What is Trust

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Abstract. We will try to define all the abstract properties that we would like "Trust" to have.

1 Introduction

Consider the UC setting, with an environment $\mathcal{E}$, an adversary $\mathcal{A}$ and a set of ITIs that follow a given protocol Π.

Definition 1 (Player). *A player is an ITM that follows Π. Let $\mathcal{P}$ be the set of all players.*

Intuitively, players spontaneously feel different desires of varying intensities and seek to satisfy them, either on their own, consuming part of their input tokens in the process, or by delegating the process to other players and paying them for their help with part of their input tokens. The choice depends on the perceived difference in price. Each player plays rationally, always attempting to maximize her utility.

More precisely, let $\mathcal{D}$ be a (finite) set containing all possible desires. At arbitrary moments during execution, $\mathcal{E}$ can provide input to any player $Alice \in \mathcal{P}$ in the form (idx, d), where $idx \in \mathbb{N}, d \in \mathcal{D}, u \in \mathbb{R}^+$. idx represents an index number that is unique for each input $\mathcal{E}$ generates and d represents the desire. d is satisfied when $Alice$ learns the string $s\left(idx, d, Alice\right)$, either by directly calculating it or by receiving it as subroutine output from another player. Some of the players, given as input the tuple $(idx, d, Alice)$, can calculate $s\left(idx, d, Alice\right)$ more efficiently than $Alice$, which means that they need to consume less input tokens than $Alice$ for this calculation. $Alice$ can choose to delegate this calculation to a more efficient player Bob and provide the necessary input tokens for his computation with a surplus to compensate Bob for his effort. Both players are better off, because $Alice$ spent less tokens than she would if she had calculated $s\left(idx, d, Alice\right)$ herself, whilst Bob obtained some tokens which can in turn be used to satisfy some of his future desires.

Definition 2 (Cost of desire). *The cost of Alice's indexed desire, say* $(idx, d) \in (\mathbb{N}, \mathcal{D})$*, when satisfied by Bob is equal to the input tokens that Alice is required by Bob to give to him in order for him to calculate* $s\,(idx, Alice, d)$ *and is represented by* $c\,(idx, d, Alice, Bob)$*. The cost of satisfying this desire herself is represented by* $c\,(idx, d, Alice)$ *and is equal to the number of computational steps Alice must make in order to calculate* $s\,(idx, d, Alice)$*. Let* $c\,(idx, d, Alice, Alice) = c\,(idx, d, Alice)$*.*

The game begins with all players being created by $\mathcal{E}$, each allocated a random amount of input tokens. The game ends at a moment specified by the $\mathcal{E}$, which is unknown to the players. At that moment $\mathcal{E}$ assigns a utility to each player depending on which desires were satisfied throughout the game.

2 Motivation for our Trust model

Nevertheless, one can say that at first sight it is in *Bob*'s best interest to trick *Alice* into believing that he can efficiently calculate $s\,(idx, Alice, d)$ and skip the computation entirely after obtaining *Alice*'s input, thus keeping all the tokens of the defrauded player. Evidently *Alice* would avoid further interaction with *Bob*, but without any way to signal other players of this unfortunate encounter, *Bob* can keep defrauding others until the pool of players is depleted; if the players are numerous or their number is increasing, *Bob* may keep this enterprise very profitable for an indefinite amount of time. This being a rational strategy, every player would eventually follow it, which through a "tragedy of the commons" effect invariably leads to a world where each player must satisfy all her desires by herself, entirely missing out on the prospect of the division of labor.

One answer to that undesirable turn of events is a method through which *Alice*, prior to interacting with an aspiring helper *Bob*, consults the collective knowledge of her neighborhood of the network regarding him. There are several methods to achieve this, such as star-based global ratings. This method however has several drawbacks:

- Very good ratings cost nothing, thus convey little valuable information.
- Different players may have different preferences, global ratings fail to capture this. Arrow's impossibility theorem is possibly relevant here.
- Susceptible to Sybil attacks; mitigation techniques are ad-hoc and require (partial) centralization and secrecy/obfuscation of methods to succeed, thus undermining the decentralized, transparent nature of the system, a property that we actively seek.

3 Desire Satisfaction Ideal Functionality

Following the UC paradigm, in this section we define the ideal functionality for desire satisfaction, $\mathcal{F}_{Desire}$. In this setting, all the desires that are generated by the environment and are input to the players are immediately forwarded to $\mathcal{F}_{Desire}$; the functionality decides which of them to satisfy. Since the players are dummy and all desires are satisfied by the functionality, no trust semantics are necessary. Nevertheless, given that all desires have a positive cost (at least one computational step), the cost semantics are still necessary: Consider a set of desires D with more elements than the total number of input tokens all players have. D could never be satisfied by the players because of the high total cost, but a $\mathcal{F}_{Desire}$ with no consideration for cost could in principle satisfy all desires in D.

4 Trust definitions

We define two kinds of trust: direct and indirect. Direct trust from *Alice* to *Bob* is represented by input tokens (initially belonging to *Alice*) actively put by her in a common account from which *Bob* can also take them. As long as *Bob* does not take these tokens, *Alice* directly trusts him equally to the amount of tokens deposited in the common account.

This information can be used by another player *Charlie* that directly trusts *Alice* in order to derive information regarding *Bob*'s trustworthiness, even if *Charlie* does not directly trust *Bob*. Through a transitive property, *Dean*, who directly trusts *Charlie*, can in turn derive information from the direct trust from *Alice* to *Bob*. This reasoning can be extended to an arbitrary number of players, as long as they have at least one trust path to *Alice*. This is called indirect trust.

Definition 3 (Direct Trust). *The direct trust from Alice to Bob, represented by $DTr_{Alice \rightarrow Bob}$, is equal to the total tokens that Alice has given as input to $\mathcal{F}_{Trust}$ with* `entrust(`*Bob*`, 1`$^{\texttt{tokens}}$`)` *and also equal to the available tokens count sent by $\mathcal{F}_{Trust}$ as a response to a message* `query_direct_trust(`*Alice*`, `*Bob*`)`.

Definition 4 (Indirect Trust). *The indirect trust from Alice to Bob, $Tr_{Alice \rightarrow Bob}$, is measured in input tokens and can be calculated deterministically given the existing direct trusts between all pairs of players. It is equal to the number sent by $\mathcal{F}_{Trust}$ as a response to a message* `query_indirect_trust(`*Alice*`, `*Bob*`)`.

By convention $DTr_{Alice \to Alice} = Tr_{Alice \to Alice}$ and are both equal to the quantity of input tokens $Alice$ has.

We would like to provide players with an ideal functionality where they:

1. Directly trust tokens to another player
2. Steal tokens previously directly entrusted by another player
3. Retract tokens previously directly entrusted to another player
4. Query indirect trust towards another player

The following functionality provides such an interface:

$\mathcal{F}_{Trust}$

```
Initialization:
for all $Alice, Bob \in \mathcal{P}$
  $DTr_{Alice \to Bob}$ = 0
  $Tr_{Alice \to Bob}$ = 0

Upon receiving input entrust($Bob$, 1^{tokens}) from $Alice$:
  $DTr_{Alice \to Bob}$ += tokens

Upon receiving message steal($Bob$, tokens) from $Alice$:
  If $DTr_{Bob \to Alice}$ >= tokens
    $DTr_{Bob \to Alice}$ -= tokens
    input 1^{tokens} to $Alice$
  Else discard request

Upon receiving message distrust($Bob$, tokens) from $Alice$:
  If $DTr_{Alice \to Bob}$ >= tokens
    $DTr_{Alice \to Bob}$ -= tokens
    input 1^{tokens} to $Alice$
  Else discard request

Upon receiving message query_direct_trust($Alice$, $Bob$) from
    $Charlie$:
  If $Charlie \in \{Alice, Bob\}$ # Privacy
    send message $DTr_{Alice \to Bob}$ to $Charlie$
  Else discard request

Upon receiving message query_indirect_trust($Alice$, $Bob$)
    from $Charlie$:
  If $Charlie$ == $Alice$ # Privacy
```

$Tr_{Alice \to Bob}$ `= calculate_indirect_trust(`$Alice$`,` Bob`, all direct trusts)`

`send message` $Tr_{Alice \to Bob}$ `to` $Charlie$

`Else discard request`

5 Desired Properties for Indirect Trust

1. $Tr_{Alice \to Bob} \geq DTr_{Alice \to Bob}$
2. If universe (1) and (2) are identical except for $DTr_{Alice \to Bob}$, then

$$Tr^2_{Alice \to Bob} = Tr^1_{Alice \to Bob} - DTr^1_{Alice \to Bob} + DTr^2_{Alice \to Bob} \enspace .$$

3. Consider an indexed desire (idx, d) $Alice$ has. Let $p\,(idx, d, Alice)$ be a function that returns the player that $Alice$ should rationally delegate the calculation of $s\,(idx, d, Alice)$ to.
   (a) If a player is cheaper and more trustworthy than all other players, delegate the calculation to him.

$$\begin{gathered} \exists Bob \in \mathcal{P} : \forall Charlie \in \mathcal{P} \setminus \{Bob\} \\ (c\,(idx, d, Alice, Bob) < c\,(idx, d, Alice, Charlie) \wedge \\ \wedge\ Tr_{Alice \to Bob} > Tr_{Alice \to Charlie}) \Rightarrow \\ \Rightarrow p\,(idx, d, Alice) = Bob \enspace . \end{gathered}$$

   (b) If there exists a player Bob that is both cheaper and more trustworthy than $Charlie$, do not delegate the calculation to $Charlie$.

$$\begin{gathered} \exists Bob, Charlie \in \mathcal{P} : \\ (c\,(idx, d, Alice, Bob) < c\,(idx, d, Alice, Charlie) \wedge \\ \wedge\ Tr_{Alice \to Bob} > Tr_{Alice \to Charlie}) \Rightarrow \\ \Rightarrow p\,(idx, d, Alice) \neq Charlie \enspace . \end{gathered}$$

Note that the first property can be deduced from the second.

Let $x = (idx, d, Alice)$. Several ideas exist as to what rules $p\,(x)$ should satisfy:

1. Indirect trust towards Bob is required to be greater than the cost of the calculation requested by Bob in order for $Alice$ to delegate $s\,(idx, d, Alice)$ to him. If there exist multiple players that $Alice$ indirectly trusts more than their cost, then the cheapest one is chosen.

If multiple trustworthy enough players have the same cost, the most trustworthy one is chosen.

$$\begin{aligned}
&c\left(x, Charlie\right) > Tr_{Alice \rightarrow Charlie} && \Rightarrow p\left(x\right) \neq Charlie \\
&\text{For the following, } \forall v \in \{Bob, Charlie\}\ c\left(x, v\right) \leq Tr_{Alice \rightarrow v}\ . \\
&c\left(x, Bob\right) < c\left(x, Charlie\right) && \Rightarrow p\left(x\right) \neq Charlie \\
&\left.\begin{array}{l} c\left(x, Bob\right) = c\left(x, Charlie\right) \\ Tr_{Alice \rightarrow Bob} > Tr_{Alice \rightarrow Charlie} \end{array}\right\} && \Rightarrow p\left(x\right) \neq Charlie
\end{aligned}$$

2. Similarly to the previous idea, the indirect trust must exceed the cost. In this case however, in case of multiple trustworthy and cheap players, the indirect trust is considered before the cost.

$$\begin{aligned}
&c\left(x, Charlie\right) > Tr_{Alice \rightarrow Charlie} && \Rightarrow p\left(x\right) \neq Charlie \\
&\text{For the following, } \forall v \in \{Bob, Charlie\}\ c\left(x, v\right) \leq Tr_{Alice \rightarrow v}\ . \\
&Tr_{Alice \rightarrow Bob} > Tr_{Alice \rightarrow Charlie} && \Rightarrow p\left(x\right) \neq Charlie \\
&\left.\begin{array}{l} Tr_{Alice \rightarrow Bob} = Tr_{Alice \rightarrow Charlie} \\ c\left(x, Bob\right) < c\left(x, Charlie\right) \end{array}\right\} && \Rightarrow p\left(x\right) \neq Charlie
\end{aligned}$$

3. The player with the highest difference between indirect trust and cost is chosen.

$$p\left(x\right) = \operatorname*{argmax}_{v \in \mathcal{P}} \left(Tr_{Alice \rightarrow v} - c\left(x, v\right)\right)$$

This aims to maximize trust and minimize cost and, contrary to the previous two approaches, attaches equal importance to these two metrics.

4. The player with the lowest difference between indirect trust and cost is chosen.

$$p\left(x\right) = \operatorname*{argmin}_{v \in \mathcal{P}} \left(Tr_{Alice \rightarrow v} - c\left(x, v\right)\right)$$

Note that this last approach constitutes another direction, which departs from choosing the cheapest and most trustworthy vendor, opting for the one whose price and trustworthiness match. It evidently does not follow the property (3).

References