K=(\alpha, \beta_i)$, or $K=\alpha$ in SPDZ [DPSZ12]
- α is global but unknown to anyone. Assume we have a sharing $[\alpha]$
- Authenticate value a by $MAC_K(a) = \alpha a$
- For a shared value x, store $[x]$ and $[\alpha x]$
- Add $x + y$: $[x] + [y]$, $[\alpha x] + [\alpha y]$
- Multiply xy: $[xy]$, $[\alpha x][y]$ (using passive multiplication protocol twice)
- Output phase: Commit to the values, open α, check MACs

Thank you!

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[Yao82] DBLP:conf/focs/Yao82b
[Beaver91] DBLP:conf/crypto/Beaver91a
[CDM00] DBLP:conf/eurocrypt/CramerDM00
[DPSZ12] DBLP:conf/crypto/DamgardPSZ12