1 Common Setup

We propose three different kinds of games, all of them finite (but possibly generalizable to the infinite setting). The first consists of one sole strategy where the players do not initially know whether they will be buyers, sellers or nothing at all, this being decided in the last moment. The second game consists of three strategies: buyers, sellers and middlemen. The last game consists of two strategies: creators and consumers. Before delving into the details of each game, we first describe their common elements.

The general approach taken is as follows: After a game is described in detail, each player is assigned a specific strategy and a relevant utility function. All players are considered to follow their respective strategy without deviating from it, except for one player that is allowed to follow any desired strategy; her utility function however remains unchanged. If that player is proven to have an incentive to deviate from her appointed strategy, we can deduce that the given strategies and utility functions do not constitute a Nash equilibrium. If on the other hand we can repeat the aforementioned process (keeping all players except for one honest) for all given strategies and no player has an incetive to deviate, then we will come to the conclusion that the given strategies and utility functions do constitute a Nash equilibrium. This approach is common in game theoretic analyses, given that allowing for all players to be rational and then searching for a Nash equilibrium constitutes a practically intractable problem [daskalakis citation].

A description of the structure that is common for all three games follows. The game graph has a random initial configuration where every player has a random direct trust towards every other player, as well as a random capital. These values may be uniformly distributed in an interval or may follow another distribution such as the exponential, or may have a high probability of being zero. The exact distribution however is not determined at this point, as it is not yet needed. This distribution will be common knowledge to the players. Transaction fees are not considered.

The players play simultaneously in each round and can do any of the known actions. If two actions conflict (e.g. A reduces $DTr_{A \rightarrow B}$ and B steals from $DTr_{A \rightarrow B}$ as well), then one of the two actions is chosen with equal probability (50%). To better model a player's actions and the aforementioned conflict resolution, we demand that each change explicitly mentions the source and the destination of the funds for each of her actions. Player A decides on the values of all the following variables. This constitutes a concrete round for A.

$$\forall B, C \in \mathcal{V}, \mathit{move}\left(A, \left(A, B\right), \left(A, C\right)\right) = ?$$
$$\forall B, C \in \mathcal{V}, B \neq A, \mathit{move}\left(A, \left(B, A\right), \left(A, C\right)\right) = ?$$

The first argument is the player who decides, the second argument is from which direct trust to take the funds and the third is to which direct trust to deposit the funds.

To clarify a detail, for any $B \in \mathcal{V}$ (including A), A is not allowed to set $\mathit{move}\left(A, \left(A, B\right), \left(A, B\right)\right)$ to any value different than 0. This choice is made to facilitate the analysis and because in the real-world setting this kind of behaviour has a totally different result, effectively dragging B into an arms race for the higher fee. In the real world case, the exact same reasoning goes for $\mathit{move}\left(A, \left(B, A\right), \left(B, A\right)\right)$. In our case however, the latter move is already not permitted.

Of course there are some constraints for player's A move:

- It makes no sense to deposit to and withdraw from a specific direct trust in the same round. Furthermore, such a possibility would allow for "chain reactions" in the conflict resolution phase that would add unnecessary complications. This constraint applies only to outgoing direct trusts, because incoming direct trusts cannot be increased.

$$\forall B, C, D \in \mathcal{V}, \mathit{move}\left(A, \left(A, B\right), \left(A, C\right)\right) \times \mathit{move}\left(A, \left(A, D\right), \left(A, B\right)\right) = 0$$
$$\wedge$$
$$\forall B, C, D \in \mathcal{V}, \mathit{move}\left(A, \left(A, B\right), \left(A, C\right)\right) \times \mathit{move}\left(A, \left(D, A\right), \left(A, B\right)\right) = 0$$

- One cannot use more funds than are available from a single direct trust.

$$\forall B \in \mathcal{V}, \sum_{C \in \mathcal{V}} \mathit{move}\left(A, \left(A, B\right), \left(A, C\right)\right) \leq DTr_{A \rightarrow B}$$
$$\forall B \in \mathcal{V}, \sum_{C \in \mathcal{V}} \mathit{move}\left(A, \left(B, A\right), \left(A, C\right)\right) \leq DTr_{B \rightarrow A}$$

If two players try to change the same direct trust, then set the relevant moves of one of the two players (chosen uniformly at random) to 0.

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resolveConflict(A, B) :
  sum1 = \sum_{C \in \mathcal{V}} move(A, (A, B), (A, C))
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    sum2 = $\sum_{C \in \mathcal{V}} move\left(B, (A,B), (B,C)\right)$
    if (sum1*sum2 != 0)
      choice $\xleftarrow{\$}$ $\{A, B\}$
      if (choice == $A$)
        $\forall C \in \mathcal{V}, move\left(A, (A,B), (A,C)\right)$ = 0
      else # if (choice == $B$)
        $\forall C \in \mathcal{V}, move\left(B, (A,B), (B,C)\right)$ = 0

resolveAllConflicts() :
  $\forall A, B \in \mathcal{V}$
    resolveConflict($A$, $B$)
    resolveConflict($B$, $A$)
```

`resolveAllConflicts()` is executed after all players choose their moves for a round.

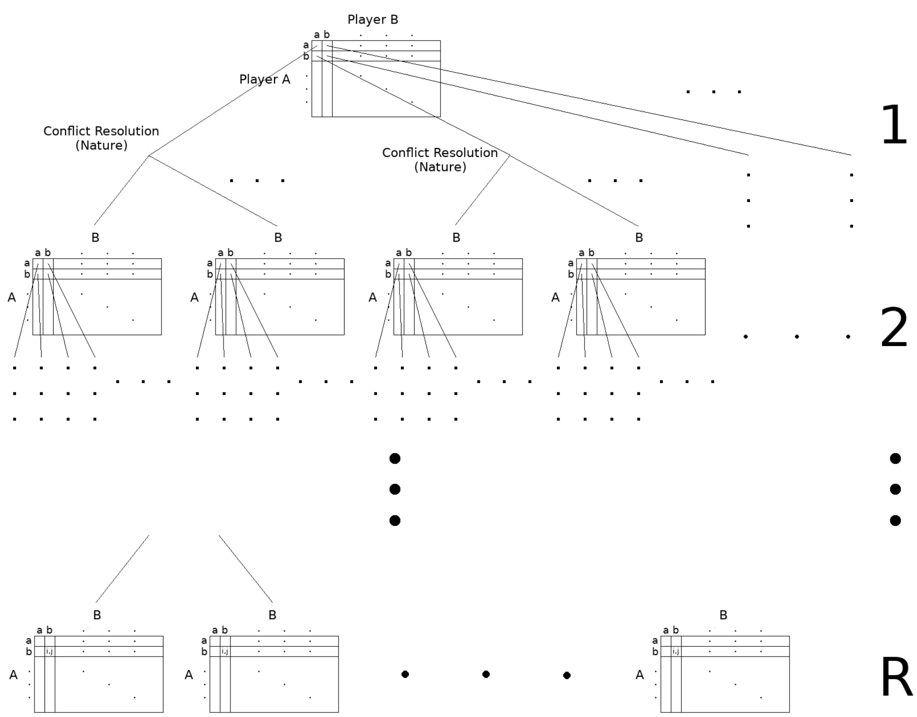


Fig. 1. The general form of the game (inspiration due here, p. 3.)

We now move on to describe the individual games.

1.1 Random Roles

All players follow the same strategy, according to which each player is permitted to freely add or steal direct trust from other players. After R rounds (blocks in bitcoin terms) exactly two players are selected at random (these choices follow the uniform distribution). One is dubbed seller and the other buyer. The seller offers a good that costs C, which she values at $C - l$ and the buyer values at $C + l$. The values R, C and l, as well as the uniform distribution with which the buyer and seller are chosen, are common knowledge from the beginning of the game. The exchange completes if and only if $DTr_{Buyer \rightarrow Seller} \geq C$. There are three variants of the game, each with a different utility for the players (the first two versions have two subvariants each):

1. If player A is not chosen to be either buyer or seller, then her utility is equal to $Cap_{A,R}$. Intuitively players do not attach any value to having (incoming or outgoing) direct trust at the end of the game. If the buyer and the seller do not manage to complete the exchange, the buyer's utility is $Cap_{Buyer,R}$. If on the other hand they manage to exchange the good, then the buyer's utility is $Cap_{Buyer,R} + l$. Intuitively these utilities signify that the buyer uses her preexisting Capital to buy. As for the seller there exist two subvariants for her utility:
   (a) If the exchange is eventually not completed, the seller's utility is $Cap_{Seller,R} - l$. If the exchange takes place, the seller's utility is $Cap_{Seller,R}$. Intuitively, the seller is first obliged to buy the good from the environment at the cost of C.
   (b) If the exchange is eventually not completed, the seller's utility is $Cap_{Seller,R} + C - l$. If the exchange takes place, the seller's utility is $Cap_{Seller,R} + C$. Intuitively, the seller is handed the good for free by the environment.
2. If player A is not chosen to be either buyer or seller, then her utility is equal to

$$\sum_{\substack{B \in \mathcal{V} \\ B \neq A}} (DTr_{A \rightarrow B,R} + DTr_{B \rightarrow A,R}) + Cap_{A,R} \ .$$

   Intuitively, players attach equal value to all the funds they can directly spend, regardless of whether others can spend them as well. If the buyer and the seller do not manage to complete the exchange, the buyer's utility is

$$\sum_{\substack{B\in\mathcal{V}\\ B\neq Buyer}} (DTr_{Buyer\to B,R} + DTr_{B\to Buyer,R}) + Cap_{Buyer,R} \ .$$

If on the other hand they manage to exchange the good, then the buyer's utility is

$$\sum_{\substack{B\in\mathcal{V}\\ B\neq Buyer}} (DTr_{Buyer\to B,R} + DTr_{B\to Buyer,R}) + Cap_{Buyer,R} + l \ .$$

Intuitively these utilities signify that the buyer uses her preexisting accessible funds to buy. As for the seller there exist two subvariants for her utility:

(a) If the exchange is not completed, the seller's utility is

$$\sum_{\substack{B\in\mathcal{V}\\ B\neq Seller}} (DTr_{Seller\to B,R} + DTr_{B\to Seller,R}) + Cap_{Seller,R} - l \ .$$

If the exchange takes place, the seller's utility is

$$\sum_{\substack{B\in\mathcal{V}\\ B\neq Seller}} (DTr_{Seller\to B,R} + DTr_{B\to Seller,R}) + Cap_{Seller,R} \ .$$

Intuitively, the seller is first obliged to buy the good from the environment at the cost of C.

(b) If the exchange is not completed, the seller's utility is

$$\sum_{\substack{B\in\mathcal{V}\\ B\neq Seller}} (DTr_{Seller\to B,R} + DTr_{B\to Seller,R}) + Cap_{Seller,R} + C - l \ .$$

If the exchange takes place, the seller's utility is

$$\sum_{\substack{B\in\mathcal{V}\\ B\neq Seller}} (DTr_{Seller\to B,R} + DTr_{B\to Seller,R}) + Cap_{Seller,R} + C \ .$$

Intuitively, the seller is handed the good for free by the environment.

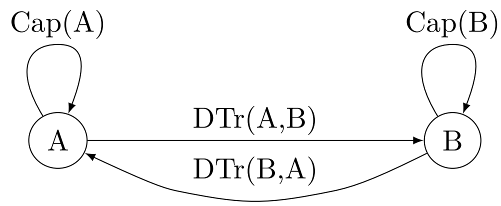
3. If player A is not chosen to be either buyer or seller, then her utility is 0. If the buyer and the seller do not manage to complete the exchange, their utility is 0 as well. If on the other hand they manage to exchange the good, then the utility is l for both of them. Intuitively these utilities signify that in this game there is gain only for those who exchange goods and the gain is exactly the difference between the objective value and the subjective value that the relevant parties perceive.

1.2 Buyers, Sellers, Middlemen

Buyers only desire to buy products and do not have incoming trust. Sellers only desire to accumulate capital through selling products and do not have outgoing trust. Middlemen

1.3 Creators and Consumers

Complete 2-player graph:



A's possible actions:

$$\begin{array}{cc} & \begin{array}{ccc} & from & \\ DTr(B,A) & DTr(A,B) & Cap(A) \end{array} \\ Action(A) = & \left[\begin{array}{ccc} x_1 & x_2 & x_3 \\ y_1 & 0 & y_3 \\ z_1 & z_2 & 0 \end{array} \right] \begin{array}{l} Purchase \\ DTr(A,B)\ to \\ Cap(A) \end{array} \end{array}$$

Results:

$$\begin{aligned} DTr'(A,B) &= DTr(A,B) + y_1 + y_3 - x_2 - z_2 \\ DTr'(B,A) &= DTr(B,A) - x_1 - y_1 - z_1 \\ Cap'(A) &= Cap(A) + z_1 + z_2 - x_3 - y_3 \\ Bought &= \frac{x_1 + x_2 + x_3}{cost(b)} \end{aligned}$$

No funds destroyed/created rule:

$$\begin{gathered} DTr'(A,B) + DTr'(B,A) + Cap(A) + Bought \times cost(b) \\ = \\ DTr(A,B) + DTr(B,A) + Cap(A) \end{gathered}$$

Individual no funds created rules:

$$\begin{aligned} x_1 + y_1 + z_1 &\leq DTr(B,A) \\ x_2 + z_2 &\leq DTr(A,B) \\ x_3 + y_3 &\leq Cap(A) \end{aligned}$$

No adding and reducing from same place rules:

$$\begin{array}{cccc} y_1 x_2 = 0 & z_1 x_3 = 0 & z_2 x_3 = 0 & y_3 x_2 = 0 \\ y_1 z_2 = 0 & z_1 y_3 = 0 & z_2 y_3 = 0 & y_3 z_2 = 0 \end{array}$$