

What is Trust

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Abstract. We will try to define all the abstract properties that we would like "Trust" to have.

1 High-level idea

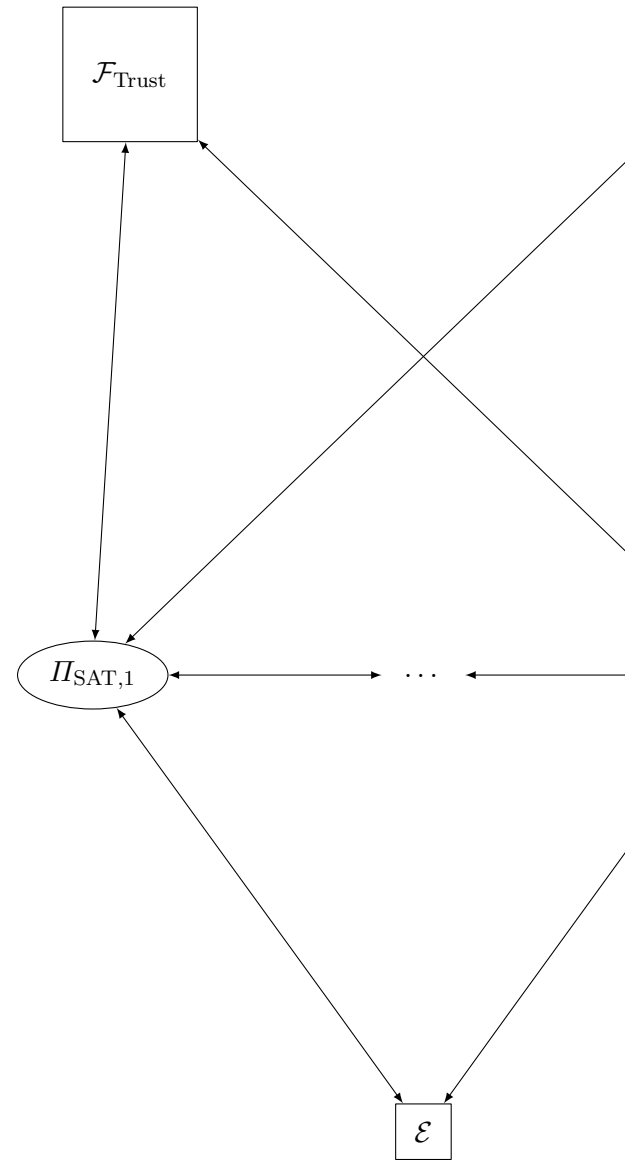
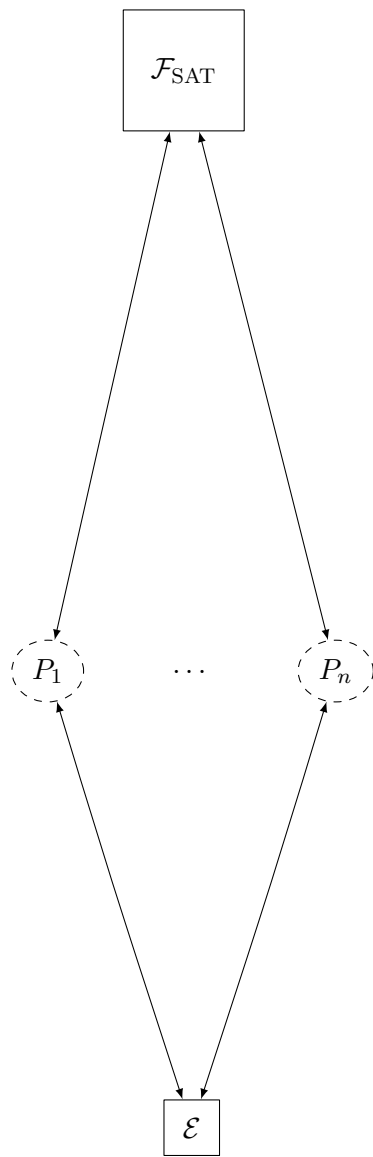


Fig. 1: (Almost) all functionalities

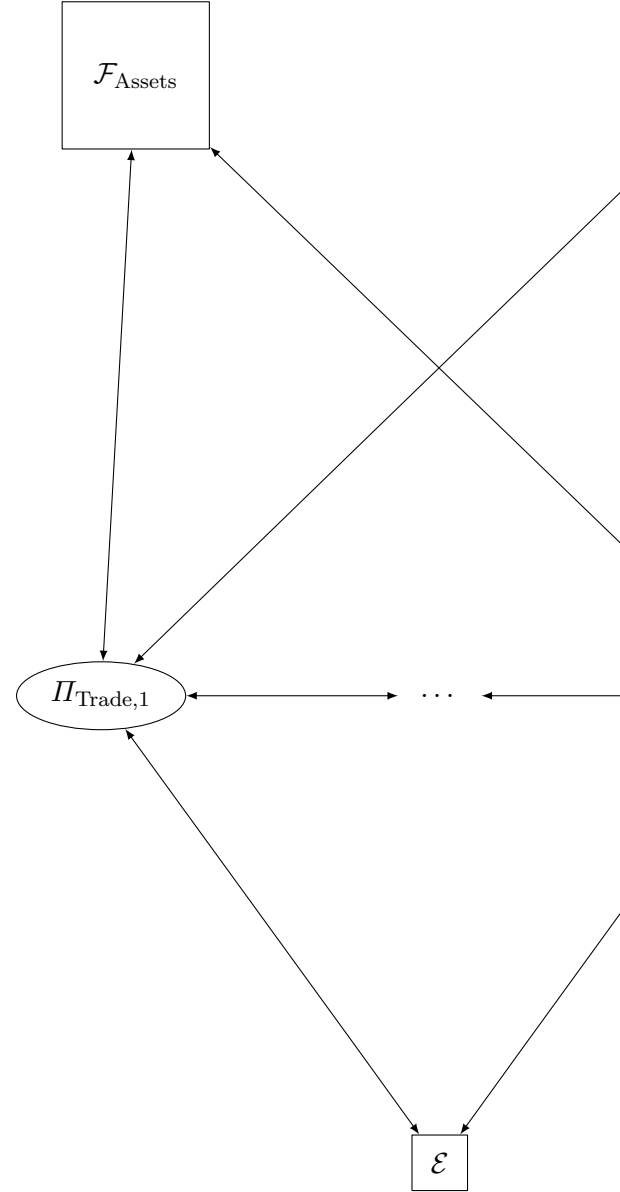
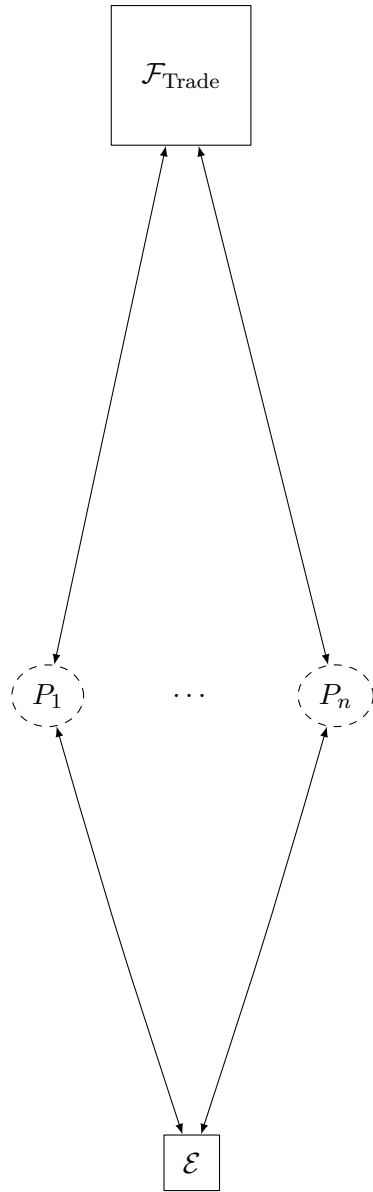


Fig. 2: Trade functionality and protocol

2 Utility Function Properties

Before *Alice* (an ITM that executes Π_{SAT}) can take any action, she must be assigned a utility function, U_{Alice} , and a discount function, λ_{Alice} , by \mathcal{E} .

Definition 1 (Utility Function). $U_{\text{Alice}} : \text{Money} \times \text{multiset}(\text{Asset}) \times \text{Rep} \rightarrow \text{Utility}$, where $\text{Money} \in \mathbb{N}, \text{Utility} \in \mathbb{R}$.

$$\text{Rep}_{\text{Alice}}(t) = \{\Pr[\text{Alice receives message } (\text{canYouSatisfy}, d) \text{ from } \text{Bob at time } t] : \text{Bob} \in \mathcal{P}, d \in 2^{\text{Asset}}\}$$

The utility function is strictly increasing with respect to the integer *Money* and may do anything with respect to the quantity of any single *Asset* or combination of *Assets*. It is also (not necessarily strictly) increasing with respect to each probability $\in \text{Rep}$.

Definition 2 (Reputation function). $\text{Reputation} : \mathcal{P} \times \text{Time} \times 2^{\text{Asset}} \rightarrow \text{Rep}$, where $\text{Time}, \text{Rep} \in \mathbb{N}$

$$\begin{aligned} \text{Rep}(\text{Alice}, t, \text{set}) &> \text{Rep}(\text{Bob}, t, \text{set}) \stackrel{\text{def}}{\iff} \\ \Pr[\text{Alice receives message } (\text{canYouSatisfy}, d) \text{ at time } t] &> \\ \Pr[\text{Bob receives message } (\text{canYouSatisfy}, d) \text{ at time } t] & \\ \text{where } d \in \text{set} & \end{aligned}$$

Rep is an integer that corresponds to *Alice*'s perceived rank in a partial ordering of all the players where *Alice* assumes that she has a higher rank than *Bob* if the average probability of third players choosing her before *Bob* when they want to buy something amongst a list of *Assets* that *Alice* is interested in selling is higher than *Bob*'s corresponding probability for the same *Assets*, as perceived by *Alice*.

Definition 3 (Discount function). $\lambda_{\text{Alice}} : \text{Time} \rightarrow [0, 1]$, where $\text{Time} \in \mathbb{N}$ and λ is strictly decreasing.

Definition 4 (State). Let $\text{state}_{\text{Alice}} : \text{Time} \rightarrow \text{Money} \times \text{multiset}(\text{Asset}) \times \text{Rep}$.

Alice's target is to maximise

$$\sum_{t=1}^{\infty} \lambda_{Alice}(t) \left(\sum_{\substack{(m,a,r) \in \\ Money \times mset(Asset) \times Rep}} U_{Alice}(m, a, r) \cdot \Pr[state_{Alice}(t) = (m, a, r)] \right) .$$

3 Protocol

In this section we will describe the real protocol executed by the players in the absence of the \mathcal{F}_{SAT} ideal functionality. The description of this protocol does not need any utility function; all the "important" decisions are taken by the environment.

Consider an environment \mathcal{E} , and adversary \mathcal{A} and n players executing copies of the same protocol Π_1, \dots, Π_n . \mathcal{E} can send the following messages to Π_i :

1. Manage player desires
 - (a) Satisfy $d \in \mathcal{D}$ through a player in $L \subseteq [n]$
 - (b) Abort attempt to satisfy $d \in \mathcal{D}$
2. Manage offered desires satisfaction
 - (a) Gain the ability to satisfy $d \in \mathcal{D}$ for players in $L \subseteq [n]$ for a price $x \in \mathbb{N}$
 - (b) Lose the ability to satisfy $d \in \mathcal{D}$ for players in $L \subseteq [n]$ for a price $x \in \mathbb{N}$
3. Satisfy another player's desire
 - (a) Satisfy player's $i \in [n]$ desire $d \in \mathcal{D}$ with the corresponding satisfaction string s
 - (b) Satisfy player's $i \in [n]$ desire $d \in \mathcal{D}$ with the satisfaction string s' (normally suitable for satisfying $d' \neq d, d' \in \mathcal{D}$)
 - (c) Ignore player's $i \in [n]$ desire $d \in \mathcal{D}$
4. Manage direct trusts
 - (a) Increase direct trust to player $i \in [n]$ by $x \in \mathbb{N}$
 - (b) Decrease direct trust to player $i \in [n]$ by $x \in \mathbb{N}$
 - (c) Steal direct trust $x \in \mathbb{N}$ from player $i \in [n]$

Some of these messages (e.g. 1b) are meaningful only when some other messages have been delivered previously (e.g. 1a). \mathcal{E} may send such messages even when they are not meaningful; the protocol should take care to reject/ignore such messages.

Let $i \in [n]$. Π_i can send the following messages to \mathcal{E} :

1. No player in L can satisfy my desire $d \in \mathcal{D}$
2. Desire $d \in \mathcal{D}$ made available for satisfaction amongst $L \subseteq [n]$ for price $x \in \mathbb{N}$
3. Desire $d \in \mathcal{D}$ made unavailable for satisfaction amongst $L \subseteq [n]$ for price $x \in \mathbb{N}$
4. Payment $x \in \mathbb{N}$ has been sent to player $j \in [n]$ for the satisfaction of desire $d \in \mathcal{D}$
5. Correct Payment $x \in \mathbb{N}$ from player $j \in [n]$ for the satisfaction of desire $d \in \mathcal{D}$ has been received
6. Wrong Payment $x \in \mathbb{N}$ from player $j \in [n]$ for the satisfaction of desire $d \in \mathcal{D}$ has been received
7. Player $j \in [n]$ has satisfied my desire $d \in \mathcal{D}$ with satisfaction string s
8. Player $j \in [n]$ has partially satisfied my desire $d \in \mathcal{D}$ with satisfaction string s' (normally suitable for satisfying $d' \neq d, d' \in \mathcal{D}$)
9. Player $j \in [n]$ has ignored my desire $d \in \mathcal{D}$
10. Direct trust to player $i \in [n]$ increased by $x \in \mathbb{N}$
11. Direct trust to player $i \in [n]$ decreased by $x \in \mathbb{N}$
12. Stole $x \in \mathbb{N}$ from player's $i \in [n]$ direct trust

Π_i should only send these messages when \mathcal{E} is expecting them.

Let $i, j \in [n]$ The messages that can be sent between Π_i and Π_j are the following:

1. Can you satisfy $d \in \mathcal{D}$?
2. I can satisfy $d \in \mathcal{D}$ for a price $x \in \mathbb{N}$
3. I cannot satisfy $d \in \mathcal{D}$
4. Payment of $x \in \mathbb{N}$ for satisfaction of $d \in \mathcal{D}$ sent
5. Satisfaction string s' , response to payment of $x \in \mathbb{N}$ for $d \in \mathcal{D}$

Π_i is supposed to send 4 when it has already paid through $\mathcal{F}_{\text{Ledger}}$. Similarly, it is supposed to send 5 when it has verified that the other party has sent the corresponding payment on $\mathcal{F}_{\text{Ledger}}$.

Going in more detail, the actual protocol is as follows:

```

 $\Pi_{\text{SAT}}$ 
1 Initialization:
2   util =  $\perp$ 
3   score = 0
4
5 Upon receiving (type,  $t$ ) from  $\mathcal{E}$ :
6   util =  $t$ 
7

```

```

8 Upon receiving message (satisfy,  $d, L$ ) from  $\mathcal{E}$ :
9   If util ==  $\perp$ :
10     send message (utilityNotSet) to  $\mathcal{E}$ 
11     go to Idle state
12   aux =  $L$ 
13   While (aux  $\neq \emptyset$ ):
14     send message (chooseBestSeller,  $d$ , aux) to  $\mathcal{F}_{\text{Trust}}$ 
15     wait for response1 from  $\mathcal{F}_{\text{Trust}}$ 
16     If response1 == (bestSeller,  $d, L, Bob$ ):
17       send message (canYouSatisfy,  $d$ ) to  $Bob$ 
18       wait for response2 from  $Bob$ 
19       If response2 == (IcanSatisfy,  $d, x, s$ ):
20         If util( $state \cup s - x$ ) > util( $state$ ) and  $s \in d$ 
21           send message (trade,  $x, s, Bob$ ) to  $\mathcal{F}_{\text{Trade}}$ 
22           wait for response3 from  $\mathcal{F}_{\text{Trade}}$ 
23           If response3 == (traded,  $x, s, Bob$ ):
24             send message (satisfied,  $d, x, s, Bob$ ) to  $\mathcal{F}_{\text{Trust}}$ 
25             # maybe send only utility difference to  $\mathcal{F}_{\text{Trust}}$ 
26             send message (satisfied,  $d, L$ ) to  $\mathcal{E}$ 
27             go to Idle state
28           Else If response3 == (cheated,  $x, s, Bob$ ):
29             send message (cheated,  $d, x, s, Bob$ ) to  $\mathcal{F}_{\text{Trust}}$ 
30             send message (cheated,  $d, L$ ) to  $\mathcal{E}$ 
31             go to Idle state
32         Else: # if response1 ==  $\perp$ 
33           send message (unsatisfied,  $d, L$ ) to  $\mathcal{E}$ 
34           go to Idle state
35         aux = aux  $\setminus \{Bob\}$  # only when response2 is not good
36       send message (unsatisfied,  $d, L$ ) to  $\mathcal{E}$ 
37
38 Upon receiving message (obtain,  $s$ ) from  $\mathcal{E}$ :
39   send message (obtain,  $s$ ) to  $\mathcal{F}_{\text{Trade}}$ 
40   wait for response from  $\mathcal{F}_{\text{Trade}}$ 
41   If response == (obtained,  $s$ ):
42     send message (obtained,  $s$ ) to  $\mathcal{E}$ 
43   Else:
44     send message (notObtained,  $s$ ) to  $\mathcal{E}$ 
45
46 Upon receiving message (lose,  $s$ ) from  $\mathcal{E}$ :
47   send message (lose,  $s$ ) to  $\mathcal{F}_{\text{Trade}}$ 

```

```

48   wait for response from  $\mathcal{F}_{\text{Trade}}$ 
49   If response == (lost,  $s$ ):
50       send message (lost,  $s$ ) to  $\mathcal{E}$ 
51   Else:
52       send message (notLost,  $s$ ) to  $\mathcal{E}$ 
53
54   Upon receiving message (canYouSatisfy,  $d$ ) from Alice:
55       If util ==  $\perp$ :
56           ignore request, go to Idle state
57       If (util( $state + x$  coins - 1 point) > util( $state$ ) or
58           util( $state \setminus s + x$  coins + 1 point) > util( $state$ )) and  $s \in d$ :
59           send message (IcanSatisfy,  $d, x, s$ ) to Alice #  $x$  is the
               price
60       Else:
61           send message (IcannotSatisfy,  $d$ ) to Alice
62
63   Upon receiving message (willWeCheat,  $x, s, Alice$ ) from  $\mathcal{F}_{\text{Trade}}$ :
64       If util ==  $\perp$ : # (Unreachable since we've already engaged)
65           ignore request, go to Idle state
66       If util( $state \setminus s + x$  coins + 1 point)  $\geq$  util( $state + x$  coins - 1 point):
67           send message (doNotCheat,  $x, s, Alice$ ) to  $\mathcal{F}_{\text{Trade}}$ 
68           score = score + 1
69       Else:
70           send message (cheat,  $x, s, Alice$ ) to  $\mathcal{F}_{\text{Trade}}$ 
71           score = score - 1

```

4 Desire Satisfaction Ideal Functionality

Following the UC paradigm, in this section we define the ideal functionality for desire satisfaction, \mathcal{F}_{SAT} . In this setting, all the desires that are generated by the environment and are input to the players are immediately forwarded to \mathcal{F}_{SAT} ; the functionality decides which desires to satisfy. Since the players are dummy and all desires are satisfied by the functionality, no trust semantics amongst the players are necessary.

Nevertheless, given that all desires have a minimum cost, the cost semantics are still necessary, as we show with the following example: Consider a set of desires D with more elements than the total number of tokens all players have. D could never be satisfied by the players because of the high total cost, but a \mathcal{F}_{SAT} with no consideration for cost could in principle satisfy all desires in D .

The functionality can calculate the properties and functions defined in ??, ?? and ?? for all inputs at any moment in time.

Without knowledge of the utilities the environment is going to give to each satisfied desire, the functionality may fail spectacularly. So knowledge of the utility of each desire, or at least some function of the utility given the desires is needed. We can assume that \mathcal{F}_{SAT} knows U or an approximation of it.

Going into more detail, \mathcal{F}_{SAT} is a stateful process that acts as a market and as a bank for the players. The market does not offer a particular product for the same price to all users; For some users it may be cheaper than for others, reflecting the fact that some players can realize some desires more efficiently than others.

\mathcal{F}_{SAT} stores a number for each player that represents the amount of tokens this player has and a table with the price of each desire for each player. It provides the functions $\text{cost}(u, d)$ which returns the cost of the desire d for player u with no side effects, $\text{sat}(u, d)$ that returns the string that satisfies the desire d to u and reduces the amount of the tokens belonging to u by $\text{cost}(u, d)$. There exists also the function $\text{transfer}(u_1, u_2, t)$ which reduces the amount of tokens u_1 has by t and increases the tokens of u_2 by t , given that initially the tokens belonging to u_1 were equal or more than t . This function is private to the functionality, thus can be used only internally.

\mathcal{F}_{SAT}

```

1 Initialisation:
2    $\forall \text{Alice} \in \mathcal{P}$ ,
3      $\text{util}(\text{Alice}) = \perp$ 
4      $\text{assets}(\text{Alice}) = \perp$ 
5
6 Upon receiving (type,  $t$ ) from Alice:
7    $\text{util}(\text{Alice}) = t$ 
8
9 Upon receiving (satisfy,  $d, L$ ) from Alice:
10  If  $\text{util}(\text{Alice}) == \perp$ :
11    send message (utilityNotSet) to Alice
12    go to Idle state
13  find list =  $\{(Bob, x, s) \in L \times \mathbb{R} \times \text{Assets} :$ 
14     $s \in \text{assets}(Bob) \text{ and } s \in d \text{ and } x \geq 0 \text{ and}$ 
15     $\text{Alice has at least } x \text{ coins available and}$ 
16     $\text{util}(\text{Alice})(\text{state}_{\text{Alice}} \cup s - x) > \text{util}(\text{Alice})(\text{state}_{\text{Alice}})$ 
17    and
```

```

18     util(Bob)(stateBob \ s + x) ≥ util(Bob)(stateBob) }
19     send (chooseBestSeller, d, list, Alice) to  $\mathcal{A}$ 
20     wait for response from  $\mathcal{A}$ 
21     If response ≠ (bestSeller, list, Bob, x, s),
22         Bob ∈  $\mathcal{P}$ , x ∈ Money, s ∈ Asset:
23         # e.g. Bob = ⊥
24         (Bob, x, s) = argmax(Bob, x, s) ∈ list {util(Alice)(stateAlice ∪ s - x) }
25     Else:
26         parse response as (bestSeller, list, Bob, x, s)
27         pay x from Alice to Bob
28         assets(Bob) = assets(Bob) \ {s}
29         assets(Alice) = assets(Alice) ∪ {s}
30         send message (satisfied, d, L) to Alice
31     Upon receiving (obtain, s) from Alice:
32         assets(Alice) = assets(Alice) ∪ {s}
33         send message (obtained, s) to Alice
34
35     Upon receiving (lose, s) from Alice:
36         assets(Alice) = assets(Alice) \ {s}
37         send message (lost, s) to Alice

 $\mathcal{F}_{\text{Trade}}$ 
1  Initialisation:
2      ∀ Alice ∈  $\mathcal{P}$ ,
3          assets(Alice) = ⊥
4
5  Upon receiving (trade, ours, theirs, Bob) from Alice:
6      If not isAvailable(ours, Alice):
7          send message (youDontHave, ours) to Alice
8          go to Idle state
9      If transfer(ours, Alice, Bob) == True:
10         send message (willWeCheat, ours, theirs, Alice) to Bob
11         wait for response from Bob
12         If (response == (complete, x, s, Alice) and
13             not isAvailable(s, Bob)) or
14             response ≠ (complete, x, s, Alice):
15             send message (youDontHave, s) to Bob
16             send message (cheated, ours, theirs, Bob) to Alice
17             go to Idle state

```

```

18     Else If (transfer, theirs, Bob, Alice) == True:
19         send message (traded, ours, theirs, Bob) to Alice
20     Else # failed to give (Unreachable for a good  $\mathcal{F}_{\text{Ledger}}$ )
21         send message (failed, ours, theirs, Bob) to Alice
22
23     isAvailable(object, player):
24         If object is money:
25             send (doIhaveBalance, object) to  $\mathcal{F}_{\text{Ledger}}$  as player
26             wait for response from  $\mathcal{F}_{\text{Ledger}}$ 
27             return response
28         Else: # object is asset
29             If object  $\in$  assets(player):
30                 return True
31             Else:
32                 return False
33
34     transfer(object, sender, receiver):
35         If isAvailable(object):
36             If object is money:
37                 send (pay, object, receiver) to  $\mathcal{F}_{\text{Ledger}}$  as sender
38                 wait for response from  $\mathcal{F}_{\text{Ledger}}$ 
39                 Upon receiving (paymentDone, object, receiver):
40                     return True
41             Else: # object is asset
42                 assets(sender) = assets(sender)  $\setminus$  {object}
43                 assets(receiver) = assets(receiver)  $\cup$  {object}
44                 return True
45         return False
46
47     Upon receiving (obtain, s) from Alice:
48         assets(Alice) = assets(Alice)  $\cup$  {s}
49         send message (obtained, s) to Alice
50
51     Upon receiving (lose, s) from Alice:
52         assets(Alice) = assets(Alice)  $\setminus$  {s}
53         send message (lost, s) to Alice

```

$\mathcal{F}_{\text{Assets}}$

```

1     Initialisation:
2      $\forall \text{Alice} \in \mathcal{P},$ 

```

```

3     assets(Alice) =  $\perp$ 
4
5 Upon receiving (add, asset) from Alice:
6     assets(Alice) = assets(Alice)  $\cup$  {asset}
7     send (added, asset) to Alice
8
9 Upon receiving (remove, asset) from Alice:
10    If asset  $\in$  assets(Alice):
11        assets(Alice) = assets(Alice)  $\setminus$  {asset}
12        send (removed, asset) to Alice
13    Else:
14        send (unableToRemove, asset) to Alice
15
16 Upon receiving (howManyDoIhave, asset) from Alice:
17     send (youHave, assets(Alice).count(asset)) to Alice
18
19 Upon receiving (transfer, asset, Bob) from Alice:
20    If asset  $\in$  assets(Alice):
21        assets(Alice) = assets(Alice)  $\setminus$  {asset}
22        assets(Alice) = assets(Bob)  $\cup$  {asset}
23        send (transferred, asset, Bob) to Alice
24    Else:
25        send (unableToTransfer, asset, Bob) to Alice

```

Π_{Trade}

```

1 Upon receiving (trade, ours, theirs, Bob) from  $\mathcal{E}$ :
2     Send (letsTrade, ours, theirs) to Bob
3     If transfer(ours, Bob) == True:
4         Send (transferred, ours, Bob) to Bob and  $\mathcal{E}$ 
5         Wait for response from Bob
6         If response == (transferred, theirs, Bob):
7             send message (traded, ours, theirs, Bob) to  $\mathcal{E}$ 
8         Else:
9             send message (cheated, ours, theirs, Bob) to  $\mathcal{E}$ 
10
11 Upon receiving (letsTrade, theirs, ours) from Bob:
12     Send (willWeCheat, theirs, ours, Bob) to  $\mathcal{E}$ 
13     Wait for response from  $\mathcal{E}$ 
14     If response is (doNotCheat, theirs, ours, Bob):
15         If (transfer, ours, Bob) == True:

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```

16         Send (transferred, ours, Bob) to Bob and  $\mathcal{E}$ 
17
18     transfer(object, receiver):
19         If isAvailable(object):
20             If object is money:
21                 send (pay, object, receiver) to  $\mathcal{F}_{\text{Ledger}}$ 
22                 wait for response from  $\mathcal{F}_{\text{Ledger}}$ 
23                 Upon receiving (paymentDone, object, receiver):
24                     return True
25             Else: # object is asset
26                 send (transfer, object, receiver) to  $\mathcal{F}_{\text{Assets}}$ 
27                 wait for response from  $\mathcal{F}_{\text{Assets}}$ 
28                 Upon receiving (transferDone, object, receiver):
29                     return True
30         return False
31
32     isAvailable(object):
33         If object is money:
34             send (doIHaveBalance, object) to  $\mathcal{F}_{\text{Ledger}}$ 
35             wait for response from  $\mathcal{F}_{\text{Ledger}}$ 
36             return response
37         Else: # object is asset
38             Send (doIHave, object) to  $\mathcal{F}_{\text{Assets}}$ 
39             wait for response from  $\mathcal{F}_{\text{Assets}}$ 
40             If response == (youHave, object):
41                 return True
42             Else:
43                 return False
44
45     Upon receiving message (obtain, s) from  $\mathcal{E}$ :
46         send message (add, s) to  $\mathcal{F}_{\text{Assets}}$ 
47         wait for response from  $\mathcal{F}_{\text{Assets}}$ 
48         If response == (added, s)
49             send message (obtained, s) to  $\mathcal{E}$ 
50         Else
51             send message (notObtained, s) to  $\mathcal{E}$ 
52
53     Upon receiving message (lose, s) from  $\mathcal{E}$ :
54         send message (remove, s) to  $\mathcal{F}_{\text{Assets}}$ 
55         wait for response from  $\mathcal{F}_{\text{Assets}}$ 

```

```

56   If response == (removed, s)
57     send message (lost, s) to  $\mathcal{E}$ 
58   Else
59     send message (notLost, s) to  $\mathcal{E}$ 

```

5 \mathcal{F}_{SAT} and Π_{SAT} are potentially distinguishable

Consider the hybrid world of Fig. 1 (right) with n ITMs executing Π_{SAT} , where $\mathcal{F}_{\text{Trust}}$ is replaced by $\mathcal{F}'_{\text{Trust}}$:

```

 $\mathcal{F}'_{\text{Trust}}$ 
1 Upon receiving (chooseBestSeller,  $d$ ,  $L$ ) from Alice:
2    $Bob \xleftarrow{R} L \cup \{\perp\}$ 
3   send message (bestSeller,  $d$ ,  $L$ ,  $Bob$ ) to Alice

```

We will show here that \mathcal{E} can distinguish between \mathcal{F}_{SAT} and $\Pi_{\text{SAT}}^{\mathcal{F}'_{\text{Trust}}}$.

Distinguishability. Consider the following adversary and environment:

```

 $\mathcal{A}$ 
1 Upon receiving (chooseBestSeller,  $d$ , list, Alice):
2   If  $|\text{list}| \neq 1 \vee |d| \neq 1$ :
3     go to Idle State
4    $Bob = p : (p, x, s) \in \text{list}$ 
5    $s = \text{asset} : \text{asset} \in d$ 
6   return (bestSeller, list,  $Bob$ , 1,  $s$ )

```

\mathcal{E} distinguisher

```

1  $Alice \xleftarrow{R} \mathcal{P}$ 
2  $Bob \xleftarrow{R} \mathcal{P} \setminus \{Alice\}$ 
3  $\text{util}(Alice)(\{\text{assets}\}, x, r) = 2|\{\text{assets}\}| + x$ 
4  $\text{util}(Bob)(\{\text{assets}\}, x, r) = |\{\text{assets}\}| + 2x$ 
5  $\lambda(Alice)(t) = \lambda(Bob)(t) = \frac{1}{t^2}$ 
6  $\forall p \in \{Alice, Bob\}$ :
7   send message (type,  $\text{util}(p)$ ,  $\lambda(p)$ ) to  $p$ 
8  $s \xleftarrow{R} \text{Asset}$ 
9 send message (obtain, 1 coin) to Alice
10 send message (obtain,  $s$ ) to Bob
11 send message (satisfy,  $\{s\}$ ,  $\{Bob\}$ ) to Alice
12

```

```

13 Upon receiving message  $(x, \{s\}, \{Bob\})$  from Alice:
14   If  $x == \text{satisfied}$ :
15     return 1 # functionality
16   Else: # if  $x \in \{\text{cheated}, \text{unsatisfied}\}$ 
17     return 0 # protocol

```

Because of the way \mathcal{E} is built, there always exists a seller (*Bob*, line 2) who has an asset (line 10) that can satisfy the desire (line 11) of the buyer (*Alice*, line 1).

In case \mathcal{E} interacts with \mathcal{F}_{SAT} , let \mathcal{S} simulator that tries to simulate \mathcal{A} . \mathcal{F}_{SAT} will always send the message (`chooseBestSeller`, $\{s\}$, $\{(Bob, 1, s)\}$, *Alice*) to \mathcal{S} because:

1. *Alice* has one coin according to \mathcal{E} , line 9, as required by \mathcal{F}_{SAT} , line 15.
2. It is in *Alice*'s benefit for the trade to go through, since she values acquiring one asset more than one coin ($\text{util}(\text{Alice})(\{s\}, 0) = 2 > 1 = \text{util}(\text{Alice})(\emptyset, 1)$ as can be seen in \mathcal{E} , lines 3 and 7), as required in \mathcal{F}_{SAT} , line 16.
3. It is in *Bob*'s benefit for the trade to go through, since he values acquiring one coin more than one asset ($\text{util}(\text{Bob})(\{s\}, 0) = 1 < 2 = \text{util}(\text{Bob})(\emptyset, 1)$ as can be seen in \mathcal{E} , lines 4 and 7), as required in \mathcal{F}_{SAT} , line 18.

\mathcal{S} should always match the buyer and the seller because of the way \mathcal{A} is built. More precisely, \mathcal{S} must always respond to (`chooseBestSeller`, $\{s\}$, $\{(Bob, 1, s)\}$, $_$) with (`bestSeller`, $\{(Bob, 1, s)\}$, *Bob*, 1, *s*) in order to correctly simulate \mathcal{A} (lines 4-6).

Furthermore, \mathcal{F}_{SAT} never cheats on a trade and always chooses a suitable seller, price and asset (given that there exists one, which is the case here as we saw earlier) (lines 21-29), thus the exchange will always complete correctly and \mathcal{E} will receive `satisfied` as response. \mathcal{E} will always correctly output 1 (which corresponds to the functionality, line 15).

On the other hand, in case \mathcal{E} interacts with Π_{SAT} , then we observe that $\mathcal{F}'_{\text{Trust}}$ does not choose players depending on their reputation (line 2), thus in this particular setting the utility of the players does not depend on their reputation. Thus, if $\mathcal{F}'_{\text{Trust}}$ does not respond with \perp , it is always in *Bob*'s interest to cheat, since keeping the asset is preferable to giving it (\mathcal{E} , lines 4 and 7). Thus *Alice*'s response to \mathcal{E} will always be `cheated`. If $\mathcal{F}'_{\text{Trust}}$ responds with \perp (line 2), the trade will not go through (Π_{SAT} , lines 32-33) and \mathcal{E} will receive `unsatisfied` as a response. In all cases \mathcal{E} will correctly output 0 (which corresponds to the protocol, line 17). \square

$\mathcal{F}_{\text{Trust}}$

```

1 Has oracle access to every player's Alice utility function
  util(Alice)
2
3 Upon receiving (chooseBestSeller, d, L) from Alice:
4   Bob =
5   argmaxBob ∈ L { util(Alice)(stateAlice ∪ s − x) :
6     s ∈ d ∧ s ∈ assets(Bob) ∧ Alice has x coins ∧
7     (util(Bob)(stateBob \ s + x coins + 1 point) >
8     util(Bob)(stateBob + x coins − 1 point)) ∧
9     util(Bob)(stateBob \ s + x coins + 1 point) > util(Bob)(stateBob) }
10  If util(Alice)(stateAlice) ≥ util(Alice)(stateAlice ∪ s − x):
11    Bob = ⊥
12  send message (bestSeller, d, L, Bob) to Alice

```

Assumptions:

- Trades are atomic. $\mathcal{F}_{\text{Trust}}$ can deterministically decide whether *Bob* will complete the trade.
- The internal workings of $\mathcal{F}_{\text{Trust}}$ is common knowledge to the players. Their utility depends on it.

Note: It would be interesting to see how utilities (as algorithms) and $\mathcal{F}_{\text{Trust}}$ have a "fixed point".

Π_{Trust}

```

1 Upon receiving (chooseBestSeller, d, L) from Alice:
2   Bob =
3   argmax(Bob, x) ∈ L {  $\mathcal{O}_{\text{Rep}}(\textit{Alice}, \textit{Bob}, \textit{s}) \text{util}(\textit{Alice})(\textit{state}_{\textit{Alice}} \cup \textit{s} - \textit{x}) +$ 
4      $(1 - \mathcal{O}_{\text{Rep}}(\textit{Alice}, \textit{Bob}, \textit{s})) \text{util}(\textit{Alice})(\textit{state}_{\textit{Alice}} - \textit{x}) :$ 
5     s ∈ d ∧ s ∈ assets(Bob) ∧ Alice has x coins }
6   # drop "Bob has s"?
7   If util(Alice)(stateAlice) ≥ util(Alice)(stateAlice ∪ s − x):
8     Bob = ⊥
9   send message (bestSeller, d, L, Bob) to Alice

```

6 An open source reputation management algorithm is impossible

Theorem 1. Let $U_{\text{Alice}} : \text{Money} \times \text{multiset}(\text{Assets}) \times \text{Rep} \rightarrow \mathbb{R}$, where $\text{Rep} \in \mathbb{N}$. ($\mathcal{F}_{\text{Trust}}$ does not have access to any oracle.) Then $\nexists \mathcal{F}_{\text{Trust}} : \Pi_{\text{SAT}}$ UC-realizes \mathcal{F}_{SAT} .

Proof. We will prove this by contradiction. Let $\mathcal{F}_{\text{Trust}}$ be such a functionality. If it does not allow for cheats (grim trigger for seller on buyer reporting a cheat), then a malicious buyer (also prospective seller) can try buying from a competitor, falsely report a cheat and beat the competition (thus increasing their utility). If it allows for cheats, then it does not emulate \mathcal{F}_{SAT} . \square

References