

1 Degree of Connectedness

We would like to have a measure of how well connected is the graph. Viewed as a whole, a network with a lot of disconnected cliques should score lower than a loosely connected network. From the perspective of a single player, this measure can be expressed as the expected mean indirect trust, defined incrementally as follows:

$$\begin{aligned}
Tr(A) &= \sum_{B \in \mathcal{V} \setminus \{A\}} Tr_{A \rightarrow B} \text{ (Total indirect trust for player } A) \\
ETr(A) &= \frac{Tr(A)}{|\mathcal{V} \setminus \{A\}|} = \frac{Tr(A)}{|\mathcal{V}| - 1} \text{ (Expected indirect trust for player } A) \\
EMTr &= \frac{1}{|\mathcal{V}|} \sum_{A \in \mathcal{V}} ETr(A) \text{ (Expected mean indirect trust)}
\end{aligned}$$

2 Centrality

Another important measure is the network centrality. We propose here some different measures to that end.

1 Degree Centrality

One possible approach is the degree centrality [Freeman citation], that can be broken down as in-degree and out-degree centrality. We first define the node in-degree centrality.

$$C_{in}(A) = \sum_{B \in \mathcal{V} \setminus \{A\}} DTr_{B \rightarrow A}^1 \text{ (Node in-degree centrality)}$$

Let $A^* = \operatorname{argmax}_{A \in \mathcal{V}} C_{in}(A)$. The network in-degree centrality is defined as:

$$C_{in} = \sum_{A \in \mathcal{V}} (C_{in}(A^*) - C_{in}(A)) \text{ (Network in-degree centrality)}$$

Similarly, for the out-degree centrality we have:

$$C_{out}(A) = \sum_{B \in \mathcal{V} \setminus \{A\}} DTr_{A \rightarrow B}^1 \text{ (Node out-degree centrality)}$$

¹ Maybe indirect trust is more intuitive here than direct trust.

Let $A^* = \operatorname{argmax}_{A \in \mathcal{V}} C_{out}(A)$. The network out-degree centrality is defouted as:

$$C_{out} = \sum_{A \in \mathcal{V}} (C_{out}(A^*) - C_{out}(A)) \quad (\text{Network out-degree centrality})$$

Here we can see several possible network health measures. A combination of them will probably do the job.

1. $n := |\mathcal{V}|$ How many players
2. $e := |\mathcal{E}| - |\{A : DTr_{A \rightarrow A} > 0\}|^2$ How many direct trust lines
3. $\frac{e}{n}$ Mean direct trust lines per player
4. $DTr := \sum_{\substack{A, B \in \mathcal{V} \\ A \neq B}} DTr_{A \rightarrow B}$ How much direct trust in total
5. $\mu := \frac{DTr}{n}$ Mean direct trust per player
6. $Cap := \sum_{A \in \mathcal{V}} DTr_{A \rightarrow A}$ Total capital
7. $\frac{Cap}{n}$ Mean capital per player
8. $\frac{DTr - Cap}{n}$ Mean direct trust minus capital per player
9. $\frac{1}{n} \sum_{A \in \mathcal{V}} \left(\sum_{\substack{B \in \mathcal{V} \\ B \neq A}} DTr_{A \rightarrow B} - \mu \right)^2$ Outgoing direct trust variance
10. $\frac{1}{n} \sum_{B \in \mathcal{V}} \left(\sum_{\substack{A \in \mathcal{V} \\ A \neq B}} DTr_{A \rightarrow B} - \mu \right)^2$ Incoming direct trust variance
11. $\frac{Cap}{DTr}$ Total capital to total direct trust ratio

² Maybe it makes sense to include looped direct trusts (Capital) as well.