What is Trust

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 $\bf Abstract.$ We will try to define all the abstract properties that we would like "Trust" to have.

1 High-level idea

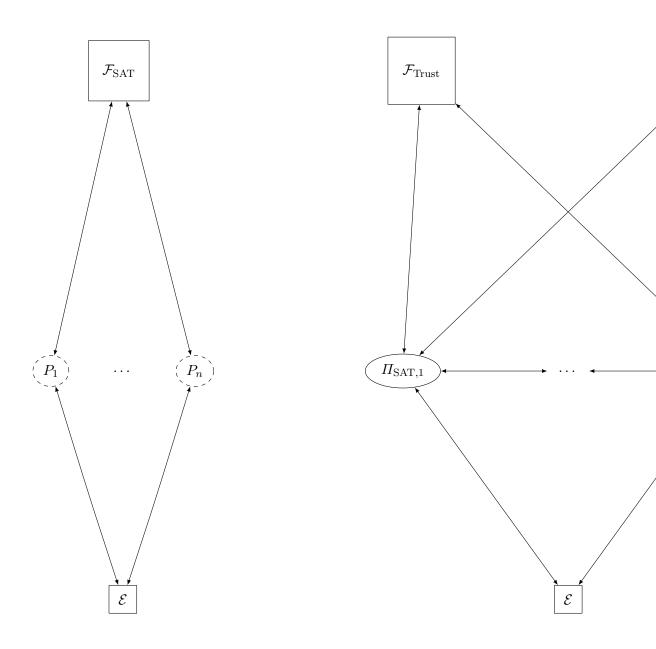


Fig. 1: (Almost) all functionalities

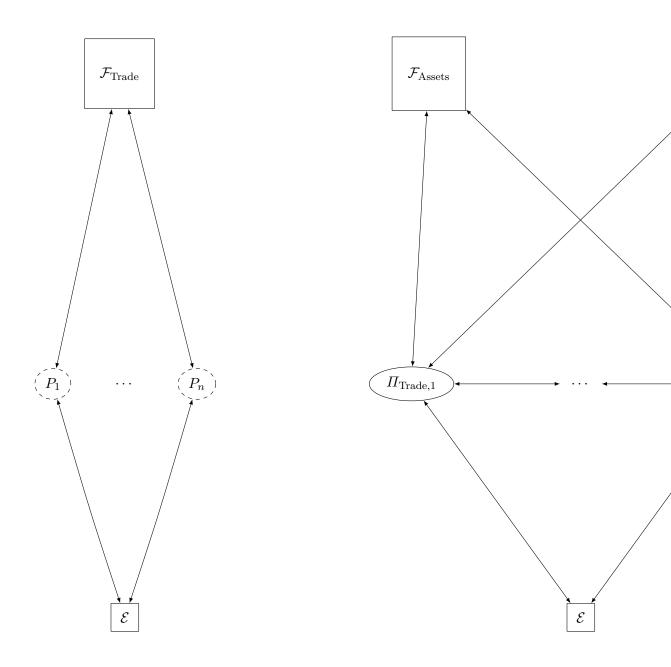


Fig. 2: Trade functionality and protocol

2 Utility Function Properties

Before Alice (an ITM that executes Π_{SAT}) can take any action, she must be assigned a utility function, U_{Alice} , and a discount function, λ_{Alice} , by \mathcal{E} .

Definition 1 (Utility Function). $U_{Alice}: Money \times multiset (Asset) \times Rep \rightarrow Utility, where <math>Money, Rep \in \mathbb{N}, Utility \in \mathbb{R}$.

The utility function is strictly increasing with respect to the integer Money and may do anything with respect to the quantity of any single Asset or combination of Assets. It is also strictly increasing with respect to the integer Rep.

Definition 2 (Reputation function). Reputation: $\mathcal{P} \times Time \times 2^{Asset} \rightarrow Rep$, where $Time, Rep \in \mathbb{N}$

$$Rep\left(Alice, t, set\right) > Rep\left(Bob, t, set\right) \stackrel{def}{\Longleftrightarrow}$$

$$\Pr\left[Alice \text{ receives message } (\text{canYouSatisfy}, d) \text{ at time t}\right] >$$

$$\Pr\left[Bob \text{ receives message } (\text{canYouSatisfy}, d) \text{ at time t}\right]$$

$$\text{where } d \in set$$

Rep is an integer that corresponds to Alice's perceived rank in a partial ordering of all the players where Alice assumes that she has a higher rank than Bob if the average probability of third players choosing her before Bob when they want to buy something amongst a list of Assets that Alice is interested in selling is higher than Bob's corresponding probability for the same Assets, as perceived by Alice.

Definition 3 (Discount function). $\lambda_{Alice} : Time \rightarrow [0, 1], where Time \in \mathbb{N}$ and λ is strictly decreasing.

Definition 4 (State). Let $state_{Alice} : Time \rightarrow Money \times multiset (Asset) \times Rep.$

Alice's target is to maximise

$$\sum_{t=1}^{\infty} \lambda_{Alice}\left(t\right) \left(\sum_{\substack{(m,a,r) \in \\ Money \times mset(Asset) \times Rep}} U_{Alice}\left(m,a,r\right) \cdot \Pr\left[state_{Alice}\left(t\right) = (m,a,r)\right] \right) \ .$$

3 Protocol

In this section we will describe the real protocol executed by the players in the absence of the \mathcal{F}_{SAT} ideal functionality. The description of this protocol does not need any utility function; all the "important" decisions are taken by the environment.

Consider an environment \mathcal{E} , and adversary \mathcal{A} and n players executing copies of the same protocol Π_1, \ldots, Π_n . \mathcal{E} can send the following messages to Π_i :

- 1. Manage player desires
 - (a) Satisfy $d \in \mathcal{D}$ through a player in $L \subseteq [n]$
 - (b) Abort attempt to satisfy $d \in \mathcal{D}$
- 2. Manage offered desires satisfaction
 - (a) Gain the ability to satisfy $d \in \mathcal{D}$ for players in $L \subseteq [n]$ for a price $x \in \mathbb{N}$
 - (b) Lose the ability to satisfy $d \in \mathcal{D}$ for players in $L \subseteq [n]$ for a price $x \in \mathbb{N}$
- 3. Satisfy another player's desire
 - (a) Satisfy player's $i \in [n]$ desire $d \in \mathcal{D}$ with the corresponding satisfaction string s
 - (b) Satisfy player's $i \in [n]$ desire $d \in \mathcal{D}$ with the satisfaction string s' (normally suitable for satisfying $d' \neq d, d' \in \mathcal{D}$)
 - (c) Ignore player's $i \in [n]$ desire $d \in \mathcal{D}$
- 4. Manage direct trusts
 - (a) Increase direct trust to player $i \in [n]$ by $x \in \mathbb{N}$
 - (b) Decrease direct trust to player $i \in [n]$ by $x \in \mathbb{N}$
 - (c) Steal direct trust $x \in \mathbb{N}$ from player $i \in [n]$

Some of these messages (e.g. 1b) are meaningful only when some other messages have been delivered previously (e.g. 1a). \mathcal{E} may send such messages even when they are not meaningful; the protocol should take care to reject/ignore such messages.

Let $i \in [n]$. Π_i can send the following messages to \mathcal{E} :

- 1. No player in L can satisfy my desire $d \in \mathcal{D}$
- 2. Desire $d \in \mathcal{D}$ made available for satisfaction amongst $L \subseteq [n]$ for price $x \in \mathbb{N}$
- 3. Desire $d \in \mathcal{D}$ made unavailable for satisfaction amongst $L \subseteq [n]$ for price $x \in \mathbb{N}$
- 4. Payment $x \in \mathbb{N}$ has been sent to player $j \in [n]$ for the satisfaction of desire $d \in \mathcal{D}$

- 5. Correct Payment $x \in \mathbb{N}$ from player $j \in [n]$ for the satisfaction of desire $d \in \mathcal{D}$ has been received
- 6. Wrong Payment $x \in \mathbb{N}$ from player $j \in [n]$ for the satisfaction of desire $d \in \mathcal{D}$ has been received
- 7. Player $j \in [n]$ has satisfied my desire $d \in \mathcal{D}$ with satisfaction string s
- 8. Player $j \in [n]$ has partially satisfied my desire $d \in \mathcal{D}$ with satisfaction string s' (normally suitable for satisfying $d' \neq d, d' \in \mathcal{D}$)
- 9. Player $j \in [n]$ has ignored my desire $d \in \mathcal{D}$
- 10. Direct trust to player $i \in [n]$ increased by $x \in \mathbb{N}$
- 11. Direct trust to player $i \in [n]$ decreased by $x \in \mathbb{N}$
- 12. Stole $x \in \mathbb{N}$ from player's $i \in [n]$ direct trust

 Π_i should only send these messages when \mathcal{E} is expecting them.

Let $i, j \in [n]$ The messages that can be sent between Π_i and Π_j are the following:

- 1. Can you satisfy $d \in \mathcal{D}$?
- 2. I can satisfy $d \in \mathcal{D}$ for a price $x \in \mathbb{N}$
- 3. I cannot satisfy $d \in \mathcal{D}$
- 4. Payment of $x \in \mathbb{N}$ for satisfaction of $d \in \mathcal{D}$ sent
- 5. Satisfaction string s', response to payment of $x \in \mathbb{N}$ for $d \in \mathcal{D}$

 Π_i is supposed to send 4 when it has already paid through \mathcal{F}_{Ledger} . Similarly, it is supposed to send 5 when it has verified that the other party has sent the corresponding payment on \mathcal{F}_{Ledger} .

Going in more detail, the actual protocol is as follows:

```
\Pi_{\text{SAT}}
    Initalization:
      util = \bot
2
   Upon receiving (type, t) from \mathcal{E}:
      util = t
    Upon receiving message (satisfy, d, L) from \mathcal{E}:
      If util == \bot:
         send message (utilityNotSet) to {\mathcal E}
         go to Idle state
10
      aux = L
11
      While (aux \neq \emptyset):
         send message (chooseBestSeller, d, aux) to \mathcal{F}_{\mathrm{Trust}}
13
         wait for response1 from \mathcal{F}_{\mathrm{Trust}}
14
```

```
If response1 == (bestSeller, d, L, Bob):
           send message (canYouSatisfy, d) to Bob
16
           wait for response2 from Bob
17
           If response2 == (IcanSatisfy, d, x, s):
18
             If util(state \cup s - x) > util(state) and s \in d
19
               send message (trade, x, s, Bob) to \mathcal{F}_{\mathrm{Trade}}
20
               wait for response3 from \mathcal{F}_{\mathrm{Trade}}
                  If response3 == (traded, x, s, Bob):
                    send message (satisfied, d, x, s, Bob) to \mathcal{F}_{Trust}
23
                    # maybe send only utility difference to \mathcal{F}_{\mathrm{Trust}}
24
                    send message (satisfied, d, L) to \mathcal{E}
25
                    go to Idle state
26
                  Else If response3 == (cheated, x, s, Bob):
27
                    send message (cheated, d, x, s, Bob) to \mathcal{F}_{Trust}
                    send message (cheated, d, L) to \mathcal{E}
                    go to Idle state
30
        Else: # if response1 == \bot
31
           send message (unsatisfied, d, L) to \mathcal{E}
32
           go to Idle state
33
         aux = aux \setminus \{Bob\} # only when response2 is not good
34
      send message (unsatisfied, d, L) to \mathcal{E}
35
36
    Upon receiving message (obtain, s) from \mathcal{E}:
37
      send message (obtain, s) to \mathcal{F}_{\mathrm{Trade}}
38
      wait for response from \mathcal{F}_{\mathrm{Trade}}
39
      If response == (obtained, s):
40
         send message (obtained, s) to {\cal E}
41
      Else:
42
         send message (notObtained, s) to {\cal E}
43
    Upon receiving message (lose, s) from \mathcal{E}:
45
      send message (lose, s) to \mathcal{F}_{\mathrm{Trade}}
46
      wait for response from \mathcal{F}_{\mathrm{Trade}}
47
      If response == (lost, s):
48
        send message (lost, s) to {\cal E}
49
      Else:
50
         send message (notLost, s) to {\cal E}
51
    Upon receiving message (canYouSatisfy, d) from Alice:
53
      If util == \bot:
54
```

```
ignore request, go to Idle state
55
      If (util(state + x + badRep) > util(state) or
56
          \mathtt{util}(state \setminus s + x + qoodRep) > \mathtt{util}(state)) and s \in d:
57
        send message (IcanSatisfy, d, x, s) to Alice \# x is the
58
             price
      Else:
59
        send message (IcannotSatisfy, d) to Alice
60
61
    Upon receiving message (willWeCheat, x, s, Alice) from \mathcal{F}_{Trade}:
62
      If util == \perp: # (Unreachable since we've already engaged)
63
        ignore request, go to Idle state
64
      If util(state \setminus s + x + goodRep) \geq util(state + x + badRep):
65
        send message (doNotCheat, x, s, Alice) to \mathcal{F}_{\mathrm{Trade}}
66
      Else:
67
        send message (cheat, x, s, Alice) to \mathcal{F}_{Trade}
```

4 Desire Satisfaction Ideal Functionality

Following the UC paradigm, in this section we define the ideal functionality for desire satisfaction, \mathcal{F}_{SAT} . In this setting, all the desires that are generated by the environment and are input to the players are immediately forwarded to \mathcal{F}_{SAT} ; the functionality decides which desires to satisfy. Since the players are dummy and all desires are satisfied by the functionality, no trust semantics amongst the players are necessary.

Nevertheless, given that all desires have a minimum cost, the cost semantics are still necessary, as we show with the following example: Consider a set of desires D with more elements than the total number of tokens all players have. D could never be satisfied by the players because of the high total cost, but a \mathcal{F}_{SAT} with no consideration for cost could in principle satisfy all desires in D.

The functionality can calculate the properties and functions defined in ??, ?? and ?? for all inputs at any moment in time.

Without knowledge of the utilities the environment is going to give to each satisfied desire, the functionality may fail spectacularly. So knowledge of the utility of each desire, or at least some function of the utility given the desires is needed. We can assume that \mathcal{F}_{SAT} knows U or an approximation of it.

Going into more detail, \mathcal{F}_{SAT} is a stateful process that acts as a market and as a bank for the players. The market does not offer a particular product for the same price to all users; For some users it may be cheaper

than for others, reflecting the fact that some players can realize some desires more efficiently than others.

 \mathcal{F}_{SAT} stores a number for each player that represents the amount of tokens this player has and a table with the price of each desire for each player. It provides the functions cost(u,d) which returns the cost of the desire d for player u with no side effects, sat(u,d) that returns the string that satisfies the desire d to u and reduces the amount of the tokens belonging to u by cost(u,d). There exists also the function $transfer(u_1,u_2,t)$ which reduces the amount of tokens u_1 has by t and increases the tokens of u_2 by t, given that initially the tokens belonging to u_1 were equal or more than t. This function is private to the functionality, thus can be used only internally.

```
\mathcal{F}_{\mathrm{SAT}}
    Initialisation:
      \forall Alice \in \mathcal{P},
2
         util(Alice) = \bot
         assets(Alice) = \bot
    Upon receiving (type, t) from Alice:
      util(Alice) = t
    Upon receiving (satisfy, d, L) from Alice:
      If util(Alice) == \bot:
10
         send message (utilityNotSet) to Alice
11
         go to Idle state
12
      Find list = \{(Bob, x, s) \in L \times \mathbb{R} \times Assets : 
13
         s \in \mathtt{assets}(Bob) and s \in d and x \ge 0 and
14
         Alice has at least x coins available and
15
         \mathtt{util}(Alice) (state_{Alice} \cup s - x) > \mathtt{util}(Alice) (state_{Alice})
         \mathtt{util}(Bob) (state_{Bob} \setminus s + x + goodRep) > \mathtt{util}(Bob) (state_{Bob})
18
19
         util(Bob)(state_{Bob} \setminus s + x + goodRep) >
20
              util(Bob)(state_{Bob} + x + badRep)
      send (chooseBestSeller, list, d, Alice) to \mathcal{A}
21
      wait for response from {\cal A}
22
      With response as (bestSeller, list, Bob, x, s), Bob \in \mathcal{P}:
23
         If x > 0:
24
           Pay x from Alice to Bob
         assets(Bob) = assets(Bob) \setminus \{s\}
26
```

```
assets(Alice) = assets(Alice) \cup \{s\}
       send message (satisfied, d, L) to Alice
28
     Else If response \neq (bestSeller, list, Bob, x, s): # e.g.
29
         Bob = \bot
       send message (unsatisfied, d, L) to Alice
30
31
   Upon receiving (obtain, s) from Alice:
     assets(Alice) = assets(Alice) \cup \{s\}
     send message (obtained, s) to Alice
34
35
   Upon receiving (lose, s) from Alice:
36
     assets(Alice) = assets(Alice) \setminus \{s\}
37
     send message (lost, s) to Alice
   \mathcal{F}_{\mathrm{Trade}}
   Initialisation:
     \forall Alice \in \mathcal{P},
       assets(Alice) = \bot
   Upon receiving (trade, ours, theirs, Bob) from Alice:
     If not isAvailable(ours, Alice):
       send message (youDontHave, ours) to Alice
       go to Idle state
     If transfer(ours, Alice, Bob) == True:
       send message (willWeCheat, ours, theirs, Alice) to Bob
10
       wait for response from Bob
11
       If (response == (complete, x, s, Alice) and
12
         not is Available (s, Bob)) or
13
         response \neq (complete, x, s, Alice):
            send message (youDontHave, s) to Bob
            send message (cheated, ours, theirs, Bob) to Alice
            go to Idle state
       Else If (transfer, theirs, Bob, Alice) == True:
18
         send message (traded, ours, theirs, Bob) to Alice
19
     Else # failed to give (Unreachable for a good \mathcal{F}_{\mathrm{Ledger}})
20
       send message (failed, ours, theirs, Bob) to Alice
21
22
   isAvailable(object, player):
     If object is money:
       send (doIhaveBalance, object) to \mathcal{F}_{\mathrm{Ledger}} as player
25
```

```
wait for response from \mathcal{F}_{	ext{Ledger}}
        return response
27
     Else: # object is asset
28
        If object \in assets(player):
29
          return True
30
        Else:
31
          return False
32
   transfer(object, sender, receiver):
34
      If isAvailable(object):
35
        If object is money:
36
          send (pay, object, receiver) to \mathcal{F}_{\mathrm{Ledger}} as sender
37
          wait for response from \mathcal{F}_{\mathrm{Ledger}}
38
          Upon receiving (paymentDone, object, receiver):
            return True
        Else: # object is asset
41
          assets(sender) = assets(sender) \ { object }
42
          assets(receiver) = assets(receiver) ∪{object}
43
          return True
44
      return False
45
46
   Upon receiving (obtain, s) from Alice:
47
      assets(Alice) = assets(Alice) \cup \{s\}
      send message (obtained, s) to Alice
49
50
   Upon receiving (lose, s) from Alice:
51
      assets(Alice) = assets(Alice) \setminus \{s\}
52
      send message (lost, s) to Alice
53
   \mathcal{F}_{\text{Assets}}
   Initialisation:
     \forall Alice \in \mathcal{P},
        assets(Alice) = \bot
   Upon receiving (add, asset) from Alice:
      assets(Alice) = assets(Alice) \cup \{asset\}
      send (added, asset) to Alice
   Upon receiving (remove, asset) from Alice:
     If asset \in assets(Alice):
```

```
assets(Alice) = assets(Alice) \setminus \{asset\}
       send (removed, asset) to Alice
12
     Else:
13
       send (unableToRemove, asset) to Alice
14
15
   Upon receiving (howManyDoIhave, asset) from Alice:
     send (youHave, assets(Alice).count(asset)) to Alice
   Upon receiving (transfer, asset, Bob) from Alice:
19
     If asset \in assets(Alice):
20
       assets(Alice) = assets(Alice) \setminus \{asset\}
21
       assets(Alice) = assets(Bob) \cup \{asset\}
22
       send (transferred, asset, Bob) to Alice
23
     Else:
       send (unableToTransfer, asset, Bob) to Alice
25
   Upon receiving (trade, ours, theirs, Bob) from \mathcal{E}:
     Send (letsTrade, ours, theirs) to Bob
     If transfer(ours, Bob) == True:
       Send (transferred, ours, Bob) to Bob and {\cal E}
       Wait for response from Bob
       If response == (transferred, theirs, Bob):
          send message (traded, ours, theirs, Bob) to {\cal E}
       Else:
         send message (cheated, ours, theirs, Bob) to {\cal E}
10
   Upon receiving (letsTrade, theirs, ours) from Bob:
11
     Send (willWeCheat, theirs, ours, Bob) to {\cal E}
12
     Wait for response from {\mathcal E}
     If response is (doNotCheat, theirs, ours, Bob):
       If (transfer, ours, Bob) == True:
         Send (transferred, ours, Bob) to Bob and {\cal E}
16
17
   transfer(object, receiver):
18
     If isAvailable(object):
19
       If object is money:
20
         send (pay, object, receiver) to \mathcal{F}_{\mathrm{Ledger}}
21
         wait for response from \mathcal{F}_{\mathrm{Ledger}}
         Upon receiving (paymentDone, object, receiver):
23
```

```
return True
         Else: # object is asset
25
           send (transfer, object, receiver) to \mathcal{F}_{\mathrm{Assets}}
26
           wait for response from \mathcal{F}_{\mathrm{Assets}}
2.7
           Upon receiving (transferDone, object, receiver):
28
              return True
29
      return False
30
31
    isAvailable(object):
32
      If object is money:
33
         send (doIhaveBalance, object) to \mathcal{F}_{\mathrm{Ledger}}
34
         wait for response from \mathcal{F}_{\mathrm{Ledger}}
35
         return response
36
      Else: # object is asset
37
         Send (doIHave, object) to \mathcal{F}_{\mathrm{Assets}}
         wait for response from \mathcal{F}_{\mathrm{Assets}}
         If response == (youHave, object):
40
           return True
41
         Else:
42
           return False
43
    Upon receiving message (obtain, s) from \mathcal{E}:
      send message (add, s) to \mathcal{F}_{\mathrm{Assets}}
      wait for response from \mathcal{F}_{\mathrm{Assets}}
      If response == (added, s)
48
         send message (obtained, s) to {\mathcal E}
49
      Else
50
         send message (notObtained, s) to {\mathcal E}
51
    Upon receiving message (lose, s) from \mathcal{E}:
      send message (remove, s) to \mathcal{F}_{\mathrm{Assets}}
      wait for response from \mathcal{F}_{\mathrm{Assets}}
55
      If response == (removed, s)
56
         send message (lost, s) to {\cal E}
57
      Else
58
         send message (notLost, s) to {\cal E}
```

5 \mathcal{F}_{SAT} and Π_{SAT} are potentially distinguishable

Consider the hybrid world of Fig. 1 (right) with n ITMs executing Π_{SAT} , where $\mathcal{F}_{\text{Trust}}$ is replaced by $\mathcal{F}'_{\text{Trust}}$: $\mathcal{F}'_{\mathrm{Trust}}$ Upon receiving (chooseBestSeller, d, L) from Alice: $Bob \stackrel{R}{\leftarrow} L \cup \{\bot\}$ send message (bestSeller, d, L, Bob) to AliceWe will show here that \mathcal{E} can distinguish between \mathcal{F}_{SAT} and a Π_{SAT} that uses $\mathcal{F}'_{\text{Trust}}$. Distinguishability. Consider the following adversary and environment: Upon receiving (chooseBestSeller, list, d, Alice): If $|\text{list}| \neq 1 \lor |d| \neq 1$: go to Idle State $Bob = p : p \in list$ $s = asset : asset \in d$ return (bestSeller, list, Bob, 1, s) \mathcal{E} distinguisher 1 Alice $\stackrel{R}{\leftarrow} \mathcal{P}$ $Bob \stackrel{R}{\leftarrow} \mathcal{P} \setminus \{Alice\}$ $util(Alice)(\{assets\}, x, r) = 2|\{assets\}| + x$ $\mathtt{util}(Bob)\left(\left\{assets\right\},x,r\right) = \left|\left\{assets\right\}\right| + 2x$ $\lambda \left(Alice\right)(t) = \lambda \left(Bob\right)(t) = \frac{1}{t^2}$ $\forall p \in \{Alice, Bob\}:$ send message (type, util(p), $\lambda\left(p\right)$) to p $s \stackrel{R}{\leftarrow} Asset$ send message (obtain, s) to Bobsend message (satisfy, $\{s\}$, $\{Bob\}$) to Alice10 11 Upon receiving message (x, $\{s\}$, $\{Bob\}$) from Alice: 12 If x == satisfied: 13 return functionality 14 Else: # if $x \in \{\text{cheated}, \text{unsatisfied}\}$

return protocol

Because of the way \mathcal{E} is built, there always exists a seller (Bob) that can satisfy the desire of the buyer (Alice).

In case \mathcal{E} interacts with \mathcal{F}_{SAT} , then \mathcal{A} will always match the buyer and the seller because of the way the former is built. Additionally, the price of 1 is cheap enough for *Alice* to want to buy the asset, since she values acquiring one asset more than one coin.

Furthermore, \mathcal{F}_{SAT} never cheats on a trade, thus the exchange will always complete correctly and \mathcal{E} will receive satisfied as response. \mathcal{E} will always correctly output functionality.

On the other hand, in case \mathcal{E} interacts with Π_{SAT} , then we observe that \mathcal{F}'_{Trust} does not choose players depending on their reputation. Thus, if \mathcal{F}'_{Trust} does not respond with \bot , it is always in Bob's interest to cheat (since keeping the asset is preferrable to giving it) and \mathcal{E} will always receive cheated as response. If \mathcal{F}'_{Trust} responds with \bot , the trade will not go through and \mathcal{E} will receive unsatisfied as a response. In all cases \mathcal{E} will correctly output protocol.

References