**Abstract.** Blockchains are slow. Layer-2 largely solves this problem. PCNs constitute the most prominent layer-2/off-chain protocols. LN is the most widely used PCN and works on Bitcoin. Opening a channel requires 1 on-chain transaction, which can at times be avoided by performing a multi-hop payment. Then however fees to the intermediaries must be paid, routing becomes an issue, payment delay is proportional to the number of intermediaries and per-payment privacy suffers.
We propose Recursive Channels, which allow for new channels to be opened on top of an arbitrarily long path of existing channels in a recursive manner (i.e. the preexisting channels may themselves be virtual), answering the question of feasibility in the affirmative.
Our construction relies on the proposed `ANYPREVOUT` signature type.

## 1 Introduction

The popularity of blockchains in recent years has stretched their performance to its limits. Due to their need for synchronisation their latency is large (e.g. Bitcoin has a latency of 1h [1]) and due to the need for massive redundancy their throughput is low (Bitcoin can handle at most 7 transactions per second [2]). To circumvent these inherent limitations of blockchains, a prominent solution is to optimistically handle payments off-chain via a Payment Channel Network (PCN) TODO: cite PCN SoK/many papers and only use the blockchain as an arbiter in case of dispute.

The most popular PCN is the Lightning Network (LN) [3], which works on top of Bitcoin. With this, parties can open a pairwise channel with a single on-chain transaction and subsequently pay each other an unlimited number of times, only limited by the speed of their internet connection. What is more, a party can pay another even if they do not have a direct channel. They can instead leverage a path of channels for a fee and perform a so-called multi-hop payment in an atomic manner. Unfortunately a multi-hop payment needs active cooperation by all intermediaries, therefore increasing the latency and the probability of failure of the payment.

To mitigate this issue, virtual payment channels have been proposed TODO: cite. These enable two parties, say Alice and Bob, to open a payment channel over two preexisting channels, one between Alice and Charlie and another between Charlie and Bob. TODO: check if recursive channels exist

However, due to the limited scripting language of Bitcoin, it has proved challenging to build a secure protocol that allows virtual channels to be opened over more than two underlying channels, TODO: delete following phrase if the previous's TODO answer is affirmative as well as to make this construction recursive in the sense that further virtual channels can be opened on top of other virtual channels.

This work fills this gap by providing a concrete protocol that allows for arbitrarily many channels to be opened on top of arbitrarily long channel paths, where the underlying channels may themselves be virtual. This is achieved using standard Bitcoin script and an elaborate transaction configuration. We formally prove the security of the protocol in the UC [4] setting. The construction relies on the `ANYPREVOUT` signature type, which does not sign the hash of the transaction it spends, therefore allowing for a single pre-signed transaction to spend any output with a suitable script. We conjecture that this primitive cannot be achieved without `ANYPREVOUT`.

## 2 Related Work

## 3 High Level Explanation

## 4 Preliminaries & Notation

## 5 Construction

## 6 Evaluation

## 7 Future work

- Add support for cooperative adding multiple virtuals to single channel (needs cooperation by all hops of all existing virtuals of current channel)
- Add support for cooperative closing
- Use eltoo instead of lightning to avoid balance restriction that prevents the revoked-griefing attack
- Allow for user-defined "leeway" timeout and timeout renegotiation
- Incorporate fees
- Prevent DoS attacks

**Functionality** $\mathcal{F}_{\text{Chan}}$ – general message handling rules

- On receiving (`msg`) by party $R$ to $P \in \{Alice, Bob\}$ by means of `mode` $\in$ {input, output, network}, handle it according to the corresponding rule in Fig. 2, 3, 5, or 4 (if any) and subsequently send (RELAY, `msg`, $P$, $\mathcal{E}$, input) $\mathcal{A}$. // all messages are relayed to $\mathcal{A}$
- On receiving (RELAY, `msg`, $P$, $R$, `mode`) by $\mathcal{A}$ (`mode` $\in$ {input, output, network}, $P \in \{Alice, Bob\}$), relay `msg` to $R$ as $P$ by means of `mode`. // $\mathcal{A}$ fully controls outgoing messages by $\mathcal{F}_{\text{Chan}}$
- On receiving (INFO, `msg`) by $\mathcal{A}$, handle (`msg`) according to the corresponding rule in Fig. 2, 3, 5, or 4 (if any). After handling the message or after an "ensure" fails, send (HANDLED, `msg`) to $\mathcal{A}$. // (INFO, `msg`) messages by $\mathcal{S}$ always return control to $\mathcal{S}$ without any side-effect to any other ITI, except if $\mathcal{F}_{\text{Chan}}$ halts
- $\mathcal{F}_{\text{Chan}}$ keeps track of two state machines, one for each of *Alice*, *Bob*. If there are more than one suitable rules for a particular message, or if a rule matches the message for both parties, then both rule versions are executed. // the two rules act on different state machines, so the order of execution does not matter

**Fig. 1.**

**Functionality** $\mathcal{F}_{\text{Chan}}$ – state machine up to OPEN for $P \in \{Alice, Bob\}$

On first activation: // before handing the message
  $pk_P \leftarrow \bot$; $\texttt{host}_P \leftarrow \bot$; $\texttt{enabler}_P \leftarrow \bot$; $\texttt{balance}_P \leftarrow 0$;
  $State_P \leftarrow$ UNINIT

On (BECAME CORRUPTED OR NEGLIGENT, $P$) by $\mathcal{A}$ or on output (ENABLER USED REVOCATION) by $\texttt{host}_P$ when in any state:
  $State_P \leftarrow$ IGNORED

On (INIT, $pk$) to $P$ by $\mathcal{E}$ when $State_P =$ UNINIT:
  $pk_P \leftarrow pk$
  $State_P \leftarrow$ INIT

On (OPEN, $x$, $\mathcal{G}_{\text{Ledger}}$, $\dots$) to $Alice$ by $\mathcal{E}$ when $State_A =$ INIT:
  store $x$
  $State_A \leftarrow$ TENTATIVE BASE OPEN

On (BASE OPEN) by $\mathcal{A}$ when $State_A =$ TENTATIVE BASE OPEN:
  $\texttt{balance}_A \leftarrow x$
  $State_A \leftarrow$ OPEN

On (BASE OPEN) by $\mathcal{A}$ when $State_B =$ INIT:
  $State_B \leftarrow$ OPEN

On (OPEN, $x$, $\texttt{hops} \neq \mathcal{G}_{\text{Ledger}}$, $\dots$) to $Alice$ by $\mathcal{E}$ when $State_A =$ INIT:
  store $x$
  $\texttt{enabler}_A \leftarrow \texttt{hops}[0]\texttt{.left}$
  add $\texttt{enabler}_A$ to $Alice$'s trusted parties
  $State_A \leftarrow$ PENDING VIRTUAL OPEN

On output (FUNDED, $\texttt{host}$, $\dots$) to $Alice$ by $\texttt{enabler}_A$ when $State_A =$ PENDING VIRTUAL OPEN:
  $\texttt{host}_A \leftarrow \texttt{host}[0]\texttt{.left}$
  $State_A \leftarrow$ TENTATIVE VIRTUAL OPEN

On output (FUNDED, $\texttt{host}$, $\dots$) to $Bob$ by ITI $R \in \{\mathcal{F}_{\text{Chan}}, \text{LN}\}$ when $State_B =$ INIT:
  $\texttt{enabler}_B \leftarrow R$
  add $\texttt{enabler}_B$ to $Bob$'s trusted parties
  $\texttt{host}_B \leftarrow \texttt{host}$
  $State_B \leftarrow$ TENTATIVE VIRTUAL OPEN

On (VIRTUAL OPEN) by $\mathcal{A}$ when $State_P =$ TENTATIVE VIRTUAL OPEN:
  **if** $P = Alice$ **then** $\texttt{balance}_P \leftarrow x$
  $State_P \leftarrow$ OPEN

**Fig. 2.**

**Functionality** $\mathcal{F}_{\text{Chan}}$ – payments state machine for $P \in \{Alice, Bob\}$

1: On $(\textsc{pay}, x)$ by $\mathcal{E}$ when $State_P = \textsc{open}$: // $P$ pays $\bar{P}$
2: store $x$
3: $State_P \leftarrow \textsc{tentative pay}$

4: On $(\textsc{pay})$ by $\mathcal{A}$ when $State_P = \textsc{tentative pay}$: // $P$ pays $\bar{P}$
5: $State_P \leftarrow (\textsc{sync pay}, x)$

6: On $(\textsc{get paid}, y)$ by $\mathcal{E}$ when $State_P = \textsc{open}$: // $\bar{P}$ pays $P$
7: store $y$
8: $State_P \leftarrow \textsc{tentative get paid}$

9: On $(\textsc{pay})$ by $\mathcal{A}$ when $State_P = \textsc{tentative get paid}$: // $\bar{P}$ pays $P$
10: $State_P \leftarrow (\textsc{sync get paid}, x)$

11: When $State_P = (\textsc{sync pay}, x)$:
12: **if** $State_{\bar{P}} \in \{\textsc{ignored}, (\textsc{sync get paid}, x)\}$ **then**
13: $\texttt{balance}_P \leftarrow \texttt{balance}_P - x$
14: // if $\bar{P}$ honest, this state transition happens simultaneously with l. 21
15: $State_P \leftarrow \textsc{open}$
16: **end if**

17: When $State_P = (\textsc{sync get paid}, x)$:
18: **if** $State_{\bar{P}} \in \{\textsc{ignored}, (\textsc{sync pay}, x)\}$ **then**
19: $\texttt{balance}_P \leftarrow \texttt{balance}_P + x$
20: // if $\bar{P}$ honest, this state transition happens simultaneously with l. 15
21: $State_P \leftarrow \textsc{open}$
22: **end if**

**Fig. 3.**

**Functionality** $\mathcal{F}_{\text{Chan}}$ – fundings state machine for $P \in \{\mathit{Alice}, \mathit{Bob}\}$

1: On input (FUND ME, $x$, …) by ITI $R \in \{\mathcal{F}_{\text{Chan}}, \text{LN}\}$ when $\mathit{State}_P = \text{OPEN}$:
2:     store $x$
3:     add $R$ to $P$'s trusted parties
4:     $\mathit{State}_P \leftarrow$ PENDING FUND

5: When $\mathit{State}_P = $ PENDING FUND:
6:     **if** we intercept the command "define new VIRT ITI `host`" by $\mathcal{A}$, routed through $P$ **then**
7:         store `host`
8:         $\mathit{State}_P \leftarrow$ TENTATIVE FUND
9:         continue executing $\mathcal{A}$'s command
10:     **end if**

11: On (FUND) by $\mathcal{A}$ when $\mathit{State}_P = $ TENTATIVE FUND:
12:     $\mathit{State}_P \leftarrow$ SYNC FUND

13: When $\mathit{State}_P = $ OPEN:
14:     **if** we intercept the command "define new VIRT ITI `host`" by $\mathcal{A}$, routed through $P$ **then**
15:         store `host`
16:         $\mathit{State}_P \leftarrow$ TENTATIVE HELP FUND
17:         continue executing $\mathcal{A}$'s command
18:     **end if**
19:     **if** we receive a RELAY message with `msg` = (INIT, …, `fundee`) addressed from $P$ by $\mathcal{A}$ **then**
20:         add `fundee` to $P$'s trusted parties
21:         continue executing $\mathcal{A}$'s command
22:     **end if**

23: On (FUND) by $\mathcal{A}$ when $\mathit{State}_P = $ TENTATIVE HELP FUND:
24:     $\mathit{State}_P \leftarrow$ SYNC HELP FUND

25: When $\mathit{State}_P = $ SYNC FUND:
26:     **if** $\mathit{State}_{\bar{P}} \in \{\text{IGNORED}, \text{SYNC HELP FUND}\}$ **then**
27:         $\texttt{balance}_P \leftarrow \texttt{balance}_P - x$
28:         $\texttt{host}_P \leftarrow \texttt{host}$
29:         // if $\bar{P}$ honest, this state transition happens simultaneously with l. 36
30:         $\mathit{State}_P \leftarrow$ OPEN
31:     **end if**

32: When $\mathit{State}_P = $ SYNC HELP FUND:
33:     **if** $\mathit{State}_{\bar{P}} \in \{\text{IGNORED}, \text{SYNC FUND}\}$ **then**
34:         $\texttt{host}_P \leftarrow \texttt{host}$
35:         // if $\bar{P}$ honest, this state transition happens simultaneously with l. 30
36:         $\mathit{State}_P \leftarrow$ OPEN
37:     **end if**

**Fig. 4.**

**Functionality $\mathcal{F}_{\text{Chan}}$ – closure state machine for $P \in \{Alice, Bob\}$**

On (CLOSE) by $\mathcal{E}$ when $State_P =$ OPEN:
    $State_P \leftarrow$ CLOSING

On input (BALANCE) to $P$ by $R$ where $R$ is trusted by $P$:
    **if** $State_P \notin \{$UNINIT, INIT, PENDING VIRTUAL OPEN, TENTATIVE VIRTUAL OPEN, TENTATIVE BASE OPEN, IGNORED, CLOSED$\}$ **then**
        reply (MY BALANCE, $\texttt{balance}_P$, $pk_P$, $\texttt{balance}_{\bar{P}}$, $pk_{\bar{P}}$)
    **else**
        reply (MY BALANCE, 0, $pk_P$, 0, $pk_{\bar{P}}$)
    **end if**

On (CLOSE, $P$) by $\mathcal{A}$ when $State_P \notin \{$UNINIT, INIT, PENDING VIRTUAL OPEN, TENTATIVE VIRTUAL OPEN, TENTATIVE BASE OPEN, IGNORED$\}$:
    input (READ) to $\mathcal{G}_{\text{Ledger}}$ as $P$ and assign ouput to $\Sigma$
    $\texttt{coins} \leftarrow$ sum of values of outputs exclusively spendable or spent by $pk_P$ in $\Sigma$
    $\texttt{balance} \leftarrow \texttt{balance}_P$
    **for all** $P$'s trusted parties $R$ **do**
        input (BALANCE) to $R$ as $P$ and extract $\texttt{balance}_R$, $pk_R$ from response
        $\texttt{balance} \leftarrow \texttt{balance} + \texttt{balance}_R$
        $\texttt{coins} \leftarrow \texttt{coins} +$ sum of values of outputs exclusively spendable or spent by $pk_R$ in $\Sigma$
    **end for**
    **if** $\texttt{coins} \geq \texttt{balance}$ **then**
        $State_P \leftarrow$ CLOSED
    **else** // balance security is broken
        halt
    **end if**

**Fig. 5.**

**Simulator $\mathcal{S}$** – general message handling rules

- On receiving (RELAY, `in_msg`, $P$, $R$, `in_mode`) by $\mathcal{F}_{\text{Chan}}$ (`in_mode` $\in$ {input, output, network}, $P \in \{Alice, Bob\}$), handle (`in_msg`) with the simulated party $P$ as if it was received from $R$ by means of `in_mode`. In case simulated $P$ does not exist yet, initialise it as an LN ITI. If there is a resulting message `out_msg` that is to be sent by simulated $P$ to $R'$ by means of `out_mode` $\in$ {input, output, network}, send (RELAY, `out_msg`, $P$, $R'$, `out_mode`) to $\mathcal{F}_{\text{Chan}}$.
- On receiving by $\mathcal{F}_{\text{Chan}}$ a message to be sent by $P$ to $R$ via the network, carry on with this action (i.e. send this message via the internal $\mathcal{A}$).
- Relay any other incoming message to the internal $\mathcal{A}$ unmodified.
- On receiving a message (`msg`) by the internal $\mathcal{A}$, if it is addressed to one of the parties that correspond to $\mathcal{F}_{\text{Chan}}$, handle the message internally with the corresponding simulated party. Otherwise relay the message to its intended recipient unmodified. // Other recipients are $\mathcal{E}$, $\mathcal{G}_{\text{Ledger}}$ or parties unrelated to $\mathcal{F}_{\text{Chan}}$

Given that $\mathcal{F}_{\text{Chan}}$ relays all messages and that we simulate the real-world machines that correspond to $\mathcal{F}_{\text{Chan}}$, the simulation is perfectly indistinguishable from the real world.

**Fig. 6.**

**Simulator $\mathcal{S}$** – notifications to $\mathcal{F}_{\text{Chan}}$

– "$P$" refers one of the parties that correspond to $\mathcal{F}_{\text{Chan}}$.
– When an action in this Figure interrupts an ITI simulation, continue simulating from the interruption location once action is over/$\mathcal{F}_{\text{Chan}}$ hands control back.

On (CORRUPT) by $\mathcal{A}$, addresed to $P$:
    // After executing this code and getting control back from $\mathcal{F}_{\text{Chan}}$ (which always happens, c.f. Fig. 1), deliver (CORRUPT) to simulated $P$ (c.f. Fig. 6.
    send (INFO, BECAME CORRUPTED OR NEGLIGENT, $P$) to $\mathcal{F}_{\text{Chan}}$

When simulated $P$ sets variable `negligent` to True (Fig. 8, l. 7/Fig. 9, l. 26):
    send (INFO, BECAME CORRUPTED OR NEGLIGENT, $P$) to $\mathcal{F}_{\text{Chan}}$

When simulated honest *Alice* receives (OPEN, $x$, `hops`, ...) by $\mathcal{E}$:
    store `hops` // will be used to inform $\mathcal{F}_{\text{Chan}}$ once the channel is open

When simulated honest *Bob* receives (OPEN, $x$, `hops`, ...) by *Alice*:
    **if** *Alice* is corrupted **then** store `hops` // if *Alice* is honest, we already have `hops`. If *Alice* became corrupted after receiving (OPEN, ...), overwrite `hops`

When the last of the honest simulated $\mathcal{F}_{\text{Chan}}$'s parties moves to the OPEN *State* for the first time (Fig. 12, l. 19/Fig. 14, l. 5/Fig. 15, l. 18):
    **if** `hops` $= \mathcal{G}_{\text{Ledger}}$ **then**
        send (INFO, BASE OPEN) to $\mathcal{F}_{\text{Chan}}$
    **else**
        send (INFO, VIRTUAL OPEN) to $\mathcal{F}_{\text{Chan}}$
    **end if**

When (both $\mathcal{F}_{\text{Chan}}$'s simulated parties are honest and complete sending and receiving a payment (Fig. 20, ll. 6 and 21 respectively), or (when only one party is honest and (completes either receiving or sending a payment)): // also send this message if both parties are honest when Fig. 20, l. 6 is executed by one party, but its counterparty is corrupted before executing Fig. 20, l. 21
    send (INFO, PAY) to $\mathcal{F}_{\text{Chan}}$

When honest $P$ executes Fig. 17, l. 20 or (when honest $P$ executes Fig. 17, l. 18 and $\bar{P}$ is corrupted): // in the first case if $\bar{P}$ is honest, it has already moved to the new host, (Fig 38, ll. 7, 23): lifting to next layer is done
    send (INFO, FUND) to $\mathcal{F}_{\text{Chan}}$

When one of the honest simulated $\mathcal{F}_{\text{Chan}}$'s parties $P$ moves to the CLOSED state (Fig. 24, l. 8 or l. 11):
    send (INFO, CLOSE, $P$) to $\mathcal{F}_{\text{Chan}}$

**Fig. 7.**

**Process** LN – init

// When not specified, input comes from and output goes to $\mathcal{E}$.
// The ITI knows whether it is *Alice* (funder) or *Bob* (fundee). The activated party is $P$ and the counterparty is $\bar{P}$.
On every activation, before handling the message:
  **if** $\texttt{last\_poll} \neq \perp$ **then** // channel has opened
    input (READ) to $\mathcal{G}_{\text{Ledger}}$ and assign ouput to $\Sigma$
    **if** $\texttt{last\_poll} + p < |\Sigma|$ **then** // $p$ is a global parameter
      $\texttt{negligent} \leftarrow$ True
    **end if**
  **end if**

On (INIT, $pk_{P,\text{out}}$):
  ensure $State = \perp$
  $State \leftarrow$ INIT
  store $pk_{P,\text{out}}$
  $(c_A, c_B, \texttt{locked}_A, \texttt{locked}_B) \leftarrow (0, 0, 0, 0)$
  $(\texttt{paid\_out}, \texttt{paid\_in}) \leftarrow (\emptyset, \emptyset)$
  $\texttt{negligent} \leftarrow$ False
  $\texttt{last\_poll} \leftarrow \perp$
  output (INIT OK)

On (TOP UP):
  ensure $P = Alice$ // activated party is the funder
  ensure $State =$ INIT
  $(sk_{P,\text{chain}}, pk_{P,\text{chain}}) \leftarrow$ KEYGEN()
  input (READ) to $\mathcal{G}_{\text{Ledger}}$ and assign ouput to $\Sigma$
  output (TOP UP TO, $pk_{P,\text{chain}}$)
  **while** $\nexists \text{tx} \in \Sigma, c_{P,\text{chain}} : (c_{P,\text{chain}}, pk_{P,\text{chain}}) \in \text{tx.outputs}$ **do**
    // while waiting, all other messages by $P$ are ignored
    wait for input (CHECK TOP UP)
    input (READ) to $\mathcal{G}_{\text{Ledger}}$ and assign ouput to $\Sigma$
  **end while**
  $State \leftarrow$ TOPPED UP
  output (TOP UP OK, $c_{P,\text{chain}}$)

On (BALANCE):
  ensure $State^P \in \{\text{OPEN}, \text{CLOSED}\}$
  output (BALANCE, $c_A, pk_{A,\text{out}}, c_B, pk_{B,\text{out}}, \texttt{locked}_A, \texttt{locked}_B$)

**Fig. 8.**

**Process** ln – methods used by virt

revokePrevious():

  ensure $State \in$ waiting for (outbound) revocation

  $R_{\bar{P},i} \leftarrow$ TX {input: $C_{P,i}$.outputs.$P$, output: ($C_{P,i}$.outputs.$P$.value, $pk_{\bar{P},\mathrm{out}}$)}

  $\mathrm{sig}_{A,R,i} \leftarrow$ sign($R_{\bar{P},i}$, $sk_{P,R}$)

  **if** $State =$ waiting for revocation **then**

    $State \leftarrow$ waiting for inbound revocation

  **else** // $State =$ waiting for outbound revocation

    $i \leftarrow i + 1$

    $State \leftarrow$ waiting for hosts ready

  **end if**

  `host`$_P \leftarrow$ `host`$'_P$ // forget old host, use new host instead

  `layer` $\leftarrow$ `layer` $+ 1$

  **return** $\mathrm{sig}_{P,R,i}$

processRemoteRevocation($\mathrm{sig}_{\bar{P},R,i}$):

  ensure $State =$ waiting for (inbound) revocation

  $R_{P,i} \leftarrow$ TX {input: $C_{\bar{P},i}$.outputs.$P$, output: ($C_{\bar{P},i}$.outputs.$\bar{P}$.value, $pk_{P,\mathrm{out}}$)}

  ensure verify($R_{P,i}$, $\mathrm{sig}_{\bar{P},R,i}$, $pk_{\bar{P},R}$) = True

  **if** $State =$ waiting for revocation **then**

    $State \leftarrow$ waiting for outbound revocation

  **else** // $State =$ waiting for inbound revocation

    $i \leftarrow i + 1$

    $State \leftarrow$ waiting for hosts ready

  **end if**

  **return** (ok)

negligent():

  `negligent` $\leftarrow$ True

  **return** (ok)

**Fig. 9.**

**Process** LN.EXCHANGEOPENKEYS()

$(sk_{A,F}, pk_{A,F}) \leftarrow$ KEYGEN(); $(sk_{A,R}, pk_{A,R}) \leftarrow$ KEYGEN()
$State \leftarrow$ WAITING FOR OPENING KEYS
send (OPEN, $c$, `hops`, $pk_{A,F}$, $pk_{A,R}$, $pk_{A,\text{out}}$) to `fundee`
// colored code is run by honest `fundee`. Validation is implicit
ensure we run the code of *Bob*
ensure $State =$ INIT
store $pk_{A,F}$, $pk_{A,R}$, $pk_{A,\text{out}}$
$(sk_{B,F}, pk_{B,F}) \leftarrow$ KEYGEN(); $(sk_{B,R}, pk_{B,R}) \leftarrow$ KEYGEN()
**if** `hops` $= \mathcal{G}_{\text{Ledger}}$ **then** // opening base channel
    `layer` $\leftarrow 0$
    $t_P \leftarrow s + p$ // $s$ is the upper bound of $\eta$ from Lemma 7.19 of [5]
    $State \leftarrow$ WAITING FOR COMM SIG
**else** // opening virtual channel
    $State \leftarrow$ WAITING FOR CHECK KEYS
**end if**
reply (ACCEPT CHANNEL, $pk_{B,F}$, $pk_{B,R}$, $pk_{B,\text{out}}$)
ensure $State =$ WAITING FOR OPENING KEYS
store $pk_{B,F}$, $pk_{B,R}$, $pk_{B,\text{out}}$
$State \leftarrow$ OPENING KEYS OK

**Fig. 10.**

**Process** LN.PREPAREBASE()

**if** `hops` $= \mathcal{G}_{\text{Ledger}}$ **then** // opening base channel
    $F \leftarrow$ TX {input: $(c, pk_{A,\text{chain}})$, output: $(c, 2/\{pk_{A,F}, pk_{B,F}\})$}
    $\texttt{host}_P \leftarrow \mathcal{G}_{\text{Ledger}}$
    `layer` $\leftarrow 0$
    $t_P \leftarrow s + p$
**else** // opening virtual channel
    input (FUND ME, *Alice*, *Bob*, `hops`, $c$, $pk_{A,F}$, $pk_{B,F}$) to `hops`[0].`left` and expect output (FUNDED, $\texttt{host}_P$, `funder_layer`, $t_P$) // ignore any other message
    `layer` $\leftarrow$ `funder_layer`
**end if**

**Fig. 11.**

**Process** LN.EXCHANGEOPENSIGS()

// $s = (2 + \lceil \texttt{maxTime}_{\texttt{window}} + \frac{\texttt{Delay}}{2} / \texttt{minTime}_{\texttt{window}} \rceil)\texttt{windowSize}$, where $\texttt{maxTime}_{\texttt{window}}$, $\texttt{Delay}$, $\texttt{minTime}_{\texttt{window}}$ and $\texttt{windowSize}$ are defined in Proposition ?? TODO: recheck and include proposition

$C_{A,0} \leftarrow \text{TX }\{\text{input: }(c, 2/\{pk_{A,F}, pk_{B,F}\}), \text{ outputs: }(c, (pk_{A,\text{out}} + (t+s)) \vee 2/\{pk_{A,R}, pk_{B,R}\}), (0, pk_{B,\text{out}})\}$

$C_{B,0} \leftarrow \text{TX }\{\text{input: }(c, 2/\{pk_{A,F}, pk_{B,F}\}), \text{ outputs: }(c, pk_{A,\text{out}}), (0, (pk_{B,\text{out}} + (t+s)) \vee 2/\{pk_{A,R}, pk_{B,R}\})\}$

$\text{sig}_{A,C,0} \leftarrow \text{SIGN}(C_{B,0}, sk_{A,F})$

$\mathit{State} \leftarrow$ WAITING FOR COMM SIG

send (FUNDING CREATED, $(c, pk_{A,\text{chain}})$, $\text{sig}_{A,C,0}$) to `fundee`

ensure $\mathit{State} =$ WAITING FOR COMM SIG // if opening virtual channel, we have received (FUNDED, `host_fundee`) by `hops`[-1].`right` (Fig 14, l. 10)

**if** `hops` $= \mathcal{G}_{\text{Ledger}}$ **then** // opening base channel

  $F \leftarrow \text{TX }\{\text{input: }(c, pk_{A,\text{chain}}), \text{ output: }(c, 2/\{pk_{A,F}, pk_{B,F}\})\}$

**end if**

$C_{B,0} \leftarrow \text{TX }\{\text{input: }(c, 2/\{pk_{A,F}, pk_{B,F}\}), \text{ outputs: }(c, pk_{A,\text{out}}), (0, (pk_{B,\text{out}} + (t+s)) \vee 2/\{pk_{A,R}, pk_{B,R}\})\}$

ensure $\text{VERIFY}(C_{B,0}, \text{sig}_{A,C,0}, pk_{A,F}) = \text{True}$

$C_{A,0} \leftarrow \text{TX }\{\text{input: }(c, 2/\{pk_{A,F}, pk_{B,F}\}), \text{ outputs: }(c, (pk_{A,\text{out}} + (t+s)) \vee 2/\{pk_{A,R}, pk_{B,R}\}), (0, pk_{B,\text{out}})\}$

$\text{sig}_{B,C,0} \leftarrow \text{SIGN}(C_{A,0}, sk_{B,F})$

**if** `hops` $= \mathcal{G}_{\text{Ledger}}$ **then** // opening base channel

  $\mathit{State} \leftarrow$ WAITING TO CHECK FUNDING

**else** // opening virtual channel

  $c_A \leftarrow c$; $c_B \leftarrow 0$; $i \leftarrow 0$

  $\mathit{State} \leftarrow$ OPEN

**end if**

reply (FUNDING SIGNED, $\text{sig}_{B,C,0}$)

ensure $\mathit{State} =$ WAITING FOR COMM SIG

ensure $\text{VERIFY}(C_{A,0}, \text{sig}_{B,C,0}, pk_{B,F}) = \text{True}$

**Fig. 12.**

**Process** LN.COMMITBASE()

$\text{sig}_F \leftarrow \text{SIGN}(F, sk_{A,\text{chain}})$

input (SUBMIT, $(F, \text{sig}_F)$) to $\mathcal{G}_{\text{Ledger}}$ // enter "while" below before sending

**while** $F \notin \Sigma$ **do**

  wait for input (CHECK FUNDING) // ignore all other messages

  input (READ) to $\mathcal{G}_{\text{Ledger}}$ and assign output to $\Sigma$

**end while**

**Fig. 13.**

**Process** LN – external open messages for *Bob*

On input (CHECK FUNDING):
    ensure *State* = WAITING TO CHECK FUNDING
    input (READ) to $\mathcal{G}_{\text{Ledger}}$ and assign output to $\Sigma$
    **if** $F \in \Sigma$ **then**
        *State* $\leftarrow$ OPEN
        reply (OPEN OK)
    **end if**

On output (FUNDED, `host`$_P$, `funder_layer`, $t_P$) by `hops`[-1].`right`:
    ensure *State* = WAITING FOR FUNDED
    store `host`$_P$ // we will talk directly to `host`$_P$
    `layer` $\leftarrow$ `funder_layer`
    *State* $\leftarrow$ WAITING FOR COMM SIG
    reply (FUND ACK)

On output (CHECK KEYS, $(pk_1, pk_2)$) by `hops`[-1].`right`:
    ensure *State* = WAITING FOR CHECK KEYS
    ensure $pk_1 = pk_{A,F} \wedge pk_2 = pk_{B,F}$
    *State* $\leftarrow$ WAITING FOR FUDNED
    reply (KEYS OK)

**Fig. 14.**

**Process** LN – On (OPEN, $c$, `hops`, `fundee`):

// `fundee` is *Bob*
ensure we run the code of *Alice* // activated party is the funder
**if** `hops` $= \mathcal{G}_{\text{Ledger}}$ **then** // opening base channel
    ensure *State* = TOPPED UP
    ensure $c = c_{A,\text{chain}}$
**else** // opening virtual channel
    ensure len(`hops`) $\geq 2$ // cannot open a virtual over 1 channel
**end if**
LN.EXCHANGEOPENKEYS()
LN.PREPAREBASE()
LN.EXCHANGEOPENSIGS()
**if** `hops` $= \mathcal{G}_{\text{Ledger}}$ **then**
    LN.COMMITBASE()
**end if**
input (READ) to $\mathcal{G}_{\text{Ledger}}$ and assign output to $\Sigma$
`last_poll` $\leftarrow |\Sigma|$
$c_A \leftarrow c$; $c_B \leftarrow 0$; $i \leftarrow 0$
*State* $\leftarrow$ OPEN
output (OPEN OK, $c$, `fundee`, `hops`)

**Fig. 15.**

**Process** LN.UPDATEFORVIRTUAL()

$C_{\bar{P},i+1} \leftarrow C_{\bar{P},i}$ with $pk'_{P,F}$ and $pk'_{\bar{P},F}$ instead of $pk_{P,F}$ and $pk_{\bar{P},F}$ respectively, reducing the input and $P$'s output by $c_{\text{virt}}$
$\text{sig}_{P,C,i+1} \leftarrow \text{SIGN}(C_{\bar{P},i+1})$ // kept by $\bar{P}$
send (UPDATE FORWARD, $\text{sig}_{P,C,i+1}$) to $\bar{P}$
// $P$ refers to payer and $\bar{P}$ to payee both in local and remote code
$C_{\bar{P},i+1} \leftarrow C_{\bar{P},i}$ with $pk'_{P,F}$ and $pk'_{\bar{P},F}$ instead of $pk_{P,F}$ and $pk_{\bar{P},F}$ respectively, reducing the input and $P$'s output by $c_{\text{virt}}$
ensure VERIFY$(C_{\bar{P},i+1}, \text{sig}_{P,C,i+1}, pk'_{P,F})$ = True
$C_{P,i+1} \leftarrow C_{P,i}$ with $pk'_{\bar{P},F}$ and $pk'_{P,F}$ instead of $pk_{\bar{P},F}$ and $pk_{P,F}$ respectively, reducing the input and $P$'s output by $c_{\text{virt}}$
$\text{sig}_{\bar{P},C,i+1} \leftarrow \text{SIGN}(C_{P,i+1}, sk'_{\bar{P},F})$ // kept by $P$
reply (UPDATE BACK, $\text{sig}_{\bar{P},C,i+1}$)
$C_{P,i+1} \leftarrow C_{P,i}$ with $pk'_{\bar{P},F}$ and $pk'_{P,F}$ instead of $pk_{\bar{P},F}$ and $pk_{P,F}$ respectively, reducing the input and $P$'s output by $c_{\text{virt}}$
ensure VERIFY$(C_{P,i+1}, \text{sig}_{\bar{P},C,i+1}, pk'_{\bar{P},F})$ = True

**Fig. 16.**

**Process** LN – virtualise start and end

On input (FUND ME, $c_{\text{virt}}$, `fundee`, `hops`, $pk_{A,V}$, $pk_{B,V}$) by `funder`:
    ensure $State$ = OPEN
    ensure $c_P - \texttt{locked}_P \geq c_{\text{virt}}$
    $State \leftarrow$ VIRTUALISING
    $(sk'_{P,F}, pk'_{P,F}) \leftarrow$ KEYGEN()
    define new VIRT ITI $\texttt{host}'_P$
    send (VIRTUALISING, $\texttt{host}'_P$, $pk'_{P,F}$, `hops`, `fundee`, $c_{\text{virt}}$, 2, len(`hops`)) to $\bar{P}$ and expect reply (VIRTUALISING ACK, $\texttt{host}'_{\bar{P}}$, $pk'_{\bar{P},F}$)
    ensure $pk'_{\bar{P},F}$ is different from $pk_{\bar{P},F}$ and all older $\bar{P}$'s funding public keys
    LN.UPDATEFORVIRTUAL()
    $State \leftarrow$ WAITING FOR REVOCATION
    input (HOST ME, `funder`, `fundee`, $\texttt{host}'_{\bar{P}}$, $\texttt{host}_P$, $c_P$, $c_{\bar{P}}$, $c_{\text{virt}}$, $pk_{A,V}$, $pk_{B,V}$, $(sk'_{P,F}, pk'_{P,F})$, $(sk_{P,F}, pk_{P,F})$, $pk_{\bar{P},F}$, $pk'_{\bar{P},F}$, $pk_{P,\text{out}}$, len(`hops`)) to $\texttt{host}'_P$

On output (HOSTS READY, $t_P$) by $\texttt{host}_P$: // $\texttt{host}_P$ is the new host, renamed in Fig. 9, l. 12
    ensure $State$ = WAITING FOR HOSTS READY
    $State \leftarrow$ OPEN
    move $pk_{P,F}$, $pk_{\bar{P},F}$ to list of old funding keys
    $(sk_{P,F}, pk_{P,F}) \leftarrow (sk'_{P,F}, pk'_{P,F})$; $pk_{\bar{P},F} \leftarrow pk'_{\bar{P},F}$
    **if** len(`hops`) = 1 **then** // we are the last hop
        output (FUNDED, $\texttt{host}_P$, `layer`, $t_P$) to `fundee` and expect reply (FUND ACK)
    **else if** we have received input FUND ME just before we moved to the VIRTUALISING state **then** // we are the first hop
        $c_P \leftarrow c_P - c_{\text{virt}}$
        output (FUNDED, $\texttt{host}_P$, `layer`, $t_P$) to `funder` // do not expect reply by `funder`
    **end if**
    reply (HOST ACK)

**Fig. 17.**

**Process** LN – virtualise hops

On (VIRTUALISING, $\texttt{host}'_{\bar{P}}$, $pk'_{\bar{P},F}$, $\texttt{hops}$, $\texttt{fundee}$, $c_{\text{virt}}$, $i$, $n$) by $\bar{P}$:

  ensure $State = $ OPEN

  ensure $c_{\bar{P}} - \texttt{locked}_{\bar{P}} \geq c$; $1 \leq i \leq n$

  ensure $pk'_{\bar{P},F}$ is different from $pk_{\bar{P},F}$ and all older $\bar{P}$'s funding public keys

  $State \leftarrow$ VIRTUALISING

  $\texttt{locked}_{\bar{P}} \leftarrow \texttt{locked}_{\bar{P}} + c$ // if $\bar{P}$ is hosting the $\texttt{funder}$, $\bar{P}$ will transfer $c_{\text{virt}}$ coins instead of locking them, but the end result is the same

  $(sk'_{P,F}, pk'_{P,F}) \leftarrow$ KEYGEN()

  **if** len($\texttt{hops}$) $> 1$ **then** // we are not the last hop

    define new VIRT ITI $\texttt{host}'_P$

    input (VIRTUALISING, $\texttt{host}'_P$, $(sk'_{P,F}, pk'_{P,F})$, $pk'_{\bar{P},F}$, $pk_{P,\text{out}}$, $\texttt{hops}[1:]$, $\texttt{fundee}$, $c_{\text{virt}}$, $c_{\bar{P}}$, $c_P$, $i$, $n$) to $\texttt{hops}[1].\texttt{left}$ and expect reply (VIRTUALISING ACK, $\texttt{host\_sibling}$, $pk_{\text{sib},\bar{P},F}$)

    input (INIT, $\texttt{host}_P$, $\texttt{host}'_{\bar{P}}$, $\texttt{host\_sibling}$, $(sk'_{P,F}, pk'_{P,F})$, $pk'_{\bar{P},F}$, $pk_{\text{sib},\bar{P},F}$, $(sk_{P,F}, pk_{P,F})$, $pk_{\bar{P},F}$, $pk_{P,\text{out}}$, $c_P$, $c_{\bar{P}}$, $c_{\text{virt}}$, $i$, $t_P$, "left", $n$) to $\texttt{host}'_P$ and expect reply (HOST INIT OK)

  **else** // we are the last hop

    input (INIT, $\texttt{host}_P$, $\texttt{host}'_{\bar{P}}$, $\texttt{fundee}$=$\texttt{fundee}$, $(sk'_{P,F}, pk'_{P,F})$, $pk'_{\bar{P},F}$, $(sk_{P,F}, pk_{P,F})$, $pk_{\bar{P},F}$, $pk_{P,\text{out}}$, $c_P$, $c_{\bar{P}}$, $c_{\text{virt}}$, $t_P$, $i$, "left", $n$) to new VIRT ITI $\texttt{host}'_P$ and expect reply (HOST INIT OK)

  **end if**

  $State \leftarrow$ WAITING FOR REVOCATION

  send (VIRTUALISING ACK, $\texttt{host}'_P$, $pk'_{P,F}$) to $\bar{P}$

On input (VIRTUALISING, $\texttt{host\_sibling}$, $(sk'_{P,F}, pk'_{P,F})$, $pk_{\text{sib},\bar{P},F}$, $pk_{\text{sib,out}}$, $\texttt{hops}$, $\texttt{fundee}$, $c_{\text{virt}}$, $c_{\text{sib,rem}}$, $c_{\text{sib}}$, $i$, $n$) by $\texttt{sibling}$:

  ensure $State = $ OPEN

  ensure $c_P - \texttt{locked}_P \geq c$

  ensure $c_{\text{sib,rem}} \geq c_P \wedge c_{\bar{P}} \geq c_{\text{sib}}$ // avoid value loss by griefing attack: one counterparty closes with old version, the other stays idle forever

  $State \leftarrow$ VIRTUALISING

  $\texttt{locked}_P \leftarrow \texttt{locked}_P + c$

  define new VIRT ITI $\texttt{host}'_P$

  send (VIRTUALISING, $\texttt{host}'_P$, $pk'_{P,F}$, $\texttt{hops}$, $\texttt{fundee}$, $c_{\text{virt}}$, $i+1$, $n$) to $\texttt{hops}[0].\texttt{right}$ and expect reply (VIRTUALISING ACK, $\texttt{host}'_{\bar{P}}$, $pk'_{\bar{P},F}$)

  ensure $pk'_{\bar{P},F}$ is different from $pk_{\bar{P},F}$ and all older $\bar{P}$'s funding public keys

  LN.UPDATEFORVIRTUAL()

  input (INIT, $\texttt{host}_P$, $\texttt{host}'_{\bar{P}}$, $\texttt{host\_sibling}$, $(sk'_{P,F}, pk'_{P,F})$, $pk'_{\bar{P},F}$, $pk_{\text{sib},\bar{P},F}$, $(sk_{P,F}, pk_{P,F})$, $pk_{\bar{P},F}$, $pk_{\text{sib,out}}$, $c_P$, $c_{\bar{P}}$, $c_{\text{virt}}$, $i$, "right", $n$) to $\texttt{host}'_P$ and expect reply (HOST INIT OK)

  $State \leftarrow$ WAITING FOR REVOCATION

  output (VIRTUALISING ACK, $\texttt{host}'_P$, $pk'_{\bar{P},F}$) to $\texttt{sibling}$

**Fig. 18.**

**Process** $\textsc{ln.signaturesRoundTrip}()$

$C_{\bar{P},i+1} \leftarrow C_{\bar{P},i}$ with $x$ coins moved from $P$'s to $\bar{P}$'s output
$\mathrm{sig}_{P,C,i+1} \leftarrow \textsc{sign}(C_{\bar{P},i+1}, sk_{P,F})$ // kept by $\bar{P}$
$State \leftarrow$ WAITING FOR COMMITMENT SIGNED
send (PAY, $x$, $\mathrm{sig}_{P,C,i+1}$) to $\bar{P}$
// $P$ refers to payer and $\bar{P}$ to payee both in local and remote code
ensure $State =$ WAITING TO GET PAID $\wedge\, x = y$
$C_{\bar{P},i+1} \leftarrow C_{\bar{P},i}$ with $x$ coins moved from $P$'s to $\bar{P}$'s output
ensure $\textsc{verify}(C_{\bar{P},i+1}, \mathrm{sig}_{P,C,i+1}, pk_{P,F}) =$ True
$C_{P,i+1} \leftarrow C_{P,i}$ with $x$ coins moved from $P$'s to $\bar{P}$'s output
$\mathrm{sig}_{\bar{P},C,i+1} \leftarrow \textsc{sign}(C_{P,i+1}, sk_{\bar{P},F})$ // kept by $P$
$R_{P,i} \leftarrow$ TX {input: $C_{\bar{P},i}.\mathrm{outputs}.P$, output: $(c_{\bar{P}}, pk_{P,\mathrm{out}})$}
$\mathrm{sig}_{\bar{P},R,i} \leftarrow \textsc{sign}(R_{P,i}, sk_{\bar{P},R})$
$State \leftarrow$ WAITING FOR PAY REVOCATION
reply (COMMITMENT SIGNED, $\mathrm{sig}_{\bar{P},C,i+1}$, $\mathrm{sig}_{\bar{P},R,i}$)
ensure $State =$ WAITING FOR COMMITMENT SIGNED
$C_{P,i+1} \leftarrow C_{P,i}$ with $x$ coins moved from $P$'s to $\bar{P}$'s output

**Fig. 19.**

**Process** LN.REVOCATIONSTRIP()

ensure VERIFY($C_{P,i+1}$, $\mathrm{sig}_{\bar{P},C,i+1}$, $pk_{\bar{P},F}$) = True
$R_{P,i} \leftarrow$ TX {input: $C_{\bar{P},i}$.outputs.$\bar{P}$, output: ($c_{\bar{P}}$, $pk_{P,\mathrm{out}}$)}
ensure VERIFY($R_{P,i}$, $\mathrm{sig}_{\bar{P},R,i}$, $pk_{\bar{P},R}$) = True
$R_{\bar{P},i} \leftarrow$ TX {input: $C_{P,i}$.outputs.$P$, output: ($c_P$, $pk_{\bar{P},\mathrm{out}}$)}
$\mathrm{sig}_{P,R,i} \leftarrow$ SIGN($R_{\bar{P},i}$, $sk_{P,R}$)
add $x$ to `paid_out`
$c_P \leftarrow c_P - x; c_{\bar{P}} \leftarrow c_{\bar{P}} + x;\ i \leftarrow i + 1$
*State* $\leftarrow$ OPEN
**if** $\texttt{host}_P \neq \mathcal{G}_{\mathrm{Ledger}} \wedge$ we have a `host_sibling` **then** // we are intermediary channel
    input (NEW BALANCE, $c_P$, $c_{\bar{P}}$) to $\texttt{host}_P$
    relay message as input to `sibling` // run by VIRT
    relay message as output to `guest` // run by VIRT
    store new sibling balance and reply (NEW BALANCE OK)
    output (NEW BALANCE OK) to `sibling` // run by VIRT
    output (NEW BALANCE OK) to `guest` // run by VIRT
**end if**
send (REVOKE AND ACK, $\mathrm{sig}_{P,R,i}$) to $\bar{P}$
ensure *State* = WAITING FOR PAY REVOCATION
$R_{\bar{P},i} \leftarrow$ TX {input: $C_{P,i}$.outputs.$\bar{P}$, output: ($c_P$, $pk_{\bar{P},\mathrm{out}}$)}
ensure VERIFY($R_{\bar{P},i}$, $\mathrm{sig}_{P,R,i}$, $pk_{P,R}$) = True
add $x$ to `paid_in`
$c_P \leftarrow c_P - x; c_{\bar{P}} \leftarrow c_{\bar{P}} + x;\ i \leftarrow i + 1$
*State* $\leftarrow$ OPEN
**if** $\texttt{host}_P \neq \mathcal{G}_{\mathrm{Ledger}} \wedge \bar{P}$ has a `host_sibling` **then** // we are intermediary channel
    input (NEW BALANCE, $c_{\bar{P}}$, $c_P$) to $\texttt{host}_{\bar{P}}$
    relay message as input to `sibling` // run by VIRT
    relay message as output to `guest` // run by VIRT
    store new sibling balance and reply (NEW BALANCE OK)
    output (NEW BALANCE OK) to `sibling` // run by VIRT
    output (NEW BALANCE OK) to `guest` // run by VIRT
**end if**

**Fig. 20.**

**Process** LN – On (PAY, $x$):

ensure $State = \text{OPEN} \wedge c_P \geq x$
**if** $\texttt{host}_P \neq \mathcal{G}_{\text{Ledger}} \wedge P$ has a `host_sibling` **then** // we are intermediary channel
    ensure $c_{\text{sib,rem}} \geq c_P - x \wedge c_{\bar{P}} + x \geq c_{\text{sib}}$ // avoid value loss by griefing attack: one counterparty closes with old version, the other stays idle forever
**end if**
LN.SIGNATURESROUNDTRIP()
LN.REVOCATIONSTRIP()
// No output is given to the caller, this is intentional

**Fig. 21.**

**Process** LN – On (GET PAID, $y$):

ensure $State = \text{OPEN} \wedge c_{\bar{P}} \geq y$
**if** $\texttt{host}_P \neq \mathcal{G}_{\text{Ledger}} \wedge P$ has a `host_sibling` **then** // we are intermediary channel
    ensure $c_P + y \leq c_{\text{sib,rem}} \wedge c_{\text{sib}} \leq c_{\bar{P}} - y$ // avoid value loss by griefing attack
**end if**
store $y$
$State \leftarrow$ WAITING TO GET PAID

**Fig. 22.**

**Process** LN – On (CHECK FOR LATERAL CLOSE):

**if** $\texttt{host}_P \neq \mathcal{G}_{\text{Ledger}}$ **then**
    input (CHECK FOR LATERAL CLOSE) to $\texttt{host}_P$
**end if**

**Fig. 23.**

**Process** LN – On (CHECK CHAIN FOR CLOSED):

ensure $\mathit{State} \notin \{\bot, \text{INIT}, \text{TOPPED UP}\}$ // channel open
// even virtual channels check $\mathcal{G}_{\text{Ledger}}$ directly. This is intentional
input (READ) to $\mathcal{G}_{\text{Ledger}}$ and assign reply to $\Sigma$
`last_poll` $\leftarrow |\Sigma|$
**if** $\exists 0 \leq j < i : C_{\bar{P},j} \in \Sigma$ **then** // counterparty has closed maliciously
    $\mathit{State} \leftarrow$ CLOSING
    LN.SUBMITANDCHECKREVOCATION$(j)$
    $\mathit{State} \leftarrow$ CLOSED
    output (CLOSED)
**else if** $C_{P,j} \in \Sigma \wedge C_{\bar{P},j} \in \Sigma$ **then**
    $\mathit{State} \leftarrow$ CLOSED
    output (CLOSED)
**end if**

**Fig. 24.**

**Process** LN.SUBMITANDCHECKREVOCATION$(j)$

$\text{sig}_{P,R,j} \leftarrow \text{SIGN}(R_{P,j}, sk_{P,R})$
input (SUBMIT, $(R_{P,j}$, $\text{sig}_{P,R,j}$, $\text{sig}_{\bar{P},R,j}))$ to $\mathcal{G}_{\text{Ledger}}$
**while** $\nexists R_{P,j} \in \Sigma$ **do**
    wait for input (CHECK REVOCATION) // ignore other messages
    input (READ) to $\mathcal{G}_{\text{Ledger}}$ and assign output to $\Sigma$
**end while**
$c_P \leftarrow c_P + c_{\bar{P}}$
**if** $\texttt{host}_P \neq \mathcal{G}_{\text{Ledger}}$ **then**
    input (USED REVOCATION) to $\texttt{host}_P$
**end if**

**Fig. 25.**

**Process** LN – On (CLOSE):

ensure *State* $\notin \{\bot$, INIT, TOPPED UP, CLOSED, BASE PUNISHED$\}$ // channel open
**if** $\texttt{host}_P \neq \mathcal{G}_{\text{Ledger}}$ **then** // we have a virtual channel
  *State* $\leftarrow$ HOST CLOSING
  input (CLOSE) to $\texttt{host}_P$ and keep relaying inputs (CHECK IF CLOSING) to $\texttt{host}_P$ until receiving output (CLOSED) by $\texttt{host}_P$
  $\texttt{host}_P \leftarrow \mathcal{G}_{\text{Ledger}}$
**end if**
*State* $\leftarrow$ CLOSING
input (READ) to $\mathcal{G}_{\text{Ledger}}$ and assign output to $\Sigma$
**if** $C_{\bar{P},i} \in \Sigma$ **then** // counterparty has closed honestly
  no-op // do nothing
**else if** $\exists 0 \leq j < i : C_{\bar{P},j} \in \Sigma$ **then** // counterparty has closed maliciously
  LN.SUBMITANDCHECKREVOCATION($j$)
**else** // counterparty is idle
  **while** $\nexists$ unspent $\texttt{output} \in \Sigma$ that $C_{P,i}$ can spend **do** // possibly due to an active timelock
    wait for input (CHECK VIRTUAL) // ignore other messages
    input (READ) to $\mathcal{G}_{\text{Ledger}}$ and assign output to $\Sigma$
  **end while**
  $\text{sig}'_{P,C,i} \leftarrow \text{SIGN}(C_{P,i}, sk_{P,F})$
  input (SUBMIT, $(C_{P,i}, \text{sig}_{P,C,i}, \text{sig}'_{P,C,i})$) to $\mathcal{G}_{\text{Ledger}}$
**end if**

**Fig. 26.**

**Process** LN – On output (ENABLER USED REVOCATION) by $\texttt{host}_P$:

*State* $\leftarrow$ BASE PUNISHED

**Fig. 27.**

**Process** VIRT

On every activation, before handling the message:
  **if** $\texttt{last\_poll} \neq \bot$ **then** // virtual layer is ready
    input (READ) to $\mathcal{G}_{\text{Ledger}}$ and assign ouput to $\Sigma$
    **if** $\texttt{last\_poll} + p < |\Sigma|$ **then**
      **for** $P \in \{\texttt{guest}, \texttt{funder}, \texttt{fundee}\}$ **do** // at most 1 of funder, fundee is defined
        ensure $P$.NEGLIGENT() returns (OK)
      **end for**
    **end if**

**end if**

// guest is trusted to give sane inputs, therefore a state machine and input verification are redundant

On input (INIT, $\texttt{host}_P$, $\bar{P}$, sibling, fundee, ($sk_{\text{loc,fund,new}}$, $pk_{\text{loc,fund,new}}$), $pk_{\text{rem,fund,new}}$, $pk_{\text{sib,rem,fund,new}}$, ($sk_{\text{loc,fund,old}}$, $pk_{\text{loc,fund,old}}$), $pk_{\text{rem,fund,old}}$, $pk_{\text{loc,out}}$, $c_P$, $c_{\bar{P}}$, $c_{\text{virt}}$, $t_P$, $i$, side, $n$) by guest:

- ensure $1 < i \leq n$ // host_funder ($i = 1$) is initialised with HOST ME
- ensure side $\in$ {"left", "right"}
- store message contents and guest // sibling, $pk_{\text{sib},\bar{P},F}$ are missing for endpoints, fundee is present only in last node
- $(sk_{i,\text{fund,new}}, pk_{i,\text{fund,new}}) \leftarrow (sk_{\text{loc,fund,new}}, pk_{\text{loc,fund,new}})$
- $pk_{\texttt{myRem},\text{fund,new}} \leftarrow pk_{\text{rem,fund,new}}$
- **if** $i < n$ **then** // we are not last hop
  - $pk_{\texttt{sibRem},\text{fund,new}} \leftarrow pk_{\text{sib,rem,fund,new}}$
- **end if**
- **if** side = "left" **then**
  - side$'$ $\leftarrow$ "right"; myRem $\leftarrow i - 1$; sibRem $\leftarrow i + 1$
- **else** // side = "right"
  - side$'$ $\leftarrow$ "left"; myRem $\leftarrow i + 1$; sibRem $\leftarrow i - 1$
- **end if**
- $(sk_{i,\texttt{side},\text{fund,old}}, pk_{i,\texttt{side},\text{fund,old}}) \leftarrow (sk_{\text{loc,fund,old}}, pk_{\text{loc,fund,old}})$
- $pk_{\text{myRem},\texttt{side}',\text{fund,old}} \leftarrow pk_{\text{rem,fund,old}}$
- **if** side = "left" **then**
  - $pk_{i,\text{out}} \leftarrow pk_{\text{loc,out}}$
- **end if** // otherwise sibling will send $pk_{i,\text{out}}$ in KEYS AND COINS FORWARD
- $(c_{i,\texttt{side}}, c_{\texttt{myRem},\texttt{side}'}, t_{i,\texttt{side}}) \leftarrow (c_P, c_{\bar{P}}, t_P)$
- last_poll $\leftarrow \bot$
- **if** side = "left" $\wedge\ i \neq n$ **then**
  - $(sk_{i,j,k}, pk_{i,j,k})_{j \in \{2,\ldots,n-1\}, k \in [n] \setminus \{j\}} \leftarrow \text{KEYGEN}()^{(n-2)(n-1)}$
- **end if**
- output (HOST INIT OK) to guest

On input (HOST ME, funder, fundee, $\bar{P}$, $\texttt{host}_P$, $c_P$, $c_{\bar{P}}$, $c_{\text{virt}}$, $pk_{\text{left,virt}}$, $pk_{\text{right,virt}}$, ($sk_{1,\text{fund,new}}$, $pk_{1,\text{fund,new}}$), ($sk_{1,\text{right,fund,old}}$, $pk_{1,\text{right,fund,old}}$), $pk_{2,\text{left,fund,old}}$, $pk_{2,\text{left,fund,new}}$, $pk_{1,\text{out}}$, $n$) by guest:

- last_poll $\leftarrow \bot$
- $i \leftarrow 1$
- $c_{1,\text{right}} \leftarrow c_P$; $c_{2,\text{left}} \leftarrow c_{\bar{P}}$
- $(sk_{1,j,k}, pk_{1,j,k})_{j \in \{2,\ldots,n-1\}, k \in [n] \setminus \{j\}} \leftarrow \text{KEYGEN}()^{(n-2)(n-1)}$
- ensure VIRT.CIRCULATEKEYSCOINSTIMES() returns (OK)
- ensure VIRT.CIRCULATEVIRTUALSIGS() returns (OK)
- ensure VIRT.CIRCULATEFUNDINGSIGS() returns (OK)
- ensure VIRT.CIRCULATEREVOCATIONS() returns (OK)
- output (HOSTS READY, $p + \sum_{j=2}^{n-1}(s - 1 + t_j)$) to guest // $p$ is every how many blocks we have to check the chain

**Fig. 28.**

**Process** VIRT.CIRCULATEKEYSCOINSTIMES(`left_data`):

**if** `left_data` is given as argument **then** // we are not `host_funder`
  parse `left_data` as $((pk_{j,\text{fund,new}})_{j\in[i-1]}, (pk_{j,\text{left,fund,old}})_{j\in\{2,\ldots,i-1\}}, (pk_{j,\text{right,fund,old}})_{j\in[i-1]}, (pk_{j,\text{out}})_{j\in[i-1]}, (c_{j,\text{left}})_{j\in\{2,\ldots,i-1\}}, (c_{j,\text{right}})_{j\in[i-1]}, (t_j)_{j\in[i-1]}, pk_{\text{left,virt}}, pk_{\text{right,virt}}, (pk_{h,j,k})_{h\in[i-1],j\in\{2,\ldots,n-1\},k\in[n]\setminus\{j\}})$
  **if** we have a **`sibling`** **then** // we are not `host_fundee`
    input (KEYS AND COINS FORWARD, (`left_data`, $(sk_{i,\text{left,fund,old}}, pk_{i,\text{left,fund,old}})$, $pk_{i,\text{out}}$, $c_{i,\text{left}}$, $t_{i,\text{left}}$, $(sk_{i,j,k}, pk_{i,j,k})_{j\in\{2,\ldots,n-1\},k\in[n]\setminus\{j\}}$) to `sibling`
    store input as `left_data` and parse it as $((pk_{j,\text{fund,new}})_{j\in[i-1]}, (pk_{j,\text{left,fund,old}})_{j\in\{2,\ldots,i\}}, (pk_{j,\text{right,fund,old}})_{j\in[i-1]}, (pk_{j,\text{out}})_{j\in[i]}, (c_{j,\text{left}})_{j\in\{2,\ldots,i\}}, (c_{j,\text{right}})_{j\in[i-1]}, (t_j)_{j\in[i-1]}, sk_{i,\text{left,fund,old}}, t_{i,\text{left}}, pk_{\text{left,virt}}, pk_{\text{right,virt}}, (pk_{h,j,k})_{h\in[i],j\in\{2,\ldots,n-1\},k\in[n]\setminus\{j\}}, (sk_{i,j,k})_{j\in\{2,\ldots,n-1\},k\in[n]\setminus\{j\}}$
    $t_i \leftarrow \max(t_{i,\text{left}}, t_{i,\text{right}})$
    replace $t_{i,\text{left}}$ in `left_data` with $t_i$
    remove $sk_{i,\text{left,fund,old}}$ and $(sk_{i,j,k})_{j\in\{2,\ldots,n-1\},k\in[n]\setminus\{j\}}$ from `left_data`
    call VIRT.CIRCULATEKEYSCOINSTIMES(`left_data`) of $\bar{P}$ and assign returned value to `right_data`
    parse `right_data` as $((pk_{j,\text{fund,new}})_{j\in\{i+1,\ldots,n\}}, (pk_{j,\text{left,fund,old}})_{j\in\{i+1,\ldots,n\}}, (pk_{j,\text{right,fund,old}})_{j\in\{i+1,\ldots,n-1\}}, (pk_{j,\text{out}})_{j\in\{i+1,\ldots,n\}}, (c_{j,\text{left}})_{j\in\{i+1,\ldots,n\}}, (c_{j,\text{right}})_{j\in\{i+1,\ldots,n-1\}}, (t_j)_{j\in\{i+1,\ldots,n\}}, (pk_{h,j,k})_{h\in\{i+1,\ldots,n\},j\in\{2,\ldots,n-1\},k\in[n]\setminus\{j\}}$
    output (KEYS AND COINS BACK, `right_data`, $(sk_{i,\text{right,fund,old}}, pk_{i,\text{right,fund,old}})$, $c_{i,\text{right}}$, $t_i$)
    store output as `right_data` and parse it as $((pk_{j,\text{fund,new}})_{j\in\{i+1,\ldots,n\}}, (pk_{j,\text{left,fund,old}})_{j\in\{i+1,\ldots,n\}}, (pk_{j,\text{right,fund,old}})_{j\in\{i,\ldots,n-1\}}, (pk_{j,\text{out}})_{j\in\{i+1,\ldots,n\}}, (c_{j,\text{left}})_{j\in\{i+1,\ldots,n\}}, (c_{j,\text{right}})_{j\in\{i,\ldots,n-1\}}, (t_j)_{j\in\{i,\ldots,n\}}, (pk_{h,j,k})_{h\in\{i+1,\ldots,n\},j\in\{2,\ldots,n-1\},k\in[n]\setminus\{j\}}, sk_{i,\text{right,fund,old}})$
    remove $sk_{i,\text{right,fund,old}}$ from `right_data`
    **return** (`right_data`, $pk_{i,\text{fund,new}}$, $pk_{i,\text{left,fund,old}}$, $pk_{i,\text{out}}$, $c_{i,\text{left}}$)
  **else** // we are `host_fundee`
    output (CHECK KEYS, $(pk_{\text{left,virt}}, pk_{\text{right,virt}})$) to `fundee` and expect reply (KEYS OK)
    **return** $(pk_{n,\text{fund,new}}, pk_{n,\text{left,fund,old}}, pk_{n,\text{out}}, c_{n,\text{left}}, t_n)$
  **end if**
**else** // we are `host_funder`
  call VIRT.CIRCULATEKEYSCOINSTIMES($pk_{1,\text{fund,new}}$, $pk_{1,\text{right,fund,old}}$, $pk_{1,\text{out}}$, $c_{1,\text{right}}$, $t_1$, $pk_{\text{left,virt}}$, $pk_{\text{right,virt}}$, $(pk_{1,j,k})_{j\in\{2,\ldots,n-1\},k\in[n]\setminus\{j\}}$) of $\bar{P}$ and assign returned value to `right_data`
  parse `right_data` as $((pk_{j,\text{fund,new}})_{j\in\{2,\ldots,n\}}, (pk_{j,\text{left,fund,old}})_{j\in\{2,\ldots,n\}}, (pk_{j,\text{right,fund,old}})_{j\in\{2,\ldots,n-1\}}, (pk_{j,\text{out}})_{j\in\{2,\ldots,n\}}, (c_{j,\text{left}})_{j\in\{2,\ldots,n\}}, (c_{j,\text{right}})_{j\in\{2,\ldots,n-1\}}, (t_j)_{j\in\{2,\ldots,n\}}, (pk_{h,j,k})_{h\in\{2,\ldots,n\},j\in\{2,\ldots,n-1\},k\in[n]\setminus\{j\}})$
  **return** (OK)
**end if**

**Fig. 29.**

**Process** VIRT

GETMIDTXS($i$, $n$, $c_{\text{virt}}$, $c_{\text{rem,left}}$, $c_{\text{loc,left}}$, $c_{\text{loc,right}}$, $c_{\text{rem,right}}$, $pk_{\text{rem,left,fund,old}}$, $pk_{\text{loc,left,fund,old}}$, $pk_{\text{loc,right,fund,old}}$, $pk_{\text{rem,right,fund,old}}$, $pk_{\text{rem,left,fund,new}}$, $pk_{\text{loc,left,fund,new}}$, $pk_{\text{loc,right,fund,new}}$, $pk_{\text{rem,right,fund,new}}$, $pk_{\text{left,virt}}$, $pk_{\text{right,virt}}$, $pk_{\text{loc,out}}$, $(pk_{p,j,k})_{p \in [n], j \in [n-1] \setminus \{1\}, k \in [n-1] \setminus \{1,j\}}$, $(pk_{p,2,1})_{p \in [n]}$, $(pk_{p,n-1,n})_{p \in [n]}$, $(t_j)_{j \in [n-1] \setminus \{1\}}$):

    ensure $1 < i < n$
    ensure $c_{\text{rem,left}} \geq c_{\text{virt}} \land c_{\text{loc,left}} \geq c_{\text{virt}}$ // left parties fund virtual channel
    ensure $c_{\text{rem,left}} \geq c_{\text{loc,right}} \land c_{\text{rem,right}} \geq c_{\text{loc,left}}$ // avoid griefing attack
    $c_{\text{left}} \leftarrow c_{\text{rem,left}} + c_{\text{loc,left}}$; $c_{\text{right}} \leftarrow c_{\text{loc,right}} + c_{\text{rem,right}}$
    `left_old_fund` $\leftarrow 2/\{pk_{\text{rem,left,fund,old}}, pk_{\text{loc,left,fund,old}}\}$
    `right_old_fund` $\leftarrow 2/\{pk_{\text{loc,right,fund,old}}, pk_{\text{rem,right,fund,old}}\}$
    `left_new_fund` $\leftarrow 2/\{pk_{\text{rem,left,fund,new}}, pk_{\text{loc,left,fund,new}}\}$
    `right_new_fund` $\leftarrow 2/\{pk_{\text{loc,right,fund,new}}, pk_{\text{rem,right,fund,new}}\}$
    `virt_fund` $\leftarrow 2/\{pk_{\text{left,virt}}, pk_{\text{right,virt}}\}$
    **for all** $j \in [n-1] \setminus \{1\}, k \in [n-1] \setminus \{1, j\}$ **do**
        $all_{j,k} \leftarrow n/\{pk_{1,j,k}, \ldots, pk_{n,j,k}\} \land$ "$k$"
    **end for**
    **if** $i = 2$ **then**
        $all_{2,1} \leftarrow n/\{pk_{1,2,1}, \ldots, pk_{n,2,1}\} \land$ "1"
    **end if**
    **if** $i = n - 1$ **then**
        $all_{n-1,n} \leftarrow n/\{pk_{1,n-1,n}, \ldots, pk_{n,n-1,n}\} \land$ "$n$"
    **end if**
    // After funding is complete, $A_j$ has the signature of all other parties for all $all_{j,k}$ inputs, but other parties do not have $A_j$'s signature for this input, therefore only $A_j$ can publish it.
    // $\text{TX}_{i,j,k} :=$ $i$-th move, $j, k$ input interval start and end. $j, k$ unneeded for $i = 1$, $k$ unneeded for $i = 2$.
    $\text{TX}_1 \leftarrow$ TX:
        inputs:
            ($c_{\text{left}}$, `left_old_fund`),
            ($c_{\text{right}}$, `right_old_fund`)
        outputs:
            ($c_{\text{left}} - c_{\text{virt}}$, `left_new_fund`),
            ($c_{\text{right}} - c_{\text{virt}}$, `right_new_fund`),
            ($c_{\text{virt}}$, $pk_{\text{loc,out}}$),
            ($c_{\text{virt}}$,
                (if $(i - 1 > 1)$ then $all_{i-1,i}$ else False)
                $\lor$ (if $(i + 1 < n)$ then $all_{i+1,i}$ else False)
                $\lor$ (
                    if $(i - 1 = 1 \land i + 1 = n)$ then `virt_fund`

          else if $(i-1>1 \wedge i+1=n)$ then $\texttt{virt\_fund} + t_{i-1}$
          else if $(i-1=1 \wedge i+1<n)$ then $\texttt{virt\_fund} + t_{i+1}$
          else /*$i-1>1 \wedge i+1<n$*/ $\texttt{virt\_fund} + \max(t_{i-1}, t_{i+1})$
        )
      )
  **if** $i=2$ **then**
    $\mathrm{TX}_{2,1} \leftarrow \mathrm{TX}$:
      inputs:
        $(c_{\mathrm{virt}},\ all_{2,1})$,
        $(c_{\mathrm{right}},\ \texttt{right\_old\_fund})$
      outputs:
        $(c_{\mathrm{right}} - c_{\mathrm{virt}},\ \texttt{right\_new\_fund})$,
        $(c_{\mathrm{virt}},\ pk_{\mathrm{loc,out}})$,
        $(c_{\mathrm{virt}}$,
          if $(n>3)$ then $(all_{3,2} \vee (\texttt{virt\_fund} + t_3))$
          else $\texttt{virt\_fund}$
        )
  **end if**
  **if** $i=n-1$ **then**
    $\mathrm{TX}_{2,n} \leftarrow \mathrm{TX}$:
      inputs:
        $(c_{\mathrm{left}},\ \texttt{left\_old\_fund})$,
        $(c_{\mathrm{virt}},\ all_{n-1,n})$
      outputs:
        $(c_{\mathrm{left}} - c_{\mathrm{virt}},\ \texttt{left\_new\_fund})$,
        $(c_{\mathrm{virt}},\ pk_{\mathrm{loc,out}})$,
        $(c_{\mathrm{virt}}$,
          if $(n-2>1)$ then $(all_{n-2,n-1} \vee (\texttt{virt\_fund} + t_{n-2}))$
          else $\texttt{virt\_fund}$
        )
  **end if**
  **for all** $k \in \{2,\dots,i-1\}$ **do** // $i-2$ txs
    $\mathrm{TX}_{2,k} \leftarrow \mathrm{TX}$:
      inputs:
        $(c_{\mathrm{virt}},\ all_{i,k})$,
        $(c_{\mathrm{right}},\ \texttt{right\_old\_fund})$
      outputs:
        $(c_{\mathrm{right}} - c_{\mathrm{virt}},\ \texttt{right\_new\_fund})$,
        $(c_{\mathrm{virt}},\ pk_{\mathrm{loc,out}})$,
        $(c_{\mathrm{virt}}$,
          (if $(k-1>1)$ then $all_{k-1,i}$ else False)
          $\vee$ (if $(i+1<n)$ then $all_{i+1,k}$ else False)
          $\vee$ (

if $(k-1=1 \wedge i+1=n)$ then `virt_fund`
else if $(k-1>1 \wedge i+1=n)$ then `virt_fund` $+ t_{k-1}$
else if $(k-1=1 \wedge i+1<n)$ then `virt_fund` $+ t_{i+1}$
else $/* k-1>1 \wedge i+1<n */$ `virt_fund` $+ \max(t_{k-1}, t_{i+1})$
)
)
**end for**
**for all** $k \in \{i+1, \ldots, n-1\}$ **do** $//\ n-i-1$ txs
$\mathrm{TX}_{2,k} \leftarrow \mathrm{TX}$:
inputs:
$(c_{\mathrm{left}},$ `left_old_fund`$)$
$(c_{\mathrm{virt}},\ all_{i,k}),$
outputs:
$(c_{\mathrm{left}} - c_{\mathrm{virt}},$ `left_new_fund`$),$
$(c_{\mathrm{virt}},\ pk_{\mathrm{loc,out}}),$
$(c_{\mathrm{virt}},$
(if $(i-1>1)$ then $all_{i-1,k}$ else False)
$\vee$ (if $(k+1<n)$ then $all_{k+1,i}$ else False)
$\vee$ (
if $(i-1=1 \wedge k+1=n)$ then `virt_fund`
else if $(i-1>1 \wedge k+1=n)$ then `virt_fund` $+ t_{i-1}$
else if $(i-1=1 \wedge k+1<n)$ then `virt_fund` $+ t_{k+1}$
else $/* i-1>1 \wedge k+1<n */$ `virt_fund` $+ \max(t_{i-1}, t_{k+1})$
)
)
**end for**
**if** $i=2$ **then** $m \leftarrow 1$ **else** $m \leftarrow 2$
**if** $i=n-1$ **then** $l \leftarrow n$ **else** $l \leftarrow n-1$
**for all** $(k_1, k_2) \in \{m, \ldots, i-1\} \times \{i+1, \ldots, l\}$ **do** $//\ (i-m)\cdot(l-i)$ txs
$\mathrm{TX}_{3,k_1,k_2} \leftarrow \mathrm{TX}$:
inputs:
$(c_{\mathrm{virt}},\ all_{i,k_1}),$
$(c_{\mathrm{virt}},\ all_{i,k_2})$
outputs:
$(c_{\mathrm{virt}},\ pk_{\mathrm{loc,out}}),$
$(c_{\mathrm{virt}},$
(if $(k_1-1>1)$ then $all_{k_1-1,\min(k_2,n-1)}$ else False)
$\vee$ (if $(k_2+1<n)$ then $all_{k_2+1,\max(k_1,2)}$ else False)
$\vee$ (
if $(k_1-1 \leq 1 \wedge k_2+1 \geq n)$ then `virt_fund`
else if $(k_1-1>1 \wedge k_2+1 \geq n)$ then `virt_fund` $+ t_{k_1-1}$
else if $(k_1-1 \leq 1 \wedge k_2+1<n)$ then `virt_fund` $+ t_{k_2+1}$
else $/* k_1-1>1 \wedge k_2+1<n */$

$\texttt{virt\_fund} + \max(t_{k_1-1}, t_{k_2+1})$
)
)
**end for**
**return** (
$\mathrm{TX}_1$,
$(\mathrm{TX}_{2,k})_{k \in \{m,\dots,l\} \setminus \{i\}}$,
$(\mathrm{TX}_{3,k_1,k_2})_{(k_1,k_2) \in \{m,\dots,i-1\} \times \{i+1,\dots,l\}}$
)

**Fig. 30.**

**Process** VIRT

// left and right refer to the two counterparties, with left being the one closer to the funder. Note difference with left/right meaning in VIRT.GETMIDTXS.
GETENDPOINTTX($i$, $n$, $c_{\text{virt}}$, $c_{\text{left}}$, $c_{\text{right}}$, $pk_{\text{left,fund,old}}$, $pk_{\text{right,fund,old}}$, $pk_{\text{left,fund,new}}$, $pk_{\text{right,fund,new}}$ $pk_{\text{left,virt}}$, $pk_{\text{right,virt}}$, $(pk_{\text{all},j})_{j \in [n]}$, $t$):
ensure $i \in \{1, n\}$
ensure $c_{\text{left}} \geq c_{\text{virt}}$ // left party funds virtual channel
$c_{\text{tot}} \leftarrow c_{\text{left}} + c_{\text{right}}$
$\texttt{old\_fund} \leftarrow 2/\{pk_{\text{left,fund,old}}, pk_{\text{right,fund,old}}\}$
$\texttt{new\_fund} \leftarrow 2/\{pk_{\text{left,fund,new}}, pk_{\text{right,fund,new}}\}$
$\texttt{virt\_fund} \leftarrow 2/\{pk_{\text{left,virt}}, pk_{\text{right,virt}}\}$
**if** $i = 1$ **then** // funder's tx
$all \leftarrow n/\{pk_{\text{all},1}, \dots, pk_{\text{all},n}\} \wedge \texttt{"1"}$
**else** // fundee's tx
$all \leftarrow n/\{pk_{\text{all},1}, \dots, pk_{\text{all},n}\} \wedge \texttt{"}n\texttt{"}$
**end if**
$\mathrm{TX}_1 \leftarrow$ TX: // endpoints only have an "initiator" tx
inputs:
$(c_{\text{tot}}, \texttt{old\_fund})$
outputs:
$(c_{\text{tot}} - c_{\text{virt}}, \texttt{new\_fund})$,
$(c_{\text{virt}}, all \vee (\texttt{virt\_fund} + t))$
**return** $\mathrm{TX}_1$

**Fig. 31.**

**Process** $\textsc{virt.siblingSigs}()$

parse input as $\text{sigs}_{\text{byLeft}}$

**if** $i = 2$ **then** $m \leftarrow 1$ **else** $m \leftarrow 2$

**if** $i = n - 1$ **then** $l \leftarrow n$ **else** $l \leftarrow n - 1$

$(\text{TX}_{i,1},\ (\text{TX}_{i,2,k})_{k \in \{m,\dots,l\} \setminus \{i\}},\ (\text{TX}_{i,3,k_1,k_2})_{(k_1,k_2) \in \{m,\dots,i-1\}\{i+1,\dots,l\}}) \leftarrow$ $\textsc{virt.getMidTXs}(i,\ n,\ c_{\text{virt}},\ c_{i-1,\text{right}},\ c_{i,\text{left}},\ c_{i,\text{right}},\ c_{i+1,\text{left}},$ $pk_{i-1,\text{right},\text{fund},\text{old}},\ pk_{i,\text{left},\text{fund},\text{old}},\ pk_{i,\text{right},\text{fund},\text{old}},\ pk_{i+1,\text{left},\text{fund},\text{old}},$ $pk_{i-1,\text{fund},\text{new}},\ pk_{i,\text{fund},\text{new}},\ pk_{i,\text{fund},\text{new}},\ pk_{i+1,\text{fund},\text{new}},\ pk_{\text{left},\text{virt}},\ pk_{\text{right},\text{virt}},$ $pk_{i,\text{out}},\ (pk_{i,j,k})_{i \in [n], j \in [n-1] \setminus \{1\}, k \in [n-1] \setminus \{1,j\}},\ (pk_{i,2,1})_{i \in [n]},\ (pk_{i,n-1,n})_{i \in [n]},$ $(t_i)_{i \in [n-1] \setminus \{1\}})$

// notation: $\text{sig}(\text{TX}, pk) :=$ sig with `ANYPREVOUT` flag such that $\textsc{verify}(\text{TX}, \text{sig}, pk) = \text{True}$

ensure that the following signatures are present in $\text{sigs}_{\text{byLeft}}$ and store them:

– // $(l - m) \cdot (i - 1)$ signatures

$\quad \forall k \in \{m, \dots, l\} \setminus \{i\}, \forall j \in [i - 1] :$

$\qquad \text{sig}(\text{TX}_{i,2,k}, pk_{j,i,k})$

– // $2 \cdot (i - m) \cdot (l - i) \cdot (i - 1)$ signatures

$\quad \forall k_1 \in \{m, \dots, i - 1\}, \forall k_2 \in \{i + 1, \dots, l\}, \forall j \in [i - 1] :$

$\qquad \text{sig}(\text{TX}_{i,3,k_1,k_2}, pk_{j,i,k_1}),\ \text{sig}(\text{TX}_{i,3,k_1,k_2}, pk_{j,i,k_2})$

$\text{sigs}_{\text{toRight}} \leftarrow \text{sigs}_{\text{byLeft}}$

**for all** $j \in \{2, \dots, n - 1\} \setminus \{i\}$ **do**

  **if** $j = 2$ **then** $m' \leftarrow 1$ **else** $m' \leftarrow 2$

  **if** $j = n - 1$ **then** $l' \leftarrow n$ **else** $l' \leftarrow n - 1$

  $(\text{TX}_{j,1},\ (\text{TX}_{j,2,k})_{k \in \{m',\dots,l'\} \setminus \{i\}},\ (\text{TX}_{j,3,k_1,k_2})_{(k_1,k_2) \in \{m',\dots,i-1\}\{i+1,\dots,l'\}}) \leftarrow$ $\textsc{getMidTXs}(j,\ n,\ c_{\text{virt}},\ c_{j-1,\text{right}},\ c_{j,\text{left}},\ c_{j,\text{right}},\ c_{j+1,\text{left}},\ pk_{j-1,\text{right},\text{fund},\text{old}},$ $pk_{j,\text{left},\text{fund},\text{old}},\ pk_{j,\text{right},\text{fund},\text{old}},\ pk_{j+1,\text{left},\text{fund},\text{old}},\ pk_{j-1,\text{fund},\text{new}},\ pk_{j,\text{fund},\text{new}},$ $pk_{j,\text{fund},\text{new}},\ pk_{j+1,\text{fund},\text{new}},\ pk_{\text{left},\text{virt}},\ pk_{\text{right},\text{virt}},\ pk_{j,\text{out}},$ $(pk_{k,p,s})_{k \in [n], p \in [n-1] \setminus \{1\}, s \in [n-1] \setminus \{1,p\}},\ (pk_{k,2,1})_{k \in [n]},\ (pk_{k,n-1,n})_{k \in [n]},$ $(t_k)_{k \in [n-1] \setminus \{1\}})$

  **if** $j < i$ **then** $\text{sigs} \leftarrow \text{sigs}_{\text{toLeft}}$ **else** $\text{sigs} \leftarrow \text{sigs}_{\text{toRight}}$

  **for all** $k \in \{m', \dots, l'\} \setminus \{j\}$ **do**

    add $\textsc{sign}(\text{TX}_{j,2,k},\ sk_{i,j,k},$ `ANYPREVOUT`$)$ to sigs

  **end for**

  **for all** $k_1 \in \{m', \dots, j - 1\}, k_2 \in \{j + 1, \dots, l'\}$ **do**

    add $\textsc{sign}(\text{TX}_{j,3,k_1,k_2},\ sk_{i,j,k_1},$ `ANYPREVOUT`$)$ to sigs

    add $\textsc{sign}(\text{TX}_{j,3,k_1,k_2},\ sk_{i,j,k_2},$ `ANYPREVOUT`$)$ to sigs

  **end for**

**end for**

**if** $i + 1 = n$ **then** // next hop is `host_fundee`

  $\text{TX}_{n,1} \leftarrow \textsc{virt.getEndpointTX}(n,\ n,\ c_{\text{virt}},\ c_{n-1,\text{right}},\ c_{n,\text{left}},$ $pk_{n-1,\text{right},\text{fund},\text{old}},\ pk_{n,\text{left},\text{fund},\text{old}},\ pk_{n-1,\text{fund},\text{new}},\ pk_{n,\text{fund},\text{new}},\ pk_{\text{left},\text{virt}},$ $pk_{\text{right},\text{virt}},\ (pk_{j,n-1,n})_{j \in [n]},\ t_{n-1})$

**end if**

call $\bar{P}$.circulateVirtualSigs($\text{sigs}_{\text{toRight}}$) and assign returned value to $\text{sigs}_{\text{byRight}}$
ensure that the following signatures are present in $\text{sigs}_{\text{byRight}}$ and store them:
– // $(l-m)\cdot(n-i)$ signatures
    $\forall k \in \{m,\dots,l\} \setminus \{i\}, \forall j \in \{i+1,\dots,n\}$ :
        $\text{sig}(\text{TX}_{i,2,k}, pk_{j,i,k})$
– // $2\cdot(i-m)\cdot(l-i)\cdot(n-i)$ signatures
    $\forall k_1 \in \{m,\dots,i-1\}, \forall k_2 \in \{i+1,\dots,l\}, \forall j \in \{i+1,\dots,n\}$ :
        $\text{sig}(\text{TX}_{i,3,k_1,k_2}, pk_{j,i,k_1})$, $\text{sig}(\text{TX}_{i,3,k_1,k_2}, pk_{j,i,k_2})$
output (virtualSigsBack, $\text{sigs}_{\text{toLeft}}$, $\text{sigs}_{\text{byRight}}$)

**Fig. 32.**

**Process** virt.intermediarySigs()

**if** $i = 2$ **then** $m \leftarrow 1$ **else** $m \leftarrow 2$
**if** $i = n-1$ **then** $l \leftarrow n$ **else** $l \leftarrow n-1$
$(\text{TX}_{i,1}, (\text{TX}_{i,2,k})_{k\in\{m,\dots,l\}\setminus\{i\}}, (\text{TX}_{i,3,k_1,k_2})_{(k_1,k_2)\in\{m,\dots,i-1\}\{i+1,\dots,l\}}) \leftarrow$ virt.getMidTXs($i$, $n$, $c_{\text{virt}}$, $c_{i-1,\text{right}}$, $c_{i,\text{left}}$, $c_{i,\text{right}}$, $c_{i+1,\text{left}}$, $pk_{i-1,\text{right},\text{fund},\text{old}}$, $pk_{i,\text{left},\text{fund},\text{old}}$, $pk_{i,\text{right},\text{fund},\text{old}}$, $pk_{i+1,\text{left},\text{fund},\text{old}}$, $pk_{i-1,\text{fund},\text{new}}$, $pk_{i,\text{fund},\text{new}}$, $pk_{i,\text{fund},\text{new}}$, $pk_{i+1,\text{fund},\text{new}}$, $pk_{\text{left},\text{virt}}$, $pk_{\text{right},\text{virt}}$, $pk_{i,\text{out}}$, $(pk_{i,j,k})_{i\in[n],j\in[n-1]\setminus\{1\},k\in[n-1]\setminus\{1,j\}}$, $(pk_{i,2,1})_{i\in[n]}$, $(pk_{i,n-1,n})_{i\in[n]}$, $(t_i)_{i\in[n-1]\setminus\{1\}}$)
// not verifying our signatures in $\text{sigs}_{\text{byLeft}}$, our (trusted) `sibling` will do that
input (virtual sigs forward, $\text{sigs}_{\text{byLeft}}$) to `sibling`
virt.siblingSigs()
$\text{sigs}_{\text{toLeft}} \leftarrow \text{sigs}_{\text{byRight}} + \text{sigs}_{\text{toLeft}}$
**if** $i = 2$ **then** // previous hop is `host_funder`
    $\text{TX}_{1,1} \leftarrow$ virt.getEndpointTX($1$, $n$, $c_{\text{virt}}$, $c_{1,\text{right}}$, $c_{2,\text{left}}$, $pk_{1,\text{right},\text{fund},\text{old}}$, $pk_{2,\text{left},\text{fund},\text{old}}$, $pk_{1,\text{fund},\text{new}}$, $pk_{2,\text{fund},\text{new}}$, $pk_{\text{left},\text{virt}}$, $pk_{\text{right},\text{virt}}$, $(pk_{j,2,1})_{j\in[n]}$, $t_2$)
**end if**
**return** $\text{sigs}_{\text{toLeft}}$

**Fig. 33.**

**Process** virt.hostFundeeSigs()

$\text{TX}_{n,1} \leftarrow$ virt.getEndpointTX($n$, $n$, $c_{\text{virt}}$, $c_{n-1,\text{right}}$, $c_{n,\text{left}}$, $pk_{n-1,\text{right},\text{fund},\text{old}}$, $pk_{n,\text{right},\text{fund},\text{old}}$, $pk_{n-1,\text{fund},\text{new}}$, $pk_{n,\text{fund},\text{new}}$, $pk_{\text{left},\text{virt}}$, $pk_{\text{right},\text{virt}}$, $(pk_{j,n-1,n})_{j\in[n]}$, $t_{n-1}$)
**for all** $j \in [n-1] \setminus \{1\}$ **do**

**if** $j = 2$ **then** $m \leftarrow 1$ **else** $m \leftarrow 2$
**if** $j = n-1$ **then** $l \leftarrow n$ **else** $l \leftarrow n-1$
$(\mathrm{TX}_{j,1},\ (\mathrm{TX}_{j,2,k})_{k\in\{m,\dots,l\}\setminus\{j\}},\ (\mathrm{TX}_{i,3,k_1,k_2})_{(k_1,k_2)\in\{m,\dots,i-1\}\{i+1,\dots,l\}}) \leftarrow$ $\textsc{virt.getMidTXs}(j,\ n,\ c_{\mathrm{virt}},\ c_{j-1,\mathrm{right}},\ c_{j,\mathrm{left}},\ c_{j,\mathrm{right}},\ c_{j+1,\mathrm{left}},$ $pk_{j-1,\mathrm{right,fund,old}},\ pk_{j,\mathrm{left,fund,old}},\ pk_{j,\mathrm{right,fund,old}},\ pk_{j+1,\mathrm{left,fund,old}},$ $pk_{j-1,\mathrm{fund,new}},\ pk_{j,\mathrm{fund,new}},\ pk_{j,\mathrm{fund,new}},\ pk_{j+1,\mathrm{fund,new}},\ pk_{\mathrm{left,virt}},\ pk_{\mathrm{right,virt}},$ $pk_{j,\mathrm{out}},\ (pk_{j,s,k})_{j\in[n],s\in[n-1]\setminus\{1\},k\in[n-1]\setminus\{1,s\}},\ (pk_{j,2,1})_{j\in[n]},\ (pk_{j,n-1,n})_{j\in[n]},$ $(t_j)_{j\in[n-1]\setminus\{1\}})$
$\mathrm{sigs}_{\mathrm{toLeft}} \leftarrow \emptyset$
**for all** $k \in \{m,\dots,l\}\setminus\{j\}$ **do**
add $\textsc{sign}(\mathrm{TX}_{j,2,k},\ sk_{n,j,k},\ \texttt{ANYPREVOUT})$ to $\mathrm{sigs}_{\mathrm{toLeft}}$
**end for**
**for all** $k_1 \in \{m,\dots,j-1\}, k_2 \in \{j+1,\dots,l\}$ **do**
add $\textsc{sign}(\mathrm{TX}_{j,3,k_1,k_2},\ sk_{n,j,k_1},\ \texttt{ANYPREVOUT})$ to $\mathrm{sigs}_{\mathrm{toLeft}}$
add $\textsc{sign}(\mathrm{TX}_{j,3,k_1,k_2},\ sk_{n,j,k_2},\ \texttt{ANYPREVOUT})$ to $\mathrm{sigs}_{\mathrm{toLeft}}$
**end for**
**end for**
**return** $\mathrm{sigs}_{\mathrm{toLeft}}$

**Fig. 34.**

**Process** $\textsc{virt.hostFunderSigs}()$

**for all** $j \in [n-1]\setminus\{1\})$ **do**
**if** $j = 2$ **then** $m \leftarrow 1$ **else** $m \leftarrow 2$
**if** $j = n-1$ **then** $l \leftarrow 1$ **else** $l \leftarrow 2$
$(\mathrm{TX}_{j,1},\ (\mathrm{TX}_{j,2,k})_{k\in\{m,\dots,l\}\setminus\{j\}},\ (\mathrm{TX}_{i,3,k_1,k_2})_{(k_1,k_2)\in\{m,\dots,i-1\}\{i+1,\dots,l\}}) \leftarrow$ $\textsc{virt.getMidTXs}(j,\ n,\ c_{\mathrm{virt}},\ c_{j-1,\mathrm{right}},\ c_{j,\mathrm{left}},\ c_{j,\mathrm{right}},\ c_{j+1,\mathrm{left}},$ $pk_{j-1,\mathrm{right,fund,old}},\ pk_{j,\mathrm{left,fund,old}},\ pk_{j,\mathrm{right,fund,old}},\ pk_{j+1,\mathrm{left,fund,old}},$ $pk_{j-1,\mathrm{fund,new}},\ pk_{j,\mathrm{fund,new}},\ pk_{j,\mathrm{fund,new}},\ pk_{j+1,\mathrm{fund,new}},\ pk_{\mathrm{left,virt}},\ pk_{\mathrm{right,virt}},$ $pk_{j,\mathrm{out}},\ (pk_{j,s,k})_{j\in[n],s\in[n-1]\setminus\{1\},k\in[n-1]\setminus\{1,s\}},\ (pk_{j,2,1})_{j\in[n]},\ (pk_{j,n-1,n})_{j\in[n]},$ $(t_j)_{j\in[n-1]\setminus\{1\}})$
$\mathrm{sigs}_{\mathrm{toRight}} \leftarrow \emptyset$
**for all** $k \in \{m,\dots,l\}\setminus\{j\}$ **do**
add $\textsc{sign}(\mathrm{TX}_{j,2,k},\ sk_{1,j,k},\ \texttt{ANYPREVOUT})$ to $\mathrm{sigs}_{\mathrm{toRight}}$
**end for**
**for all** $k_1 \in \{m,\dots,j-1\}, k_2 \in \{j+1,\dots,l\}$ **do**
add $\textsc{sign}(\mathrm{TX}_{j,3,k_1,k_2},\ sk_{1,j,k_1},\ \texttt{ANYPREVOUT})$ to $\mathrm{sigs}_{\mathrm{toRight}}$
add $\textsc{sign}(\mathrm{TX}_{j,3,k_1,k_2},\ sk_{1,j,k_2},\ \texttt{ANYPREVOUT})$ to $\mathrm{sigs}_{\mathrm{toRight}}$
**end for**
**end for**
call $\textsc{virt.circulateVirtualSigs}(\mathrm{sigs}_{\mathrm{toRight}})$ of $\bar{P}$ and assign output to $\mathrm{sigs}_{\mathrm{byRight}}$
$\mathrm{TX}_{1,1} \leftarrow \textsc{virt.getEndpointTX}(1,\ n,\ c_{\mathrm{virt}},\ c_{1,\mathrm{right}},\ c_{2,\mathrm{left}},\ pk_{1,\mathrm{right,fund,old}},$ $pk_{2,\mathrm{left,fund,old}},\ pk_{1,\mathrm{fund,new}},\ pk_{2,\mathrm{fund,new}},\ pk_{\mathrm{left,virt}},\ pk_{\mathrm{right,virt}},\ (pk_{j,2,1})_{j\in[n]},\ t_2)$

**return** (OK)

**Fig. 35.**

**Process** VIRT.CIRCULATEVIRTUALSIGS($\text{sigs}_{\text{byLeft}}$)

**if** $1 < i < n$ **then** // we are not `host_funder` nor `host_fundee`
  **return** VIRT.INTERMEDIARYSIGS()
**else if** $i = 1$ **then** // we are `host_funder`
  **return** VIRT.HOSTFUNDERSIGS()
**else if** $i = n$ **then** // we are `host_fundee`
  **return** VIRT.HOSTFUNDEESIGS()
**end if** // it is always $1 \leq i \leq n$ – c.f. Fig. 28, l. 12 and l. 39

**Fig. 36.**

**Process** VIRT.CIRCULATEFUNDINGSIGS($\text{sigs}_{\text{byLeft}}$)

**if** $1 < i < n$ **then** // we are not endpoint
  **if** $i = 2$ **then** $m \leftarrow 1$ **else** $m \leftarrow 2$
  **if** $i = n - 1$ **then** $l \leftarrow n$ **else** $l \leftarrow n - 1$
  ensure that the following signatures are present in $\text{sigs}_{\text{byLeft}}$ and store them:
  – // 1 signature
    $\text{sig}(\text{TX}_{i,1}, pk_{i-1,\text{right},\text{fund},\text{old}})$
  – // $n - 3 + \chi_{i=2} + \chi_{i=n-1}$ signatures
    $\forall k \in \{m, \dots, l\} \setminus \{i\}$
      $\text{sig}(\text{TX}_{i,2,k}, pk_{i-1,\text{right},\text{fund},\text{old}})$
  input (VIRTUAL BASE SIG FORWARD, $\text{sigs}_{\text{byLeft}}$) to `sibling`
  extract and store $\text{sig}(\text{TX}_{i,1}, pk_{i-1,\text{right},\text{fund},\text{old}})$ and $\forall k \in \{m, \dots, l\} \setminus \{i\}$ $\text{sig}(\text{TX}_{i,2,k}, pk_{i-1,\text{right},\text{fund},\text{old}})$ from $\text{sigs}_{\text{byLeft}}$ // same signatures as `sibling`
  $\text{sigs}_{\text{toRight}} \leftarrow \{\text{SIGN}(\text{TX}_{i+1,1}, sk_{i,\text{right},\text{fund},\text{old}}, \texttt{ANYPREVOUT})\}$
  **if** $i + 1 < n$ **then**
    **if** $i + 1 = n - 1$ **then** $l' \leftarrow n$ **else** $l' \leftarrow n - 1$
    **for all** $k \in \{2, \dots, l'\}$ **do**
      add $\text{SIGN}(\text{TX}_{i+1,2,k}, sk_{i,\text{right},\text{fund},\text{old}}, \texttt{ANYPREVOUT})$ to $\text{sigs}_{\text{toRight}}$
    **end for**
  **end if**
  call VIRT.CIRCULATEFUNDINGSIGS($\text{sigs}_{\text{toRight}}$) of $\bar{P}$ and assign returned values to $\text{sigs}_{\text{byRight}}$
  ensure that the following signatures are present in $\text{sigs}_{\text{byRight}}$ and store them:

– // 1 signature

  $\mathrm{sig}(\mathrm{TX}_{i,1}, pk_{i+1,\mathrm{left,fund,old}})$

– // $n - 3 + \chi_{i=2} + \chi_{i=n-1}$ signatures

  $\forall k \in \{m, \dots, l\} \setminus \{i\}$

    $\mathrm{sig}(\mathrm{TX}_{i,2,k}, pk_{i+1,\mathrm{right,fund,old}})$

  output (VIRTUAL BASE SIG BACK, $\mathrm{sigs}_{\mathrm{byRight}}$)

  extract and store $\mathrm{sig}(\mathrm{TX}_{i,1}, pk_{i+1,\mathrm{right,fund,old}})$ and $\forall k \in \{m, \dots, l\} \setminus \{i\}$ $\mathrm{sig}(\mathrm{TX}_{i,2,k}, pk_{i+1,\mathrm{right,fund,old}})$ from $\mathrm{sigs}_{\mathrm{byRight}}$ // same signatures as `sibling`

  $\mathrm{sigs}_{\mathrm{toLeft}} \leftarrow \{\mathrm{SIGN}(\mathrm{TX}_{i-1,1}, sk_{i,\mathrm{left,fund,old}}, \texttt{ANYPREVOUT})\}$

  **if** $i - 1 > 1$ **then**

    **if** $i - 1 = 2$ **then** $m' \leftarrow 1$ **else** $m' \leftarrow 2$

    **for all** $k \in \{m', \dots, n - 1\}$ **do**

      add $\mathrm{SIGN}(\mathrm{TX}_{i-1,2,k}, sk_{i,\mathrm{left,fund,old}}, \texttt{ANYPREVOUT})$ to $\mathrm{sigs}_{\mathrm{toLeft}}$

    **end for**

  **end if**

  **return** $\mathrm{sigs}_{\mathrm{toLeft}}$

**else if** $i = 1$ **then** // we are `host_funder`

  $\mathrm{sigs}_{\mathrm{toRight}} \leftarrow \{\mathrm{SIGN}(\mathrm{TX}_{2,1}, sk_{1,\mathrm{right,fund,old}}, \texttt{ANYPREVOUT})\}$

  **if** $2 = n - 1$ **then** $l' \leftarrow n$ **else** $l' \leftarrow n - 1$

  **for all** $k \in \{3, \dots, l'\}$ **do**

    add $\mathrm{SIGN}(\mathrm{TX}_{2,2,k}, sk_{1,\mathrm{right,fund,old}}, \texttt{ANYPREVOUT})$ to $\mathrm{sigs}_{\mathrm{toRight}}$

  **end for**

  call VIRT.CIRCULATEFUNDINGSIGS($\mathrm{sigs}_{\mathrm{toRight}}$) of $\bar{P}$ and assign returned value to $\mathrm{sigs}_{\mathrm{byRight}}$

  ensure that $\mathrm{sig}(\mathrm{TX}_{1,1}, pk_{2,\mathrm{left,fund,old}})$ is present in $\mathrm{sigs}_{\mathrm{byRight}}$ and store it

  **return** (OK)

**else if** $i = n$ **then** // we are `host_fundee`

  ensure $\mathrm{sig}(\mathrm{TX}_{n,1}, pk_{n-1,\mathrm{right,fund,old}})$ is present in $\mathrm{sigs}_{\mathrm{byLeft}}$ and store it

  $\mathrm{sigs}_{\mathrm{toLeft}} \leftarrow \{\mathrm{SIGN}(\mathrm{TX}_{n-1,1}, sk_{n,\mathrm{left,fund,old}}, \texttt{ANYPREVOUT})\}$

  **if** $n - 1 = 2$ **then** $m' \leftarrow 1$ **else** $m' \leftarrow 2$

  **for all** $k \in \{m', \dots, n - 2\}$ **do**

    add $\mathrm{SIGN}(\mathrm{TX}_{n-1,2,k}, sk_{n,\mathrm{left,fund,old}}, \texttt{ANYPREVOUT})$ to $\mathrm{sigs}_{\mathrm{toLeft}}$

  **end for**

  **return** $\mathrm{sigs}_{\mathrm{toLeft}}$

**end if** // it is always $1 \leq i \leq n$ – c.f. Fig. 28, l. 12 and l. 39

**Fig. 37.**

**Process** VIRT.CIRCULATEREVOCATIONS(`revoc_by_prev`)

**if** `revoc_by_prev` is given as argument **then** // we are not `host_funder`

  ensure `guest`.PROCESSREMOTEREVOCATION(`revoc_by_prev`) returns (OK)

**else** // we are `host_funder`

  `revoc_for_next` ← `guest`.REVOKEPREVIOUS()

input (READ) to $\mathcal{G}_{\text{Ledger}}$ and assign ouput to $\Sigma$
`last_poll` $\leftarrow |\Sigma|$
call VIRT.CIRCULATEREVOCATIONS(`revoc_for_next`) of $\bar{P}$ and assign returned value to `revoc_by_next`
ensure `guest`.PROCESSREMOTEREVOCATION(`revoc_by_next`) returns (OK) // If the "ensure" fails, the opening process freezes, this is intentional. The channel can still close via (CLOSE)
**return** (OK)
**end if**
**if** we have a `sibling` **then** // we are not `host_fundee` nor `host_funder`
input (VIRTUAL REVOCATION FORWARD) to `sibling`
`revoc_for_next` $\leftarrow$ `guest`.REVOKEPREVIOUS()
input (READ) to $\mathcal{G}_{\text{Ledger}}$ and assign ouput to $\Sigma$
`last_poll` $\leftarrow |\Sigma|$
call VIRT.CIRCULATEREVOCATIONS(`revoc_for_next`) of $\bar{P}$ and assign output to `revoc_by_next`
ensure `guest`.PROCESSREMOTEREVOCATION(`revoc_by_next`) returns (OK)
output (HOSTS READY, $t_i$) to `guest` and expect reply (HOST ACK)
output (VIRTUAL REVOCATION BACK)
**end if**
`revoc_for_prev` $\leftarrow$ `guest`.REVOKEPREVIOUS()
**if** $1 < i < n$ **then** // we are intermediary
output (HOSTS READY, $t_i$) to `guest` and expect reply (HOST ACK) // $p$ is every how many blocks we have to check the chain
**else** // we are `host_fundee`, case of `host_funder` covered earlier
output (HOSTS READY, $p + \sum\limits_{j=2}^{n-1}(s - 1 + t_j)$) to `guest` and expect reply (HOST ACK)
**end if**
**return** `revoc_for_prev`

**Fig. 38.**

**Process** VIRT – poll

On input (CHECK FOR LATERAL CLOSE) by $R \in \{$`guest`, `funder`, `fundee`$\}$:
input (READ) to $\mathcal{G}_{\text{Ledger}}$ and assign output to $\Sigma$
$k_1 \leftarrow 0$
**if** $\mathrm{TX}_{i-1,1}$ is defined and $\mathrm{TX}_{i-1,1} \in \Sigma$ **then**
$k_1 \leftarrow i - 1$
**end if**
**for all** $k \in [i-2]$ **do**
**if** $\mathrm{TX}_{i-1,2,k}$ is defined and $\mathrm{TX}_{i-1,2,k} \in \Sigma$ **then**
$k_1 \leftarrow k$
**end if**

**end for**
$k_2 \leftarrow 0$
**if** $\mathrm{TX}_{i+1,1}$ is defined and $\mathrm{TX}_{i+1,1} \in \Sigma$ **then**
  $k_2 \leftarrow i+1$
**end if**
**for all** $k \in \{i+2, \dots, n\}$ **do**
  **if** $\mathrm{TX}_{i+1,2,k}$ is defined and $\mathrm{TX}_{i+1,2,k} \in \Sigma$ **then**
    $k_2 \leftarrow k$
  **end if**
**end for**
`last_poll` $\leftarrow |\Sigma|$
**if** $k_1 > 0 \vee k_2 > 0$ **then** // at least one neighbour has published its TX
  ignore all messages except for (CHECK IF CLOSING) by $R$
  *State* $\leftarrow$ CLOSING
  sigs $\leftarrow \emptyset$
**end if**
**if** $k_1 > 0 \wedge k_2 > 0$ **then** // both neighbours have published their TXs
  add $(\mathrm{sig}(\mathrm{TX}_{i,3,k_1,k_2}, pk_{p,i,k_1}))_{p \in [n] \setminus \{i\}}$ to sigs
  add $(\mathrm{sig}(\mathrm{TX}_{i,3,k_1,k_2}, pk_{p,i,k_2}))_{p \in [n] \setminus \{i\}}$ to sigs
  add SIGN($\mathrm{TX}_{i,3,k_1,k_2}$, $sk_{i,i,k_1}$, `ANYPREVOUT`) to sigs
  add SIGN($\mathrm{TX}_{i,3,k_1,k_2}$, $sk_{i,i,k_2}$, `ANYPREVOUT`) to sigs
  input (SUBMIT, $\mathrm{TX}_{i,3,k_1,k_2}$, sigs) to $\mathcal{G}_{\mathrm{Ledger}}$
**else if** $k_1 > 0$ **then** // only left neighbour has published its TX
  add $(\mathrm{sig}(\mathrm{TX}_{i,2,k_1}, pk_{p,i,k_1}))_{p \in [n] \setminus \{i\}}$ to sigs
  add SIGN($\mathrm{TX}_{i,2,k_1}$, $sk_{i,i,k_1}$, `ANYPREVOUT`) to sigs
  add SIGN($\mathrm{TX}_{i,2,k_1}$, $sk_{i,\mathrm{left,fund,old}}$, `ANYPREVOUT`) to sigs
  input (SUBMIT, $\mathrm{TX}_{i,2,k_1}$, sigs) to $\mathcal{G}_{\mathrm{Ledger}}$
**else if** $k_2 > 0$ **then** // only right neighbour has published its TX
  add $(\mathrm{sig}(\mathrm{TX}_{i,2,k_2}, pk_{p,i,k_2}))_{p \in [n] \setminus \{i\}}$ to sigs
  add SIGN($\mathrm{TX}_{i,2,k_2}$, $sk_{i,i,k_2}$, `ANYPREVOUT`) to sigs
  add SIGN($\mathrm{TX}_{i,2,k_2}$, $sk_{i,\mathrm{right,fund,old}}$, `ANYPREVOUT`) to sigs
  input (SUBMIT, $\mathrm{TX}_{i,2,k_2}$, sigs) to $\mathcal{G}_{\mathrm{Ledger}}$
**end if**

**Fig. 39.**

**Process** VIRT – On input (CLOSE) by $R \in \{$`guest`, `funder`, `fundee`$\}$:

// At most one of `funder`, `fundee` is defined
**if** *State* = CLOSED **then** output (CLOSED) to $R$
**if** *State* = GUEST PUNISHED **then** output (GUEST PUNISHED) to $R$
ensure *State* = OPEN
**if** `host`$_P \neq \mathcal{G}_{\mathrm{Ledger}}$ **then** // `host`$_P$ is a VIRT
  ignore all messages except for output (CLOSED) by `host`$_P$. Also relay to `host`$_P$ any (CHECK IF CLOSING) input received

    input (CLOSE) to $\texttt{host}_P$
**end if**
// if we have a $\texttt{host}_P$, continue from here on output (CLOSED) by it
send (READ) to $\mathcal{G}_{\text{Ledger}}$ as $R$ and assign reply to $\Sigma$
**if** $i \in \{1, n\} \wedge (\text{TX}_{(i-1)+\frac{2}{n-1}(n-i),1} \in \Sigma \vee \exists k \in [n] : \text{TX}_{(i-1)+\frac{2}{n-1}(n-i),2,k} \in \Sigma)$ **then** // we are an endpoint and our counterparty has closed – 1st subscript of TX is 2 if $i = 1$ and $n - 1$ if $i = n$
    ignore all messages except for (CHECK IF CLOSING) by $R$
    $State \leftarrow$ CLOSING
    give up execution token // control goes to $\mathcal{E}$
**end if**
let $\texttt{tx}$ be the unique TX among $\text{TX}_{i,1}$, $(\text{TX}_{i,2,k})_{k\in[n]}$, $(\text{TX}_{i,3,k_1,k_2})_{k_1,k_2\in[n]}$ that can be appended to $\Sigma$ in a valid way // ignore invalid subscript combinations
let $\texttt{sigs}$ be the set of stored signatures that sign $\texttt{tx}$
add $\text{SIGN}(\texttt{tx}, sk_{i,\text{left},\text{fund},\text{old}}, \texttt{ANYPREVOUT})$, $\text{SIGN}(\texttt{tx}, sk_{i,\text{right},\text{fund},\text{old}}, \texttt{ANYPREVOUT})$, $(\text{SIGN}(\texttt{tx}, sk_{i,j,k}, \texttt{ANYPREVOUT}))_{j,k\in[n]}$ to $\texttt{sigs}$ // ignore invalid signatures
ignore all messages except for (CHECK IF CLOSING) by $R$
$State \leftarrow$ CLOSING
send (SUBMIT, $\texttt{tx}$, $\texttt{sigs}$) to $\mathcal{G}_{\text{Ledger}}$

**Fig. 40.**

**Process** VIRT – On input (CHECK IF CLOSING) by $R \in \{\texttt{guest}, \texttt{funder}, \texttt{fundee}\}$:

ensure $State =$ CLOSING
send (READ) to $\mathcal{G}_{\text{Ledger}}$ as $R$ and assign reply to $\Sigma$
**if** $i = 1$ **then** // we are $\texttt{host\_funder}$
    ensure that there exists an output with $c_P + c_{\bar{P}} - c_{\text{virt}}$ coins and a $2/\{pk_{1,\text{fund},\text{new}}, pk_{2,\text{fund},\text{new}}\}$ spending method with expired/non-existent timelock in $\Sigma$ // new base funding output
    ensure that there exists an output with $c_{\text{virt}}$ coins and a $2/\{pk_{\text{left},\text{virt}}, pk_{\text{right},\text{virt}}\}$ spending method with expired/non-existent timelock in $\Sigma$ // virtual funding output
**else if** $i = n$ **then** // we are $\texttt{host\_fundee}$
    ensure that there exists an output with $c_P + c_{\bar{P}} - c_{\text{virt}}$ coins and a $2/\{pk_{n-1,\text{fund},\text{new}}, pk_{n,\text{fund},\text{new}}\}$ spending method with expired/non-existent timelock in $\Sigma$ // new base funding output
    ensure that there exists an output with $c_{\text{virt}}$ coins and a $2/\{pk_{\text{left},\text{virt}}, pk_{\text{right},\text{virt}}\}$ spending method with expired/non-existent timelock in $\Sigma$ // virtual funding output
**else** // we are intermediary
    **if** $\texttt{side}$ = "left" **then** $j \leftarrow i - 1$ **else** $j \leftarrow i + 1$ // $\texttt{side}$ is defined for all intermediaries – c.f. Fig. 28, l. 11

ensure that there exists an output with $c_P + c_{\bar{P}} - c_{\text{virt}}$ coins and a $2/\{pk_{i,\text{fund,new}}, pk_{j,\text{fund,new}}\}$ spending method with expired/non-existent timelock and an output with $c_{\text{virt}}$ coins and a $pk_{i,\text{out}}$ spending method with expired/non-existent timelock in $\Sigma$

**end if**

*State* ← CLOSED

output (CLOSED) to $R$

**Fig. 41.**

**Process** VIRT – punishment handling

On input (USED REVOCATION) by `guest`: // (USED REVOCATION) by `funder`/`fundee` is ignored

  *State* ← GUEST PUNISHED

  input (USED REVOCATION) to $\texttt{host}_P$, expect reply (USED REVOCATION OK)

  **if** `funder` or `fundee` is defined **then**

    output (ENABLER USED REVOCATION) to it

  **else** // `sibling` is defined

    output (ENABLER USED REVOCATION) to `sibling`

  **end if**

On input (ENABLER USED REVOCATION) by `sibling`:

  *State* ← GUEST PUNISHED

  output (ENABLER USED REVOCATION) to `guest`

On output (USED REVOCATION) by $\texttt{host}_P$:

  *State* ← GUEST PUNISHED

  **if** `funder` or `fundee` is defined **then**

    output (ENABLER USED REVOCATION) to it

  **else** // `sibling` is defined

    output (ENABLER USED REVOCATION) to `sibling`

  **end if**

**Fig. 42.**

**Lemma 1 (Real world balance security).** *Consider a real world execution with $P \in \{Alice, Bob\}$ honest* LN *ITI and $\bar{P}$ the counterparty ITI. Assume that all of the following are true:*

- *the internal variable* `negligent` *of $P$ has value "False",*
- *$P$ has transitioned to the* OPEN *State for the first time after having received* (OPEN, $c, \ldots$) *by either $\mathcal{E}$ or $\bar{P}$,*
- *$P$ [has received* (FUND ME, $f_i, \ldots$) *as input by another* LN *ITI while State was* OPEN *and subsequently $P$ transitioned to* OPEN *State] $n$ times,*

– $P$ *[has received* $(\text{PAY}, d_i)$ *by* $\mathcal{E}$ *while State was* OPEN *and* $P$ *subsequently transitioned to* OPEN *State]* $m$ *times,*
– $P$ *[has received* $(\text{GET PAID}, e_i)$ *by* $\mathcal{E}$ *while State was* OPEN *and* $P$ *subsequently transitioned to* OPEN *State]* $l$ *times.*

*Let* $\phi = 1$ *if* $P = Alice$*, or* $\phi = 0$ *if* $P = Bob$*. If* $P$ *receives* (CLOSE) *by* $\mathcal{E}$ *and, if* $\texttt{host}_P \neq \mathcal{G}_{\text{Ledger}}$ *the output of* $\texttt{host}_P$ *is* (CLOSED)*, then eventually the state obtained when* $P$ *inputs* (READ) *to* $\mathcal{G}_{\text{Ledger}}$ *will contain* $h$ *outputs each of value* $c_i$ *and that has been spent or is exclusively spendable by* $pk_{R,\text{out}}$*) such that*

$$\sum_{i=1}^{h} c_i \geq \phi \cdot c - \sum_{i=1}^{n} f_i - \sum_{i=1}^{m} d_i + \sum_{i=1}^{l} e_i \tag{1}$$

*with overwhelming probability in the security parameter, where* $R$ *is a local, trusted machine (i.e. either* $P$*,* $P$*'s* `sibling`*, the party to which* $P$ *sent* FUND ME *if such a message has been sent, or the* `sibling` *of one of the transitive closure of hosts of* $P$*).*

*Proof.* We first note that, as signature forgeries only happen with negligible probability and only a polynomial number of signatures are verified by honest parties throughout an execution, the event in which any forged signature passes the verification of an honest party or of $\mathcal{G}_{\text{Ledger}}$ happens only with negligible probability. We can therefore ignore this event throughout this proof and simply add a computationally negligible distance between $\mathcal{E}$'s outputs in the real and the ideal world at the end.

We also note that $pk_{P,\text{out}}$ has been provided by $\mathcal{E}$, therefore it can freely use coins spendable by this key. This is why we allow for any of the $pk_{P,\text{out}}$ outputs to have been spent.

Define the *history* of a channel as $H = (F, C)$, where each of $F, C$ is a list of lists of integers. A party $P$ which satisfies the Lemma conditions has a unique, unambiguously and recursively defined history: If the value `hops` in the (OPEN, $c$, `hops`, ...) message was equal to $\mathcal{G}_{\text{Ledger}}$, then $F$ is the empty list, otherwise $F$ is the concatenation of the $F$ and $C$ lists of the party that sent (FUNDED, ...) to $P$, as they were at the moment the latter message was sent. After initialised, $F$ remains immutable. Observe that, if $\texttt{hops} \neq \mathcal{G}_{\text{Ledger}}$, both aforementioned messages must have been received before $P$ transitions to the OPEN state.

The list $C$ of party $P$ is initialised to $[[g]]$ when $P$'s *State* transitions for the first time to OPEN, where $g = c$ if $P = Alice$, or $g = 0$ if $P = Bob$; this represents the initial channel balance. The value $x$ or $-x$ is appended to the last list in $C$ when a payment is received (Fig. 20, l. 21) or sent (Fig. 20, l. 6) respectively by $P$. Moving on to the funding of new virtual channels, whenever $P$ funds a new virtual channel (Fig. 17, l. 20), $[-c_{\text{virt}}]$ is appended to $C$ and whenever $P$ helps with the opening of a new virutal channel, but does not fund it (Fig. 17, l. 23), $[0]$ is appended to $C$. Therefore $C$ consists of one list of integers for each sequence of inbound and outbound payments that have not been interrupted by a virtualisation step and a new list is added for every new virtual layer. We also observe that a non-negligent party with history $(F, C)$ satisfies the Lemma conditions and that the value of the right hand side of the inequality (1) is equal to $\sum\limits_{s \in C} \sum\limits_{x \in s} x$, as all inbound and outbound payment values and new channel funding values that appear in the Lemma conditions are recorded in $C$.

Let party $P$ with a particular history. We will inductively prove that $P$ satisfies the Lemma. The base case is when a channel is opened with `hops` $= \mathcal{G}_{\text{Ledger}}$ and is closed right away, therefore $H = ([], [g])$, where $g = c$ if $P = \mathit{Alice}$ and $g = 0$ if $P = \mathit{Bob}$. $P$ can transition to the OPEN *State* for the first time only if all of the following have taken place:

- It has received (OPEN, $c$, ...) while in the INIT *State*. In case $P = \mathit{Alice}$, this message must have been received as input by $\mathcal{E}$ (Fig. 15, l. 1), or in case $P = \mathit{Bob}$, this message must have been received via the network by $\bar{P}$ (Fig. 10, l. 3).
- It has received $pk_{\bar{P},F}$. In case $P = \mathit{Bob}$, $pk_{\bar{P},F}$ must have been contained in the (OPEN, ...) message by $\bar{P}$ (Fig. 10, l. 3), otherwise if $P = \mathit{Alice}$ $pk_{\bar{P},F}$ must have been contained in the (ACCEPT CHANNEL, ...) message by $\bar{P}$ (Fig. 10, l. 16).
- It internally holds a signature on the commitment transaction $C_{P,0}$ that is valid when verified with public key $pk_{\bar{P},F}$ (Fig. 12, ll. 12 and 23).
- It has the transaction $F$ in the $\mathcal{G}_{\text{Ledger}}$ state (Fig. 13, l. 3 or Fig. 14, l. 5).

We observe that $P$ satisfies the Lemma conditions with $m = n = l = 0$. Before transitioning to the OPEN *State*, $P$ has produced only one valid signature for the "funding" output $(c, 2/\{pk_{P,F}, pk_{\bar{P},F}\})$ of $F$ with $sk_{P,F}$, namely for $C_{\bar{P},0}$ (Fig. 12, ll. 4 or 14), and sent it to $\bar{P}$ (Fig. 12, ll. 6 or 21), therefore the only two ways to spend $(c, 2/\{pk_{P,F}, pk_{\bar{P},F}\})$ are by either publishing $C_{P,0}$ or $C_{\bar{P},0}$. We observe that $C_{P,0}$ has a $(g, (pk_{P,\text{out}}{+}(t{+}s)) \vee 2/\{pk_{P,R}, pk_{\bar{P},R}\})$ output (Fig. 12, l. 2 or 3). The spending method $2/\{pk_{P,R}, pk_{\bar{P},R}\}$ cannot be used since $P$ has not produced a signature for it with $sk_{P,R}$, therefore the alternative spending method, $pk_{P,\text{out}} + (t+s)$, is the only one that will be spendable if $C_{P,0}$ is included in $\mathcal{G}_{\text{Ledger}}$, thus contributing $g$ to the sum of outputs that contribute to inequality (1). Likewise, if $C_{\bar{P},0}$ is included in $\mathcal{G}_{\text{Ledger}}$, it will contribute at least one $(g,\ pk_{P,\text{out}})$ output to this inequality, as $C_{\bar{P},0}$ has a $(g,\ pk_{P,\text{out}})$ output (Fig. 12, l. 2 or 3). Additionally, if $P$ receives (CLOSE) by $\mathcal{E}$ while $H = ([], [g])$, it attempts to publish $C_{P,0}$ (Fig. 26, l. 19), and will either succeed or $C_{\bar{P},0}$ will be published instead. We therefore conclude that in every case $\mathcal{G}_{\text{Ledger}}$ will eventually have a state $\Sigma$ that contains at least one $(g, pk_{P,\text{out}})$ output, therefore satisfying the Lemma consequence.

Let $P$ with history $H = (F, C)$. The induction hypothesis is that the Lemma holds for $P$. Let $c_P$ the sum in the right hand side of inequality (1). In order to perform the induction step, assume that $P$ is in the OPEN state. We will prove all the following (the facts to be proven are shown with emphasis for clarity):

- If $P$ receives (FUND ME, $f$, ...) by a (local, trusted) LN ITI $R$, subsequently transitions back to the OPEN state (therefore moving to history $(F, C')$ where $C' = C + [-f]$) and finally receives (CLOSE) by $\mathcal{E}$ and (CLOSED) by `host`$_P$

before any further change to its history, then *eventually $P$'s $\mathcal{G}_{\text{Ledger}}$ state will contain $h$ transaction outputs each of value $c_i$ exclusively spendable or already spent by $pk_{P,\text{out}}$) that are descendants of an output with spending method $2/\{pk_{P,F}, pk_{\bar{P},F}\}$ such that $\sum_{i=1}^{h} c_i \geq \sum_{s \in C'} \sum_{x \in s} x$.* Furthermore, given that $P$ moves to the OPEN state after the (FUND ME, ...) message, it also sends (FUNDED, ...) to $R$ (Fig. 17, l. 21). If subsequently the state of $R$ transitions to OPEN (therefore obtaining history $(F_R, C_R)$ where $F_R = F + C$ and $C_R = [[f]]$), and finally receives (CLOSE) by $\mathcal{E}$ and (CLOSED) by $\texttt{host}_R$ ($\texttt{host}_R = \texttt{host}_P$ – Fig. 14, l. 10) before any further change to its history, then *eventually $R$'s $\mathcal{G}_{\text{Ledger}}$ state will contain $k$ transaction outputs each of value $c_i^R$ exclusively spendable or already spent by $pk_{R,\text{out}}$) that are descendants of an output with spending method $2/\{pk_{R,F}, pk_{\bar{R},F}\}$ such that $\sum_{i=1}^{k} c_i^R \geq \sum_{s \in C_R} \sum_{x \in s} x$.*

– If $P$ receives (VIRTUALISING, ...) by $\bar{P}$, subsequently transitions back to OPEN (therefore moving to history $(F, C')$ where $C' = C + [0]$) and finally receives CLOSE by $\mathcal{E}$ and (CLOSED) by $\texttt{host}_P$ before any further change to its history, then *eventually $P$'s $\mathcal{G}_{\text{Ledger}}$ state will contain $h$ transaction outputs each of value $c_i$ exclusively spendable or already spent by $pk_{P,\text{out}}$) that are descendants of an output with spending method $2/\{pk_{P,F}, pk_{\bar{P},F}\}$ such that $\sum_{i=1}^{h} c_i \geq \sum_{s \in C} \sum_{x \in s} x$.* Furthermore, given that $P$ moves to the OPEN state after the (VIRTUALISING, ...) message and in case it sends (FUNDED, ...) to some party $R$ (Fig. 17, l. 18), the latter party is the (local, trusted) $\texttt{fundee}$ of a new virtual channel. If subsequently the state of $R$ transitions to OPEN (therefore obtaining history $(F_R, C_R)$ where $F_R = F + C$ and $C_R = [[0]]$), and finally receives (CLOSE) by $\mathcal{E}$ and (CLOSED) by $\texttt{host}_R$ ($\texttt{host}_R = \texttt{host}_P$ – Fig. 14, l. 10) before any further change to its history, then *eventually $R$'s $\mathcal{G}_{\text{Ledger}}$ state will contain an output with a $2/\{pk_{R,F}, pk_{\bar{R},F}\}$ spending method.*

– If $P$ receives (PAY, $d$) by $\mathcal{E}$, subsequently transitions back to OPEN (therefore moving to history $(F, C')$ where $C'$ is $C$ with $-d$ appended to the last list of $C$) and finally receives CLOSE by $\mathcal{E}$ and (CLOSED) by $\texttt{host}_P$ (the latter only if $\texttt{host}_P \neq \mathcal{G}_{\text{Ledger}}$ or equivalently $F \neq []$) before any further change to its history, then *eventually $P$'s $\mathcal{G}_{\text{Ledger}}$ state will contain $h$ transaction outputs each of value $c_i$ exclusively spendable or already spent by $pk_{P,\text{out}}$) that are descendants of an output with a $2/\{pk_{P,F}, pk_{\bar{P},F}\}$ spending method such that $\sum_{i=1}^{h} c_i \geq \sum_{s \in C'} \sum_{x \in s} x$.*

– If $P$ receives (GET PAID, $e$) by $\mathcal{E}$, subsequently transitions back to OPEN (therefore moving to history $(F, C')$ where $C'$ is $C$ with $e$ appended to the last list of $C$) and finally receives CLOSE by $\mathcal{E}$ and (CLOSED) by $\texttt{host}_P$ (the latter only if $\texttt{host}_P \neq \mathcal{G}_{\text{Ledger}}$ or equivalently $F = []$) before any further change to its history, then *eventually* $P$*'s* $\mathcal{G}_{\text{Ledger}}$ *state will contain* $h$ *transaction outputs each of value* $c_i$ *exclusively spendable or already spent by* $pk_{P,\text{out}}$) *that are descendants of an output with a* $2/\{pk_{P,F}, pk_{\bar{P},F}\}$ *spending method such that* $\sum_{i=1}^{h} c_i \geq \sum_{s \in C'} \sum_{x \in s} x$.

By the induction hypothesis, before the funding procedure started $P$ could close the channel and end up with on-chain transaction outputs exclusively spendable or already spent by $pk_{P,\text{out}}$ with a sum value of $c_P$. When $P$ is in the OPEN state and receives (FUND ME, $f$, ...), it can only move again to the OPEN state after doing the following state transitions: OPEN $\rightarrow$ VIRTUALISING $\rightarrow$ WAITING FOR REVOCATION $\rightarrow$ WAITING FOR INBOUND REVOCATION $\rightarrow$ WAITING FOR HOSTS READY $\rightarrow$ OPEN. During this sequence of events, a new $\texttt{host}_P$ is defined (Fig. 17, l. 6), new commitment transactions are negotiated with $\bar{P}$ (Fig. 17, l. 9), control of the old funding output is handed over to $\texttt{host}_P$ (Fig. 17, l. 11), $\texttt{host}_P$ negotiates with its counterparty a new set of transactions and signatures that spend the aforementioned funding output and make available a new funding output with the keys $pk'_{P,F}, pk'_{\bar{P},F}$ as $P$ instructed (Fig. 35 and 37) and the previous valid commitment transactions of both $P$ and $\bar{P}$ are invalidated (Fig. 9, l. 1 and l. 14 respectively). We note that the use of the `ANYPREVOUT` flag in all signatures that correspond to transaction inputs that may spend various different transaction outputs ensures that this is possible, as it avoids tying each input to a specific, predefined output. When $P$ receives (CLOSE) by $\mathcal{E}$, it inputs (CLOSE) to $\texttt{host}_P$ (Fig. 26, l. 4). As per the Lemma conditions, $\texttt{host}_P$ will output (CLOSED). This can happen only when $\mathcal{G}_{\text{Ledger}}$ contains a suitable output for both $P$'s and $R$'s channel (Fig. 41, and 4 ll. 5 respectively).

If the `host` of $\texttt{host}_P$ is $\mathcal{G}_{\text{Ledger}}$, then the funding output $o_{1,2} = (c_P + c_{\bar{P}}, 2/\{pk_{P,F}, pk_{\bar{P},F}\})$ for the $P, \bar{P}$ channel is already on-chain. Regarding the case in which $\texttt{host}_P \neq \mathcal{G}_{\text{Ledger}}$, after the funding procedure is complete, the new $\texttt{host}_P$ will have as its `host` the old $\texttt{host}_P$ of $P$. If the (CLOSE) sequence is initiated, the new $\texttt{host}_P$ will follow the same steps that will be described below once the old $\texttt{host}_P$ succeeds in closing the lower layer (Fig. 40, l. 5). The old $\texttt{host}_P$ however will see no difference in its interface compared to what would happen if $P$ had received (CLOSE) before the funding procedure, therefore it will successfully close by the induction hypothesis. Thereafter the process is identical to the one when the old $\texttt{host}_P = \mathcal{G}_{\text{Ledger}}$.

Moving on, $\texttt{host}_P$ is either able to publish its $\text{TX}_{1,1}$ (it has necessarily received a valid signature $\text{sig}(\text{TX}_{1,1}, pk_{\bar{P},F})$ (Fig. 37, l. 39) by its counterparty before it moved to the OPEN state for the first time), or the output $(c_P + c_{\bar{P}}, 2/\{pk_{P,F}, pk_{\bar{P},F}\})$ needed to spend $\text{TX}_{1,1}$ has already been spent. The only other transactions that can spend it are $\text{TX}_{2,1}$ and any of $(\text{TX}_{2,2,k})_{k>2}$, since these are the only transactions that spend the aforementioned output and that $\texttt{host}_P$ has signed with $sk_{P,F}$ (Fig. 37, ll. 33-37). The output can be also spent by old, revoked commitment transactions, but in that case $\texttt{host}_P$ would not have output (CLOSED); $P$ would have instead detected this triggered by a (CHECK CHAIN FOR CLOSED) message by $\mathcal{E}$ (Fig. 24) and would have moved to the

CLOSED state on its own accord (lack of such a message by $\mathcal{E}$ would lead $P$ to become `negligent`, something that cannot happen according to the Lemma conditions). Every transaction among $\mathrm{TX}_{1,1}$, $\mathrm{TX}_{2,1}$, $(\mathrm{TX}_{2,2,k})_{k>2}$ has a $(c_P + c_{\bar{P}} - f, 2/\{pk'_{P,F}, pk'_{\bar{P},F}\})$ output (Fig. 31, l. 18 and Fig. 30, ll. 27 and 91) which will end up in $\mathcal{G}_{\text{Ledger}}$ – call this output $o_P$. We will prove that at most $\sum\limits_{i=2}^{n-1}(t_i + p + s - 1)$ blocks after (CLOSE) is received by $P$, an output $o_R$ with $c_{\text{virt}}$ coins and a $2/\{pk_{R,F}, pk_{\bar{R},F}\}$ spending condition without or with an expired timelock will be included in $\mathcal{G}_{\text{Ledger}}$. In case party $\bar{P}$ is idle, then $o_{1,2}$ is consumed by $\mathrm{TX}_{1,1}$ and the timelock on its virtual output expires, therefore the required output $o_R$ is on-chain. In case $\bar{P}$ is active, exactly one of $\mathrm{TX}_{2,1}$, $(\mathrm{TX}_{2,2,k})_{k>2}$ or $(\mathrm{TX}_{2,3,1,k})_{k>2}$ is a descendant of $o_{1,2}$; if the transaction belongs to one of the two last transaction groups then necessarily $\mathrm{TX}_{1,1}$ is on-chain in some block height $h$ and given the timelock on the virtual output of $\mathrm{TX}_{1,1}$, $\bar{P}$'s transaction can be at most at block height $h + t_2 + p + s - 1$. If $n = 3$ or $k = n - 1$, then $\bar{P}$'s unique transaction has the required output $o_R$ (without a timelock). The rest of the cases are covered by the following sequence of events:

**Closing sequence**

`maxDel` $\leftarrow t_2 + p + s - 1$ // $A_2$ is active and the virtual output of $\mathrm{TX}_{1,1}$ has a timelock of $t_2$
$i \leftarrow 3$
**loop**
    **if** $A_i$ is idle **then**
        The timelock on the virtual output of the transaction published by $A_{i-1}$ expires and therefore the required $o_R$ is on-chain
    **else** // $A_i$ publishes a transaction that is a descendant of $o_{1,2}$
        `maxDel` $\leftarrow$ `maxDel` $+ t_i + p + s - 1$
        The published transaction can be of the form $\mathrm{TX}_{i,2,2}$ or $(\mathrm{TX}_{i,3,2,k})_{k>i}$ as it spends the virtual output which is encumbered with a public key controlled by $R$ and $R$ has only signed these transactions
        **if** $i = n - 1$ or $k \geq n - 1$ **then** // The interval contains all intermediaries
            The virtual output of the transaction is not timelocked and has only a $2/\{pk_{R,F}, pk_{\bar{R},F}\}$ spending method, therefore it is the required $o_R$
        **else** // At least one intermediary is not in the interval
            **if** the transaction is $\mathrm{TX}_{i,3,2,k}$ **then** $i \leftarrow k$ **else** $i \leftarrow i + 1$
        **end if**
    **end if**
**end loop**
// `maxDel` $\leq \sum\limits_{i=2}^{n-1}(t_i + p + s - 1)$

**Fig. 43.**

In every case $o_P$ and $o_R$ end up on-chain in at most $s$ and $\sum\limits_{i=2}^{n-1}(t_i + p + s - 1)$ blocks respectively from the moment (CLOSE) is received. The output $o_P$ an be spent either by $C_{P,i}$ or $C_{\bar{P},i}$. Both these transactions have a $(c_P - f, pk_{P,\mathrm{out}})$ output. This output of $C_{P,i}$ is timelocked, but the alternative spending method cannot be used as $P$ never signed a transaction that uses it (as it is reserved for revocation, which has not taken place yet in this virtualisation layer). We have now proven that if $P$ completes the funding of a new channel then it can close its channel for a $(c_P - f,\ pk_{P,\mathrm{out}})$ output that is a descendant of an output with spending method $2/\{pk_{P,F}, pk_{\bar{P},F}\}$ and that lower bound of value holds for the duration of the funding procedure, i.e. we have proven the first claim of the first bullet.

We will now prove that the newly funded party $R$ can close its channel securely. After $R$ receives (FUNDED, $\texttt{host}_P$, ...) by $P$ and before moving to the OPEN state, it receives $\mathrm{sig}_{\bar{R},C,0} = \mathrm{sig}(C_{R,0},\ pk_{\bar{R},F})$ and sends $\mathrm{sig}_{R,C,0} = \mathrm{sig}(C_{\bar{R},0},\ pk_{R,F})$. Both these transactions spend $o_R$. As we showed before, if $R$ receives (CLOSE) by $\mathcal{E}$ then $o_R$ eventually ends up on-chain. After receiving (CLOSED) from $\texttt{host}_P$, $R$ attempts to add $C_{R,0}$ to $\mathcal{G}_{\mathrm{Ledger}}$, which may only fail if $C_{\bar{R},0}$ ends up on-chain instead. Similar to the case of $P$, both these transactions have an $(f, pk_{R,\mathrm{out}})$ output. This output of $C_{R,0}$ is timelocked, but the alternative spending method cannot be used as $R$ never signed a transaction that uses it (as it is reserved for revocation, which has not taken place yet) so the timelock will expire and the desired spending method will be available. We have now proven that if $R$'s channel is funded to completion (i.e. $R$ moves to the OPEN state for the first time) then it can close its channel for a $(f,\ pk_{R,\mathrm{out}})$ output that is a descendant of $o_R$. We have therefore proven the first bullet.

We now move on to the second bullet. In case $P$ is the `funder` (i.e. $i = n$), then the same arguments as in the previous bullet hold here with "WAITING FOR INBOUND REVOCATION" replaced with "WAITING FOR OUTBOUND REVOCATION", $o_{1,2}$ with $o_{n-1,n}$, $\mathrm{TX}_{1,1}$ with $\mathrm{TX}_{n,1}$, $\mathrm{TX}_{2,1}$ with $\mathrm{TX}_{n-1,1}$, $(\mathrm{TX}_{2,2,k})_{k>2}$ with $(\mathrm{TX}_{n-1,2,k})_{k<n-1}$, $(\mathrm{TX}_{2,3,1,k})_{k>2}$ with $(\mathrm{TX}_{n-1,3,n,k})_{k<n-1}$, $t_2$ with $t_{n-1}$, $\mathrm{TX}_{i,3,2,k}$ with $\mathrm{TX}_{i,3,n-1,k}$, $i$ is initialized to $n-2$ in l. 2 of Fig. 43, $i$ is decremented instead of incremented in l. 12 of the same Figure and $f$ is replaced with 0. This is so because these two cases are symmetric.

In case $P$ is not the `funder` ($1 < i < n$), then we only need to prove the first statement of the second bullet. By the induction hypothesis and since `sibling` is trusted, we know that both $P$'s and `sibling`'s funding outputs either are or can be eventually put on-chain and that $P$'s funding output has at least $c_P = \sum\limits_{s \in C} \sum\limits_{x \in s} x$ coins. If $P$ is on the "left" of its `sibling` (i.e. there is an untrusted party that sent the (VIRTUALISING, ...) message to $P$ which triggered the latter to move to the VIRTUALISING state and to send a (VIRTUALISING, ...) message to its own `sibling`), the "left" funding output $o_{\mathrm{left}}$ (the one held with the untrusted party to the left) can be spent by one of $\mathrm{TX}_{i,1}$, $(\mathrm{TX}_{i,2,k})_{k>i}$, $\mathrm{TX}_{i-1,1}$, or $(\mathrm{TX}_{i-1,2,k})_{k<i-1}$, as these are the only transactions that $P$ has signed with $sk_{P,F}$. All these transactions have a $(c_P + c_{\bar{P}} - f,\ 2/\{pk_{P,F'}, pk_{\bar{P},F'}\})$ output that can in turn be spent by either $C_{P,0}$ or $C_{\bar{P},0}$, both of which have an output of value $c_P$ and a $pk_{P,\mathrm{out}}$ spending method and no other spending method can be used (as $P$ has not signed the "revocation" spending method of $C_{P,0}$).

In the case that $P$ is to the right of its `sibling` (i.e. $P$ receives by `sibling` the (VIRTUALISING, …) message that causes $P$'s transition to the VIRUTALISING state), the "right" funding output $o_{\mathrm{right}}$ (the one held with the untrusted party to the right) can be spent by one of $\mathrm{TX}_{i,1}$, $(\mathrm{TX}_{i,2,k})_{k<i}$, $\mathrm{TX}_{i+1,1}$, or $(\mathrm{TX}_{i+1,2,k})_{k>i+1}$, as these are the only transactions that $P$ has signed with $sk_{P,F}$. All these transactions have a $(c_P + c_{\bar{P}} - f,\ 2/\{pk_{P,F'}, pk_{\bar{P},F'}\})$ output that can in turn be spent by either $C_{P,0}$ or $C_{\bar{P},0}$, both of which have an output of value $c_P - f$ and a $pk_{P,\mathrm{out}}$ spending method and no other spending method can be used (as $P$ has not signed the "revocation" spending method of $C_{P,0}$). $P$ can get the remaining $f$ coins as follows: $\mathrm{TX}_{i,1}$ and all of $(\mathrm{TX}_{i,2,k})_{k<i}$ already have an $(f,\ pk_{P,\mathrm{out}})$ output. If instead $\mathrm{TX}_{i+1,1}$ or one of $(\mathrm{TX}_{i+1,2,k_2})_{k_2>i+1}$ spends $o_{\mathrm{right}}$, then $P$ will publish $\mathrm{TX}_{i,2,i+1}$ or $\mathrm{TX}_{i,2,k_2}$ respectively if $o_{\mathrm{left}}$ is unspent, otherwise $o_{\mathrm{left}}$ is spent by one of $\mathrm{TX}_{i-1,1}$ or $(\mathrm{TX}_{i-1,2,k_1})_{k_1<i-1}$ in which case $P$ will publish one of $\mathrm{TX}_{i,3,k_1,i+1}$, $\mathrm{TX}_{i,3,i-1,k_2}$, $\mathrm{TX}_{i,3,i-1,i+1}$ or $\mathrm{TX}_{i,3,k_1,k_2}$. In particular, $\mathrm{TX}_{i,3,k_1,i+1}$ is published if $\mathrm{TX}_{i-1,2,k_1}$ and $\mathrm{TX}_{i+1,1}$ are on-chain, $\mathrm{TX}_{i,3,i-1,k_2}$ is published if $\mathrm{TX}_{i-1,1}$ and $\mathrm{TX}_{i+1,2,k_2}$ are on-chain, $\mathrm{TX}_{i,3,i-1,i+1}$ is published if $\mathrm{TX}_{i-1,1}$ and $\mathrm{TX}_{i+1,1}$ are on-chain, or $\mathrm{TX}_{i,3,k_1,k_2}$ is published if $\mathrm{TX}_{i-1,2,k_1}$ and $\mathrm{TX}_{i+1,2,k_2}$ are on-chain. All these transactions include an $(f,\ pk_{P,\mathrm{out}})$ output. We have therefore covered all cases and proven the second bullet.

Regarding now the third bullet, once again the induction hypothesis guarantees that before (PAY, $d$) was received, $P$ could close the channel resulting in on-chain outputs exclusively spendable or already spent by $pk_{P,\mathrm{out}}$ that are descendants of an output with a $2/\{pk_{P,F}, pk_{\bar{P},F}\}$ spending method that have a sum value of $c_P = \sum\limits_{s \in C} \sum\limits_{x \in s} x$. (Note that $\sum\limits_{s \in C'} \sum\limits_{x \in s} x = d + \sum\limits_{s \in C} \sum\limits_{x \in s} x$.) When $P$ receives (PAY, $d$) while in the OPEN state, it moves to the WAITING FOR COMMITMENT SIGNED state before returning to the OPEN state. It signs (Fig. 19, l. 2) the new commitment transaction $C_{\bar{P},i+1}$ in which the counterparty owns $d$ more coins than before that moment (Fig. 19, l. 1), sends the signature to the counterparty (Fig. 19, l. 4) and expects valid signatures on its own updated commitment transaction (Fig. 20, l. 1) and the revocation transaction for the old commitment transaction of the counterparty (Fig. 20, l. 3). Note that if the counterparty does not respond or if it responds with missing/invalid signatures, either $P$ can close the channel with the old commitment transaction $C_{P,i}$ exactly like before the update started (as it has not yet sent the signature for the old revocation transaction), or the counterparty will close the channel either with the new or with the old commitment transaction. In all cases in which validation fails and the channel closes, there is an output with a $pk_{P,\mathrm{out}}$ spending method and no other useable spending method that carries at least $c_P - d$ coins. Only if the verification succeeds does $P$ sign (Fig. 20, l. 5) and send (Fig. 20, l. 17) the counterparty's revocation transaction for $P$'s previous commitment transaction.

Similarly to previous bullets, if $\texttt{host}_P \neq \mathcal{G}_{\text{Ledger}}$ the funding output can be put on-chain, otherwise the funding output is already on-chain. In both cases, since the closing procedure continues, one of $C_{P,i+1}$ $(C_{\bar{P},j})_{0 \leq j \leq i+1}$ will end up on-chain. If $C_{\bar{P},j}$ for some $j < i+1$ is on-chain, then $P$ submits $R_{P,j}$ (we discussed how $P$ obtained $R_{P,i}$ and the rest of the cases are covered by induction) and takes the entire value of the channel which is at least $c_P - d$. If $C_{\bar{P},i+1}$ is on-chain, it has a $(c_P - d,\, pk_{P,\text{out}})$ output. If $C_{P,i+1}$ is on-chain, it has an output of value $c_P - d$, a timelocked $pk_{P,\text{out}}$ spending method and a non-timelocked spending method that needs the signature made with $sk_{P,R}$ on $R_{\bar{P},i+1}$. $P$ however has not generated that signature, therefore this spending method cannot be used and the timelock will expire, therefore in all cases outputs that descend from the funding output, can be spent exclusively by $pk_{P,\text{out}}$ and carry at least $c_P - d$ coins are put on-chain. We have proven the third bullet.

For the fourth and last bullet, again by the induction hypothesis, before (GET PAID, $e$) was received $P$ could close the channel resulting in on-chain outputs exclusively spendable or already spent by $pk_{P,\text{out}}$ that are descendants of an output $o_F$ with a $2/\{pk_{P,F}, pk_{\bar{P},F}\}$ spending method and have a sum value of $c_P = \sum\limits_{s \in C} \sum\limits_{x \in s} x$. (Note that $e + \sum\limits_{s \in C'} \sum\limits_{x \in s} x = \sum\limits_{s \in C} \sum\limits_{x \in s} x$ and that $o_F$ either is already on-chain or can be eventually put on-chain as we have argued in the previous bullets by the induction hypothesis.) When $P$ receives (GET PAID, $e$) while in the OPEN state, if the balance of the counterparty is enough it moves to the WAITING TO GET PAID state (Fig. 22, l. 6). If subsequently it receives a valid signature for $C_{P,i+1}$ (Fig. 19, l. 8) which is a commitment transaction that can spend the $o_F$ output and gives to $P$ an additional $e$ coins compared to $C_{P,i}$. Subsequently $P$'s state transitions to WAITING FOR PAY REVOCATION and sends signatures for $C_{\bar{P},i+1}$ and $R_{\bar{P},i}$ to $\bar{P}$. If the $o_F$ output is spent while $P$ is in the latter state, it can be spent by one of $C_{P,i+1}$ or $(C_{\bar{P},j})_{0 \leq j \leq i+1}$. If it is spent by $C_{P,i+1}$ or $C_{\bar{P},i+1}$, then these two transactions have a $(c_P + e,\, pk_{P,\text{out}})$ output. (Note that the former is encumbered with a timelock, but the alternative spending method cannot be used as $P$ has not signed $R_{\bar{P},i+1}$.) If it is spent by $C_{\bar{P},i}$ then a $(c_P,\, pk_{P,\text{out}})$ output becomes available instead, therefore $P$ can still get the $c_P$ coins that correspond to the previous state. If any of $(C_{\bar{P},j})_{0 \leq j < i}$ spends $o_F$ then it makes available a $pk_{P,\text{out}}$ output with the coins that $P$ had at state $j$ and additionally $P$ can publish $R_{P,j}$ that spends $\bar{P}$'s output of $C_{\bar{P},j}$ and obtain the entirety of $\bar{P}$'s coins at state $j$ for a total of $c_P + c_{\bar{P}}$ coins. Therefore in every case $P$ can claim at least $c_P$ coins. In the case that $P$ instead subsequently receives a valid signature to $R_{P,i}$ (Fig. 20, l. 20) it finally moves to the OPEN state once again. In this state the above analysis of what can happen when $o_F$ holds similarly, with the difference that if $\bar{P}$ spends $o_F$ with $C_{\bar{P},i}$ now $P$ can publish $R_{P,i}$ which gives $P$ the coins of $\bar{P}$. Therefore with this difference $P$ is now guaranteed to gain at least $c_P + e$ coins upon channel closure. We have therefore proven the fourth bullet. $\square$

**Lemma 2 (Ideal world balance).** *Consider an ideal world execution with functionality $\mathcal{F}_{\text{Chan}}$ and simulator $\mathcal{S}$. Let $P \in \{Alice, Bob\}$ one of the two parties of $\mathcal{F}_{\text{Chan}}$. Assume that all of the following are true:*

- *$State_P \neq$ IGNORED,*
- *$P$ has transitioned to the OPEN State at least once. Additionally, if $P = Alice$, it has received $($OPEN$, c, \dots)$ by $\mathcal{E}$ prior to transitioning to the OPEN State,*
- *$P$ [has received $($FUND ME$, f_i, \dots)$ as input by another $\mathcal{F}_{\text{Chan}}$/LN ITI while $State_P =$ OPEN and $P$ subsequently transitioned to OPEN State] $n \geq 0$ times,*
- *$P$ [has received $($PAY$, d_i)$ by $\mathcal{E}$ while $State_P =$ OPEN and $P$ subsequently transitioned to OPEN State] $m \geq 0$ times,*
- *$P$ [has received $($GET PAID$, e_i)$ by $\mathcal{E}$ while $State_P =$ OPEN and $P$ subsequently transitioned to OPEN State] $l \geq 0$ times.*

*Let $\phi = 1$ if $P = Alice$, or $\phi = 0$ if $P = Bob$. If $\mathcal{F}_{\text{Chan}}$ receives $($CLOSE$, P)$ by $\mathcal{S}$, then the following holds with overwhelming probability on the security parameter:*

$$\mathtt{balance}_P = \phi \cdot c - \sum_{i=1}^{n} f_i - \sum_{i=1}^{m} d_i + \sum_{i=1}^{l} e_i \quad (2)$$

*Proof.* We will prove the Lemma by following the evolution of the $\mathtt{balance}_P$ variable.

- When $\mathcal{F}_{\text{Chan}}$ is activated for the first time, it sets $\mathtt{balance}_P \leftarrow 0$ (Fig. 2, l. 1).
- If $P = Alice$ and it receives (OPEN, $c$, ...) by $\mathcal{E}$, it stores $c$ (Fig. 2, l. 10). If later $State_P$ becomes OPEN, $\mathcal{F}_{\text{Chan}}$ sets $\mathtt{balance}_P \leftarrow c$ (Fig. 2, ll. 13 or 31). In contrast, if $P = Bob$, it is $\mathtt{balance}_P = 0$ until at least the first transition of $State_P$ to OPEN (Fig. 2).
- Every time $P$ receives input $($FUND ME$, f_i, \dots)$ by another party while $State_P =$ OPEN, $P$ stores $f_i$ (Fig. 4, l. 1). The next time $State_P$ transitions to OPEN (if such a transition happens), $\mathtt{balance}_P$ is decremented by $f_i$ (Fig. 4, l. 27). Therefore, if this cycle happens $n \geq 0$ times, $\mathtt{balance}_P$ will be decremented by $\sum_{i=1}^{n} f_i$ in total.
- Every time $P$ receives input $($PAY$, d_i)$ by $\mathcal{E}$ while $State_P =$ OPEN, $d_i$ is stored (Fig. 3, l. 2). The next time $State_P$ transitions to OPEN (if such a transition happens), $\mathtt{balance}_P$ is decremented by $d_i$ (Fig. 3, l. 13). Therefore, if this cycle happens $m \geq 0$ times, $\mathtt{balance}_P$ will be decremented by $\sum_{i=1}^{m} d_i$ in total.
- Every time $P$ receives input $($GET PAID$, e_i)$ by $\mathcal{E}$ while $State_P =$ OPEN, $e_i$ is stored (Fig. 3, l. 7). The next time $State_P$ transitions to OPEN (if such a transition happens) $\mathtt{balance}_P$ is incremented by $e_i$ (Fig. 3, l. 19). Therefore, if this cycle happens $l \geq 0$ times, $\mathtt{balance}_P$ will be incremented by $\sum_{i=1}^{l} e_i$ in total.

On aggregate, after the above are completed and then $\mathcal{F}_{\text{Chan}}$ receives (CLOSE, $P$) by $\mathcal{S}$, it is $\texttt{balance}_P = c - \sum\limits_{i=1}^{n} f_i - \sum\limits_{i=1}^{m} d_i + \sum\limits_{i=1}^{l} e_i$ if $P = \mathit{Alice}$, or else if $P = \mathit{Bob}$, $\texttt{balance}_P = - \sum\limits_{i=1}^{n} f_i - \sum\limits_{i=1}^{m} d_i + \sum\limits_{i=1}^{l} e_i$. □

**Lemma 3 (No halt).** *In an ideal execution with $\mathcal{F}_{\text{Chan}}$ and $\mathcal{S}$, if the trusted parties of the honest parties of $\mathcal{F}_{\text{Chan}}$ are themselves honest, then the functionality halts with negligible probability in the security parameter (i.e. l. 21 of Fig. 5 is executed negligibly often).*

*Proof.* We prove the Lemma in two steps. We first show that if the conditions of Lemma 2 hold, then the conditions of Lemma 1 for the real world execution with protocol LN and the same $\mathcal{E}$ and $\mathcal{A}$ hold as well for the same $m, n$ and $l$ values.

For $\mathit{State}_P$ to become IGNORED, either $\mathcal{S}$ has to send (BECAME CORRUPTED OR NEGLIGENT, $P$) or $\texttt{host}_P$ must output (ENABLER USED REVOCATION) to $\mathcal{F}_{\text{Chan}}$ (Fig. 2, l. 4). The first case only happens when either $P$ receives (CORRUPT) by $\mathcal{A}$ (Fig. 7, l. 1), which means that the simulated $P$ is not honest anymore, or when $P$ becomes $\texttt{negligent}$ (Fig. 7, l. 4), which means that the first condition of Lemma 1 is violated. In the second case, it is $\texttt{host}_P \neq \mathcal{G}_{\text{Ledger}}$ and the state of $\texttt{host}_P$ is GUEST PUNISHED (Fig. 42, ll. 1 or 12), so in case $P$ receives (CLOSE) by $\mathcal{E}$ the output of $\texttt{host}_P$ will be (GUEST PUNISHED) (Fig. 40, l. 3). In all cases, some condition of Lemma 1 is violated.

For $\mathit{State}_P$ to become OPEN at least once, the following sequence of events must take place (Fig. 2): If $P = \mathit{Alice}$, it must receive (INIT, $pk$) by $\mathcal{E}$ when $\mathit{State}_P =$ UNINIT, then either receive (OPEN, $c$, $\mathcal{G}_{\text{Ledger}}$, $\dots$) by $\mathcal{E}$ and (BASE OPEN) by $\mathcal{S}$ or (OPEN, $c$, $\texttt{hops}$ ($\neq \mathcal{G}_{\text{Ledger}}$), $\dots$) by $\mathcal{E}$, (FUNDED, HOST, $\dots$) by $\texttt{hops}[0]\texttt{.left}$ and (VIRTUAL OPEN) by $\mathcal{S}$. In either case, $\mathcal{S}$ only sends its message only if all its simulated honest parties move to the OPEN state (Fig. 7, l. 10), therefore if the second condition of Lemma 2 holds and $P = \mathit{Alice}$, then the second condition of Lemma 1 holds as well. The same line of reasoning can be used to deduce that if $P = \mathit{Bob}$, then $\mathit{State}_P$ will become OPEN for the first time only if all honest simulated parties move to the OPEN *state*, therefore once more the second condition of Lemma 2 holds only if the second condition of Lemma 1 holds as well. We also observe that, if both parties are honest, they will transition to the OPEN state simultaneously.

Regarding the third Lemma 2 condition, we assume (and will later show) that if both parties are honest and the state of one is OPEN, then the state of the other is also OPEN. Each time $P$ receives input (FUND ME, $f$, $\dots$) by $R \in \{\mathcal{F}_{\text{Chan}}, \text{LN}\}$, $\mathit{State}_P$ transitions to PENDING FUND, subsequently when a command to define a new VIRT ITI through $P$ is intercepted by $\mathcal{F}_{\text{Chan}}$, $\mathit{State}_P$ transitions to TENTATIVE FUND and afterwards when $\mathcal{S}$ sends (FUND) to $\mathcal{F}_{\text{Chan}}$, $\mathit{State}_P$ transitions to SYNC FUND. In parallel, if $\mathit{State}_{\bar{P}} =$ IGNORED, then $\mathit{State}_P$ transitions directly back to OPEN. If on the other hand $\mathit{State}_{\bar{P}} =$ OPEN and $\mathcal{F}_{\text{Chan}}$ intercepts a similar VIRT ITI definition command through $\bar{P}$, $\mathit{State}_{\bar{P}}$ transitions to TENTATIVE HELP FUND. On receiving the aforementioned (FUND) message by $\mathcal{S}$ and given that $State_{\bar{P}}$ = TENTATIVE HELP FUND, $\mathcal{F}_{\text{Chan}}$ also sets $State_{\bar{P}}$ to SYNC HELP FUND. Then both $State_{\bar{P}}$ and $State_P$ transition simultaneously to OPEN (Fig. 4). This sequence of events may repeat any $n \geq 0$ times. We observe that throughout these steps, honest simulated $P$ has received (FUND ME, $f$, ...) and that $\mathcal{S}$ only sends (FUND) when all honest simulated parties have transitioned to the OPEN state (Fig. 7, l. 18 and Fig. 17, l. 12), so the third condition of Lemma 1 holds with the same $n$ as that of Lemma 2.

Regarding the fourth Lemma 2 condition, we again assume that if both parties are honest and the state of one is OPEN, then the state of the other is also OPEN. Each time $P$ receives input (PAY, $d$) by $\mathcal{E}$, $State_P$ tranisitions to TENTATIVE PAY and subsequently when $\mathcal{S}$ sends (PAY) to $\mathcal{F}_{\text{Chan}}$, $State_P$ transitions to (SYNC PAY, $d$). In parallel, if $State_{\bar{P}}$ = IGNORED, then $State_P$ transitions directly back to OPEN. If on the other hand $State_{\bar{P}}$ = OPEN and $\mathcal{F}_{\text{Chan}}$ receives (GET PAID, $d$) by $\mathcal{E}$ addressed to $\bar{P}$, $State_{\bar{P}}$ transitions to TENTATIVE GET PAID. On receiving the aforementioned (PAY) message by $\mathcal{S}$ and given that $State_{\bar{P}}$ = TENTATIVE GET PAID, $\mathcal{F}_{\text{Chan}}$ also sets $State_{\bar{P}}$ to SYNC GET PAID. Then both $State_P$ and $State_{\bar{P}}$ transition simultaneously to OPEN (Fig. 3). This sequence of events may repeat any $m \geq 0$ times. We observe that throughout these steps, honest simulated $P$ has received (PAY, $d$) and that $\mathcal{S}$ only sends (PAY) when all honest simulated parties have completed sending or receiving the payment (Fig. 7, l. 16), so the fourth condition of Lemma 1 holds with the same $m$ as that of Lemma 2. As far as the fifth condition of Lemma 2 goes, we observe that this case is symmetric to the one discussed for its fourth condition above if we swap $P$ and $\bar{P}$, therefore we deduce that if Lemma 2 holds with some $l$, then Lemma 1 holds with the same $l$.

As promised, we here argue that if both parties are honest and one party moves to the OPEN state, then the other party will move to the OPEN state as well. We already saw that the first time one party moves to the OPEN state, it will happen simultaneously with the same transition for the other party. We also saw that, when a party transitions from the SYNC HELP FUND or the SYNC FUND state to the OPEN state, then the other party will also transition to the OPEN state simultaneously. Furthermore, we saw that if one party transitions from the SYNC PAY or the SYNC GET PAID state to the OPEN state, the other party will also transition to the OPEN state simultaneously. Lastly we notice that we have exhausted all manners in which a party can transition to the OPEN state, therefore we have proven that transitions of honest parties to the OPEN state happen simultaneously.

Now, given that $\mathcal{S}$ internally simulates faithfully both LN parties and that $\mathcal{F}_{\text{Chan}}$ relinquishes to $\mathcal{S}$ complete control of the external communication of the parties as long as it does not halt, we deduce that $\mathcal{S}$ replicates the behaviour of the aforementioned real world. By combining these facts with the consequences of the two Lemmas and the check that leads $\mathcal{F}_{\text{Chan}}$ to halt if it fails (Fig. 5, l. 18), we deduce that if the conditions of Lemma 2 hold for the honest parties of

$\mathcal{F}_{\text{Chan}}$ and their trusted parties, then the functionality halts only with negligible probability.

In the second proof step, we show that if the conditions of Lemma 2 do not hold, then the check of Fig. 5, l. 18 never takes place. We first discuss the $State_P =$ IGNORED case. We observe that the IGNORED *State* is a sink state, as there is no way to leave it once in. Additionally, for the balance check to happen, $\mathcal{F}_{\text{Chan}}$ must receive (CLOSED, $P$) by $\mathcal{S}$ when $State_P \neq$ IGNORED (Fig. 5, l. 9). We deduce that, once $State_P =$ IGNORED, the balance check will not happen. Moving to the case where $State_P$ has never been OPEN, we observe that it is impossible to move to any of the states required by l. 9 of Fig. 5 without first having been in the OPEN state. Moreover if $P = Alice$, it is impossible to reach the OPEN state without receiving input (OPEN, $c, \dots$) by $\mathcal{E}$. Lastly, as we have observed already, the three last conditions of Lemma 2 are always satisfied. We conclude that if the conditions to Lemma 2 do not hold, then the check of Fig. 5, l. 18 does not happen and therefore $\mathcal{F}_{\text{Chan}}$ does not halt.

On aggregate, $\mathcal{F}_{\text{Chan}}$ may only halt with negligible probability in the security parameter. $\square$

**Theorem 1 (Recursive Virtual Payment Channel Security).** *The protocol $\Pi_{\text{Chan}}$ realises $\mathcal{F}_{\text{Chan}}$ given a global functionality $\mathcal{G}_{\text{Ledger}}$ and assuming the security of the underlying digital signature. Specifically,*

$$\forall \text{ } PPT \text{ } \mathcal{A}, \exists \text{ } PPT \text{ } \mathcal{S} : \forall \text{ } PPT \text{ } \mathcal{E} \text{ } it \text{ } is \text{ } \text{EXEC}^{\mathcal{G}_{\text{Ledger}}}_{\Pi_{\text{Chan}},\mathcal{A},\mathcal{E}} \approx \text{EXEC}^{\mathcal{F}_{\text{Chan}},\mathcal{G}_{\text{Ledger}}}_{\mathcal{S},\mathcal{E}}$$

*Proof.* By inspection of Figs. 1 and 6 we can deduce that for a particular $\mathcal{E}$, in the ideal world execution $\text{EXEC}^{\mathcal{F}_{\text{Chan}},\mathcal{G}_{\text{Ledger}}}_{\mathcal{S}_{\mathcal{A}},\mathcal{E}}$, $\mathcal{S}_{\mathcal{A}}$ simulates internally the two $\Pi_{\text{Chan}}$ parties exactly as they would execute in the real world execution, $\text{EXEC}^{\mathcal{G}_{\text{Ledger}}}_{\Pi_{\text{Chan}},\mathcal{A},\mathcal{E}}$ in case $\mathcal{F}_{\text{Chan}}$ does not halt. Indeed, $\mathcal{F}_{\text{Chan}}$ only halts with negligible probability according to Lemma 3, therefore the two executions are computationally indistinguishable. $\square$

We now generalise Theorem 1 to prove the indistinguishability of multiple $\mathcal{F}_{\text{Chan}}$ instances from multiple $\Pi_{\text{Chan}}$ instances, leveraging the definition of the multi-session extension of an ideal functionality [6].

**Definition 1 (Multi-Session Extension of a Protocol).** *Let protocol $\pi$. Its* multi-session extension $\widehat{\pi}$ *has the same code as $\pi$ and has 2 session ids: the "sub-session id"* ssid *which replaces the session id of $\pi$ and the usual session id* sid *which has no further function apart from what is prescribed by the UC framework.*

**Theorem 2 (Indistinguishability of multiple sessions).** *Let $\widehat{\mathcal{F}}_{\text{Chan}}$ the multi-session extension of $\mathcal{F}_{\text{Chan}}$ and $\widehat{\Pi}_{\text{Chan}}$ the protocol-multi-session extension of $\Pi_{\text{Chan}}$.*

$$\forall \text{ } PPT \text{ } \mathcal{A}, \exists \text{ } PPT \text{ } \mathcal{S} : \forall \text{ } PPT \text{ } \mathcal{E} \text{ } it \text{ } is \text{ } \text{EXEC}^{\mathcal{G}_{\text{Ledger}}}_{\widehat{\Pi}_{\text{Chan}},\mathcal{A},\mathcal{E}} \approx \text{EXEC}^{\widehat{\mathcal{F}}_{\text{Chan}},\mathcal{G}_{\text{Ledger}}}_{\mathcal{S},\mathcal{E}}$$

*Proof.* We observe that $\widehat{\mathcal{F}}_{\text{Chan}}$ uses $\mathcal{F}_{\text{Chan}}$ internally. According to the UC theorem [4] and given that $\Pi_{\text{Chan}}$ UC-realises $\mathcal{F}_{\text{Chan}}$ (Theorem 1), $\widehat{\mathcal{F}}_{\text{Chan}}^{\mathcal{F}_{\text{Chan}} \to \Pi_{\text{Chan}}}$ UC-emulates $\widehat{\mathcal{F}}_{\text{Chan}}$. We now observe that $\widehat{\mathcal{F}}_{\text{Chan}}^{\mathcal{F}_{\text{Chan}} \to \Pi_{\text{Chan}}}$ behaves identically to a session with $\widehat{\Pi}_{\text{Chan}}$ protocols, as the former routes each message to the same internal $\Pi_{\text{Chan}}$ instance that would handle the same message in the latter case, therefore $\widehat{\mathcal{F}}_{\text{Chan}}^{\mathcal{F}_{\text{Chan}} \to \Pi_{\text{Chan}}}$ UC-emulates $\widehat{\Pi}_{\text{Chan}}$. By the transitivity of UC-emulation, we deduce that $\widehat{\mathcal{F}}_{\text{Chan}}$ UC-emulates $\widehat{\Pi}_{\text{Chan}}$. □

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