TODO: Add support for cooperative adding multiple virtuals to single channel as future work (needs cooperation by all hops of all existing virtuals of current channel) TODO: Add support for cooperative closing as future work

### Functionality $\mathcal{F}_{Chan}$ —general message handling rules

- On receiving input (msg) by  $\mathcal{E}$  to  $P \in \{Alice, Bob\}$ , handle it according to the corresponding rule in Fig 2 or Fig 3 (if any) and subsequently send (RELAY, msg, P,  $\mathcal{E}$ , input)  $\mathcal{A}$ . // all messages by  $\mathcal{E}$  are relayed to  $\mathcal{A}$
- On receiving (msg) by  $R \neq \mathcal{E}$  to  $P \in \{Alice, Bob\}$  by means of mode  $\in \{\text{input}, \text{output}, \text{network}\}$ , send (RELAY, msg, P, R, mode) to A. // all messages by machines other than  $\mathcal{E}$  are relayed to A
- On receiving (RELAY, msg, P, R, mode) by  $\mathcal{A}$  (mode  $\in$  {input, output, network},  $P \in \{Alice, Bob\}$ ), relay msg to R as P by means of mode. //  $\mathcal{A}$  fully controls outgoing messages by  $\mathcal{F}_{Chan}$
- On receiving (INFO, msg) by  $\mathcal{A}$ , handle (msg) according to the corresponding rule in Fig 2 or Fig 3 (if any). After handling the message or after an "ensure" fails, send (HANDLED, msg) to  $\mathcal{A}$ . // (INFO, msg) messages by  $\mathcal{S}$  always return control to  $\mathcal{S}$  without any side-effect to any other ITI, except if  $\mathcal{F}_{Chan}$  halts

```
Functionality \mathcal{F}_{Chan} state machine, Pt. 1
     1: On initalisation:
     2:
                                pk_A \leftarrow \bot; pk_B \leftarrow \bot
     3:
                                balance_A \leftarrow 0; balance_B \leftarrow 0
                                 \texttt{is\_corrupted\_or\_negligent}_A \leftarrow \texttt{False}; \ \texttt{is\_corrupted\_or\_negligent}_B \leftarrow \texttt{False}_B \leftarrow \texttt{Fa
                 False;
     5:
                                 State \leftarrow \bot
     6: On (BECAME CORRUPTED OR NEGLIGENT, P) by \mathcal{A}:
                                 \texttt{is\_corrupted\_or\_negligent}_P \leftarrow \mathsf{True}
    8: On (INIT, pk) to P by \mathcal{E}:
                                if P = Bob then \bar{P} \leftarrow Alice else \bar{P} \leftarrow Bob
                                 ensure State \in \{\bot, INIT_{\bar{P}}\}
11:
                                 pk_P \leftarrow pk
12:
                                 if State = \bot then State \leftarrow \texttt{TENTATIVE-INIT}_P else State \leftarrow \texttt{TENTATIVE-INIT}_P
13: On (INIT, P) by A:
14:
                                 if State = \text{TENTATIVE-INIT}_P then State \leftarrow \text{INIT}_P
15:
                                 if State = \text{TENTATIVE-INIT} then State \leftarrow \text{INIT}
16: On (OPEN, x, y) to P by \mathcal{E}:
17:
                                 ensure State = INIT
18:
                                 balance_A \leftarrow x; balance_B \leftarrow y
19:
                                 State \leftarrow \texttt{TENTATIVE-OPEN}
20: On (OPEN) by A:
21:
                                 ensure State = \text{TENTATIVE-OPEN}
22:
                                   State \leftarrow \text{OPEN}
23: On (PAY, x) to P by \mathcal{E}:
                                 ensure State = OPEN
24:
25:
                                 if P = Alice then S \leftarrow Alice; \bar{S} \leftarrow Bob else S \leftarrow Bob; \bar{S} \leftarrow Alice
26:
27:
                                 State \leftarrow \text{Paying}
28: On (PAY OK) by A:
                                 ensure State = PAYING
30:
                                 balance_S \leftarrow balance_S - c; balance_{\bar{S}} \leftarrow balance_{\bar{S}} + c
                                 forget c, S, \bar{S}
31:
32:
                                 State \leftarrow \text{OPEN}
```

Fig. 2.

```
Functionality \mathcal{F}_{Chan} state machine, Pt. 2
 1: On (PAY FAIL) by A:
 2:
          ensure State = PAYING
 3:
          forget c, S, \bar{S}
 4:
          State \leftarrow \text{OPEN}
 5: On (FUND, x) to P by R:
 6:
          ensure R is an ITI that runs \mathcal{F}_{Chan} or \Pi_{Chan} code
 7:
          ensure State = OPEN
 8:
          c \leftarrow x
          if P = Alice then S \leftarrow Alice else S \leftarrow Bob
 9:
10:
          State \leftarrow \texttt{FUNDING}
11: On (fund ok) by A:
          ensure State = FUNDING
          balance_S \leftarrow balance_S - c
13:
14:
          forget c, S
15:
          State \leftarrow \text{OPEN}
16: On (FUND FAIL) by A:
          ensure State = FUNDING
17:
18:
          forget c, S
19:
          State \leftarrow \text{OPEN}
20: On (CLOSE) by \mathcal{E}:
          ensure State = OPEN
21:
22:
          State \leftarrow \texttt{CLOSING}
23: On (CLOSE) by A:
24:
          ensure State \in \{OPEN, CLOSING\}
25:
          for P \in \{A, B\} do
26:
               \label{eq:first-problem} \textbf{if} \ \texttt{is\_corrupted\_or\_negligent}_P = False \ \textbf{then}
27:
                    input (READ) to \mathcal{G}_{Ledger} as P and assign outut to \Sigma_P
28:
                    \mathtt{coins}_P \leftarrow \mathtt{sum} \ \mathtt{of} \ \mathtt{coins} \ \mathtt{exclusively} \ \mathtt{spendable} \ \mathtt{by} \ pk_P \ \mathtt{in} \ \Sigma_P
29:
               end if
          end for
30:
          \mathbf{if}\ (\mathtt{is\_corrupted\_or\_negligent}_A \lor \mathtt{coins}_A \ge \mathtt{balance}_A)
     \land (\mathtt{is\_corrupted\_or\_negligent}_B \lor \mathtt{coins}_B \geq \mathtt{balance}_B) \ \mathbf{then}
32:
               State \leftarrow \text{CLOSED}
33:
          else // balance security is broken
34:
               halt
          end if
35:
```

Fig. 3.

## Simulator $\mathcal{S}$

- On receiving (RELAY, in\_msg, P, R, in\_mode) by  $\mathcal{F}_{Chan}$  (in\_mode  $\in$  {input, output, network},  $P \in \{Alice, Bob\}$ ), handle (in\_msg) with the simulated party P as if it was received from R by means of in\_mode. In case simulated P does not exist yet, initialise it as an LN ITI. If there is a resulting message out\_msg that is to be sent by simulated P to R' by means of out\_mode  $\in$  {input, output, network}, send (RELAY, out\_msg, P, R', out\_mode) to  $\mathcal{F}_{Chan}$ .
- On receiving by  $\mathcal{F}_{Chan}$  a message to be sent by P to R via the network, carry on with this action (i.e. send this message via the internal  $\mathcal{A}$ ).
- Relay any other incoming message to the internal  ${\mathcal A}$  unmodified.
- On receiving a message (msg) by the internal  $\mathcal{A}$ , if it is addressed to one of the parties that correspond to  $\mathcal{F}_{Chan}$ , handle the message internally with the corresponding simulated party. Otherwise relay the message to its intended recipient unmodified. // Other recipients are  $\mathcal{E}$ ,  $\mathcal{G}_{Ledger}$  or parties unrelated to  $\mathcal{F}_{Chan}$

Given that  $\mathcal{F}_{Chan}$  relays all messages and that we simulate the real-world machines that correspond to  $\mathcal{F}_{Chan}$ , the simulation is perfectly indistinguishable from the real world.

```
Simulator S-Pt. 1
 1: On (OPEN, c_F, pk_{A,out}, pk_{B,out}, F, sig_FAlice) by \mathcal{F}_{Chan}: // both honest
         simulate Alice receiving input (OPEN, c_F, pk_{A, \text{out}}, pk_{B, \text{out}}) by \mathcal{E}
 3:
         ensure simulated Alice inputs (SUBMIT, (F', \operatorname{sig}_{F'})) to \mathcal{G}_{Ledger}
         input (SUBMIT, (F, \operatorname{sig}_F)) to \mathcal{G}_{\operatorname{Ledger}}
 5: On (OPEN, c_F, pk_{A,out}, pk_{B,out}, pk_{B,F}, Bob) by \mathcal{F}_{Chan}: // Alice corrupted
         send LN message (OPEN, pk_{B,F}) to Alice and relay reply to \mathcal{F}_{Chan} TODO:
    change msg to fit LN, ensure Alice doesn't see a difference from real world
 7: On (PAY, x, Dave) by \mathcal{F}_{Chan}:
         if both channel parties are honest then
 9:
             simulate Dave receiving input (PAY, x) by \mathcal{E}
             ensure simulated Dave outputs (OK)
10:
             send (OK) to \mathcal{F}_{Chan}
11:
12:
         else if only Dave's counterparty is corrupted then // else just relay to A
13:
             simulate Dave receiving input (PAY, x) by \mathcal{E}
             ensure simulated Dave outputs (OK)
14:
15:
             extract the latest commitment transaction C and its signature by
     \mathit{Dave} 's counterparty \mathrm{sig}_{\bar{D},C} from simulated \mathit{Dave} 's state
16:
             send (C, \operatorname{sig}_{\bar{D},C}) to \mathcal{F}_{\operatorname{Chan}}
17:
         end if
18: On (FUND YOU, c, Bob, Charlie, Alice) by \mathcal{F}_{Chan}:
         simulate Alice receiving input (FUND YOU, c, Bob) by Charlie
19:
20:
         ensure simulated Alice outputs (OK) to Charlie
21:
         send (OK) to \mathcal{F}_{Chan}
22: On (FUND c, hops, sub_parties = (fundee, counterparty), outer_parties =
     (Charlie, Dave), funder = Alice, id) by \mathcal{F}_{Chan}:
23:
         add the message data to virtual_opening
         simulate execution of line ?? of Fig. ?? with Alice// S knows Bob (Alice's
24:
    counterparty) through opening procedure
25:
         send (OK) to \mathcal{F}_{Chan}
26: On (ALLOW FUND, c, sub_parties, local_funder = L_i, id, i \stackrel{?}{=} |\text{hops}|) by
     \mathcal{F}_{Chan}'s Alice to Charlie:
         simulate receiving message with Charlie by Alice and all subsequent
27:
    communication
28:
         ensure the simulated Charlie sends (OK) to the simulated Alice
29:
         intercept this message and send it to \mathcal{F}_{Chan}'s Alice
```

Fig. 4.

### Simulator S- Pt. 2

- 1: On (IS OPEN SUCCESSFUL, id) by  $\mathcal{F}_{Chan}$ :
- 2: retrieve and remove from virtual\_opening the data marked with id
- 3: simulate line ?? of Fig. ?? with Alice using this data
- 4: ensure Alice completes execution of VChan() successfully
- 5: send (OK) to  $\mathcal{F}_{Chan}$
- 6: On (update to virtual ) by  $\mathcal{F}_{\text{Chan}}$ :
- 7: retrieve and remove from virtual\_opening the data marked with id
- 8: simulate line ?? of Fig. ?? with Alice using this data
- 9: ensure Alice completes execution of VChan() successfully
- 10: extract from Alice's state the new virtual funding TX V for pre-existing channel
- 11: extract from Alice's state the new commitment TX C that spends the on-chain funding TX
- 12: send (V, C) to  $\mathcal{F}_{Chan}$
- 13: On (fund done, id) by  $\mathcal{F}_{Chan}$ 's Alice to Charlie:
- 14: simulate receiving message with *Charlie* by *Alice* and all subsequent communication
- 15: ensure the simulated Charlie sends (OK) to the simulated Alice
- 16: intercept this message and send it to  $\mathcal{F}_{Chan}$ 's Alice

Fig. 5.

```
Process LN - init
 1: // When not specified, input comes from and output goes to \mathcal E in the real or
      P \in \{Alice, Bob\} in the ideal world.
 2:\ //\ \mbox{In} the real world, the player knows whether it is \mbox{\it Alice} (funder) or \mbox{\it Bob}
     (fundee). The activated party is P and the counterparty is \bar{P}.
 3: On (INIT, pk_{P,\text{out}}):
          \begin{array}{l} \text{ensure } \mathit{State}^P = \bot \\ \mathit{State}^P \leftarrow \mathsf{INIT} \end{array}
 4:
 5:
          store pk_{P,\mathrm{out}}
 6:
 7:
          (c_A, c_B, \mathtt{locked}_A, \mathtt{locked}_B) \leftarrow (0, 0, 0, 0)
          (paid\_out, paid\_in) \leftarrow (\emptyset, \emptyset)
 9:
          output (INIT OK)
10: On (TOP UP):
           ensure P = Alice // activated party is the funder
11:
           ensure State^P = INIT
12:
13:
           (sk_{P,\text{chain}}, pk_{P,\text{chain}}) \leftarrow \text{KEYGEN}()
          input (READ) to \mathcal{G}_{\mathrm{Ledger}} as P and assign outut to \Sigma
14:
15:
          output (top up to, pk_{P,\text{chain}})
16:
           while \nexists \operatorname{tx} \in \Sigma, c_{P,\operatorname{chain}} : (c_{P,\operatorname{chain}}, pk_{P,\operatorname{chain}}) \in \operatorname{tx.outputs} \operatorname{\mathbf{do}}
17:
                // while waiting, all other messages by P are ignored
18:
                wait for input (CHECK TOP UP)
                input (READ) to \mathcal{G}_{\texttt{Ledger}} as P and assign ouput to \varSigma
19:
20:
           end while
           State^P \leftarrow \texttt{TOPPED} \ \texttt{UP}
21:
          output (top up ok, c_{P,\mathrm{chain}})
22:
23: On (Balance):
          ensure State^P \in \{\text{OPEN}, \text{CLOSED}\}
24:
25:
           output (BALANCE, c_A, c_B, locked_A, locked_B)
```

Fig. 6.

```
Process LN – methods used by VIRT
 1: REVOKEPREVIOUS():
 2:
          ensure State^P \in WAITING FOR (OUTBOUND) REVOCATION
          R_{\bar{P},i} \leftarrow \text{TX } \{\text{input: } C_{P,i}.\text{outputs.} P, \text{output: } (C_{P,i}.\text{outputs.} P.\text{value,} \}
 3:
     pk_{\bar{P},\mathrm{out}})\}
          \operatorname{sig}_{A,R,i} \leftarrow \operatorname{SIGN}(R_{\bar{P},i},sk_{P,R})

if \operatorname{State}^P = \operatorname{WAITING} FOR REVOCATION then
 5:
               State^P \leftarrow \text{Waiting for inbound revocation}
 6:
          else // State<sup>P</sup> = WAITING FOR OUTBOUND REVOCATION
 7:
 8:
               i \leftarrow i + 1
               State^P \leftarrow \text{Waiting for hosts ready}
10:
          end if
11:
          \mathbf{return}\ \mathrm{sig}_{P,R,i}
12: PROCESSREMOTEREVOCATION(\operatorname{sig}_{\bar{P},R,i}):
13:
          ensure State^P = WAITING FOR (INBOUND) REVOCATION
14:
           R_{P,i} \leftarrow \text{TX \{input: } C_{\bar{P},i}.\text{outputs.}P, \text{ output: } (C_{\bar{P},i}.\text{outputs.}\bar{P}.\text{value,}
     pk_{P,\mathrm{out}})
15:
          ensure VERIFY(R_{P,i}, \operatorname{sig}_{\bar{P},R,i}, pk_{\bar{P},R}) = \operatorname{True}
16:
          if State^P = WAITING FOR REVOCATION then
               State^P \leftarrow \text{Waiting for outbound revocation}
17:
           else // State^P = Waiting for inbound revocation
18:
19:
               State^P \leftarrow \text{Waiting for hosts ready}
20:
21:
          end if
22:
          return (OK)
```

Fig. 7.

```
Process LN.EXCHANGEOPENKEYS()
 1: (sk_{A,F}, pk_{A,F}) \leftarrow \texttt{KEYGEN}(); (sk_{A,R}, pk_{A,R}) \leftarrow \texttt{KEYGEN}()
 2: if ideal world then
         ensure State^B = INIT
3:
         send (open channel, c, hops, pk_{A,F}, \, pk_{A,R}, \, pk_{A,\mathrm{out}}) to \mathcal A and expect reply
     (OPEN CHANNEL OK)
         (sk_{B,F}, pk_{B,F}) \leftarrow \text{KEYGEN}(); (sk_{B,R}, pk_{B,R}) \leftarrow \text{KEYGEN}()
 6: else // real world
         State^A \leftarrow \text{Waiting for opening keys}
 7:
         send (OPEN CHANNEL, c,\, \mathsf{hops},\, pk_{A,F},\, pk_{A,R},\, pk_{A,\mathrm{out}}) to fundee
 8:
         // colored code is run by honest fundee. Validation is implicit
9:
         ensure State^B = INIT
10:
         store pk_{A,F},\;pk_{A,R},\;pk_{A,\mathrm{out}}
11:
         (sk_{B,F}, pk_{B,F}) \leftarrow \text{KEYGEN}(); (sk_{B,R}, pk_{B,R}) \leftarrow \text{KEYGEN}()
12:
         if hops = \mathcal{G}_{Ledger} then // opening base channel
13:
              State^B \leftarrow \text{WAITING FOR COMM SIG}
14:
         else // opening virtual channel
15:
              State^B \leftarrow \text{Waiting for Check Keys}
16:
17:
18:
         reply (ACCEPT CHANNEL, pk_{B,F}, pk_{B,R}, pk_{B,out})
         ensure State^A = WAITING FOR OPENING KEYS
19:
         store pk_{B,F},\;pk_{B,R},\;pk_{B,\mathrm{out}}
20:
         State^A \leftarrow \text{Opening keys ok}
21:
22: end if
```

Fig. 8.

```
Process LN.PREPAREBASE()
 1: if hops = \mathcal{G}_{Ledger} then // opening base channel
 2:
          F \leftarrow \text{TX \{input: } (c, pk_{A, \text{chain}}), \text{ output: } (c, 3 \land 2/\{pk_{A,F}, pk_{B,F}\})\}
          \mathtt{host}_A \leftarrow \mathcal{G}_{\mathrm{Ledger}}
 3:
 4:
          if \ {\rm ideal} \ {\rm world} \ {\bf then}
 5:
               \mathtt{host}_B \leftarrow \mathcal{G}_{\mathrm{Ledger}}
               State^B \leftarrow \text{Waiting to Check funding}
 6:
 7:
          end if
 8: else // opening virtual channel
9:
          if ideal world then
     input (FUND ME, \bar{P}, hops, c, pk_{A,F}, pk_{B,F}) to hops[0].left and expect output (FUNDED, host_{\bar{P}}) to fundee by hops[-1].right // ignore any other
10:
     message
               input (FUND ACK) as fundee to hops[-1].right and expect output
11:
     (FUNDED, host_P) to funder by hops[0].left
12:
          else // real world
     input (FUND ME, Alice, Bob, hops, c, pk_{A,F}, pk_{B,F}) to hops[0].left and expect output (FUNDED, host_P) // ignore any other message
13:
          end if
15: end if
```

Fig. 9.

```
Process LN.EXCHANGEOPENSIGS()
   1: C_{A,0} \leftarrow \text{TX (input: } (c, 3 \land 2/\{pk_{A,F}, pk_{B,F}\}), \text{ outputs: } (c, (pk_{A,\text{out}} + t) \lor c
  \begin{array}{l} 2/\{pk_{A,R},pk_{B,R}\}),\,(0,\,pk_{B,\text{out}})\}\\ 2:\;C_{B,0}\leftarrow\text{TX \{input:}\;(c,\mathbf{3}\wedge2/\{pk_{A,F},pk_{B,F}\}),\,\text{outputs:}\;(c,\,pk_{A,\text{out}}),\,(0,\,pk_{A,B})\}\\ \end{array}
              (pk_{B,\text{out}} + t) \vee 2/\{pk_{A,R}, pk_{B,R}\})
   3: \operatorname{sig}_{A,C,0} \leftarrow \operatorname{SIGN}(C_{B,0},sk_{A,F})
   4: State^{A} \leftarrow Waiting for comm sig
   5: send (funding created, (c, pk_{A, \text{chain}}), \text{sig}_{A,C,0}) to fundee
   6: ensure State^B = WAITING FOR COMM SIG // if opening virtual channel, we
             have received (FUNDED, host_fundee) by hops[-1].right (c.f. Fig 12, line ??)
   7: if hops = \mathcal{G}_{Ledger} then // opening base channel
                          F \leftarrow \text{TX } \{\text{input: } (c, pk_{A, \text{chain}}), \text{ output: } (c, 3 \land 2/\{pk_{A,F}, pk_{B,F}\})\}
  9: end if
10: C_{B,0} \leftarrow \text{TX {input: }} (c, 3 \land 2/\{pk_{A,F}, pk_{B,F}\}), \text{ outputs: } (c, pk_{A,\text{out}}), (0, pk_
              (pk_{B,\text{out}} + t) \vee 2/\{pk_{A,R}, pk_{B,R}\})\}
11: ensure VERIFY(C_{B,0}, \operatorname{sig}_{A,C,0}, pk_{A,F}) = True 12: C_{A,0} \leftarrow \operatorname{TX} {input: (c, 3 \land 2/\{pk_{A,F}, pk_{B,F}\}), outputs: (c, (pk_{A,\mathrm{out}} + t) \lor pk_{B,F})
              2/\{pk_{A,R}, pk_{B,R}\}), (0, pk_{B,\text{out}})\}
13: \operatorname{sig}_{B,C,0} \leftarrow \operatorname{SIGN}(C_{A,0}, sk_{B,F})
14: if hops = \mathcal{G}_{Ledger} then // opening base channel
                           State^B \leftarrow \text{Waiting to Check funding}
15:
16: else // opening virtual channel
                          c_A \leftarrow c; c_B \leftarrow 0; i \leftarrow 0
17:
                           State^B \leftarrow \text{OPEN}
18:
19: end if
20: reply (FUNDING SIGNED, sig_{B,C,0})
21: ensure State^A = WAITING FOR COMM SIG
22: ensure VERIFY(C_{A,0}, \operatorname{sig}_{B,C,0}, pk_{B,F}) = \operatorname{True}
```

Fig. 10.

```
Process LN.COMMITBASE()

1: \operatorname{sig}_F \leftarrow \operatorname{SIGN}(F, sk_{A,\operatorname{chain}})
2: \operatorname{send} (\operatorname{OPEN}, c, pk_{A,\operatorname{out}}, pk_{B,\operatorname{out}}, F, \operatorname{sig}_F, Alice, Bob) to \mathcal{A}
3: while F \notin \Sigma do
4: wait for input (CHECK FUNDING) // ignore all other messages
5: input (READ) to \mathcal{G}_{\operatorname{Ledger}} as P and assign output to \Sigma
6: end while
```

Fig. 11.

```
Process LN – external open messages for Bob
 1: On input (CHECK FUNDING): // real or ideal world
        ensure State^B = WAITING TO CHECK FUNDING
 2:
        input (READ) to \mathcal{G}_{\mathrm{Ledger}} and assign output to \Sigma
3:
        if F \in \Sigma then
 4:
           State^B \leftarrow \text{OPEN}
 5:
            reply (OPEN OK)
 6:
 7:
        end if
8: On output (FUNDED, host_P) by hops[-1].right: // real world
        ensure \mathit{State}^B = \mathtt{WAITING} for funded
10:
        store host_P // we will talk directly to host_P
        State^B \leftarrow \text{Waiting for comm sig}
11:
12:
        reply (FUND ACK)
13: On output (CHECK KEYS, (pk_1, pk_2)) by hops[-1].right:
        ensure \mathit{State}^B = \mathtt{WAITING} FOR CHECK KEYS
        ensure pk_1 = pk_{A,F} \wedge pk_2 = pk_{B,F}
15:
        State^B \leftarrow \text{Waiting for fudned}
16:
17:
        reply (KEYS OK)
```

Fig. 12.

```
Process LN – On (OPEN, c, fundee, hops):
 1: // fundee is Bob
 2: ensure P = Alice // activated party is the funder
 3: if hops = \mathcal{G}_{Ledger} then // opening base channel
        ensure State^A = \text{TOPPED UP}
        ensure c = c_{A,\text{chain}}
 6: else // opening virtual channel
        ensure len(hops) \geq 2 // cannot open a virtual over 1 channel
 8: end if
 9: LN.EXCHANGEOPENKEYS()
10: LN.PREPAREBASE()
11: if real world then
        LN.EXCHANGEOPENSIGS()
13: else // ideal world
        send (FUNDING CREATED, (c, pk_{A,\text{chain}})) to \mathcal{A} and expect reply (FUNDING
    CREATED OK)
15: end if
16: if hops = \mathcal{G}_{Ledger} then
        LN.COMMITBASE()
18: else if ideal world then // opening virtual channel in ideal world
19:
        State^B \leftarrow \text{OPEN}
20: end if
21: c_A \leftarrow c; c_B \leftarrow 0; i \leftarrow 0
22: State^A \leftarrow \text{OPEN}
23: output (OPEN OK, c, fundee, hops)
```

Fig. 13.

```
Process LN.UPDATEFORVIRTUAL()

1: C_{\bar{P},i+1} \leftarrow C_{\bar{P},i} with pk'_{P,F} and pk'_{\bar{P},F} instead of pk_{P,F} and pk_{\bar{P},F} respectively

2: \operatorname{sig}_{P,C,i+1} \leftarrow \operatorname{SIGN}(C_{\bar{P},i+1}) // kept by \bar{P}

3: \operatorname{send} (UPDATE FORWARD, \operatorname{sig}_{P,C,i+1}) to \bar{P}

4: // P refers to payer and \bar{P} to payee both in local and remote code

5: C_{\bar{P},i+1} \leftarrow C_{\bar{P},i} with pk'_{P,F} and pk'_{\bar{P},F} instead of pk_{P,F} and pk_{\bar{P},F} respectively

6: \operatorname{ensure} VERIFY(C_{\bar{P},i+1}, \operatorname{sig}_{P,C,i+1}, pk'_{P,F}) = True

7: C_{P,i+1} \leftarrow C_{P,i} with pk'_{\bar{P},F} and pk'_{P,F} instead of pk_{\bar{P},F} and pk_{P,F} respectively

8: \operatorname{sig}_{\bar{P},C,i+1} \leftarrow \operatorname{SIGN}(C_{P,i+1},sk'_{\bar{P},F}) // kept by P

9: \operatorname{reply} (UPDATE BACK, \operatorname{sig}_{\bar{P},C,i+1})

10: C_{P,i+1} \leftarrow C_{P,i} with pk'_{\bar{P},F} and pk'_{P,F} instead of pk_{\bar{P},F} and pk_{P,F} respectively

11: \operatorname{ensure} VERIFY(C_{P,i+1}, \operatorname{sig}_{\bar{P},C,i+1}, pk'_{\bar{P},F}) = True
```

Fig. 14.

```
Process LN - virtualise start and end
 1: On input (fund me, fundee, hops, c_{\text{guest}}, \, pk_{A,V}, \, pk_{B,V}) by funder:
         ensure State^P = OPEN
 3:
         ensure c_P - \mathsf{locked}_P \ge c
         State^P \leftarrow \text{VIRTUALISING}
 4:
 5:
         (sk'_{P,F}, pk'_{P,F}) \leftarrow \text{KEYGEN}()
 6:
         define new VIRT ITI host'_P
         send (VIRTUALISING, \mathsf{host}_P',\ pk_{P,F}',\ \mathsf{hops},\ \mathsf{fundee},\ c_{\mathsf{guest}}) to \bar{P} and expect
    reply (virtualising ack, \mathtt{host}_{\bar{P}}',\ pk_{\bar{P},F}')
         ensure pk'_{\bar{P},F} is different from pk_{\bar{P},F} and all older \bar{P}'s funding public keys
         LN.UPDATEFORVIRTUAL()
 9:
         State^P \leftarrow \text{Waiting for Revocation}
         input (HOST ME, funder, fundee, host'_{\bar{P}}, host_{P}, c_{guest}, pk_{A,V}, pk_{B,V},
     (sk'_{P,F}, pk'_{P,F}), (sk_{P,F}, pk_{P,F}), pk_{\bar{P},F}, pk'_{\bar{P},F}) to host'
12: On output (HOSTS READY) by host'_P: // real or ideal world
         ensure State^P = WAITING FOR HOSTS READY
         State^P \leftarrow \text{OPEN}
14:
         host_P \leftarrow host'_P // forget old host, use new host instead
15:
         move pk_{P,F},\ pk_{\bar{P},F} to list of old funding keys
16:
         (sk_{P,F}, pk_{P,F}) \leftarrow (sk'_{P,F}, pk'_{P,F}); pk_{\bar{P},F} \leftarrow pk'_{\bar{P},F}
17:
         if len(hops) = 1 then // we are the last hop
18:
19:
             output (FUNDED, host_P) to fundee and expect reply (FUND ACK)
20:
         else if we have received input FUND ME just before we moved to the
     VIRTUALISING state then // we are the first hop
21:
             output (FUNDED, host<sub>P</sub>) to funder // do not expect reply by funder
22:
         end if
         reply (HOST ACK)
23:
24: On output (SIGN TXS, TXs) by host'<sub>P</sub>:
25:
         sigs \leftarrow \emptyset
26:
         for TX in TXs do
27:
             add SIGN(TX, sk_{P,F}, ANYPREVOUT) to sigs
28:
         end for
29:
         reply (TXs signed, sigs)
```

Fig. 15.

```
Process LN – virtualise hops
 1: On (VIRTUALISING, \operatorname{host}_{\bar{P}}', pk_{\bar{P}.F}', \operatorname{hops}, \operatorname{fundee}, c_{\operatorname{guest}}) by P:
          ensure State^P = OPEN
 2:
 3:
          ensure c_{\bar{P}} - \mathsf{locked}_{\bar{P}} \geq c
          ensure pk'_{\bar{P},F} is different from pk_{\bar{P},F} and all older \bar{P}'s funding public keys
          State^P \leftarrow VIRTUALISING
 5:
          locked_{\bar{P}} \leftarrow locked_{\bar{P}} + c // if \bar{P} is hosting the funder, \bar{P} will transfer c_{guest}
     coins instead of locking them, but the end result is the same
 7:
          (sk'_{P,F}, pk'_{P,F}) \leftarrow \text{KEYGEN}()
 8:
          if len(hops) > 1 then // we are not the last hop
 9:
               define new VIRT ITI host'<sub>P</sub>
10:
               input (VIRTUALISING, host'<sub>P</sub>, (sk'_{P,F}, pk'_{P,F}), pk'_{\bar{P},F}, \text{hops}[1:], fundee,
     c_{\text{guest}}, c_{\bar{P}}, c_{P}) to hops[1].left and expect reply (VIRTUALISING ACK,
     \mathtt{host\_sibling},\ pk_{\mathrm{sib},\bar{P},F})
               input (INIT, host<sub>P</sub>, host<sub>\bar{P}</sub>, host_sibling, (sk'_{P,F}, pk'_{P,F}), pk'_{\bar{P},F},
     pk_{\text{sib},\bar{P},F},~(sk_{P,F},~pk_{P,F}),~pk_{\bar{P},F},~c_{\text{guest}}) to \texttt{host}_P' and expect reply (HOST INIT
12:
          else // we are the last hop
13:
               input (INIT, host<sub>P</sub>, host'<sub>\bar{P}</sub>, fundee=fundee, (sk'_{P,F}, pk'_{P,F}), pk'_{\bar{P}|F},
     (sk_{P,F}, pk_{P,F}), pk_{\bar{P},F}, c_{\text{guest}}) to new VIRT ITI host'<sub>P</sub> and expect reply (HOST
          end if
14:
          State^P \leftarrow \text{Waiting for Revocation}
15:
          send (VIRTUALISING ACK, host'<sub>P</sub>, pk'_{P,F}) to \bar{P}
16:
17: On input (VIRTUALISING, host_sibling, (sk'_{P,F}, pk'_{P,F}), pk_{\text{sib},\bar{P},F}, \text{hops},
     fundee, c_{\text{guest}}, c_{\text{sib,rem}}, sib) by sibling:
          ensure State^P = OPEN
18:
          ensure c_P - \mathsf{locked}_P \ge c
19:
20:
          ensure c_{\rm sib,rem} \geq c_P \wedge c_{\bar{P}} \geq c_{\rm sib} // avoid value loss by griefing attack: one
     counterparty closes with old version, the other stays idle forever
          State^P \leftarrow VIRTUALISING
21:
22:
          locked_P \leftarrow locked_P + c
23:
          define new VIRT ITI host'P
          send (VIRTUALISING, host'_P, pk'_{P,F}, hops, fundee, c_{guest}) to hops[0].right
     and expect reply (VIRTUALISING ACK, host'_{\bar{P}}, pk'_{\bar{P},F})
25:
          ensure pk'_{\bar{P},F} is different from pk_{\bar{P},F} and all older \bar{P}'s funding public keys
26:
          LN.UPDATEFORVIRTUAL()
          \text{input (INIT, host}_P, \text{host}_{\bar{P}}', \text{host\_sibling, } (sk_{P,F}', \textit{pk}_{P,F}'), \textit{pk}_{\bar{P},F}', \textit{pk}_{\text{sib},\bar{P},F}',
27:
     (sk_{P,F}, pk_{P,F}), pk_{\bar{P},F}, c_{\text{guest}}) to \mathsf{host}_P' and expect reply (HOST INIT OK)
          State^P \leftarrow \text{Waiting for Revocation}
28:
29:
          output (VIRTUALISING ACK, host'_P, pk'_{\bar{P}|F}) to sibling
```

Fig. 16.

```
Process LN.SIGNATURESROUNDTRIP()

1: C_{\bar{P},i+1} \leftarrow C_{\bar{P},i} with x coins moved from P's to \bar{P}'s output

2: \operatorname{sig}_{P,C,i+1} \leftarrow \operatorname{SIGN}(C_{\bar{P},i+1},sk_{P,F}) \ / / \text{ kept by } \bar{P}

3: \operatorname{send} (PAY, x, \operatorname{sig}_{P,C,i+1}) to \bar{P}

4: //P refers to payer and \bar{P} to payee both in local and remote code

5: \operatorname{if} host _{\bar{P}} \neq \mathcal{G}_{\operatorname{Ledger}} \wedge \bar{P} has a host_sibling then // we are intermediary channel

6: \operatorname{ensure} c_{\operatorname{sib,rem}} \geq c_P - x \wedge c_{\bar{P}} + x \geq c_{\operatorname{sib}} \ / / \text{ avoid value loss by griefing attack}

7: \operatorname{end} if

8: C_{\bar{P},i+1} \leftarrow C_{\bar{P},i} with x coins moved from P's to \bar{P}'s output

9: \operatorname{ensure} \operatorname{VERIFY}(C_{\bar{P},i+1}, \operatorname{sig}_{P,C,i+1}, pk_{P,F}) = \operatorname{True}

10: C_{P,i+1} \leftarrow C_{P,i} with x coins moved from P's to \bar{P}'s output

11: \operatorname{sig}_{\bar{P},C,i+1} \leftarrow \operatorname{SIGN}(C_{P,i+1},sk_{\bar{P},F}) \ / / \text{ kept by } P

12: R_{P,i} \leftarrow \operatorname{TX} {input: C_{\bar{P},i}.outputs.P, output: (c_{\bar{P}}, pk_{P,\text{out}})}

13: \operatorname{sig}_{\bar{P},R,i} \leftarrow \operatorname{SIGN}(R_{P,i},sk_{\bar{P},R})

14: \operatorname{reply} (COMMITMENT SIGNED, \operatorname{sig}_{\bar{P},C,i+1}, \operatorname{sig}_{\bar{P},R,i})

15: C_{P,i+1} \leftarrow C_{P,i} with x coins moved from P's to \bar{P}'s output
```

Fig. 17.

```
Process LN.REVOCATIONSTRIP()
 1: ensure VERIFY(C_{P,i+1}, \operatorname{sig}_{\bar{P},C,i+1}, pk_{\bar{P},F}) = \operatorname{True}
 2: R_{P,i} \leftarrow \text{TX {input: }} C_{\bar{P},i}.\text{outputs.}P, \text{ output: } (c_{\bar{P}}, pk_{P,\text{out}})}
 3: ensure VERIFY(R_{P,i}, \operatorname{sig}_{\bar{P},R,i}, pk_{\bar{P},R}) = \operatorname{True}
 4: R_{\bar{P},i} \leftarrow \text{TX {input: }} C_{P,i}.\text{outputs.}\bar{P}, \text{ output: } (c_P, pk_{\bar{P},\text{out}})}
 5: \operatorname{sig}_{P,R,i}^{'} \leftarrow \operatorname{SIGN}(R_{\bar{P},i}, sk_{P,R})
 6: add x to paid_out
 7: c_P \leftarrow c_P - x; c_{\bar{P}} \leftarrow c_{\bar{P}} + x; i \leftarrow i + 1
 8: if host_P \neq \mathcal{G}_{Ledger} \land we have a host_sibling then // we are intermediary
 9:
          input (EW BALANCE, c_P, c_{\bar{P}}) to host<sub>P</sub>
          relay message as input to sibling // run by VIRT
10:
11:
          relay message as output to guest // run by VIRT
12:
          store new sibling balance and reply (NEW BALANCE OK)
13:
          output (NEW BALANCE OK) to sibling // run by VIRT
          output (NEW BALANCE OK) to guest // run by VIRT
15: end if
16: send (REVOKE AND ACK, \operatorname{sig}_{P,R,i}) to \bar{P}
17: R_{\bar{P},i} \leftarrow \text{TX {input: }} C_{P,i}.\text{outputs.}\bar{P}, \text{ output: }} (c_P, pk_{\bar{P},\text{out}})
18: ensure VERIFY(R_{\bar{P},i}, sig_{P,R,i}, pk_{P,R}) = True
19: add x to paid_in
20: c_P \leftarrow c_P - x; c_{\bar{P}} \leftarrow c_{\bar{P}} + x; i \leftarrow i + 1
21: if host_P \neq \mathcal{G}_{Ledger} \wedge \bar{P} has a host_sibling then // we are intermediary
22:
          input (NEW BALANCE, c_{\bar{P}}, c_{P}) to host _{\bar{P}}
23:
          relay message as input to sibling // run by VIRT
          relay message as output to guest // run by VIRT
24:
25:
          store new sibling balance and reply (NEW BALANCE OK)
26:
          output (NEW BALANCE OK) to sibling // run by VIRT
27:
          output (NEW BALANCE OK) to guest // run by VIRT
28: end if
```

Fig. 18.

```
Process LN – On (PAY, x):
 1: ensure State^P = OPEN \land c_P \ge x
 2: if host_P \neq \mathcal{G}_{Ledger} \wedge P has a host_sibling then // we are intermediary
    channel
        ensure c_{\rm sib,rem} \geq c_P - x \wedge c_{\bar{P}} + x \geq c_{\rm sib} // avoid value loss by griefing
    attack: one counterparty closes with old version, the other stays idle forever
 4: end if
 5: if ideal world then
        send (PAY, x) to \mathcal A and expect reply (PAY OK)
        ensure State^{\tilde{P}} = OPEN
        c_P \leftarrow c_B - x; c_{\bar{P}} \leftarrow c_{\bar{P}} + x; i \leftarrow i + 1
        if host_P \neq \mathcal{G}_{Ledger} \wedge P has a host_sibling then // we are intermediary
    channel
10:
            input (NEW BALANCE, c_P, c_{\bar{P}}) to \mathtt{host}_P
            relay message as input to sibling // run by VIRT
11:
12:
            relay message as output to \mathsf{guest} // run by \mathsf{VIRT}
            store new sibling balance and reply (NEW BALANCE OK)
13:
14:
            output (NEW BALANCE OK) to sibling // run by VIRT
15:
            output (NEW BALANCE OK) to guest // run by VIRT
16:
17: else
18:
        LN.SIGNATURESROUNDTRIP()
19:
        LN.REVOCATIONSTRIP()
20: end if
21: // No output is given to the caller, this is intentional
```

Fig. 19.

```
Process LN – On (CHECK FOR LATERAL CLOSE):

1: if host_P \neq \mathcal{G}_{Ledger} then State input (CHECK FOR LATERAL CLOSE) to host_P

2: end if
```

Fig. 20.

```
Process LN – On (CHECK CHAIN FOR OLD COMM):

1: ensure State^P \notin \{\bot, \text{INIT}, \text{TOPPED UP}\} // channel open

2: // even virtual channels check \mathcal{G}_{\text{Ledger}} directly. This is intentional

3: send (READ) to \mathcal{G}_{\text{Ledger}} as P and assign reply to \Sigma

4: if \exists 0 \leq j < i : C_{\bar{P},j} \in \Sigma then // counterparty has closed maliciously

5: State^P \leftarrow \text{CLOSING}

6: LN.SUBMITANDCHECKREVOCATION(j)

7: State^P \leftarrow \text{CLOSED}

8: end if
```

Fig. 21.

```
Process LN.SUBMITANDCHECKREVOCATION(j)

1: \operatorname{sig}_{P,R,j} \leftarrow \operatorname{SIGN}(R_{P,j}, sk_{P,R})
2: input (SUBMIT, (R_{P,j}, \operatorname{sig}_{P,R,j}, \operatorname{sig}_{\bar{P},R,j})) to \mathcal{G}_{\operatorname{Ledger}} as P
3: while \nexists R_{P,j} \in \Sigma do
4: wait for input (CHECK REVOCATION) // ignore other messages
5: input (READ) to \mathcal{G}_{\operatorname{Ledger}} as P and assign output to \Sigma
6: end while
```

Fig. 22.

```
Process LN - On (CLOSE):
 1: ensure State^P \notin \{\bot, INIT, TOPPED UP\} // channel open
 2: if host_P \neq \mathcal{G}_{Ledger} then // we have a virtual channel
        State^P \leftarrow \text{HOST CLOSING}
        input (CLOSE) to host_P and keep relaying inputs (CHECK CHAIN FOR
    CLOSING) to host_P until receiving output (CLOSED) by host_P
 5: end if
 6: State^P \leftarrow CLOSING
 7: input (READ) to \mathcal{G}_{Ledger} as P and assign output to \Sigma
 8: if C_{\bar{P},i} \in \Sigma then // counterparty has closed honestly
        no-op // do nothing
10: else if \exists 0 \leq j < i : C_{\bar{P},j} \in \Sigma then // counterparty has closed maliciously
         LN.SUBMITANDCHECKREVOCATION(j)
12: else // counterparty is idle
         while \nexists unspent output \in \Sigma that C_{P,i} can spend do // possibly due to an
    active timelock
14:
             wait for input (CHECK VIRTUAL) // ignore other messages
             input (READ) to \mathcal{G}_{Ledger} as P and assign output to \Sigma
15:
16:
         end while
         // provably reachable – c.f. TODO: ref
17:
        \operatorname{sig}_{P,C,i}' \leftarrow \operatorname{SIGN}(C_{P,i}, sk_{P,F})
18:
        input (SUBMIT, (C_{P,i}, \operatorname{sig}_{P,C,i}, \operatorname{sig}'_{P,C,i})) to \mathcal{G}_{\operatorname{Ledger}} as P
19:
20:
         while C_{P,i} \notin \Sigma do
21:
             wait for input (CHECK CLOSED) // ignore other messages
22:
             input (READ) to \mathcal{G}_{\text{Ledger}} as P and assign output to \Sigma
23:
         end while
24:
         // provably reachable - c.f. TODO: ref
25: end if
26: State^P \leftarrow CLOSED
27: output (Closed)
```

Fig. 23.

TODO: explicitly ensure that all hosts share the same sid – only if we decide that a functionality is needed after all

```
Process VIRT
 1: On input (INIT, host_P, \bar{P}, sibling, fundee, (sk_{loc,virt}, pk_{loc,virt}), pk_{rem,virt},
     pk_{\text{sib,rem,virt}}, (sk_{\text{loc},F}, pk_{\text{loc},F}), pk_{\text{rem},F}, c_{\text{guest}}) by guest:
         store message contents and guest // sibling, pk_{\mathrm{sib},\bar{P},F} are missing for
     edge nodes, fundee is present only in last node
         output (HOST INIT OK) to guest
 4: On input (HOST ME, funder, fundee, \bar{P}, host_P, c_{\mathrm{guest}}, pk_{\mathrm{left,guest}}, pk_{\mathrm{right,guest}},
     (sk_{\text{loc,virt}}, pk_{\text{loc,virt}}), (sk_{\text{loc,F}}, pk_{\text{loc,F}}), pk_{\text{rem,F}}, pk_{\text{rem,virt}}) by guest:
         ensure VIRT.CIRCULATEKEYSANDCOINS() returns (OK)
         ensure VIRT.CIRCULATEVIRTUALSIGS() returns (OK)
 6:
 7:
         ensure VIRT.CIRCULATEFUNDINGSIGS() returns (OK)
         ensure VIRT.CIRCULATEREVOCATIONS() returns (OK)
 9:
         output (HOSTS READY) to guest
10: CIRCULATEKEYSANDCOINS(left_data):
         if \ \texttt{left\_data} \ is \ given \ as \ argument \ \mathbf{then} \ // \ we \ are \ not \ \texttt{host\_funder}
11:
12:
              if we have a sibling then // we are not host_fundee
                   input (KEYS AND COINS FORWARD, (left_data, (sk_{loc,virt}, pk_{loc,virt}))
13:
     (sk_{\text{loc},F}, pk_{\text{loc},F}), pk_{\text{rem},F}, c_P, c_{\bar{P}}) to sibling
14:
                   store input as left_data
15:
                   parse left_data as far_left_data, (sk_{loc,virt}, pk_{loc,virt}), (sk_{sib,F}, pk_{loc,virt})
     pk_{\mathrm{sib},F}),\;pk_{\mathrm{sib,rem},F},\;c_{\mathrm{sib}},\;c_{\mathrm{sib,rem}} // remove parentheses as necessary
                   call virt.circulateKeysAndCoins(left_data) of \bar{P} and assign
16:
     returned value to right_data
                   parse right_data as far_right_data, pk_{\text{rem,virt}}
17:
                   output (KEYS AND COINS BACK, right_data, (sk_{loc,F}, pk_{loc,F}),
18:
19:
                   store output as right_data
20:
                   parse right_data as far_right_data, (sk_{\text{sib},F}, pk_{\text{sib},F}), pk_{\text{sib},\text{rem},F},
     c_{\rm sib}, c_{\rm sib, rem}
21:
                   \mathbf{return}\ (\mathtt{right\_data},\ pk_{\mathtt{loc,virt}})
22:
              else // we are host_fundee
23:
                   extract (pk_{\text{left,guest}}, pk_{\text{right,guest}}) from left_data
24:
                   output (CHECK KEYS, (pk_{\text{left,guest}}, pk_{\text{right,guest}})) to fundee and
     expect reply (KEYS OK)
25:
                  \mathbf{return}\ \mathit{pk}_{\mathrm{loc,virt}}
26:
27:
                // we are host_funder
28:
              call virt.circulate
KeysAndCoins(pk_{\text{loc,virt}}, (pk_{\text{left,guest}}, pk_{\text{right,guest}}))
     of \bar{P} and assign returned value to right_data
29:
              return (OK)
30:
          end if
```

Fig. 24.

```
Process VIRT
  1: GETMIDTXS(c_{\mathrm{guest}}, c_{\mathrm{loc}}, c_{\mathrm{rem}}, c_{\mathrm{sib}}, c_{\mathrm{sibRem}}, pk_{\mathrm{left,fund}}, pk_{\mathrm{loc,fund}}, pk_{\mathrm{sib,fund}},
           pk_{\text{right,fund}}, pk_{\text{left,virt}}, pk_{\text{loc,virt}}, pk_{\text{sib,virt}}, pk_{\text{right,virt}}, pk_{\text{left,guest}}, pk_{\text{right,guest}},
           pk_{\text{loc,out}}, \{pk_{\text{sec},i}\}_{i \in 1...n}):
  2:
                     ensure c_{\text{sibRem}} \ge c_{\text{guest}} \land c_{\text{loc}} \ge c_{\text{guest}}
  3:
                     c_{\text{left}} \leftarrow c_{\text{sib}} + c_{\text{sibRem}}; c_{\text{right}} \leftarrow c_{\text{loc}} + c_{\text{rem}}
                    \texttt{left\_fund} \leftarrow 2/\{pk_{\text{left,fund}}, pk_{\text{loc,fund}}\}
  4:
  5:
                     \texttt{right\_fund} \leftarrow 2/\{pk_{\texttt{sib},\texttt{fund}}, pk_{\texttt{right},\texttt{fund}}\}
                     \texttt{left\_virt} \leftarrow 2/\{pk_{\text{left,virt}}, pk_{\text{loc,virt}}\}
  6:
                     \texttt{left\_virt\_checked} \leftarrow 4/\{pk_{\text{left,virt}}, pk_{\text{loc,virt}}, pk_{\text{left,guest}}, pk_{\text{right,guest}}\}
  7:
                     \texttt{right\_virt} \leftarrow 2/\{pk_{\texttt{sib,virt}}, pk_{\texttt{right,virt}}\}
                     \texttt{right\_virt\_checked} \leftarrow 4/\{pk_{\text{sib,virt}}, pk_{\text{right,virt}}, pk_{\text{left,guest}}, pk_{\text{right,guest}}\}
  9:
10:
                     \texttt{left\_out\_checked} \leftarrow (2 \land \texttt{left\_virt\_checked}) \lor (3 \land \texttt{left\_virt} + t)
                     right_out \leftarrow (1 \land right_virt) \lor (3 \land right_virt + t)
11:
12:
                     \texttt{right\_out\_checked} \leftarrow (\texttt{1} \land \texttt{right\_virt\_checked}) \lor (\texttt{3} \land \texttt{right\_virt} + t)
13:
                      \texttt{guest\_all} \leftarrow 5 \land n / \{pk_{\text{left,guest}}, pk_{\text{right,guest}}, \{pk_{\text{sec},1...n}\}\}
                      \texttt{guest\_out} \leftarrow 4 \land 2/\{pk_{\text{left,guest}}, pk_{\text{right,guest}}\}
14:
                      \texttt{guest} \leftarrow (\texttt{guest\_out} + t) \lor \texttt{guest\_all}
15:
                      TX_{none} \leftarrow TX \text{ {inputs: }} ((c_{left}, left_fund), (c_{right}, right_fund)), outputs:
           ((c_{\text{left}} - c_{\text{guest}}, \, \texttt{left\_out\_checked}), \, (c_{\text{right}} - c_{\text{guest}}, \, \texttt{right\_out\_checked}),
           (c_{\mathrm{guest}},\,pk_{\mathrm{loc,out}}),\,(c_{\mathrm{guest}},\,\mathtt{guest}))\}
17:
                      TX_{left} \leftarrow TX \{left - C_{guest}, 1 \land left\_virt\_checked\}, (c_{right}, c_{right}, c_{ri
           right_fund)), outputs: ((c_{\text{left}} - c_{\text{guest}}, 3 \land \text{left\_virt}), (c_{\text{right}} - c_{\text{guest}},
          \texttt{right\_out\_checked}),\,(c_{\texttt{guest}},\,pk_{\texttt{loc},\texttt{out}}))\}
                      TX_{right} \leftarrow TX \text{ {inputs: }} ((c_{left}, left\_fund), (c_{right} - c_{guest}, 2 \land c_{left})
          \verb|right_virt_checked||, (c_{\text{guest}}, \verb|guest_all||)|, outputs: ((c_{\text{left}} - c_{\text{guest}},
          left_out_checked), (c_{right} - c_{guest}, 3 \land right_virt), (c_{guest}, pk_{loc,out}), (c_{guest}, pk_{loc,out})
           guest))}
19:
                     TX_{both} \leftarrow TX \text{ [inputs: } ((c_{left} - c_{guest}, 1 \land left\_virt\_checked),
           (c_{\text{right}} - c_{\text{guest}}, 2 \land \text{right\_virt\_checked}), (c_{\text{guest}}, \text{guest\_all})), \text{ outputs:}
           ((c_{\text{left}} - c_{\text{guest}}, 3 \land \text{left\_virt}), (c_{\text{right}} - c_{\text{guest}}, 3 \land \text{right\_virt}),
           (c_{\text{guest}}, pk_{\text{loc,out}})
20:
                     \mathbf{return}\ (TX_{\mathrm{none}},\ TX_{\mathrm{left}},\ TX_{\mathrm{right}},\ TX_{\mathrm{both}})
```

Fig. 25.

#### Process VIRT 1: // left and right refer to the two counterparties, with left being the one closer to the funder. Note difference with left/right meaning in VIRT.GETMIDTXS. 2: GETEDGETXS( $c_{\mathrm{guest}}, \, c_{\mathrm{left}}, \, c_{\mathrm{right}}, \, pk_{\mathrm{left,fund}}, \, pk_{\mathrm{right,fund}}, \, pk_{\mathrm{left,virt}}, \, pk_{\mathrm{right,virt}},$ $pk_{\text{left,guest}}, pk_{\text{right,guest}}, \{pk_{\text{sec},i}\}_{i \in 1...n}, \text{ is\_funder})$ : 3: ensure $c_{\text{left}} \ge c_{\text{guest}}$ 4: $c_{\text{tot}} \leftarrow c_{\text{left}} + c_{\text{right}}$ $\texttt{fund} \leftarrow 2/\{pk_{\text{left,fund}}, pk_{\text{right,fund}}\}$ 5: $\texttt{virt} \leftarrow 2/\{pk_{\text{left,virt}}, pk_{\text{right,virt}}\}$ 7: $\texttt{virt\_checked} \leftarrow 4/\{pk_{\text{left,virt}}, pk_{\text{right,virt}}, pk_{\text{left,guest}}, pk_{\text{right,guest}}\}$ $if is\_funder = True then$ 9: $\mathtt{out} \leftarrow (\texttt{1} \land \mathtt{virt\_checked}) \lor (\texttt{3} \land \mathtt{virt} + t)$ else // TXs belong to fundee 10: $\mathtt{out} \leftarrow (\mathtt{2} \land \mathtt{virt\_checked}) \lor (\mathtt{3} \land \mathtt{virt} + t)$ 11: 12: end if $\texttt{guest\_all} \leftarrow \texttt{5} \land n/\{pk_{\text{left,guest}}, pk_{\text{right,guest}}, \{pk_{\text{sec},1...n}\}\}$ 13: $\texttt{guest\_out} \leftarrow \texttt{4} \land 2/\{pk_{\text{left,guest}}, pk_{\text{right,guest}}\}$ 14: 15: $\texttt{guest} \leftarrow (\texttt{guest\_out} + t) \lor \texttt{guest\_all}$ $TX_{base} \leftarrow TX \text{ (input: } (c_{tot}, \text{ fund), outputs: } ((c_{tot} - c_{guest}, \text{ out), } (c_{guest}, \text{ out), } (c_{guest}, \text{ out), } (c_{guest}, \text{ out})$ 16: $guest))\}$ $\mathbf{return}\ \mathrm{TX}_{\mathrm{base}}$ 17:

Fig. 26.

```
Process VIRT.SIBLINGSIGS()
  1: parse input as sigs<sub>byLeft</sub>
  c_{\bar{P}}, c_{\rm sib}, c_{\rm sib,rem}, pk_{\rm sib,rem,F}, pk_{\rm sib,F}, pk_{\rm loc,F}, pk_{\rm rem,F}, pk_{\rm sib,rem,virt}, pk_{\rm loc,virt}
         pk_{\text{loc,virt}}, pk_{\text{rem,virt}}, pk_{\text{left,guest}}, pk_{\text{right,guest}}, pk_{\text{loc,virt}}, \{pk_{\text{sec},i}\}_{i \in 1...n})
  3: store all signatures in sigs_{byLeft} that sign any of TX_{loc,none}, TX_{loc,left},
         TX_{loc,right}, TX_{loc,both} and remove these signatures from sigs_{byLeft}
  4: ensure that the stored signatures contain one valid signature for TX_{loc,right}
         and TX<sub>loc,both</sub> which sign the guest_all input by each one of the previous
        i-1 hops
  5: ensure that there are exactly 4 more valid signatures in the stored signatures,
         which sign the 1 \land left\_virt\_checked inputs of TX_{loc,left} and TX_{loc,both} with
         pk_{\rm sib,rem,virt} and pk_{\rm left,guest}
  6: sigs_{toRight} \leftarrow sigs_{bvLeft}
  7: for each hop apart from the first, the last and ours (i \in [2, ..., n-1] \setminus \{j\}) do
         //~j is our hop number, hop data encoded in left_data and right_data
                extract data needed for GETMIDTXs() from left_data (if i < j) or
        right_data (if i > j) and assign it to data, and \{pk_{\text{sec},i}\}_{i \in 1...n} // P and
         comm_keys are missing, that is OK. \{pk_{\text{sec},i}\}_{i\in 1...n} contains each party's pk_{i,\text{virt}}
                (TX_{i,\text{none}}, TX_{i,\text{left}}, TX_{i,\text{right}}, TX_{i,\text{both}}) \leftarrow VIRT.GETMIDTXS(data_i,
         \{pk_{\text{sec},i}\}_{i\in 1...n}
                 add SIGN(TX_{i,right}, sk_{loc,virt}, ANYPREVOUT) and SIGN(TX_{i,both}, sk_{loc,virt},
         Anyprevout) to \operatorname{sigs}_{\text{toLeft}} if i < j, or \operatorname{sigs}_{\text{toRight}} if i > j // if i-th hop is
        adjacent, 2 signatures will be produced by each SIGN() invocation: one for the
         guest_all and one for the 2 ∧ right_virt_checked input
                if i - j = 1 then // hop is our next
11:
                         add SIGN(TX_{i,left}, sk_{loc,virt}, ANYPREVOUT) to sigs_{toRight}
12:
13:
                 else if j - i = 1 then // hop is our previous
                         add SIGN(TX<sub>i,left</sub>, sk_{loc,virt}, ANYPREVOUT) to sigs<sub>toLeft</sub>
14:
15:
                 end if
16: end for
17: if right_data does not contain data from a second-next hop then // next
         hop is host_fundee
                \mathrm{TX}_{\mathrm{next,none}} \leftarrow \mathrm{VIRT.GETEDGETXS}(c_{\mathrm{guest}},\, c_P,\, c_{\bar{P}},\, pk_{\mathrm{loc},F},\, pk_{\mathrm{rem},F},\, pk_{\mathrm{loc},\mathrm{virt}},\, pk_{\mathrm{loc},F},\, pk_{\mathrm{loc},F
18:
         pk_{\text{rem,virt}}, pk_{\text{left,guest}}, pk_{\text{right,guest}}, \text{False})
20: call P.CIRCULATEVIRTUALSIGS(sigs<sub>toRight</sub>) and assign returned value to
         sigs_{byRight}
21: store all signatures in sigs<sub>byRight</sub> that sign any of TX<sub>loc,none</sub>, TX<sub>loc,left</sub>,
         TX<sub>loc,right</sub>, TX<sub>loc,both</sub> and remove these signatures from sigs<sub>byRight</sub>
        ensure that the stored signatures contain one valid signature for TX<sub>loc,right</sub> and
         TX_{loc,both} which sign the guest_all input by each one of the next n-i hops
23: ensure that there are exactly 4 more valid signatures in the stored signatures,
         which sign the 2 \land right\_virt\_checked inputs of TX_{loc,right} and TX_{loc,both}
         with pk_{\text{rem.virt}} and pk_{\text{right,guest}}
24: output (VIRTUALSIGSBACK, sigs_{toLeft}, sigs_{byRight})
```

Fig. 27.

```
Process VIRT.INTERMEDIARYSIGS()

1: (TX_{loc,none}, TX_{loc,left}, TX_{loc,right}, TX_{loc,both}) \leftarrow VIRT.GETMIDTXS(c_{guest}, c_P, c_{\bar{P}}, c_{sib}, c_{sib,rem}, pk_{loc,F}, pk_{rem,F}, pk_{sib,rem}, pk_{rem,virt}, pk_{loc,virt}, pk_{loc,virt}
```

Fig. 28.

```
Process VIRT.HOSTFUNDEESIGS()
    1: \text{TX}_{\text{loc,none}} \leftarrow \text{VIRT.GETEDGETXS}(c_{\text{guest}}, c_P, c_{\bar{P}}, pk_{\text{loc},F}, pk_{\text{rem},F}, pk_{\text{loc,virt}}, c_{\bar{P}}, pk_{\text{loc},F}, pk_{\text{rem},F}, pk_{\text{loc},F}, pk_{\text{lo
               pk_{\text{rem,virt}}, pk_{\text{left,guest}}, pk_{\text{right,guest}}, \text{False}
   2: for each hop apart from the first and ours (i \in [2, ..., n-1]) do // hop data
              encoded in left_data
                           extract data needed for GETMIDTXs() from left_data and assign it to
              \mathtt{data}_i and \{pk_{\sec,i}\}_{i\in 1...n} // \{pk_{\sec,i}\}_{i\in 1...n} contains each party's pk_{i,\text{virt}}
                           (TX_{i,none}, TX_{i,left}, TX_{i,right}, TX_{i,both}) \leftarrow VIRT.GETMIDTXS(data_i, TX_{i,none}, TX_{i,left}, TX_{i,right}, TX_{i,both})
               \{pk_{\text{sec},i}\}_{i\in 1...n}
                           add SIGN(TX_{i,right}, sk_{loc,virt}, ANYPREVOUT) and SIGN(TX_{i,both}, sk_{loc,virt},
              Anyprevout) to sigs_{toLeft} // if i-th hop is adjacent, 2 signatures will be
              produced by each SIGN() invocation: one for the guest_all and one for the
               2 ∧ right_virt_checked input
                           output (SIGN TXS, TX_{i,left}, TX_{i,right}, TX_{i,both}) to fundee and expect reply
               (TXS SIGNED, sigs_{guest})
   7:
                           add \mathrm{sigs}_{\mathrm{guest}} to \mathrm{sigs}_{\mathrm{toLeft}}
   8:
                           if i = n - 1 then // hop is our previous
  9:
                                         add SIGN(TX_{i,left}, sk_{loc,virt}, ANYPREVOUT) to sigs_{toLeft}
10:
                             end if
11: end for
12: \mathbf{return} \ \mathrm{sigs}_{\mathrm{toLeft}}
```

Fig. 29.

```
Process VIRT.HOSTFUNDERSIGS()
     1: for each hop apart from the last and ours (i \in [2, ..., n-1]) do // hop data
                  encoded in right_data
                                   extract data needed for GETMIDTXs() from right_data and assign it to
                   \mathtt{data}_i and \{pk_{\mathtt{sec},i}\}_{i\in 1...n} // \{pk_{\mathtt{sec},i}\}_{i\in 1...n} contains each party's pk_{i,\mathrm{virt}}
                                   (TX_{i,none}, TX_{i,left}, TX_{i,right}, TX_{i,both}) \leftarrow VIRT.GETMIDTXS(data_i, TX_{i,none}, TX_{i,left}, TX_{i,right}, TX_{i,both})
                    \{pk_{\text{sec},i}\}_{i\in 1...n}
                                  add SIGN(TX_{i,right}, sk_{loc,virt}, ANYPREVOUT) and SIGN(TX_{i,both}, sk_{loc,virt}, sk_{l
                  ANYPREVOUT) to \operatorname{sigs_{toRight}} // if i-th hop is adjacent, 2 signatures will be
                   produced by each SIGN() invocation: one for the guest_all and one for the
                  2 \land \texttt{right\_virt\_checked} \ input
                                  output (SIGN TXS, TX_{i,left}, TX_{i,right}, TX_{i,both}) to fundee and expect reply
                   (TXS SIGNED, sigs_{guest})
    6:
                                   add sigs_{guest} to sigs_{toRight}
    7:
                                   if i = 2 then // hop is our next
    8:
                                                     add SIGN(TX_{i,left}, sk_{loc,virt}, ANYPREVOUT) to sigs_{toRight}
                                   end if
   9:
10: end for
11: call VIRT.CIRCULATEVIRTUALSIGS(sigs_{toRight}) of P and assign output to
12: TX_{loc,none} \leftarrow VIRT.GETEDGETXS(c_{guest}, c_P, c_{\bar{P}}, pk_{loc,F}, pk_{rem,F}, pk_{loc,virt}, pk_{rem,F}, pk_{rem,F},
                    pk_{\text{rem,virt}}, pk_{\text{left,guest}}, pk_{\text{right,guest}}, \text{True})
13: return (OK)
```

Fig. 30.

```
Process VIRT.CIRCULATEVIRTUALSIGS(sigs<sub>byLeft</sub>)

1: if sigs<sub>byLeft</sub> is given as argument then // we are not host_funder

2: if we have a sibling then // we are not host_fundee

3: return VIRT.INTERMEDIARYSIGS()

4: else // we are host_fundee

5: return VIRT.HOSTFUNDEESIGS()

6: end if

7: else // we are host_funder

8: return VIRT.HOSTFUNDERSIGS()

9: end if
```

Fig. 31.

```
\textbf{Process} \ \operatorname{VIRT.CIRCULATEFUNDINGSigs}(\operatorname{sig}_{\operatorname{loc,none}})
 1: if sig_{loc,none} is given as argument then // we are not host_funder
 2:
             ensure VERIFY(TX_{loc,none}, sig_{loc,none}, pk_{prev,F}) = True // pk_{prev,F}, found in
      left_data
 3:
             sigs_{loc,none} \leftarrow \{sig_{loc,none}\}
             if we have a sibling then // we are not host_fundee
                  input (VIRTUAL BASE SIG FORWARD, \mathrm{sig_{loc,none}}) to \mathtt{sibling} // \mathtt{sibling}
 5:
       needs sig_{loc,none} for closing
 6:
                   \operatorname{sigs}_{\operatorname{loc,none}} \leftarrow \{\operatorname{sig}_{\operatorname{loc,none}}\}
                   \operatorname{sig}_{\operatorname{next,none}} \leftarrow \operatorname{SIGN}(\operatorname{TX}_{\operatorname{next,none}}, sk_{\operatorname{loc},F})
 7:
                   call virt.circulate
VirtualSigs(sig_{next,none}) of \bar{P} and assign returned
                  ensure VERIFY(TX<sub>loc,none</sub>, sig<sub>loc,none</sub>, pk_{\text{next},F}) = True // pk_{\text{next},F},
       found in right_data
                   add \mathrm{sig}_{\mathrm{loc,none}} to \mathrm{sigs}_{\mathrm{loc,none}}
10:
                   output (VIRTUAL BASE SIG BACK, sig<sub>loc,none</sub>) // sibling needs sig<sub>loc,none</sub>
       for closing
12:
                  add \mathrm{sig}_{\mathrm{loc,none}} to \mathrm{sigs}_{\mathrm{loc,none}}
13:
             end if
             \mathrm{sig}_{\mathrm{prev},\mathrm{none}} \leftarrow \mathrm{SIGN}(\mathrm{TX}_{\mathrm{prev},\mathrm{none}},\mathit{sk}_{\mathrm{loc},\mathit{F}})
14:
             \mathbf{return} \ \mathrm{sig}_{\mathrm{prev},\mathrm{none}}
15:
16: else // we are host_funder
             \mathrm{sig}_{\mathrm{next,none}} \leftarrow \mathrm{SIGN}(\mathrm{TX}_{\mathrm{next,none}}, \mathit{sk}_{\mathrm{loc},\mathit{F}})
17:
             call VIRT.CIRCULATEFUNDINGSIGS(sig_{next,none}) of \bar{P} and assign returned
18:
       value to sig_{loc,none}
             ensure Verify(TX<sub>loc,none</sub>, \operatorname{sig}_{\operatorname{loc,none}}, \operatorname{pk}_{\operatorname{next},F}) = True // \operatorname{pk}_{\operatorname{next},F} found in
19:
20:
             sigs_{loc,none} \leftarrow \{sig_{loc,none}\}
21:
             return (OK)
22: end if
```

Fig. 32.

```
Process VIRT.CIRCULATEREVOCATIONS(revoc_by_prev)
1: if revoc_by_prev is given as argument then // we are not host_funder
      ensure guest.PROCESSREMOTEREVOCATION(revoc_by_prev) returns (OK)
3: else // we are host_funder
      revoc_for_next ← guest.REVOKEPREVIOUS()
      call virt.circulateRevocations(revoc_for_next) of \bar{P} and assign
   returned value to revoc_by_next
      ensure guest.PROCESSREMOTEREVOCATION(revoc_by_next) returns (OK)
7:
      return (OK)
8: end if
9: if we have a sibling then // we are not host_funder
      input (VIRTUAL REVOCATION FORWARD) to sibling
      revoc_for_next \( \to \) guest.REVOKEPREVIOUS()
12:
      call virt.circulateRevocations(revoc_for_next) of \bar{P} and assign
   output to revoc_by_next
      ensure guest.PROCESSREMOTEREVOCATION(revoc_by_next) returns (OK)
13:
14:
      output (VIRTUAL REVOCATION BACK)
15: end if
16: revoc_for_prev ← guest.REVOKEPREVIOUS()
17: output (HOSTS READY) to guest and expect reply (HOST ACK)
18: return revoc_for_prev // we are not host_fundee nor host_funder
```

Fig. 33.

```
Process VIRT - poll
 1: On input (CHECK FOR LATERAL CLOSE) by R \in \{\text{guest}, \text{funder}, \text{fundee}\}:
         input (READ) to \mathcal{G}_{\mathrm{Ledger}} and assign output to \Sigma
 2:
 3:
         \texttt{prev\_went\_on\_chain} \leftarrow TX_{\texttt{prev}, \texttt{left}} \in \varSigma \land TX_{\texttt{prev}, \texttt{both}} \in \varSigma
 4:
         next\_went\_on\_chain \leftarrow TX_{next,right} \in \Sigma \land TX_{next,both} \in \Sigma
 5:
         if prev_went_on_chain \( \times \) next_went_on_chain then
 6:
              ignore all messages except for (CHECK CHAIN FOR CLOSING) by R
 7:
              State \leftarrow \text{CLOSING}
 8:
         end if
 9:
         \mathbf{if} \ \mathtt{prev\_went\_on\_chain} \land \mathtt{next\_went\_on\_chain} \ \mathbf{then}
              virt.signAndSubmit(TX_{loc,both},\,sigs_{loc,both})
10:
11:
          else if prev_went_on_chain then
              {\tt VIRT.SIGNANDSUBMIT}(TX_{\tt loc,left},\,sigs_{\tt loc,left})
12:
13:
          else if next_went_on_chain then
              {\tt VIRT.SIGNANDSUBMIT}(TX_{\rm loc,right},\,sigs_{\rm loc,right})
14:
15:
          end if
16: VIRT.SIGNANDSUBMIT(tx, sigs):
17:
          add SIGN(tx, sk_{loc,F}) to sigs
18:
          input (SUBMIT, tx, sigs) to \mathcal{G}_{\mathrm{Ledger}}
```

Fig. 34.

```
Process VIRT - close
 1: On input (CLOSE) by R \in \{\text{guest}, \text{funder}, \text{fundee}\}: // At most one of funder,
    fundee is defined
 2:
        if State = CLOSED then
 3:
            output (CLOSED) to R
        end if
 4:
 5:
        ensure State = OPEN
 6:
        if host_P \neq \mathcal{G}_{Ledger} then // host is a VIRT
 7:
            ignore all messages except for output (CLOSED) by host. Also relay to
    host<sub>P</sub> any (CHECK CHAIN FOR CLOSING) input received
 8:
            input (CLOSE) to host_P
 9:
        end if
         // if we have a host_P, continue from here on output (CLOSED) by it
10:
        send (READ) to \mathcal{G}_{\text{Ledger}} as R and assign reply to \Sigma
11:
         let tx be the unique valid TX for \Sigma among (TX_{loc,none}, TX_{loc,left},
12:
    TX_{loc,right}, TX_{loc,both}) // if we are not an intermediary, only the first exists
13:
        let sigs be the corresponding set of signatures among (sigs<sub>loc,none</sub>,
    sigs_{loc,left}, sigs_{loc,right}, sigs_{loc,both}
         add SIGN(tx, sk_{A,F}) and SIGN(tx, sk_{loc,virt}) to sigs // one of the two
14:
    signatures may be empty, as some transactions don't need a signature by both
    keys. This is not a problem.
15:
        ignore all messages except for (CHECK CHAIN FOR CLOSING) by R
         State \leftarrow \text{closing}
16:
17:
        send (SUBMIT, (tx, sigs)) to \mathcal{G}_{Ledger}
18: On (CHECK CHAIN FOR CLOSING) by R \in \{\text{guest}, \text{funder}, \text{fundee}\}:
        ensure State = CLOSING
19:
20:
        send (READ) to \mathcal{G}_{\text{Ledger}} as R and assign reply to \Sigma
21:
        if R = guest then
22:
            pk_1 \leftarrow pk_{\text{left,guest}}; pk_2 \leftarrow pk_{\text{right,guest}}
23:
         else //R \in \{\text{funder}, \text{fundee}\}
24:
            pk_1 \leftarrow pk_{\text{loc,virt}}; pk_2 \leftarrow pk_{\text{rem,virt}}
25:
         if \Sigma has an unspent output that can be spent exclusively by a 2-of-\{pk_1,
    pk_2 multisig then // if there is a timelock, it must have expired
27:
             State \leftarrow CLOSED
28:
            output (CLOSED) to R
29:
         end if
```

Fig. 35.

# 1 Security Proof

When  $\mathcal{E}$  sends (FUND, c, hops, (fundee, counterparty), (*Charlie*, *Dave*),  $pk_{VA,out}$ ,  $pk_{VB,out}$ ) to *Alice* in the real world, lines ??-?? of Fig. ?? are executed and then control is handed over to the "fundee" ITI, which executes lines ??-?? of Fig. ??. This ITI will output (OK) if and only if line ?? of Fig. ?? succeeds.

When  $\mathcal{E}$  sends (FUND, c, hops, (fundee, counterparty), (*Charlie*, *Dave*)) to *Alice* in the ideal world, lines ??-?? of Fig. ?? are executed and then control is handed over to the functionality that controls the "fundee", which executes lines ??-?? of Fig. ?? and then hands control over to  $\mathcal{E}$ . The latter in turn simulates lines ??-?? of Fig. ??, thus following the exact same steps as in the real world, therefore it will send (OK) to  $\mathcal{F}_{Chan}$  if and only if the simulated line ?? of Fig. ?? succeeds. From this and the previous paragraph, we see that, up to this point, the two worlds are perfectly indistinguishable.

Moving on, in the ideal world subsequently lines ??-?? of Fig. ?? are executed, which results in S executing lines 22-25 of Fig. 4. During the latter steps, S simulates executing line ?? of Fig. ?? with *Alice*.

Similarly in the real world, *Alice* executes lines ?? and ?? of Fig. ??, therefore the two worlds still are perfectly indistinguishable.

The "for" loop of lines ??-?? of Fig. ?? is then executed in both the real and the ideal worlds. The message of line ?? results in the execution of lines ??-?? of Fig. ?? by  $L_i$  in both worlds: in the real world directly, in the ideal world simulated by S.

In the ideal world, line ?? in Fig. ?? prompts  $\mathcal{S}$  to simulate line ?? of Fig. ?? with Alice, which is exactly the code that would be directly run by Alice in the real world. Therefore the two worlds remain perfectly indistinguishable.

The "for" loop of lines ??-?? of Fig. ?? is also perfectly indistinguishable in the two worlds. With argumentation similar to that of the previous "for" loop, we conclude that the FUND message does not induce any chance of distinguishability between the two worlds.

**Theorem 1.** Assume that at the end of the execution,  $\mathcal{G}_{Ledger}$  contains exactly one "groups" transaction that precedes all "funding" transactions and contains as payload a partition  $\mathcal{G}$  into groups of all VChan parties, with each group containing the parties that belong to the same (human) owner. Then the following holds:

$$\forall G \in \mathcal{G} \ such \ that \ all \ parties \ in \ G \ are \ honest,$$
 
$$\sum_{P \in G} \operatorname{logged-coins}(P) = \sum_{P \in G} \operatorname{ledger-coins}(P) =$$
 
$$= \sum_{P \in G} (\operatorname{top-up}(P) + \sum_{m \in \mathcal{T}} \operatorname{pay-in}(m,P) - \sum_{m \in \mathcal{T}} \operatorname{pay-out}(m,P)) \ ,$$

where T is the execution transcript and:

$$\label{eq:coins} \begin{split} \operatorname{logged-coins}(P) &= c_P, \text{ as recorded in } \mathcal{F}_{\operatorname{Chan}}/\Pi_{\operatorname{Chan}} \\ \operatorname{ledger-coins}(P) &= \operatorname{coins spendable with the secret key sk of } P \text{ if the closing} \\ \operatorname{transactions of all open channels are submitted to } \mathcal{G}_{\operatorname{Ledger}} \\ \operatorname{and added to the state of all parties and then } t \text{ new blocks} \\ \operatorname{enter the state of all honest parties} \\ \operatorname{top-up}(P) &= \begin{cases} c_{\operatorname{on}}, \text{ as determined on message (CHECK TOP UP),} \\ \operatorname{if such a message was handled} \\ 0, & \operatorname{otherwise} \end{cases} \\ \operatorname{pay-in}(m,P) &= \begin{cases} x, & \text{if message m updated the channel to} \\ a \text{ state in which } P \text{ had } x \text{ more coins } TODO: \text{ improve prev} \\ 0, & \operatorname{otherwise} \end{cases} \\ \operatorname{pay-out}(m,P) &= \begin{cases} x, & \text{if } m = (\operatorname{PAY}, x) \text{ was received by } P \text{ and} \\ P \text{ output (PAY SUCCESS) as a result} \\ 0, & \operatorname{otherwise} \end{cases} \\ \end{split}$$

# References