## Payment Channels Overview

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**Abstract.** This is an overview of the existing literature on virtual payment channels. Lightning [?], Perun [?] and TeeChan [?] are considered.

#### 1 Introduction

Payment channels are constructions that permit the secure exchange of assets between remote agents without the need for each transaction to be recorded in a global database. They are constructed in a way that gives the opportunity to the cheated agents to report the latest valid state to a global database (i.e. blockchain) and reclaim their assets.

For example, imagine that Alice works in Bob's pin factory. They have agreed that Alice be paid right after she makes each pin a small amount x [?]. This can add up to hundreds, even thousands small of payments each day. Since most cryptocurrencies impose fees per transaction, it would be a waste to broadcast a new transaction for each small payment. For this reason, they turn to payment channels.

At the beginning of each month, *Bob* creates a transaction that pays e.g. 100 coins (a bit more than *Alice*'s expected pay for the month) to himself. He builds it in a way that needs both his and *Alice*'s signature to be spent (i.e. 2-of-2 multisig). This is the "bond" transaction. *Alice* confirms that the "bond" looks fine and gives *Bob* a special transaction that spends the "bond". This transaction is the "refund" transaction. *Bob* broadcasts the "bond" (but not yet the "refund") to the blockchain. The channel is now open.

Every time Alice makes a pin, Bob pays x to Alice as follows: He creates a new "refund" that pays to Alice the amount she already owned according to the previous "refund" plus x; accordingly, his payment is reduced by x. The total coins in the refund are always the same. He signs the new "refund" and sends it to Alice. She in turn signs the new "refund" and sends it back to Bob. The channel is now updated.

Finally, the end of the month comes and *Alice* wants to cash out on the blockchain, so that she can use her coins elsewhere. In order to do so,

she simply broadcasts the latest "refund". The "bond" is spent according to the latest update, so she takes her rightful payment and *Bob* takes the rest of the 100 initial coins. The channel is now closed.

Note that exactly two transactions have been broadcast on the blockchain no matter how many payments were made, so the fees are kept low. Furthermore, both parties can unilaterally close the channel at any given point and claim the coins of their latest "refund", thus no trust is required between the two parties.

As an extension of the previous model, let Charlie be a colleague of Alice, who also has a payment channel with Bob. It is reasonable to imagine a system where Alice can pay Charlie without touching the blockchain, by leveraging the two pre-existing channels ( $Alice \Leftrightarrow Bob, Bob \Leftrightarrow Charlie$ ) with minimal interaction with Bob and without having to trust him at all.

In the following sections we will summarise and compare various specific constructions that realise the high-level ideas described above. We will use the original terminology used in each paper.

### 2 Lightning Network

This construction is the first to achieve a functional model for payment channels. It is designed for bitcoin and requires some new opcodes and removing the malleability of transactions to function properly [?].

#### 2.1 Simple two-party channel

The basic construction is as follows. Suppose that *Alice* and *Bob* want to create a payment channel that contains 1 BTC consisting of 0.5 BTC from each party. To achieve this, they follow these steps (see also section 3.1.2 and Figure 4 in 3.3.2 in [?]):

- 1. Either party (say Alice) creates a "Funding" transaction (F) with an input of 0.5 BTC from her and 0.5 BTC from Bob, and a 2-of- $\{Alice, Bob\}$  multisig as output; she then sends F to Bob. This transaction is not yet signed nor broadcast. F needs to be signed by both parties to be valid.
- 2. Alice creates, signs and sends to Bob a "Commitment" transaction (C1b) that spends F and has the following outputs:
  - (a) 0.5 BTC that can be spent by Alice immediately when C1b is broadcast.

(b) 0.5 BTC that can be spent by either party, but *Bob* can spend it only after a specified amount of blocks (say *n*) have been mined on top of *C1b*, whereas *Alice* can spend it only if *Bob* provides her with a "Breach Remedy" transaction (explained later) signed by him. This output is called "Revocable Sequence Maturity Contract" (RSMC).

Furthermore, Alice creates, signs and sends a "Revocable Delivery" transaction (RD1b) that pays the first of the two outputs of C1b to Bob, but will be accepted by the network if it is in the mempool only after n blocks have been mined on top of C1b.

Bob similarly creates, signs and sends C1a and RD1a to Alice.

3. After *Alice* receives the signed C1a and RD1a from Bob, she verifies that they are both valid and correctly spend F. Given that everything works out right, she signs F and sends it to Bob.

Bob similarly verifies that C1b and RD1b have the correct structure, along with Alice's signature on F. He then signs F and broadcasts it. Note that he does not have to trust Alice in any way.

The fact that Alice holds C1a and RD1a, already signed by Bob, ensures her that her 0.5 BTC cannot be locked in the 2-of-2 multisig of F in case Bob stops cooperating. If she decides that Bob stopped cooperating, she can broadcast C1a, wait for it to be confirmed n times and broadcast RD1a to get her money back. Thus Alice need not trust Bob either.

Observe that if Bob refuses to cooperate in signing F, then the blockchain has not been changed and no funds are at risk. In such case, to ensure that Bob cannot lock her funds in the future, she should immediately transfer her funds to a new address or periodically check the blockchain for F and broadcast C1a and RD1a in case she finds F on the ledger.

After initially setting up the channel, *Alice* and *Bob* can update it as follows (see also section 3.3.4 and Figures 7, 8 in [?]):

- 1. Both Alice and Bob follow exactly the same steps as before to create C2a, C2b, RD2a and RD2b; the only difference these transactions have to their counterparts from the previous state of the channel is that, instead of 0.5 BTC for each player, they contain the new agreed balance of the channel (e.g. 0.4 BTC for Alice and 0.6 BTC for Bob).
- 2. Alice creates, signs and sends to Bob a so-called "Breach Remedy" transaction (BR1a). This transaction lets Bob redeem the RSMC output of C1a as soon as C1a is broadcast. Bob similarly creates, signs and sends BR1b to Alice.

Note that this effectively disincentivises Alice from ever broadcasting C1a, since in such case Bob will have a window of n blocks during which he can claim the entire sum in C1a, 1 BTC, for himself. Alice had better purge C1a after BR1a is sent to Bob. Similarly Bob is incentivised to refrain from ever broadcasting C1b.

This arrangement creates a situation where both players can be confident that the state of the channel is the one expressed by C2a, C2b, RD2a and RD2b, thus they can assume that Alice has just paid Bob 0.1 BTC. No trust between the two players was needed all along. There are only two caveats: First, both players must periodically check the blockchain to ensure that the other party has not broadcast an old Commitment transaction. Second, in case of an uncooperative counterparty, one has to wait a prespecified amount of time before releasing their funds, which may be undesirable.

Thus, the necessary number of blocks mined on top of a Confirmation transaction for a subsequent Revocable Delivery to be valid (previously called n) must be carefully chosen in a way that does not lock up the funds for a long time in case of a dispute and at the same time does not require that the parties check the blockchain too often for a malicious broadcast of an already invalidated Commitment transaction.

Alice can outsource the task of the periodic check to a dedicated service by sending it all the previous Breach Remedy transactions. To incentivise the service to cooperate, Alice can pay a fee to it as an output of these transactions. Note that Alice does not need to trust the service, since the only thing it can do is to broadcast a Branch Remedy transaction that was created by Alice; she never discloses any of her private keys to it.

Finally, the parties can cooperatively close the channel without having to wait n blocks as follows: When both parties have agreed to closing the channel, Alice creates, signs and sends to Bob an "Exercise Settlement" transaction (ES) that spends the Funding transaction and has two simple outputs, each paying to the respective party the sum of the last agreed Commitment transaction. Following the previous example, this transaction would pay  $0.4~\mathrm{BTC}$  to Alice and  $0.6~\mathrm{BTC}$  to Bob. Bob can then also sign and broadcast the transaction to close the channel.

Once Alice has sent ES, she considers the channel as closed. If Bob does not broadcast ES, we have a dispute and she has to broadcast the latest Commitment transaction and wait for her funds to be unlocked.

#### 2.2 Payments depending on preimage knowledge (HTLC)

Multi-hop payments can take place between players (e.g. *Alice* and *Dave*) who do not share a simple channel (i.e. an on-chain Funding transaction), but share simple channels with intermediate nodes (e.g. *Alice* with *Bob*, *Bob* with *Carol* and *Carol* with *Dave*).

To enable the creation of multi-hop channels, so-called "Hashed Time-lock Contracts" (HTLC) are used. An HTLC is an additional output in a Commitment transaction which can be redeemed by either Alice or Bob; Alice can redeem it after a specified number of additional blocks, say m, have been mined after the creation (not the broadcast) of the Commitment transaction, whereas Bob can redeem it at any time, but only if he produces the preimage R of a hash specified in the HTLC output (see also section 4.2 and Figure 12 in [?]).

More specifically, consider C2a, C2b where, contrary to the example in the previous subsection, Alice has paid the 0.1 BTC to an HTLC instead of directly to Bob. Bob should be able to redeem the 0.1 BTC only if he knows the preimage R before the m blocks have been mined. In addition to RD2a and RD2b, six additional transactions have to be signed and exchanged.

- 1. Alice signs and sends an "HTLC Execution Delivery" transaction (HED1a) to Bob. HED1a pays the HTLC output of C2a to Bob, only if he knows the required preimage R. Only Bob can broadcast the transaction.
- 2. Bob signs and sends a so-called "HTLC Timeout Delivery" transaction (HTD1b) to Alice. HTD1b pays the HTLC output of C2b to Alice, only after m blocks have been mined from the time C2b was created. Only Alice can broadcast this transaction.
- 3. Alice signs and sends an "HTLC Execution" transaction (HE1b) to Bob. HE1b pays the HTLC output of C2b to Bob, only if he knows the required preimage R. Only Bob can broadcast this transaction. Its single output is an RSMC with duration n, spendable by Bob.
- 4. Alice signs and sends an "HTLC Execution Revocable Delivery" transaction (HERD1b) to Bob. This transaction spends the RSMC output of HE1b. Bob can broadcast this transaction after n blocks have been mined on top of HE1b.
- 5. Bob signs and sends an "HTLC Timeout" transaction (HT1a) to Alice. HT1a pays the HTLC output of C2a to Alice, only after m blocks have been mined from the time HT1a was created. Its single output is an RSMC with duration n, spendable by Alice.

6. Bob signs and sends an "HTLC Timeout Revocable Delivery" transaction (HTRD1b) to Alice. This transaction spends the RSMC output of HT1b. Alice can broadcast this transaction after n blocks have been mined on top of HE1b.

Note that once again, no trust is necessary in the process described above. The RSMC outputs of HT1a and HE1b are necessary for future invalidation according to the "Breach Remedy" method. More details can be found in Figure 14 of section 4.3. In case of common desire to close the channel, they can be cooperatively closed using the "Exercise Settlement" method.

#### 2.3 Multi-hop channels

With the use of HTLC outputs, it is possible to execute multi-hop payments as follows. Suppose *Alice* wants to pay *Dave* 0.001 BTC and they find out that they are connected through the preexisting channels  $Alice \Leftrightarrow Bob, Bob \Leftrightarrow Carol$  and  $Carol \Leftrightarrow Dave$ . This payment can be completed with the following steps:

- 1. Dave generates a random number R and sends hash(R) to Alice, Bob and Carol.
- 2. Alice and Bob update their channel with an e.g. 300-block HTLC that transfers 0.001 BTC from Alice to Bob.
- 3. Bob and Carol update their channel with an e.g. 200-block HTLC that transfers 0.001 BTC from Bob to Carol.
- 4. Carol and Dave update their channel with an e.g. 100-block HTLC that transfers 0.001 BTC from Carol to Dave.
- 5. Dave discloses R to Carol; he obtains 0.001 BTC from the 100-block HTLC transaction.
- 6. Carol discloses R to Bob; she obtains 0.001 BTC from the 200-block HTLC transaction.
- 7. Bob discloses R to Alice; he obtains 0.001 BTC from the 300-block HTLC transaction.

Thus Alice has paid Dave 0.001. No party can be defrauded: For example, Carol will pay 0.001 BTC to Dave if he shows her R within 100 blocks but then she can take the 0.001 BTC back by disclosing R to Bob; she has at least 100 more blocks to do so. In case Dave does not disclose R, all parties can take their funds back by settling on-chain.

On the other hand, assume that Bob does not cooperate after the establishment of the HTLC transactions, but keeps R hidden. In this

case *Bob* will lose his 0.001 BTC to *Carol* and no other player will be negatively affected; *Carol* and *Dave* can fulfill their part without *Bob*'s cooperation, albeit *Carol* will have to wait for her channel with *Bob* to expire, since she has to settle on-chain. Likewise *Alice* can take back her 0.001 after the 300-block HTLC lock has expired. Thus no trust between parties is needed.

One can note three things: Firstly, there is no such thing as a persistent multi-hop channel. The whole procedure must be repeated for each subsequent multi-hop payment and the successful completion of one such payment does not facilitate the creation of future payments along the same route as far as the techniques described above are concerned. Nevertheless, previous cooperation between players can obviate the need of exploring the network anew for a connecting series of preexisting channels.

Secondly, merely the existence of a channel is not enough to ensure that multi-hop payments can be achieved through it. It must be the case that the correct player holds at least as much funds as the desired payment, which can only be verified by asking the players of the channel, since the latest state is not public. Thus, in the previous example, Bob must own at least 0.001 BTC in the  $Bob \Leftrightarrow Charlie$  channel in order for the payment to be possible. Alice (or Dave) must ask Bob and Charlie whether this is the case before initiating the multi-hop payment process.

Finally, all intermediate players have to actively engage for a multihop payment to go through. This means that a multi-hop payment's latency increases linearly with the length of the chain, as well as the waiting time if on-chain settlement is needed (given that the same margin of security is desired irrespective of the payment length). This reduces the scalability of the design and fosters the creation of centralized, heavily connected players that ensure that short chains are available instead of distributed, loosely connected players that exchange funds through long chains.

### 3 Perun

Perun [?] is a payment network designed for Turing-complete smart contract scripting languages. It has been implemented for Ethereum. Its main contribution is *multistate channels* that allow the dynamic deployment of virtual contracts, known as *nanocontracts*. Contracts of this type do not have to enter the blockchain if all parties are cooperative and only do so in case of a dispute.

The paper describes specifically the use of such multistate channels for creating virtual payment channels between parties that do not have a basic payment channel between them, but both have basic multistate channels with an intermediary. Then the intermediary could substitute for the blockchain and thus a virtual payment channel on top of the two basic multistate channels can be created. The parties need the intermediary only for setting up the channel and to close it fast. If the intermediary refuses to close the channel, they can always fall back to the blockchain in order to close it.

#### 3.1 Basic payment channels

A basic payment channel is a tuple

$$\gamma = (\gamma.id, \gamma.Alice, \gamma.Bob, \gamma.cash, \gamma.ver-num, \gamma.sign)$$

Versions of this tuple are held by Alice and Bob.  $\gamma$ .id is a unique identifier for the channel,  $\gamma$ . Alice and  $\gamma$ . Bob are the end-users of  $\gamma$  and  $\gamma$ . cash is a function from the end-users to a real non-negative value that denotes the amount of cash the user has in the channel.  $\gamma$ . ver-num is a number that is incremented with each channel update (so that the latest state of the channel is known in case of dispute) and  $\gamma$ .sign is the singature of the other party on  $(\gamma.id, \gamma.cash, \gamma.ver-num)$ .

A payment channel has a corresponding PaymentContract $_{\gamma.id}$  on the ledger. End-users interact with the contract only to set up and close the channel, whereas updating the channel happens off-chain. The contract does not contain the fields  $\gamma.ver-num$  and  $\gamma.sign$ ; the two fields are kept only by the end-users.

#### Channel creation

The procedure of creating a channel is as follows:

- 1. Alice creates a PaymentContract  $(\gamma)$ , pays it  $\gamma$ .cash  $(\gamma$ .Alice) coins and broadcasts it on the ledger. The fields  $\gamma$ .ver-num and  $\gamma$ .sign are not included.
- 2. The contract sends the message (initialising,  $\gamma$ ) to both end-users ( $\gamma$ .Alice and  $\gamma$ .Bob).
- 3. Bob calls the confirm() function of the contract and pays it the already specified amount of  $\gamma$ .cash ( $\gamma$ .Bob) coins.
- 4. The contract sends the message (initialised,  $\gamma$ ) to both end-users.

5. If Alice does not receive (initialised,  $\gamma$ ) after a predefined period  $\Delta$  has passed from receiving (initialising,  $\gamma$ ), she calls the contract function refund() and gets her deposit back.

Note that *Alice* can get her money back if *Bob* does not cooperate and *Bob* only pays the contract after he verifies that *Alice* has set up everything correctly. The contract code is public and thus end-users do not engage with it if it does not correspond to the expected code; no trust towards the contract is needed.

### Channel update

Assume that the end-users want to update an existing channel balance from  $\gamma$ .cash to cash', where the total channel balance has remained unchanged:

$$\gamma. \operatorname{cash}(\gamma. \operatorname{Alice}) + \gamma. \operatorname{cash}(\gamma. \operatorname{Bob}) = \operatorname{cash}'(\gamma. \operatorname{Alice}) + \operatorname{cash}'(\gamma. \operatorname{Bob})$$

The procedure of updating to the new balance is as follows:

- 1. Alice builds a new channel tuple  $\gamma^{Alice}$  where
  - the fields id and users are as in  $\gamma$ ,
  - $-\gamma^{Alice}$ .cash = cash',
  - $-\gamma^{Alice}$ .ver-num  $=\gamma$ .ver-num +1 and
  - $-\gamma^{Alice}. \mathtt{sign} \text{ is } Alice\text{'s signature on } \\ \left(\gamma^{Alice}. \mathtt{id}, \gamma^{Alice}. \mathtt{cash}, \gamma^{Alice}. \mathtt{ver-num}\right).$
- 2. Alice sends  $\gamma^{Alice}$  to Bob and waits for his response.
- 3. Bob checks that all fields are as expected and replaces the old channel tuple,  $\gamma$ , with the newly tuple,  $\gamma^{Alice}$ . From his point of view, the payment has gone through.
- 4. Bob sends to Alice the updated channel,  $\gamma^{Bob}$ , of which all fields are the same as  $\gamma^{Alice}$  except for  $\gamma^{Bob}.sign$ , which is Bob's signature on  $\left(\gamma^{Bob}.id,\gamma^{Bob}.cash,\gamma^{Bob}.ver-num\right)$ .
- 5. If Alice receives the expected  $\gamma^{Bob}$ , she replaces the old channel tuple with  $\gamma^{Bob}$ . From her point of view, the payment has gone through.

The above description holds symmetrically if *Bob* initiates the channel update. If any player diverges from these steps, the other player can assume that the first has been corrupted and should close the channel immediately.

Note that after the first update, the channel tuples held by the two players are not the same, their only difference being in the signature field.

Strictly speaking, this means that the description of updating a channel above abuses the notation when it refers to  $\gamma$  as the common previous channel state.

Also note that the following scenario may arise: Alice sends the updated version of the channel along with her signature, but Bob does not reply. In this case, Alice wants to close the channel since Bob is assumed to be corrupt, but the latest state of which she has Bob's signature is one version earlier than Bob's latest state. The only way Alice can retrieve her funds is by broadcasting this older state. Bob can then broadcast his latest state, which supersedes Alice's state. From the point of view of the blockchain, Alice has tried to close the channel with an older state.

Since there is a situation where the blockchain cannot say which player was corrupt, *Alice* cannot be punished for broadcasting an older state of the channel by losing all her funds in the channel. She should be entitled to her share, as defined by the latest channel state that has been broadcast. Thus the punishment scheme of Lightning cannot be applied here.

#### Closing the channel

Finally, we present the procedure of closing a channel.

- 1. Alice calls the function  $\operatorname{close}\left(\gamma^{Alice}\right)$  of PaymentContract<sub> $\gamma$ .id</sub>.
- 2. PaymentContract $_{\gamma.id}$  checks that  $\gamma^{Alice}$  is correctly formed and holds the same total balance as the initial channel recorded in the contract. If so, it accepts  $\gamma^{Alice}$  as the channel state. Additionally, Bob can call close () at any time and either Alice or Bob can call finalize () after time  $\Delta$  has passed. If  $\gamma^{Alice}$  is does not pass the checks, the contract ignores the call.
- 3. If Bob disagrees with the channel state published by Alice, he calls  $close(\gamma^{Bob})$  of PaymentContract<sub> $\gamma$ .id</sub>.
- 4. Upon receiving a close  $(\gamma^{Bob})$  call from Bob, PaymentContract $_{\gamma.id}$  checks that  $\gamma^{Bob}$  is correctly formed, holds the same total balance as the initial channel recorded in the contract and additionally has a higher version number than  $\gamma^{Alice}$ . If so, it accepts  $\gamma^{Bob}$  as the channel state. Either Alice or Bob can still call finalize () after time  $\Delta$  from Alice's original close () call has passed. If  $\gamma^{Bob}$  does not pass the checks, the contract ignores the call.
- 5. After time  $\Delta$  has passed, either end-user can call finalize() of PaymentContract<sub> $\gamma$ .id</sub>.

6. Upon receiving a finalize() call from either end-user, the contract PaymentContract $_{\gamma.id}$  checks that time  $\Delta$  has passed since the original close(). If so, it sends closed and  $\gamma.cash(P)$  to each end-user P. If not, it ignores the finalize() call.

The above closing sequence gives Bob a window of duration at least  $\Delta$  to dispute the closing channel state reported by Alice.

Note that, in contrast to Lightning, there is no provision for cooperative closing of a channel, thus a delay of  $\Delta$  must always be incurred between initiating a channel closure and getting access to the funds. The parameter  $\Delta$  is decided by the parties when the channel is created and presents the same tradeoffs as the parameter n of Lightning.

#### 3.2 Multistate channels

A basic multistate channel is a tuple

$$\gamma = (\gamma.id, \gamma.Alice, \gamma.Bob, \gamma.cash, \gamma.nspace)$$
,

where  $\gamma$ .id,  $\gamma$ .Alice,  $\gamma$ .Bob and  $\gamma$ .cash are as in a payment channel and  $\gamma$ .nspace is a set of nanocontracts, or *nanocontracts space*.

The multistate channel  $\gamma$  has a corresponding contract MSContract $_{\gamma.id}$  on the ledger. The end-users have to interact with this contract upon channel creation, channel closure and in case of dispute over the state of a nanocontract. Note that the end-users can create new nanocontracts, as well as cooperatively update them, without touching the ledger.

A nanocontract  $\nu \in \gamma$ .nspace is a tuple

$$\nu = (\nu.\text{nid}, \nu.\text{blocked}, \nu.\text{storage}, \nu.\text{ver-num}, \nu.\text{sign})$$
,

where  $\nu.nid$  is a globally unique identifier of the nanocontract,  $\nu.blocked$  is a function from the end-users of the multistate channel to a real nonnegative value that denotes the amount of cash the end-user has in the nanocontract and  $\nu.storage$  contains the storage of the nanocontract. Like simple payment channels, the nanocontract with the highest  $\nu.ver-num$  and a valid  $\nu.sign$  will be accepted by the blockchain in case of registration of the state of the nanocontract on the ledger.

#### Nanocontract creation and update

The update mechanism for a nanocontract is similar to the update mechanism of a simple payment channel and thus will not be explained in detail. The only substantial differences are the following:

- 1. After Alice proposes a nanocontract update, Bob has time  $\Upsilon$  to reply whether he agrees with this update or not. If he agrees the update goes through, else the state of the nanocontract is not updated (apart from increasing the version number). This is not considered a dispute, so (on-chain) nanocontract state registration does not need to take place.
- 2. In case of a successful update, the cash balance of both end-users in the underlying multistate channel  $(\gamma.\mathtt{cash}\,(Alice))$  and  $\gamma.\mathtt{cash}\,(Alice))$  are updated to reflect the fact that the nanocontract update has consumed or returned some funds to the end-users.

Each nanocontract has its own  $\nu$ -ver-num and  $\nu$ -sign field, so that several nanocontracts of the same multistate channel can be updated in parallel. Let  $\nu'$  be the state of the nanocontract  $\nu$  after an update. The only requirement is that

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\nu'.blocked (Alice) + \nu'.blocked (Bob) \le \nu.blocked (Alice) + \nu.blocked (Bob).
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This ensures that no two nanocontracts will together require more funds than are available in the multistate channel and thus that all nanocontracts can be updated in parallel. If it is the case that  $\nu$ .blocked (Alice)+ $\nu$ .blocked (Bob) = 0, we say that the nanocontract  $\nu$  is terminated.

To create a new nanocontract  $\nu$ , users simply apply the update mechanism. They have to ensure that  $\nu$ .nid is a new, globally unique identifier and that the  $\nu$ .ver-num = 0.

#### Nanocontract registration

Registration of the state of a nanocontract on MSContract happens in case of dispute with regard to the state of the nanocontract or when the parties want to close the multistate channel. The registration mechanism is very similar to that of closing a simple payment channel, so will not be described in detail.

Given that a nanocontract  $\nu$  is registered on the MSContract of the underlying multistate channel of  $\nu$ , any end-user can unilaterally execute a nanocontract function fun by calling the MSContract function execute  $(\nu.nid, fun, z)$ . MSContract then updates the state of the nanocontract on the ledger and returns the output to the end-users.

#### Nanocontract termination

Finally, when the end-users wish to close the multistate channel, they have to update all nanocontracts such that they are terminated, register their state (or alternatively register their state and then execute functions on the ledger until they are terminated) and then initiate a procedure similar to the closing of basic payment channels, which gives the opportunity to both end-users to publish their latest version of all nanocontracts of the multistate channel. The nanocontract states with the highest version number are accepted by the ledger as valid. Each end-user receives coins equal to the initial coins they contributed to the multistate channel, amended by the changes introduced by the nanocontracts. These coins are now available to use with other users of the ledger.

It may be the case that some nanocontracts cannot be updated to a terminated state due to dispute between end-users or design problems of the nanocontract. The end-users can have special provision in MSContract for such cases to be able to kill such misbehaving nanocontracts and distribute the funds in a predefined manner. Such a mechanism is not explicitly specified.

#### 3.3 Virtual payment channels

Virtual payment channels are channels created on top of suitable preexisting multistate channels that facilitate trustless funds exchange between parties that do not share an on-chain channel. Going into more detail, a virtual payment channel  $\gamma$  is a tuple:

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(\gamma.\mathtt{id}, \gamma.\mathtt{Alice}, \gamma.\mathtt{Ingrid}, \gamma.\mathtt{Bob}, \gamma.\mathtt{cash}, \gamma.\mathtt{subchan}, \\ \gamma.\mathtt{validity}, \gamma.\mathtt{ver-num}, \gamma.\mathtt{sign})
```

Let Alice have a multistate channel with Ingrid (Alice  $\stackrel{\gamma_a}{\Leftrightarrow} Ingrid$ ); also let Bob have a multistate channel with Ingrid ( $Bob \stackrel{\gamma_b}{\Leftrightarrow} Ingrid$ ).

#### Channel creation

To build  $\gamma$ , two nanocontracts  $\nu_a$  and  $\nu_b$  are created, each on the corresponding multistate channel.  $\nu_a$  has  $\gamma.\mathtt{cash}\,(Alice)$  blocked by Alice and  $\gamma.\mathtt{cash}\,(Bob)$  blocked by Ingrid. Similarly,  $\nu_b$  has  $\gamma.\mathtt{cash}\,(Alice)$  blocked by Ingrid and  $\gamma.\mathtt{cash}\,(Bob)$  blocked by Bob. At a high level, a virtual payment channel creation protocol is as follows:

0. Alice and Bob discover that Ingrid is an intermediary. They also agree on the initial balance of  $\gamma$ .

- 1. Alice sends a signed  $\nu_a$  to Ingrid.
- 2. Ingrid sends a signed  $\nu_b$  to Bob.
- 3. Bob replies to Ingrid with  $\nu_b$ , signed by the former.
- 4. Ingrid replies to Alice with  $\nu_a$ , signed by the former.

We now say that there exists the virtual payment channel  $\gamma$  between Alice and Bob (Alice  $\stackrel{\gamma}{\leftrightarrow}$  Bob).

#### Channel update

Updating the cash balance of the channel can be accomplished in the same way as for the basic payment channels, with both *Alice* and *Bob* signing the new state with an incremented version number. Note that, in contrast to Lightning, the end-users do not need to interact with *Ingrid* at all to update the channel. This decreases the number of rounds needed for an update to 2 in the optimistic case that both *Alice* and *Bob* are honest. Furthermore, it somewhat increases the privacy of the end-users.

### Closing the channel

In case all three parties are honest, they agree that they want to close  $\gamma$  and  $\gamma$ -validity time has not yet passed, then both Alice and Bob attempt to terminate their respective nanocontract with Ingrid,  $\nu_a$  and  $\nu_b$ . The end-users send their latest version of  $\gamma$  to the intermediary, who expects a tuple with valid signatures by the initially registered end-users and a total balance equal to that of the initial state of the channel. If both end-users send different valid tuples, then Ingrid chooses the one with the higher version number as valid. Thus both nanocontracts can be terminated with each of the end-users unblocking their respective balance, as defined by the valid  $\gamma$ , on their multistate channel with Ingrid. The sum of coins Ingrid will unblock in both multistate channels will be equal to the sum of the original coins she blocked during the virtual payment channel creation, only redistributed between the two channels as defined by the latest  $\gamma$  state.

Alice can unilaterally register the  $\nu_a$  nanocontract state on the ledger and provide her latest  $\gamma$  version, thus she does not need Ingrid's cooperation to unblock her funds. If Ingrid learns a newer valid  $\gamma$  version from Bob (which means that Alice tried to cheat), then Ingrid can publish it within a predetermined timeframe and claim her rightful funds.

Furthermore, if the channel hasn't closed after  $\gamma$ .validity time has passed, any of the three parties can unilaterally finalize the nanocontract(s) she has access to and unblock the respective funds.

Thus we have seen that no trust between the parties is necessary. Similarly to Lightning though, cooperating parties can unblock their funds faster and with less interaction with the ledger (and thus lower fees).

### 4 Sprites

Sprites [?] constitute an improvement upon Lightning [?] regarding the worst-case time needed to settle in case of a dispute. Consider a channel of l hops, where  $\Delta$  is the time given to each participant to publish their state after a counterparty has unilaterally broadcast theirs. The worst-case time to settle in the case of Lightning is  $\Theta(l\Delta)$ , whereas in Sprites it is  $\Theta(l+\Delta)$ .

To achieve this, Sprites propose a smart contract called Preimage Manager (PM). Let  $\mathcal{H}(\cdot)$  be a suitable hash function. Parties can interact with PM in the following way:

- Call publish(x) at time T: PM stores timestamp  $[\mathcal{H}(x)] = T$ .
- Call published(h,T): PM returns True if
  - $h \in \mathtt{timestamp}$  and
  - timestamp  $[h] \leq T$ ,

False otherwise.

In case all parties are honest, PM is not invoked, the entire interaction happens off-chain and needs l+1 rounds to complete. In case a party misbehaves by delaying sending the preimage until the last possible moment (i.e. time  $\Delta$  after she received the preimage from the previous link), she will have to publish the preimage to the blockchain instead of just sharing it with the next link in the chain of payments in order to ensure she gets her funds. Thus, the rest of the (honest) players can settle the channel by asking PM whether the hash they already know has been published(). This action can be completed concurrently, thus the maximum delay that can be incurred is  $l+\Delta$ .

### 5 General properties of Payment Channels

- 1. Number of participants in the channel
- 2. On-chain connection(s) between participants
- 3. Actions: open, update, execute, close

- 4. Who needs to sign for each action, who is notified, how many rounds of communication?
- 5. What information can one obtain by observing the blockchain?
- 6. Under what circumstances an operation cannot complete? (e.g. concurrency issues)
- 7. Which participants are aware of the identity of which participants?
- 8. Is there an upper bound to the amount of updates? How is this number decided?
- 9. Can a participant unilaterally commit on-chain?
- 10. Up to how much money can a participant unilaterally obtain?
- 11. What can a malicious party do? If it corrupts more participants it can do more?
- 12. Can a malicious/honest-but-curious party that is a participant learn who is transacting with who?
- 13. How much slower is the process in case of a malicious party?
- 14. How expensive are the actions? (CPU, memory, storage)
- 15. How expensive are interactions with the blockchain? (fees, time, etc.)

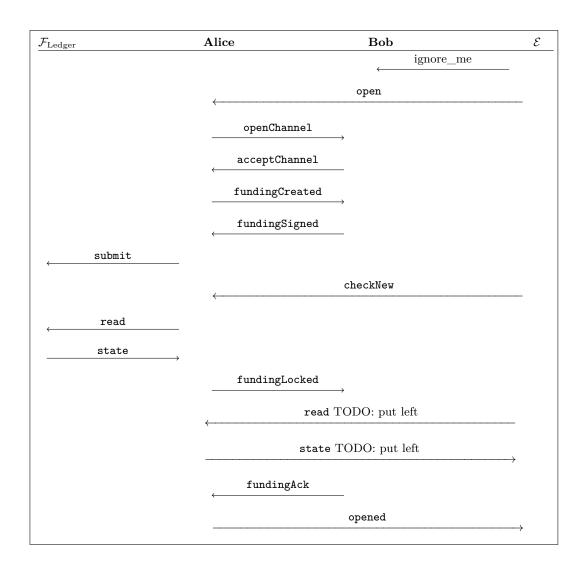


Fig. 1. Protocol for opening a Lightning channel

?,	Alice	Bob	Charlie
$\begin{array}{c} \texttt{pay}(Bob\&Charlie) \\ \longrightarrow \end{array}$			
	updateAddHTLC		
	commitmentSigned		
	revokeAndAck		
commit			
,	commitmentSigned		
	revokeAndAck		
	<u> </u>		
		push	
		updateAddHTLC	<del></del>
		-	•
		$\xleftarrow{\texttt{commitmentSigned}}$	
		revokeAndAck	
		commit	
		commitmentSigned	•
		revokeAndAck	
	-	push	
		$\overset{\text{updateFulfillHTLC}}{\leftarrow}$	
		commitmentSigned	
		revokeAndAck	•
		<del>(</del>	-
		commit	A
		commitmentSigned	
	18	<del></del>	
		revokeAndAck	
		push	
		F 200.0	

#### 6 State of a channel

Consider a channel between *Alice* and *Bob*. Both parties hold some data locally that correspond to ownership of some funds in the channel. Here we define a concise way of representing this data.

What *Alice* really holds, specific for this channel:

```
local {payment, htlc, delayed_payment}_basepoint_secret
 - remote {payment, htlc, delayed_payment}_basepoint
 - local per_commitment_secret[1,...,n+2]
 - remote per commitment point[1,...,n]
 - more stuff
\Pi_{\rm LN} (self is Alice)
 1: Initialisation:
        channels, pendingOpen, pendingPay, pendingClose, newChannels,
    \verb|closedChannels|, \verb|unclaimedOfferedHTLCs|, \verb|unclaimedRece|| ivedHTLCs|, \verb|pendingGetPaid| \leftarrow |
 3:
 4: Upon receiving (REGISTER, delay) from \mathcal{E}:
        delay \leftarrow delay
 5:
        send (READ) to \mathcal{G}_{\mathrm{Ledger}} and assign largest block number to lastPoll
 6:
        (pk_{Alice}, sk_{Alice}) \leftarrow \text{genKey}()
 7:
        send (REGISTER, Alice, delay, pk_{Alice}) to A [Orfeas: delay to A
    needed?
 9:
10: Upon receiving (REGISTERED) from A:
        send (READ) to \mathcal{G}_{\text{Ledger}} and assign reply to \Sigma_{Alice}
11:
        assign the sum of all output values that are exclusively spendable
12:
    by Alice to onChainBalance
        send (REGISTERED) to {\cal E}
13:
14:
    Upon receiving any message (M) except for (REGISTER):
        if if haven't received (REGISTER) from \mathcal{E} then
16:
            send (INVALID, M) to \mathcal{A} and ignore message
17:
        end if
18:
19:
20: Upon receiving (OPENCHANNEL, pid<sub>Alice</sub>, pid<sub>Bob</sub>, x) from \mathcal{E}:
```

```
((ph_F, sh_F), (ph_base_{pay}, sh_base_{pay}), (ph_base_{dpay}, sh_base_{dpay}),
21:
      (ph\_base_{htlc}, sh\_base_{htlc}), (ph_{com,1}, sh_{com,1}), (ph\_base_{rev}, sh\_base_{rev})) \leftarrow
     GETKEYS () [TODO: change font]
22:
          choose unique temporary ID tid

    □ unique for the two parties

23:
          associate keys with tid
          \operatorname{add}\left(Alice, Bob, x, \mathtt{delay}, (ph_F, sh_F), (ph\_\operatorname{base}_{\operatorname{pay}}, sh\_\operatorname{base}_{\operatorname{pay}}\right), (ph\_\operatorname{base}_{\operatorname{dpay}}, sh_F)
24:
      sh\_base_{dpav}, (ph\_base_{htlc}, sh\_base_{htlc}), (ph\_base_{com,1}, sh\_base_{com,1}),
      (ph\_base_{rev}, sh\_base_{rev}), tid) to pendingOpen
          send (OPENCHANNEL, x, delay, ph_F, ph_base_{pay}, ph_base_{dpay}, ph_base_{htle},
25:
     ph_{\text{com},1}, ph \text{ base}_{\text{rev}}, tid) to Bob and A
26:
27: Upon receiving (OPENCHANNEL, x, BobDelay, pt_F, pt_{base_{pay}}, pt_{base_{dpay}},
     pt\_base_{htlc}, pt_{com.1}, pt\_base_{rev}, tid) from Bob:
          ensure tid has not been used yet with Bob
28:
          ((ph_F, sh_F), (ph_base_{pay}, sh_base_{pay}), (ph_base_{dpay}, sh_base_{dpay}),
29:
      (ph\_base_{htlc}, sh\_base_{htlc}), (ph_{com,1}, sh_{com,1}), (ph\_base_{rev}, sh\_base_{rev})) \leftarrow
     GETKEYS () [TODO: change font]
          associate keys with tid and store in pendingOpen
30:
          send (ACCEPTCHANNEL, delay, ph_F, ph_base_{pay}, ph_base_{dpay}, ph_base_{htlc},
31:
     ph_{\text{com},1}, ph\_\text{base}_{\text{rev}}, tid) to Bob and A
32:
33: Upon receiving (ACCEPTCHANNEL, delay, pt_F, pt_base_{pay}, pt_base_{dpay},
     pt\_base_{htlc}, pt_{com,1}, pt\_base_{rev}, tid) from Bob:
34:
          ensure there is a temporary ID tid with Bob in pendingOpen on
     which ACCEPTCHANNEL hasn't been received
          associate received keys with tid
35:
          send (READ) to \mathcal{G}_{\text{Ledger}} and assign reply to \Sigma_{Alice}
36:
          assign to prevout a transaction output found in \Sigma_{Alice} that is
37:
     currently exclusively spendable by Alice and has value y \geq x
          F \leftarrow TX {input spends prevout with a signature(TX, sk_{Alice}),
38:
     output 0 pays y-x to pk_{Alice} now, output 1 pays x to tid.ph_F \wedge pt_F
          pchid \leftarrow \mathcal{H}(F)
39:
          replace tid with pchid in storage
40:
          pt_{\text{rev},1} \leftarrow pt\_\text{base}_{\text{rev}} \cdot \mathcal{H} \left( pt\_\text{base}_{\text{rev}} || pt_{\text{com},1} \right) + pt_{\text{com},1} \cdot \mathcal{H} \left( pt_{\text{com},1} || pt\_\text{base}_{\text{rev}} \right)
41:
         ph_{\text{dpay},1} \leftarrow ph_{\text{base}_{\text{dpay}}} + \mathcal{H}\left(ph_{\text{com},1}||ph_{\text{base}_{\text{dpay}}}\right)
42:
43:
          ph_{\text{pay},1} \leftarrow ph\_\text{base}_{\text{pay}} + \mathcal{H}\left(ph_{\text{com},1}||ph\_\text{base}_{\text{pay}}\right)
          remoteCom, remoteCom<sub>1</sub> \leftarrow TX {input: output 1 of F, output:
44:
     (x, ph_{\text{pav},1})
```

```
localCom \leftarrow TX \{localCom to TX \} (input: output 1 of F, output: (x, pt_{rev, 1} \lor (ph_{doay, 1}, delay relative))
45:
     [TODO: check delays]
          add remoteCom and localCom to channel entry in pendingOpen
46:
47:
          \operatorname{sig} \leftarrow \operatorname{signature}(\operatorname{remoteCom}_1, sh_F)
48:
          lastRemoteSigned \leftarrow 0
          send (FUNDINGCREATED, tid, pchid, sig) to Bob and A
49:
50:
51: Upon receiving (FUNDINGCREATED, tid, pchid, BobSig<sub>1</sub>) from Bob:
52:
          ensure there is a temporary ID tid with Bob in pendingOpen on
     which we have sent up to ACCEPTCHANNEL
          ph_{\text{rev},1} \leftarrow ph\_\text{base}_{\text{rev}} \cdot \mathcal{H} (ph\_\text{base}_{\text{rev}} || ph_{\text{com},1}) + ph_{\text{com},1} \cdot \mathcal{H} (ph_{\text{com},1} || ph\_\text{base}_{\text{rev}})
53:
         pt_{\text{dpay},1} \leftarrow pt\_\text{base}_{\text{dpay}} + \mathcal{H}\left(pt_{\text{com},1}||pt\_\text{base}_{\text{dpay}}\right)
54:
          pt_{\text{pay},1} \leftarrow pt\_\text{base}_{\text{pay}} + \mathcal{H}\left(pt_{\text{com},1} || pt\_\text{base}_{\text{pay}}\right)
55:
          localCom, localCom_1 \leftarrow TX \{input: output 1 of F, output: (x, pt_{pay,1})\}
56:
          ensure verify (localCom<sub>1</sub>, BobSig<sub>1</sub>, pt_F) = True
57:
          remoteCom, remoteCom<sub>1</sub> \leftarrow TX {input: output 1 of F, output:
58:
     (x, ph_{rev} \lor (pt_{dpay}, delay relative)) [Orfeas: check delays]
          add BobSig<sub>1</sub>, remoteCom<sub>1</sub> and localCom<sub>1</sub> to channel entry in pendingOpen
59:
60:
          sig \leftarrow signature (remoteCom_1, sh_F)
          mark channel as "broadcast, no fundingLocked"
61:
          lastRemoteSigned, lastLocalSigned \leftarrow 0
62:
          send (FUNDINGSIGNED, pchid, sig) to Bob and A
63:
64:
65: Upon receiving (FUNDINGSIGNED, pchid, BobSig<sub>1</sub>) from Bob:
          ensure there is a channel ID pchid with Bob in pendingOpen on
66.
     which we have sent up to FUNDINGCREATED
          ensure verify (localCom, BobSig<sub>1</sub>, pb_F) = True
67:
          \texttt{localCom}_1 \leftarrow \texttt{localCom}
68:
          lastLocalSigned \leftarrow 0
69:
          add BobSig<sub>1</sub> to channel entry in pendingOpen
70:
71:
          sig \leftarrow signature(F, sk_{Alice})
          mark pchid in pendingOpen as "broadcast, no FUNDINGLOCKED"
72:
73:
          send (SUBMIT, (\text{sig}, F)) to \mathcal{G}_{\text{Ledger}}
74:
    function Getkeys [Todo: change font]
75:
          (p_F, s_F) \leftarrow \text{genKey}()
                                                                               \triangleright For F output
76:
                                                             ▶ For com output to remote
77:
          (p_{\text{pay}}, s_{\text{pay}}) \leftarrow \text{genKey}()
78:
          (p_{\text{dpay}}, s_{\text{dpay}}) \leftarrow \text{genKey}()
                                                                  ▶ For com output to self
          (p_{\text{htlc}}, s_{\text{htlc}}) \leftarrow \text{genKey}()
                                                                   ▶ For htlc output to self
79:
```

```
(p_{\text{com},1}, s_{\text{com},1}) \leftarrow \text{genKey}()
                                                             ▶ For deriving all keys
80:
81:
         (p_{\text{rev}}, s_{\text{rev}}) \leftarrow \text{genKey}()
                                                           ▶ For revocation in com
         return ((p_F, s_F), (p_{\text{pay}}, s_{\text{pay}}), (p_{\text{dpay}}, s_{\text{dpay}}),
82:
             (p_{\text{htlc}}, s_{\text{htlc}}), (p_{\text{com},1}, s_{\text{com},1}), (p_{\text{rev}}, s_{\text{rev}}))
83:
    end function
84:
85:
     [TODO: explicitly add keys et al to channel]
87: Upon receiving (CHECKNEW, Alice, Bob, x) from \mathcal{E}: \triangleright new message:
    represents lnd polling daemon
         ensure there is a matching channel in pendingOpen with id pchid
88:
    with a "broadcast" mark
         send (READ) to \mathcal{G}_{\text{Ledger}} and assign reply to \Sigma_{Alice}
89:
         ensure \exists unspent TX in \Sigma_{Alice} with ID pchid and a (x, ph_F \wedge pt_F)
90:
    output
         (ph\_base_{com,2}, sh\_base_{com,2}) \leftarrow genKey()
91:
         add TX to channel data
92:
         replace "broadcast" mark in channel with "in state"
93:
         if channel is marked as "in state, FUNDINGLOCKED" then
94:
             move channel data from pendingOpen to channels
95:
             add receipt of channel to newChannels
96:
         end if
97:
         send (fundingLocked, pchid, ph_base<sub>com,2</sub>) to Bob
98:
99:
100: Upon receiving (FUNDINGLOCKED, pchid, pt_base<sub>com.2</sub>) from Bob:
         ensure there is a channel with ID pchid with Bob in pendingOpen
101:
    with a "no fundingLocked" mark
         ensure pk\left(st_{\text{com},n}\right) = pt_{\text{com},n}
102:
         replace "no FundingLocked" mark in channel with "FundingLocked"
103:
         ensure channel has an "in state" mark
104:
         generate 2nd remote delayed payment, htlc, payment keys
105:
         add TX to channel data
106:
         move channel data from pendingOpen to channels
107:
         add receipt of channel to newChannels
108:
109:
110: Upon receiving (POLL) from \mathcal{E}:
         send (READ) to \mathcal{G}_{\text{Ledger}} and assign reply to \Sigma_{Alice}
111:
         assign largest block number in \Sigma_{Alice} to lastPoll
112:
         \texttt{toSubmit} \leftarrow \emptyset
113:
114:
         for all \tau \in \mathtt{unclaimedOfferedHTLCs} do
```

```
if input of \tau has been spent then \triangleright by remoteHTLC-success
115:
116:
                  remove \ 	au \ from \ unclaimedOfferedHTLCs
             else if input of \tau has not been spent and timelock is over
117:
    then
                  remove \tau from unclaimedOfferedHTLCs
118:
119:
                  add \tau to toSubmit
             end if
120:
         end for
121:
         for all remoteCom_n \in \Sigma_{Alice} that spend F of a channel \in channels
122:
    do
123:
             if we do not have sh_{rev,n} then
                                                                    ▶ Honest closure
                  for all received HTLC outputs i of remoteCom<sub>n</sub> do
124:
                      if we know the preimage R then
125:
                          TX \leftarrow \{\text{input: } i \text{ HTLC output of } remoteCom_n \text{ with } \}
126:
    (ph_{\text{htlc},n}, R) as method, output: pk_{Alice}
                          sig \leftarrow signature(TX, sh_{htlc,n})
127:
                          add (sig, TX) to toSubmit
128:
129:
130:
                          \operatorname{add}\left(\operatorname{channel},\operatorname{remoteCom}_n,h,sh_{\operatorname{htlc},n}\right) to unclaimedReceivedHTLCs
                      end if
131:
                  end for
132:
                  for all unspent offered HTLC outputs i of remoteCom<sub>n</sub> do
133:
                      TX \leftarrow \{\text{input: } i \text{ HTLC output of remoteCom}_n \text{ with } \}
134:
    ph_{\text{htlc},n} as method, output: pk_{Alice}
                      sig \leftarrow signature(TX, sh_{htlc,n})
135:
                      if timelock has not expired then
136:
                          add (sig, TX) to unclaimedOfferedHTLCs
137:
138:
                      else if timelock has expired then
                          add (sig, TX) to toSubmit
139:
                      end if
140:
                  end for
141:
142:
             else
                                                                 ▶ malicious closure
                  rev \leftarrow TX {inputs: all remoteCom<sub>n</sub> outputs, choosing ph_{rev,n}
143:
    method, output: pk_{Alice}
                  sig \leftarrow signature(rev, sh_{rev,n})
144:
145:
                  add (sig, rev) to toSubmit
              end if
146:
             move channel from channels to closedChannels
147:
148:
         send (SUBMIT, toSubmit) to \mathcal{G}_{\mathrm{Ledger}}
149:
```

```
150:
     [Orfeas: maybe leak queries to global functionalities to A?]
152: Upon receiving (PAY, Bob, x, path, receipt) from \mathcal{E}:
         ensure that path consists of valid pchids
153:
         ensure that the first pchid \in path has the same pchid as in
154:
    receipt
         ensure that receipt corresponds to the latest version of an open
155:
    channel \in channels in which we have at least x.
         choose unique payment ID payid
                                                   \triangleright unique for Alice and Bob
156:
         add (Bob, x, \overline{path}, receipt, payid, "waiting for invoice") to pendingPay
157:
         send (SENDINVOICE, payid) to Bob and A
158:
159:
160: Upon receiving (SENDINVOICE, payid) from Bob:
         ensure there is no (Bob, payid) entry in pendingGetPaid
161:
        choose random, unique preimage R
162:
         add (Bob, R, payid) to pendingGetPaid
163:
         send (INVOICE, \mathcal{H}(R), payid) to Bob and \mathcal{A}
164:
165:
166: Upon receiving (INVOICE, h, payid) from Bob:
         ensure there is a (Bob, x, \overrightarrow{path}, receipt, payid, "waiting for in-
167:
    voice") entry in pendingPay
168:
         ensure h is valid (in the range of \mathcal{H})
         send (READ) to \mathcal{G}_{\text{Ledger}} and assign largest block number to t
169:
         m \leftarrow \text{the concatenation of length } \overrightarrow{\text{path}} (x, \text{absDelay}_i) \text{ pairs},
170:
    where the last absDelay is t + [Orfeas: TODECIDE] and every
    previous absDelay is incremented by [Orfeas: TODECIDE]
         (\mu_0, \delta_0) \leftarrow \text{SphinxCreate}(m, \text{ public keys of } \overrightarrow{\text{path}} \text{ parties})
171:
        increment \mathrm{HTLCNo}_{pchid} by one and associate x, h, pchid with it
172:
         mark HTLCNo as "sender"
173:
        mark channel as "waiting for COMMITMENTSIGNED"
174:
         send (UPDATEADDHTLC, first pchid of path, HTLCNo<sub>pchid</sub>, x, h, largest
175:
    absDelay, (\mu_0, \delta_0)) to pchid channel counterparty and A
176:
177: Upon receiving (UPDATEADDHTLC, pchid, HTLCNo, x, h, absDelay, M)
         ensure pchid corresponds to an open channel in channels where
178:
    Bob has at least x
         ensure \mathtt{HTLCNo} = \mathtt{HTLCNo}_{pchid} + 1
179:
         ensure absDelay [Orfeas: TODECIDE]
180:
```

```
(pchid', x', absDelay', \delta) \leftarrow SphinxPeel(sk_{Alice}, M)
181:
182:
        if \delta = \text{receiver then}
            ensure pchid' = \bot, x = x', absDelay = absDelay'
183:
            increment \mathtt{HTLCNo}_{pchid} by one
184:
            mark HTLCNo as "receiver'
185:
         else
                                                     ▶ We are an intermediary
186:
            ensure x = x', absDelay = absDelay'+ [Orfeas: TODECIDE]
187:
            ensure pchid' corresponds to an open channel in channels
188:
    where we have at least x
            increment HTLCNo_{pchid} by one
189:
            mark HTLCNo as "intermediary"
190:
         end if
191:
        let remoteCom_n the latest signed remote commitment tx
192:
        reduce delayed payment output in remoteCom by x
193:
194:
        add an (x, ph_{\text{rev},n+1} \lor (ph_{\text{htlc},n+1} \land pt_{\text{htlc},n+1}, \text{cltv\_expiry} \text{ absolute}) \lor
    ph_{\text{htlc},n+1}, on preimage of h) htlc output (all with n+1 keys) to
    remoteCom, marked with HTLCNo
        if \delta = \text{receiver then}
195:
196:
            retrieve R: \mathcal{H}(R) = h from pendingGetPaid
             \operatorname{add} (\operatorname{HTLCNo}, R) \text{ to pendingFulfills}_{pchid}
197:
         else if \delta \neq receiver then
                                                    ▶ Send HTLC to next hop
198:
            retrieve pchid' data
199:
200:
            let remoteCom_n the latest signed remote commitment tx
            reduce simple payment output in remoteCom by x
201:
            add an additional (x, ph_{\text{rev},n+1} \lor (ph_{\text{htlc},n+1} \land pt_{\text{htlc},n+1}, \text{ on preimage})
202:
     of h) \vee ph_{\text{htlc},n+1}cltv_expiry absolute) output (all with n+1 keys)
    to remoteCom, marked with HTLCNo
203:
            calculate absDelay [TODO: ]
            increment HTLCNo by 1
204:
             M' \leftarrow \text{SphinxPrepare}(M, \delta, sk_{Alice})
205:
            send (UPDATEADDHTLC, pchid', HTLCNo, x, h, absDelay, M)
206:
    to pchid' counterparty
         end if
207:
208:
209: Upon receiving (UPDATEFULFILLHTLC, pchid, HTLC_no, R) from
    Bob:
         ensure HTLC\_no \le lastRemoteSigned, HTLC\_no \le lastLocalSigned
210:
         ensure HTLC_no is an offered HTLC (localCom has h tied to a
211:
    public key that we own)
```

```
ensure \mathcal{H}(R) = h, where h is the hash in the HTLC with number
212:
    HTLC no
         add value of HTLC to delayed payment of remoteCom
213:
214:
         remove HTLC output with number HTLC_no from remoteCom
         if we have a channel phcid' that has a received HTLC with hash
215:
    h with number HTLCNo' then
                                                           ▶ We are intermediary
             if HTLCNo' ≤ lastRemoteSigned' then ▷ HTLC committed
216:
                 send (READ) to \mathcal{G}_{\text{Ledger}} and assign reply to \Sigma_{Alice}
217:
                 if latest remoteCom'_n \in \Sigma_{Alice} then
                                                               ▷ counterparty has
218:
    gone on-chain
                     TX \leftarrow \{\text{input: remoteCom' HTLC output with number}\}
219:
    HTLCNo', output: pk_{Alice}
                     sig \leftarrow signature(TX, sh_{htlc,n})
220:
                     send (SUBMIT, (sig, R, TX)) to \mathcal{G}_{Ledger} \triangleright \text{shouldn't be}
221:
    already spent by remote HTLCTimeout
                                                    ▷ counterparty still off-chain
                 else
222:
                     send (UPDATEFULFILLHTLC, pchid', HTLCNo, R) to coun-
223:
    terparty
224:
                 end if
225:
             else
                                       ▶ we haven't received REVOKEANDACK
                 add (HTLCNo', R) to pendingFulfills<sub>pchid'</sub> [TODO: send
226:
    all after rAA]
             end if
227:
         end if
228:
229:
230: Upon receiving (COMMIT, pchid) from \mathcal{E}:
231:
         ensure that there is a channel \in channels with ID pchid
         retrieve latest remote commitment tx remoteCom, in channel
232:
         ensure remoteCom \neq remoteCom<sub>n</sub>

    b there are uncommitted

233:
    updates
         ensure channel is not marked as "waiting for REVOKEANDACK"
234:
         remoteCom_{n+1} \leftarrow remoteCom
235:
236:
         ComSig \leftarrow signature (remoteCom_{n+1}, sh_F)
         HTLCSigs \leftarrow \emptyset
237:
238:
         {f for}\ i\ {
m from}\ {
m lastRemoteSigned}\ {
m to}\ {
m HTLCNo}\ {f do}
             \texttt{remoteHTLC}_{n+1.i} \leftarrow \texttt{TX} \left\{ \texttt{input: HTLC output} \ i \ \texttt{of remoteCom}_{n+1}, \right.
239:
    output: (c_{\text{htlc,i}}, ph_{\text{rev},n+1} \lor (pt_{\text{dpay},n+1}, \text{delay relative}))
             add signature (remoteHTLC_{n+1,i}, sh_{\text{htlc},n+1}) to HTLCSigs
240:
241:
         add signature (remoteHTLC_{n+1,m+1}, sh_{\text{htlc},n+1}) to HTLCSigs
242:
```

```
\texttt{lastRemoteSigned} \leftarrow \texttt{HTLCNo}
243:
244:
            mark channel as "waiting for REVOKEANDACK"
            send (COMMITMENTSIGNED, pchid, ComSig, HTLCSigs) to pchid
245:
      counterparty
246:
247: Upon receiving (COMMITMENT SIGNED, pchid, comSig<sub>n+1</sub>, HTLCSigs<sub>n+1</sub>)
      from Bob:
            ensure that there is a channel \in channels with ID pchid with
248:
      Bob
249:
            retrieve latest local commitment tx localCom_n in channel
            ensure localCom \neq localCom, and localCom \neq pendingLocalCom
250:

    b there are uncommitted updates

            ensure verify (localCom, comSig_{n+1}, pt_F) = true
251:
252:
            {f for}\ i\ {
m from}\ {
m lastLocalSigned}\ {
m to}\ {
m HTLCNo}\ {f do}
                 localHTLC_{n+1,i} \leftarrow TX  {input: HTLC output i of localCom,
253:
      output: (c_{\text{htlc,i}}, ph_{\text{rev},n+1} \lor (pt_{\text{dpay},n+1}, \text{delay relative}))
                 ensure verify (localHTLC_{n+1,i}, HTLCSigs_{n+1,i}, pt_{htlc,n+1}) =
254:
      true
            end for
255:
            pendingLocalCom \leftarrow localCom
256:
            mark pendingLocalCom as "irrevocably committed"
257:
            (ph_{\text{com},n+2}, sh_{\text{com},n+2}) \leftarrow \text{genKey}()
258:
            send (REVOKEANDACK, pchid, sh_{com,n}, ph_{com,n+2}) to Bob
259:
260:
261: Upon receiving (REVOKEANDACK, pchid, st_{com,n}, pt_{com,n+2}) from Bob:
            ensure there is a channel \in channels with Bob with ID pchid
262:
      marked as "waiting for REVOKEANDACK"
            ensure pk\left(st_{\text{com},n}\right) = pt_{\text{com},n}
263:
            mark remoteCom_{n+1} as "irrevocably committed"
264:
            \texttt{localCom}_{n+1} \gets \texttt{pendingLocalCom}
265:
            unmark channel
266:
            sh_{\text{rev},n} \leftarrow sh\_\text{base}_{\text{rev}} \cdot \mathcal{H} \left( ph\_\text{base}_{\text{rev}} || ph_{\text{com},n} \right) + st_{\text{com},n} \cdot \mathcal{H} \left( ph_{\text{com},n} || ph\_\text{base}_{\text{rev}} \right)
267:
268:
            ph_{\text{rev},n+2} \leftarrow ph\_\text{base}_{\text{rev}} \cdot \mathcal{H} \left( ph\_\text{base}_{\text{rev}} || ph_{\text{com},n+2} \right) + pt_{\text{com},n+2} \cdot
      \mathcal{H}(ph_{\text{com},n+2}||ph_{\text{base}_{\text{rev}}})
            pt_{\text{rev},n+2} \leftarrow pt\_\text{base}_{\text{rev}} \cdot \mathcal{H}(pt\_\text{base}_{\text{rev}}||pt_{\text{com},n+2}) + pt_{\text{com},n+2}
269:
      \mathcal{H}\left(pt_{\text{com},n+2}||pt\_\text{base}_{\text{rev}}\right)
            ph_{\text{dpay},n+2} \leftarrow ph_{\text{base}_{\text{dpay}}} + \mathcal{H}\left(ph_{\text{com},n+2}||ph_{\text{base}_{\text{dpay}}}\right)
270:
            pt_{\text{dpay},n+2} \leftarrow pt\_\text{base}_{\text{dpay}} + \mathcal{H}\left(pt_{\text{com},n+2} || pt\_\text{base}_{\text{dpay}}\right)
271:
            ph_{\text{pay},n+2} \leftarrow ph\_\text{base}_{\text{pay}} + \mathcal{H}\left(ph_{\text{com},n+2} || ph\_\text{base}_{\text{pay}}\right)
272:
```

```
pt_{\text{pay},n+2} \leftarrow pt\_\text{base}_{\text{pay}} + \mathcal{H}\left(pt_{\text{com},n+2} || pt\_\text{base}_{\text{pay}}\right)
273:
274:
           ph_{\text{htlc},n+2} \leftarrow ph\_\text{base}_{\text{htlc}} + \mathcal{H}\left(ph_{\text{com},n+2}||ph\_\text{base}_{\text{htlc}}\right)
           pt_{\text{htlc},n+2} \leftarrow pt\_\text{base}_{\text{htlc}} + \mathcal{H}\left(pt_{\text{com},n+2}||pt\_\text{base}_{\text{htlc}}\right)
275:
276:
277: Upon receiving (PUSH, pchid) from \mathcal{E}:
           ensure that there is a channel \in channels with ID pchid
278:
           choose a member (HTLC_no, R) of pendingFulfills<sub>pchid</sub> that
279:
     is both in an "irrevocably committed" remoteCom<sub>n</sub> and localCom<sub>n</sub>
           remove (HTLC_no, R) from pendingFulfills<sub>pchid</sub>
280:
           send (UPDATEFULFILLHTLC, pchid, HTLC_no, R) to pchid coun-
281:
     terparty
282: Upon receiving (CLOSECHANNEL, receipt) from \mathcal{E}:
           ensure receipt corresponds to an open channel \in channels
283:
           assign latest channel sequence number to n
284:
           HTLCs \leftarrow \emptyset
285:
           for every HTLC output \in localCom_n with number i do
286:
                sig \leftarrow signature(localHTLC_{n,i}, sh_{htlc,n})
287:
                add \left(\text{sig}, \text{HTLCSigs}_{n,i}, \text{localHTLC}_{n,i}\right) to HTLCs
288:
           end for
289:
290:
           \operatorname{sig} \leftarrow \operatorname{signature} (\operatorname{localCom}_n, sh_F)
           remove channel from channels
291:
292:
           send (SUBMIT, (sig, remoteSig<sub>n</sub>, localCom<sub>n</sub>), HTLCs) to \mathcal{G}_{Ledger}
```

A well-formed transaction contains:

- A list of inputs
- A list of outputs
- An arbitrary payload (optional)

Each input must be connected to a single valid, previously unconnected (unspent) output in the state.

We assume a one-way, collision-free hash function  $\mathcal H$  that creates the id of each transaction.

A well-formed output contains:

- A value in coins
- A list of spending methods. An input that spends this output must specify exactly one of the available spending methods.

A well-formed spending method contains any combination of the following:

- Public keys in disjunctive normal form. An input that spends using this spending method must contain signatures made with the private keys that correspond to the public keys of one of the conjunctions. If empty, no signatures are needed.
- Absolute locktime in block height, transaction height or time. The output can be spent by an input to a transaction that is added to the state after the specified block height, transaction height or time. If empty, output can be spent immediately after being added to the state.
- Relative locktime in block height, transaction height or time. The output can be spent by an input that is added to the state after the current output has been part of the state for the specified number of blocks, transactions or time. If empty, output can be spent immediately after being added to the state.
- Hashlock value. The output can be spent by an input that contains a
  preimage that hashes to the hashlock value. If empty, the input does
  not need to specify a preimage.

A well-formed input contains:

- A reference to the output and the spending method it spends
- A set of signatures that correspond to one of the conjunctions of public keys in the referred spending method (if needed)
- A preimage that hashes to the hashlock value of the referred spending method (if needed)

Lastly, the sum of coins of the outputs referenced by the inputs of the transaction (to-be-spent outputs) should be greater than or equal to the sum of coins of the outputs of the transaction.

We say that an unspent output is currently exclusively spendable by a player Alice with a public key pk and a hash list hl if for each spending method one of the following two holds:

- It still has a locktime that has not expired and thus is currently unspendable, or
- The only specified public key is pk and if there is a hashlock, its hash is contained in hl.

If an output is exclusively spendable, we say that its coins are exclusively spendable.

# Functionality $\mathcal{F}_{PayNet}$

### Parameters: - one-way, collision-free hash function $\mathcal{H}$ (for generating transaction IDs) Interface (messages from $\mathcal{E}$ ): - (REGISTER) - (SETDELAY, delay) - (OPENCHANNEL, self, peer, selfCoins) - (CLOSECHANNEL, receipt) - (PAY, peer, coins, path, receipt) Initialisation:1: Initialisation: 2: channels, pendingPay, corrupted $\leftarrow \emptyset$ 3: Responses to messages: 1: Upon receiving (REGISTER, delay) from Alice: $delay(Alice) \leftarrow delay$ 3: $\texttt{pendingUpdates} \ (Alice) \leftarrow \emptyset$ 4: $\texttt{blindTime}\left(Alice\right) \leftarrow \emptyset$ send (READ) to $\mathcal{G}_{Ledger}$ as Alice and largest block number to lastPoll(Alice) 5: send (REGISTER, Alice, delay) to ${\mathcal S}$ 6: 7: 8: Upon receiving (REGISTERDONE, Alice, x, pubKey) from S: 9: $\mathtt{onChainBalance}\left(Alice\right) \leftarrow x$ 10: $\texttt{offChainBalance}\left(Alice\right) \leftarrow 0$ $pubKey(Alice) \leftarrow pubKey$ 11: send (REGISTERED) to Alice 12: 13: 14: Upon receiving any message except for (REGISTER) from Alice: 15: ignore message if Alice has not registered 16: 17: Upon receiving (CORRUPTED, Alice) from S:

18:

19:

 $\operatorname{add} Alice \ \operatorname{to} \ \operatorname{corrupted}$ 

```
20: Upon receiving (OPENCHANNEL, Alice, Bob, x) from Alice:
21:
        choose unique channel ID fchid
22:
        pendingOpen(fchid) \leftarrow (Alice, Bob, x)
        send (OPENCHANNEL, Alice, Bob, x, fchid) to S
23:
24:
25: Upon receiving is (CHANNELOPENED, Alice, Bob, x, fchid, pchid) from S
        ensure that there is no channel \in channels with id pchid or fchid
26:
27:
        send (READ) to \mathcal{G}_{\text{Ledger}} as Alice and assign reply to \Sigma_{Alice}
        for some payer \in \{Alice, Bob\}, ensure that there exists a transaction \mathtt{tx} \in
28:
    \Sigma_{Alice} with a (x, pubKey (payer)) input, a (x, pubKey (Alice) \wedge pubkey (Bob))
    output and a transaction ID pchid.
29:
        if payer is honest then
                                              \triangleright otherwise \mathcal{A} decided to open channel
           ensure that pendingOpen (fchid) = (Alice, Bob, x)
30:
31:
           pendingOpen (fchid) \leftarrow \bot
32:
33:
        offChainBalance (payer) \leftarrow offChainBalance (payer)+x [Orfeas: remove
    on/offChainBalance?]
        onChainBalance (payer) \leftarrow offChainBalance (payer) -x
34:
        channel \leftarrow (Alice, Bob, x, 0, delay (Alice), delay (Bob), 0, fchid, pchid)
35:
36:
        add channel to channels and receipt to newChannels(\{Alice, Bob\})
37:
38: Upon receiving (PAY, Bob, x, \overrightarrow{path}, receipt) from Alice:
39:
        ensure that receipt corresponds to a channel where Alice has at least x
40:
        ensure that path consists of channels
        ensure that each consecutive pair of channels in path has a common mem-
41:
42:
        choose unique payment ID payid
        add (Alice, Bob, x, \overline{path}, payid) to pendingPay
43:
44:
        send (PAY, Alice, Bob, x, \overrightarrow{\mathtt{path}}, \mathtt{receipt}, payid) to \mathcal S
45:
46: Upon receiving (RESOLVEPAY, payid, Charlie) from S
        retrieve (Alice, Bob, x, \overrightarrow{path}) pending payment with ID payid and remove
47:
    it from pendingPay
        if (Charlie \neq Alice \text{ and } Charlie \notin \text{corrupted}) \text{ or } Charlie \notin \overrightarrow{\text{path then}}
48:
            halt [Aggelos: or ignore?]
49:
             Orfeas: In the protocol, even negligent players cannot lose
50:
    money here because the next hop is done by the protocol (without
    permission from the environment). So I removed negligent from the
    check above.]
        else
51:
52:
            for all channels \in \overrightarrow{path} starting from the one where Charlie pays do
53:
               in the first iteration, Charlieis payer. In subsequent iterations,
    payer is the unique player that has been paid but has not paid. The other
    channel party is payee
54:
               if payer has x or more in channel then
```

```
update channel to the next version and transfer x from payer
55:
    to payee
                   add channel receipt to both parties' pendingUpdates
56:
57:
                else
58:
                   revert all updates and remove them from pendingUpdates
59:
                    Orfeas: entire if may be avoided by simply not sending
    a message to Alice
                   if all players on path from Alice up to and including failed payer
60:
    are honest then
                       send (NOTPAID, Bob, x, \overrightarrow{path}, \text{receipt}) to Alice
61:
62:
                   else
                       send (REPORTNOTPAID, payid) to S [Orfeas: TODO: split
63:
    in two messages]
                       if reply from S is (DOREPORT, payid) then
64:
                           send (NOTPAID, Bob, x, \overrightarrow{path}, \text{receipt}) to Alice
65:
66:
                       end if
67:
                   end if
                    [Orfeas: end of possibly avoidable if]
68:
69:
               end if
70:
            end for
71:
            offChainBalance (Charlie) \leftarrow offChainBalance (Charlie) -x
72:
            offChainBalance(Bob) \leftarrow offChainBalance(Bob) + x
            if Charlie = Alice then
73:
74:
               send (PAID, Bob, x, \overline{path}, \overline{receipt}) to Alice
            end if
75:
        end if
76:
77:
78: Upon receiving (CLOSECHANNEL, receipt) from Alice [Aggelos: (or S)??
       [Orfeas: @Aggelos: why S?]
79:
        ensure that receipt corresponds to an open channel that contains Alice
80:
        retrieve fchid from channel
        \texttt{pendingClose}\left(fchid\right) \leftarrow Alice
81:
        send (CLOSECHANNEL, fchid, Alice) to S
82:
83:
84: Upon receiving (CHANNELCLOSED, fchid) from S:
        Alice \leftarrow \mathtt{pendingClose}\left(fchid\right)
85:
        retrieve Charlie, Bob, x, y, pchid from channel with ID fchid
86:
87:
        ensure that Charlie = Alice
        send (READ) to \mathcal{G}_{Ledger} as Alice and assign reply to \Sigma_{Alice}
88:
89:
        ensure that transaction with ID pchid is in \Sigma_{Alice}, is spent, x of its coins
    are spendable or will be spendable exclusively by \mathit{Alice} and y of its coins are
    spendable exclusively by Bob
90:
        pendingClose (fchid) \leftarrow \bot
91:
        add receipt of channel to closedChannels(Bob)
92:
        remove channel from channels
        onChainBalance(Alice) \leftarrow onChainBalance(Alice) + x
93:
94:
        \mathtt{onChainBalance}\left(Bob\right) \leftarrow \mathtt{onChainBalance}\left(Bob\right) + y
95:
        \texttt{offChainBalance}\left(Alice\right) \leftarrow \texttt{offChainBalance}\left(Alice\right) - x
```

```
96:
       offChainBalance(Bob) \leftarrow offChainBalance(Bob) - y
97:
       send (CHANNELCLOSED, receipt from channel) to Alice
98:
99: Upon receiving (POLL) from Alice:
100:
         toReport(Alice) \leftarrow \emptyset
         send (READ) to \mathcal{G}_{Ledger} as Alice and assign reply to \Sigma_{Alice}
101:
102:
         assign largest block number in \Sigma_{Alice} to t
103:
         if lastPoll (Alice) + delay (Alice) < t then
104:
            add [lastpoll (Alice), t - delay (Alice) - 1] to blindTime(Alice)
105:
         end if
         lastPoll(Alice) \leftarrow t
106:
107:
         scan \Sigma_{Alice} for honestly closed channels that contain Alice and exist in
    channels (txs that spend funding txs that have the same channel version
    as stored), remove them from channels and add them to toReport(Alice)
    (marked as "honest")
108:
         scan \Sigma_{Alice} for maliciously closed channels that contain Alice and exist
   in channels (txs that spend funding txs that have an older channel version
    than stored)
109:
         for all maliciously closed channel of which the spending txs can still be
   spent by Alice do
                              ▶ If Alice is negligent, she may be unable to punish
110:
            if Alice cannot spend those spending txs then
111:
                ensure that Alice has been negligent in the last interval
112:
            end if
            add channel to toReport(Alice) (marked as "malicious")
113:
114:
115:
         send (GETCLOSEDFUNDS, toReport(Alice), Alice) to S
116:
117: Upon receiving (CHANNELSCLOSED, details, Alice) from S:
118:
         send (READ) to \mathcal{G}_{Ledger} as Alice and assign reply to \Sigma_{Alice}
119:
         \mathbf{for} \ \mathbf{all} \ \mathbf{channel} \in \mathtt{details} \ \mathbf{do}
120:
            ensure channel \in toReport (Alice)
            if channel is marked as "malicious" then
121:
122:
                ensure that transactions that spend the funding tx of channel and
    pay Alice the entire channel value exist in \Sigma_{Alice}
123:
            else
                                                  ▷ channel is marked as "honest"
124:
                ensure that transactions that spend the funding tx of channel and
    pay Alice the her part in channel exist in \Sigma_{Alice}
125:
            end if
126:
            add the receipt of channel to closedChannels(Alice)
127:
            remove channel from channels and toReport(Alice)
128:
         end for
129:
130: Upon receiving (CHECKNEW) from Alice:
131:
         clear newChannels(Alice), closedChannels(Alice), pendingUpdates(Alice)
    and send them to Alice
132:
133: At the end of each activation: [Orfeas: can this part completely go?]
```

134: verify on Chainbalance() for all parties is consistent with ledger (if not roll back the state and ignore command of activation).

### Notes on Lightning Specification

- The relevant part of the specification can be found at https://github. com/lightningnetwork/lightning-rfc/blob/master/02-peer-protocol.
- In the current LND implementation and BOLT specification, only the opener (sender) of a channel funds it initially. In the present work, we allow both sides to fund the channel.

#### 8 Model for Payment Channels

A payment channel is a tuple

$$PC = (\{(P_1, c_1), \dots, (P_n, c_n)\}, \{(e_1, b_1), \dots, (e_m, b_m)\}, f : A^n \to PC)$$

where 
$$\sum_{i=1}^{n} c_i \leq \sum_{i=1}^{m} b_i$$
.

where  $\sum_{i=1}^{n} c_i \leq \sum_{i=1}^{m} b_i$ .  $(P_i, c_i)$  represents the *i*-th player and her available funds on settling.  $(e_j,b_j)$  represents the j-th on-chain endpoint and the corresponding funds that will be released for use in the blockchain if this endpoint is settled.

f is a function from player actions to a new payment channel. The new payment channel must have at most as much funds as the old.