MIDTERM ASSESSMENT

UNIVERSITY OF LONDON

CM1020

BSc EXAMINATION

COMPUTER SCIENCE

Discrete Mathematics

INSTRUCTIONS TO CANDIDATES:

This assignment consists of five questions. You should answer all five questions.

Full marks will be awarded for complete answers to a total of Five questions. The marks for each part of a question are indicated at the end of the part in [.] brackets.

There are 100 marks available on this paper.

Question 1

(a) Let $A = \{1, 2, 3, 4\}, B = \{3k|k \in \mathbb{Z} - \{-1, 1\}\}$ and $C = \{n \in \mathbb{Z}|n^2 + n = 0\}$. Which one of the following statements is True? Explain your answer

i. $A \cap B = \emptyset$

ii. $B - C = \emptyset$

iii. $B \cap C = \emptyset$

iv. $A \cup C = \emptyset$

[4]

(b) Let A and B be two sets. Show that if $A \subseteq B$, then

i. $A \cup B = B$

ii. $A \cap B = A$

[6]

(c) Let A, B and C be three sets. Draw the Venn diagrams for each of these combinations:

i. $\overline{A} \cap \overline{B} \cap \overline{C}$

ii. $(A - B) \cup (A - C) \cup (B - C)$

[6]

(d) Let A and B be subsets of a universal set U. Show that $A \subseteq B$ if and only if $\overline{B} \subseteq \overline{A}$. [4]

Question 2

(a) Let f and g be two functions defined from $\mathbb{R} \to \mathbb{R}$ with $f(x) = x^3 + 1$ and $g(x) = \sqrt[3]{x-1}$. Find $(gof)(1),\ (gof)(2)$ and $(gof)(x)$. [4]

(b) One of the following function $f : \mathbb{R} \to \mathbb{R}$ below is a bijection. Which one is it? Explain your answer.

i. $f(x) = x^2 + x$ for all $x \in \mathbb{R}$

ii. $f(x) = x^3 + x$ for all $x \in \mathbb{R}$

iii. $f(x) = x^2 - x$ for all $x \in \mathbb{R}$

iv. $f(x) = x^3 - x$ for all $x \in \mathbb{R}$

[6]

(c) Solve the following logarithmic equation $2\log_9(\sqrt{x}) - \log_9(6x-1) = 0$. Show your work.

[2]

(d) Let $f : \mathbb{R} - \{1\} \to \mathbb{R}$ with $f(x) = \frac{5x-3}{x-1}$.

i. What is the image of f?

ii. Find $d \in \mathbb{R}$ such that the function $f : \mathbb{R} - \{1\} \to \mathbb{R} - \{d\}$ is a bijection. Explain your answer

iii. Construct the inverse function of f.

[8]

Question 3

(a) Let p, q and r be three propositions.

i. Construct a truth table for the compound proposition

$$((p \to q) \wedge (r \to \neg p) \wedge r) \to \neg q$$

ii. Is the compound proposition in question (i) a tautology? Explain your answer.

[5]

(b) Let p, q, r and s be four propositions. Assuming that p and r are false and that q and s are true, find the truth value of the following proposition

$((p \vee q) \wedge (q \vee s)) \to ((\neg r \vee p) \wedge (q \vee s))$

[2]

(c) Define a set of at most three atomic propositions. Then use them to translate all of these sentences.

i. It's time for bed only if it is after 9 PM and I'm tired

ii. I'm not tired unless it is after 9 PM.

iii. It's time for bed and after 9 PM if and only if I am tired

[6]

(d) Give the contrapositive, the converse and the inverse of the following statement:

$$\forall x \in \mathbb{R}, \text{ if } x(x+2) > 0 \text{ then } x > 0 \text{ or } x < -2$$

[3]

(e) A tautology is a proposition that is always true. Let p, q and r be three propositions. Using the laws of propositions or the truth table, show that $(p \to (q \vee r)) \Leftrightarrow ((p \wedge \neg q) \to r)$ is a tautology. [4]

Question 4

(a) Given the following propositions:

d: it is day

n: it is night

s: the sun shines

m: the moon shines

Give propositions that respectively resemble the meaning of the following two sentences:

i. The sun only shines when it is day, but the sun doesn't shine now although it is day.

ii. Either it is day, or it is night, but when the sun shines it is not night.

[6]

(b) Indicate which of the following statements are true and which are false. Justify your answer.

i. $\forall x \in \mathbb{Z}, \exists y \in \mathbb{R} - \{0\}$ such that $xy < 1$

ii. $\forall x \in \mathbb{R} - \{0\}, \forall y \in \mathbb{Z}$ such that $xy < 1$

iii. $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}$ such that $xy = 1$

[6]

(c) Let p, q, r and s be three propositions: Say whether or not the following argument is a valid argument. Explain your answer.

$s \to r$
$(p \vee q) \to \neg r$
$(\neg s) \to (\neg q \to r)b$
p

$\therefore q$ [4]

(d) Let $P(x)$ and $Q(x)$ be two predicates and suppose D is the the domain of x.Simplify each of the following propositions. In your answer, the $\neg$ operator should be applied only to individual predicates.

i. $\neg\exists x \in D(P(x) \wedge Q(x))$

ii. $\neg\forall x \in D(P(x) \to Q(x))$

[4]

Question 5

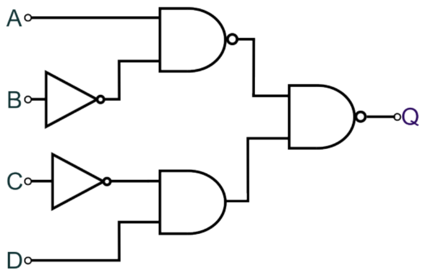
(a) Use DeMorgan's laws to simplify the following expressions:

i. $\overline{(\overline{a.b.\overline{c}}) + (\overline{\overline{c}.d})}$

ii. $\overline{\overline{a} + b}\ .\ \overline{b + \overline{c}}\ .\ \overline{\overline{c} + d}$

[6]

(b) Given the following logic circuit:



[4]

i. Find a Boolean expression for the output Q for this logic circuit

ii. Use the laws of boolean algebra and give to simplify this expression

(c) Use the duality principle to find out the dual of the following equation:

$$a.b + c.\overline{d} = (a + c).(a + \overline{d}).(b + c).(b + \overline{d})$$

[2]

(d) For the following Boolean expression, give:

$$F(a, b, c, d) = (\overline{a} + b.\overline{d}).(c.b.a. + \overline{c}.d)$$

i. The truth table

ii. The Karnaugh map

iii. The minimal sum of products expression

[8]

END OF PAPER

.