Trust is Risk: Generalized Max Flow for Strategies between Idle and Conservative

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Abstract. Previous versions of Trust is Risk present the Conservative and the Idle strategy as distinct and unrelated. This work is an attempt to generalize this idea into a continuous spectrum of strategies, the two ends of which correspond to the two previously defined strategies. Prior to querying the system for an indirect trust towards Bob, Alice can attribute a specific expected strategy to each one of the participating players, or even fine-tune the response of each player to a steal action from each one of the players she directly trusts. The system then executes the generalized MaxFlow [1] to determine the indirect trust from Alice to Bob, given the specified strategies of the rest of the players.

1 Introduction

In our previous work, we presented three distinct strategies for the players: the Idle, the Conservative and the Evil strategy. The process of determining the indirect trust from *Alice* to *Bob* involved assigning the Idle strategy to *Alice*, the Evil strategy to *Bob* and the Conservative strategy to all other players. The indirect trust from *Alice* to *Bob* would then be the worst case scenario for *Alice* when *Bob* initiates a "chain reaction" of steal actions. This value was proven to be equal to the maximum flow from *Alice* to *Bob*. Consider however the two following cases:

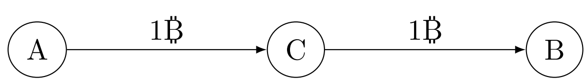

Fig. 1: $\mathcal{G}_1$, Few hops

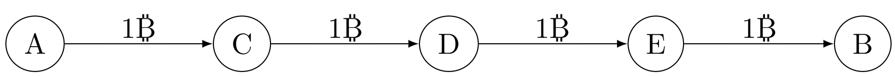

Fig. 2: $\mathcal{G}_2$, Many hops

One could argue that intuitively it should be $Tr_{\mathcal{G}_1,A\rightarrow B} > Tr_{\mathcal{G}_2,A\rightarrow B}$, since the longer chain of players connecting A and B in $\mathcal{G}_2$ introduces more uncertainty as to whether B is trustworthy. Nevertheless, according to our prior approach, it is $Tr_{\mathcal{G}_1, A \to B} = Tr_{\mathcal{G}_2, A \to B}$ since $maxFlow_{\mathcal{G}_1}(A, B) = maxFlow_{\mathcal{G}_2}(A, B)$.

To mitigate this problem, we introduce a generalization of the previous approach that can handle these cases more robustly. The mechanism that we propose is analogous to an attenuation factor that accounts for the number of hops, but is better suited for the ambience of maximum flow. Each edge $e = (Alice, Bob) \in \mathcal{E}$ of the graph is supplemented with an additional number called *loss factor*, which intuitively represents the "leakage" ratio on this edge, or the percentage of the damage incurred by *Bob* that is carried over to *Alice* through this edge, in case he is stolen some funds. The player who initiates the maxFlow calculation is the one that should specify the loss factors, according to her beliefs about the other players. It should be explicitly noted that the loss factor expresses *Bob*'s expected attitude towards *Alice*, not the other way around.

2 Strategy Generalization

TODO: Switch to $\gamma \geq 1$.

TODO: Prove (or disprove) that graphs with $\gamma \geq 1$ have the same outgoing flow from the source in the maxFlow as the same graphs with $\gamma' = \frac{1}{\gamma}$ have incoming flow to the sink in the corresponding maxFlow.

More specifically, apart from the capacity c_e, each edge $e \in \mathcal{E}$ is assigned a gain factor $\gamma_e \in (0, 1]$.

Definition 1 (Extended Strategy). *Player A follows the Extended strategy if she partly replenishes the value Dmg_A she lost since the previous turn by stealing from each player $v \in N^{-}(A)_{j-1}$ value y_v₿ in a way which ensures that*

$$\sum_{v \in N(A)_{j-1}} \frac{y_v}{\gamma_{(v,A)}} = \min \left(Dmg_A, \sum_{v \in N(A)_{j-1}} \frac{DTr_{v \to A}}{\gamma_{(v,A)}} \right) . \qquad (1)$$

Given a graph with capacities, loss factors, a source A and a sink B, the generalized MaxFlow algorithm yields a generalized flow that corresponds to the steal actions in the worst case scenario for A, given that A follows the Idle strategy, B follows the Evil strategy and all other players follow the Extended strategy [proof needed]. For the sake of example, consider the following two graphs:

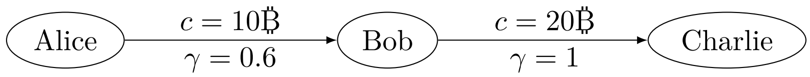

Fig. 3: $Tr_{Alice \rightarrow Charlie} = 6$₿

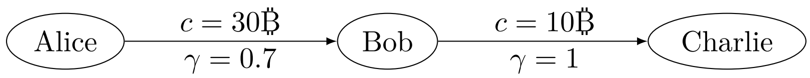

Fig. 4: $Tr_{Alice \rightarrow Charlie} = 7$₿

We will now attempt to obtain some insight as to how players behave on the two ends of the spectrum of the loss factor γ. Consider an edge $e = (Alice, Bob) \in \mathcal{E}$. If $\gamma_e \rightarrow 0$, then *Bob* will not steal any funds from *Alice*, no matter how big a loss he has suffered. On the other hand, if $\gamma_e = 1$, then *Bob* can replenish any amount of stolen funds through stealing from *Alice*, given that she directly trusts him with sufficient funds.

The following two lemmas show the relation of the Extended strategy with the Idle and the Conservative strategies.

Lemma 1 (Extended Strategy special case: Idle).
Let $A \in \mathcal{V}$. If A is following the Extended strategy where $\forall v \in N^{-}(A)$, $\gamma_{(v,A)} \rightarrow 0$, then A is following the Idle strategy.

Proof. We know that all direct trusts are finite. Combining this with the fact that $\forall v \in N^{-}(A), \gamma_{(v,A)} \rightarrow 0$ and that $Dmg_A \geq 0$, we deduce that the right part of (1) is always equal to 0. In order for the left part of (1) to be 0, it must hold that $\forall v \in N^{-}(A), y_v = 0$, or equivalently that player A will not steal any value from any player that directly trusts her. This means that player A passes her turn, or equivalently that follows the Idle strategy. □

Lemma 2 (Extended Strategy special case: Conservative).
Let $A \in \mathcal{V}$. If A is following the Extended strategy where $\forall v \in N^{-}(A)$, $\gamma_{(v,A)} = 1$, then A is following the Conservative strategy.

Proof. The definition of the Conservative strategy could be equivalently rewritten as follows: Player A follows the Conservative strategy if she attempts to replenish the value Dmg_A she lost since the previous turn by stealing from each player $v \in N^{-}(A)_{j-1}$ value y_v₿ in a way which ensures either that

$$\sum_{v \in N(A)_{j-1}} y_v = Dmg_A \wedge \forall v \in N(A)_{j-1}, y_v \leq DTr_{v \rightarrow A} \ ,$$

if $\sum\limits_{v \in N^{-}(A)_{j-1}} DTr_{v \to A} \geq Dmg_A$ or that $\forall v \in N^{-}(A)_{j-1}, y_v = DTr_{v \to A}$ else. Thus put, the definition of the Conservative strategy implies that

$$\sum_{v \in N^{-}(A)_{j-1}} y_v = \min\left(Dmg_A, \sum_{v \in N^{-}(A)_{j-1}} DTr_{v \to A}\right) .$$

We observe that this is the special case of the Extended strategy when $\forall v \in N^{-}(A)_{j-1}, \gamma_{(v,A)} = 1$, thus the proof is complete. $\square$

The incoming direct trusts to the Evil player should all have a gain factor equal to 1, since we consider that she steals all her incoming direct trust.

References

1. Wayne K. D., Tardos E.: Generalized Maximum Flow Algorithms. Ph.D. thesis: Cornell University (1999)