TODO: Add support for cooperative adding multiple virtuals to single channel as future work (needs cooperation by all hops of all existing virtuals of current channel) TODO: Add support for cooperative closing as future work

Functionality \mathcal{F}_{Chan} – general message handling rules

- On receiving input (msg) by \mathcal{E} to $P \in \{Alice, Bob\}$, handle it according to the corresponding rule in Fig 2 or Fig 3 (if any) and subsequently send (RELAY, msg, P, \mathcal{E} , input) \mathcal{A} . // all messages by \mathcal{E} are relayed to \mathcal{A}
- On receiving (msg) by $R \neq \mathcal{E}$ to $P \in \{Alice, Bob\}$ by means of mode $\in \{\text{input}, \text{output}, \text{network}\}$, send (RELAY, msg, P, R, mode) to A. // all messages by machines other than \mathcal{E} are relayed to A
- On receiving (RELAY, msg, P, R, mode) by \mathcal{A} (mode \in {input, output, network}, $P \in \{Alice, Bob\}$), relay msg to R as P by means of mode. // \mathcal{A} fully controls outgoing messages by \mathcal{F}_{Chan}
- On receiving (INFO, msg) by \mathcal{A} , handle (msg) according to the corresponding rule in Fig 2 or Fig 3 (if any). After handling the message or after an "ensure" fails, send (HANDLED, msg) to \mathcal{A} . // (INFO, msg) messages by \mathcal{S} always return control to \mathcal{S} without any side-effect to any other ITI, except if \mathcal{F}_{Chan} halts

```
Functionality \mathcal{F}_{Chan} – state machine, Pt. 1
     1: On initalisation:
     2:
                                 pk_A \leftarrow \bot; pk_B \leftarrow \bot
     3:
                                 balance_A \leftarrow 0; balance_B \leftarrow 0
                                  \texttt{is\_corrupted\_or\_negligent}_A \leftarrow \texttt{False}; \ \texttt{is\_corrupted\_or\_negligent}_B \leftarrow \texttt{False}_B \leftarrow \texttt{Fa
                  False;
     5:
                                  State \leftarrow \bot
     6: On (BECAME CORRUPTED OR NEGLIGENT, P) by \mathcal{A}:
                                  \texttt{is\_corrupted\_or\_negligent}_P \leftarrow \mathsf{True}
     8: On (INIT, pk) to P by \mathcal{E}:
                                 if P = Bob then \bar{P} \leftarrow Alice else \bar{P} \leftarrow Bob
                                  ensure State \in \{\bot, INIT_{\bar{P}}\}
 11:
                                  pk_P \leftarrow pk
 12:
                                  if State = \bot then State \leftarrow \texttt{TENTATIVE-INIT}_P else State \leftarrow \texttt{TENTATIVE-INIT}_P
 13: On (INIT, P) by A:
 14:
                                  if State = \text{TENTATIVE-INIT}_P then State \leftarrow \text{INIT}_P
 15:
                                  if State = \text{TENTATIVE-INIT} then State \leftarrow \text{INIT}
 16: On (OPEN, x, \ldots) to Alice by \mathcal{E}:
 17:
                                  ensure State = INIT
 18:
                                  balance_A \leftarrow x; balance_B \leftarrow 0
                                  State \leftarrow \texttt{TENTATIVE-OPEN}
 19:
 20: On (OPEN) by A:
 21:
                                   ensure State = \text{TENTATIVE-OPEN}
 22:
                                   State \leftarrow \text{OPEN}
 23: On (PAY, x) to P by \mathcal{E}:
                                  ensure State = OPEN
24:
25:
                                  if P = Alice then S \leftarrow Alice; \bar{S} \leftarrow Bob else S \leftarrow Bob; \bar{S} \leftarrow Alice
26:
27:
                                  State \leftarrow \text{Paying}
 28: On (PAY OK) by A:
                                  ensure State = PAYING
 30:
                                  balance_S \leftarrow balance_S - c; balance_{\bar{S}} \leftarrow balance_{\bar{S}} + c
                                  forget c, S, \bar{S}
31:
32:
                                  State \leftarrow \text{OPEN}
```

Fig. 2.

```
Functionality \mathcal{F}_{Chan} – state machine, Pt. 2
 1: On (FUND ME, x, \ldots) to P by R:
           ensure R is an ITI that runs \mathcal{F}_{Chan} or \Pi_{Chan} code
 2:
 3:
           ensure State = OPEN
 4:
           c \leftarrow x
           if P = Alice then S \leftarrow Alice else S \leftarrow Bob
           State \leftarrow \texttt{FUNDING}
 7: On (fund ok) by A:
           ensure State = FUNDING
 9:
           \mathtt{balance}_S \leftarrow \mathtt{balance}_S - c
10:
           forget c, S
11:
           State \leftarrow \text{OPEN}
12: On (CLOSE) by \mathcal{E}:
           ensure State = OPEN
14:
           State \leftarrow \text{CLOSING}
15: On (CLOSE) by A:
           ensure State \in \{\text{OPEN}, \text{CLOSING}, \text{PAYING}, \text{FUNDING}\}
16:
17:
           for P \in \{A, B\} do
18:
                \label{eq:fisher_problem} \textbf{if} \  \, \textbf{is\_corrupted\_or\_negligent}_P = \textbf{False then}
19:
                     input (READ) to \mathcal{G}_{Ledger} as P and assign outut to \Sigma_P
20:
                     \mathtt{coins}_P \leftarrow \mathtt{sum} \ \mathrm{of} \ \mathrm{coins} \ \mathrm{exclusively} \ \mathrm{spendable} \ \mathrm{by} \ pk_P \ \mathrm{in} \ \Sigma_P
21:
                end if
           end for
22:
23:
           \mathtt{check}_A \leftarrow \mathtt{is\_corrupted\_or\_negligent}_A \lor \mathtt{coins}_A \geq \mathtt{balance}_A
24:
           \mathtt{check}_B \leftarrow \mathtt{is\_corrupted\_or\_negligent}_B \lor \mathtt{coins}_B \geq \mathtt{balance}_B
25:
           if \mathrm{check}_A \wedge \mathrm{check}_B then
26:
                 State \leftarrow CLOSED
27:
           else // balance security is broken
28:
                halt
29:
           end if
```

Fig. 3.

Simulator ${\cal S}$ – general message handling rules

- On receiving (RELAY, in_msg, P, R, in_mode) by \mathcal{F}_{Chan} (in_mode \in {input, output, network}, $P \in \{Alice, Bob\}$), handle (in_msg) with the simulated party P as if it was received from R by means of in_mode. In case simulated P does not exist yet, initialise it as an LN ITI. If there is a resulting message out_msg that is to be sent by simulated P to R' by means of out_mode \in {input, output, network}, send (RELAY, out_msg, P, R', out_mode) to \mathcal{F}_{Chan} .
- On receiving by \mathcal{F}_{Chan} a message to be sent by P to R via the network, carry on with this action (i.e. send this message via the internal \mathcal{A}).
- Relay any other incoming message to the internal ${\mathcal A}$ unmodified.
- On receiving a message (msg) by the internal \mathcal{A} , if it is addressed to one of the parties that correspond to $\mathcal{F}_{\text{Chan}}$, handle the message internally with the corresponding simulated party. Otherwise relay the message to its intended recipient unmodified. // Other recipients are \mathcal{E} , $\mathcal{G}_{\text{Ledger}}$ or parties unrelated to $\mathcal{F}_{\text{Chan}}$

Given that \mathcal{F}_{Chan} relays all messages and that we simulate the real-world machines that correspond to \mathcal{F}_{Chan} , the simulation is perfectly indistinguishable from the real world.

Fig. 4.

Simulator ${\cal S}$ – notifications to ${\cal F}_{\rm Chan}$

- When referring to a player P, it must be one of the parties that correspond to \mathcal{F}_{Chap} .
- When an action in the current Figure interrupts the simulation of a party, the latter is continued from the interruption location after control is handed back by \mathcal{F}_{Chan} .
- 1: On (CORRUPT) by A, addresed to P:
- // After executing this code, deliver (CORRUPT) to simulated P as detailed in Fig. 4. Given that F_{Chan} returns control directly to us after it handles this message, we will always deliver (CORRUPT) successfully.
- 3: send (INFO, BECAME CORRUPTED OR NEGLIGENT, P) to $\mathcal{F}_{\text{Chan}}$
- 4: When simulated P sets its internal variable negligent to True (Fig. 7, l. 26/Fig. 6, l. 7):
- 5: send (INFO, BECAME CORRUPTED OR NEGLIGENT, P) to $\mathcal{F}_{\operatorname{Chan}}$
- 6: When simulated P moves to the INIT state (Fig. 6, l. 21):
- 7: send (INFO, INIT, P) to \mathcal{F}_{Chan}
- 8: When the last of the honest simulated \mathcal{F}_{Chan} 's parties moves to the OPEN state for the first time (Fig. 10, l. 19/Fig. 12, l. 5/Fig. 13, l. 18):
- 9: send (INFO, OPEN) to \mathcal{F}_{Chan}
- 10: When (both \$\mathcal{F}_{Chan}\$'s simulated parties are honest and one party completes sending a payment (Fig. 18, l. 6) and the counterparty completes receiving that payment (Fig. 18, l. 19)), or (when only one of the two is honest and (completes either receiving or sending a payment)): // also send this message if both parties are honest when Fig. 18, l. 6 is executed by one party, but its counterparty is corrupted before executing Fig. 18, l. 19
- 11: send (INFO, PAY OK) to \mathcal{F}_{Chan}
- 12: When (both \mathcal{F}_{Chan} 's simulated parties are honest and they both complete changing to a new host (Fig. 7, l. 12)), or (when only one of the two is honest and it completes changing to a new host): // also send this message if both parties are honest when the change of host is executed by one party, but its counterparty is corrupted before changing host
- 13: send (INFO, FUND OK) to \mathcal{F}_{Chan}
- 14: When one of the honest simulated \mathcal{F}_{Chan} 's parties moves to the CLOSED state (Fig. 21, l. 8/Fig. 23, l. 26):
- 15: send (INFO, CLOSE) to \mathcal{F}_{Chan}

Fig. 5.

```
Process LN - init
 1: // When not specified, input comes from and output goes to \mathcal{E}.
 2: // The ITI knows whether it is Alice (funder) or Bob (fundee). The activated
     party is P and the counterparty is \bar{P}.
 3: On every activation, before handling the message:
         if last_poll \neq \perp then // channel has opened
 5:
              input (READ) to \mathcal{G}_{Ledger} and assign outut to \Sigma
 6:
              if last_poll + t < |\Sigma| then
 7:
                  negligent \leftarrow True
              end if
 8:
 9:
         end if
10: On (INIT, pk_{P,\text{out}}):
11:
         ensure State = \bot
12:
          State \leftarrow \text{INIT}
13:
         store pk_{P,\text{out}}
14:
          (c_A, c_B, \mathtt{locked}_A, \mathtt{locked}_B) \leftarrow (0, 0, 0, 0)
          (paid\_out, paid\_in) \leftarrow (\emptyset, \emptyset)
15:
16:
         negligent \leftarrow False
17:
         \texttt{last\_poll} \leftarrow \bot
         output (INIT OK)
18:
19: On (TOP UP):
         ensure P = Alice // activated party is the funder
20:
21:
         ensure State = INIT
22:
          (sk_{P, \mathrm{chain}}, pk_{P, \mathrm{chain}}) \leftarrow \mathtt{KEYGEN}()
          input (READ) to \mathcal{G}_{\mathrm{Ledger}} and assign ouput to \varSigma
23:
24:
          output (top up to, pk_{P,\text{chain}})
25:
          while \nexists tx \in \Sigma, c_{P,\text{chain}} : (c_{P,\text{chain}}, pk_{P,\text{chain}}) \in \text{tx.outputs do}
26:
               // while waiting, all other messages by P are ignored
27:
              wait for input (CHECK TOP UP)
28:
              input (READ) to \mathcal{G}_{\mathrm{Ledger}} and assign ouput to \Sigma
29:
          end while
30:
          State \leftarrow \texttt{TOPPED} \ \texttt{UP}
         output (top up ok, c_{P,\text{chain}})
31:
32: On (BALANCE):
         ensure State^P \in \{\text{OPEN}, \text{CLOSED}\}
33:
34:
         output (BALANCE, c_A, c_B, locked<sub>A</sub>, locked<sub>B</sub>)
```

Fig. 6.

```
\textbf{Process} \ \mathtt{LN-methods} \ \mathtt{used} \ \mathtt{by} \ \mathtt{VIRT}
 1: REVOKEPREVIOUS():
 2:
          ensure State \in WAITING FOR (OUTBOUND) REVOCATION
 3:
          R_{\bar{P},i} \leftarrow \text{TX {input: }} C_{P,i}.\text{outputs.}P, \text{ output: } (C_{P,i}.\text{outputs.}P.\text{value,}
     pk_{\bar{P},\mathrm{out}})\}
          \operatorname{sig}_{A,R,i} \leftarrow \operatorname{SIGN}(R_{\bar{P},i}, sk_{P,R})
 5:
          if State = WAITING FOR REVOCATION then
 6:
              State \leftarrow \text{Waiting for inbound revocation}
          else // State = WAITING FOR OUTBOUND REVOCATION
 7:
 8:
              i \leftarrow i + 1
 9:
              State \leftarrow \text{Waiting for hosts ready}
10:
          end if
          host_P \leftarrow host_P' // forget old host, use new host instead
12:
          \texttt{layer} \leftarrow \texttt{layer} + 1
13:
          return sig_{P,R,i}
14: PROCESSREMOTEREVOCATION(\operatorname{sig}_{\bar{P},R,i}):
          ensure State = WAITING FOR (INBOUND) REVOCATION
15:
16:
          R_{P,i} \leftarrow \text{TX {input: }} C_{\bar{P},i}.\text{outputs.}P, \text{ output: } (C_{\bar{P},i}.\text{outputs.}\bar{P}.\text{value,}
     pk_{P,\mathrm{out}})
          ensure VERIFY(R_{P,i}, \operatorname{sig}_{\bar{P},R,i}, pk_{\bar{P},R}) = \operatorname{True}
17:
18:
          if State = WAITING FOR REVOCATION then
               State \leftarrow \text{Waiting for outbound revocation}
20:
          else // State = WAITING FOR INBOUND REVOCATION
21:
               i \leftarrow i + 1
22:
               State \leftarrow \text{Waiting for hosts ready}
23:
          end if
24:
          return (OK)
25: NEGLIGENT():
26:
          negligent \leftarrow True
27:
          return (OK)
```

Fig. 7.

```
Process LN.EXCHANGEOPENKEYS()
 1: (sk_{A,F}, pk_{A,F}) \leftarrow \text{KEYGEN}(); (sk_{A,R}, pk_{A,R}) \leftarrow \text{KEYGEN}()
 2: State \leftarrow \text{Waiting for opening keys}
 3: send (OPEN, c, hops, pk_{A,F},\ pk_{A,R},\ pk_{A,\mathrm{out}}) to fundee
 4: // colored code is run by honest fundee. Validation is implicit
 5: ensure we run the code of Bob
 6: ensure State = INIT
 7: store pk_{A,F}, pk_{A,R}, pk_{A,out}
 8: (sk_{B,F}, pk_{B,F}) \leftarrow \text{KEYGEN}(); (sk_{B,R}, pk_{B,R}) \leftarrow \text{KEYGEN}()
9: if hops = \mathcal{G}_{Ledger} then // opening base channel
10:
         \texttt{layer} \leftarrow 0
         State \leftarrow \text{Waiting for comm sig}
11:
12: else // opening virtual channel
         State \leftarrow \text{Waiting for check keys}
14: end if
15: reply (accept channel, pk_{B,F}, pk_{B,R}, pk_{B,\text{out}})
16: ensure State = Waiting for opening keys
17: store pk_{B,F}, pk_{B,R}, pk_{B,out}
18: State \leftarrow \text{Opening keys ok}
```

Fig. 8.

```
Process LN.PREPAREBASE()

1: if hops = \mathcal{G}_{Ledger} then // opening base channel

2: F \leftarrow TX {input: (c, pk_{A, chain}), output: (c, 3 \land 2/\{pk_{A,F}, pk_{B,F}\})}

3: host<sub>A</sub> \leftarrow \mathcal{G}_{Ledger}

4: layer \leftarrow 0

5: else // opening virtual channel

6: input (FUND ME, Alice, Bob, hops, c, pk_{A,F}, pk_{B,F}) to hops[0].left and expect output (FUNDED, host<sub>P</sub>, funder_layer) // ignore any other message

7: layer \leftarrow funder_layer

8: end if
```

Fig. 9.

```
Process LN.EXCHANGEOPENSIGS()
     1: //s = (2 + \lceil \max Time_{window} + \frac{Delay}{2} / \min Time_{window} \rceil) windowSize, where
                 maxTimewindow, Delay, minTimewindow and windowSize are defined in
                 Proposition ?? TODO: recheck and include proposition
    2: C_{A,0} \leftarrow \text{TX (input: } (c, 3 \land 2/\{pk_{A,F}, pk_{B,F}\}), \text{ outputs: } (c, (pk_{A,\text{out}} + (t+s)) \lor c
                 2/\{pk_{A,R}, pk_{B,R}\}), (0, pk_{B,out})\}
    3: C_{B,0} \leftarrow \text{TX \{input: } (c, 3 \land 2/\{pk_{A,F}, pk_{B,F}\}), \text{ outputs: } (c, pk_{A,\text{out}}), (0, pk_{A
                  (pk_{B,\text{out}} + (t+s)) \vee 2/\{pk_{A,R}, pk_{B,R}\})\}
    4: \operatorname{sig}_{A,C,0} \leftarrow \operatorname{SIGN}(C_{B,0}, sk_{A,F})
    5: State \leftarrow \text{Waiting for comm sig}
    6: send (FUNDING CREATED, (c, pk_{A,\text{chain}}), sig_{A,C,0}) to fundee
    7: ensure State = WAITING FOR COMM SIG // if opening virtual channel, we have
                 received (FUNDED, host_fundee) by hops[-1].right (c.f. Fig 12, line 10)
    8: if hops = \mathcal{G}_{Ledger} then // opening base channel
                                  F \leftarrow \text{TX } \{\text{input: } (c, pk_{A, \text{chain}}), \text{ output: } (c, 3 \land 2/\{pk_{A,F}, pk_{B,F}\})\}
11: C_{B,0} \leftarrow \text{TX (input: } (c, 3 \land 2/\{pk_{A,F}, pk_{B,F}\}), \text{ outputs: } (c, pk_{A,\text{out}}), (0, pk_{
                  (pk_{B,\text{out}} + (t+s)) \vee 2/\{pk_{A,R}, pk_{B,R}\})\}
12: ensure VERIFY(C_{B,0}, \operatorname{sig}_{A,C,0}, pk_{A,F}) = True
13: C_{A,0} \leftarrow \text{TX {input: }} (c, 3 \land 2/\{pk_{A,F}, pk_{B,F}\}), \text{ outputs: }} (c, (pk_{A,\text{out}} + (t+s)) \lor
                  2/\{pk_{A,R}, pk_{B,R}\}), (0, pk_{B,out})\}
14: \operatorname{sig}_{B,C,0} \leftarrow \operatorname{SIGN}(C_{A,0}, sk_{B,F})
15: if hops = \mathcal{G}_{Ledger} then // opening base channel
                                  State \leftarrow \text{Waiting to check funding}
17: else // opening virtual channel
18:
                                  c_A \leftarrow c; c_B \leftarrow 0; i \leftarrow 0
19:
                                  State \leftarrow \text{OPEN}
20: end if
21: reply (funding signed, sig_{B,C,0})
22: ensure State = WAITING FOR COMM SIG
23: ensure VERIFY(C_{A,0}, \operatorname{sig}_{B,C,0}, pk_{B,F}) = \operatorname{True}
```

Fig. 10.

```
Process LN.COMMITBASE()

1: \operatorname{sig}_F \leftarrow \operatorname{SIGN}(F, sk_{A, \operatorname{chain}})
2: \operatorname{send} (OPEN, c, pk_{A, \operatorname{out}}, pk_{B, \operatorname{out}}, F, \operatorname{sig}_F, Alice, Bob) to \mathcal A
3: while F \notin \mathcal D do
4: wait for input (CHECK FUNDING) // ignore all other messages
5: input (READ) to \mathcal G_{\operatorname{Ledger}} and assign output to \mathcal D
6: end while
```

Fig. 11.

```
Process LN – external open messages for Bob
 1: On input (CHECK FUNDING):
2:
        ensure \mathit{State} = \mathtt{WAITING}\ \mathtt{TO}\ \mathtt{CHECK}\ \mathtt{FUNDING}
3:
        input (READ) to \mathcal{G}_{\mathrm{Ledger}} and assign output to \Sigma
 4:
        if F \in \Sigma then
 5:
             State \leftarrow \text{OPEN}
             reply (OPEN OK)
 6:
7:
        end if
8: On output (FUNDED, host<sub>P</sub>, funder_layer) by hops[-1].right:
        ensure \mathit{State} = \mathtt{WAITING} \ \mathtt{FOR} \ \mathtt{FUNDED}
         store host_P // we will talk directly to host_P
10:
         \texttt{layer} \leftarrow \texttt{funder\_layer}
11:
12:
         State \leftarrow \texttt{WAITING FOR COMM SIG}
13:
         reply (FUND ACK)
14: On output (CHECK KEYS, (pk_1,\ pk_2)) by hops[-1].right:
         ensure State = WAITING FOR CHECK KEYS
15:
16:
         ensure pk_1 = pk_{A,F} \wedge pk_2 = pk_{B,F}
         State \leftarrow \text{WAITING FOR FUDNED}
17:
18:
         reply (KEYS OK)
```

Fig. 12.

```
Process LN - On (OPEN, c, hops, fundee):
 1: // fundee is Bob
 2: ensure we run the code of Alice // activated party is the funder
 3: if hops = \mathcal{G}_{Ledger} then // opening base channel
        ensure State = \text{TOPPED UP}
        ensure c = c_{A,\text{chain}}
 6: else // opening virtual channel
        ensure len(hops) \geq 2 // cannot open a virtual over 1 channel
8: end if
9: LN.EXCHANGEOPENKEYS()
10: LN.PREPAREBASE()
11: LN.EXCHANGEOPENSIGS()
12: if hops = \mathcal{G}_{Ledger} then
        LN.COMMITBASE()
14: end if
15: input (READ) to \mathcal{G}_{Ledger} and assign output to \Sigma
16: last_poll \leftarrow |\Sigma|
17: c_A \leftarrow c; c_B \leftarrow 0; i \leftarrow 0
18: State \leftarrow \text{OPEN}
19: output (OPEN OK, c, fundee, hops)
```

Fig. 13.

```
Process LN.UPDATEFORVIRTUAL()

1: C_{\bar{P},i+1} \leftarrow C_{\bar{P},i} with pk'_{P,F} and pk'_{\bar{P},F} instead of pk_{P,F} and pk_{\bar{P},F} respectively

2: \mathrm{sig}_{P,C,i+1} \leftarrow \mathrm{SIGN}(C_{\bar{P},i+1}) // kept by \bar{P}

3: \mathrm{send} (UPDATE FORWARD, \mathrm{sig}_{P,C,i+1}) to \bar{P}

4: // P refers to payer and \bar{P} to payee both in local and remote code

5: C_{\bar{P},i+1} \leftarrow C_{\bar{P},i} with pk'_{P,F} and pk'_{\bar{P},F} instead of pk_{P,F} and pk_{\bar{P},F} respectively

6: \mathrm{ensure} VERIFY(C_{\bar{P},i+1}, \mathrm{sig}_{P,C,i+1}, pk'_{P,F}) = True

7: C_{P,i+1} \leftarrow C_{P,i} with pk'_{\bar{P},F} and pk'_{P,F} instead of pk_{\bar{P},F} and pk_{P,F} respectively

8: \mathrm{sig}_{\bar{P},C,i+1} \leftarrow \mathrm{SIGN}(C_{P,i+1},sk'_{\bar{P},F}) // kept by P

9: \mathrm{reply} (UPDATE BACK, \mathrm{sig}_{\bar{P},C,i+1})

10: C_{P,i+1} \leftarrow C_{P,i} with pk'_{\bar{P},F} and pk'_{P,F} instead of pk_{\bar{P},F} and pk_{P,F} respectively

11: \mathrm{ensure} VERIFY(C_{P,i+1}, \mathrm{sig}_{\bar{P},C,i+1}, pk'_{\bar{P},F}) = True
```

Fig. 14.

```
\textbf{Process} \,\, \mathtt{LN-virtualise} \,\, \mathtt{start} \,\, \mathtt{and} \,\, \mathtt{end}
 1: On input (FUND ME, c_{guest}, fundee, hops, pk_{A,V}, pk_{B,V}) by funder:
         ensure State = OPEN
 3:
         ensure c_P - \mathsf{locked}_P \ge c
 4:
         State \leftarrow \textit{Virtualising}
         (sk'_{P,F}, pk'_{P,F}) \leftarrow \text{KEYGEN}()
 5:
 6:
         define new VIRT ITI host'_P
         send (VIRTUALISING, \mathsf{host}_P', \mathit{pk}_{P,F}', \mathsf{hops}, \mathsf{fundee}, c_{\mathsf{guest}}) to \bar{P} and expect
     reply (virtualising ack, \mathtt{host}_{\bar{P}}',\ pk_{\bar{P},F}')
         ensure pk'_{\bar{P},F} is different from pk_{\bar{P},F} and all older \bar{P}'s funding public keys
         LN.UPDATEFORVIRTUAL()
 9:
10:
          State \leftarrow \text{Waiting for Revocation}
          input (HOST ME, funder, fundee, \mathsf{host}_{\bar{P}}', \mathsf{host}_{P}, \, c_{\mathsf{guest}}, \, pk_{A,V}, \, pk_{B,V},
     (sk'_{P,F}, pk'_{P,F}), (sk_{P,F}, pk_{P,F}), pk_{\bar{P},F}, pk'_{\bar{P},F}) to host'
12: On output (HOSTS READY) by host<sub>P</sub>: // host<sub>P</sub> is the new host, renamed in
     Fig. 7, l. 12
         ensure State = WAITING FOR HOSTS READY
13:
14:
          State \leftarrow \text{OPEN}
          move pk_{P,F}, pk_{\bar{P},F} to list of old funding keys
15:
          (sk_{P,F}, pk_{P,F}) \leftarrow (sk'_{P,F}, pk'_{P,F}); pk_{\bar{P},F} \leftarrow pk'_{\bar{P},F}
16:
          if len(hops) = 1 then // we are the last hop
17:
18:
              output (FUNDED, host<sub>P</sub>, layer) to fundee and expect reply (FUND
     ACK)
19:
         else if we have received input fund me just before we moved to the
     VIRTUALISING state then // we are the first hop
20:
              output (FUNDED, host<sub>P</sub>, layer) to funder // do not expect reply by
     funder
21:
          end if
22:
          reply (HOST ACK)
23: On output (SIGN TXS, TXs) by host'_P:
24:
         sigs \leftarrow \emptyset
25:
          for TX in TXs do
26:
              add SIGN(TX, sk_{P,F}, ANYPREVOUT) to sigs
27:
          end for
28:
          reply (TXs signed, sigs)
```

Fig. 15.

```
Process LN – virtualise hops
 1: On (VIRTUALISING, \operatorname{host}_{\bar{P}}', pk_{\bar{P}.F}', \operatorname{hops}, \operatorname{fundee}, c_{\operatorname{guest}}) by P:
 2:
          ensure State = OPEN
 3:
          ensure c_{\bar{P}} - \mathsf{locked}_{\bar{P}} \geq c
          ensure pk'_{\bar{P},F} is different from pk_{\bar{P},F} and all older \bar{P}'s funding public keys
          State \leftarrow VIRTUALISING
 5:
          locked_{\bar{P}} \leftarrow locked_{\bar{P}} + c // if \bar{P} is hosting the funder, \bar{P} will transfer c_{guest}
     coins instead of locking them, but the end result is the same
          (sk'_{P,F}, pk'_{P,F}) \leftarrow \text{KEYGEN}()
 7:
          if len(hops) > 1 then // we are not the last hop
 8:
 9:
               define new VIRT ITI host'_P
10:
               input (VIRTUALISING, host'<sub>P</sub>, (sk'_{P,F}, pk'_{P,F}), pk'_{\bar{P},F}, \text{hops}[1:], fundee,
     c_{\mathrm{guest}},\,c_{\bar{P}},\,c_{P}) to hops[1].left and expect reply (VIRTUALISING ACK,
     host\_sibling, pk_{sib,\bar{P},F}
               input (INIT, host<sub>P</sub>, host<sub>\bar{P}</sub>, host_sibling, (sk'_{P,F}, pk'_{P,F}), pk'_{\bar{P},F}
11:
     pk_{\text{sib},\bar{P},F}, (sk_{P,F}, pk_{P,F}), pk_{\bar{P},F}, c_{\text{guest}}) to host' and expect reply (HOST INIT
12:
          else // we are the last hop
               input (INIT, host<sub>P</sub>, host'<sub>\bar{P}</sub>, fundee=fundee, (sk'_{P,F}, pk'_{P,F}), pk'_{\bar{P},F},
13:
     (sk_{P,F},\ pk_{P,F}),\ pk_{\bar{P},F},\ c_{\mathrm{guest}}) to new VIRT ITI host'_P and expect reply (HOST
     INIT OK)
          end if
14:
          State \leftarrow \text{Waiting for revocation}
15:
          send (VIRTUALISING ACK, host'<sub>P</sub>, pk'_{PF}) to \bar{P}
16:
17: On input (VIRTUALISING, host_sibling, (sk'_{P,F}, pk'_{P,F}), pk_{\text{sib},\bar{P},F}, hops,
     fundee, c_{\text{guest}}, c_{\text{sib,rem}}, _{\text{sib}}) by sibling:
18:
          ensure State = OPEN
19:
          ensure c_P - \mathsf{locked}_P \ge c
20:
          ensure c_{\text{sib,rem}} \geq c_P \wedge c_{\bar{P}} \geq c_{\text{sib}} // avoid value loss by griefing attack: one
     counterparty closes with old version, the other stays idle forever
21:
          State \leftarrow VIRTUALISING
22:
          locked_P \leftarrow locked_P + c
23:
          define new VIRT ITI host'P
24:
          send (VIRTUALISING, host'<sub>P</sub>, pk'_{P,F}, hops, fundee, c_{guest}) to hops[0].right
     and expect reply (VIRTUALISING ACK, \operatorname{host}_{\bar{P}}', pk_{\bar{P},F}')
25:
          ensure pk'_{\bar{P},F} is different from pk_{\bar{P},F} and all older \bar{P}'s funding public keys
26:
          LN.UPDATEFORVIRTUAL()
27:
          \text{input (INIT, host}_P, \text{host}_{\bar{P}}', \text{host\_sibling}, (\mathit{sk}_{P,F}', \mathit{pk}_{P,F}'), \mathit{pk}_{\bar{P},F}', \mathit{pk}_{\mathrm{sib},\bar{P},F}',
     (sk_{P,F}, pk_{P,F}), pk_{\bar{P},F}, c_{\text{guest}}) to \mathsf{host}_P' and expect reply (HOST INIT OK)
28:
          State \leftarrow \text{Waiting for Revocation}
29:
          output (VIRTUALISING ACK, host'_P, pk'_{\bar{P},F}) to sibling
```

Fig. 16.

```
Process LN.SIGNATURESROUNDTRIP()
 1: C_{\bar{P},i+1} \leftarrow C_{\bar{P},i} with x coins moved from P's to \bar{P}'s output
 2: \operatorname{sig}_{P,C,i+1} \leftarrow \operatorname{SIGN}(C_{\bar{P},i+1},sk_{P,F}) // kept by \bar{P}
 3: send (PAY, x, \operatorname{sig}_{P,C,i+1}) to \bar{P}
 4: // P refers to payer and \bar{P} to payee both in local and remote code
 5: ensure State = OPEN
 6: if host<sub>\bar{P}</sub> \neq \mathcal{G}_{Ledger} \land \bar{P} has a host_sibling then // we are intermediary
            ensure c_{\rm sib,rem} \geq c_P - x \wedge c_{\bar{P}} + x \geq c_{\rm sib} // avoid value loss by griefing attack
 8: end if
9: C_{\bar{P},i+1} \leftarrow C_{\bar{P},i} with x coins moved from P's to \bar{P}'s output 10: ensure \operatorname{VERIFY}(C_{\bar{P},i+1},\operatorname{sig}_{P,C,i+1},pk_{P,F}) = \operatorname{True}_{\underline{\phantom{A}}}
11: C_{P,i+1} \leftarrow C_{P,i} with x coins moved from P's to \bar{P}'s output
12: \sup_{\bar{P},C,i+1} \leftarrow \operatorname{SIGN}(C_{P,i+1},sk_{\bar{P},F}) // kept by P
13: R_{P,i} \leftarrow \text{TX \{input: } C_{\bar{P},i}.\text{outputs.}P, \text{ output: } (c_{\bar{P}}, pk_{P,\text{out}})\}
14: \operatorname{sig}_{\bar{P},R,i} \leftarrow \operatorname{SIGN}(R_{P,i}, sk_{\bar{P},R})
15: reply (COMMITMENT SIGNED, \operatorname{sig}_{\bar{P},C,i+1}, \operatorname{sig}_{\bar{P},R,i})
16: C_{P,i+1} \leftarrow C_{P,i} with x coins moved from P's to \bar{P}'s output
```

Fig. 17.

```
Process LN.REVOCATIONSTRIP()
 1: ensure VERIFY(C_{P,i+1}, \operatorname{sig}_{\bar{P},C,i+1}, pk_{\bar{P},F}) = \operatorname{True}
 2: R_{P,i} \leftarrow \text{TX {input: }} C_{\bar{P},i}.\text{outputs.}P, \text{ output: } (c_{\bar{P}}, pk_{P,\text{out}})}
 3: ensure VERIFY(R_{P,i}, \operatorname{sig}_{\bar{P},R,i}, pk_{\bar{P},R}) = \operatorname{True}
 4: R_{\bar{P},i} \leftarrow \text{TX {input: }} C_{P,i}.\text{outputs.} \bar{P}, \text{ output: } (c_P, pk_{\bar{P},\text{out}})}
 5: \operatorname{sig}_{P,R,i}^{'} \leftarrow \operatorname{SIGN}(R_{\bar{P},i}, sk_{P,R})
 6: add x to paid_out
 7: c_P \leftarrow c_P - x; c_{\bar{P}} \leftarrow c_{\bar{P}} + x; i \leftarrow i + 1
 8: if host_P \neq \mathcal{G}_{Ledger} \land we have a host_sibling then // we are intermediary
 9:
          input (EW BALANCE, c_P, c_{\bar{P}}) to host<sub>P</sub>
          relay message as input to sibling // run by VIRT
10:
11:
          relay message as output to guest // run by VIRT
12:
          store new sibling balance and reply (NEW BALANCE OK)
13:
          output (NEW BALANCE OK) to sibling // run by VIRT
          output (NEW BALANCE OK) to guest // run by VIRT
15: end if
16: send (REVOKE AND ACK, \operatorname{sig}_{P,R,i}) to \bar{P}
17: R_{\bar{P},i} \leftarrow \text{TX {input: }} C_{P,i}.\text{outputs.}\bar{P}, \text{ output: }} (c_P, pk_{\bar{P},\text{out}})
18: ensure \operatorname{VERIFY}(R_{\bar{P},i},\operatorname{sig}_{P,R,i},pk_{P,R})=\operatorname{True}
19: add x to paid_in
20: c_P \leftarrow c_P - x; c_{\bar{P}} \leftarrow c_{\bar{P}} + x; i \leftarrow i + 1
21: if host_P \neq \mathcal{G}_{Ledger} \wedge \bar{P} has a host_sibling then // we are intermediary
22:
          input (NEW BALANCE, c_{\bar{P}}, c_{P}) to host _{\bar{P}}
23:
          relay message as input to sibling // run by VIRT
          relay message as output to guest // run by VIRT
24:
25:
          store new sibling balance and reply (NEW BALANCE OK)
26:
          output (NEW BALANCE OK) to sibling // run by VIRT
27:
          output (NEW BALANCE OK) to guest // run by VIRT
28: end if
```

Fig. 18.

```
Process LN − On (PAY, x):

1: ensure State = OPEN ∧ c<sub>P</sub> ≥ x

2: if host<sub>P</sub> ≠ G<sub>Ledger</sub> ∧ P has a host_sibling then // we are intermediary channel

3: ensure c<sub>sib,rem</sub> ≥ c<sub>P</sub> − x ∧ c<sub>P̄</sub> + x ≥ c<sub>sib</sub> // avoid value loss by griefing attack: one counterparty closes with old version, the other stays idle forever

4: end if

5: LN.SIGNATURESROUNDTRIP()

6: LN.REVOCATIONSTRIP()

7: // No output is given to the caller, this is intentional
```

Fig. 19.

```
Process LN – On (CHECK FOR LATERAL CLOSE):

1: if host_P \neq \mathcal{G}_{Ledger} then

2: input (CHECK FOR LATERAL CLOSE) to host_P

3: end if
```

Fig. 20.

```
Process LN – On (CHECK CHAIN FOR OLD COMM):

1: ensure State \notin \{\bot, \text{INIT}, \text{TOPPED UP}\} // \text{ channel open}

2: // \text{ even virtual channels check } \mathcal{G}_{\text{Ledger}} \text{ directly. This is intentional}

3: input (READ) to \mathcal{G}_{\text{Ledger}} and assign reply to \Sigma

4: last_poll \leftarrow |\varSigma|

5: if \exists 0 \le j < i : C_{\bar{P},j} \in \Sigma \text{ then } // \text{ counterparty has closed maliciously}

6: State \leftarrow \text{CLOSING}

7: LN.SUBMITANDCHECKREVOCATION(j)

8: State \leftarrow \text{CLOSED}

9: end if
```

Fig. 21.

```
Process LN.SUBMITANDCHECKREVOCATION(j)

1: \operatorname{sig}_{P,R,j} \leftarrow \operatorname{SIGN}(R_{P,j}, sk_{P,R})
2: input (SUBMIT, (R_{P,j}, \operatorname{sig}_{P,R,j}, \operatorname{sig}_{\bar{P},R,j})) to \mathcal{G}_{\operatorname{Ledger}}
3: while \nexists R_{P,j} \in \Sigma do
4: wait for input (CHECK REVOCATION) // ignore other messages
5: input (READ) to \mathcal{G}_{\operatorname{Ledger}} and assign output to \Sigma
6: end while
```

Fig. 22.

```
Process LN - On (CLOSE):
 1: ensure State \notin \{\bot, INIT, TOPPED UP\} // channel open
 2: if host_P \neq \mathcal{G}_{Ledger} then // we have a virtual channel
         State \leftarrow \text{HOST CLOSING}
         input (CLOSE) to host_P and keep relaying inputs (CHECK CHAIN FOR
    CLOSING) to host_P until receiving output (CLOSED) by host_P
 5: end if
 6: State \leftarrow CLOSING
 7: input (READ) to \mathcal{G}_{Ledger} and assign output to \Sigma
 8: if C_{\bar{P},i} \in \Sigma then // counterparty has closed honestly
         no-op // do nothing
10: else if \exists 0 \leq j < i : C_{\bar{P},j} \in \Sigma then // counterparty has closed maliciously
         LN.SUBMITANDCHECKREVOCATION(j)
11:
12: else // counterparty is idle
         while \nexists unspent output \in \Sigma that C_{P,i} can spend do // possibly due to an
13:
    active timelock
14:
             wait for input (CHECK VIRTUAL) // ignore other messages
             input (READ) to \mathcal{G}_{\mathrm{Ledger}} and assign output to \Sigma
15:
         end while
16:
         // provably reachable – c.f. TODO: ref
17:
         \operatorname{sig}'_{P,C,i} \leftarrow \operatorname{SIGN}(C_{P,i}, sk_{P,F})
18:
         input (SUBMIT, (C_{P,i}, \operatorname{sig}_{P,C,i}, \operatorname{sig}'_{P,C,i})) to \mathcal{G}_{\operatorname{Ledger}}
19:
20:
         while C_{P,i} \notin \Sigma do
21:
             wait for input (CHECK CLOSED) // ignore other messages
22:
             input (READ) to \mathcal{G}_{\mathrm{Ledger}} and assign output to \Sigma
23:
         end while
         // provably reachable - c.f. TODO: ref
24:
25: end if
26: State \leftarrow CLOSED
27: output (closed)
```

Fig. 23.

```
Process VIRT
 1: On every activation, before handling the message:
         if last_poll \neq \perp then // virtual layer is ready
 2:
 3:
             input (READ) to \mathcal{G}_{Ledger} and assign outut to \Sigma
 4:
             if last_poll + t < |\Sigma| then
                 for P \in \{\text{guest}, \text{funder}, \text{fundee}\}\ do // \text{ at most } 1 \text{ of funder}, \text{ fundee}\}
 5:
    is defined
                      ensure P.NEGLIGENT() returns (OK)
 6:
 7:
                 end for
 8:
             end if
9:
         end if
10: // guest is trusted to give sane inputs, therefore a state machine and input
    verification is redundant
11: On input (INIT, host_P, \bar{P}, sibling, fundee, (sk_{loc,virt},\ pk_{loc,virt}),\ pk_{rem,virt},
    pk_{\text{sib,rem,virt}}, (sk_{\text{loc,}F}, pk_{\text{loc,}F}), pk_{\text{rem,}F}, c_{\text{guest}}) by guest:
         store message contents and guest // sibling, pk_{\mathrm{sib},\bar{P},F} are missing for
12:
    edge nodes, fundee is present only in last node
         \texttt{last\_poll} \leftarrow \bot
13:
         output (HOST INIT OK) to guest
14:
15: On input (HOST ME, funder, fundee, \bar{P}, host_P, c_{\mathrm{guest}}, pk_{\mathrm{left,guest}}, pk_{\mathrm{right,guest}},
     (sk_{\text{loc,virt}}, pk_{\text{loc,virt}}), (sk_{\text{loc,F}}, pk_{\text{loc,F}}), pk_{\text{rem,F}}, pk_{\text{rem,virt}}) by guest:
         \texttt{last\_poll} \leftarrow \bot
16:
17:
         ensure VIRT.CIRCULATEKEYSANDCOINS() returns (OK)
18:
         ensure Virt.circulateVirtualSigs() returns (OK)
19:
         ensure VIRT.CIRCULATEFUNDINGSIGS() returns (OK)
20:
         ensure VIRT.CIRCULATEREVOCATIONS() returns (OK)
21:
         output (HOSTS READY) to guest
```

Fig. 24.

```
Process VIRT.CIRCULATEKEYSANDCOINS(left_data):
 1: if left_data is given as argument then // we are not host_funder
 2:
         if we have a sibling then // we are not host_fundee
              input (KEYS AND COINS FORWARD, (left_data, (sk_{loc,virt}, pk_{loc,virt}),
 3:
     (sk_{\text{loc},F}, pk_{\text{loc},F}), pk_{\text{rem},F}, c_P, c_{\bar{P}}) to sibling
              store input as left_data
              parse left_data as far_left_data, (sk_{loc,virt}, pk_{loc,virt}), (sk_{sib,F}, pk_{loc,virt})
 5:
     pk_{\mathrm{sib},F}),\;pk_{\mathrm{sib,rem},F},\;c_{\mathrm{sib}},\;c_{\mathrm{sib,rem}} // remove parentheses as necessary
              call VIRT.CIRCULATEKEYSANDCOINS(left_data) of \bar{P} and assign
    returned value to right_data
              parse right_data as far_right_data, pk_{\text{rem,virt}}
 7:
              output (KEYS AND COINS BACK, right_data, (sk_{\text{loc},F},\ pk_{\text{loc},F}),\ pk_{\text{rem},F},
 8:
9:
              store output as right_data
              \texttt{parse right\_data} \text{ as far\_right\_data}, \ (sk_{\texttt{sib},F}, \ pk_{\texttt{sib},F}), \ pk_{\texttt{sib},\texttt{rem},F}, \ c_{\texttt{sib}},
10:
     c_{
m sib,rem}
              \mathbf{return}\ (\mathtt{right\_data},\ pk_{\mathtt{loc,virt}})
11:
12:
          else // we are host_fundee
13:
              extract (pk_{\text{left,guest}}, pk_{\text{right,guest}}) from left_data
              output (CHECK KEYS, (pk_{\text{left,guest}}, pk_{\text{right,guest}})) to fundee and expect
14:
    reply (KEYS OK)
              \mathbf{return}\ \mathit{pk}_{\mathrm{loc,virt}}
15:
16:
          end if
17: else // we are host_funder
          call virt.circulate
KeysAndCoins(pk_{\rm loc,virt},\,(pk_{\rm left,guest},\,pk_{\rm right,guest})) of
18:
     \bar{P} and assign returned value to right_data
         return (OK)
19:
20: end if
```

Fig. 25.

```
Process VIRT
    1: GETMIDTXS(c_{\mathrm{guest}}, c_{\mathrm{loc}}, c_{\mathrm{rem}}, c_{\mathrm{sib}}, c_{\mathrm{sibRem}}, pk_{\mathrm{left,fund}}, pk_{\mathrm{loc,fund}}, pk_{\mathrm{sib,fund}},
                 pk_{\text{right,fund}}, pk_{\text{left,virt}}, pk_{\text{loc,virt}}, pk_{\text{sib,virt}}, pk_{\text{right,virt}}, pk_{\text{left,guest}}, pk_{\text{right,guest}},
                 pk_{\text{loc,out}}, \{pk_{\text{sec},i}\}_{i \in 1...n}):
    2:
                               ensure c_{\text{sibRem}} \ge c_{\text{guest}} \land c_{\text{loc}} \ge c_{\text{guest}}
    3:
                               c_{\text{left}} \leftarrow c_{\text{sib}} + c_{\text{sibRem}}; \, c_{\text{right}} \leftarrow c_{\text{loc}} + c_{\text{rem}}
                               \texttt{left\_fund} \leftarrow 2/\{pk_{\text{left,fund}}, pk_{\text{loc,fund}}\}
    4:
    5:
                               \texttt{right\_fund} \leftarrow 2/\{pk_{\texttt{sib},\texttt{fund}}, pk_{\texttt{right},\texttt{fund}}\}
                               \texttt{left\_virt} \leftarrow 2/\{pk_{\text{left,virt}}, pk_{\text{loc,virt}}\}
                               \texttt{left\_virt\_checked} \leftarrow 4/\{pk_{\text{left,virt}}, pk_{\text{loc,virt}}, pk_{\text{left,guest}}, pk_{\text{right,guest}}\}
    7:
                               \texttt{right\_virt} \leftarrow 2/\{pk_{\texttt{sib,virt}}, pk_{\texttt{right,virt}}\}
                               \texttt{right\_virt\_checked} \leftarrow 4/\{pk_{\text{sib,virt}}, pk_{\text{right,virt}}, pk_{\text{left,guest}}, pk_{\text{right,guest}}\}
                                left_out_checked \leftarrow (2 \land left_virt_checked) \lor (3 \land left_virt_+(t+s))
10:
                               right_out \leftarrow (1 \land right_virt) \lor (3 \land right_virt + (t+s))
11:
12:
                right_out_checked \leftarrow (1 \land right_virt_checked) \lor (3 \land right_virt + (t+s))
                                 \texttt{guest\_all} \leftarrow \texttt{5} \land n/\{pk_{\text{left,guest}}, pk_{\text{right,guest}}, \{pk_{\text{sec},1...n}\}\}
13:
                                 \texttt{guest\_out} \leftarrow \texttt{4} \land 2/\{pk_{\text{left,guest}}, pk_{\text{right,guest}}\}
14:
15:
                                 \texttt{guest} \leftarrow (\texttt{guest\_out} + (t + s)) \lor \texttt{guest\_all}
                                 TX_{none} \leftarrow TX \text{ {inputs: }} ((c_{left}, \texttt{left\_fund}), (c_{right}, \texttt{right\_fund})), \text{ outputs: }}
16:
                 ((c_{\text{left}} - c_{\text{guest}}, \, \texttt{left\_out\_checked}), \, (c_{\text{right}} - c_{\text{guest}}, \, \texttt{right\_out\_checked}),
                 (c_{\text{guest}}, pk_{\text{loc,out}}), (c_{\text{guest}}, \text{guest}))
                                 TX_{left} \leftarrow TX \{left - C_{guest}, 1 \land left\_virt\_checked\}, (c_{right}, c_{right}, c_{ri
                right_fund)), outputs: ((c_{\text{left}} - c_{\text{guest}}, 3 \land \text{left\_virt}), (c_{\text{right}} - c_{\text{guest}},
                \texttt{right\_out\_checked}),\,(c_{\texttt{guest}},\,pk_{\texttt{loc},\texttt{out}}))\}
                                 TX_{right} \leftarrow TX \text{ [inputs: ((}c_{left}, left\_fund), (}c_{right} - c_{guest}, 2 \land 
                right_virt_checked), (c_{guest}, guest_all)), outputs: ((c_{left} - c_{guest}, guest_all))
                \texttt{left\_out\_checked}), \ (c_{\texttt{right}} - c_{\texttt{guest}}, \ \texttt{3} \land \texttt{right\_virt}), \ (c_{\texttt{guest}}, \ pk_{\texttt{loc,out}}), \ (c
19:
                                TX_{both} \leftarrow TX \{inputs: ((c_{left} - c_{guest}, 1 \land left\_virt\_checked),\}
                 (c_{\text{right}} - c_{\text{guest}}, 2 \land \text{right\_virt\_checked}), (c_{\text{guest}}, \text{guest\_all})), \text{ outputs:}
                 ((c_{\text{left}} - c_{\text{guest}}, 3 \land \text{left\_virt}), (c_{\text{right}} - c_{\text{guest}}, 3 \land \text{right\_virt}),
                 (c_{\text{guest}}, pk_{\text{loc,out}})
20:
                                return (TX<sub>none</sub>, TX<sub>left</sub>, TX<sub>right</sub>, TX<sub>both</sub>)
```

Fig. 26.

```
Process VIRT
 1: // left and right refer to the two counterparties, with left being the one closer
       to the funder. Note difference with left/right meaning in VIRT.GETMIDTXS.
 2: GETEDGETXS(c_{\mathrm{guest}}, \, c_{\mathrm{left}}, \, c_{\mathrm{right}}, \, pk_{\mathrm{left,fund}}, \, pk_{\mathrm{right,fund}}, \, pk_{\mathrm{left,virt}}, \, pk_{\mathrm{right,virt}},
       pk_{\text{left,guest}}, pk_{\text{right,guest}}, \{pk_{\text{sec},i}\}_{i \in 1...n}, \text{ is\_funder}):
 3:
              ensure c_{\text{left}} \ge c_{\text{guest}}
 4:
              c_{\text{tot}} \leftarrow c_{\text{left}} + c_{\text{right}}
              \texttt{fund} \leftarrow 2/\{pk_{\text{left,fund}}, pk_{\text{right,fund}}\}
 5:
              \texttt{virt} \leftarrow 2/\{pk_{\text{left,virt}}, pk_{\text{right,virt}}\}
 7:
              \texttt{virt\_checked} \leftarrow 4/\{pk_{\text{left,virt}}, pk_{\text{right,virt}}, pk_{\text{left,guest}}, pk_{\text{right,guest}}\}
              \quad \textbf{if is\_funder} = True \; \textbf{then} \\
                     \mathtt{out} \leftarrow (\texttt{1} \land \mathtt{virt\_checked}) \lor (\texttt{3} \land \mathtt{virt} + (t+s))
 9:
              else // TXs belong to fundee
10:
                     \mathtt{out} \leftarrow (\mathtt{2} \land \mathtt{virt\_checked}) \lor (\mathtt{3} \land \mathtt{virt} + (t+s))
11:
12:
               end if
               \texttt{guest\_all} \leftarrow \texttt{5} \land n / \{pk_{\text{left,guest}}, pk_{\text{right,guest}}, \{pk_{\text{sec},1...n}\}\}
13:
14:
               \texttt{guest\_out} \leftarrow \texttt{4} \land 2/\{pk_{\text{left,guest}}, pk_{\text{right,guest}}\}
15:
               \texttt{guest} \leftarrow (\texttt{guest\_out} + (t+s)) \lor \texttt{guest\_all}
               TX_{base} \leftarrow TX \text{ {input: }} (c_{tot}, \text{ fund}), \text{ outputs: } ((c_{tot} - c_{guest}, \text{ out}), (c_{guest}, \text{ out}), (c_{guest}, \text{ out}), (c_{guest}, \text{ out}), (c_{guest}, \text{ outputs: })
16:
       \mathtt{guest}))\}
17:
              \mathbf{return}\ \mathrm{TX}_{\mathrm{base}}
```

Fig. 27.

```
Process VIRT.SIBLINGSIGS()
   1: parse input as sigs<sub>byLeft</sub>
  c_{\bar{P}}, c_{\rm sib}, c_{\rm sib,rem}, pk_{\rm sib,rem,F}, pk_{\rm sib,F}, pk_{\rm loc,F}, pk_{\rm rem,F}, pk_{\rm sib,rem,virt}, pk_{\rm loc,virt}
            pk_{\text{loc,virt}}, pk_{\text{rem,virt}}, pk_{\text{left,guest}}, pk_{\text{right,guest}}, pk_{\text{loc,virt}}, \{pk_{\text{sec},i}\}_{i \in 1...n})
  3: store all signatures in sigs_{byLeft} that sign any of TX_{loc,none}, TX_{loc,left},
            TX_{loc,right}, TX_{loc,both} and remove these signatures from sigs_{byLeft}
  4: ensure that the stored signatures contain one valid signature for TX_{loc,right}
            and TX<sub>loc,both</sub> which sign the guest_all input by each one of the previous
           i-1 hops
  5: ensure that there are exactly 4 more valid signatures in the stored signatures,
            which sign the 1 \land left\_virt\_checked inputs of TX_{loc,left} and TX_{loc,both} with
            pk_{\rm sib,rem,virt} and pk_{\rm left,guest}
   6: sigs_{toRight} \leftarrow sigs_{bvLeft}
  7: for each hop apart from the first, the last and ours (i \in [2, ..., n-1] \setminus \{j\}) do
            //~j is our hop number, hop data encoded in left_data and right_data
                      extract data needed for GETMIDTXs() from left_data (if i < j) or
           right_data (if i > j) and assign it to data, and \{pk_{\text{sec},i}\}_{i \in 1...n} // P and
            comm_keys are missing, that is OK. \{pk_{\text{sec},i}\}_{i\in 1...n} contains each party's pk_{i,\text{virt}}
                      (TX_{i,none}, TX_{i,left}, TX_{i,right}, TX_{i,both}) \leftarrow VIRT.GETMIDTXS(data_i, TX_{i,left}, TX
            \{pk_{\text{sec},i}\}_{i\in 1...n}
                       add SIGN(TX_{i,right}, sk_{loc,virt}, ANYPREVOUT) and SIGN(TX_{i,both}, sk_{loc,virt},
            Anyprevout) to \operatorname{sigs}_{\text{toLeft}} if i < j, or \operatorname{sigs}_{\text{toRight}} if i > j // if i-th hop is
           adjacent, 2 signatures will be produced by each SIGN() invocation: one for the
            guest_all and one for the 2 ∧ right_virt_checked input
                      if i - j = 1 then // hop is our next
11:
                                 add SIGN(TX_{i,left}, sk_{loc,virt}, ANYPREVOUT) to sigs_{toRight}
12:
13:
                       else if j - i = 1 then // hop is our previous
                                 add SIGN(TX<sub>i,left</sub>, sk_{loc,virt}, ANYPREVOUT) to sigs<sub>toLeft</sub>
14:
15:
                       end if
16: end for
17: if right_data does not contain data from a second-next hop then // next
            hop is host_fundee
                      \mathrm{TX}_{\mathrm{next,none}} \leftarrow \mathrm{VIRT.GETEDGETXS}(c_{\mathrm{guest}},\, c_P,\, c_{\bar{P}},\, pk_{\mathrm{loc},F},\, pk_{\mathrm{rem},F},\, pk_{\mathrm{loc},\mathrm{virt}};\, c_{\mathrm{loc},F},\, pk_{\mathrm{loc},F},\, pk_{\mathrm{loc},F}
18:
            pk_{\text{rem,virt}}, pk_{\text{left,guest}}, pk_{\text{right,guest}}, \text{False})
20: call P.CIRCULATEVIRTUALSIGS(sigs<sub>toRight</sub>) and assign returned value to
            sigs_{byRight}
21: store all signatures in sigs<sub>byRight</sub> that sign any of TX<sub>loc,none</sub>, TX<sub>loc,left</sub>,
            TX<sub>loc,right</sub>, TX<sub>loc,both</sub> and remove these signatures from sigs<sub>byRight</sub>
           ensure that the stored signatures contain one valid signature for TX<sub>loc,right</sub> and
            TX_{loc,both} which sign the guest_all input by each one of the next n-i hops
23: ensure that there are exactly 4 more valid signatures in the stored signatures,
            which sign the 2 \land right\_virt\_checked inputs of TX_{loc,right} and TX_{loc,both}
            with pk_{\text{rem.virt}} and pk_{\text{right,guest}}
24: output (VIRTUALSIGSBACK, sigs_{toLeft}, sigs_{byRight})
```

Fig. 28.

```
Process VIRT.INTERMEDIARYSIGS()

1: (TX_{loc,none}, TX_{loc,left}, TX_{loc,right}, TX_{loc,both}) \leftarrow VIRT.GETMIDTXS(c_{guest}, c_P, c_{\bar{p}}, c_{sib}, c_{sib,rem}, pk_{loc,F}, pk_{rem,F}, pk_{sib,F}, pk_{sib,remF}, pk_{rem,virt}, pk_{loc,virt}, pk_{loc,virt},
```

Fig. 29.

```
Process VIRT.HOSTFUNDEESIGS()
    1: \text{TX}_{\text{loc,none}} \leftarrow \text{VIRT.GETEDGETXS}(c_{\text{guest}}, c_P, c_{\bar{P}}, pk_{\text{loc},F}, pk_{\text{rem},F}, pk_{\text{loc,virt}}, c_{\bar{P}}, pk_{\text{loc},F}, pk_{\text{rem},F}, pk_{\text{loc},F}, pk_{\text{lo
               pk_{\text{rem,virt}}, pk_{\text{left,guest}}, pk_{\text{right,guest}}, \text{False}
   2: for each hop apart from the first and ours (i \in [2, ..., n-1]) do // hop data
              encoded in left_data
                           extract data needed for GETMIDTXs() from left_data and assign it to
              \mathtt{data}_i and \{pk_{\sec,i}\}_{i\in 1...n} // \{pk_{\sec,i}\}_{i\in 1...n} contains each party's pk_{i,\text{virt}}
                           (TX_{i,none}, TX_{i,left}, TX_{i,right}, TX_{i,both}) \leftarrow VIRT.GETMIDTXS(data_i, TX_{i,none}, TX_{i,left}, TX_{i,right}, TX_{i,both})
               \{pk_{\text{sec},i}\}_{i\in 1...n}
                           add SIGN(TX_{i,right}, sk_{loc,virt}, ANYPREVOUT) and SIGN(TX_{i,both}, sk_{loc,virt},
              Anyprevout) to sigs_{toLeft} // if i-th hop is adjacent, 2 signatures will be
              produced by each SIGN() invocation: one for the guest_all and one for the
               2 ∧ right_virt_checked input
                           output (SIGN TXS, TX_{i,left}, TX_{i,right}, TX_{i,both}) to fundee and expect reply
               (TXS SIGNED, sigs_{guest})
   7:
                           add \mathrm{sigs}_{\mathrm{guest}} to \mathrm{sigs}_{\mathrm{toLeft}}
   8:
                           if i = n - 1 then // hop is our previous
  9:
                                         add SIGN(TX_{i,left}, sk_{loc,virt}, ANYPREVOUT) to sigs_{toLeft}
10:
                             end if
11: end for
12: \mathbf{return} \ \mathrm{sigs}_{\mathrm{toLeft}}
```

Fig. 30.

```
Process VIRT.HOSTFUNDERSIGS()
     1: for each hop apart from the last and ours (i \in [2, ..., n-1]) do // hop data
                  encoded in right_data
                                   extract data needed for GETMIDTXs() from right_data and assign it to
                   \mathtt{data}_i and \{pk_{\mathtt{sec},i}\}_{i\in 1...n} // \{pk_{\mathtt{sec},i}\}_{i\in 1...n} contains each party's pk_{i,\mathrm{virt}}
                                   (TX_{i,none}, TX_{i,left}, TX_{i,right}, TX_{i,both}) \leftarrow VIRT.GETMIDTXS(data_i, TX_{i,none}, TX_{i,left}, TX_{i,right}, TX_{i,both})
                    \{pk_{\text{sec},i}\}_{i\in 1...n}
                                   add SIGN(TX_{i,right}, sk_{loc,virt}, ANYPREVOUT) and SIGN(TX_{i,both}, sk_{loc,virt}, sk_{l
                  ANYPREVOUT) to \operatorname{sigs_{toRight}} // if i-th hop is adjacent, 2 signatures will be
                   produced by each SIGN() invocation: one for the guest_all and one for the
                  2 \land \texttt{right\_virt\_checked} \ input
                                   output (SIGN TXS, TX_{i,left}, TX_{i,right}, TX_{i,both}) to fundee and expect reply
                   (TXS SIGNED, sigs_{guest})
                                   add \mathrm{sigs}_{\mathrm{guest}} to \mathrm{sigs}_{\mathrm{toRight}}
    6:
    7:
                                   if i = 2 then // hop is our next
    8:
                                                     add SIGN(TX_{i,left}, sk_{loc,virt}, ANYPREVOUT) to sigs_{toRight}
                                   end if
   9:
10: end for
11: call Virt.circulate
VirtualSigs(sigs_toright) of P and assign output to
12: TX_{loc,none} \leftarrow VIRT.GETEDGETXS(c_{guest}, c_P, c_{\bar{P}}, pk_{loc,F}, pk_{rem,F}, pk_{loc,virt}, pk_{rem,F}, pk_{rem,F},
                    pk_{\text{rem,virt}}, pk_{\text{left,guest}}, pk_{\text{right,guest}}, \text{True})
13: return (OK)
```

Fig. 31.

```
Process VIRT.CIRCULATEVIRTUALSIGS(sigs<sub>byLeft</sub>)

1: if sigs<sub>byLeft</sub> is given as argument then // we are not host_funder

2: if we have a sibling then // we are not host_fundee

3: return VIRT.INTERMEDIARYSIGS()

4: else // we are host_fundee

5: return VIRT.HOSTFUNDEESIGS()

6: end if

7: else // we are host_funder

8: return VIRT.HOSTFUNDERSIGS()

9: end if
```

Fig. 32.

```
\textbf{Process} \ \operatorname{VIRT.CIRCULATEFUNDINGSigs}(\operatorname{sig}_{\operatorname{loc,none}})
 1: if sig_{loc,none} is given as argument then // we are not host_funder
 2:
             ensure VERIFY(TX_{loc,none}, sig_{loc,none}, pk_{prev,F}) = True // pk_{prev,F}, found in
      left_data
 3:
             sigs_{loc,none} \leftarrow \{sig_{loc,none}\}
             if we have a sibling then // we are not host_fundee
                   input (VIRTUAL BASE SIG FORWARD, \mathrm{sig_{loc,none}}) to \mathtt{sibling} // \mathtt{sibling}
 5:
       needs sig_{loc,none} for closing
 6:
                   \operatorname{sigs}_{\operatorname{loc,none}} \leftarrow \{\operatorname{sig}_{\operatorname{loc,none}}\}
                   \operatorname{sig}_{\operatorname{next,none}} \leftarrow \operatorname{SIGN}(\operatorname{TX}_{\operatorname{next,none}}, sk_{\operatorname{loc},F})
 7:
                   call virt.circulate
VirtualSigs(sig_{next,none}) of \bar{P} and assign returned
                   ensure VERIFY(TX<sub>loc,none</sub>, sig<sub>loc,none</sub>, pk_{\text{next},F}) = True // pk_{\text{next},F},
       found in right_data
                   add \mathrm{sig}_{\mathrm{loc,none}} to \mathrm{sigs}_{\mathrm{loc,none}}
10:
                   output (VIRTUAL BASE SIG BACK, \operatorname{sig_{loc,none}}) // sibling needs \operatorname{sig_{loc,none}}
       for closing
12:
                   add \mathrm{sig}_{\mathrm{loc,none}} to \mathrm{sigs}_{\mathrm{loc,none}}
13:
             end if
             \mathrm{sig}_{\mathrm{prev},\mathrm{none}} \leftarrow \mathrm{SIGN}(\mathrm{TX}_{\mathrm{prev},\mathrm{none}},\mathit{sk}_{\mathrm{loc},\mathit{F}})
14:
             \mathbf{return} \ \mathrm{sig}_{\mathrm{prev},\mathrm{none}}
15:
16: else // we are host_funder
             \mathrm{sig}_{\mathrm{next,none}} \leftarrow \mathrm{SIGN}(\mathrm{TX}_{\mathrm{next,none}}, \mathit{sk}_{\mathrm{loc},\mathit{F}})
17:
             call VIRT.CIRCULATEFUNDINGSIGS(sig_{next,none}) of \bar{P} and assign returned
18:
       value to sig_{loc,none}
             ensure Verify(TX<sub>loc,none</sub>, \operatorname{sig}_{\operatorname{loc,none}}, \operatorname{pk}_{\operatorname{next},F}) = True // \operatorname{pk}_{\operatorname{next},F} found in
19:
20:
             sigs_{loc,none} \leftarrow \{sig_{loc,none}\}
21:
             return (OK)
22: end if
```

Fig. 33.

```
Process VIRT.CIRCULATEREVOCATIONS(revoc_by_prev)
1: if revoc_by_prev is given as argument then // we are not host_funder
       ensure guest.PROCESSREMOTEREVOCATION(revoc_by_prev) returns (OK)
3: else // we are host_funder
       revoc_for_next \leftargue guest.REVOKEPREVIOUS()
5:
       input (READ) to \mathcal{G}_{Ledger} and assign outut to \Sigma
       \texttt{last\_poll} \leftarrow |\varSigma|
       call virt.circulateRevocations(revoc_for_next) of \bar{P} and assign
   returned value to revoc_by_next
       ensure guest.PROCESSREMOTEREVOCATION(revoc_by_next) returns (OK)
    // If the "ensure" fails, the opening process freezes, this is intentional. The
   channel can still close via (CLOSE)
       return (OK)
10: end if
11: if we have a sibling then // we are not host_fundee nor host_funder
       input (VIRTUAL REVOCATION FORWARD) to sibling
13:
       revoc_for_next ← guest.REVOKEPREVIOUS()
14:
       input (READ) to \mathcal{G}_{\mathrm{Ledger}} and assign outut to \Sigma
15:
       last_poll \leftarrow |\Sigma|
       call virt.circulateRevocations(revoc_for_next) of \bar{P} and assign
   output to revoc_by_next
17:
       ensure guest.PROCESSREMOTEREVOCATION(revoc_by_next) returns (OK)
18:
       output (VIRTUAL REVOCATION BACK)
19: end if
20: revoc_for_prev ← guest.REVOKEPREVIOUS()
21: output (HOSTS READY) to guest and expect reply (HOST ACK)
22: return revoc_for_prev // we are not host_fundee nor host_funder
```

Fig. 34.

```
Process VIRT - poll
 1: On input (CHECK FOR LATERAL CLOSE) by R \in \{\text{guest}, \text{funder}, \text{fundee}\}:
          input (READ) to \mathcal{G}_{\mathrm{Ledger}} and assign output to \Sigma
 2:
 3:
          \texttt{prev\_went\_on\_chain} \leftarrow TX_{\texttt{prev}, \texttt{left}} \in \varSigma \land TX_{\texttt{prev}, \texttt{both}} \in \varSigma
 4:
          \texttt{next\_went\_on\_chain} \leftarrow TX_{next,right} \in \varSigma \land TX_{next,both} \in \varSigma
 5:
          last_poll \leftarrow |\Sigma|
 6:
          if prev_went_on_chain \lor next_went_on_chain then
 7:
               ignore all messages except for (CHECK CHAIN FOR CLOSING) by R
 8:
               State \leftarrow \text{CLOSING}
          end if
9:
10:
          \mathbf{if} \ \mathtt{prev\_went\_on\_chain} \land \mathtt{next\_went\_on\_chain} \ \mathbf{then}
               \operatorname{virt.signAndSubmit}(TX_{\operatorname{loc,both}}, \operatorname{sigs}_{\operatorname{loc,both}})
11:
12:
          else if prev_went_on_chain then
               {\tt VIRT.SIGNANDSUBMIT}(TX_{\tt loc,left},\,sigs_{\tt loc,left})
13:
14:
          else if next_went_on_chain then
               {\tt VIRT.SIGNANDSUBMIT}(TX_{\rm loc,right},\,sigs_{\rm loc,right})
15:
16:
          end if
17: VIRT.SIGNANDSUBMIT(tx, sigs):
18:
          add SIGN(tx, sk_{loc,F}) to sigs
19:
          input (SUBMIT, tx, sigs) to \mathcal{G}_{Ledger}
```

Fig. 35.

```
Process VIRT - close
 1: On input (CLOSE) by R \in \{\text{guest}, \text{funder}, \text{fundee}\}: // At most one of funder,
     fundee is defined
 2:
         if State = CLOSED then
 3:
             output (CLOSED) to R
         end if
 4:
 5:
         ensure State = OPEN
 6:
         if host_P \neq \mathcal{G}_{Ledger} then // host is a VIRT
 7:
             ignore all messages except for output (CLOSED) by host. Also relay to
    host<sub>P</sub> any (CHECK CHAIN FOR CLOSING) input received
 8:
             input (CLOSE) to host_P
 9:
         end if
         // if we have a host_P, continue from here on output (CLOSED) by it
10:
         send (READ) to \mathcal{G}_{\text{Ledger}} as R and assign reply to \Sigma
11:
         let tx be the unique valid TX for \Sigma among (TX_{loc,none}, TX_{loc,left},
12:
    TX<sub>loc,right</sub>, TX<sub>loc,both</sub>) // if we are not an intermediary, only the first exists
13:
         let sigs be the corresponding set of signatures among (sigs<sub>loc,none</sub>,
    sigs_{loc,left}, sigs_{loc,right}, sigs_{loc,both})
         add SIGN(tx, sk_{A,F}) and SIGN(tx, sk_{loc,virt}) to sigs // one of the two
    signatures may be empty, as some transactions don't need a signature by both
    keys. This is not a problem.
15:
         ignore all messages except for (CHECK CHAIN FOR CLOSING) by R
16:
         State \leftarrow \text{closing}
17:
         send (SUBMIT, (tx, sigs)) to \mathcal{G}_{Ledger}
18: On (CHECK CHAIN FOR CLOSING) by R \in \{\text{guest}, \text{funder}, \text{fundee}\}:
         ensure State = CLOSING
19:
20:
         send (READ) to \mathcal{G}_{\text{Ledger}} as R and assign reply to \Sigma
21:
         if R = guest then
22:
             pk_1 \leftarrow pk_{\text{left,guest}}; pk_2 \leftarrow pk_{\text{right,guest}}
23:
         else //R \in \{\text{funder}, \text{fundee}\}
24:
             pk_1 \leftarrow pk_{\text{loc,virt}}; pk_2 \leftarrow pk_{\text{rem,virt}}
25:
         if \Sigma has an unspent output that can be spent exclusively by a 2-of-\{pk_1,
    pk_2 multisig then // if there is a timelock, it must have expired
27:
             State \leftarrow CLOSED
28:
             output (CLOSED) to R
29:
         end if
```

Fig. 36.

Lemma 1 (Real world balance security). Let P honest LN ITI such that all of the following are true:

- the internal variable negligent of P has value "False",
- P has transitioned to the OPEN State for the first time after having received (OPEN, c, \ldots) by either \mathcal{E} or another LN ITI \bar{P} ,

- P [has received (FUND ME, f_i, \ldots) as input by another LN ITI while State was OPEN and subsequently P transitioned to OPEN State] n times.
- P [has received (PAY, d_i) by \mathcal{E} while State was OPEN and P subsequently transitioned to OPEN State] m times,
- P [has received (PAY, e_i, \ldots) by \bar{P} while State was OPEN and P subsequently transitioned to OPEN State] l times.

Let $\phi = 1$ if P = Alice, or $\phi = 0$ if P = Bob. If P receives (CLOSE) by \mathcal{E} , then eventually the state obtained when P inputs (READ) to \mathcal{G}_{Ledger} will contain h $(c_i, pk_{P,out})$ outputs such that

$$\sum_{i=1}^{h} c_i \ge \phi \cdot c - \sum_{i=1}^{n} f_i - \sum_{i=1}^{m} d_i + \sum_{i=1}^{l} e_i . \tag{1}$$

Lemma 2 (Ideal world balance). Consider an ideal execution with functionality \mathcal{F}_{Chan} and simulator \mathcal{S} . Let $P \in \{Alice, Bob\}$ one of the two parties of \mathcal{F}_{Chan} and \bar{P} the other party. Assume that all of the following are true:

- the internal variable is_corrupted_or_negligent_P of \mathcal{F}_{Chan} has the value "False".
- If P = Alice, P has received (OPEN, c, ...) by \mathcal{E} ,
- P [has received (FUND ME, $f_i,...$) as input by another \mathcal{F}_{Chan} or Π_{Chan} ITI while State was OPEN and subsequently \mathcal{F}_{Chan} transitioned to OPEN State] n times.
- P [has received (PAY, d_i) by \mathcal{E} while State was OPEN and \mathcal{F}_{Chan} subsequently transitioned to OPEN State] m times,
- P [has received (PAY, e_i) by \mathcal{E} while State was OPEN and \mathcal{F}_{Chan} subsequently transitioned to OPEN State] l times.

Let $\phi = 1$ if P = Alice, or $\phi = 0$ if P = Bob. If P or \bar{P} receives (CLOSE) by S, then

$$balance_{B} = \phi \cdot c - \sum_{i=1}^{n} f_{i} - \sum_{i=1}^{m} d_{i} + \sum_{i=1}^{l} e_{i}$$
 (2)

Lemma 3 (No halt). In an ideal execution with \mathcal{F}_{Chan} and \mathcal{S} , the functionality never halts (i.e. l. 28 of Fig. 3 is never executed).

Proof. The only way for \mathcal{F}_{Chan} to halt is if either $check_A$ or $check_B$ fails. For these checks to be run, \mathcal{F}_{Chan} must have received (CLOSE) by \mathcal{S} and must have been in the OPEN State (as any other state of Fig. 3, l. 16 can only be reached after \mathcal{F}_{Chan} has been in the OPEN State). Additionally, \mathcal{F}_{Chan} can only reach the OPEN State if all honest simulated parties transition to the OPEN State as well (Fig. 5, l. 9), which in turn happens for simulated Alice only if she has received (OPEN, ...) by \mathcal{E} . Observe further that \mathcal{E} notifies \mathcal{F}_{Chan} right away when either simulated party becomes corrupted or negligent (Fig. 5, ll. 3 and 5 respectively) and the balance of each party is checked by \mathcal{F}_{Chan} only if this party is not corrupted nor negligent. These facts in combination mean that for each party,

whenever the prerequisites for Lemma 2 are true the prerequisites for Lemma 1 are also true and therefore the check for this party will succeed (c.f. Fig 3, l. 20). Furthermore, when the prerequisites for Lemma 2 do not hold, the check for the respective party will be true. On aggregate, when an ideal execution of \mathcal{F}_{Chan} and \mathcal{S} take place, in no case will \mathcal{F}_{Chan} halt.

Theorem 1 (Recursive Virtual Payment Channel Security). The protocol LN realises \mathcal{F}_{Chan} with simulator \mathcal{S} given a global functionality \mathcal{G}_{Ledger} and assuming the security of the underlying digital signature. Specifically,

$$\forall \ PPT \ \mathcal{A}, \mathcal{E} \ \ it \ is \ \mathrm{EXEC}_{\mathrm{LN},\mathcal{A},\mathcal{E}}^{\mathcal{G}_{\mathrm{Ledger}}} = \mathrm{EXEC}_{\mathcal{S}_{\mathcal{A}},\mathcal{E}}^{\mathcal{F}_{\mathrm{Chan}},\mathcal{G}_{\mathrm{Ledger}}}$$

Proof. By inspection of Figs. 1 and 4 we can deduce that for a particular \mathcal{E} , in the ideal world execution $\text{EXEC}_{\mathcal{S}_{\mathcal{A}},\mathcal{E}}^{\mathcal{F}_{\text{Chan}},\mathcal{G}_{\text{Ledger}}}$, $\mathcal{S}_{\mathcal{A}}$ simulates internally the two LN parties exactly as they would execute in the real world execution, $\text{EXEC}_{\text{LN},\mathcal{A},\mathcal{E}}^{\mathcal{G}_{\text{Ledger}}}$ in case $\mathcal{F}_{\text{Chan}}$ never halts. The latter is true according to Lemma 3, therefore the two executions are information-theoretically indistinguishable.

Theorem 2. Assume that at the end of the execution, \mathcal{G}_{Ledger} contains exactly one "groups" transaction that precedes all "funding" transactions and contains as payload a partition \mathcal{G} into groups of all VChan parties, with each group containing the parties that belong to the same (human) owner. Then the following holds:

 $\forall G \in \mathcal{G} \text{ such that all parties in } G \text{ are honest},$

$$\begin{split} &\sum_{P \in G} \text{logged-coins}(P) = \sum_{P \in G} \text{ledger-coins}(P) = \\ &= \sum_{P \in G} (\text{top-up}(P) + \sum_{m \in \mathcal{T}} \text{pay-in}(m,P) - \sum_{m \in \mathcal{T}} \text{pay-out}(m,P)) \enspace, \end{split}$$

where \mathcal{T} is the execution transcript and:

logged-coins $(P) = c_P$, as recorded in $\mathcal{F}_{Chan}/\Pi_{Chan}$

 $\label{eq:coins} \mbox{ledger-coins}(P) = \mbox{coins spendable with the secret key sk of P if the closing} \\ \mbox{transactions of all open channels are submitted to \mathcal{G}_{Ledger}} \\ \mbox{and added to the state of all parties and then t new blocks} \\ \mbox{enter the state of all honest parties}$

$$\operatorname{top-up}(P) = \begin{cases} c_{\operatorname{on}}, \ as \ determined \ on \ message \ (\operatorname{CHECK} \ \operatorname{TOP} \ \operatorname{UP}), \\ \text{ if such a message was handled} \\ 0, & \text{ otherwise} \end{cases}$$

$$\operatorname{pay-in}(m,P) = \begin{cases} x, & \text{ if message } m \ \operatorname{updated \ the \ channel \ to} \\ a \ state \ in \ which \ P \ had \ x \ more \ coins \ TODO: \ improve \ prev} \\ 0, & \text{ otherwise} \end{cases}$$

$$\operatorname{pay-out}(m,P) = \begin{cases} x, \ \text{ if } m = (\operatorname{PAY},x) \ was \ received \ by \ P \ and} \\ P \ \operatorname{output} \ (\operatorname{PAY} \ \operatorname{SUCCESS}) \ as \ a \ result} \\ 0, & \text{ otherwise} \end{cases}$$

References