# Payment Channels Overview

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**Abstract.** We provide a payment network functionality and prove that the Lightning Network [1] UC-realizes it.

# 1 State of a channel

Consider a channel between *Alice* and *Bob*. Both parties hold some data locally that correspond to ownership of some funds in the channel. Here we define a concise way of representing this data.

What Alice has to hold, specific for this channel:

- keys:
  - local funding secret key
  - remote funding public key
  - local {payment, htlc, delayed payment, revocation} basepoint secret
  - remote {payment, htlc, delayed\_payment, revocation}\_basepoint
  - seed (for local per\_commitment\_secrets)
  - remote per\_commitment\_secret $_{1,\dots,m-1}$
  - remote per\_commitment\_point $_{m,m+1}$
- Alice's coins
- Bob's coins
- every HTLC that is included in the latest irrevocably committed (local or remote) commitment:
  - direction (Alice  $\rightarrow Bob$  or  $Bob \rightarrow Alice$ )
  - hash
  - preimage (or ⊥ if still unresolved)
  - coins
  - Is it included in local commitment<sub>n</sub>?
  - HTLC number
- signatures:
  - signature of local commitment $_n$  with secret key corresponding to remote funding public key

• for every HTLC included in local commitment<sub>n</sub>, one signature of HTLC-Timeout if outgoing, HTLC-Success if incoming with secret key corresponding to remote htlc\_pubkey<sub>n</sub> (= htlc\_basepoint +  $\mathcal{H}$ (remote per\_commitment\_point<sub>n</sub> ||remote htlc\_basepoint) · G)

The rest of the things used in the protocol can be derived by the above.

Representation of a channel's state (from the point of view of Alice):

- Alice's coins  $c_{Alice}$
- Bob's coins  $c_{Bob}$
- list of (coins, state ∈ {proposed, committed}) preimage, whether we have a signature), HTLCs
  - negative coins are outgoing, positive are incoming
  - HTLCs can either be simply proposed (not in an irrevocably committed remote transaction) or committed (the opposite). After the preimage is supplied (no matter the direction), the HTLC is considered settled and is discarded.

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I.e. State<sub>Alice,pchid</sub> = (c_{Alice}, c_{Bob}, ((c_1, \text{state}_1), \dots, (c_k, \text{state}_k)))
E.g. State<sub>Alice,pchid</sub> = (4, 5, ((0.1, \text{proposed}), (-0.2, \text{signed})))
```

We do not include in the state elements whose contents are irrelevant (e.g. sigs, keys, hashes).

#### 2 UC conventions

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– send (READ) to \mathcal{G}_{Ledger} and assign reply to \Sigma ... = { send (READ) to \mathcal{G}_{Ledger} upon receiving delayed output \Sigma ... }
```

– every output that is returned by  $\mathcal{F}_{PayNet}$  or a player to  $\mathcal{E}$  is in fact a delayed output: It is handed over to  $\mathcal{A}$ , who in turn decides when to give it to  $\mathcal{E}$ .

#### 3 Differences from LND

- They use an ad-hoc construction for generating progressive secrets from seed and index, we use a PRF.

- To generate several public keys from one piece of info, they use the basepoint and the per commitment point and take advantage of EC homomorphic properties. We use an Identity Based Signature scheme.
- They also provide a way to cooperatively close a channel. we should do this as well
- In LND there are more messages that cover errors in transmission etc. There are also rules that govern message retransmission upon connection failure.
- We don't use the concept of "dust transactions/outputs".
- In our case, the delay of a player is set once, at her registration. In contrast to LN, it can't be changed later.

#### 4 Transaction Structure

A well-formed transaction contains:

- A list of inputs
- A list of outputs
- An arbitrary payload (optional)

Each input must be connected to a single valid, previously unconnected (unspent) output in the state.

We assume a one-way, collision-free hash function  $\mathcal{H}$  that creates the id of each transaction.

A well-formed output contains:

- A value in coins
- A list of spending methods. An input that spends this output must specify exactly one of the available spending methods.

A well-formed spending method contains any combination of the following:

- Public keys in disjunctive normal form. An input that spends using this spending method must contain signatures made with the private keys that correspond to the public keys of one of the conjunctions. If empty, no signatures are needed.
- Absolute locktime in block height, transaction height or time. The
  output can be spent by an input to a transaction that is added to the
  state after the specified block height, transaction height or time.
- Relative locktime in block height, transaction height or time. The
  output can be spent by an input that is added to the state after the
  current output has been part of the state for the specified number of
  blocks, transactions or time.

Hashlock value. The output can be spent by an input that contains a
preimage that hashes to the hashlock value. If empty, the input does
not need to specify a preimage.

If both the absolute and the relative locktime are empty, output can be spent immediately after being added to the state.

A well-formed input contains:

- A reference to the output and the spending method it spends
- A set of signatures that correspond to one of the conjunctions of public keys in the referred spending method (if needed)
- A preimage that hashes to the hashlock value of the referred spending method (if needed)

Lastly, the sum of coins of the outputs referenced by the inputs of the transaction (to-be-spent outputs) should be greater than or equal to the sum of coins of the outputs of the transaction.

We say that an unspent output is currently exclusively spendable by a player Alice with a public key pk and a hash list hl if for each spending method one of the following two holds:

- It still has a locktime that has not expired and thus is currently unspendable, or
- The only specified public key is pk and if there is a hashlock, its hash is contained in hl.

If an output is exclusively spendable, we say that its coins are exclusively spendable.

#### 5 Lightning Protocol

find out what fu (\tochain) must be

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Protocol \Pi_{LN} (self is Alice always) - support
 1: Initialisation:
 2:
          \texttt{channels}, \texttt{pendingOpen}, \texttt{pendingPay}, \texttt{pendingClose} \leftarrow \emptyset
 3:
          \texttt{newChannels}, \texttt{closedChannels} \leftarrow \emptyset
          unclaimedOfferedHTLCs, unclaimedReceivedHTLCs, pendingGetPaid \leftarrow \emptyset
 5: Upon receiving (REGISTER, delay, relayDelay) from \mathcal{E}:
          delay ← delay // Must check chain at least once every delay blocks
 6:
 7:
          relayDelay \leftarrow relayDelay
          send (READ) to \mathcal{G}_{\mathrm{Ledger}} and assign largest block number to lastPoll
 8:
          (pk_{Alice}, sk_{Alice}) \leftarrow \text{KeyGen}()
10:
          send (REGISTER, Alice, delay, relayDelay, pk_{Alice}) to \mathcal{E}
11: Upon receiving (REGISTERED) from \mathcal{E}:
          send (READ) to \mathcal{G}_{Ledger} and assign reply to \Sigma_{Alice}
13:
          assign the sum of all output values that are exclusively spendable by Alice
     {\it to} on Chain Balance
14:
          send (REGISTERED) to {\cal E}
15: Upon receiving any message (M) except for (REGISTER):
          if if haven't received (REGISTER) from \mathcal{E} then
16:
17:
               send (INVALID, M) to \mathcal{E} and ignore message
18:
          end if
19: function GetKeys
          (p_F, s_F) \leftarrow \text{KeyGen}() // \text{ For } F \text{ output}
20:
          (p_{\text{pay}}, s_{\text{pay}}) \leftarrow \text{MKeyGen}() // \text{ For com output to remote}
21:
22:
          (p_{\text{dpay}}, s_{\text{dpay}}) \leftarrow \text{MKeyGen}() // \text{ For com output to self}
          (p_{\mathrm{htlc}}, s_{\mathrm{htlc}}) \leftarrow \mathrm{MKeyGen}\,() // For htlc output to self
23:
          \mathtt{seed} \xleftarrow{\$} U(k) \mathrel{//} \mathsf{For per com point}
24:
          (p_{\text{rev}}, s_{\text{rev}}) \leftarrow \text{MKeyGen}() // \text{ For revocation in com}
25:
26:
          return ((p_F, s_F), (p_{\text{pay}}, s_{\text{pay}}), (p_{\text{dpay}}, s_{\text{dpay}}),
27:
               (p_{\text{htlc}}, s_{\text{htlc}}), seed, (p_{\text{rev}}, s_{\text{rev}}))
28: end function
```

Fig. 1.

```
Protocol \Pi_{\text{LN}} - OPENCHANNEL from \mathcal{E}

1: Upon receiving (OPENCHANNEL, Alice, Bob, x, tid) from \mathcal{E}:

2: ensure tid hasn't been used for opening another channel before

3: ((ph_F, sh_F), (phb_{\text{pay}}, shb_{\text{pay}}), (phb_{\text{dpay}}, shb_{\text{dpay}}),
(phb_{\text{htlc}}, shb_{\text{htlc}}), \text{seed}, (phb_{\text{rev}}, shb_{\text{rev}})) \leftarrow \text{GetKeys}()

4: prand<sub>1</sub> \leftarrow PRF (seed, 1)

5: (sh_{\text{com},1}, ph_{\text{com},1}) \leftarrow \text{KeyShareGen}(1^k; \text{prand}_1)

6: associate keys with tid

7: add (Alice, Bob, x, \text{tid}, (ph_F, sh_F), (phb_{\text{pay}}, shb_{\text{pay}}), (phb_{\text{dpay}}, shb_{\text{dpay}})
(phb_{\text{htlc}}, shb_{\text{htlc}}), (phb_{\text{com},1}, shb_{\text{com},1}), (phb_{\text{rev}}, shb_{\text{rev}}), tid) to pendingOpen

8: send (OPENCHANNEL, x, \text{delay} + k + fu, ph_F, phb_{\text{pay}}, phb_{\text{dpay}}, phb_{\text{htlc}}, ph_{\text{com},1}, phb_{\text{rev}}, tid) to Bob
```

Fig. 2.

```
Protocol \Pi_{\text{LN}} - OPENCHANNEL from Bob

1: Upon receiving (OPENCHANNEL,
x, remoteDelay, pt_F, ptb_{\text{pay}}, ptb_{\text{dpay}}, ptb_{\text{htlc}}, pt_{\text{com},1}, ptb_{\text{rev}}, tid) from Bob:

2: ensure tid has not been used yet with Bob

3: ((ph_F, sh_F), (phb_{\text{pay}}, shb_{\text{pay}}), (phb_{\text{dpay}}, shb_{\text{dpay}}), (phb_{\text{htlc}}, shb_{\text{htlc}}), seed,
(phb_{\text{rev}}, shb_{\text{rev}})) \leftarrow GetKeys ()

4: prand<sub>1</sub> \leftarrow PRF (seed, 1)

5: (sh_{\text{com},1}, ph_{\text{com},1}) \leftarrow KeyShareGen (1<sup>k</sup>; prand<sub>1</sub>)

6: associate keys with tid and store in pendingOpen

7: send (ACCEPTCHANNEL, delay + k + fu, ph_F, phb_{\text{pay}}, phb_{\text{dpay}}, phb_{\text{htlc}}, ph_{\text{com},1}, phb_{\text{rev}}, tid) to Bob
```

Fig. 3.

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Protocol \Pi_{\mathrm{LN}} - Accept Channel
 1: Upon receiving (ACCEPTCHANNEL, remoteDelay, pt_F, ptb_{pay}, ptb_{dpay}, ptb_{htlc},
     pt_{\text{com},1}, ptb_{\text{rev}}, \ tid) from Bob:
         ensure there is a temporary ID tid with Bob in pendingOpen on which
     ACCEPTCHANNEL hasn't been received
 3:
         associate received keys with tid
         send (READ) to \mathcal{G}_{Ledger} and assign reply to \Sigma_{Alice}
 4:
         assign to prevout a transaction output found in \Sigma_{Alice} that is currently
     exclusively spendable by Alice and has value y \geq x
          F \leftarrow TX (input spends prevout with a signature(TX, sk_{Alice}), output 0
     pays y - x to pk_{Alice}, output 1 pays x to tid.ph_F \wedge pt_F
 7:
         pchid \leftarrow \mathcal{H}(F)
         add pchid to pendingOpen entry with id tid
         pt_{\text{rev},1} \leftarrow \texttt{CombinePubKey}\left(ptb_{\text{rev}}, ph_{\text{com},1}\right)
10:
         ph_{\text{dpay},1} \leftarrow \text{PubKeyGen}\left(phb_{\text{dpay}}, ph_{\text{com},1}\right)
         ph_{\text{pay},1} \leftarrow \texttt{PubKeyGen}\left(phb_{\text{pay}}, ph_{\text{com},1}\right)
11:
12:
         remoteCom \leftarrow remoteCom_1 \leftarrow TX \{input: output 1 of F, outputs:
     (x, ph_{\text{pay},1}), (0, ph_{\text{rev},1} \lor (pt_{\text{dpay},1}, \text{delay} + k + fu \text{ relative}))
13:
          localCom \leftarrow TX \{ input: output 1 of F, outputs: \}
     (x, pt_{rev,1} \lor (ph_{dpay,1}, remoteDelay relative)), (0, pt_{pay,1})
14:
          add remoteCom and localCom to channel entry in pendingOpen
15:
          sig \leftarrow signature (remoteCom_1, sh_F)
16:
          \texttt{lastRemoteSigned} \leftarrow 0
17:
          send (FUNDINGCREATED, tid, pchid, sig) to Bob
```

Fig. 4.

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Protocol \Pi_{\mathrm{LN}} - fundingCreated
 1: Upon receiving (FUNDINGCREATED, tid, pchid, BobSig<sub>1</sub>) from Bob:
 2:
          ensure there is a temporary ID tid with Bob in pendingOpen on which we
     have sent up to ACCEPTCHANNEL
          ph_{\text{rev},1} \leftarrow \texttt{CombinePubKey}\left(phb_{\text{rev}}pt_{\text{com},1}\right)
          pt_{\text{dpay},1} \leftarrow \text{PubKeyGen}\left(ptb_{\text{dpay}}, pt_{\text{com},1}\right)
          pt_{\text{pay},1} \leftarrow \texttt{PubKeyGen}\left(ptb_{\text{pay}}, pt_{\text{com},1}\right)
 5:
          localCom \leftarrow localCom_1 \leftarrow TX  {input: output 1 of F, outputs:
     (x, pt_{\text{pay},1}), (0, pt_{\text{rev},1} \lor (ph_{\text{dpay},1}, \text{remoteDelay relative}))\}
 7:
          {\rm ensure} \ {\tt verify} \ ({\tt localCom}_1, {\tt BobSig}_1, pt_F) = {\tt True}
          remoteCom \leftarrow remoteCom_1 \leftarrow TX  {input: output 1 of F, outputs:
 8:
     (x, ph_{\text{rev},1} \lor (pt_{\text{dpay},1}, \text{delay} + k + fu \text{ relative})), (0, ph_{\text{pay},1})\}
          add BobSig<sub>1</sub>, remoteCom<sub>1</sub> and localCom<sub>1</sub> to channel entry in pendingOpen
          sig \leftarrow signature (remoteCom_1, sh_F)
10:
          mark channel as "broadcast, no fundingLocked"
11:
12:
          {\tt lastRemoteSigned}, {\tt lastLocalSigned} \leftarrow 0
13:
          send (FUNDINGSIGNED, pchid, sig) to Bob
```

Fig. 5.

```
Protocol \varPi_{\mathrm{LN}} - \mathtt{FUNDINGSIGNED}
1: Upon receiving (FUNDINGSIGNED, pchid, BobSig<sub>1</sub>) from Bob:
       ensure there is a channel ID pchid with Bob in pendingOpen on which we
   have sent up to FUNDINGCREATED
       ensure verify (localCom, BobSig<sub>1</sub>, pb_F) = True
       \texttt{localCom}_1 \leftarrow \texttt{localCom}
       \texttt{lastLocalSigned} \leftarrow 0
5:
6:
       add BobSig_1 to channel entry in pendingOpen
7:
       sig \leftarrow signature(F, sk_{Alice})
       mark pchid in pendingOpen as "broadcast, no FUNDINGLOCKED"
8:
9:
       send (SUBMIT, (sig, F)) to \mathcal{G}_{Ledger}
```

Fig. 6.

#### Protocol $\Pi_{\mathrm{LN}}$ - CheckNew 1: Upon receiving (CHECKNEW, Alice, Bob, tid) from $\mathcal{E}$ : // new message: represents lnd polling daemon ensure there is a matching channel in pendingOpen with id pchid, with a "broadcast" mark, funded with x coins send (READ) to $\mathcal{G}_{Ledger}$ and assign reply to $\Sigma_{Alice}$ ensure $\exists$ unspent TX in $\Sigma_{Alice}$ with ID pchid and a $(x, ph_F \wedge pt_F)$ output 4: 5: $\operatorname{prand}_2 \leftarrow \operatorname{PRF}\left(\operatorname{\mathtt{seed}},2\right)$ $(sh_{\text{com},2}, ph_{\text{com},2}) \leftarrow \text{KeyShareGen}\left(1^k; \text{prand}_2\right)$ 6: 7: add TX to channel data replace "broadcast" mark in channel with "in state" 8: 9: if channel is marked as "in state, FUNDINGLOCKED" then 10: move channel data from pendingOpen to channels add receipt of channel to newChannels 11: 12: end if 13: send (FundingLocked, pchid, $ph_{com,2}$ ) to Bob

Fig. 7.

#### Protocol $\Pi_{\mathrm{LN}}$ - fundingLocked 1: Upon receiving (FUNDINGLOCKED, pchid, $pt_{com,2}$ ) from Bob: ensure there is a channel with ID pchid with Bob in pendingOpen with a "no fundingLocked" mark ensure $pk\left(st_{\text{com},n}\right) = pt_{\text{com},n}$ replace "no fundingLocked" mark in channel with "fundingLocked" 4: ensure channel has an "in state" mark 5: 6: generate 2nd remote delayed payment, htlc, payment keys add TX to channel data 7: move channel data from ${\tt pendingOpen}$ to ${\tt channels}$ 8: add receipt of channel to newChannels

Fig. 8.

```
Protocol \Pi_{\mathrm{LN}} - poll
 1: Upon receiving (POLL) from \mathcal{E}:
 2:
        send (READ) to \mathcal{G}_{\text{Ledger}} and assign reply to \Sigma_{Alice}
        assign largest block number in \Sigma_{Alice} to lastPoll
 3:
        \texttt{toSubmit} \leftarrow \emptyset
 4:
        for all 	au\in \mathtt{unclaimedOfferedHTLCs}\ \mathbf{do}
 5:
 6:
            if input of \tau has been spent then // by remote HTLC-success
 7:
                remove \tau from unclaimedOfferedHTLCs
 8:
                remember preimage - hash combination
 9:
            else if input of \tau has not been spent and timelock is over then
10:
                remove \ 	au \ from \ unclaimedOfferedHTLCs
11:
                add \tau to toSubmit
12:
            end if
13:
         end for
         for all remoteCom_n \in \Sigma_{Alice} that spend F of a channel \in channels do
14:
15:
            if we do not have sh_{rev,n} then // Honest closure
                for all unspent offered HTLC outputs i of remoteCom_n do
16:
17:
                    TX \leftarrow \{\text{input: } i \text{ HTLC output of remoteCom}_n \text{ with } ph_{\text{htlc},n} \text{ as} \}
    method, output: pk_{Alice}
18:
                    sig \leftarrow signature(TX, sh_{htlc.n})
19:
                    if timelock has not expired then
20:
                        add (sig, TX) to unclaimedOfferedHTLCs
21:
                    else if timelock has expired then
22:
                        add (sig, TX) to toSubmit
23:
                    end if
24:
                end for
25:
                for all spent offered HTLC output i of remoteCom_n do
26:
                    if we are intermediary then
                        retrieve preimage R, pchid of previous channel on the path
27:
    of the HTLC, and HTLCNo' of the corresponding HTLC' in pchid'
28:
                        add (HTLCNo', R) to pendingFulfills<sub>nchid'</sub>
29:
                    end if
30:
                end for
31:
            else // malicious closure
32:
                rev \leftarrow TX  {inputs: all remoteCom<sub>n</sub> outputs, choosing ph_{rev,n}
    method, output: pk_{Alice}}
33:
                sig \leftarrow signature(rev, sh_{rev,n})
34:
                add (sig, rev) to toSubmit
35:
            end if
36:
            move channel from channels to closedChannels
37:
38:
         for all honestly closed remoteCom_n that were processed above, with
    channel id pchid do
39:
            for all received HTLC outputs i of remoteCom_n do
                {f if} there is an entry in {f pendingFulfills}_{pchid} with the same HTLCNo
40:
    and R then
41:
                    TX \leftarrow \{\text{input: } i \text{ HTLC output of remoteCom}_n \text{ with } (ph_{\text{htlc},n}, R)\}
    as method, output: pk_{Alice}
42:
                    sig \leftarrow signature(TX, sh_{htlc,n})
                    add (sig, TX) to toSubmit
43:
44:
                    remove entry from pendingFulfills<sub>nchid</sub>
45:
                end if
                                              10
46:
            end for
47:
        send (SUBMIT, toSubmit) to \mathcal{G}_{\mathrm{Ledger}}
48:
49: Upon receiving (GETNEW) from Alice:
50:
        clear newChannels, closedChannels, pendingUpdates and send them to
    Alice
```

```
Protocol \Pi_{LN} - invoice
 1: Upon receiving (PAY, Bob, x, \overrightarrow{path}, receipt) from \mathcal{E}:
        ensure that path consists of valid pchids and CltvExpiryDeltas //
    Payment completes only if \forall i \in \overline{\mathtt{path}}, \mathtt{CltvExpiryDelta}_i \geq 3k + \mathtt{RelayDelay}_i
        ensure that the first pchid \in \overrightarrow{path} has the same pchid as in receipt
        ensure that receipt corresponds to the latest version of an open
    channel \in channels in which we have at least x.
        choose unique payment ID payid // unique for Alice and Bob
        add (Bob, x, path, receipt, payid, "waiting for invoice") to pendingPay
 6:
 7:
        send (SENDINVOICE, payid) to Bob
8: Upon receiving (SENDINVOICE, payid) from Bob:
 9:
        ensure there is no (Bob, payid) entry in pendingGetPaid
10:
        choose random, unique preimage R
11:
        add (Bob, R, payid) to pendingGetPaid
12:
         send (INVOICE, \mathcal{H}(R), relayDelay +3k+2fu-1, payid) to Bob
13: Upon receiving (INVOICE, h, payid) from Bob:
         ensure there is a (Bob, x, \overrightarrow{path}, receipt, payid, "waiting for invoice") entry
    in pendingPay
15:
        ensure h is valid (in the range of \mathcal{H})
         remove entry from pendingPay
16:
         send (READ) to \mathcal{G}_{Ledger} and assign largest block number to t
17:
18:
        l \leftarrow | (\mathtt{path})|
         m \leftarrow \text{the concatenation of } l\left(x, \texttt{OutgoingCltvExpiry}\right) \text{ pairs, where}
19:
    \texttt{OutgoingCltvExpiry}_l \leftarrow t, \forall i \in \{1, \dots, l-1\}, \texttt{OutgoingCltvExpiry}_{l-i} \leftarrow
    {\tt OutgoingCltvExpiry}_{l-i+1} + {\tt CltvExpiryDelta}_{l-i+1}
20:
         (\mu_0, \delta_0) \leftarrow \text{SphinxCreate}(m, \text{ public keys of } \overline{\text{path parties}})
21:
         let remoteCom_n the latest signed remote commitment tx
22:
        \texttt{CltvExpiry} \leftarrow \texttt{OutgoingCltvExpiry}_1 + \texttt{relayDelay} + 2k + fu - 1
23:
         reduce simple payment output in remoteCom by x
         add an additional (x, ph_{rev,n+1} \lor (ph_{htlc,n+1} \land pt_{htlc,n+1}, on preimage)
     of h) \vee ph_{\text{htlc},n+1}, CltvExpiry absolute) output (all with n+1 keys) to
    remoteCom, marked with HTLCNo
25:
         reduce delayed payment output in localCom by x
26:
         add an additional (x, pt_{rev, n+1} \vee (pt_{htlc, n+1}, on preimage)
     of h) \vee (ph_{\mathrm{htlc},n+1} \wedge pt_{\mathrm{htlc},n+1}, \mathtt{CltvExpiry} \ absolute)) output (all with n+1
    keys) to localCom, marked with HTLCNo
27:
        increment HTLCNo<sub>pchid</sub> by one and associate x, h, pchid with it
28:
        mark HTLCNo as "sender"
29:
        send (UPDATEADDHTLC, first pchid of
    \overline{\text{path}}, HTLCNo<sub>pchid</sub>, x, h, CltvExpiry, (\mu_0, \delta_0)) to pchid channel counterparty
```

Fig. 10.

```
Protocol \Pi_{\mathrm{LN}} - updateAddHtlc
 1: Upon receiving (UPDATEADDHTLC, pchid, HTLCNo, x, h, CltvExpiry, M) from
    Bob:
 2:
        ensure pchid corresponds to an open channel in channels where Bob has
    at least x
 3:
        ensure \mathtt{HTLCNo} = \mathtt{HTLCNo}_{pchid} + 1
         (pchid', x', \texttt{CltvExpiry}', \delta) \leftarrow \texttt{SphinxPeel}(sk_{Alice}, M)
        if \delta = \text{receiver then}
 5:
             ensure
 6:
    pchid' = \bot, x = x', \mathtt{CltvExpiry} \ge \mathtt{CltvExpiry}' + \mathtt{relayDelay} + 2k + fu - 1
             mark HTLCNo as "receiver"
 7:
        else // We are an intermediary
 8:
 9:
             ensure x = x', CltvExpiry \geq CltvExpiry' + relayDelay + 3k + 2fu - 1
10:
             ensure pchid' corresponds to an open channel in channels where we
    have at least x
11:
             mark HTLCNo as "intermediary"
12:
         end if
13:
         increment HTLCNo<sub>pchid</sub> by one
14:
         let remoteCom_n the latest signed remote commitment tx
15:
         reduce delayed payment output in remoteCom by x
16:
         add an (x, ph_{\text{rev},n+1} \lor (ph_{\text{htlc},n+1} \land pt_{\text{htlc},n+1}, \text{CltvExpiry absolute}) \lor
    ph_{\text{htlc},n+1}, on preimage of h) htlc output (all with n+1 keys) to remoteCom,
    marked with HTLCNo
17:
         reduce simple payment output in localCom by x
18:
         add an (x, pt_{rev,n+1} \lor pt_{htlc,n+1}, CltvExpiry absolute) \lor
    ((pt_{\text{htlc},n+1} \land ph_{\text{htlc},n+1}, \text{ on preimage of } h)) htlc output (all with n+1 keys)
    to remoteCom, marked with HTLCNo
19:
        if \delta = \text{receiver then}
20:
             retrieve R:\mathcal{H}\left(R\right)=h from pendingGetPaid and clear entry
             \operatorname{add} \; (\mathtt{HTLCNo}, R) \; \operatorname{to} \; \mathtt{pendingFulfills}_{pchid}
21:
22:
         else if \delta \neq receiver then // Send HTLC to next hop
23:
             retrieve pchid' data
24:
             let remoteCom'_n the latest signed remote commitment tx
25:
             reduce simple payment output in remoteCom' by x
26:
             add an additional (x, ph_{\text{rev},n+1} \lor (ph_{\text{htlc},n+1} \land pt_{\text{htlc},n+1}, \text{ on preimage})
     of h) \vee ph_{\mathrm{htlc},n+1}\mathsf{CltvExpiry}' absolute) output (all with n+1 keys) to
    remoteCom', marked with HTLCNo'
27:
             reduce delayed payment output in localCom' by x
28:
             add an additional (x, pt_{rev, n+1} \lor (pt_{htlc, n+1}, on preimage)
     of h) \vee (pt_{\text{htlc},n+1} \wedge ph_{\text{htlc},n+1}CltvExpiry' absolute)) output (all with n+1
    keys) to remoteCom', marked with HTLCNo'
29:
             increment HTLCNo' by 1
             M' \leftarrow \mathtt{SphinxPrepare}\left(M, \delta, sk_{Alice}\right)
30:
             add (HTLCNo', x, h, CltvExpiry', M') to pendingAdds<sub>nchid'</sub>
31:
32:
         end if
```

Fig. 11.

```
Protocol \Pi_{\mathrm{LN}} - updateFulfillHtlc
 1: Upon receiving (UPDATEFULFILLHTLC, pchid, HTLCNo, R) from Bob:
       if \mathtt{HTLCNo} > \mathtt{lastRemoteSigned} \lor \mathtt{HTLCNo} > \mathtt{lastLocalSigned} \lor \mathcal{H}\left(R\right) \neq h,
    where h is the hash in the HTLC with number HTLCNo then
           close channel (as in Fig. 17)
 3:
           return
 4:
       end if
 5:
       ensure HTLCNo is an offered HTLC (localCom has h tied to a public key
    that we own)
 7:
       add value of HTLC to delayed payment of remoteCom
       remove HTLC output with number HTLCNo from remoteCom
 8:
9:
       add value of HTLC to simple payment of localCom
       remove HTLC output with number HTLCNo from localCom
10:
11:
        if we have a channel phcid' that has a received HTLC with hash h with
    number HTLCNo' then // We are intermediary
12:
           send (READ) to \mathcal{G}_{\text{Ledger}} and assign reply to \Sigma_{Alice}
           if latest remoteCom'_n \in \Sigma_{Alice} then // counterparty has gone on-chain
13:
               TX \leftarrow \{\text{input: (remoteCom' HTLC output with number HTLCNo'}, R),}
14:
    output: pk_{Alice}}
               sig \leftarrow signature(TX, sh_{htlc.n})
15:
16:
               send (SUBMIT, (sig, TX)) to \mathcal{G}_{Ledger} // shouldn't be already spent by
    remote HTLCTimeout
17:
           else // counterparty still off-chain
               // Not having the HTLC irrevocably committed is impossible
18:
    (Fig. 16, l. 15)
               send (UPDATEFULFILLHTLC, pchid', HTLCNo', R) to counterparty
19:
20:
           end if
21:
        end if
```

Fig. 12.

```
Protocol \Pi_{LN} - Commit
 1: Upon receiving (COMMIT, pchid) from \mathcal{E}:
 2:
         ensure that there is a channel \in channels with ID pchid
         retrieve latest remote commitment tx remoteCom_n in channel
 3:
         ensure remoteCom \neq remoteCom<sub>n</sub> // there are uncommitted updates
 4:
         ensure channel is not marked as "waiting for REVOKEANDACK"
 5:
 6:
         \texttt{remoteCom}_{n+1} \leftarrow \texttt{remoteCom}
 7:
         \operatorname{ComSig} \leftarrow \operatorname{signature} (\operatorname{remoteCom}_{n+1}, sh_F)
 8:
         \mathrm{HTLCSigs} \leftarrow \emptyset
9:
         \mathbf{for}\ i\ \mathrm{from}\ \mathtt{lastRemoteSigned}\ \mathrm{to}\ \mathtt{HTLCNo}\ \mathbf{do}
10:
              \texttt{remoteHTLC}_{n+1,i} \leftarrow \texttt{TX} \; \{ \texttt{input: HTLC output} \; i \; \texttt{of remoteCom}_{n+1}, \\
     \text{output: } (c_{\text{htlc,i}}, ph_{\text{rev},n+1} \lor (pt_{\text{dpay},n+1}, \texttt{delay} + k + fu \text{ relative}))\}
              add signature (remoteHTLC_{n+1,i}, sh_{\text{htlc},n+1}) to HTLCSigs
11:
12:
13:
          add signature (remoteHTLC_{n+1,m+1}, sh_{\text{htlc},n+1}) to HTLCSigs
14:
          {\tt lastRemoteSigned} \leftarrow {\tt HTLCNo}
          mark channel as "waiting for REVOKEANDACK"
15:
16:
         send (COMMITMENTSIGNED, pchid, ComSig, HTLCSigs) to pchid
     counterparty
```

Fig. 13.

```
Protocol \Pi_{\mathrm{LN}} - CommitmentSigned
 1: Upon receiving (COMMITMENTSIGNED, pchid, comSig_{n+1}, HTLCSigs_{n+1}) from
     Bob:
 2:
          ensure that there is a channel \in channels with ID pchid with Bob
          retrieve latest local commitment \operatorname{tx} \operatorname{localCom}_n in channel
 3:
          ensure localCom \neq localCom, and localCom \neq pendingLocalCom // there
     are uncommitted updates
          \textbf{if verify}\left(\texttt{localCom}, \texttt{comSig}_{n+1}, pt_F\right) = \texttt{false} \lor |\texttt{HTLCSigs}_{n+1}| \neq
     {\tt HTLCNo-lastLocalSigned+1\ then}
               close channel (as in Fig. 17)
 7:
              return
 8:
          end if
 9:
          \mathbf{for}\ i\ \mathrm{from}\ \mathtt{lastLocalSigned}\ \mathrm{to}\ \mathtt{HTLCNo}\ \mathbf{do}
10:
               localHTLC_{n+1,i} \leftarrow TX  {input: HTLC output i of localCom, output:
     (c_{\text{htlc,i}}, ph_{\text{rev},n+1} \lor (pt_{\text{dpay},n+1}, \texttt{remoteDelay} \text{ relative}))
11:
                \text{if verify} \left( \texttt{localHTLC}_{n+1,i}, \texttt{HTLCSigs}_{n+1,i}, pt_{\texttt{htlc},n+1} \right) = \texttt{false then} 
12:
                    close channel (as in Fig. 17)
13:
                    return
               end if
14:
15:
          end for
16:
          \texttt{pendingLocalCom} \leftarrow \texttt{localCom}
          mark\ {\tt pendingLocalCom}\ as\ "irrevocably\ committed"
17:
          \operatorname{prand}_{n+2} \leftarrow \mathtt{PRF}\left(\mathtt{seed}, n+2\right)
18:
          (sh_{\text{com},n+2}, ph_{\text{com},n+2}) \leftarrow \text{KeyShareGen}\left(1^k; \text{prand}_{n+2}\right)
19:
20:
          send (REVOKEANDACK, pchid, prand_n, ph_{com,n+2}) to Bob
```

Fig. 14.

```
Protocol \Pi_{\mathrm{LN}} - RevokeAndAck
 1: Upon receiving (REVOKEANDACK, pchid, st_{com,n}, pt_{com,n+2}) from Bob:
            ensure there is a channel \in channels with Bob with ID pchid marked as
      "waiting for REVOKEANDACK"
            if pk(st_{com,n}) \neq pt_{com,n} then // wrong st_{com,n} - closing
 4:
                 close channel (as in Fig. 17)
 5:
                 return
 6:
            end if
 7:
            mark remoteCom_{n+1} as "irrevocably committed"
 8:
            \texttt{localCom}_{n+1} \gets \texttt{pendingLocalCom}
 9:
            unmark channel
10:
            sh_{\text{rev},n} \leftarrow \texttt{CombineKey}\left(shb_{\text{rev}}, phb_{\text{rev}}, st_{\text{com}n}, pt_{\text{com},n}\right)
            ph_{\text{rev},n+2} \leftarrow \texttt{CombinePubKey}\left(phb_{\text{rev}},pt_{\text{com},n+2}\right)
11:
            pt_{\mathrm{rev},n+2} \leftarrow \mathtt{CombinePubKey}\left(ptb_{\mathrm{rev}},ph_{\mathrm{com},n+2}\right)
12:
13:
            ph_{\text{dpay},n+2} \leftarrow \texttt{PubKeyGen}\left(phb_{\text{dpay}},ph_{\text{com},n+2}\right)
14:
            pt_{\texttt{dpay},n+2} \leftarrow \texttt{PubKeyGen}\left(ptb_{\texttt{dpay}}, pt_{\texttt{com},n+2}\right)
            ph_{\text{pay},n+2} \leftarrow \texttt{PubKeyGen}\left(phb_{\text{pay}},ph_{\text{com},n+2}\right)
15:
            pt_{\text{pay},n+2} \leftarrow \texttt{PubKeyGen}\left(ptb_{\text{pay}},pt_{\text{com},n+2}\right)
16:
17:
            ph_{\text{htlc},n+2} \leftarrow \texttt{PubKeyGen}\left(phb_{\text{htlc}},ph_{\text{com},n+2}\right)
18:
            pt_{\text{htlc},n+2} \leftarrow \texttt{PubKeyGen}\left(ptb_{\text{htlc}}, pt_{\text{com},n+2}\right)
```

Fig. 15.

```
Protocol \Pi_{\mathrm{LN}} - Push
 1: Upon receiving (PUSHFULFILL, pchid) from \mathcal{E}:
 2:
        ensure that there is a channel \in channels with ID pchid
        choose a member (HTLCNo, R) of pendingFulfills_{pchid} that is both in an
    "irrevocably committed" remoteCom_n and localCom_n
        send (READ) to \mathcal{G}_{\text{Ledger}} and assign reply to \Sigma_{Alice}
        remove (HTLCNo, R) from pendingFulfills _{pchid}
        if remoteCom<sub>n</sub> \notin \Sigma_{Alice} then // counterparty cooperative
 6:
            send (UPDATEFULFILLHTLC, pchid, HTLCNo, R) to pchid counterparty
 7:
 8:
        else // counterparty gone on-chain
            TX \leftarrow \{\text{input: (remoteCom}_n \ HTLC \ output \ with \ number \ HTLCNo, R),}
9:
    output: pk_{Alice}}
10:
            sig \leftarrow signature(TX, sh_{htlc,n})
            send (SUBMIT, (sig, TX)) to \mathcal{G}_{Ledger} // shouldn't be already spent by
    remote HTLCTimeout
        end if
12:
13: Upon receiving (PUSHADD, pchid) from \mathcal{E}:
        ensure that there is a channel \in channels with ID pchid
        choose a member (HTLCNo, x, h, CltvExpiry, M) of pendingAdds<sub>pchid</sub> that is
    both in an "irrevocably committed" remoteCom<sub>n</sub> and localCom<sub>n</sub>
        remove chosen entry from pendingAdds_{pchid}
        send (UPDATEADDHTLC, pchid, HTLCNo, x, h, CltvExpiry, M) to pchid
    counterparty
18: Upon receiving (FULFILLONCHAIN) from \mathcal{E}:
19:
        send (READ) to \mathcal{G}_{\text{Ledger}} and assign largest block number to t
20:
        \mathtt{toSubmit} \leftarrow \emptyset
21:
        for all channels do
22:
            if there exists an HTLC in latest localCom_n for which we have sent
    both UPDATEFULFILLHTLC and COMMITMENTSIGNED to a transaction without
    that HTLC to counterparty, but have not received the corresponding
    REVOKEANDACK AND the HTLC expires within [t, 2?k + fu - 1 + t] then
23:
               add localCom_n of the channel and all corresponding valid
    HTLC-successes and HTLC-timeouts (for both localCom_n and remoteCom_n<sup>a</sup>),
    along with their signatures to toSubmit
24:
            end if
25:
        end for
        send (SUBMIT, toSubmit) to \mathcal{G}_{Ledger}
26:
 <sup>a</sup> Ensures funds retrieval if counterparty has gone on-chain
```

Fig. 16.

```
Protocol \Pi_{\mathrm{LN}} - close
 1: Upon receiving (CLOSECHANNEL, receipt) from \mathcal{E}:
 2:
              ensure receipt corresponds to an open channel \in channels
 3:
              assign latest {\tt channel} sequence number to n
              \mathrm{HTLCs} \leftarrow \emptyset
 4:
 5:
              \mathbf{for} \ \mathrm{every} \ \mathrm{HTLC} \ \mathrm{output} \ \in \mathtt{localCom}_n \ \mathrm{with} \ \mathrm{number} \ i \ \mathbf{do}
                     \begin{array}{l} \operatorname{sig} \leftarrow \operatorname{\mathtt{signature}} \left( \operatorname{\mathtt{localHTLC}}_{n,i}, sh_{\operatorname{htlc},n} \right) \\ \operatorname{\mathtt{add}} \left( \operatorname{sig}, \operatorname{\mathtt{HTLCSigs}}_{n,i}, \operatorname{\mathtt{localHTLC}}_{n,i} \right) \text{ to } \operatorname{HTLCs} \end{array}
 7:
              end for
 8:
 9:
              \mathrm{sig} \leftarrow \mathtt{signature}\left(\mathtt{localCom}_n, sh_F\right)
10:
               remove channel from channels
              send (SUBMIT, (sig, remoteSig_n, localCom_n), HTLCs) to \mathcal{G}_{\texttt{Ledger}}
11:
```

Fig. 17.

# 6 Payment Network Functionality

```
Functionality \mathcal{F}_{PayNet}- support
Parameters:
- one-way, collision-free hash function \mathcal{H} (for generating transaction IDs)
Interface (messages from \mathcal{E}): check
- (register)
- (SETDELAY, delay)
- (OPENCHANNEL, self, peer, selfCoins)

    (CLOSECHANNEL, receipt)

   (PAY, peer, coins, path, receipt)
Initialisation:
 1: Initialisation:
        channels, pendingPay, pendingOpen, corrupted \leftarrow \emptyset
 3: Upon receiving (REGISTER, delay, relayDelay) from Alice:
        delay(Alice) \leftarrow delay // Must check chain at least once every <math>delay(Alice)
 4:
   blocks
       relayDelay(Alice) \leftarrow relayDelay
 5:
        pendingUpdates (Alice), newChannels (Alice) \leftarrow \emptyset
 6:
 7:
        polls(Alice) \leftarrow \emptyset
        focs(Alice) \leftarrow \emptyset
        send (READ) to \mathcal{G}_{\text{Ledger}} as Alice and add largest block number to
        send (REGISTER, Alice, delay, relayDelay, lastPoll) to \mathcal{S}
11: Upon receiving (REGISTERDONE, Alice, pubKey) from S:
        pubKey(Alice) \leftarrow pubKey
13:
        send (REGISTER, Alice, delay(Alice), relayDelay(Alice), pubKey) to Alice
14: Upon receiving (REGISTERED) from Alice:
15:
        send (READ) to \mathcal{G}_{Ledger} as Alice and assign reply to \Sigma_{Alice}
16:
        assign the sum of all output values that are exclusively spendable by Alice
    to onChainBalance
17:
        send (REGISTERED) to Alice
18: Upon receiving any message except for (REGISTER) from Alice:
        ignore message if Alice has not registered 20
20: Upon receiving (CORRUPTED, Alice) from S:
        add Alice to corrupted
22: At the end of each activation: [Orfeas: can this part completely go?]
        verify on Chainbalance() for honest parties is consistent with ledger (if not
    roll back the state and ignore command of activation).
```

```
Functionality \mathcal{F}_{\mathrm{PayNet}}- open
 1: Upon receiving (OPENCHANNEL, Alice, Bob, x, tid) from Alice:
 2:
        ensure tid hasn't been used by Alice for opening another channel before
 3:
        choose unique channel ID fchid
        pendingOpen (fchid) \leftarrow (Alice, Bob, x, tid)
 4:
 5:
        send (OPENCHANNEL, Alice, Bob, x, fchid, tid) to S
 6: Upon receiving (CHANNELANNOUNCED, Alice, p_{Alice,F}, p_{Bob,F}, fchid, pchid, tid)
 7:
        ensure that there is a pendingOpen(fchid) entry with temporary id tid
        add "Alice announced", p_{Alice,F}, p_{Bob,F}, pchid to pendingOpen(fchid)
 9: Upon receiving (CHECKNEW, Alice, Bob, tid) from Alice:
        ensure there is a matching channel in pendingOpen(fchid), marked with
    "Alice announced"
11:
        (funder, fundee, x, p_{Alice,F}, p_{Bob,F}) \leftarrow pendingOpen(fchid)
        send (READ) to \mathcal{G}_{Ledger} as Alice and assign reply to \Sigma_{Alice}
12:
        ensure that there is a TX F \in \Sigma_{Alice} with a (x, (p_{\text{funder},F} \land p_{\text{fundee},F}))
13:
    output such that \mathcal{H}(F) = pchid
14:
        mark pendingOpen(fchid) with "Alice checked"
15:
        if pendingOpen(fchid) is not marked with "noted" then
16:
           mark pendingOpen(fchid) with "noted"
17:
           if funder = Alice then
               offChainBalance (Alice) \leftarrow offChainBalance (Alice) + x [Orfeas:
18:
    remove on/offChainBalance?]
19:
               onChainBalance (Alice) \leftarrow offChainBalance (Alice) -x
20:
               channel \leftarrow (Alice, Bob, x, 0, 0, fchid, pchid)
21:
           else//Bob is the funder
               offChainBalance (Bob) \leftarrow offChainBalance (Bob) + x [Orfeas:
22:
    remove on/offChainBalance?
               on
ChainBalance (Bob) \leftarrow off
ChainBalance (Bob) -x
23:
24:
               channel \leftarrow (Bob, Alice, x, 0, 0, fchid, pchid)
25:
           end if
           add channel to channels
26:
27:
        end if
28:
        add receipt(channel) to newChannels(Alice)
29:
        if pendingOpen(fchid) is marked with "Alice checked" and "Bob checked"
    then
30:
           clear pendingOpen(fchid) entry
31:
        end if
32:
        send (Channel Opened, Alice, fchid) to S
```

Fig. 19.

```
Functionality \mathcal{F}_{PayNet}- pay
 1: Upon receiving (PAY, Bob, x, \overline{path}, receipt) from Alice:
2:
       ensure that (Alice, c) \in \texttt{receipt} and c \geq x
3:
       ensure that there is a channel \in channels : receipt (channel) = receipt
       ensure that path consists of (channel, CltvExpiryDelta) pairs where
    channel \in channels
       ensure that each consecutive pair of channels in path has a common
5:
   member
       choose unique payment ID payid
6:
7:
       add (Alice, Bob, x, \overrightarrow{path}, payid) to pendingPay
       send (PAY, Alice, Bob, x, \overrightarrow{path}, receipt, payid) to S
9: Upon receiving (RESOLVEPAY, payid, charged) from S:
        retrieve (Alice, Bob, x, path) with ID payid and remove it from pendingPay
11:
        calculate IncomingCltvExpiry, OutgoingCltvExpiry of charged (as in
   Fig. 10, l. 19)
12:
       if charged = \bot then
           return
13:
14:
        else if charged \neq Alice \vee charged \notin corrupted \vee (polls(charged)
    contains an element in
    [OutgoingCltvExpiry + 2k + fu - 2, IncomingCltvExpiry - 2?k - fu - 1] \land
   polls(charged) contains OutgoingCltvExpiry -2k - fu - 1) then
15:
           halt
       end if
16:
        for all channels \in \overrightarrow{path} starting from the one where charged pays do
17:
18:
           in the first iteration, payer is charged. In subsequent iterations, payer
    is the unique player that has received but has not given. The other channel
   party is payee
19:
           if payer has x or more in channel then
20:
               update channel to the next version and transfer x from payer to
   payee
21:
               add receipt(channel) to both parties' pendingUpdates
22:
23:
               revert all updates and remove them from pendingUpdates
24:
           end if
        end for
25:
26:
        if charged ∉ corrupted then
27:
           offChainBalance(charged) \leftarrow offChainBalance(charged) - x
28:
        end if
29:
        offChainBalance(Bob) \leftarrow offChainBalance(Bob) + x
```

Fig. 20.

```
Functionality \mathcal{F}_{\mathrm{PayNet}}- close
 1: Upon receiving (CLOSECHANNEL, receipt) from Alice
 2:
         ensure that there is a channel \in channels : receipt (channel) = receipt
 3:
         retrieve fchid from channel
         \texttt{pendingClose}\left(fchid\right) \leftarrow Alice
 4:
 5:
         send (CLOSECHANNEL, fchid, Alice) to S
 6: Upon receiving (CHANNELCLOSED, fchid) from S:
 7:
         Alice \leftarrow \mathtt{pendingClose}\left(fchid\right)
 8:
         retrieve Charlie, Bob, x, y, pchid from channel with ID fchid
9:
         ensure that Charlie = Alice
10:
         send (READ) to \mathcal{G}_{Ledger} as Alice and assign reply to \Sigma_{Alice}
11:
         ensure that transaction with ID pchid is in \Sigma_{Alice}, is spent, x of its coins
    are spendable or will be spendable exclusively by Alice and y of its coins are
    spendable exclusively by Bob
12:
         \texttt{pendingClose}\left(fchid\right) \leftarrow \bot
         add receipt of channel to closedChannels(Bob)
13:
14:
         remove channel from channels
15:
         \texttt{onChainBalance} \ (Alice) \leftarrow \texttt{onChainBalance} \ (Alice) + x
         \mathtt{onChainBalance}\left(Bob\right) \leftarrow \mathtt{onChainBalance}\left(Bob\right) + y
16:
17:
         \texttt{offChainBalance}\left(Alice\right) \leftarrow \texttt{offChainBalance}\left(Alice\right) - x
18:
         \texttt{offChainBalance}\left(Bob\right) \leftarrow \texttt{offChainBalance}\left(Bob\right) - y
19:
         send (CHANNELCLOSED, receipt from channel) to Alice
```

Fig. 21.

```
Functionality \mathcal{F}_{\mathrm{PayNet}}- poll
 1: Upon receiving (POLL) from Alice:
       send (READ) to \mathcal{G}_{\text{Ledger}} as Alice and assign reply to \Sigma_{Alice}
2:
3:
       add largest block number in \Sigma_{Alice} to polls(Alice)
       scan \Sigma_{Alice} for honestly closed channels that contain Alice and exist in
    channels (txs that spend funding txs that have the same channel version as
   stored), remove them from channels and add them to toReport (marked as
    "honest")
       scan \Sigma_{Alice} for maliciously closed channels that contain Alice and exist in
    channels (txs that spend funding txs that have an older channel version than
       for all maliciously closed channels by a remote committment tx in block
    with height h_{tx} do
           if the delayed output (of the counterparty) has been spent AND
   polls(Alice) has an element in [h_{tx} + k, h_{tx} + k + delay(Alice) - 1] then
               halt // Alice wasn't negligent but couldn't punish - bad event
8:
9:
           end if
           add channel to toReport (marked as "malicious")
10:
11:
        end for
12:
        generate unique reportid and store toReport with it
13:
        send (GETCLOSEDFUNDS, toReport, Alice, reported) to S
14: // Expected after resolutions are visible on-chain
15: Upon receiving (CHANNELSCLOSED, details, Alice, reported) from S:
        retrieve toReport associated with reportid (if none, ignore message) and
16:
    remove pair from storage
17:
       send (READ) to \mathcal{G}_{\text{Ledger}} as Alice and assign reply to \Sigma_{Alice}
18:
        \mathbf{for} \ \mathbf{all} \ \mathbf{channel} \in \mathtt{details} \ \mathbf{do}
           ensure channel \in toReport
19:
20:
           if channel is marked as "malicious" then
21:
               ensure that transactions that spend the funding tx of channel and
    pay Alice the entire channel value exist in \Sigma_{Alice}, otherwise halt
22:
           else // channel is marked as "honest"
23:
               ensure that transactions that spend the funding tx of channel and
   pay Alice her part in the latest state of channel exist in \Sigma_{Alice} AND she
   received funds from all "received HTLCs" for which she knew the preimage
    when she received the associated POLL AND received funds from all "offered
   HTLCs" that had timed out when she received the associated POLL, otherwise
   halt
24:
           end if
25:
           add the receipt of channel to closedChannels(Alice)
26:
           remove channel from channels
       end for
27:
```

Fig. 22.

# Functionality $\mathcal{F}_{PayNet}$ - miscalleanous 1: Upon receiving (PUSHFULFILL, pchid) from Alice: 2: send (PUSHFULFILL, pchid, Alice) to $\mathcal{S}$ 3: Upon receiving (PUSHADD, pchid) from Alice: 4: send (PUSHADD, pchid, Alice) to $\mathcal{S}$ 5: Upon receiving (COMMIT, pchid) from Alice: 6: send (COMMIT, pchid, Alice) to $\mathcal{S}$

- 7: Upon receiving (fulfillOnChain) from Alice:
- 8: send (READ) to  $\mathcal{G}_{\text{Ledger}}$  as Alice and assign largest block number to t
- 9: add t to focs(Alice)
- 10: send (FULFILLONCHAIN, t, Alice) to S
- 11: Upon receiving (GETNEW) from Alice:
- 12: clear newChannels(Alice), closedChannels(Alice), pendingUpdates(Alice) and send them to Alice

Fig. 23.

# 7 Security Proof

```
Functionality \mathcal{F}_{\text{PayNet,dummy}}

1: Upon receiving any message M from Alice:

2: if M is a valid \mathcal{F}_{\text{PayNet}} message from a player then

3: send (M, Alice) to \mathcal{S}

4: end if

5: Upon receiving any message (M, Alice) from \mathcal{S}:

6: if M is a valid \mathcal{F}_{\text{PayNet}} message from \mathcal{S} then

7: send M to Alice

8: end if
```

Fig. 24.

#### Simulator $\mathcal{S}_{\mathrm{LN}}$

Expects the same messages as the protocol, but messages that the protocol expects to receive from  $\mathcal{E}$ , the simulator expects to receive from  $\mathcal{F}_{\text{PayNet,dummy}}$  with the name of the player appended. The simulator internally executes one copy of the protocol per player. Upon receiving any message, the simulator runs the relevant code of the protocol copy tied to the appended player name. Mimicking the real-world case, if a protocol copy sends a message to another player, that message is passed to  $\mathcal{A}$  as if sent by the player and if  $\mathcal{A}$  allows the message to reach the receiver, then the simulator reacts by acting upon the message with the protocol copy corresponding to the recipient player. A message sent by a protocol copy to  $\mathcal{E}$  will be routed by  $\mathcal{S}$  to  $\mathcal{F}_{\text{PayNet,dummy}}$  instead. To distinguish which player it comes from,  $\mathcal{S}$  also appends the player name to the message.

Fig. 25.

Lemma 1. 
$$\text{Exec}_{\Pi_{\text{LN}}, \mathcal{A}_{\text{d}}, \mathcal{E}}^{\mathcal{G}_{\text{Ledger}}} = \text{Exec}_{\mathcal{S}_{\text{LN}}, \mathcal{E}}^{\mathcal{F}_{\text{PayNet,dummy}}, \mathcal{G}_{\text{Ledger}}}$$

*Proof.* Consider a message that  $\mathcal{E}$  sends. In the real world, the protocol ITIs produce an output. In the ideal world, the message is given to  $\mathcal{S}_{LN}$  through  $\mathcal{F}_{PayNet,dummy}$ . The former simulates the protocol ITIs of the real world (along with their coin flips) and so produces an output from the exact same distribution, which is given to  $\mathcal{E}$  through  $\mathcal{F}_{PayNet,dummy}$ . Thus the two outputs are indistinguishable.

```
1: For messages REGISTER, REGISTERDONE and REGISTERED, act like \mathcal{F}_{PayNet}.

2: Upon receiving any other message M from Alice:

3: if M is a valid \mathcal{F}_{PayNet} message from a player then

4: send (M, Alice) to \mathcal{S}

5: end if

6: Upon receiving any other message (M, Alice) from \mathcal{S}:

7: if M is a valid \mathcal{F}_{PayNet} message from \mathcal{S} then

8: send M to Alice

9: end if
```

Fig. 26.

# Simulator $\mathcal{S}_{\mathrm{LN-Reg}}$

Like  $\mathcal{S}_{LN}$ , but it does not accept (REGISTERED) from  $\mathcal{F}_{PayNet,Reg}$ . Additional differences:

- 1: Upon receiving (REGISTER, Alice, delay, relayDelay, lastPoll) from  $\mathcal{F}_{PayNet,Reg}$ :
- 2: delay of Alice ITI  $\leftarrow$  delay
- 3: relayDelay of  $Alice ITI \leftarrow relayDelay$
- 4: lastPoll of  $Alice ITI \leftarrow lastPoll$
- 5:  $(pk_{Alice}, sk_{Alice})$  of  $Alice ITI \leftarrow \texttt{KeyGen}()$
- 6: send (REGISTERDONE, Alice,  $pk_{Alice}$ ) to  $\mathcal{F}_{\text{PayNet,Reg}}$

Fig. 27.

$$\mathbf{Lemma~2.}~\mathrm{Exec}_{\mathcal{S}_{\mathrm{LN}},\mathcal{E}}^{\mathcal{F}_{\mathrm{PayNet,dummy}},\mathcal{G}_{\mathrm{Ledger}}} = \mathrm{Exec}_{\mathcal{S}_{\mathrm{LN-Reg}},\mathcal{E}}^{\mathcal{F}_{\mathrm{PayNet,Reg}},\mathcal{G}_{\mathrm{Ledger}}}$$

*Proof.* When  $\mathcal{E}$  sends (REGISTER, delay, relayDelay) to *Alice*, it receives as a response (REGISTER, *Alice*, delay, relayDelay,  $pk_{Alice}$ ) where  $pk_{Alice}$  is a public key generated by KeyGen() both in the real (c.f. Fig. 1, line 9) and in the ideal world (c.f. Fig. 27, line 5).

Furthermore, one (READ) is sent to  $\mathcal{G}_{Ledger}$  from *Alice* in both cases (Fig. 1, line 8 and Fig. 18, line 9).

Additionally,  $S_{\text{LN-Reg}}$  ensures that the state of *Alice* ITI is exactly the same as what would have been in the case of  $S_{\text{LN}}$ , as lines 6-9 of Fig. 1 change the state of *Alice* ITI in the same way as lines 2-5 of Fig. 27.

Lastly, the fact that the state of the Alice ITIs are changed in the same way in both worlds, along with the same argument as in the proof of Lemma 1 ensures that the rest of the messages are responded in an indistinguishable way in both worlds.

```
Functionality \mathcal{F}_{\mathrm{PayNet},\mathrm{Open}}
1: For messages register, registerDone, registered, openChannel,
   CHANNELANNOUNCED and CHECKNEW, act like \mathcal{F}_{\text{PayNet}}.
2: Upon receiving any other message M from Alice:
       if M is a valid \mathcal{F}_{\text{PayNet}} message from a player then
           send (M, Alice) to S
4:
       end if
5:
6: Upon receiving any other message (M, Alice) from S:
7:
       if M is a valid \mathcal{F}_{\text{PayNet}} message from \mathcal{S} then
8:
           send M to Alice
9:
       end if
```

Fig. 28.

```
Simulator \mathcal{S}_{\mathrm{LN-Reg-Open}}
Like S_{LN-Reg}. Differences:
 1: Upon receiving (OPENCHANNEL, Alice, Bob, x, fchid, tid) from \mathcal{F}_{PavNet,Open}:
        if both Alice and Bob are honest then
           Simulate the interaction between Alice and Bob in their respective ITI,
    as defined in Figures 2-6. All messages should be handed to and received from
    \mathcal{A}, as in the real world execution.
           After sending (FUNDINGSIGNED) as Bob to Alice, send
    (CHANNELANNOUNCED, Bob, p_{Alice,F}, p_{Bob,F}, fchid, pchid, tid) to \mathcal{F}_{PayNet,Open}.
 5:
           After submitting F to \mathcal{G}_{Ledger} as Alice, send
    (CHANNELANNOUNCED, Alice, p_{Alice,F}, p_{Bob,F}, fchid, pchid) to \mathcal{F}_{PayNet,Open}.
 6:
        else if Alice is honest, Bob is corrupted then
 7:
           Simulate Alice's part of the interaction between Alice and Bob in
    Alice's ITI, as defined in Figures 2, 4, and 6.All messages should be handed to
    and received from A, as in the real world execution.
           After submitting F to \mathcal{G}_{Ledger} as Alice, send
    (CHANNELANNOUNCED, Alice, p_{Alice,F}, p_{Bob,F}, fchid, pchid) to \mathcal{F}_{PayNet,Open}.
        else if Alice is corrupted, Bob is honest then
9:
           send (OPENCHANNEL, Alice, Bob, x, fchid, tid) to simulated (corrupted)
10:
    Alice
11:
           Simulate Bob's part of the interaction between Alice and Bob in Bob's
    ITI, as defined in Figures 3 and 5. All messages should be handed to and
    received from \mathcal{A}, as in the real world execution.
12:
            After sending (FUNDINGSIGNED) as Bob to Alice, send
    (CHANNELANNOUNCED, Bob, p_{Alice,F}, p_{Bob,F}, fchid, pchid) to \mathcal{F}_{PayNet,Open}.
13:
        else if both Alice and Bob are corrupted then
14:
            forward message to \mathcal{A} // \mathcal{A} may open the channel or not
15:
        end if
16: Upon receiving (CHANNELOPENED, Alice, fchid) from \mathcal{F}_{\text{PayNet,Open}}:
        execute lines 5-13 of Fig. 7 with Alice's ITI
17:
18:
        if Bob is honest then
19:
           expect the delivery of Alice's (FundingLocked) message from A
20:
           simulate Fig. 8 with received message in Bob's ITI
21:
        end if
22: Upon receiving (CHECKNEW, Alice, Bob, tid) from \mathcal{F}_{PayNet,Open}: // Alice
    should be corrupted
23:
        send (CHECKNEW, Alice, Bob, tid) as \mathcal{E} to \mathcal{A}
24:
        if Bob is honest then
25:
           expect a (FUNDINGLOCKED) message from A
26:
           simulate Fig. 8 with received message in Bob's ITI
27:
        end if
```

Fig. 29.

# $\mathbf{Lemma~3.}~\mathrm{Exec}_{\mathcal{S}_{\mathrm{LN-Reg}},\mathcal{E}}^{\mathcal{F}_{\mathrm{PayNet,Reg}},\mathcal{G}_{\mathrm{Ledger}}} = \mathrm{Exec}_{\mathcal{S}_{\mathrm{LN-Reg-Open}},\mathcal{E}}^{\mathcal{F}_{\mathrm{PayNet,Open}},\mathcal{G}_{\mathrm{Ledger}}}$

Proof. When  $\mathcal{E}$  sends (OPENCHANNEL, Alice, Bob, x, fchid, tid) to Alice, the interaction of Figures 2-6 will be executed in both the real and the ideal world. In more detail, in the ideal world the execution of the honest parties will be simulated by the respective ITIs run by  $\mathcal{S}_{\text{LN-Reg-Open}}$ , so their state will be identical to that of the parties in the real execution. Furthermore, since  $\mathcal{S}_{\text{LN-Reg-Open}}$  executes faithfully the protocol code, it generates the same messages as would be generated by the parties themselves in the real-world setting.

We observe that the input validity check executed by  $\mathcal{F}_{PayNet,Open}$  (Fig. 19, line 2) filters only messages that would be ignored by the real protocol as well and would not change its state either (Fig. 2, line 2).

We also observe that, upon receiving OPENCHANNEL or CHANNELAN-NOUNCED,  $\mathcal{F}_{PayNet,Open}$  does not send any messages to parties other than  $\mathcal{S}_{LN-Reg-Open}$ , so we don't have to simulate those.

When  $\mathcal{E}$  sends (CHECKNEW, Alice, Bob, tid) to Alice in the real world, line 2 of Fig. 7 will allow execution to continue if there exists an entry with temporary id tid in pendingOpen marked as "broadcast". Such an entry can be added either in Fig. 2, line 7 or in Fig. 3, line 6. The former event can happen only in case Alice received a valid OPENCHANNEL message from Bob with temporary id tid, which in turn can be triggered only by a valid OPENCHANNEL message with the same temporary id from  $\mathcal{E}$  to Bob, whereas the latter only in case Alice received a valid OPENCHANNEL message from  $\mathcal{E}$  with the same temporary id. Furthermore, in the first case the "broadcast" mark can be added only before Alice sends (FUNDINGSIGNED, pchid, sig) to Bob (Fig. 5, line 11) (which needs a valid Alice-Bob interaction up to that point more in-depth?), and in the second case the "broadcast" mark can be added only before Alice sends (SUBMIT, (sig,  $\mathcal{F}$ )) to  $\mathcal{G}_{\text{Ledger}}$  (Fig. 6, line 8) (which also needs a valid Alice-Bob interaction up to that point more in-depth?)

When  $\mathcal{E}$  sends (CHECKNEW, Alice, Bob, tid) to Alice in the ideal world, line 10 of Fig. 19 will allow execution to continue if there exists an entry with temporary id tid and member Alice marked as "Alice announced" in pendingOpen(fchid) for some fchid. This can only happen if line 8 of Fig. 19 is executed, where pendingOpen(fchid) contains tid as temporary id. This line in turn can only be executed if  $\mathcal{F}_{PayNet,Open}$  received (CHANNELANNOUNCED, Alice,  $p_{Alice,F}$ ,  $p_{Bob,F}$ , fchid, pchid, tid) from  $\mathcal{S}_{LN-Reg-Open}$  such that pendingOpen(fchid) exists and has temporary id tid, as mandated by line 7 of Fig. 19. Such a message is sent by  $\mathcal{S}_{LN-Reg-Open}$  of Fig. 29 either in lines 5/8, or in lines 4/12. One of the first

pair of lines is executed only if  $S_{\text{LN-Reg-Open}}$  receives (OPENCHANNEL, Alice, Bob, x, fchid, tid) from  $\mathcal{F}_{\text{PayNet,Open}}$  and the simulated  $\mathcal{A}$  allows a valid Alice-Bob interaction up to the point where Alice sends (SUBMIT) to  $\mathcal{G}_{\text{Ledger}}$ , whereas one of the second pair of lines is executed only if  $S_{\text{LN-Reg-Open}}$  receives (OPENCHANNEL, Bob, Alice, x, fchid, tid) from  $\mathcal{F}_{\text{PayNet,Open}}$  and the simulated  $\mathcal{A}$  allows a valid Alice-Bob interaction up to the point where Alice sends (FUNDINGSIGNED) to Bob.

The last two points lead us to deduce that line 10 of Fig. 19 in the ideal and line 2 of Fig. 7 in the real world will allow execution to continue in the exact same cases with respect to the messages that  $\mathcal{E}$  and  $\mathcal{A}$  send. Given that execution continues, *Alice* subsequently sends (READ) to  $\mathcal{G}_{Ledger}$  and performs identical checks in both the ideal (Fig. 19, lines 12-13) and the real world (Fig. 7, lines 3-4).

Moving on, in the real world lines 5-13 of Fig. 7 are executed by *Alice* and, given that  $\mathcal{A}$  allows it, the code of Fig. 8 is executed by *Bob*. Likewise, in the ideal world, the functionality executes lines 14-32 and as a result it (always) sends (CHANNELOPENED, *Alice*, *fchid*) to  $\mathcal{S}$ . In turn  $\mathcal{S}$  simulates lines 5-13 of Fig. 7 with *Alice*'s ITI and, if  $\mathcal{A}$  allows it,  $\mathcal{S}$  simulates the code of Fig. 8 with *Bob*'s ITI. Once more we conclude that both worlds appear to behave identically to both  $\mathcal{E}$  and  $\mathcal{A}$  under the same inputs from them.

```
Functionality \mathcal{F}_{PavNet,Pav}
1: For messages register, registerDone, registered, openChannel,
   CHANNELANNOUNCED, CHECKNEW, POLL, PAY, PUSHADD, PUSHFULFILL,
   FULFILLONCHAIN and COMMIT, act like \mathcal{F}_{PavNet}.
2: Upon receiving any other message M from Alice:
      if M is a valid \mathcal{F}_{\text{PayNet}} message from a player then
4:
          send (M, Alice) to S
5:
       end if
6: Upon receiving any other message (M, Alice) from S:
      if M is a valid \mathcal{F}_{\text{PayNet}} message from \mathcal{S} then
7:
          send M to Alice
8:
      end if
```

Fig. 30.

```
Simulator \mathcal{S}_{\mathrm{LN-Reg-Open-Pay}}
Like \mathcal{S}_{\text{LN-Reg-Open}}. Differences:
 1: Upon receiving (FULFILLONCHAIN, t, Alice) from \mathcal{F}_{PayNet,Pay}:
        Execute lines 20-26 of Fig. 16 as Alice, using t from message
 3: Upon receiving (PAY, Alice, Bob, x, \overrightarrow{path}, \texttt{receipt}, payid) from \mathcal{F}_{PayNet, Pay}:
        strip payid, simulate receiving the message with Alice ITI and further
    execute the parts of \Pi_{LN} that correspond to honest parties (Fig. 10- Fig. 12)
        if any "ensure" in \Pi_{LN} fails until receiver processes UPDATEADDHTLC
    then // payment failed
            \texttt{charged} \leftarrow \bot
        else if honest Charlie ∈ path that sent UPDATEFULFILLHTLC to a
    corrupted player either (gets the fulfillment of the HTLC irrevocably
    committed OR fulfills the HTLC on-chain), AND the next honest player Dave
    down the line successfully times out the HTLC on-chain (i.e. adds the relevant
    HTLC-Timeout to \mathcal{G}_{Ledger}) (due to no or bad communication from the
    corrupted player) then
            set charged to one of the corrupted parties between Charlie and Dave
        else if player before Charlie (closer to payer) goes on-chain with an older
    version (that doesn't contain the HTLC) at block h_{\mathrm{com}} and Charlie doesn't
    POLL in time to revoke (is negligent during
    [h_{\text{com}} + k, h_{\text{com}} + \text{CharlieDelay} + k - 1]) OR player after Charlie (closer to
    receiver) fulfills the HTLC on-chain and Charlie doesn't POLL in time to learn
    preimage and send UPDATEFULFILLHTLC (is relay-negligent during
    [TODO, cltvExpiry -2k-1]) then
10:
            charged \leftarrow Charlie
11:
        else
12:
            charged \leftarrow Alice
13:
        end if
14:
        send (RESOLVEPAY, payid, charged) to \mathcal{F}_{PavNet,Pav}
```

Fig. 31.

```
Lemma 4. \operatorname{Exec}_{\mathcal{S}_{\operatorname{LN-Reg-Open}},\mathcal{E}}^{\mathcal{F}_{\operatorname{PayNet},\operatorname{Open}},\mathcal{G}_{\operatorname{Ledger}}} \overset{c}{\approx} \operatorname{Exec}_{\mathcal{S}_{\operatorname{LN-Reg-Open-Pay}},\mathcal{E}}^{\mathcal{F}_{\operatorname{PayNet},\operatorname{Pay}},\mathcal{G}_{\operatorname{Ledger}}}

Proof. continue When \mathcal{E} sends (PAY, Bob, x, \overrightarrow{\operatorname{path}}, \operatorname{receipt})

When \mathcal{E} sends (POLL)

When \mathcal{E} sends (PUSHADD, pchid)

When \mathcal{E} sends (PUSHFULFILL, pchid)

When \mathcal{E} sends (FULFILLONCHAIN)

When \mathcal{E} sends (COMMIT, pchid)
```

# 8 Combined Sign primitive

# 8.1 Algorithms

```
- (mpk, msk) \leftarrow \text{MasterKeyGen} \left( 1^k \right) 
- (pk, sk) \leftarrow \text{KeyShareGen} \left( 1^k \right) 
- (cpk_l, csk_l) \leftarrow \text{CombineKey} \left( msk, mpk, sk, pk \right) 
- cpk_l \leftarrow \text{CombinePubKey} \left( mpk, pk \right) 
- \sigma \leftarrow \text{Sign} \left( csk, m \right) 
- \left\{ 0, 1 \right\} \leftarrow \text{Verify} \left( cpk, m, \sigma \right)
```

#### 8.2 Correctness

```
 \begin{array}{l} - \ \forall k \in \mathbb{N}, \\ \Pr[(mpk, msk) \leftarrow \operatorname{MasterKeyGen}\left(1^k\right), \\ (pk, sk) \leftarrow \operatorname{KeyShareGen}\left(1^k\right), \\ (cpk_1, csk_1) \leftarrow \operatorname{CombineKey}\left(msk, mpk, sk, pk\right), \\ cpk_2 \leftarrow \operatorname{CombinePubKey}\left(mpk, pk\right), \\ cpk_1 = cpk_2] = 1 \\ - \ \forall k \in \mathbb{N}, m \in \mathcal{M}, \\ \Pr[(mpk, msk) \leftarrow \operatorname{MasterKeyGen}\left(1^k\right), \\ (pk, sk) \leftarrow \operatorname{KeyShareGen}\left(1^k\right), \\ (cpk, csk) \leftarrow \operatorname{CombineKey}\left(mpk, msk, pk, sk\right), \\ \operatorname{Verify}(cpk, m, \operatorname{Sign}(csk, m)) = 1] = 1 \end{array}
```

#### 8.3 Security

```
Game share-EUF^{\mathcal{A}} (1^k)

1: (\mathsf{aux}, mpk, n) \leftarrow \mathcal{A} (INIT)

2: for i \leftarrow 1 to n do

3: (pk_i, sk_i) \leftarrow \mathsf{KEYSHAREGEN} (1^k)

4: end for

5: (cpk^*, pk^*, m^*, \sigma^*) \leftarrow \mathcal{A} (KEYS, \mathsf{aux}, pk_1, \ldots, pk_n)

6: if pk^* \in \{pk_1, \ldots, pk_n\} \land cpk^* = \mathsf{COMBINEPUBKEY} (mpk, pk^*) \land \mathsf{VERIFY} (cpk^*, m^*, \sigma^*) = 1 then

7: return 1

8: else

9: return 0

10: end if
```

Fig. 32.

**Definition 1.** A Combined Sign scheme is share-EUF-secure if

$$\forall k \in \mathbb{N}, \forall \mathcal{A} \in \mathsf{PPT}, \Pr\left[\mathsf{share}\text{-}\mathsf{EUF}^{\mathcal{A}}\left(1^{k}\right) = 1\right] < negl\left(k\right)$$

```
1: (mpk, msk) \leftarrow \text{MasterKeyGen}(1^k)
 3: (aux_i, response) \leftarrow \mathcal{A}(INIT, mpk)
 4: while response can be parsed as (pk, sk, m) do
         i \leftarrow i+1
 6:
         store pk, sk, m as pk_i, sk_i, m_i
          (cpk_i, csk_i) \leftarrow \text{CombineKey}(mpk, msk, pk_i, sk_i)
 7:
          \sigma_i \leftarrow \text{Sign}\left(csk_i, m_i\right)
          (\mathtt{aux}_i, \mathtt{response}) \leftarrow \mathcal{A}(\mathtt{SIGNATURE}, \mathtt{aux}_{i-1}, \sigma_i)
10: end while
11: parse response as (cpk^*, pk^*, m^*, \sigma^*)
12: if m^* \notin \{m_1, \dots, m_i\} \land cpk^* = \text{CombinePubKey}(mpk, pk^*) \land
     Verify (cpk^*, m^*, \sigma^*) = 1 then
13:
          return 1
14: else
          return 0
15:
16: end if
```

Fig. 33.

**Definition 2.** A Combined Sign scheme is master-EUF-CMA-secure if

$$\forall k \in \mathbb{N}, \forall \mathcal{A} \in \mathtt{PPT}, \Pr\left[\mathsf{master-EUF-CMA}^{\mathcal{A}}\left(1^{k}\right) = 1\right] < negl\left(k\right)$$

**Definition 3.** A Combined Sign scheme is combine-EUF-secure if it is both share-EUF-secure and master-EUF-CMA-secure.

#### 8.4 Construction

output standard signing keypairs to avoid duplication? Parameters:  $\mathcal{H}, G$ function MasterKeyGen( $1^k$ , rand)
Return (rand,  $G \cdot \text{rand}$ )
end function
function KeyShareGen( $1^k$ , rand)
Return (rand,  $G \cdot \text{rand}$ )

#### end function

function CombineKey(msk, mpk, sk, pk)

**return**  $msk \cdot \mathcal{H}(mpk \parallel pk) + sk \cdot \mathcal{H}(pk \parallel mpk)$ 

end function

function CombinePubKey(mpk, pk)

**return**  $mpk \cdot \mathcal{H}(mpk \parallel pk) + pk \cdot \mathcal{H}(pk \parallel mpk)$ 

end function

function Sign(csk, m)

like standard sign

end function

function  $Verify(cpk, m, \sigma)$ 

like standard verify

#### end function

just to remember

 $sh_{\text{rev},n} \leftarrow shb_{\text{rev}} \cdot \mathcal{H}\left(phb_{\text{rev}}||pt_{\text{com},n}\right) + st_{\text{com},n} \cdot \mathcal{H}\left(pt_{\text{com},n}||phb_{\text{rev}}\right)$  $pt_{\text{rev},n+2} \leftarrow ptb_{\text{rev}} \cdot \mathcal{H}\left(ptb_{\text{rev}}||ph_{\text{com},n+2}\right) + ph_{\text{com},n+2} \cdot \mathcal{H}\left(ph_{\text{com},n+2}||ptb_{\text{rev}}\right)$  $ph_{\text{rev},n+2} \leftarrow phb_{\text{rev}} \cdot \mathcal{H}\left(phb_{\text{rev}}||pt_{\text{com},n+2}\right) + pt_{\text{com},n+2} \cdot \mathcal{H}\left(pt_{\text{com},n+2}||phb_{\text{rev}}\right)$ 

**Lemma 5.** The construction above is share-EUF-secure in the Random Oracle model under the assumption that the underlying signature scheme is strongly EUF-CMA-secure and the range of the Random Oracle coincides with that of the underlying signature scheme signing keys.

*Proof.* Let  $k \in \mathbb{N}, \mathcal{B}$  PPT algorithm such that

$$\Pr\left[\mathsf{share}\mathsf{-EUF}^{\mathcal{B}}\left(1^{k}\right)=1\right]=a>\operatorname{negl}\left(k\right)$$
 .

We construct a PPT distinguisher  $\mathcal{A}$  (Fig. 34) such that

$$\Pr\left[\mathsf{EUF\text{-}CMA}^{\mathcal{A}}\left(1^{k}\right)=1\right]>\operatorname{negl}\left(k\right)$$

that breaks the assumption, thus proving Lemma 5.

```
Algorithm \mathcal{A}\left(vk\right)
 1: j \stackrel{\$}{\leftarrow} U[1, T(\mathcal{B})] // T(M) is the maximum running time of M
          Random Oracle: for every first-seen query q from \mathcal{B} set \mathcal{H}(q) to a random
          return \mathcal{H}(q) to \mathcal{B}
 4: (aux, mpk, n) \leftarrow \mathcal{A}(INIT)
 5: for i \leftarrow 1 to n do
          (pk_i, sk_i) \leftarrow \text{KeyShareGen}(1^k)
 7: end for
          Random Oracle: Let q be the jth first-seen query from \mathcal{B}:
 9:
          if q = (mpk \parallel x) then
               if \mathcal{H}(x \parallel mpk) unset then
                    set \mathcal{H}(x || mpk) to a random value
11:
12:
               set \mathcal{H}(mpk \parallel x) to (vk - x \cdot \mathcal{H}(x \parallel mpk)) \cdot mpk^{-1}
13:
14:
          else if q = (x \parallel mpk) then
               if \mathcal{H}(mpk \parallel x) unset then
15:
16:
                    set \mathcal{H}(mpk || x) to a random value
17:
               set \mathcal{H}(x \parallel mpk) to (vk - mpk \cdot \mathcal{H}(mpk \parallel x)) \cdot x^{-1}
18:
19:
          else
20:
               set \mathcal{H}(q) to a random value
21:
          end if
22:
          return \mathcal{H}(q) to \mathcal{B}
23: (cpk^*, pk^*, m^*, \sigma^*) \leftarrow \mathcal{B}(\text{KEYS}, \text{aux}, pk_1, \dots, pk_n)
24: if vk = cpk^* \wedge \mathcal{B} wins the share-EUF game then //\mathcal{A} won the EUF-CMA game
25:
          return (m^*, \sigma^*)
26: else
27:
          return FAIL
28: end if
```

Fig. 34.

Let Y be the range of the random oracle. The modified random oracle used in Fig. 34 is indistinguishable from the standard random oracle by PPT algorithms since the statistical distance of the standard random oracle from the modified one is at most  $\frac{1}{2|Y|} < negl(k)$  as they differ in at most one element.

Let E denote the event in which  $\mathcal{B}$  does not invoke CombinePubKey to produce  $cpk^*$ . In that case the values  $\mathcal{H}(pk^* \parallel mpk)$  and  $\mathcal{H}(mpk \parallel pk^*)$ 

are decided after  $\mathcal{B}$  terminates (Fig. 34, line 24) and thus

$$\Pr\left[cpk^* = \text{CombinePubKey}\left(mpk, pk^*\right) | E\right] = \frac{1}{|Y|} < negl\left(k\right) \Rightarrow$$

$$\Pr\left[cpk^* = \text{CombinePubKey}\left(mpk, pk^*\right) \land E\right] < negl\left(k\right) .$$
(1)

It is

$$(\mathcal{B} \text{ wins}) \to (cpk^* = \text{CombinePubKey}(mpk, pk^*)) \Rightarrow$$

$$\Pr\left[\mathcal{B} \text{ wins}\right] \leq \Pr\left[cpk^* = \text{CombinePubKey}(mpk, pk^*)\right] \Rightarrow$$

$$\Pr\left[\mathcal{B} \text{ wins} \land E\right] \leq \Pr\left[cpk^* = \text{CombinePubKey}(mpk, pk^*) \land E\right] \stackrel{(1)}{\Rightarrow}$$

$$\Pr\left[\mathcal{B} \text{ wins} \land E\right] < negl(k) .$$

But we know that  $\Pr[\mathcal{B} \text{ wins}] = \Pr[\mathcal{B} \text{ wins} \wedge E] + \Pr[\mathcal{B} \text{ wins} \wedge \neg E]$ and  $\Pr[\mathcal{B} \text{ wins}] = a$  by the assumption, thus

$$\Pr\left[\mathcal{B} \text{ wins } \wedge \neg E\right] > a - negl(k) \quad . \tag{2}$$

We now focus at the event  $\neg E$ . Let F the event in which the call of  $\mathcal{B}$  to CombinePubKey to produce  $cpk^*$  results in the jth invocation of the Random Oracle. Since j is chosen uniformly at random,  $\Pr[F|\neg E] = \frac{1}{T(\mathcal{B})}$ . Observe that  $\Pr[F|E] = 0 \Rightarrow \Pr[F] = \Pr[F|\neg E] = \frac{1}{T(\mathcal{B})}$ .

In the case where the event  $(F \wedge \mathcal{B} \text{ wins } \land \neg E)$  holds, it is

$$cpk^* = \text{CombinePubKey}(mpk, pk^*) = mpk \cdot \mathcal{H}(mpk \parallel pk^*) + pk^* \cdot \mathcal{H}(pk^* \parallel mpk)$$

Since F holds, the jth invocation of the Random Oracle queried either  $\mathcal{H}(mpk \parallel pk^*)$  or  $\mathcal{H}(pk^* \parallel mpk)$ . In either case (Fig. 34, lines 9-18), it is  $cpk^* = vk$ . This means that VERIFY  $(vk, m^*, \sigma^*) = 1$ . We conclude that in the event  $(F \wedge \mathcal{B} \text{ wins } \wedge \neg E)$ ,  $\mathcal{A}$  wins the EUF-CMA game. A final observation is that the probability that the events  $(\mathcal{B} \text{ wins } \wedge \neg E)$  and F are almost independent, thus

$$\Pr[F \land \mathcal{B} \text{ wins } \land \neg E] = \Pr[F] \Pr[\mathcal{B} \text{ wins } \land \neg E] \pm negl(k) \stackrel{(2)}{=} \frac{a - negl(k)}{T(\mathcal{B})} \pm negl(k) > negl(k)$$

**Lemma 6.** The construction above is master-EUF-CMA-secure in the Random Oracle model under the assumption that the underlying signature scheme is strongly EUF-CMA-secure and the range of the Random Oracle coincides with that of the underlying signature scheme signing keys.

*Proof.* Let  $k \in \mathbb{N}, \mathcal{B}$  PPT algorithm such that

$$\Pr\left[\mathsf{master}\text{-}\mathsf{EUF}\text{-}\mathsf{CMA}^{\mathcal{B}}\left(1^{k}\right)=1\right]=a>\operatorname{negl}\left(k\right)\ .$$

We construct a PPT distinguisher  $\mathcal{A}$  (Fig. 35) such that

$$\Pr\left[\mathsf{EUF\text{-}CMA}^{\mathcal{A}}\left(1^{k}\right)=1\right]>\operatorname{negl}\left(k\right)$$

that breaks the assumption, thus proving Lemma 6.

```
Algorithm \mathcal{A}(vk)
 1: j \stackrel{\$}{\leftarrow} U[1, T(\mathcal{B}) + T(\mathcal{A})] // T(M) is the maximum running time of M
 2:
          Random Oracle: for every first-seen query q from \mathcal{B} set \mathcal{H}(q) to a random
          return \mathcal{H}(q) to \mathcal{B}
 3:
 4: (mpk, msk) \leftarrow \text{MasterKeyGen}(1^k)
          Random Oracle: Let q be the jth first-seen query from \mathcal B or \mathcal A:
          if q = (mpk \parallel x) then
 6:
 7:
               if \mathcal{H}(x \parallel mpk) unset then
                    set \mathcal{H}(x \parallel mpk) to a random value
 8:
 9:
               end if
               set \mathcal{H}(mpk \parallel x) to (vk - x \cdot \mathcal{H}(x \parallel mpk)) \cdot mpk^{-1}
10:
          else if q = (x \parallel mpk) then
11:
12:
               if \mathcal{H}(mpk \parallel x) unset then
13:
                     set \mathcal{H}(mpk \parallel x) to a random value
14:
15:
               set \mathcal{H}(x \parallel mpk) to (vk - mpk \cdot \mathcal{H}(mpk \parallel x)) \cdot x^{-1}
16:
17:
               set \mathcal{H}(q) to a random value
18:
          end if
          return \mathcal{H}(q) to \mathcal{B} or \mathcal{A}
19:
20: i \leftarrow 0
21: (aux_i, response) \leftarrow \mathcal{B}(INIT, mpk)
22: while response can be parsed as (pk, sk, m) do
23:
          i \leftarrow i + 1
24:
          store pk, sk, m as pk_i, sk_i, m_i
          (cpk_i, csk_i) \leftarrow \text{CombineKey}(mpk, msk, pk_i, sk_i)
25:
26:
          \sigma_i \leftarrow \text{Sign}\left(csk_i, m_i\right)
27:
           (\mathtt{aux}_i, \mathtt{response}) \leftarrow \mathcal{B}\left(\mathtt{SIGNATURE}, \mathtt{aux}_{i-1}, \sigma_i\right)
28: end while
29: parse response as (cpk^*, pk^*, m^*, \sigma^*)
30: (cpk^*, pk^*, m^*, \sigma^*) \leftarrow \mathcal{B}(\text{KEYS}, \text{aux}, pk_1, \dots, pk_n)
31: if vk = cpk^* \wedge \mathcal{B} wins the master-EUF-CMA game then //\mathcal{A} won the
      EUF-CMA game
32:
          return (m^*, \sigma^*)
33: else
34:
          return FAIL
35: end if
```

Fig. 35.

The modified random oracle used in Fig. 35 is indistinguishable from the standard random oracle for the same reasons as in the proof of Lemma 5.

Let E denote the event in which CombinePubKey is not invoked to produce  $cpk^*$ . In that case the values  $\mathcal{H}(pk^* \parallel mpk)$  and  $\mathcal{H}(mpk \parallel pk^*)$ are decided after  $\mathcal{B}$  terminates (Fig. 35, line 31) and thus

$$\Pr\left[cpk^* = \text{CombinePubKey}\left(mpk, pk^*\right) | E\right] < negl\left(k\right) \Rightarrow \\ \Pr\left[cpk^* = \text{CombinePubKey}\left(mpk, pk^*\right) \land E\right] < negl\left(k\right) . \end{cases}$$
(3)

We can reason like in the proof of Lemma 5 to deduce that

$$\Pr\left[\mathcal{B} \text{ wins } \wedge \neg E\right] > a - negl\left(k\right) . \tag{4}$$

We now focus at the event  $\neg E$ . Let F the event in which the call of to CombinePubKey that produces  $cpk^*$  results in the jth invocation of the Random Oracle. Since j is chosen uniformly at random,  $\Pr[F|\neg E] =$  $\frac{1}{T(\mathcal{B})+T(\mathcal{A})}$ . Observe that  $\Pr[F|E]=0 \Rightarrow \Pr[F]=\Pr[F|\neg E]=\frac{1}{T(\mathcal{B})+T(\mathcal{A})}$ . Once more we can reason in the same fashion as in the proof of

Lemma 5 to deduce that

$$\Pr[F \land \mathcal{B} \text{ wins } \land \neg E] = \Pr[F] \Pr[\mathcal{B} \text{ wins } \land \neg E] \pm negl(k) \stackrel{(4)}{=} \frac{a - negl(k)}{T(\mathcal{B}) + T(\mathcal{A})} \pm negl(k) > negl(k)$$

**Theorem 1.** The construction above is combine-EUF-secure in the Random Oracle model under the assumption that the underlying signature scheme is strongly EUF-CMA-secure.

*Proof.* The construction is combine-EUF-secure as a direct consequence of Lemma 5, Lemma 6 and the definition of combine-EUF-security. 

#### Notes on Lightning Specification

The relevant part of the specification can be found at https://github. com/lightningnetwork/lightning-rfc/blob/master/02-peer-protocol. md.

#### References

1. Poon J., Dryja T.: The Bitcoin Lightning Network: Scalable Off-Chain Instant Payments