

## 1 Degree of Connectedness

We would like to have a measure of how well connected is the graph. Viewed as a whole, a network with a lot of disconnected cliques should score lower than a loosely connected network. From the perspective of a single player, this measure can be expressed as the expected mean indirect trust, defined incrementally as follows:

$$\begin{aligned} Tr(A) &= \sum_{B \in \mathcal{V} \setminus \{A\}} Tr_{A \rightarrow B} \text{ (Total indirect trust for player } A) \\ ETr(A) &= \frac{Tr(A)}{|\mathcal{V} \setminus \{A\}|} = \frac{Tr(A)}{|\mathcal{V}| - 1} \text{ (Expected indirect trust for player } A) \\ EMTr &= \frac{1}{|\mathcal{V}|} \sum_{A \in \mathcal{V}} ETr(A) \text{ (Expected mean indirect trust)} \end{aligned}$$

## 2 Centrality

Another important measure is the network centrality. We propose here some different measures to that end.

### 2.1 Degree Centrality

One possible approach is the degree centrality [Freeman citation], that can be broken down as in-degree and out-degree centrality. We first define the node in-degree centrality.

$$C_{in}(A) = \sum_{B \in \mathcal{V} \setminus \{A\}} DTr_{B \rightarrow A}^1 \text{ (Node in-degree centrality)}$$

Let  $A^* = \operatorname{argmax}_{A \in \mathcal{V}} C_{in}(A)$ . The network in-degree centrality is defined as:

$$C_{in} = \sum_{A \in \mathcal{V}} (C_{in}(A^*) - C_{in}(A)) \text{ (Network in-degree centrality)}$$

Similarly, for the out-degree centrality we have:

$$C_{out}(A) = \sum_{B \in \mathcal{V} \setminus \{A\}} DTr_{A \rightarrow B}^1 \text{ (Node out-degree centrality)}$$

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<sup>1</sup> Maybe indirect trust is more intuitive here than direct trust.

Let  $A^* = \operatorname{argmax}_{A \in \mathcal{V}} C_{out}(A)$ . The network out-degree centrality is defined as:

$$C_{out} = \sum_{A \in \mathcal{V}} (C_{out}(A^*) - C_{out}(A)) \quad (\text{Network out-degree centrality})$$

A problem of these centrality measures is that their unit is the currency used (i.e. Bitcoin) and thus may not always have an intuitive meaning. We would thus like to have a measure of *centralization* that has no units and can take values in the interval  $[0, 1]$ , with 0 corresponding to a network of no centralization (all nodes are equal, e.g. cycle) and 1 to a network with the maximum centralization possible (there is one central vital node for all, e.g. star). A centralization measure that achieves this target is proposed in [Freeman citation]. Here we use the following modified form. For a graph  $\mathcal{G}$ , the in- and out-centralization are defined as:

$$C'_{in} = \frac{C_{in}}{\max C_{in}} \quad \text{and} \quad C'_{out} = \frac{C_{out}}{\max C_{out}} ,$$

where  $\max C_{in}$  is defined as the maximum in-degree centrality for any graph with the same number of nodes for which the maximum direct<sup>2</sup> trust is equal to the maximum direct<sup>2</sup> direct trust of  $\mathcal{G}$ ;  $\max C_{out}$  is defined equivalently.

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<sup>2</sup> Maybe indirect trust is more intuitive here than direct trust.

Here we can see several possible network health measures. A combination of them will probably do the job.

1.  $n := |\mathcal{V}|$  How many players
2.  $e := |\mathcal{E}| - |\{A : DTr_{A \rightarrow A} > 0\}|^3$  How many direct trust lines
3.  $\frac{e}{n}$  Mean direct trust lines per player
4.  $DTr := \sum_{\substack{A, B \in \mathcal{V} \\ A \neq B}} DTr_{A \rightarrow B}$  How much direct trust in total
5.  $\mu := \frac{DTr}{n}$  Mean direct trust per player
6.  $Cap := \sum_{A \in \mathcal{V}} DTr_{A \rightarrow A}$  Total capital
7.  $\frac{Cap}{n}$  Mean capital per player
8.  $\frac{DTr - Cap}{n}$  Mean direct trust minus capital per player
9.  $\frac{1}{n} \sum_{A \in \mathcal{V}} \left( \sum_{\substack{B \in \mathcal{V} \\ B \neq A}} DTr_{A \rightarrow B} - \mu \right)^2$  Outgoing direct trust variance
10.  $\frac{1}{n} \sum_{B \in \mathcal{V}} \left( \sum_{\substack{A \in \mathcal{V} \\ A \neq B}} DTr_{A \rightarrow B} - \mu \right)^2$  Incoming direct trust variance
11.  $\frac{Cap}{DTr}$  Total capital to total direct trust ratio

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<sup>3</sup> Maybe it makes sense to include looped direct trusts (Capital) as well.