Elmo: Recursive Virtual Payment Channels for Bitcoin

Anonymised Submission

ABSTRACT

A dominant approach towards the solution of the scalability problem in blockchain systems has been the development of layer 2 protocols and specifically payment channel networks (PCNs) such as the Lightning Network (LN) over Bitcoin. Routing payments over LN requires the coordination of all path intermediaries in a multi-hop round trip that encumbers the layer 2 solution both in terms of responsiveness as well as privacy. The issue is resolved by "virtual channel" protocols that, capitalizing on a suitable off-chain setup operation, enable the two endpoints to engage as if they had a direct payment channel between them. Once the channel is unneeded, it can be optimistically closed in an off-chain fashion.

Apart from communication efficiency, virtual channel constructions have three natural desiderata. A virtual channel constructor is *recursive* if it can also be applied on pre-existing virtual channels, *variadic* if it can be applied on any number of pre-existing channels and *symmetric* if it encumbers in an egalitarian fashion all channel participants both in optimistic and pessimistic execution paths. We put forth the first Bitcoin-suitable recursive variadic virtual channel construction. Furthermore our virtual channel constructor is symmetric and offers optimal round complexity for payments, optimistic closing and unilateral closing. We express and prove the security of our construction in the universal composition setting. TODO: decide if we should mention ANYPREVOUT

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1 INTRODUCTION

The popularity of blockchain protocols in recent years has stretched their performance exposing a number of scalability considerations. In particular, Bitcoin and related blockchain protocols exhibit very high latency (e.g. Bitcoin has a latency of 1h [1]) and a very low throughput (e.g., Bitcoin can handle at most 7 transactions per second [2]), both significant shortcomings that jeopardize wider use and adoption and are to a certain extent inherent [2]. To address these considerations a prominent approach is to optimistically handle transactions off-chain via a "Payment Channel Network" (PCN) (see, e.g., [3] for a survey) and only use the underlying blockchain protocol as an arbiter in case of dispute.

The key primitive of PCN protocols is a payment (or more generally, state) channel. Two parties initiate the channel by locking some funds on-chain and subsequently exchange direct messages to update the state of the channel. The key feature is that state updates are not posted on-chain and hence they remain unencumbered by the performance limitations of the underlying blockchain protocol. Given this primitive, multiple overlapping payment channels can be combined and form the PCN.

Closing a channel is an operation that involves posting the state of the channel on-chain; it is essential that any party individually can close a channel as otherwise a malicious counterparty (i.e. the other channel participant) could prevent an honest party from accessing their funds. This functionality however raises an important design consideration: how to prevent malicious parties from posting old states of the channel. Addressing this issue can be done with some suitable use of transaction "timelocks", a feature that prevents a transaction or a specific script from being processed on-chain prior to a specific time (measured in block height). For instance, diminishing transaction timelocks facilitated the Duplex Micropayment Channels (DMC) [4] at the expense of bounding the overall lifetime of a channel. Using script timelocks, the Lightning Network (LN) [5] provided a better solution that enabled channels staying open for an arbitrary duration: the key idea was to duplicate the state of the channel between the two counterparties, say Alice and Bob, and facilitate a punishment mechanism that can be triggered by Bob whenever Alice posts an old state update and vice-versa. The script timelocking is essential to allow an honest counterparty some time to act.

Interconnecting state channels in LN enables any two parties to transmit funds to each other as long as they can find a route of payment channels that connects them. The downside of this mechanism is that it requires the direct involvement of all the parties along the path for each payment. Instead, "virtual payment channels", suggest the more attractive approach of putting a one-time off-chain initialization step to set up a virtual payment channel, which subsequently can be used for direct payments with complexity —in the optimistic case— independent of the length of the path. When the virtual channel has exhausted its usefulness, it can be closed off-chain if involved parties cooperate. Initial constructions for virtual channels essentially capitalized on the extended functionality of Ethereum, e.g., Perun [6] and GSCN [7], while more recent work [8] brought them closer to Bitcoin-compatibility (by leveraging adaptor signatures [9]).

A virtual channel constructor can be thought of as an *operator* over the underlying primitive of a state channel. We can identify three natural desiderata for this operator.

- Recursive. A recursive virtual channel constructor can operate over channels that themselves could be the results of previous applications of the operator. This is important in the context of PCNs since it allows building virtual channels on top of pre-existing virtual channels, allowing the channel structure to evolve dynamically.
- Variadic. A variadic virtual channel constructor can virtualize any number of input state channels directly, i.e. without

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leveraging recursion. This is important in the context of PCNs since it enables applying the operator to build virtual channels of arbitrary length, without the undue overhead of opening, managing and closing multiple virtual channels only to use the one at the "top" of the recursion.
- Symmetric. A symmetric virtual channel constructor offers setup and closing operations that are symmetric in terms of cost between the two "endpoints" or the "intermediaries" (but not a mix of both) for the optimistic and pessimistic execution paths. This is important in the context of PCNs since it ensures that no party is worse-off or better-off after an application of the operator in terms of accessing the basic functionality of the channel.

Endpoints are the two parties that share the virtual channel, intermediaries are the parties that take part in any of underlying channels.

We note that recursiveness, while identified already as an important design property (e.g., see [7]), has not been achieved in the context of Bitcoin-compatible channels (it was achieved only for DMC-like fixed lifetime channels in [10] and left as an open question for LN-type channels in [8]). The reason behind this are the severe limitations imposed to the design by the scripting language of Bitcoin-compatible systems. With respect to the other two properties, observe that successive applications of a recursive *binary* virtual channel operator to make it variadic will break symmetry (since the sequence of operator applications will impact the participants' functions with respect to the resulting channel). This is of particular concern since most previous virtual channel constructors proposed are binary, c.f. [7, 8, 10].

Our Contributions. We present the first Bitcoin-suitable recursive virtual channel constructor that supports channels with an indefinite lifetime. In addition, our constructor, Elmo (named after St. Elmo's fire), is variadic and symmetric. In our constructor, both optimistic and pessimistic execution paths are optimal in terms of round complexity: issuing payments between the two endpoints requires just three messages of size independent of the length of the channel, closing the channel cooperatively requires at most three messages from each party while closing the channel unilaterally requires up to two on-chain transactions for any involved party (endpoint or intermediary) that can be submitted simultaneously, also independent of the channel's length. Our construction is also compatible with the current version of any blockchain that supports Turing-complete smart contracts, such as Ethereum [11].

We achieve the above by leveraging a sophisticated virtual channel setup protocol which, on the one hand, enables endpoints to use an interface that is invariant between on-chain and off-chain (i.e. virtual) channels, while on the other, parties can securely close the channel cooperatively off-chain, or instead opt for unilateral on-chain closing, following any arbitrary activation sequence. The latter is achieved by making it feasible for anyone to become an initiator towards closing the channel, while subsequent respondents, following the activation sequence, can choose the right action to successfully complete the closure process by posting a single transaction each.

We formally prove the security of the constructor protocol in the UC [12] setting; our ideal functionality is global, according to the definition of [13]. The construction relies on the `ANYPREVOUT` signature type (currently under discussion for future inclusion in Bitcoin), which does not sign the hash of the transaction it spends, therefore allowing for a single pre-signed transaction to spend any output with a suitable script. We conjecture that any virtual channel constructor protocol that has participants store transactions in their local state and offers an efficient closing operation via $O(1)$ transactions will have an exponentially large state in the number of intermediaries, unless `ANYPREVOUT` is available.

Related work The first proposal for PCNs was due to [14] which only enabled unidirectional payment channels. As mentioned previously, DMCs [4] with their decrementing timelocks have the shortcoming of limited channel lifetime. This was ameliorated by LN [5] which has become the dominant paradigm for designing PCNs for Bitcoin-compatible systems. LN is currently implemented and operational for Bitcoin. It has also been adapted for Ethereum [11], where it is known as the Raiden Network [15].

A number of attacks have been identified against LN. The wormhole attack [16] against LN allows colluding parties in a multi-hop payment to steal the fees of the intermediaries between them and Flood & Loot [17] analyses the feasibility of an attack in which too many channels are forced to close in a short amount of time, reducing the blockchain liveness and enabling a malicious party to steal off-chain funds.

Payment routing [18–20] is another research area that aims to improve the network efficiency without sacrificing privacy. Actively rebalancing channels [21] can further increase network efficiency by preventing routes from becoming unavailable due to lack of well-balanced funds.

An alternantive payment channel construction that aspires to be the successor of Lightning is eltoo [22]. It has a conceptually simpler construction, smaller on-chain footprint and a more forgiving attitude towards submitting an old channel state than Lightning, but it needs the `ANYPREVOUT` sighash flag to be added to Bitcoin. Bolt [23] constructs privacy-preserving payment channels enabling both direct payments and payments with a single untrusted intermediary. Generalized Bitcoin-Compatible Channels [9] enable the creation of state channels on Bitcoin, extending channel functionality from simple payments to arbitrary Bitcoin scripts.

Sprites [24] leverages the scripting language of Ethereum to decrease the time collateral is locked up compared to Lightning. Perun [6] and GSCN [7] exploit the Turing-complete scripting language of Ethereum to provide virtual state channels, i.e. channels that can open without an on-chain transaction and that allow for arbitrary scripts to be executed off-chain. Similar features are provided by Celer [25]. Hydra [26] provides state channels for the Cardano [27] blockchain which combines a UTXO type of model with general purpose smart contract functionality that are also isomorphic, i.e. Hydra channels can accommodate any script that is compatible with the underlying blockchain.

BDW [28] shows how pairwise channels over Bitcoin can be funded with no on-chain transactions by allowing parties to form groups that can pool their funds together off-chain and then use those funds to open channels. ACMU [29] allows for multi-path atomic payments with reduced collateral, enabling new applications such as crowdfunding conditional on reaching a funding target.

TEE-based [30] solutions [20, 31–33] improve the throughput and efficiency of PCNs by an order of magnitude or more, at the cost of having to trust TEEs. Brick [34] uses a partially trusted committee to extend PCNs to fully asynchronous networks.

Solutions alternative to PCNs include sidechains (e.g., [35–37]), non-custodial chains (e.g., [38–41]), and partially centralised payment networks that entirely avoid using a blockchain [42–45].

Last but not least, a number of works propose virtual channel constructions for Bitcoin. Lightweight Virtual Payment Channels [10] enables a virtual channel to be opened on top of two preexisting channels and uses a technique similar to DMC, unfortunately inheriting the fixed lifetime limitation. Let "simple channels" be those built directly on-chain, i.e. channels that are not virtual. Bitcoin-Compatible Virtual Channels [8] also enables virtual channels on top of two preexisting simple channels and offers two protocols, the first of which guarantees that the channel will stay off-chain for an agreed period, while the second allows the single intermediary to turn the virtual into a simple channel. This strategy has the shortcoming that even if it is made recursive (a direction left open in [8]) after k applications of the constructor the virtual channel participant will have to publish on-chain k transactions in order to close the channel if all intermediaries actively monitor the blockchain.

Furthermore, Donner [46] is the first work to achieve variadic virtual channels without the need for recursion nor features that are not yet available in Bitcoin. This is achieved by having the funder use funds that are external to the "base channels" (i.e. the channels that the virtual channel is based on), so a party that has all its coins in channels cannot fund a Donner channel; additionally, we conjecture that using external coins precludes variadic virtual channel designs that are not encumbered with limited lifetime. Furthermore, due to its incentive structure, all of the base channels are forced to close if the virtual channel is closed. Donner also relies on placeholder outputs which, due to the minimum coins they need to carry to exceed Bitcoin's "dust limit", may skew the incentives of rational players and adds to the opportunity cost of maintaining a channel. Furthermore, its design complicates future iterations that lift its current restriction that only one of the two channel parties can fund the virtual channel. Donner is more efficient than the present work in terms of storage, computation and communication complexity, and boasts a simpler design, but has less room for optimisations and is not recursive.

We refer the reader to Table 1 for a comparison of the features and limitations of virtual channel protocols, including the one put forth in the current work.

2 PROTOCOL DESCRIPTION

Conceptually, Elmo is split into four main actions: channel opening, payments, cooperative closing and unilateral closing. A channel (P_1, P_n) between parties P_1 and P_n may be opened directly on-chain, in which case the two parties follow an opening procedure similar to that of LN; such a channel is called "simple". Otherwise it can be opened on top of a path of preexisting "base" channels (P_1, P_2), (P_2, P_3), …, (P_{n-2}, P_{n-1}), (P_{n-1}, P_n), in which case (P_1, P_n) is a "virtual" channel (each base channel may itself be simple or virtual). For a virtual channel, all parties P_i on the path follow our novel protocol, setting aside funds in their channels as collateral for the new virtual channel that is being opened; this is done by creating so called "virtual" transactions that essentially tie the spending of two adjacent base channels into a single atomic action. Once all intermediaries are done, P_1 and P_n finally create (and keep off-chain) their initial "commitment" transaction, following a logic similar to Lightning: their channel is now open.

A payment over an established channel follows a procedure heavily inspired by LN, but without the use of HTLCs. To be completed, a payment needs three messages to be exchanged by the two parties.

A virtual channel can be optimistically closed completely off-chain. Put simply, the parties that control the base channels revoke their virtual transactions and the related commitment transactions, effectively peeling one layer of virtualisation. Balances are redistributed so that intermediaries "break even", while P_1 and P_n get their rightful coins as reflected in the last state of their virtual channel.

Finally, the unilateral closing procedure of a channel C does not need cooperation and consists of signing and publishing a number of transactions on-chain. As we will discuss later, the exact transactions that a party will publish vary depending on the actions of the parties controlling the channels that form the base of C and the channels that are based on C.

In a nutshell, a virtual channel is built on top of two or more "base" channels, which, due to the recursive property, may themselves be simple or virtual. The parties that control the base channels are called "base parties". The fact that more than two base channels can be used by a single virtual channel is ensured by the variadic property.

In more detail, to open a channel (c.f. Figure 31) the two counterparties (a.k.a. "endpoints") first create new keypairs and exchange the resulting public keys (2 messages), then prepare the underlying base channels if the new channel is virtual ($12 \cdot (n-1)$ total messages, i.e. 6 outgoing messages per endpoint and 12 outgoing messages per intermediary, for $n-2$ intermediaries), next they exchange signatures for their respective initial commitment transactions (2 messages) and lastly, if the channel is to be opened directly on-chain, the "funder" signs and publishes the "funding" transaction to the ledger. As we alluded to earlier, a channel with its funding transaction on-chain is called "simple". A channel is either simple or virtual, not both. We here note that like LN, only one of the two parties, the funder, provides coins for a new channel. This limitation simplifies the execution model and the analysis, but can be lifted at the cost of additional protocol complexity.

Let us now introduce some notation used in figures with transactions. Reflecting the UTXO model, each transaction is represented by a circular, named node with one incoming edge per input and one outgoing edge per output. Each output can be connected with at most one input of another transaction; cycles are not allowed. Above an input or an output edge we note the number of coins it carries. In some figures the coins are omitted. Below an input we place the data carried and below an output its spending conditions (a.k.a script). For a connected input-output pair, we omit the data carried by the input. σ_K is a signature on the transaction by sk_K; in all cases, signatures are carried by inputs. An output marked with pk_K needs a signature by sk_K to be spent. $m/\{pk_1, \ldots, pk_n\}$ is an m-of-n multisig ($m \leq n$) that needs signatures from m distinct

Table 1: Features & requirements comparison of virtual channel protocols

	Unlimited lifetime	Recursive	Variadic	Script requirements
LVPC [10]	x	D[a]	x	Bitcoin
BCVC [8]	v	x	x	Bitcoin
Perun [6]	v	x	x	Ethereum
GSCN [7]	v	v	x	Ethereum
Donner [46]	x	x	v	Bitcoin
this work	v	v	v	Bitcoin + ANYPREVOUT

[a]lacks security analysis

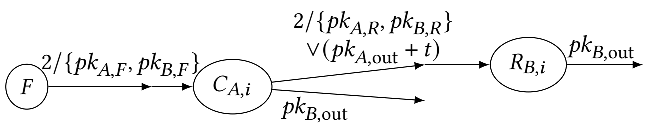

Figure 1: Funding, commitment and revocation transactions

keys among $sk_1, \dots, sk_n$. If k is a spending condition, then $k + t$ is the same spending condition but with a relative timelock of t. Spending conditions or data can be combined with logical "AND" ($\wedge$) and "OR" ($\vee$), so an output $a \vee b$ can be spent either by matching the condition a or the condition b, and an input $\sigma_a \wedge \sigma_b$ carries signatures from sk_a and sk_b.

Note that all signatures for all multisig outputs make use of the ANYPREVOUT hash type.

2.1 Simple Channels

In a similar vein to earlier UTXO-based PCN proposals, having an open channel essentially means having very specific keys, transactions and signatures at hand, as well as checking the ledger periodically and being ready to take action if misbehaviour is detected. Let us first consider a simple channel that has been established between *Alice* and *Bob* where the former owns c_A and the latter c_B coins. There are three sets of transactions at play: A "funding" transaction that is put on-chain, "commitment" transactions that are stored off-chain and spend the funding output on channel closure and off-chain "revocation" transactions that spend commitment outputs in case of misbehaviour (c.f. Figure 1).

In particular, there is a single on-chain funding transaction that spends $c_A + c_B$ coins (originally belonging to the funder), with a single output that is encumbered with a $2/\{pk_{A,F}, pk_{B,F}\}$ multisig and carries $c_A + c_B$ coins.

Next, there are two commitment transactions, one for each party, each of which can spend the funding tx and produce two outputs with c_A and c_B coins each. The two txs differ in the outputs' spending conditions: The c_A output in *Alice*'s commitment tx can be spent either by *Alice* after it has been on-chain for a pre-agreed period (i.e. it is encumbered with a "timelock"), or by a "revocation" transaction (discussed below) via a 2-of-2 multisig between the counterparties, whereas the c_B output can be spent only by *Bob* without a timelock. *Bob*'s commitment tx is symmetric: the c_A output can be spent only by *Alice* without timelock and the c_B output can be spent either by *Bob* after the timelock expiration or by a revocation tx. When a new pair of commitment txs are created (either during channel opening or on each update) *Alice* signs *Bob*'s commitment tx and sends him the signature (and vice-versa), therefore *Alice* can later unilaterally sign and publish her commitment tx but not *Bob*'s (and vice-versa).

Last, there are $2m$ revocation transactions, where m is the total number of updates of the channel. The jth revocation tx held by an endpoint spends the output carrying the counterparty's funds in the counterparty's jth commitment tx. It has a single output spendable immediately by the aforementioned endpoint. Each endpoint stores m revocation txs, one for each superseded commitment tx. This creates a disincentive for an endpoint to cheat by using any other commitment transaction than its most recent one to close the channel: the timelock on the commitment output permits its counterparty to use the corresponding revocation transaction and thus claim the cheater's funds. Endpoints do not have a revocation tx for the last commitment transaction, therefore these can be safely published. For a channel update to be completed, the endpoints must exchange the signatures for the revocation txs that spend the commitment txs that just became obsolete.

Observe that the above logic is essentially a simplification of LN.

2.2 Virtual Channels

In order to gain intuition on how virtual channels function, consider $n - 1$ simple channels established between n honest parties as before. P_1, the funder, and P_n, the fundee, want to open a virtual channel over these base channels. Before opening the virtual, each base channel is entirely independent, having different unique keys, separate on-chain funding outputs, a possibly different balance and number of updates. After the n parties follow our novel virtual channel opening protocol, they will all hold off-chain a number of new, "virtual" transactions that spend their respective funding transactions and can themselves be spent by new commitment transactions in a manner that ensures fair funds allocation for all honest parties.

In particular, apart from the transactions of simple channels (i.e. commitment and revocation txs), each of the two endpoints also has an "initiator" transaction that spends the funding output of its only base channel and produces two outputs: one new funding output for the base channel and one "virtual" output (c.f. Figures 2, 52). If the initiator transaction ends up on-chain honestly, the latter output carries coins that will directly or indirectly fund the funding output of the virtual channel. This virtual funding output can in turn be spent by a commitment transaction that is negotiated and

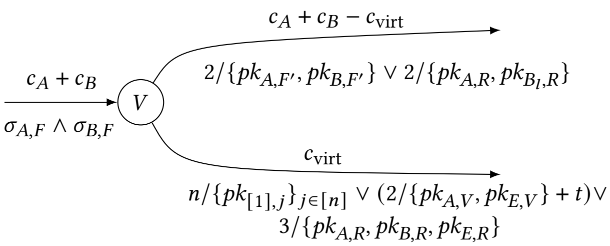

Figure 2: $A - E$ virtual channel: A's initiator transaction. Spends the funding output of the $A - B$ channel. Can be used if B has not published a virtual transaction yet.

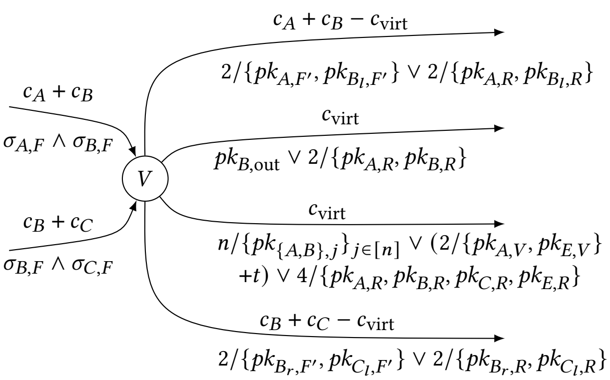

Figure 3: $A - E$ virtual channel: B's initiator transaction. Spends the funding outputs of the $A - B$ and $B - C$ channels. Can be used if neither A nor C have published a virtual transaction yet.

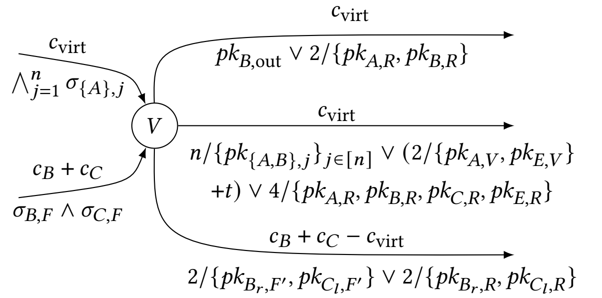

Figure 4: $A - E$ virtual channel: One of B's extend interval transactions. σ is the signature. Spends the virtual output of A's initiator transaction and the funding output of the B-C channel. Can be used if A has already published its initiator transaction and C has not published a virtual transaction yet.

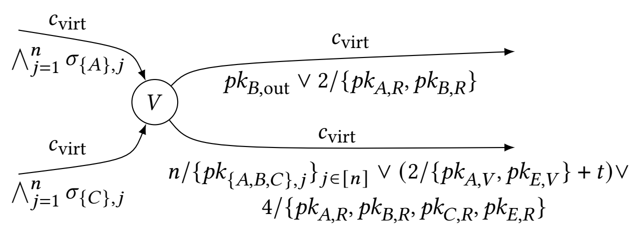

Figure 5: A - E virtual channel: One of B's merge intervals transactions. Spends the virtual outputs of A's and C's virtual transactions. Can be used if both A and C have already published their initiator transactions. Notice that the interval of C's virtual output only contains C, which can only happen if C has published its initiator and not any other of its virtual transactions.

updated with direct communication between the two endpoints in exactly the same manner as the payments of simple channels.

Intermediaries on the other hand store three sets of virtual transactions (Figure 51): "initiator" (Figure 3), "extend-interval" (Figure 4) and "merge-intervals" (Figure 5). Each intermediary has one initiator tx, which spends the party's two funding outputs and produces four: one funding output for each base channel, one output that directly pays the intermediary coins equal to the total value in the virtual channel, and one "virtual output", which carries coins that can potentially fund the virtual channel. If both funding outputs are still unspent, publishing its initiator tx is the only way for an honest intermediary to close either of its channels.

Furthermore, each intermediary has $O(n)$ extend-interval transactions. Being an intermediary, the party is involved in two base channels, each having its own funding output. In case exactly one of these two outputs has been spent honestly and the other is still unspent, publishing an extend-interval transaction is the only way for the party to close the base channel corresponding to the unspent output and take its funds. Such a transaction consumes two outputs: the only available funding output and a suitable virtual output, as discussed below. An extend-interval tx has three outputs: A funding output replacing the one just spent, one output that directly pays the intermediary coins equal to the total value of the virtual channel, and one virtual output.

Last, each intermediary has $O(n^2)$ merge-intervals transactions. If both base channels' funding outputs of the party have been spent honestly, publishing a merge-intervals transaction is the only way for the party to close either base channel. Such a transaction consumes two suitable virtual outputs, as discussed below. It has two outputs: One that directly pays the intermediary coins equal to the total value of the virtual channel, and one virtual output.

Note that each output of a virtual transaction has a "revocation" spending method that needs a signature from every party that may end up owning the output coins: each funding output is signed by the two parties of the corresponding channel, each refund output is signed by the transaction owner and the party to the left (giving c_{virt} coins to the left party if the owner acts maliciously), whereas each virtual output is signed by the transaction owner, the right party and the two virtual channel parties. If the owner acts maliciously, c_{virt} are given to the right party. The virtual channel parties have to sign as well since this output may end up funding their channel – lack of such signatures would allow two colluding intermediaries to claim the virtual output for themselves.

To understand why this multitude of virtual transactions is needed, we now zoom out from the individual party and discuss the dynamic of unilateral closing as a whole. The first party P_i that wishes to close a base channel observes that its funding output(s) remain(s) unspent and publishes its initiator transaction. First, this allows P_i to use its commitment transaction to close the base channel. Second, in case P_i is an intermediary, it directly regains the coins it has locked for the virtual channel. Third, it produces a virtual output that can only be consumed by P_{i-1} and P_{i+1}, the parties adjacent to P_i (if any) with specific extend-interval transactions. The virtual output of this extend-interval transaction can in turn be spent by specific extend-interval transactions of P_{i-2} or P_{i+2} that have not published a virtual transaction yet (if any) and so on for the next neighbours. The idea is that each party only needs to publish a single virtual transaction to "collapse" the virtual layer and each virtual output uniquely defines the continuous interval of parties that have already published a virtual transaction and only allows parties at the edges of this interval to extend it. This prevents malicious parties from indefinitely replacing a virtual output with a new one. As the name suggests, merge-intervals transactions are published by parties that are adjacent to two parties that have already published their virtual transactions and in effect joins the two intervals into one.

Each virtual output can also be used as the funding output for the virtual channel after a timelock, to protect from unresponsive parties blocking the virtual channel indefinitely. This in turn means that if an intermediary observes either of its funding outputs being spent, it has to publish its suitable virtual transaction before the timelock expires to avoid losing funds. What is more, all virtual outputs need the signature of all parties to be spent before the timelock (i.e. they have an n-of-n multisig) in order to prevent colluding parties from faking the intervals progression. To ensure that parties have an opportunity to react, the timelock of a virtual output is the maximum of the required timelocks of the intermediaries that can spend it. Let p be a global constant representing the maximum number of blocks a party is allowed to stay offline between activations without becoming negligent (the latter term is explained in detail later), and s the maximum number of blocks needed for an honest transaction to enter the blockchain after being published, as in Proposition F.1 of Section F. The required timelock of a party is $p + s$ if its channel is simple, or $p + \sum_{j=2}^{n-1} (s - 1 + t_j)$ if the channel is virtual, where t_j is the required timelock of the base channel of the jth intermediary's channel. The only exception are virtual outputs with an interval that includes all parties, which are just funding outputs for the virtual channel: an interval with all parties cannot be further extended, therefore one spending method and the timelock are dropped.

Many extend-interval and merge-intervals transactions have to be able to spend different outputs, depending on the order other base parties publish their virtual transactions. For example, P_3's extend-interval tx that extends the interval $\{P_1, P_2\}$ to $\{P_1, P_2, P_3\}$ must be able to spend both the virtual output of P_2's initiator transaction and P_2's extend-interval transaction which has spent P_1's initiator transaction. The same issue is faced by commitment transactions of a virtual channel, as any virtual output can potentially be used as the funding ouput for the channel. In order for the received signatures for virtual and commitment txs to be valid for multiple previous outputs, the previously proposed `ANYPREVOUT` sighash flag [47] is needed to be added to Bitcoin. We conjecture that variadic recursive virtual channels with $O(1)$ on-chain and subexponential number of off-chain transactions for each party cannot be constructed in Bitcoin without this flag. We hope this work provides additional motivation for this flag to be included in the future.

Note also that the newly established virtual channel can itself act as a base for further virtual channels, as its funding output can be unilaterally put on-chain in a pre-agreed maximum number of blocks. This in turn means that, as we discussed above, a further virtual channel must take the delay of its virtual base channels into account to determine the timelocks needed for its own virtual outputs.

Let a single *channel round* be a series of messages starting from the funder and hop by hop reaching the fundee and back again. For the actual protocol that establishes a virtual channel 6 channel rounds are needed (c.f. Figure 27). The first communicates parties' identities, their funding keys, revocation keys and their neighbours' channel balances, the second creates new commitment transactions, the third communicates keys for virtual transactions (a.k.a. virtual keys), all parties' coins and desired timelocks, the fourth and the fifth communicate signatures for the virtual transactions (signatures for virtual outputs and funding outputs respectively) and the sixth shares revocation signatures for the old channel states.

Cooperative closing is quite intuitive (c.f. Figures 44, 45, 46, 47 and 63). It can be initiated by any party, one and a half communication rounds are needed. The funder builds new commitment txs, which once again spend the funding outputs that the virtual txs spent before, just like prior to opening the virtual channel. The funder sends their signatures to the first intermediary; the latter signs and sends the same to the second intermediary and so on until the fundee. The fundee responds with its own commitment tx signatures, along with signatures revoking the previous commitment tx and virtual txs. This is repeated backwards until revocations reach the funder. Finally the funder sends its revocation to its neighbour and it to the next, until the revocations reach the fundee. The channel has now closed cooperatively.

As for the unilateral closing, let us now turn to an example in order to better grasp how our construction plays out on-chain in practice (Figure 6). Consider an established virtual channel on top of 4 preexisting simple base channels. Let A, B, C, D and E be the relevant parties, which control the (A, B), (B, C), (C, D) and (D, E) base channels, along with the (A, E) virtual channel. After carrying out some payments, A decides to unilaterally close the virtual channel. It therefore publishes its initiator transaction, thus consuming the funding output of (A, B) and producing (among others) a virtual output with the interval $\{A\}$. B notices this before the timelock of the virtual output expires and publishes its extend-interval transaction that consumes the aforementioned virtual output and the funding output of (B, C), producing a virtual output with the interval $\{A, B\}$. C in turn publishes the corresponding extend-interval transaction, consuming the virtual output of B and the funding output of (C, D) while producing a virtual output with the interval $\{A, B, C\}$. Finally D publishes the last extend-interval transaction, thus producing an interval with all players. Instead of a virtual output, it produces the funding output for the virtual channel (A, E). Now A can spend this funding output with its latest commitment transaction. Note that if any of B, C or D does not act within the timelock prescribed in their consumed virtual output, then A or E can spend the virtual output with their latest commitment transaction, thus eventually A can close its virtual channel in all cases.

3 MODEL

3.1 $\mathcal{G}_{\text{Ledger}}$ Functionality

In this work we embrace the Universal Composition (UC) framework [12] together with its global subroutines extension, UCGS [13], to model parties, network interactions, adversarial influence and corruptions, as well as formalise and prove security. We choose to model the Bitcoin ledger with the $\mathcal{G}_{\text{Ledger}}$ functionality as defined in [48] and further refined in [49]. $\mathcal{G}_{\text{Ledger}}$ formalizes an ideal data structure that is distributed and append-only, akin to a blockchain. Participants can read from $\mathcal{G}_{\text{Ledger}}$, which returns an ordered list of transactions. Additionally a party can submit a new transaction which, if valid, will eventually be added to the ledger when the adversary decides, but necessarily within a predefined time window. This property is named liveness. Once a transaction becomes part of the ledger, it then becomes visible to all parties at the discretion of the adversary, but necessarily within another predefined time window, and it cannot be reordered or removed. This is named persistence.

Moreover, $\mathcal{G}_{\text{Ledger}}$ needs the $\mathcal{G}_{\text{CLOCK}}$ functionality [50], which models the notion of time. Any $\mathcal{G}_{\text{CLOCK}}$ participant can request to read the current time (which is initially 0) and inform $\mathcal{G}_{\text{CLOCK}}$ that her round is over. $\mathcal{G}_{\text{CLOCK}}$ increments the time by one once all parties have declared the end of their round. We further note that both $\mathcal{G}_{\text{Ledger}}$ and $\mathcal{G}_{\text{CLOCK}}$ are global functionalities [13] and therefore can be accessed directly by the environment.

3.2 Modelling time

The protocol and functionality defined in this work do not use $\mathcal{G}_{\text{CLOCK}}$ directly. Indeed, the only notion of time in our work is provided by the blockchain height, as reported by $\mathcal{G}_{\text{Ledger}}$. We therefore omit it in the statement of our lemmas and theorems for simplicity of notation; it should normally appear as a hybrid together with $\mathcal{G}_{\text{Ledger}}$.

Our protocol is fully asynchronous, i.e., the adversary can delay any network message arbitrarily long. The protocol is robust against such delays, as an honest party can unilaterally prevent loss of funds even if some of its incoming and outgoing network messages are dropped by $\mathcal{A}$, as long as the party has input-output communication with the ledger. We also note that, following the conventions of single-threaded UC execution model, the duration of local computation is not taken into account in any way (as long as it does not exceed its polynomial bound).

4 PROTOCOL PSEUDOCODE

We here present a simplified version of the protocol in pseudocode form. We omit complications imposed by UC. We refer the reader to Appendix E for the complete protocol and to Appendix D for an in-depth description of the protocol in prose.

Process Π_{Chan} – self is P

- Before handling each message:
  - **if** we have not been activated since more than p blocks **then**
    - Mark ourselves as negligent // no balance security guarantees anymore
  - **end if**
- Initialisation:
  - Receive $pk_{P,\text{out}}$ from $\mathcal{E}$ // all outputs owned by P pay $pk_{P,\text{out}}$
  - Generate own keypair
  - Wait for $\mathcal{E}$ to give own keypair some starting coins
- Opening:
  - Generate funding and revocation keypairs
  - Exchange funding, revocation and out public keys with counterparty
  - **if** opening virtual (off-chain) channel **then**
    - Ask our host channel to prepare, passing them our funding keys // c.f. next bullet, "Hosting a virtual channel"
    - Get t_P from host // timelock to ensure our balance security
  - **end if**
  - Exchange and verify signatures on commitment transactions with counterparty
  - **if** opening simple (on-chain) channel **then**
    - Prepare and submit funding transaction to ledger and wait for its inclusion // only one party funds the channel, so the funding transaction needs only the funder's signature
    - $t_P \leftarrow s + p$ // timelock to ensure balance security for simple channels
  - **end if**
- Hosting a virtual channel of c_{virt} coins:
  - Ensure we have enough coins to host such a virtual channel
  - Generate one new funding keypair, $O(n^2)$ virtual keypairs ($O(n)$ per hop) and one virtual revocation keypair // all keypairs are generated normally, using KEYGEN()
  - Exchange generated public keys among all base channel parties
  - Generate and sign new commitment transactions with our counterparties. The new funding keys and the latest revocation keys are used and the balance of the party "closer" to the funder is reduced by c_{virt} // 1 counterparty if we are endpoint, 2 counterparties if we are intermediary
  - Exchange signatures with counterparties and verify them
  - Generate and sign all $O(n^3)$ virtual transactions // one signature for each virtual input – each virtual input needs one signature from each party. Only "extend-interval" and "merge-intervals" transactions need these signatures
  - Exchange all signatures among all base channel parties and verify that all our virtual transactions have fully signed virtual inputs
  - Exchange with counterparties and verify signatures for the funding inputs of our virtual transactions // only "initiator" and "extend-interval" transactions need these signatures
  - Exchange with counterparties and verify signatures for the revocation transactions of the previous channel state
  - **if** P is intermediary **then**
    - $t_P \leftarrow \max\{t \text{ of left channel}, t \text{ of right channel}\}$
  - **else** // P is endpoint
    - $t_P \leftarrow p + \sum_{j=2}^{n-1} (s - 1 + t_j)$ // worst case delay is if counterparty uses initiator tx and every intermediary uses its

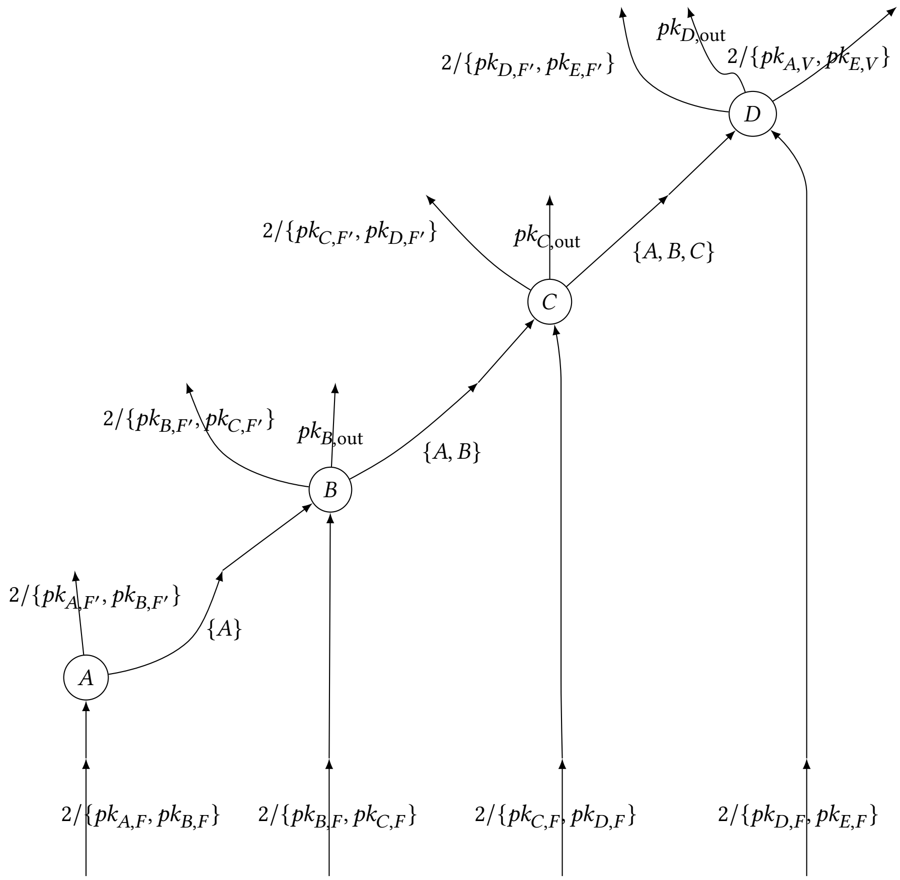

Figure 6: 4 **simple channels supporting a virtual.** A **initiates the closing procedure and no party is negligent. Virtual outputs are marked with their interval.**

extend-interval tx sequentially – the maximum possible delay is O(sum of intermediaries' delays)
end if

- Reacting if counterparty publishes virtual transaction:
  if both our counterparties have published a virtual transaction **then**
  Publish our merge-intervals transaction that has an interval equal to the union of the intervals of the two virtual transactions plus ourselves
  else // exactly one of our counterparties has published a virtual transaction
  Publish our extend-interval transaction that has an interval equal to the interval of the virtual transaction plus ourselves
  end if
- Paying x coins:
  Ensure we have enough coins to pay
  if we host a virtual channel **then**
  Ensure balance after payment will not allow griefing attack // c.f. Subsubsection D.3
  end if
  Generate and sign new commitment transactions, with x coins less for the payer and x coins more for the payee and using the latest revocation keys
  Exchange and verify signatures
  Sign revocation transactions that correspond to the old commitment transactions
  Generate next revocation keypairs
  Exchange and verify commitment signatures and revocation public keys
- Unilaterally closing:
  Publish all initiator transactions that are needed to put our funding output on the ledger
  Publish our latest commitment transaction to the ledger
- Cooperatively closing:
  // Only a virtual channel which does not host any further virtual channel may close cooperatively
  Both endpoints sign and broadcast the final virtual channel balance (c_1, c_2)
  Every party verifies both signatures, ensures that the two opinions agree and that the balance sum is equal to c_{virt}

Generate and sign new commitment transactions with:
- the most recent old funding keys (the ones used before hosting the virtual channel)
- the new revocation keys
- c_1 additional coins for the party closest to the virtual channel funder and c_2 for the counterparty

Generate new revocation keypairs
Exchange and verify signatures and revocation public keys
Generate and sign revocation transactions for both the old virtual transactions (with the virtual revocation keys) and the old commitment transactions (with the normal revocation keys)
Exchange and verify signatures // if a party publishes a revoked virtual transaction, its various outputs can be spent by revocation transactions so that its (1 or 2) counterparties can claim all base channel funds

- Punishing malicious counterparties:

// Executed at least every p blocks
if the ledger contains an old commitment transaction **then**
Sign and publish the corresponding revocation transaction
end if
if the ledger contains an old virtual transaction **then**
Sign and publish the corresponding revocation transaction(s)
end if

Figure 7: High level pseudocode of the Elmo protocol

5 SECURITY

The first step to formally arguing about the security of Elmo is to clearly delineate the exact security guarantees it provides. To that end, we first prove two similar claims regarding the conservation of funds in the real and ideal world, Lemmas G.1 and G.2 respectively. Informally, the first establishes that if an honest, non-negligent party was implicated in a channel on which a number of payments took place and has now been unilaterally closed, then the party will have at least the expected funds on-chain.

Lemma 5.1 (Real world balance security (informal)). *Consider a real world execution with $P \in \{Alice, Bob\}$ honest LN ITI. Assume that all of the following are true:*

- *P is not negligent,*
- *P successfully opened the channel, with initial balance c ($c = 0$ if P is a fundee)*
- *P is the host of n channels, where the i-th channel is funded with f_i coins,*
- *P has cooperatively closed k channels, where the i-th channel transferred r_i coins from the hosted virtual channel to P,*
- *P has successfully sent m payments, where the i-th payment involved d_i coins,*
- *P has successfully received l payments, where the i-th payment involved c_i coins.*

If P unilaterally closes its channel, eventually its view of $\mathcal{G}_{\text{Ledger}}$ will contain h outputs spendable only by P or another kindred party, each of value c_i, such that

$$\sum_{i=1}^{h} c_i \geq c - \sum_{i=1}^{n} f_i - \sum_{i=1}^{m} d_i + \sum_{i=1}^{l} e_i + \sum_{i=1}^{k} r_i \ . \tag{1}$$

The formal statement, Lemma G.1, is deferred to the Appendix.

The second lemma states that for an ideal party in a similar situation, the balance that $\mathcal{G}_{\text{Chan}}$ has stored for it is at least equal to the expected funds.

Lemma 5.2 (Ideal world balance (informal)). *Consider an ideal world execution with functionality $\mathcal{G}_{\text{Chan}}$ and simulator $\mathcal{S}$. Let $P \in \{Alice, Bob\}$ one of the two parties of $\mathcal{G}_{\text{Chan}}$. Assume that all of the following are true:*

- *P is not corrupted or negligent, nor the any member of the transitive closure of its hosts has published a revocation transaction,*
- *P successfully opened the channel, with initial balance c ($c = 0$ if P is a fundee)*
- *P is the host of n channels, where the i-th channel is funded with f_i coins,*
- *P has cooperatively closed k channels, where the i-th channel transferred r_i coins from the hosted virtual channel to P,*
- *P has successfully sent m payments, where the i-th payment involved d_i coins,*
- *P has successfully received l payments, where the i-th payment involved c_i coins.*

Let $\texttt{balance}_P$ be the balance that $\mathcal{G}_{\text{Chan}}$ stores internally for P. If the channel is closed (either unilaterally or cooperatively), then the following holds with overwhelming probability on the security parameter:

$$\texttt{balance}_P = c - \sum_{i=1}^{n} f_i - \sum_{i=1}^{m} d_i + \sum_{i=1}^{l} e_i + \sum_{i=1}^{k} r_i + \sum_{i=1}^{k} r_i \ . \tag{2}$$

The formal statement, Lemma G.2, is deferred to the Appendix.

In both cases the expected funds are [initial balance - funds for hosted virtuals + funds returned from hosted virtuals - outbound payments + inbound payments]. Note that the funds for hosted virtuals only refer to those funds used by the funder of the virtual channel, not the rest of the base parties.

Both proofs follow the various possible execution paths, keeping track of the resulting balance in each case and coming to the conclusion that balance is secure in all cases, except if signatures are forged.

It is important to note that in fact Π_{Chan} provides a stronger guarantee, namely that an honest, non-negligent party with an open channel can unilaterally close it and obtain the expected funds on-chain within a known number of blocks, given that $\mathcal{E}$ sends the necessary “daemon” messages. This stronger guarantee is sufficient to make this construction reliable enough for real-world applications. However a corresponding ideal world functionality with such guarantees would have to be aware of the specific transactions and signatures, therefore it would be essentially as complicated as the protocol, thus violating the spirit of the simulation-based security paradigm.

Subsequently we prove Lemma 5.3, which informally states that if an ideal party and all its kindred parties are honest, then $\mathcal{G}_{\text{Chan}}$ does not halt with overwhelming probability.

Lemma 5.3 (No halt). *In an ideal execution with $\mathcal{G}_{\text{Chan}}$ and $\mathcal{S}$, if the kindred parties of the honest parties of $\mathcal{G}_{\text{Chan}}$ are themselves honest, then the functionality halts with negligible probability in the security parameter (i.e. l. 21 of Fig. 12 is executed negligibly often).*

This is proven by first arguing that if the conditions of Lemma G.2 for the ideal world hold, then the conditions of Lemma G.1 also hold for the equivalent real world execution, therefore in this case $\mathcal{G}_{\text{Chan}}$ does not halt. We then argue that also in case the conditions of Lemma G.2 do not hold, $\mathcal{G}_{\text{Chan}}$ may never halt as well, therefore concluding the proof.

A salient observation regarding an instance s of Π_{Chan} is that, in order to open a virtual channel, it passes inputs to another Π_{Chan} instance s' that belongs to a different extended session. This means that s (and therefore Π_{Chan}) is not *subroutine respecting*, as defined in [12]. To address this issue, we first annotate Π_{Chan} with a numeric superscript, i.e. Π^n_{Chan}. Π^1_{Chan} is always a simple (i.e. on-chain) channel. To achieve this, Π_{Chan} undergoes a modification under which it ignores all (open, x, hops $\neq$ "ledger", ...) messages. Likewise we define $\mathcal{G}^1_{\text{Chan}}$ as a version of $\mathcal{G}_{\text{Chan}}$ that ignores (open, x, hops $\neq$ "ledger", ...) messages. As for the rest of the superscripts, $\forall n \in \mathbb{N}^*, \Pi^{n+1}_{\text{Chan}}$ is a virtual channel protocol Π_{Chan} of which the base channels have a maximum superscript n. It then holds that $\forall n \in \mathbb{N}^*, \Pi^n_{\text{Chan}}$ is $(\mathcal{G}_{\text{Ledger}}, \Pi^1_{\text{Chan}}, \dots, \Pi^{n-1}_{\text{Chan}})$-subroutine respecting, as defined in [13]. Likewise, $\mathcal{G}^{n+1}_{\text{Chan}}$ is a virtual channel functionality $\mathcal{G}_{\text{Chan}}$ of which the base channels have a maximum superscript n. It then holds that $\forall n \in \mathbb{N}^*, \mathcal{G}^n_{\text{Chan}}$ is $(\mathcal{G}_{\text{Ledger}}, \mathcal{G}^1_{\text{Chan}}, \dots, \mathcal{G}^{n-1}_{\text{Chan}})$-subroutine respecting.

We now formulate and prove Theorem 5.4, which states that Π^1_{Chan} UC-realises $\mathcal{G}^1_{\text{Chan}}$.

Theorem 5.4 (Simple Payment Channel Security). *The protocol Π^1_{Chan} UC-realises $\mathcal{G}^1_{\text{Chan}}$ in the presence of a global functionality $\mathcal{G}_{\text{Ledger}}$ and assuming the security of the underlying digital signature. Specifically,*

$$\forall\ PPT\ \mathcal{A}, \exists\ PPT\ \mathcal{S} : \forall\ PPT\ \mathcal{E}\ \textit{it is}\ \textsc{exec}^{\mathcal{G}_{\text{Ledger}}}_{\Pi^1_{\text{Chan}}, \mathcal{A}, \mathcal{E}} \approx \textsc{exec}^{\mathcal{G}^1_{\text{Chan}}, \mathcal{G}_{\text{Ledger}}}_{\mathcal{S}, \mathcal{E}}$$

The corresponding proof is a simple application of Lemma 5.3, the fact that $\mathcal{G}_{\text{Chan}}$ is a simple relay and that $\mathcal{S}$ faithfully simulates Π_{Chan} internally.

Proof of Theorem 5.4. By inspection of Figures 8 and 22 we can deduce that for a particular $\mathcal{E}$, in the ideal world execution $\textsc{exec}^{\mathcal{G}^1_{\text{Chan}}, \mathcal{G}_{\text{Ledger}}}_{\mathcal{S}_{\mathcal{A}}, \mathcal{E}}$, $\mathcal{S}_{\mathcal{A}}$ simulates internally the two Π^1_{Chan} parties exactly as they would execute in $\textsc{exec}^{\mathcal{G}_{\text{Ledger}}}_{\Pi^1_{\text{Chan}}, \mathcal{A}, \mathcal{E}}$, the real world execution, in case $\mathcal{G}^1_{\text{Chan}}$ does not halt. Indeed, $\mathcal{G}^1_{\text{Chan}}$ only halts with negligible probability according to Lemma 5.3, therefore the two executions are computationally indistinguishable. $\square$

Lastly we prove that $\forall$ integers $n \geq 2, \Pi^n_{\text{Chan}}$ UC-realises $\mathcal{G}^n_{\text{Chan}}$ in the presence of $\mathcal{G}^1_{\text{Chan}}, \dots, \mathcal{G}^{n-1}_{\text{Chan}}$ (leveraging the relevant definition from [13]).

Theorem 5.5 (Recursive Virtual Payment Channel Security). *$\forall n \in \mathbb{N}^* \setminus \{1\}$, the protocol Π^n_{Chan} UC-realises $\mathcal{G}^n_{\text{Chan}}$ in the presence of $\mathcal{G}^1_{\text{Chan}}, \dots, \mathcal{G}^{n-1}_{\text{Chan}}$ and $\mathcal{G}_{\text{Ledger}}$, assuming the security of the underlying digital signature. Specifically,*

$$\forall n \in \mathbb{N}^* \setminus \{1\}, \forall\ PPT\ \mathcal{A}, \exists\ PPT\ \mathcal{S} : \forall\ PPT\ \mathcal{E}\ \textit{it is}$$
$$\textit{EXEC}^{\mathcal{G}_{\text{Ledger}}, \mathcal{G}^1_{\text{Chan}}, \dots, \mathcal{G}^{n-1}_{\text{Chan}}}_{\Pi^n_{\text{Chan}}, \mathcal{A}, \mathcal{E}} \approx \textit{EXEC}^{\mathcal{G}^n_{\text{Chan}}, \mathcal{G}_{\text{Ledger}}}_{\mathcal{S}, \mathcal{E}}$$

Proof of Theorem 5.5. The proof is exactly the same as that of Theorem 5.4, replacing superscripts 1 for n. $\square$

Formal proofs for the three lemmas can be found in Section G.

6 EFFICIENCY EVALUATION

We offer here a comparison of this work with LVPC [10], BCVC [8] and Donner [46] in terms of communication efficiency when opening and updating. We also compare the maximum on-chain cost for an endpoint to unilaterally close its virtual channel. Furthermore, we compare the maximum on-chain cost for an intermediary to close its base channel. In order to only show the costs caused by supporting a virtual channel, we subtract the cost the intermediary would pay to close its channel if it was not supporting any virtual channel. Lastly, we compare the maximum total on-chain cost, aggregated over all parties. On-chain cost is measured in terms of size[1] and number of transactions. The comparison is performed both for channels of length 2 and for channels of length n. In the second comparison BCVC is not included, since only LVPC and Donner offer a way to open virtual channels of length greater than 2 (since LVPC is recursive and Donner is variadic). For a virtual channel between P_1 and P_n over $n-1$ base channels via LVPC, we consider the case in which the funder P_1 initially has a channel with P_2 and then opens one virtual channel with party P_i on top of its channel with party P_{i-1} for $i \in \{3, \dots, n\}$. We choose this topology, as P_1 cannot assume that there exist any virtual channels between other parties (which could be used as shortcuts).

BCVC proposes 4 configurations, each with different efficiency characteristics: over Generalised Channels [9] (GC) or Lightning, and with or without validity (V or NV respectively); we include all of them here. We refer the reader to [8] for more details.

Since all constructions produce pairwise channels, updates (a.k.a. payments) always implicate exactly two parties (the payer and the payee) and their interaction is independent of the number of underlying channels. Therefore the "Update" entries in Table 2 also cover the general case of n underlying channels. *Party rounds* are calculated as [#incoming messages + #outgoing messages]/2, for each party separately. Note that the update-related storage requirements of Elmo-related entries in Table 2 do not take into account savings that may arise by deleting old unneeded data at the end of an update.

Some results disagree slightly with those of the tables found in related work, due to differences in the counting method. Also note that the data for Elmo are derived assuming a virtual channel opened directly on top of $n-1$ base channels, in other words the channel considered is opened without the help of recursion.

[1]For Table 4, transaction size is calculated in so-called "virtual bytes", which map directly to on-chain fees and thus are preferred to raw bytes. For Table 2, transaction size is calculated in raw bytes instead, to better align with the counting method used in the efficiency evaluation found in [8]. We used the tool found in https://jlopp.github.io/bitcoin-transaction-size-calculator/ to aid size calculation.

	Open		Update		Cooperative Close		Unilateral Close	
	#txs	size	#txs	size	#txs	size	#txs	size
BCVC-GC-NV	7	2829	2	695	4	1390	7	2829
BCVC-GC-V	8	2803	2	695	4	1390	8	2803
BCVC-LN-NV	16	7704	8	2824	4	1412	4	2153
BCVC-LN-V	14	5722	4	1412	4	1412	5	2131
Elmo	7	5245	4	1517	20	8002	4	2222.75

Table 2: Efficiency comparison of Elmo and BCVC [8]

In Table 3 we compare the resources needed to open a new virtual channel both for each party individually and for all parties in total, in terms of party rounds and amount of data sent and stored. The data is counted as the sum of the relevant channel identifiers (8 bytes each, as defined by the Lightning Network specification[2]), transaction output identifiers (36 bytes), secret keys (32 bytes each), public keys (33 bytes each, compressed form – these double as party identifiers), signatures (71 bytes each), coins (8 bytes each), times and timelocks (both 4 bytes each). UC-specific data is ignored. Optimisations are not considered.

In LVPC, every intermediary, apart from the first one, acts both as a fundee in a new virtual channel with the funder and as an intermediary in the funder's virtual channel with the party after said intermediary. In Table 3 the intermediary columns contain the total cost of any intermediary that is not the first one, therefore the first intermediary (the party after the funder) incurs [intermediary's costs - fundee's costs] for all three measured quantities.

The on-chain number of transactions to close a virtual channel in the case of LVPC is calculated as follows: One "split" transaction is needed for each base channel ($n-1$ in total), plus one "merge" transaction per virtual channel ($n-2$ in total), plus a single "refund" transaction for the virtual channel, for a total of $2n-2$ transactions.

We note that the linear size factor when the intermediary closes can, in the case of our work, be eliminated with the aid of Schnorr signatures (recently made available on Bitcoin), bringing its cost below that of LVPC. This is so because the n signatures that are needed to spend each virtual output can be shrunk down to a single aggregate signature without compromising security. For the same reason, Schnorr signatures help eliminate the quadratic term and reduce the linear term of the total on-chain cost. The same cannot be said for the linear factor in Donner, since there is currently no way to optimise away the n outputs of the funder's transaction $\mathtt{tx}^{\mathtt{vc}}$. Likewise LVPC cannot obtain a linear improvement with this optimisation, since each of its relevant transactions ("split", "merge" and "refund") only needs a constant number of signatures. We deduce that, using this optimisation, Elmo has the smallest worst-case total on-chain footprint compared to LVPC and Donner.

7 DISCUSSION AND FUTURE WORK

A number of features can be added to our protocol for additional efficiency, usability and flexibility. First of all, in our current construction, each time a particular channel C acts as a base channel for a new virtual channel, one more "virtualisation layer" is added. When one of its owners wants to close C, it has to put on-chain as many transactions as there are virtualisation layers. Also the timelocks associated with closing a virtual channel increase with the number of virtualisation layers of its base channels. Both these issues can be alleviated by extending the opening and cooperative closing subprotocol with the ability to cooperatively open and close multiple virtual channels in the same layer, either simultaneously or as an amendment to an existing virtualisation layer.

Due to the possibility of the griefing attack discussed in Subsection D.3, the range of balances a virtual channel can support is limited by the balances of neighbouring channels. We believe that this limitation can be lifted if instead of using a Lightning-based construction for the payment layer, we instead replace it with an eltoo-based [22] construction. Since in eltoo a maliciously published old state can be simply re-spent by the honest latest state, the griefing attack is completely avoided. What is more, our protocol shares with eltoo the need for the `ANYPREVOUT` sighash flag, therefore no additional requirements from the Bitcoin protocol would be added by this change. Lastly, due to the separation of intermediate layers with the payment layer in our pseudocode implementation as found in Section E (i.e. the distinction between the LN and the VIRT protocols), this change should in principle not need extensive changes in all parts of the protocol.

As it currently stands, the timelocks calculated for the virtual channels are based on p (Figure 24) and s (Figure 28), which are global constants that are immutable and common to all parties. The parameter s stems from the liveness guarantees of Bitcoin, as discussed in Proposition F.1 and therefore cannot be tweaked. However, p represents the maximum time (in blocks) between two activations of a non-negligent party, so in principle it is possible for the parties to explicitly negotiate this value when opening a new channel and even renegotiate it after the channel has been opened if the counterparties agree. We leave this usability-augmenting protocol feature as future work.

Our protocol is not designed to "gracefully" recover from a situation in which halfway through a subprotocol, one of the counterparties starts misbehaving. Currently the only solution is to unilaterally close the channel. This however means that DoS attacks (that still do not lead to channel fund losses) are possible. A practical implementation of our protocol would need to expand the available actions and states to be able to transparently and gracefully recover from such problems, avoiding closing the channel where possible, especially when the problem stems from network issues and not from malicious behaviour.

Furthermore, any deployment of the protocol has to explicitly handle the issue of transaction fees. These include miner fees for

[2]https://github.com/lightning/bolts/blob/master/07-routing-gossip.md#definition-of-short_channel_id

Open											
	Funder			Fundee			ith Intermediary			Total	
	party rounds	size		party rounds	size		party rounds	size		size	
		sent	stored		sent	stored		sent	stored	sent	stored
LVPC	$8(n-2)$	$1381(n-2)$	$3005(n-2)$	7	1254	2936	16	2989	6385	$4370n - 8740$	$9390n - 18780$
Donner	2	$184n + 828.25$	$1332.5k+$ $43n + 125.5$	1	$43n + 192.5$	$1332.5k+$ $43n + 125.5$	1	546.75	$1332.5k+$ $43n + 125.5$	$773.75n - 72.75$	$1332.5kn+$ $43n^2 + 125.5n$
Elmo	6	$\frac{7}{3}n^3 - 109n^2+$ $\frac{1435}{3}n - 325$	$33n + 444$	6	$\frac{7}{3}n^3 - 109n^2+$ $\frac{1336}{3}n - 401$	$33n + 444$	12	$\frac{106}{3}n^3 - 208n^2$ $+\frac{2302}{3}n - 551$	$175(i-2)n^2-$ $(175i^2 - 64i - 572)n$ $+111(i^2 - i - 2)$	$\frac{106}{3}n^4 - 274n^3+$ $688n^2 - 1162n + 376$	$\frac{175}{6}n^4 - 281n^3+$ $\frac{5549}{6}n^2 - 1181n + 444$

Table 3: Open efficiency comparison of virtual channel protocols with n parties and k payments

Unilateral Close								
	Intermediary		Funder		Fundee		Total	
	#txs	size	#txs	size	#txs	size	#txs	size
LVPC	3	626.25	2	383	2	359	$2n - 2$	$434.75n - 510.5$
Donner	1	204.5	4	$704 + 43n$	1	136.5	$2n$	$458n - 26$
Elmo	1	$593.75 + 26.75n$	2	503	2	503	n	$26.75n^2 + 540.25n$ -684.5

Table 4: On-chain worst-case closing efficiency comparison of virtual channel protocols with n parties

on-chain transactions and intermediary fees for the parties that own base channels and facilitate opening virtual channels. These fees should take into account the fact that each intermediary has quadratic storage requirements, whereas endpoints only need constant storage, creating an opportunity for amplification attacks. Our protocol is compatible with any such fee parameterization and we leave for future work the incentive analyses that can determine concrete values for such intermediary fees.

In order to increase readability and to keep focus on the salient points of the construction, our protocol does not exploit a number of possible optimisations. These include a number of techniques employed in Lightning that drastically reduce storage requirements, along with a variety of possible improvements to our novel virtual subprotocol. Most notably, the Taproot [51] feature that has been recently added to Bitcoin will allow for a drastic reduction in the size of transactions, as in the optimistic case only the hash of the Script has to be added to the blockchain and the n signatures needed to spend a virtual output can be replaced with their aggregate, resulting in constant size storage. As this work is mainly a proof of feasibility, we leave these optimisations as future work.

Additionally, our protocol does not feature one-off multi-hop payments like those possible in Lightning. This however is a useful feature in case two parties know that they will only transact once, as opening a virtual channel needs substantially more network communication than performing an one-off multi-hop payment. It would be therefore fruitful to also enable the multi-hop payment technique used in Lightning and allow human users to choose which method to use in each case. Likewise, optimistic cooperative on-chain closing of simple channels could be done just like in Lightning, obviating the need to wait for the revocation timelock to expire and reducing on-chain costs if the counterparty is cooperative.

Moreover, our need for `ANYPREVOUT` prevents our protocol from being deployable on Bitcoin today. Even though it is one of the prime contenders for inclusion in a future update, there are no guarantees that it will ever be added. Nevertheless, as we mentioned before, we conjecture that a variadic virtual channel protocol with unlimited lifetime needs each party to store an exponential number of signatures if `ANYPREVOUT` is not available. We leave proof of this conjecture as future work.

Last but not least, the current analysis gives no privacy guarantees for the protocol, as it does not employ onion packets [52] like Lightning. Furthermore, $\mathcal{G}_{\text{Chan}}$ leaks all messages to the ideal adversary therefore theoretically no privacy is offered at all. Nevertheless, onion packets can be incorporated in the current construction and intuitively our construction leaks less data than Lightning for the same multi-hop payments, as intermediaries in our case are not notified on each payment, contrary to multi-hop payments in Lightning. Therefore a future extension can improve the privacy of the construction and formally demonstrate exact privacy guarantees.

8 CONCLUSION

In this work we presented Recursive Virtual Payment Channels for Bitcoin, a construction which enables the establishment and optimistic teardown of pairwise payment channels without the need for posting on-chain transactions. Such a channel can be opened over a path of consecutive base channels of arbitrary length, i.e., the virtual channel constructor is variadic.

The base channels themselves can be virtual, therefore the novel recursive nature of the construction. A key performance characteristic of our construction is that it has optimal round complexity for on-chain channel closing: a single transaction is required by any participant to turn the virtual channel into a simple one and one more transaction is needed to close it, be it an end-point or an intermediary. The two transactions can be submitted simultaneously.

We formally described the protocol in the UC setting, provided a corresponding ideal functionality and simulator and finally proved the indistinguishability of the protocol and functionality, along with the balance security properties that ensure no loss of funds for honest, non-negligent parties. This is achieved through the use of the ANYPREVOUT sighash flag, which is a proposed feature for Bitcoin, also required by the eltoo improvement to lightning [22].

A ON THE NECESSITY OF ANYPREVOUT

As our protocol relies on the ANYPREVOUT sighash flag, it cannot be deployed on Bitcoin until it is introduced. We here argue that any efficient protocol that achieves goals similar to ours and has parties maintain Bitcoin transactions in their local state requires the proposed sighash flag.

Definition A.1 (Off-chain base protocol). An *off-chain base protocol of $n \geq 2$ parties* is a generalisation of pairwise channels to n participants, in which a number of coins are locked in one or more outputs, each of which requires an n-of-n multisig in order to be spent (with 1 signature per participant) and where each party can unilaterally spend these outputs with one or more alternative transactions specified by the protocol, thus terminating (closing) the protocol.

Theorem A.2 (ANYPREVOUT is necessary). *Consider n independent, ordered off-chain base protocols such that every pair of consecutive protocols (Π_{i-1}, Π_i) for $i \in \{2, \ldots, n-1\}$ has a common party P_i. Also consider a protocol that establishes a virtual channel (i.e. a payment channel that does not need to add any txs on-chain when opening) between two parties P_1, P_n that take part in the first and last off-chain protocols respectively. If this protocol:*

(1) guarantees that each honest protocol party (either endpoint or intermediary) needs to put at most $O(1)$ transactions on-chain for unilateral closure,
(2) ensures that for each honest party P, if after establishing the virtual channel, no other party communicates off-chain with P, then in all scenarios P will regain its fair share of coins exclusively from one or more transactions that are descendants of one or more of the multisig outputs of the base protocol(s) in which it is implicated and
(3) needs to have at most a subexponential (in n) number of transactions available off-chain,

then the protocol needs the ANYPREVOUT sighash flag.

Proof of Theorem A.2. **[Orfeas: the next paragraph is wrong]** When an off-chain protocol is closed, there has to be some form of on-chain enforceable information and coin flow to at least one of its neighbouring protocols. This is to ensure that the virtual channel will be funded exactly once if at least one of its participants is honest and that no honest intermediary will be charged. If such information flow is lacking, then we have a partition of the path, making it possible to have no common party in the two partitions. In that case, there is no party that has to either provide its signature to both partitions in order for the protocol to progress or risk losing coins. This in turn, combined with the fact that all payments in the (P_1, P_n) virtual channel happen without the need to inform any intermediary, means that the participants of the partition that contains P_1 (w.l.o.g.) can collude and give to P_1 any sum of money they agree on, without giving an opportunity to P_n to object in case this sum does not correspond to the (P_1, P_n) channel balance, therefore violating the rules of the virtual channel.

[Orfeas: the next sentence is wrong: it would be more correct if it asked for atomic spending of the two base protocol outputs *or* their "descendants"] Due to the way the UTXO model works and respecting the second theorem rule, such information or coin flow can happen only by having intermediaries atomically spend the outputs of one of their base protocols together with the outputs of the other base protocol. This need for atomic spending also holds for any outputs that carry the relevant information or coins and are created when other protocol participants spend the base protocol outputs – such "successor" outputs must exist in order to permit the required information/-coin flow. Such atomic spends can only be carried out via a single transaction that consumes all relevant outputs. There is no other possible manner of on-chain enforceable information and coin flow that is compatible with the theorem requirements. Indeed, coins can only cross from one base protocol to the next via a transaction that involves both protocols. Note that adaptor signatures [9] do not constitute an exception, as they facilitate coin exchange only if the parties and all base protocols for this particular virtual channel were known when the off-chain protocols were opened (contradicting off-chain protocol independence) or if new on-chain transactions are introduced when opening the virtual channel (contradicting off-chain opening).

Therefore each party must have different transactions available to close its off-chain protocol(s), each corresponding to a different order of actions taken by participants of other off-chain protocols. This is true because if a party could close its protocol in an identical way whether one of its neighbouring protocols had already closed or not, it would then fail to make use of and possibly propagate to the other side the relevant coins and information. We will now prove by induction in the number $m = n - 1$ of base protocols that the number of these transactions T_m is exponential if ANYPREVOUT is not available, by calculating a lower bound, specifically, that $T_m \geq 2^{m-1}$.

If $m = 2$, then there is a single intermediary P_2. It needs at least 2 different transactions: one if it moves first and one if it moves second, after a member in the off-chain protocol to its right, e.g. P_3. From this it follows immediately that $T_2 \geq 2$.

If $m = k > 2$, then assume that P_2 needs to have $f \geq 2^{m-1}$ transactions available to be able to unilaterally close its protocols in all scenarios in which all parties P_i for $i \in \{3, \ldots, k+1\}$ act before P_2. Each of those transactions corresponds to one or more orderings of the closing actions of the parties of the other base protocols. No two transactions correspond to the same ordering.

For the induction step, consider a virtual channel over $m = k + 1$ base protocols. P_2 would still need f different transactions, each corresponding to the same orderings of parties' actions as in the induction hypothesis. These transactions are possibly different to the ones they correspond to in the case of the induction hypothesis, but their total number is the same. For each of these orderings we produce two new orderings: one in which the new party P_{k+2} acts right before and one in which it acts right after P_{k+1}. Given such an ordering o, consider the neighbor relation between the set of parties that have been activated and take its reflexive and transitive closure $\sim_o$. Now consider any party P_i with the following properties: (i) it acts after P_{k+2} and P_{k+1} (e.g., P_2 is such a party), and (ii) at least one of its neighbours belongs to the equivalence class of $\sim_o$ that contains P_{k+1}. Observe that such party P_i is always well defined. Since P_{k+1} must necessarily use a different transaction for each of the two orderings with P_{k+2}, and since there is a continuous chain of parties between P_{k+1} and P_i that have already acted, it is the case that P_i must have a different transaction for each of these two cases as well, as without ANYPREVOUT, an input of a transaction can only spend a specific output of a specific transaction. Finally, given that P_2 will have to act in response to at least as many of the above options, we deduce that P_2 needs to have at least $2f \geq 2^m$ transactions available. This completes the induction step.

As a result, we conclude that party P_2 needs at least $2^{m-1} \in O(2^n)$ transactions to be able to unilaterally close its protocol. □

Note that in case of a protocol that resembles ours but does not make use of ANYPREVOUT, the situation is further complicated in two distinct ways: First, virtual channel parties would have to generate and sign an at least exponential number of new commitment transactions on each update, one for each possible virtual output, therefore making virtual channel payments unrealistic. Second, if one of the base channels of a virtual channel is itself virtual, then the new channel needs a different set of virtual transactions for each of the (exponentially many) possible funding outputs of the base virtual channel, thus further compounding the issue.

B UNIVERSAL COMPOSITION FRAMEWORK

In this work we embrace the Universal Composition (UC) framework [12] to model parties, network interactions, adversarial influence and corruptions, as well as formalise and prove security.

UC closely follows and expands upon the paradigm of simulation-based security [53]. For a particular real world protocol, the main goal of UC is allow us to provide a simple "interface", the ideal world functionality, that describes what the protocol achieves in an ideal way. The functionality takes the inputs of all protocol parties and knows which parties are corrupted, therefore it normally can achieve the intention of the protocol in a much more straightforward manner. At a high level, once we have the protocol and the functionality defined, our goal is to prove that no probabilistic polynomial-time (PPT) Interactive Turing Machine (ITM) can distinguish whether it is interacting with the real world protocol or the ideal world functionality. If this is true we then say that the protocol UC-realises the functionality.

The principal contribution of UC is the following: Once a functionality that corresponds to a particular protocol is found, any other higher level protocol that internally uses the former protocol can instead use the functionality. This allows cryptographic proofs to compose and obviates the need for re-proving the security of every underlying primitive in every new application that uses it, therefore vastly improving the efficiency and scalability of the effort of cryptographic proofs.

An Interactive Turing Instance (ITI) is a single instantiation of an ITM. In UC, a number of ITIs execute and send messages to each other. At each moment only one ITI is executing (has the "execution token") and when it sends a message to another ITI, it transfers the execution token to the receiver. Messages can be sent either locally (inputs, outputs) or over the network. There is no notion of time built in UC – the only requirement is that the total number of execution steps each ITI takes throughout the experiment is polynomial in the security parameter.

The first ITI to be activated is the environment $\mathcal{E}$. This can be an instance of any PPT ITM. This ITI encompasses everything that happens around the protocol under scrutiny, including the players that send instructions to the protocol. It also is the ITI that tries to distinguish whether it is in the real or the ideal world. Put otherwise, it plays the role of the distinguisher.

After activating and executing some code, $\mathcal{E}$ may input a message to any party. If this execution is in the real world, then each party is an ITI running the protocol Π. Otherwise if the execution takes place in the ideal world, then each party is a dummy that simply relays messages to the functionality $\mathcal{F}$. An activated real world party then follows its code, which may instruct it to parse its input and send a message to another party via the network.

In UC the network is fully controlled by the so called adversary $\mathcal{A}$, which may be any PPT ITI. Once activated by any network message, this machine can read the message contents and act adaptively, freely communicate with $\mathcal{E}$ bidirectionally, choose to deliver the message right away, delay its delivery arbitrarily long, even corrupt it or drop it entirely. Crucially, it can also choose to corrupt any protocol party (in other words, UC allows adaptive corruptions). Once a party is corrupted, its internal state, inputs, outputs and execution comes under the full control of $\mathcal{A}$ for the rest of the execution. Corruptions take place covertly, so other parties do not necessarily learn which parties are corrupt. Furthermore, a corrupted party cannot become honest again.

The fact that $\mathcal{A}$ controls the network in the real world is modelled by providing direct communication channels between $\mathcal{A}$ and every other machine. This however poses an issue for the ideal world, as $\mathcal{F}$ is a single party that replaces all real world parties, so the interface has to be adapted accordingly. Furthermore, if $\mathcal{F}$ is to be as simple as possible, simulating internally all real world parties is not the way forward. This however may prove necessary in order to faithfully simulate the messages that the adversary expects to see in the real world. To solve these issues an ideal world adversary, also known as simulator $\mathcal{S}$, is introduced. This party can communicate freely with $\mathcal{F}$ and completely engulfs the real world $\mathcal{A}$. It can therefore internally simulate real world parties and generate suitable messages so that $\mathcal{A}$ remains oblivious to the fact that this is the ideal world. Normally messages between $\mathcal{A}$ and $\mathcal{E}$ are just relayed by $\mathcal{S}$, without modification or special handling.

From the point of view of the functionality, $\mathcal{S}$ is untrusted, therefore any information that $\mathcal{F}$ leaks to $\mathcal{S}$ has to be carefully monitored by the designer. Ideally it has to be as little as possible so that $\mathcal{S}$ does not learn more than what is needed to simulate the real world. This facilitates modelling privacy.

At any point during one of its activations, $\mathcal{E}$ may return a binary value (either 0 or 1). The entire execution then halts. Informally, we say that Π UC-realises $\mathcal{F}$, or equivalently that the ideal and the real worlds are indistinguishable, if $\forall$ PPT $\mathcal{A}$, $\exists$ PPT $\mathcal{S}$: $\forall$ PPT $\mathcal{E}$, the distance of the distributions over the machines' random tapes of the outputs of $\mathcal{E}$ in the two worlds is negligibly small. Note the order of quantifiers: $\mathcal{S}$ depends on $\mathcal{A}$, but not on $\mathcal{E}$.

C FUNCTIONALITY & SIMULATOR

Functionality $\mathcal{G}_{\text{Chan}}$ – general message handling rules

- On receiving input (`msg`) by $\mathcal{E}$ addressed to $P \in \{Alice, Bob\}$, handle it according to the corresponding rule in Fig. 9, 10, 11, 12 or 13 (if any) and subsequently send (RELAY, `msg`, P, $\mathcal{E}$, input) to $\mathcal{A}$.
- On receiving (`msg`) by party R addressed to $P \in \{Alice, Bob\}$ by means of `mode` $\in$ {output, network}, handle it according to the corresponding rule in Fig. 9, 10, 11, 12 or 13 (if any) and subsequently send (RELAY, `msg`, P, $\mathcal{E}$, `mode`) to $\mathcal{A}$. // all messages are relayed to $\mathcal{A}$
- On receiving (RELAY, `msg`, P, R, `mode`) by $\mathcal{A}$ (`mode` $\in$ {input, output, network}, $P \in \{Alice, Bob\}$), relay `msg` to R as P by means of `mode`. // $\mathcal{A}$ fully controls outgoing messages by $\mathcal{G}_{\text{Chan}}$
- On receiving (INFO, `msg`) by $\mathcal{A}$, handle (`msg`) according to the corresponding rule in Fig. 9, 10, 11, 12 or 13 (if any). After handling the message or after an "ensure" fails, send (HANDLED, `msg`) to $\mathcal{A}$. // (INFO, `msg`) messages by $\mathcal{S}$ always return control to $\mathcal{S}$ without any side-effect to any other ITI, except if $\mathcal{G}_{\text{Chan}}$ halts
- $\mathcal{G}_{\text{Chan}}$ keeps track of two state machines, one for each of *Alice*, *Bob*. If there are more than one suitable rules for a particular message, or if a rule matches the message for both parties, then both rule versions are executed. // the two rules act on different state machines, so the order of execution does not matter

Figure 8

Note that in UCGS [13], just like in UC, every message to an ITI may arrive via one of three channels: input, output and network. In the session of interest, input messages come from the environment $\mathcal{E}$ in the real world, whereas in the ideal world each input message comes from the corresponding dummy party, which forwards it as received by $\mathcal{E}$. Outputs may be received from any subroutine (local or global). This means that the "sender field" of inputs and outputs cannot be tampered with by $\mathcal{E}$ or $\mathcal{A}$. Network messages only come from $\mathcal{A}$; they may have been sent from any machine but are relayed (and possibly delayed, reordered, modified or even dropped) by $\mathcal{A}$. Therefore, in contrast to inputs and outputs, network messages may have a tampered "sender field".

Functionality $\mathcal{G}_{\text{Chan}}$ – open state machine, $P \in \{Alice, Bob\}$

On first activation: // before handing the message
  $pk_P \leftarrow \bot$; $\texttt{balance}_P \leftarrow 0$; $State_P \leftarrow$ UNINIT
  $\texttt{enabler}_P \leftarrow \bot$ // if we are a virtual channel, the ITI of P's base channel
  $\texttt{host}_P \leftarrow \bot$ // if we are a virtual channel, the ITI of the common host of this channel and P's base channel

On (BECAME CORRUPTED OR NEGLIGENT, P) by $\mathcal{A}$ or on output (ENABLER USED REVOCATION) by $\texttt{host}_P$ when in any state:
  $State_P \leftarrow$ IGNORED

On (INIT, pk) by P when $State_P$ = UNINIT:
  $pk_P \leftarrow pk$
  $State_P \leftarrow$ INIT

On (OPEN, x, "`ledger`", ...) by *Alice* when $State_A$ = INIT:
  store x

12: $State_A \leftarrow$ TENTATIVE BASE OPEN

13: On (BASE OPEN) by $\mathcal{A}$ when $State_A$ = TENTATIVE BASE OPEN:
14: $\quad$ `balance`$_A \leftarrow x$
15: $\quad$ `layer`$_A \leftarrow 0$
16: $\quad State_A \leftarrow$ OPEN

17: On (BASE OPEN) by $\mathcal{A}$ when $State_B$ = INIT:
18: $\quad$ `layer`$_B \leftarrow 0$
19: $\quad State_B \leftarrow$ OPEN

20: On (OPEN, x, `hops` ≠ "`ledger`", . . .) by *Alice* when $State_A$ = INIT:
21: $\quad$ store x
22: $\quad$ `enabler`$_A \leftarrow$ `hops[0].left`
23: $\quad$ add `enabler`$_A$ to *Alice*'s kindred parties
24: $\quad State_A \leftarrow$ PENDING VIRTUAL OPEN

25: On output (FUNDED, `host`, . . .) to *Alice* by `enabler`$_A$ when $State_A$ = PENDING VIRTUAL OPEN:
26: $\quad$ `host`$_A \leftarrow$ `host[0].left`
27: $\quad State_A \leftarrow$ TENTATIVE VIRTUAL OPEN

28: On output (FUNDED, `host`, . . .) to *Bob* by ITI $R \in \{\mathcal{G}_{\text{Chan}}, \text{LN}\}$ when $State_B$ = INIT:
29: $\quad$ `enabler`$_B \leftarrow R$
30: $\quad$ add `enabler`$_B$ to *Bob*'s kindred parties
31: $\quad$ `host`$_B \leftarrow$ `host`
32: $\quad State_B \leftarrow$ TENTATIVE VIRTUAL OPEN

33: On (VIRTUAL OPEN) by $\mathcal{A}$ when $State_P$ = TENTATIVE VIRTUAL OPEN:
34: $\quad$ **if** $P = Alice$ **then** `balance`$_P \leftarrow x$
35: $\quad$ `layer`$_P \leftarrow 0$
36: $\quad State_P \leftarrow$ OPEN

Figure 9: State machine in Fig. 14, 15, 16 and 21

Functionality $\mathcal{G}_{\text{Chan}}$ – payment state machine, $P \in \{Alice, Bob\}$

1: On (PAY, x) by P when $State_P$ = OPEN: // P pays $\bar{P}$
2: $\quad$ store x
3: $\quad State_P \leftarrow$ TENTATIVE PAY

4: On (PAY) by $\mathcal{A}$ when $State_P$ = TENTATIVE PAY: // P pays $\bar{P}$
5: $\quad State_P \leftarrow$ (SYNC PAY, x)

6: On (GET PAID, y) by P when $State_P$ = OPEN: // $\bar{P}$ pays P
7: $\quad$ store y
8: $\quad State_P \leftarrow$ TENTATIVE GET PAID

9: On (PAY) by $\mathcal{A}$ when $State_P$ = TENTATIVE GET PAID: // $\bar{P}$ pays P
10: $\quad State_P \leftarrow$ (SYNC GET PAID, x)

11: When $State_P$ = (SYNC PAY, x):
12: $\quad$ **if** $State_{\bar{P}} \in$ {IGNORED, (SYNC GET PAID, x)} **then**
13: $\quad\quad$ `balance`$_P \leftarrow$ `balance`$_P - x$
14: $\quad\quad$ // if $\bar{P}$ honest, this state transition happens simultaneously with l. 21
15: $\quad\quad State_P \leftarrow$ OPEN
16: $\quad$ **end if**

17: When $State_P$ = (SYNC GET PAID, x):
18: $\quad$ **if** $State_{\bar{P}} \in$ {IGNORED, (SYNC PAY, x)} **then**
19: $\quad\quad$ `balance`$_P \leftarrow$ `balance`$_P + x$
20: $\quad\quad$ // if $\bar{P}$ honest, this state transition happens simultaneously with l. 15
21: $\quad\quad State_P \leftarrow$ OPEN
22: $\quad$ **end if**

Figure 10: State machine in Fig. 17

Functionality $\mathcal{G}_{\text{Chan}}$ – funding state machine, $P \in \{Alice, Bob\}$

1: On input (FUND ME, x, . . .) by ITI $R \in \{\mathcal{G}_{\text{Chan}}, \text{LN}\}$ when $State_P$ = OPEN:
2: $\quad$ store x
3: $\quad$ add R to P's kindred parties
4: $\quad State_P \leftarrow$ PENDING FUND

5: When $State_P$ = PENDING FUND:
6: $\quad$ **if** we intercept the command "define new VIRT ITI `host`" by $\mathcal{A}$, routed through P **then**
7: $\quad\quad$ store `host`
8: $\quad\quad State_P \leftarrow$ TENTATIVE FUND
9: $\quad\quad$ continue executing $\mathcal{A}$'s command
10: $\quad$ **end if**

11: On (FUND) by $\mathcal{A}$ when $State_P$ = TENTATIVE FUND:
12: $\quad State_P \leftarrow$ SYNC FUND

13: When $State_P$ = OPEN:
14: $\quad$ **if** we intercept the command "define new VIRT ITI `host`" by $\mathcal{A}$, routed through P **then**
15: $\quad\quad$ store `host`
16: $\quad\quad State_P \leftarrow$ TENTATIVE HELP FUND
17: $\quad\quad$ continue executing $\mathcal{A}$'s command
18: $\quad$ **end if**
19: $\quad$ **if** we receive a RELAY message with `msg` = (INIT, . . . , `fundee`) addressed from P by $\mathcal{A}$ **then**
20: $\quad\quad$ add `fundee` to P's kindred parties
21: $\quad\quad$ continue executing $\mathcal{A}$'s command
22: $\quad$ **end if**

23: On (FUND) by $\mathcal{A}$ when $State_P$ = TENTATIVE HELP FUND:
24: $\quad State_P \leftarrow$ SYNC HELP FUND

25: When $State_P$ = SYNC FUND:
26: $\quad$ **if** $State_{\bar{P}} \in$ {IGNORED, SYNC HELP FUND} **then**
27: $\quad\quad$ `balance`$_P \leftarrow$ `balance`$_P - x$
28: $\quad\quad$ `host`$_P \leftarrow$ `host`

29: // if $\bar{P}$ honest, this state transition happens simultaneously with l. 38
30: $\texttt{layer}_P \leftarrow \texttt{layer}_P + 1$
31: $State_P \leftarrow$ OPEN
32: **end if**

33: When $State_P =$ SYNC HELP FUND:
34: **if** $State_{\bar{P}} \in \{$IGNORED, SYNC FUND$\}$ **then**
35: $\texttt{host}_P \leftarrow \texttt{host}$
36: // if $\bar{P}$ honest, this state transition happens simultaneously with l. 31
37: $\texttt{layer}_P \leftarrow \texttt{layer}_P + 1$
38: $State_P \leftarrow$ OPEN
39: **end if**

Figure 11: State machine in Fig. 18

Functionality $\mathcal{G}_{\text{Chan}}$ – force close state machine, $P \in \{Alice, Bob\}$

1: On (FORCECLOSE) by P when $State_P =$ OPEN:
2: $State_P \leftarrow$ CLOSING

3: On input (BALANCE) by R addressed to P where R is kindred with P:
4: **if** $State_P \notin \{$UNINIT, INIT, PENDING VIRTUAL OPEN, TENTATIVE VIRTUAL OPEN, TENTATIVE BASE OPEN, IGNORED, CLOSED$\}$ **then**
5: reply (MY BALANCE, $\texttt{balance}_P$, pk_P, $\texttt{balance}_{\bar{P}}$, $pk_{\bar{P}}$)
6: **else**
7: reply (MY BALANCE, 0, pk_P, 0, $pk_{\bar{P}}$)
8: **end if**

9: On (FORCECLOSE, P) by $\mathcal{A}$ when $State_P \notin \{$UNINIT, INIT, PENDING VIRTUAL OPEN, TENTATIVE VIRTUAL OPEN, TENTATIVE BASE OPEN, IGNORED$\}$:
10: input (READ) to $\mathcal{G}_{\text{Ledger}}$ as P and assign ouput to Σ
11: $\texttt{coins} \leftarrow$ sum of values of outputs exclusively spendable or spent by pk_P in Σ
12: $\texttt{balance} \leftarrow \texttt{balance}_P$
13: **for all** P's kindred parties R **do**
14: input (BALANCE) to R as P and extract $\texttt{balance}_R$, pk_R from response
15: $\texttt{balance} \leftarrow \texttt{balance} + \texttt{balance}_R$
16: $\texttt{coins} \leftarrow \texttt{coins} +$ sum of values of outputs exclusively spendable or spent by pk_R in Σ
17: **end for**
18: **if** $\texttt{coins} \geq \texttt{balance}$ **then**
19: $State_P \leftarrow$ CLOSED
20: **else** // balance security is broken
21: halt
22: **end if**

Figure 12

Functionality $\mathcal{G}_{\text{Chan}}$ – cooperative close state machine, $P \in \{Alice, Bob\}$

1: On (COOP CLOSING, P, x) by $\mathcal{A}$ when $State_P =$ OPEN:
2: store x
3: $State_P \leftarrow$ COOP CLOSING

4: On (COOP CLOSED, P) by $\mathcal{A}$ when $State_P =$ COOP CLOSING:
5: **if** $\texttt{layer}_P = 0$ **then** // P's channel, which is virtual, is cooperatively closed
6: $State_P \leftarrow$ COOP CLOSED
7: **else** // the virtual channel for which P's channel is base is cooperatively closed
8: $\texttt{layer}_P \leftarrow \texttt{layer}_P - 1$
9: $\texttt{balance}_P \leftarrow \texttt{balance}_P + x$
10: $State_P \leftarrow$ OPEN
11: **end if**

Figure 13

D MODEL & CONSTRUCTION

D.1 Model

In this section we will examine the architecture and the details of our model, along with possible attacks and their mitigations. We follow the UCGS framework [13] to formulate the protocol and its security. We list the ideal-world global functionality $\mathcal{G}_{\text{Chan}}$ in Section C (Figures 8-12) and a simulator $\mathcal{S}$ (Figures 22-23), along with a real-world protocol Π_{Chan} (Figures 24-64) that UC-realizes $\mathcal{G}_{\text{Chan}}$ (Theorem 5.5). We give a self-contained description in this section, while pointing to figures in Sections C and E, in case the reader is interested in a pseudocode style specification.

As in previous formulations, (e.g., [54]), the role of $\mathcal{E}$ corresponds to two distinct actors in a real world implementation. On the one hand $\mathcal{E}$ passes inputs that correspond to the desires of human users (e.g. open a channel, pay, close), on the other hand $\mathcal{E}$ is responsible with periodically waking up parties to check the ledger and act upon any detected counterparty misbehaviour, similar to an always-on "daemon" of real-life software that periodically nudges the implementation to perform these checks.

Since it is possible that $\mathcal{E}$ fails to wake up a party often enough, Π_{Chan} explicitly checks whether it has become "negligent" every time it is activated and all security guarantees are conditioned on the party not being negligent. A party is deemed negligent if more than p blocks have been added to $\mathcal{G}_{\text{Ledger}}$ between any consecutive pair of activations. The need for explicit negligence checking stems from the fact that party activation is entirely controlled by $\mathcal{E}$ and no synchrony limitations are imposed (e.g. via the use of $\mathcal{G}_{\text{CLOCK}}$), therefore it can happen that an otherwise honest party is not activated in time to prevent a malicious counterparty from successfully using an old commitment transaction. If a party is marked as negligent, no balance security guarantees are given (c.f. Lemma 5.1). Note that in realistic software the aforementioned daemon is local and trustworthy, therefore it would never allow Π_{Chan} to become negligent, as long as the machine is powered on and in good order.

D.2 Ideal world functionality $\mathcal{G}_{\text{Chan}}$

Our ideal world functionality $\mathcal{G}_{\text{Chan}}$ represents a single channel, either simple or virtual. It acts as a relay between $\mathcal{A}$ and $\mathcal{E}$, leaking all messages. This simplifies the functionality and facilitates the indistinguishability argument by having $\mathcal{S}$ simply running internally the real world protocols of the channel parties Π_{Chan} with no modifications. Furthermore, the communication of parties with $\mathcal{G}_{\text{Ledger}}$ is handled by $\mathcal{G}_{\text{Chan}}$: when a simulated honest party in $\mathcal{S}$ needs to send a message to $\mathcal{G}_{\text{Ledger}}$, $\mathcal{S}$ instructs $\mathcal{G}_{\text{Chan}}$ to send this message to $\mathcal{G}_{\text{Ledger}}$ on this party's behalf. $\mathcal{G}_{\text{Chan}}$ internally maintains two state machines, one per channel party (c.f. Figures 14, 15, 16, 18, 17, 19, 21) that keep track of whether the parties are corrupted or negligent, whether the channel has opened, whether a payment is underway, which ITIs are to be considered *kindred* parties (as they correspond to other channels owned by the same human user, discussed below) and whether the channel is currently closing collaboratively or has already closed. The single security check performed is *whether the on-chain coins are at least equal to the expected balance once the channel closes.* If this check fails, $\mathcal{G}_{\text{Chan}}$ halts. Since the protocol Π_{Chan} (which realises $\mathcal{G}_{\text{Chan}}$, c.f. Theorems 5.4 and 5.5) never halts, this ideal world check corresponds to the security guarantee offered by Π_{Chan}. Note that this check is not performed for negligent parties, as $\mathcal{S}$ notifies $\mathcal{G}_{\text{Chan}}$ if a party becomes negligent and the latter omits the check. Thus indistinguishability between the real and the ideal world is not violated in case of negligence.

Observe that a human user may participate in various channels, therefore it corresponds to more than one ITMs. This is the case for example for the funder of a virtual channel and the corresponding party of the first base channel. Such parties are called *kindred*. They communicate locally (i.e. via inputs and outputs, without using the adversarially controlled network) and balance guarantees concern their aggregate coins. Formally this communication is modelled by having a virtual channel using its base channels as global subroutines, as defined in [13].

If we were using plain UC, the above would constitute a violation of the subroutine respecting property that functionalities have to fulfill. We leverage the concept of global functionalities put forth in [13] to circumvent the issue. More specifically, we say that a simple channel functionality is of "level" 1, which is written as $\mathcal{G}^1_{\text{Chan}}$. Inductively, a virtual channel functionality that is based on channels of any "level" up to and including $n-1$ has a "level" n, which write as $\mathcal{G}^n_{\text{Chan}}$. Then $\mathcal{G}^n_{\text{Chan}}$ is $(\mathcal{G}_{\text{Ledger}}, \mathcal{G}^1_{\text{Chan}}, \dots, \mathcal{G}^{n-1}_{\text{Chan}})$-subroutine respecting, according to the definition of [13]. The same structure is used in the real world between protocols. This technique ensures that the necessary conditions for the validity of the functionality and the protocol are met and that the realisability proof can go through, as we will see in Section 5 in more detail.

We could instead contain all the channels in a single, monolithic functionality (following the approach of [54]) and we believe that we could still carry out the security proof. Nevertheless, having the functionality correspond to a single channel has no drawbacks, as all desired security guarantees are provided by our modular architecture, and instead brings two benefits. Firstly, the functionality is easier to intuitively grasp, as it handles less tasks. Having a simple and intuitive functionality aids in its reusability and is an informal goal of the simulation-based paradigm. Secondly, this approach permits our functionality to be global, as defined in [13]. We note that the ideal functionality defined in [9] is unsuitable for our case, as it requires direct access to the ledger, which is not the case for a $\mathcal{G}_{\text{Chan}}$ corresponding to a virtual channel.

D.3 Real world protocol Π_{Chan}

Our real world protocol Π_{Chan}, ran by party P, consists of two subprotocols: the Lightning-inspired part, dubbed LN (Figures 24-43) and the novel virtual layer subprotocol, named VIRT (Figures 49-64). A simple channel that is not the base of any virtual channel leverages only LN, whereas a simple channel that is the base of at least one virtual channel does leverage both LN and VIRT. A virtual channel uses both LN and VIRT.

LN subprotocol The LN subprotocol has two variations depending on whether P is the channel funder (*Alice*) or the fundee (*Bob*). It performs a number of tasks: Initialisation takes a single step for fundees and two steps for funders. LN first receives a public key $pk_{P,\text{out}}$ from $\mathcal{E}$. This is the public key that should eventually own all P's coins after the channel is closed. LN also initialises its internal variables. If P is a funder, LN waits for a second activation to generate a keypair and then waits for $\mathcal{E}$ to endow it with some coins, which will be subsequently used to open the channel (Figure 24).

After initialisation, the funder *Alice* is ready to open the channel. Once $\mathcal{E}$ gives to *Alice* the identity of *Bob*, the initial channel balance c and, (looking forward to the VIRT subprotocol description) in case it is a virtual channel, the identities of the base channel owners (Figure 31), *Alice* generates and sends *Bob* her funding and revocation public keys ($pk_{A,F}$, $pk_{A,R}$, used for the funding and revocation outputs respectively) along with c, $pk_{A,\text{out}}$, and the base channel identities (only for virtual channels). Given that *Bob* has been initialised, it generates funding and revocation keys and replies to *Alice* with $pk_{B,F}$, $pk_{B,R}$, and $pk_{B,\text{out}}$ (Figure 26).

The next step prepares the base channels (Figure 27) if needed. If our channel is a simple one, then *Alice* simply generates the funding tx. If it is a virtual and assuming all base parties (running LN) cooperate, a chain of messages from *Alice* to *Bob* and back via all base parties is initiated (Figures 33 and 34). These messages let each successive neighbour know the identities of all the base parties. Furthermore each party instantiates a new "host" party that runs VIRT. It also generates new funding keys and communicates them, along with its "out" key $pk_{P,\text{out}}$ and its leftward and rightward balances. If this circuit of messages completes, *Alice* delegates the creation of the new virtual layer transactions to its new VIRT host, which will be discussed later in detail. If the virtual layer is successful, each base party is informed by its host accordingly, intermediaries return to the OPEN state (i.e., they have completed their part and are in standby, ready to accept instructions for, e.g., new payments) and *Alice* and *Bob* continue the opening procedure. In particular, *Alice* and *Bob* exchange signatures on the initial commitment transactions, therefore ensuring that the funding output can be spent (Figure 28). After that, in case the channel is simple the funding transaction is put on-chain (Figure 29) and finally $\mathcal{E}$ is informed of the successful channel opening.

There are two facts that should be noted: Firstly, in case the opened channel is virtual, each intermediary necessarily partakes in two channels. However each protocol instance only represents a party in a single channel, therefore each intermediary is in practice realised by two kindred Π_{Chan} instances that communicate locally, called "siblings". Secondly, our protocol is not designed to gracefully recover if other parties do not send an expected message at any point in the opening or payment procedure. Such anti-Denial-of-Service measures would greatly complicate the protocol and are left as a task for a real world implementation. It should however be stressed that an honest party with an open channel that has fallen victim to such an attack can still unilaterally close the channel, therefore no coins are lost in any case.

Once the channel is open, *Alice* and *Bob* can carry out an unlimited number of payments in either direction, only needing to exchange 3 direct network messages with each other per payment, therefore avoiding the slow and costly on-chain validation. The payment procedure is identical for simple and virtual channels and crucially it does not implicate the intermediaries (and therefore *Alice* and *Bob* do not incur any delays such an interaction with intermediaries would introduce). For a payment to be carried out, the payee is first notified by $\mathcal{E}$ (Figure 38) and subsequently the payer is instructed by $\mathcal{E}$ to commence the payment (Figure 37).

If the channel is virtual, each party also checks that its upcoming balance is lower than the balance of its sibling's counterparty and that the upcoming balance of the counterparty is higher than the balance of its own sibling, otherwise it rejects the payment. This is to mitigate a "griefing" attack (i.e. one that does not lead to financial gain) where a malicious counterparty uses an old commitment transaction to spend the base funding output, therefore blocking the honest party from using its initiator virtual transaction. This check ensures that the coins gained by the punishment are sufficient to cover the losses from the blocked initiator transaction. If the attack takes place, other local channels based directly or indirectly on it are informed and are moved to a failed state. Note that this does not bring a risk of losing any of the total coins of all local channels. We conjecture that this balance constraint can be lifted if the current Lightning-inspired payment method is replaced with an eltoo-inspired one [22].

Subsequently each of the two parties builds the new commitment transaction of its counterparty and signs it. It also generates a new revocation keypair for the next update and sends over the generated signature and public key. Then the revocation transactions for the previously valid commitment transactions are generated, signed and the signatures are exchanged. To reduce the number of messages, the payee sends the two signatures and the public key in one message. This does not put it at risk of losing funds, since the new commitment transaction (for which it has already received a signature and therefore can spend) gives it more funds than the previous one.

Π_{Chan} also checks the chain for outdated commitment transactions by the counterparty and publishes the corresponding revocation transaction in case one is found (Figure 40). It also keeps track of whether the party is activated often enough and marks it as negligent otherwise (Figure 24). In particular, at the beginning of every activation while the channel is open, LN checks if the party has been activated within the last p blocks (where p is an implementation-dependent global constant) by reading from $\mathcal{G}_{\text{Ledger}}$ and comparing the current block height with that of the last activation.

Cooperative closing involves both LN (Figures 44-47) and VIRT (Figure 63) subprotocols. Any party can initiate it by asking the virtual channel fundee. The latter signs the last coin balance and sends it to the funder, who first ensures the fundee signed the correct balance, then signs it as well. Its enabler (i.e. the kindred party that is a member of the 1st base channel) generates and signs a new commitment tx in which it adds the funder's coins to its own and the fundee's coins to its counterparty's, while using the funding keys that were used before opening the virtual channel. It also generates a new revocation keypair for the next channel update and sends the revocation public key with the signature and the final virtual channel balance to its counterparty. The latter verifies the signature and that the two virtual channel parties agree on their final balance. If all goes well, it passes control to its kindred party that is a member of the next channel in sequence. If no verification fails, the process repeats until the fundee is reached. Now a backwards sequence of messages begins, in which each party that previously did verification now provides a signature for the new commitment tx, along with a revocation signature for the old commitment tx and a new revocation public key for the next update. Eacch receiver verifies the signatures and "passes the baton" to its kindred party closer to the funder. When the funder is reached, the last series of messages begins. Now each party that has not yet sent a revocation does so. Once the chain of messages reaches the fundee, the channel has successfully closed cooperatively. In total, each LN party sends and stores 2 signatures, 1 private key and 1 public key. The associated behaviour of the VIRT subprotocol is discussed later.

Alternatively, when either party is instructed by $\mathcal{E}$ to unilaterally close the channel (Figure 42), it first asks its host to unilaterally close (details on the exact steps are discussed later) and once that is done, the ledger is checked for any transaction spending the funding output. In case the latest remote commitment tx is on-chain, then the channel is already closed and no further action is necessary. If an old committment transaction is on-chain, the corresponding revocation transaction is used for punishment. If the funding output is still unspent, the party attempts to publish the latest commitment transaction after waiting for any relevant timelock to expire. Until the funding output is irrevocably spent, the party still has to periodically check the blockchain and again be ready to use a revocation transaction if an old commitment transaction spends the funding output after all (Figure 40).

VIRT subprotocol This subprotocol acts as a mediator between the base channels and the Lightning-based logic. Put simply, its main responsibility is putting on-chain the funding output of the channel when needed. When first initialised by a machine that executes the LN subprotocol (Figure 49), it learns and stores the identities, keys, and balances of various relevant parties, along with the required timelock and other useful data regarding the base channels. It then generates a number of keys as needed for the rest of the base preparation. If the initialiser is also the channel funder, then the VIRT machine initiates 4 "circuits" of messages. Each circuit consists of one message from the funder P_1 to its neighbour P_2, one message from each intermediary P_i to the "next" neighbour P_{i+1}, one message from the fundee P_n to its neighbour P_{n-1} and one more message from each intermediary P_i to the "previous" neighbour P_{i-1}, for a total of $2 \cdot (n-1)$ messages per circuit.

The first circuit (Figure 50) communicates all "out", virtual, revocation and funding keys (both old and new), all balances and all timelocks among all parties. In the second circuit (Figure 57) every party receives and verifies all signatures for all inputs of its virtual transactions that spend a virtual output. It also produces and sends its own such signatures to the other parties. Each party generates and circulates $S = \sum_{i=2}^{n-2} (n-3+\chi_{i=2}+\chi_{i=n-1}+2(i-2+\chi_{i=2})(n-i-1+\chi_{i=n-1})) \in O(n^3)$ signatures (where χ_A is the characteristic function that equals 1 if A is true and 0 else), which is derived by calculating the total number of virtual outputs of all parties' virtual transactions – we remind that each virtual output can be spent by a n-of-n multisig. On a related note, the number of virtual transactions for which each party needs to store signatures is 1 for the two endpoints (Figure 52) and $n-2+\chi_{i=2}+\chi_{i=n-1}+(i-2+\chi_{i=2})(n-i-1+\chi_{i=n-1}) \in O(n^2)$ for the i-th intermediary (Figure 51). The latter is derived by counting the number of extend-interval and merge-intervals transactions held by the intermediary, which are equal to the number of distinct intervals that the party can extend and the number of distinct pairs of intervals that the party can merge respectively, plus 1 for the unique initiator transaction of the party. The third circuit concerns sharing signatures for the funding outputs (Figure 58). Each party signs all transactions that spend a funding output relevant to the party, i.e. the initiator transaction and some of the extend-interval transactions of its neighbours. The two endpoints send 2 signatures each when $n = 3$ and $n-2$ signatures each when $n > 3$, whereas each intermediary sends $2+\chi_{i+1<n}(n-2+\chi_{i=n-2})+\chi_{i-1>1}(n-2+\chi_{i=3}) \in O(n)$ signatures each. The last circuit of messages (Figure 59) carries the revocations of the previous states of all base channels. After this, base parties can only use the newly created virtual transactions to spend their funding outputs. In this step each party exchanges a single signature with each of its neighbours.

In case of a cooperative closing, VIRT orchestrates the hosted LN ITIs, instructing them to perform the actions discussed previously. It also is responsible for sending the actual messages to the host of the next counterparty and receiving its responses. Apart from controlling the flow of messages, a VIRT ITI also generates revocation signatures to invalidate its virtual transactions and verifies the respective revocation signatures generated by its counterparty VIRT ITI, thereby ensuring that, moving forward, the use of an old virtual transaction can be punished.

On the other hand, when VIRT is instructed to unilaterally close by party R (Figure 61), it first notifies its VIRT host (if any) and waits for it to unilaterally close. After that, it signs and publishes the unique valid virtual transaction. It then repeatedly checks the chain to see if the transaction is included (Figure 62). If it is included, the virtual layer is closed and VIRT informs (i.e. outputs (CLOSED) to) R. The instruction to close has to be received potentially many times, because a number of virtual transactions (the ones that spend the same output) are mutually exclusive and therefore if another base party publishes an incompatible virtual transaction contemporaneously and that remote transaction wins the race to the chain, then our VIRT party has to try again with another, compatible virtual transaction.

Simulator $\mathcal{S}$ – general message handling rules

- On receiving (RELAY, `in_msg`, P, R, `in_mode`) by $\mathcal{G}_{\text{Chan}}$ (`in_mode` $\in$ {input, output, network}, $P \in \{Alice, Bob\}$), handle (`in_msg`) with the simulated party P as if it was received from R by means of `in_mode`. In case simulated P does not exist yet, initialise it as an LN ITI. If there is a resulting message `out_msg` that is to be sent by simulated P to R' by means of `out_mode` $\in$ {input, output, network}, send (RELAY, `out_msg`, P, R', `out_mode`) to $\mathcal{G}_{\text{Chan}}$.
- On receiving by $\mathcal{G}_{\text{Chan}}$ a message to be sent by P to R via the network, carry on with this action (i.e. send this message via the internal $\mathcal{A}$).
- Relay any other incoming message to the internal $\mathcal{A}$ unmodified.
- On receiving a message (`msg`) by the internal $\mathcal{A}$, if it is addressed to one of the parties that correspond to $\mathcal{G}_{\text{Chan}}$, handle the message internally with the corresponding simulated party. Otherwise relay the message to its intended recipient unmodified. // Other recipients are $\mathcal{E}$, $\mathcal{G}_{\text{Ledger}}$ or parties unrelated to $\mathcal{G}_{\text{Chan}}$

Given that $\mathcal{G}_{\text{Chan}}$ relays all messages and that we simulate the real-world machines that correspond to $\mathcal{G}_{\text{Chan}}$, the simulation is perfectly indistinguishable from the real world.

Figure 22

Simulator $\mathcal{S}$ – notifications to $\mathcal{G}_{\text{Chan}}$

- "P" refers one of the parties that correspond to $\mathcal{G}_{\text{Chan}}$.
- When an action in this Figure interrupts an ITI simulation, continue simulating from the interruption location once action is over/$\mathcal{G}_{\text{Chan}}$ hands control back.

On (CORRUPT) by $\mathcal{A}$, addresed to P:
    // After executing this code and getting control back from $\mathcal{G}_{\text{Chan}}$ (which always happens, c.f. Fig. 8), deliver (CORRUPT) to simulated P (c.f. Fig. 22).
    send (INFO, BECAME CORRUPTED OR NEGLIGENT, P) to $\mathcal{G}_{\text{Chan}}$

When simulated P sets variable `negligent` to True (Fig. 24, l. 7/Fig. 25, l. 26):
    send (INFO, BECAME CORRUPTED OR NEGLIGENT, P) to $\mathcal{G}_{\text{Chan}}$

When simulated honest *Alice* receives (OPEN, x, `hops`, . . .) by $\mathcal{E}$:
    store `hops` // will be used to inform $\mathcal{G}_{\text{Chan}}$ once the channel is open

When simulated honest *Bob* receives (OPEN, x, `hops`, . . .) by *Alice*:
    if *Alice* is corrupted **then** store `hops` // if *Alice* is honest, we already have `hops`. If *Alice* became corrupted after receiving (OPEN, . . .), overwrite `hops`

When the last of the honest simulated $\mathcal{G}_{\text{Chan}}$'s parties moves to the OPEN *State* for the first time (Fig. 28, l. 19/Fig. 30, l. 16/Fig. 31, l. 18):
    if `hops` = "`ledger`" **then**
        send (INFO, BASE OPEN) to $\mathcal{G}_{\text{Chan}}$
    else
        send (INFO, VIRTUAL OPEN) to $\mathcal{G}_{\text{Chan}}$
    end if

When (both $\mathcal{G}_{\text{Chan}}$'s simulated parties are honest and complete sending and receiving a payment (Fig. 36, ll. 6 and 21 respectively), or (when only one party is honest and (completes either receiving or sending a payment)): // also send this message if both parties are honest when Fig. 36, l. 6 is executed by one party, but its counterparty is corrupted before executing Fig. 36, l. 21

  send (INFO, PAY) to $\mathcal{G}_{\text{Chan}}$

When honest P executes Fig. 33, l. 21 or (when honest P executes Fig. 33, l. 19 and $\bar{P}$ is corrupted): // in the first case if $\bar{P}$ is honest, it has already moved to the new host, (Fig 59, ll. 7, 23): lifting to next layer is done

  send (INFO, FUND) to $\mathcal{G}_{\text{Chan}}$

When one of the honest simulated $\mathcal{G}_{\text{Chan}}$'s parties P moves to the COOP CLOSING state (Fig. 46, l. 4, Fig. 47, ll. 6, 12, Fig. 63, ll. 11, 24):

  if triggered by Fig. 46, l. 4 or Fig. 47, l. 6 **then** // P is `funder` or `fundee`

    send (INFO, COOP CLOSING, P, $-c_P$) to $\mathcal{G}_{\text{Chan}}$ // coin value extracted from simulated P

  else if triggered by Fig. 47, l. 12 **then** // P is `funder`'s base

    send (INFO, COOP CLOSING, P, c'_1) to $\mathcal{G}_{\text{Chan}}$

  else if triggered by Fig. 63, l. 11 **then** // P is an intermediary farther from `funder` than $\bar{P}$

    send (INFO, COOP CLOSING, P, c'_2) to $\mathcal{G}_{\text{Chan}}$

  else if triggered by Fig. 63, l. 24 **then** // P is an intermediary closer to `funder` than $\bar{P}$

    send (INFO, COOP CLOSING, P, $c'_1 - c_{\text{virt}}$) to $\mathcal{G}_{\text{Chan}}$

  end if

When one of the honest simulated $\mathcal{G}_{\text{Chan}}$'s parties P completes cooperative closing (Fig. 47, l. 45, Fig. 63, l. 134, Fig. 63, l. 119, Fig. 63, or l. 103):

  send (INFO, COOP CLOSED, P) to $\mathcal{G}_{\text{Chan}}$

When one of the honest simulated $\mathcal{G}_{\text{Chan}}$'s parties P moves to the CLOSED state (Fig. 40, l. 8 or l. 11):

  send (INFO, FORCECLOSE, P) to $\mathcal{G}_{\text{Chan}}$

Figure 23

E PROTOCOL

Process LN – init

// When not specified, input comes from and output goes to $\mathcal{E}$.

// The ITI knows whether it is *Alice* (funder) or *Bob* (fundee). The activated party is P and the counterparty is $\bar{P}$.

On every activation, before handling the message:

  if `last_poll` $\neq \bot \wedge$ *State* $\neq$ CLOSED **then** // channel is open

    input (READ) to $\mathcal{G}_{\text{Ledger}}$ and assign ouput to Σ

    if `last_poll` $+ p < |\Sigma|$ **then** // p is a global parameter

      `negligent` $\leftarrow$ True

    end if

  end if

  if *State* = WAITING FOR NOTHING REVOKED $\wedge$ activation is not caused by output (NOTHING REVOKED), received by a member of the list of old hosts **then** // the only way for this case to be true is if the old host punished a misbehaving counterparty

    State $\leftarrow$ BASE PUNISHED

  end if

On (INIT, $pk_{P,\text{out}}$):

  ensure *State* = $\bot$

  State $\leftarrow$ INIT

  `hosting` $\leftarrow$ False

  store $pk_{P,\text{out}}$

  $(c_A, c_B, \texttt{locked}_A, \texttt{locked}_B) \leftarrow (0, 0, 0, 0)$

  $(\texttt{paid\_out}, \texttt{paid\_in}) \leftarrow (\emptyset, \emptyset)$

  `negligent` $\leftarrow$ False

  `last_poll` $\leftarrow \bot$

  output (INIT OK)

On (TOP UP):

  ensure P = *Alice* // activated party is the funder

  ensure *State* = INIT

  $(sk_{P,\text{chain}}, pk_{P,\text{chain}}) \leftarrow$ KEYGEN()

  input (READ) to $\mathcal{G}_{\text{Ledger}}$ and assign ouput to Σ

  output (TOP UP TO, $pk_{P,\text{chain}}$)

  while $\neg\exists \text{tx} \in \Sigma, c_{P,\text{chain}} : (c_{P,\text{chain}}, pk_{P,\text{chain}}) \in \text{tx.outputs}$ **do**

    // while waiting, all other messages by P are ignored

    wait for input (CHECK TOP UP)

    input (READ) to $\mathcal{G}_{\text{Ledger}}$ and assign ouput to Σ

  end while

  State $\leftarrow$ TOPPED UP

  output (TOP UP OK, $c_{P,\text{chain}}$)

On (BALANCE):

  ensure *State* $\in$ {OPEN, CLOSED}

  output (BALANCE, c_A, $pk_{A,\text{out}}$, c_B, $pk_{B,\text{out}}$, $\texttt{locked}_A$, $\texttt{locked}_B$)

Figure 24

Process LN – methods used by VIRT

REVOKEPREVIOUS():

  ensure *State* $\in$ WAITING FOR (OUTBOUND) REVOCATION

  $R_{\bar{P},i} \leftarrow$ TX {input: $C_{P,i}$.outputs.P, output: ($C_{P,i}$.outputs.P.value, $pk_{\bar{P},\text{out}}$)}

  $\text{sig}_{A,R,i} \leftarrow \text{SIGN}(R_{\bar{P},i}, sk_{P,R,i})$

  if *State* = WAITING FOR REVOCATION **then**

    State $\leftarrow$ WAITING FOR INBOUND REVOCATION

  else // *State* = WAITING FOR OUTBOUND REVOCATION

    $i \leftarrow i + 1$

    State $\leftarrow$ WAITING FOR HOSTS READY

  end if

  $\texttt{host}_P \leftarrow \texttt{host}'_P$ // forget old host, use new host instead

  `layer` $\leftarrow$ `layer` + 1

  return $\text{sig}_{P,R,i}$

PROCESSREMOTEREVOCATION($\text{sig}_{\bar{P},R,i}$):
ensure *State* = WAITING FOR (INBOUND) REVOCATION
$R_{P,i} \leftarrow$ TX {input: $C_{\bar{P},i}$.outputs.$\bar{P}$, output: ($C_{\bar{P},i}$.outputs.$\bar{P}$.value, $pk_{P,\text{out}}$)}
ensure VERIFY($R_{P,i}$, $\text{sig}_{\bar{P},R,i}$, $pk_{\bar{P},R,i}$) = True
if *State* = WAITING FOR REVOCATION **then**
State $\leftarrow$ WAITING FOR OUTBOUND REVOCATION
else // *State* = WAITING FOR INBOUND REVOCATION
$i \leftarrow i + 1$
State $\leftarrow$ WAITING FOR HOSTS READY
end if
return (OK)

NEGLIGENT():
`negligent` $\leftarrow$ True
return (OK)

Figure 25

Process LN.EXCHANGEOPENKEYS()

$(sk_{A,F}, pk_{A,F}), (sk_{A,R,1}, pk_{A,R,1}), (sk_{A,R,2}, pk_{A,R,2}) \leftarrow \text{KEYGEN}()^3$
State $\leftarrow$ WAITING FOR OPENING KEYS
send (OPEN, c, `hops`, $pk_{A,F}$, $pk_{A,R,1}$, $pk_{A,R,2}$, $pk_{A,\text{out}}$) to `fundee`
// colored code is run by honest `fundee`. Validation is implicit
ensure we run the code of *Bob*
ensure *State* = INIT
store $pk_{A,F}$, $pk_{A,R,1}$, $pk_{A,R,2}$, $pk_{A,\text{out}}$
$(sk_{B,F}, pk_{B,F}), (sk_{B,R,1}, pk_{B,R,1}), (sk_{B,R,2}, pk_{B,R,2}) \leftarrow \text{KEYGEN}()^3$
if `hops` = "`ledger`" **then** // opening base channel
`layer` $\leftarrow 0$
$t_P \leftarrow s + p$ // s is the upper bound of η from Lemma 7.19 of [48]
State $\leftarrow$ WAITING FOR COMM SIG
else // opening virtual channel
State $\leftarrow$ WAITING FOR CHECK KEYS
end if
reply (ACCEPT CHANNEL, $pk_{B,F}$, $pk_{B,R,1}$, $pk_{B,R,2}$, $pk_{B,\text{out}}$)
ensure *State* = WAITING FOR OPENING KEYS
store $pk_{B,F}$, $pk_{B,R,1}$, $pk_{B,R,2}$, $pk_{B,\text{out}}$
State $\leftarrow$ OPENING KEYS OK

Figure 26

Process LN.PREPAREBASE()

if `hops` = "`ledger`" **then** // opening base channel
$F \leftarrow$ TX {input: $(c, pk_{A,\text{chain}})$, output: $(c, 2/\{pk_{A,F}, pk_{B,F}\})$}
$\texttt{host}_P \leftarrow$ "`ledger`"
`layer` $\leftarrow 0$
$t_P \leftarrow s + p$
else // opening virtual channel
input (FUND ME, *Bob*, `hops`, c, $pk_{A,F}$, $pk_{B,F}$) to `hops[0].left` and expect output (FUNDED, $\texttt{host}_P$, `funder_layer`, t_P) // ignore any other message
`layer` $\leftarrow$ `funder_layer`
end if

Figure 27

Process LN.EXCHANGEOPENSIGS()

// $s = (2 + q)$`windowSize`, where q and `windowSize` are defined in Proposition F.1
$C_{A,0} \leftarrow$ TX {input: $(c, 2/\{pk_{A,F}, pk_{B,F}\})$, outputs: ($c$, $(pk_{A,\text{out}} + (p + s)) \vee 2/\{pk_{A,R,1}, pk_{B,R,1}\}$), (0, $pk_{B,\text{out}}$)}
$C_{B,0} \leftarrow$ TX {input: $(c, 2/\{pk_{A,F}, pk_{B,F}\})$, outputs: ($c$, $pk_{A,\text{out}}$), (0, $(pk_{B,\text{out}} + (p + s)) \vee 2/\{pk_{A,R,1}, pk_{B,R,1}\}$)}
$\text{sig}_{A,C,0} \leftarrow \text{SIGN}(C_{B,0}, sk_{A,F})$
State $\leftarrow$ WAITING FOR COMM SIG
send (FUNDING CREATED, $(c, pk_{A,\text{chain}})$, $\text{sig}_{A,C,0}$) to `fundee`
ensure *State* = WAITING FOR COMM SIG // if opening virtual channel, we have received (FUNDED, `host_fundee`) by `hops[-1].right` (Fig 30, l. 3)
if `hops` = "`ledger`" **then** // opening base channel
$F \leftarrow$ TX {input: $(c, pk_{A,\text{chain}})$, output: $(c, 2/\{pk_{A,F}, pk_{B,F}\})$}
end if
$C_{B,0} \leftarrow$ TX {input: $(c, 2/\{pk_{A,F}, pk_{B,F}\})$, outputs: ($c$, $pk_{A,\text{out}}$), (0, $(pk_{B,\text{out}} + (p + s)) \vee 2/\{pk_{A,R,1}, pk_{B,R,1}\}$)}
ensure VERIFY($C_{B,0}$, $\text{sig}_{A,C,0}$, $pk_{A,F}$) = True
$C_{A,0} \leftarrow$ TX {input: $(c, 2/\{pk_{A,F}, pk_{B,F}\})$, outputs: ($c$, $(pk_{A,\text{out}} + (p + s)) \vee 2/\{pk_{A,R,1}, pk_{B,R,1}\}$), (0, $pk_{B,\text{out}}$)}
$\text{sig}_{B,C,0} \leftarrow \text{SIGN}(C_{A,0}, sk_{B,F})$
if `hops` = "`ledger`" **then** // opening base channel
State $\leftarrow$ WAITING TO CHECK FUNDING
else // opening virtual channel
$c_A \leftarrow c$; $c_B \leftarrow 0$; $i \leftarrow 0$
State $\leftarrow$ OPEN
end if
reply (FUNDING SIGNED, $\text{sig}_{B,C,0}$)
ensure *State* = WAITING FOR COMM SIG
ensure VERIFY($C_{A,0}$, $\text{sig}_{B,C,0}$, $pk_{B,F}$) = True

Figure 28

Process LN.COMMITBASE()

$\text{sig}_F \leftarrow \text{SIGN}(F, sk_{A,\text{chain}})$
input (SUBMIT, (F, sig_F)) to $\mathcal{G}_{\text{Ledger}}$ // enter "while" below before sending
while $F \notin \Sigma$ **do**
wait for input (CHECK FUNDING) // ignore all other messages
input (READ) to $\mathcal{G}_{\text{Ledger}}$ and assign output to Σ
end while

Figure 29

Process LN – external open messages for *Bob*

On output (FUNDED, $\texttt{host}_P$, `funder_layer`, t_P) by `hops[-1].right`:
ensure *State* = WAITING FOR FUNDED

store $\texttt{host}_P$ // we will talk directly to $\texttt{host}_P$
$\texttt{layer} \leftarrow \texttt{funder\_layer}$
State ← WAITING FOR COMM SIG
reply (FUND ACK)

On output (CHECK KEYS, (pk_1, pk_2)) by hops[-1].right:
ensure *State* = WAITING FOR CHECK KEYS
ensure $pk_1 = pk_{A,F} \land pk_2 = pk_{B,F}$
State ← WAITING FOR FUDNED
reply (KEYS OK)

On input (CHECK FUNDING):
ensure *State* = WAITING TO CHECK FUNDING
input (READ) to $\mathcal{G}_{\text{Ledger}}$ and assign output to Σ
if $F \in \Sigma$ **then**
State ← OPEN
reply (OPEN OK)
end if

Figure 30

Process LN – On (OPEN, c, hops, fundee):

// fundee is *Bob*
ensure we run the code of *Alice* // activated party is the funder
if hops = "ledger" **then** // opening base channel
ensure *State* = TOPPED UP
ensure $c = c_{A,\text{chain}}$
else // opening virtual channel
ensure len(hops) ≥ 2 // cannot open a virtual over 1 channel
end if
LN.EXCHANGEOPENKEYS()
LN.PREPAREBASE()
LN.EXCHANGEOPENSIGS()
if hops = "ledger" **then**
LN.COMMITBASE()
end if
input (READ) to $\mathcal{G}_{\text{Ledger}}$ and assign output to Σ
last_poll ← $|\Sigma|$
$c_A \leftarrow c$; $c_B \leftarrow 0$; $i \leftarrow 0$
State ← OPEN
output (OPEN OK, c, fundee, hops)

Figure 31

Process LN.UPDATEFORVIRTUAL()

$C_{\bar{P},i+1} \leftarrow C_{\bar{P},i}$ with $pk'_{P,F}$, $pk'_{\bar{P},F}$, $pk_{\bar{P},R,i+1}$ and $pk_{P,R,i+1}$ instead of $pk_{P,F}$, $pk_{\bar{P},F}$, $pk_{\bar{P},R,i}$ and $pk_{P,R,i}$ respectively, reducing the input and P's output by c_{virt}
$\text{sig}_{P,C,i+1} \leftarrow \text{SIGN}(C_{\bar{P},i+1})$ // kept by $\bar{P}$
$(sk_{P,R,i+2}, pk_{P,R,i+2}) \leftarrow \text{KEYGEN}()$
send (UPDATE FORWARD, $\text{sig}_{P,C,i+1}$, $pk_{P,R,i+2}$) to $\bar{P}$
// P refers to payer and $\bar{P}$ to payee both in local and remote code

$C_{\bar{P},i+1} \leftarrow C_{\bar{P},i}$ with $pk'_{P,F}$, $pk'_{\bar{P},F}$, $pk_{P,R,i+1}$ and $pk_{\bar{P},R,i+1}$ instead of $pk_{P,F}$, $pk_{\bar{P},F}$, $pk_{P,R,i}$ and $pk_{\bar{P},R,i}$ respectively, reducing the input and P's output by c_{virt}
ensure VERIFY($C_{\bar{P},i+1}$, $\text{sig}_{P,C,i+1}$, $pk'_{P,F}$) = True
$C_{P,i+1} \leftarrow C_{P,i}$ with $pk'_{\bar{P},F}$, $pk'_{P,F}$, $pk_{\bar{P},R,i+1}$ and $pk_{P,R,i+1}$ instead of $pk_{\bar{P},F}$, $pk_{P,F}$, $pk_{\bar{P},R,i}$ and $pk_{P,R,i}$ respectively, reducing the input and P's output by c_{virt}
$\text{sig}_{\bar{P},C,i+1} \leftarrow \text{SIGN}(C_{P,i+1}, sk'_{\bar{P},F})$ // kept by P
$(sk_{\bar{P},R,i+2}, pk_{\bar{P},R,i+2}) \leftarrow \text{KEYGEN}()$
reply (UPDATE BACK, $\text{sig}_{\bar{P},C,i+1}$, $pk_{\bar{P},R,i+2}$)
$C_{P,i+1} \leftarrow C_{P,i}$ with $pk'_{\bar{P},F}$, $pk'_{P,F}$, $pk_{\bar{P},R,i+1}$ and $pk_{P,R,i+1}$ instead of $pk_{\bar{P},F}$, $pk_{P,F}$, $pk_{\bar{P},R,i}$ and $pk_{P,R,i}$ respectively, reducing the input and P's output by c_{virt}
ensure VERIFY($C_{P,i+1}$, $\text{sig}_{\bar{P},C,i+1}$, $pk'_{\bar{P},F}$) = True

Figure 32

Process LN – virtualise start and end

On input (FUND ME, funder, fundee, hops, c_{virt}, $pk_{A,V}$, $pk_{B,V}$) by funder:
ensure *State* = OPEN
ensure $c_P - \texttt{locked}_P \geq c_{\text{virt}}$
State ← VIRTUALISING
$(sk'_{P,F}, pk'_{P,F}) \leftarrow \text{KEYGEN}()$
define new VIRT ITI $\texttt{host}'_P$
send (VIRTUALISING, $\texttt{host}'_P$, $pk'_{P,F}$, hops, fundee, c_{virt}, 2, len(hops)) to $\bar{P}$ and expect reply (VIRTUALISING ACK, $\texttt{host}'_{\bar{P}}$, $pk'_{\bar{P},F}$)
ensure $pk'_{\bar{P},F}$ is different from $pk_{\bar{P},F}$ and all older $\bar{P}$'s funding public keys
LN.UPDATEFORVIRTUAL()
State ← WAITING FOR REVOCATION
input (HOST ME, funder, fundee, $\texttt{host}'_{\bar{P}}$, $\texttt{host}_P$, c_P, $c_{\bar{P}}$, c_{virt}, $pk_{A,V}$, $pk_{B,V}$, $(sk'_{P,F}, pk'_{P,F})$, $(sk_{P,F}, pk_{P,F})$, $pk_{\bar{P},F}$, $pk'_{\bar{P},F}$, $pk_{P,\text{out}}$, len(hops)) to $\texttt{host}'_P$

On output (HOSTS READY, t_P) by $\texttt{host}_P$: // $\texttt{host}_P$ is the new host, renamed in Fig. 25, l. 12
ensure *State* = WAITING FOR HOSTS READY
State ← OPEN
hosting ← True
move $sk_{P,F}$, $pk_{P,F}$, $pk_{\bar{P},F}$ to list of old funding keys
$(sk_{P,F}, pk_{P,F}) \leftarrow (sk'_{P,F}, pk'_{P,F})$; $pk_{\bar{P},F} \leftarrow pk'_{\bar{P},F}$
if len(hops) = 1 **then** // we are the last hop
output (FUNDED, $\texttt{host}_P$, layer, t_P) to fundee and expect reply (FUND ACK)
else if we have received input FUND ME just before we moved to the VIRTUALISING state **then** // we are the first hop
$c_P \leftarrow c_P - c_{\text{virt}}$
output (FUNDED, $\texttt{host}_P$, layer, t_P) to funder // do not expect reply by funder
end if
reply (HOST ACK)

Figure 33

Process LN – virtualise hops

On (VIRTUALISING, $\texttt{host}'_{\bar{P}}$, $pk'_{\bar{P},F}$, $\texttt{hops}$, $\texttt{fundee}$, c_{virt}, i, n) by $\bar{P}$:

ensure $State$ = OPEN

ensure $c_{\bar{P}} - \texttt{locked}_{\bar{P}} \geq c_{\text{virt}}$; $1 \leq i \leq n$

ensure $pk'_{\bar{P},F}$ is different from $pk_{\bar{P},F}$ and all older $\bar{P}$'s funding public keys

$State \leftarrow$ VIRTUALISING

$\texttt{locked}_{\bar{P}} \leftarrow \texttt{locked}_{\bar{P}} + c_{\text{virt}}$ // if $\bar{P}$ is hosting the funder, $\bar{P}$ will transfer c_{virt} coins instead of locking them, but the end result is the same

$(sk'_{P,F}, pk'_{P,F}) \leftarrow$ KEYGEN()

if len(hops) > 1 **then** // we are not the last hop

define new VIRT ITI $\texttt{host}'_P$

input (VIRTUALISING, $\texttt{host}'_P$, $(sk'_{P,F}, pk'_{P,F})$, $pk'_{\bar{P},F}$, $pk_{P,\text{out}}$, $\texttt{hops[1:]}$, $\texttt{fundee}$, c_{virt}, $c_{\bar{P}}$, c_P, i, n) to $\texttt{hops[1].left}$ and expect reply (VIRTUALISING ACK, $\texttt{host\_sibling}$, $pk_{\text{sib},\bar{P},F}$)

input (INIT, $\texttt{host}_P$, $\texttt{host}'_{\bar{P}}$, $\texttt{host\_sibling}$, $(sk'_{P,F}, pk'_{P,F})$, $pk'_{\bar{P},F}$, $pk_{\text{sib},\bar{P},F}$, $(sk_{P,F}, pk_{P,F})$, $pk_{\bar{P},F}$, $pk_{P,\text{out}}$, c_P, $c_{\bar{P}}$, c_{virt}, i, t_P, "left", n) to $\texttt{host}'_P$ and expect reply (HOST INIT OK)

else // we are the last hop

input (INIT, $\texttt{host}_P$, $\texttt{host}'_{\bar{P}}$, $\texttt{fundee=fundee}$, $(sk'_{P,F}, pk'_{P,F})$, $pk'_{\bar{P},F}$, $(sk_{P,F}, pk_{P,F})$, $pk_{\bar{P},F}$, $pk_{P,\text{out}}$, c_P, $c_{\bar{P}}$, c_{virt}, t_P, i, "left", n) to new VIRT ITI $\texttt{host}'_P$ and expect reply (HOST INIT OK)

end if

$State \leftarrow$ WAITING FOR REVOCATION

send (VIRTUALISING ACK, $\texttt{host}'_P$, $pk'_{P,F}$) to $\bar{P}$

On input (VIRTUALISING, $\texttt{host\_sibling}$, $(sk'_{P,F}, pk'_{P,F})$, $pk_{\text{sib},\bar{P},F}$, $pk_{\text{sib,out}}$, $\texttt{hops}$, $\texttt{fundee}$, c_{virt}, $c_{\text{sib,rem}}$, c_{sib}, i, n) by $\texttt{sibling}$:

ensure $State$ = OPEN

ensure $c_P - \texttt{locked}_P \geq c_{\text{virt}}$

ensure $c_{\text{sib,rem}} \geq c_P \wedge c_{\bar{P}} \geq c_{\text{sib}}$ // avoid value loss by griefing attack: one counterparty closes with old version, the other stays idle forever

$State \leftarrow$ VIRTUALISING

$\texttt{locked}_P \leftarrow \texttt{locked}_P + c_{\text{virt}}$

define new VIRT ITI $\texttt{host}'_P$

send (VIRTUALISING, $\texttt{host}'_P$, $pk'_{P,F}$, $\texttt{hops}$, $\texttt{fundee}$, c_{virt}, $i+1$, n) to $\texttt{hops[0].right}$ and expect reply (VIRTUALISING ACK, $\texttt{host}'_{\bar{P}}$, $pk'_{\bar{P},F}$)

ensure $pk'_{\bar{P},F}$ is different from $pk_{\bar{P},F}$ and all older $\bar{P}$'s funding public keys

LN.UPDATEFORVIRTUAL()

input (INIT, $\texttt{host}_P$, $\texttt{host}'_{\bar{P}}$, $\texttt{host\_sibling}$, $(sk'_{P,F}, pk'_{P,F})$, $pk'_{\bar{P},F}$, $pk_{\text{sib},\bar{P},F}$, $(sk_{P,F}, pk_{P,F})$, $pk_{\bar{P},F}$, $pk_{\text{sib,out}}$, c_P, $c_{\bar{P}}$, c_{virt}, i, "right", n) to $\texttt{host}'_P$ and expect reply (HOST INIT OK)

$State \leftarrow$ WAITING FOR REVOCATION

output (VIRTUALISING ACK, $\texttt{host}'_P$, $pk'_{\bar{P},F}$) to $\texttt{sibling}$

Figure 34

Process LN.SIGNATURESROUNDTRIP()

$C_{\bar{P},i+1} \leftarrow C_{\bar{P},i}$ with $pk_{P,R,i+1}$ and $pk_{\bar{P},R,i+1}$ instead of $pk_{P,R,i}$ and $pk_{\bar{P},R,i}$ respectively, and x coins moved from P's to $\bar{P}$'s output

$\text{sig}_{P,C,i+1} \leftarrow$ SIGN$(C_{\bar{P},i+1}, sk_{P,F})$ // kept by $\bar{P}$

$(sk_{P,R,i+2}, pk_{P,R,i+2}) \leftarrow$ KEYGEN()

$State \leftarrow$ WAITING FOR COMMITMENT SIGNED

send (PAY, x, $\text{sig}_{P,C,i+1}$, $pk_{P,R,i+2}$) to $\bar{P}$

// P refers to payer and $\bar{P}$ to payee both in local and remote code

ensure $State$ = WAITING TO GET PAID $\wedge\ x = y$

$C_{\bar{P},i+1} \leftarrow C_{\bar{P},i}$ with $pk_{P,R,i+1}$ and $pk_{\bar{P},R,i+1}$ instead of $pk_{P,R,i}$ and $pk_{\bar{P},R,i}$ respectively, and x coins moved from P's to $\bar{P}$'s output

ensure VERIFY$(C_{\bar{P},i+1}, \text{sig}_{P,C,i+1}, pk_{P,F})$ = True

$C_{P,i+1} \leftarrow C_{P,i}$ with $pk_{\bar{P},R,i+1}$ and $pk_{P,R,i+1}$ instead of $pk_{\bar{P},R,i}$ and $pk_{P,R,i}$ respectively, and x coins moved from P's to $\bar{P}$'s output

$\text{sig}_{\bar{P},C,i+1} \leftarrow$ SIGN$(C_{P,i+1}, sk_{\bar{P},F})$ // kept by P

$R_{P,i} \leftarrow$ TX {input: $C_{\bar{P},i}$.outputs.$\bar{P}$, output: $(c_{\bar{P}}, pk_{P,\text{out}})$}

$\text{sig}_{\bar{P},R,i} \leftarrow$ SIGN$(R_{P,i}, sk_{\bar{P},R,i})$

$(sk_{\bar{P},R,i+2}, pk_{\bar{P},R,i+2}) \leftarrow$ KEYGEN()

$State \leftarrow$ WAITING FOR PAY REVOCATION

reply (COMMITMENT SIGNED, $\text{sig}_{\bar{P},C,i+1}$, $\text{sig}_{\bar{P},R,i}$, $pk_{\bar{P},R,i+2}$)

ensure $State$ = WAITING FOR COMMITMENT SIGNED

$C_{P,i+1} \leftarrow C_{P,i}$ with $pk_{\bar{P},R,i+1}$ and $pk_{P,R,i+1}$ instead of $pk_{\bar{P},R,i}$ and $pk_{P,R,i}$ respectively, and x coins moved from P's to $\bar{P}$'s output

Figure 35

Process LN.REVOCATIONSTRIP()

ensure VERIFY$(C_{P,i+1}, \text{sig}_{\bar{P},C,i+1}, pk_{\bar{P},F})$ = True

$R_{P,i} \leftarrow$ TX {input: $C_{\bar{P},i}$.outputs.$\bar{P}$, output: $(c_{\bar{P}}, pk_{P,\text{out}})$}

ensure VERIFY$(R_{P,i}, \text{sig}_{\bar{P},R,i}, pk_{\bar{P},R,i})$ = True

$R_{\bar{P},i} \leftarrow$ TX {input: $C_{P,i}$.outputs.P, output: $(c_P, pk_{\bar{P},\text{out}})$}

$\text{sig}_{P,R,i} \leftarrow$ SIGN$(R_{\bar{P},i}, sk_{P,R,i})$

add x to $\texttt{paid\_out}$

$c_P \leftarrow c_P - x$; $c_{\bar{P}} \leftarrow c_{\bar{P}} + x$; $i \leftarrow i + 1$

$State \leftarrow$ OPEN

if $\texttt{host}_P \neq$ "$\texttt{ledger}$" $\wedge$ we have a $\texttt{host\_sibling}$ **then** // we are intermediary channel

input (NEW BALANCE, c_P, $c_{\bar{P}}$) to $\texttt{host}_P$

relay message as input to $\texttt{sibling}$ // run by VIRT

relay message as output to $\texttt{guest}$ // run by VIRT

store new sibling balance and reply (NEW BALANCE OK)

output (NEW BALANCE OK) to $\texttt{sibling}$ // run by VIRT

output (NEW BALANCE OK) to $\texttt{guest}$ // run by VIRT

end if

send (REVOKE AND ACK, $\text{sig}_{P,R,i}$) to $\bar{P}$

ensure $State$ = WAITING FOR PAY REVOCATION

$R_{\bar{P},i} \leftarrow$ TX {input: $C_{P,i}$.outputs.P, output: $(c_P, pk_{\bar{P},\text{out}})$}

ensure VERIFY$(R_{\bar{P},i}, \text{sig}_{P,R,i}, pk_{P,R,i})$ = True

add x to $\texttt{paid\_in}$

$c_P \leftarrow c_P - x$; $c_{\bar{P}} \leftarrow c_{\bar{P}} + x$; $i \leftarrow i + 1$

$State \leftarrow$ OPEN

if $\texttt{host}_P \neq$ "$\texttt{ledger}$" $\wedge\ \bar{P}$ has a $\texttt{host\_sibling}$ **then** // we are intermediary channel

input (NEW BALANCE, $c_{\bar{P}}$, c_P) to $\texttt{host}_{\bar{P}}$

relay message as input to $\texttt{sibling}$ // run by VIRT

relay message as output to $\texttt{guest}$ // run by VIRT

store new sibling balance and reply (NEW BALANCE OK)

output (NEW BALANCE OK) to sibling // run by VIRT
output (NEW BALANCE OK) to guest // run by VIRT
end if

Figure 36

Process LN – On (PAY, x):

ensure $State$ = OPEN $\wedge\ c_P \geq x$
if host$_P \neq$ "ledger" $\wedge$ P has a host_sibling **then** // we are intermediary channel
  ensure $c_{\text{sib,rem}} \geq c_P - x \wedge c_{\bar{P}} + x \geq c_{\text{sib}}$ // avoid value loss by griefing attack: one counterparty closes with old version, the other stays idle forever
end if
LN.SIGNATURESROUNDTRIP()
LN.REVOCATIONSTRIP()
// No output is given to the caller, this is intentional

Figure 37

Process LN – On (GET PAID, y):

ensure $State$ = OPEN $\wedge\ c_{\bar{P}} \geq y$
if host$_P \neq$ "ledger" $\wedge$ P has a host_sibling **then** // we are intermediary channel
  ensure $c_P + y \leq c_{\text{sib,rem}} \wedge c_{\text{sib}} \leq c_{\bar{P}} - y$ // avoid value loss by griefing attack
end if
store y
$State \leftarrow$ WAITING TO GET PAID

Figure 38

Process LN – On (CHECK FOR LATERAL CLOSE):

if host$_P \neq$ "ledger" **then**
  input (CHECK FOR LATERAL CLOSE) to host$_P$
end if

Figure 39

Process LN – On (CHECK CHAIN FOR CLOSED):

ensure $State \notin \{\perp$, INIT, TOPPED UP$\}$ // channel open
// even virtual channels check $\mathcal{G}_{\text{Ledger}}$ directly. This is intentional
input (READ) to $\mathcal{G}_{\text{Ledger}}$ and assign reply to Σ
last_poll $\leftarrow |\Sigma|$
if $\exists 0 \leq j < i : C_{\bar{P},j} \in \Sigma$ **then** // counterparty has closed maliciously
  $State \leftarrow$ CLOSING
  LN.SUBMITANDCHECKREVOCATION(j)
  $State \leftarrow$ CLOSED
  output (CLOSED)
else if $C_{P,i} \in \Sigma \vee C_{\bar{P},i} \in \Sigma$ **then**
  $State \leftarrow$ CLOSED
  output (CLOSED)
else
  state_before_checking_revoked $\leftarrow State$
  for each host in list of old hosts **do**
    $State \leftarrow$ WAITING FOR NOTHING REVOKED
    input (CHECK FOR REVOKED) to host and expect output (NOTHING REVOKED)
    $State \leftarrow$ state_before_checking_revoked
  end for
end if

Figure 40

Process LN.SUBMITANDCHECKREVOCATION(j)

$\text{sig}_{P,R,j} \leftarrow \text{SIGN}(R_{P,j}, sk_{P,R,j})$
input (SUBMIT, $(R_{P,j}, \text{sig}_{P,R,j}, \text{sig}_{\bar{P},R,j})$) to $\mathcal{G}_{\text{Ledger}}$
while $\neg\exists R_{P,j} \in \Sigma$ **do**
  wait for input (CHECK REVOCATION) // ignore other messages
  input (READ) to $\mathcal{G}_{\text{Ledger}}$ and assign output to Σ
end while
$c_P \leftarrow c_P + c_{\bar{P}}$
if host$_P \neq$ "ledger" **then**
  input (USED REVOCATION) to host$_P$
end if

Figure 41

Process LN – On (FORCECLOSE):

ensure $State \notin \{\perp$, INIT, TOPPED UP, CLOSED, BASE PUNISHED$\}$ // channel open
if host$_P \neq$ "ledger" **then** // we have a virtual channel
  $State \leftarrow$ HOST CLOSING
  input (FORCECLOSE) to host$_P$ and keep relaying any (CHECK IF CLOSING) or (FORCECLOSE) input to host$_P$ until receiving output (CLOSED) by host$_P$
  host$_P \leftarrow$ "ledger"
end if
$State \leftarrow$ CLOSING
input (READ) to $\mathcal{G}_{\text{Ledger}}$ and assign output to Σ
if $C_{\bar{P},i} \in \Sigma$ **then** // counterparty has closed honestly
  no-op // do nothing
else if $\exists 0 \leq j < i : C_{\bar{P},j} \in \Sigma$ **then** // counterparty has closed maliciously
  LN.SUBMITANDCHECKREVOCATION(j)
else // counterparty is idle
  while $\nexists$ unspent output $\in \Sigma$ that $C_{P,i}$ can spend **do** // possibly due to an active timelock
    wait for input (CHECK VIRTUAL) // ignore other messages
    input (READ) to $\mathcal{G}_{\text{Ledger}}$ and assign output to Σ
  end while

18: $\quad \mathrm{sig}'_{P,C,i} \leftarrow \textsc{sign}(C_{P,i}, sk_{P,F})$
19: $\quad$ input ($\textsc{submit}$, $(C_{P,i}, \mathrm{sig}_{P,C,i}, \mathrm{sig}'_{P,C,i})$) to $\mathcal{G}_{\text{Ledger}}$
20: **end if**

Figure 42

Process LN – On output (ENABLER USED REVOCATION) by $\texttt{host}_P$:

1: *State* $\leftarrow$ BASE PUNISHED

Figure 43

Process LN – On (COOPCLOSE):

// any endpoint or intermediary can initiate virtual channel closing
1: ensure $\texttt{host}_P \neq$ "ledger"
2: ensure *State* = OPEN
3: `we_are_close_initiator` $\leftarrow$ True
4: **if** `hosting` = True $\vee$ we have received OPEN from $\mathcal{E}$ while *State* was TOPPED UP **then** // we are not the fundee of a channel that is not the base of any other channel
5: $\quad$ **if** `hosting` = True **then** // we are not the `funder` of the channel to be closed
6: $\quad\quad$ the next time we are activated, if we are not activated by output (CHECK COOP CLOSE, ...) from $\texttt{host}_P$, set `we_are_close_initiator` $\leftarrow$ False
7: $\quad$ **else** // we are the funder of the channel to be closed
8: $\quad\quad$ the next time we are activated, if we are not activated by output (COOP CLOSE, ...) from $\bar{P}$, set `we_are_close_initiator` $\leftarrow$ False
9: $\quad$ **end if**
10: $\quad$ send (COOP CLOSE) to `fundee`
11: **else** // we are the `fundee` of a channel that is not the base of any other channel
12: $\quad$ the next time we are activated, if we are not activated by output (CHECK COOP CLOSE FUNDEE, ...) from $\texttt{host}_P$, set `we_are_close_initiator` $\leftarrow$ False
13: $\quad$ `close_initiator` $\leftarrow P$
14: $\quad$ execute code of Fig. 46
15: **end if**

Figure 44

Process LN – On (COOPCLOSED) by R:

1: **if** `hosting` = True **then** // we are intermediary
2: $\quad$ ensure *State* = OPEN
3: **else** // we are endpoint
4: $\quad$ ensure *State* = COOP CLOSED
5: **end if**
6: ensure `we_are_close_initiator` = True
7: ensure that the last cooperatively closed channel in which we acted as a base had R as its `fundee`
8: `we_are_close_initiator` $\leftarrow$ False
9: output (COOPCLOSED)

Figure 45

Process LN – On (COOP CLOSE) by R:

// also executed when we are instructed to close a channel cooperatively by $\mathcal{E}$– c.f. Fig. 44, l. 14
1: ensure we are `fundee`
2: ensure `hosting` $\neq$ True
3: ensure *State* = OPEN
4: *State* $\leftarrow$ COOP CLOSING
5: `close_initiator` $\leftarrow R$
6: $\texttt{sig\_bal} \leftarrow ((c_{\bar{P}}, c_P), \textsc{sign}((c_{\bar{P}}, c_P), sk_{P,F}))$
7: *State* $\leftarrow$ WAITING TO REVOKE VIRT COMM
8: send (COOP CLOSE, `sig_bal`) to $\bar{P}$

Figure 46

Process LN – On (COOP CLOSE, $\texttt{sig\_bal}_{\bar{P}}$) by $\bar{P}$:

1: ensure we are `funder`
2: ensure *State* = OPEN
3: parse $\texttt{sig\_bal}_{\bar{P}}$ as $((c'_1, c'_2), \mathrm{sig}_{\bar{P}})$
4: ensure $c_P = c'_1 \wedge c_{\bar{P}} = c'_2 \wedge \textsc{verify}((c'_1, c'_2), \mathrm{sig}_{\bar{P}}, pk_{\bar{P},F}) = \text{True}$
5: $\texttt{sig\_bal} \leftarrow ((c_P, c_{\bar{P}}), \textsc{sign}((c_P, c_{\bar{P}}), sk_{P,F}), \mathrm{sig}_{\bar{P}})$
6: *State* $\leftarrow$ COOP CLOSING
7: input (COOP CLOSE, `sig_bal`) to $\texttt{host}_P$
8: ensure *State* = OPEN // executed by $\texttt{host}_P$
9: *State* $\leftarrow$ COOP CLOSING
10: output (COOP CLOSE SIGN COMM FUNDER, (c'_1, c'_2)) to `guest`
11: ensure *State* = OPEN // executed by `guest` of $\texttt{host}_P$
12: *State* $\leftarrow$ COOP CLOSING
13: remove most recent keys from list of old funding keys and assign them to $sk'_{P,F}, pk'_{P,F}, pk'_{\bar{P},F}$
14: $C_{\bar{P},i+1} \leftarrow$ TX {input: $(c_P + c_{\bar{P}} + c'_1 + c'_2, 2/\{pk'_{P,F}, pk'_{\bar{P},F}\})$, outputs: $(c_P + c'_1, pk_{P,\text{out}})$, $(c_{\bar{P}} + c'_2, (pk_{\bar{P},\text{out}} + (p + s)) \vee 2/\{pk_{P,R,i+1}, pk_{\bar{P},R,i+1}\})$}
15: $\mathrm{sig}_{P,C,i+1} \leftarrow \textsc{sign}(C_{\bar{P},i+1}, sk'_{P,F})$
16: $(sk_{P,R,i+2}, pk_{P,R,i+2}) \leftarrow \textsc{keyGen}()$
17: input (NEW COMM TX, $\mathrm{sig}_{P,C,i+1}$, $pk_{P,R,i+2}$) to $\texttt{host}_P$
18: rename received signature to $\mathrm{sig}_{1,\text{right},C}$ // executed by $\texttt{host}_P$
19: rename received public key to $pk_{1,\text{right},R}$
20: send (COOP CLOSE, `sig_bal`, $\mathrm{sig}_{1,\text{right},C}$, $pk_{1,\text{right},R}$) to $\bar{P}$ and expect reply (COOP CLOSE BACK, (`right_comms_revkeys`, `right_revocations`)
21: $R_{\text{loc,virt}} \leftarrow$ TX {input: $(c_{\text{virt}}, 2/\{pk_{1,\text{rev}}, pk_{2,\text{rev}}\})$, output: $(c_{\text{virt}}, pk_{1,\text{out}})$}
22: extract $\mathrm{sig}_{2,\text{right,rev,virt}}$ from `right_revocations`
23: ensure $\textsc{verify}(R_{\text{loc,virt}}, \mathrm{sig}_{2,\text{right,rev,virt}}, pk_{2,\text{rev}}) = \text{True}$
24: $R_{\text{loc,fund}} \leftarrow$ TX {input: $(c_P + c_{\bar{P}}, 2/\{pk_{1,\text{rev}}, pk_{2,\text{rev}}\})$, output: $(c_P + c_{\bar{P}}, pk_{1,\text{out}})$}
25: extract $\mathrm{sig}_{2,\text{right,rev,fund}}$ from `right_revocations`
26: ensure $\textsc{verify}(R_{\text{loc,fund}}, \mathrm{sig}_{2,\text{right,rev,fund}}, pk_{2,\text{rev}}) = \text{True}$
27: extract $\mathrm{sig}_{2,\text{right},R}$ from `right_revocations`
28: extract $\mathrm{sig}_{2,\text{right},C}$ from `right_comms_revkeys`
29: extract $pk_{2,R}$ from `right_comms_revkeys`

output (VERIFY REVOKE, $\mathrm{sig}_{2,\mathrm{right},C}$, $\mathrm{sig}_{2,\mathrm{right},R}$, $pk_{2,R}$, $\mathtt{host}_P$) to `guest`
store $\mathrm{sig}_{2,\mathrm{right},C}$ as $\mathrm{sig}_{\bar{P},C,i+1}$ // executed by `guest` of $\mathtt{host}_P$
store $\mathrm{sig}_{2,\mathrm{right},R}$ as $\mathrm{sig}_{\bar{P},R,i}$
store received public key as $pk_{\bar{P},R,i+2}$
$C_{P,i+1} \leftarrow$ TX {input: $(c_P + c_{\bar{P}} + c_1' + c_2')$, outputs: $(c_P + c_1', (pk_{P,\mathrm{out}} + (p+s)) \vee 2/\{pk_{P,R,i+1}, pk_{\bar{P},R,i+1}\})$, $(c_{\bar{P}} + c_2', pk_{\bar{P},\mathrm{out}})$}
ensure VERIFY$(C_{P,i+1}, \mathrm{sig}_{\bar{P},C,i+1}, pk'_{\bar{P},F})$ = True
$R_{P,i} \leftarrow$ TX {input: $C_{\bar{P},i}$.outputs.$\bar{P}$, output: $(c_{\bar{P}}, pk_{P,\mathrm{out}})$}
ensure VERIFY$(R_{P,i}, \mathrm{sig}_{\bar{P},R,i}, pk_{\bar{P},R,i})$ = True
input (VERIFIED) to $\mathtt{host}_P$
extract $\mathrm{sig}_{n,\mathrm{left},R}$ from `right_revocations` // executed by $\mathtt{host}_P$
output (VERIFY REVOCATION, $\mathrm{sig}_{n,\mathrm{left},R}$) to `funder`
$R_{P,i} \leftarrow$ TX {input: $C_{\bar{P},i}$.outputs.$\bar{P}$, output: $(c_{\bar{P}}, pk_{P,\mathrm{out}})$}
ensure VERIFY$(R_{P,i}, \mathrm{sig}_{\bar{P},R,i}, pk_{\bar{P},R,i})$ = True
$R_{\bar{P},i} \leftarrow$ TX {input: $C_{P,i}$.outputs.P, output: $(c_P, pk_{\bar{P},\mathrm{out}})$}
$\mathrm{sig}_{P,R,i} \leftarrow$ SIGN$(R_{\bar{P},i}, sk_{P,R,i})$
State $\leftarrow$ COOP CLOSED // in LN, only virtual channels can end up in this state
input (COOP CLOSE REVOCATION, $\mathrm{sig}_{P,R,i}$) to $\mathtt{host}_P$
output (COOP CLOSE REVOCATIONS, $\mathtt{host}_P$) to `guest` // executed by $\mathtt{host}_P$
$R_{\bar{P},i} \leftarrow$ TX {input: $C_{P,i}$.outputs.P, output: $(c_P, pk_{\bar{P},\mathrm{out}})$} // executed by `guest` of $\mathtt{host}_P$
$\mathrm{sig}_{P,R,i} \leftarrow$ SIGN$(R_{\bar{P},i}, sk_{P,R,i})$
add $sk_{P,F}, pk_{P,F}, pk_{\bar{P},F}$ to list of old enable channel funding keys
add $\mathtt{host}_P$ to list of old hosts
assign received host to $\mathtt{host}_P$
$c_P \leftarrow c_P + c_1'$; $c_{\bar{P}} \leftarrow c_{\bar{P}} + c_2'$
`layer` $\leftarrow$ `layer` $- 1$
$\mathtt{locked}_P \leftarrow \mathtt{locked}_P - c_{\mathrm{virt}}$
State $\leftarrow$ OPEN
input (REVOCATION, $\mathrm{sig}_{P,R,i}$) to last old host
rename received signature to $\mathrm{sig}_{1,\mathrm{right},R}$ // executed by $\mathtt{host}_P$
$R_{\mathrm{rem,virt}} \leftarrow$ TX {input: $(c_{\mathrm{virt}}, 4/\{pk_{1,\mathrm{rev}}, pk_{1,\mathrm{rev}}, pk_{2,\mathrm{rev}}, pk_{n,\mathrm{rev}}\})$, output: $(c_{\mathrm{virt}}, pk_{2,\mathrm{out}})$}
$\mathrm{sig}_{1,\mathrm{right,rev,virt}} \leftarrow$ SIGN$(R_{\mathrm{rem,virt}}, sk_{1,\mathrm{rev}})$
$R_{\mathrm{rem,fund}} \leftarrow$ TX {input: $(c_P + c_{\bar{P}}, 2/\{pk_{1,\mathrm{rev}}, pk_{2,\mathrm{rev}}\})$, output: $(c_P + c_{\bar{P}}, pk_{2,\mathrm{out}})$}
$\mathrm{sig}_{1,\mathrm{right,rev,fund}} \leftarrow$ SIGN$(R_{\mathrm{rem,fund}}, sk_{1,\mathrm{rev}})$
for all $j \in \{2, \ldots, n\}$ **do**
    $R_{j,\mathrm{left}} \leftarrow$ TX {input: $(c_{\mathrm{virt}}, 4/\{pk_{1,\mathrm{rev}}, pk_{j-1,\mathrm{rev}}, pk_{j,\mathrm{rev}}, pk_{n,\mathrm{rev}}\})$, output: $(c_{\mathrm{virt}}, pk_{j,\mathrm{out}})$}
    $\mathrm{sig}_{1,j,\mathrm{left,rev}} \leftarrow$ SIGN$(R_{j,\mathrm{left}}, sk_{1,\mathrm{rev}})$
end for
State $\leftarrow$ COOP CLOSED
send (COOP CLOSE REVOCATIONS, ($\mathrm{sig}_{1,\mathrm{right},R}$, $\mathrm{sig}_{1,\mathrm{right,rev,virt}}$, $\mathrm{sig}_{1,\mathrm{right,rev,fund}}$, $(\mathrm{sig}_{1,j,\mathrm{left,rev}})_{j\in\{2,\ldots,n\}}$)) to $\bar{P}$

Figure 47

Process LN – On (CORRUPT) by $\mathcal{A}$ or kindred party R:

// This is executed by the shell – c.f. [12]
if *State* $\neq$ CORRUPTED **then**
    State $\leftarrow$ CORRUPTED
    for $S \in$ set of kindred parties **do**
        input (CORRUPT) to S and expect reply (OK)
    end for
end if
reply (OK)

Figure 48

Process VIRT

On every activation, before handling the message:
    if `last_poll` $\neq \perp$ **then** // virtual layer is ready
        input (READ) to $\mathcal{G}_{\mathrm{Ledger}}$ and assign ouput to Σ
        if `last_poll` $+ p < |\Sigma|$ **then**
            for $P \in \{$`guest`, `funder`, `fundee`$\}$ **do** // at most 1 of `funder`, `fundee` is defined
                ensure P.NEGLIGENT() returns (OK)
            end for
        end if
    end if

// `guest` is trusted to give sane inputs, therefore a state machine and input verification are redundant
On input (INIT, $\mathtt{host}_P$, $\bar{P}$, `sibling`, `fundee`, $(sk_{\mathrm{loc,fund,new}}, pk_{\mathrm{loc,fund,new}})$, $pk_{\mathrm{rem,fund,new}}$, $pk_{\mathrm{sib,rem,fund,new}}$, $(sk_{\mathrm{loc,fund,old}}, pk_{\mathrm{loc,fund,old}})$, $pk_{\mathrm{rem,fund,old}}$, $pk_{\mathrm{loc,out}}$, c_P, $c_{\bar{P}}$, c_{virt}, t_P, i, `side`, n) by `guest`:
    ensure $1 < i \le n$ // `host_funder` ($i = 1$) is initialised with HOST ME
    ensure `side` $\in$ {"left", "right"}
    store message contents and `guest` // `sibling`, $pk_{\mathrm{sib},\bar{P},F}$ are missing for endpoints, `fundee` is present only in last node
    $(sk_{i,\mathrm{fund,new}}, pk_{i,\mathrm{fund,new}}) \leftarrow (sk_{\mathrm{loc,fund,new}}, pk_{\mathrm{loc,fund,new}})$
    $pk_{\mathrm{myRem,fund,new}} \leftarrow pk_{\mathrm{rem,fund,new}}$
    if $i < n$ **then** // we are not last hop
        $pk_{\mathtt{sibRem},\mathrm{fund,new}} \leftarrow pk_{\mathrm{sib,rem,fund,new}}$
    end if
    if `side` = "left" **then**
        `side'` $\leftarrow$ "right"; `myRem` $\leftarrow i - 1$; `sibRem` $\leftarrow i + 1$
        $pk_{i,\mathrm{out}} \leftarrow pk_{\mathrm{loc,out}}$
        $(sk_{i,j,k}, pk_{i,j,k})_{j\in\{2,\ldots,n-1\},k\in[n]\setminus\{j\}} \leftarrow$ KEYGEN$()^{(n-2)(n-1)}$
        $(sk_{i,\mathrm{rev}}, pk_{i,\mathrm{rev}}) \leftarrow$ KEYGEN$()$
    else // `side` = "right"
        `side'` $\leftarrow$ "left"; `myRem` $\leftarrow i + 1$; `sibRem` $\leftarrow i - 1$
        // `sibling` will send keys in KEYS AND COINS FORWARD
    end if
    $(sk_{i,\mathtt{side},\mathrm{fund,old}}, pk_{i,\mathtt{side},\mathrm{fund,old}}) \leftarrow (sk_{\mathrm{loc,fund,old}}, pk_{\mathrm{loc,fund,old}})$
    $pk_{\mathtt{myRem},\mathtt{side}',\mathrm{fund,old}} \leftarrow pk_{\mathrm{rem,fund,old}}$
    $(c_{i,\mathtt{side}}, c_{\mathtt{myRem},\mathtt{side}'}, t_{i,\mathtt{side}}) \leftarrow (c_P, c_{\bar{P}}, t_P)$
    `last_poll` $\leftarrow \perp$
    output (HOST INIT OK) to `guest`

On input (HOST ME, `funder`, `fundee`, $\bar{P}$, $\mathtt{host}_P$, c_P, $c_{\bar{P}}$, c_{virt}, $pk_{\mathrm{left,virt}}$, $pk_{\mathrm{right,virt}}$, $(sk_{1,\mathrm{fund,new}}, pk_{1,\mathrm{fund,new}})$, $(sk_{1,\mathrm{right,fund,old}}, pk_{1,\mathrm{right,fund,old}})$, $pk_{2,\mathrm{left,fund,old}}$, $pk_{2,\mathrm{left,fund,new}}$, $pk_{1,\mathrm{out}}$, n) by `guest`:

last_poll $\leftarrow \bot$
$i \leftarrow 1$
$c_{1,\text{right}} \leftarrow c_P$; $c_{2,\text{left}} \leftarrow c_{\bar{P}}$
$(sk_{1,j,k}, pk_{1,j,k})_{j \in \{2,\dots,n-1\}, k \in [n] \setminus \{j\}} \leftarrow$ KEYGEN$()^{(n-2)(n-1)}$
$(sk_{1,\text{rev}}, pk_{1,\text{loc,rev}}) \leftarrow$ KEYGEN$()$
ensure VIRT.CIRCULATEKEYSCOINSTIMES() returns (OK)
ensure VIRT.CIRCULATEVIRTUALSIGS() returns (OK)
ensure VIRT.CIRCULATEFUNDINGSIGS() returns (OK)
ensure VIRT.CIRCULATEREVOCATIONS() returns (OK)
output (HOSTS READY, $p + \sum_{j=2}^{n-1} (s - 1 + t_j)$) to guest // p is every how many blocks we have to check the chain

Figure 49

Process VIRT.CIRCULATEKEYSCOINSTIMES(left_data):

if left_data is given as argument **then** // we are not host_funder
parse left_data as $((pk_{j,\text{fund,new}})_{j \in [i-1]}, (pk_{j,\text{left,fund,old}})_{j \in \{2,\dots,i-1\}}, (pk_{j,\text{right,fund,old}})_{j \in [i-1]}, (pk_{j,\text{out}})_{j \in [i-1]}, (c_{j,\text{left}})_{j \in \{2,\dots,i-1\}}, (c_{j,\text{right}})_{j \in [i-1]}, (t_j)_{j \in [i-1]}, pk_{\text{left,virt}}, pk_{\text{right,virt}}, (pk_{h,j,k})_{h \in [i-1], j \in \{2,\dots,n-1\}, k \in [n] \setminus \{j\}}, (pk_{h,\text{loc,rev}})_{h \in [i-1]}, (pk_{h,\text{rem,rev}})_{h \in [i-1]})$
if we have a sibling **then** // we are not host_fundee
input (KEYS AND COINS FORWARD, (left_data, $(sk_{i,\text{left,fund,old}}, pk_{i,\text{left,fund,old}}), pk_{i,\text{out}}, c_{i,\text{left}}, t_{i,\text{left}}, (sk_{i,j,k}, pk_{i,j,k})_{j \in \{2,\dots,n-1\}, k \in [n] \setminus \{j\}}, (sk_{i,\text{rev}}, pk_{i,\text{rev}})$)) to sibling
store input as left_data and parse it as $((pk_{j,\text{fund,new}})_{j \in [i-1]}, (pk_{j,\text{left,fund,old}})_{j \in \{2,\dots,i\}}, (pk_{j,\text{right,fund,old}})_{j \in [i-1]}, (pk_{j,\text{out}})_{j \in [i]}, (c_{j,\text{left}})_{j \in \{2,\dots,i\}}, (c_{j,\text{right}})_{j \in [i-1]}, (t_j)_{j \in [i-1]}, sk_{i,\text{left,fund,old}}, t_{i,\text{left}}, pk_{\text{left,virt}}, pk_{\text{right,virt}}, (pk_{h,j,k})_{h \in [i], j \in \{2,\dots,n-1\}, k \in [n] \setminus \{j\}}, (sk_{i,j,k})_{j \in \{2,\dots,n-1\}, k \in [n] \setminus \{j\}}, (pk_{h,\text{rev}})_{h \in [i]}, sk_{i,\text{rev}}$
$t_i \leftarrow \max(t_{i,\text{left}}, t_{i,\text{right}})$
replace $t_{i,\text{left}}$ in left_data with t_i
remove $sk_{i,\text{left,fund,old}}, (sk_{i,j,k})_{j \in \{2,\dots,n-1\}, k \in [n] \setminus \{j\}}, sk_{i,\text{loc,rev}}$ and $sk_{i,\text{rem,rev}}$ from left_data
call VIRT.CIRCULATEKEYSCOINSTIMES(left_data) of $\bar{P}$ and assign returned value to right_data
parse right_data as $((pk_{j,\text{fund,new}})_{j \in \{i+1,\dots,n\}}, (pk_{j,\text{left,fund,old}})_{j \in \{i+1,\dots,n\}}, (pk_{j,\text{right,fund,old}})_{j \in \{i+1,\dots,n-1\}}, (pk_{j,\text{out}})_{j \in \{i+1,\dots,n\}}, (c_{j,\text{left}})_{j \in \{i+1,\dots,n\}}, (c_{j,\text{right}})_{j \in \{i+1,\dots,n-1\}}, (t_j)_{j \in \{i+1,\dots,n\}}, (pk_{h,j,k})_{h \in \{i+1,\dots,n\}, j \in \{2,\dots,n-1\}, k \in [n] \setminus \{j\}}, (pk_{h,\text{rev}})_{h \in \{i+1,\dots,n\}}$
output (KEYS AND COINS BACK, right_data, $(sk_{i,\text{right,fund,old}}, pk_{i,\text{right,fund,old}}), c_{i,\text{right}}, t_i$)
store output as right_data and parse it as $((pk_{j,\text{fund,new}})_{j \in \{i+1,\dots,n\}}, (pk_{j,\text{left,fund,old}})_{j \in \{i+1,\dots,n\}}, (pk_{j,\text{right,fund,old}})_{j \in \{i,\dots,n-1\}}, (pk_{j,\text{out}})_{j \in \{i+1,\dots,n\}}, (c_{j,\text{left}})_{j \in \{i+1,\dots,n\}}, (c_{j,\text{right}})_{j \in \{i,\dots,n-1\}}, (t_j)_{j \in \{i,\dots,n\}}, (pk_{h,j,k})_{h \in \{i+1,\dots,n\}, j \in \{2,\dots,n-1\}, k \in [n] \setminus \{j\}}, (pk_{h,\text{loc,rev}})_{h \in \{i+1,\dots,n\}}, (pk_{h,\text{rem,rev}})_{h \in \{i+1,\dots,n\}}, sk_{i,\text{right,fund,old}})$
remove $sk_{i,\text{right,fund,old}}$ from right_data
return (right_data, $pk_{i,\text{fund,new}}, pk_{i,\text{left,fund,old}}, pk_{i,\text{out}}, c_{i,\text{left}}$)
else // we are host_fundee
output (CHECK KEYS, $(pk_{\text{left,virt}}, pk_{\text{right,virt}})$) to fundee and expect reply (KEYS OK)
return $(pk_{n,\text{fund,new}}, pk_{n,\text{left,fund,old}}, pk_{n,\text{out}}, c_{n,\text{left}}, t_n, (pk_{n,j,k})_{j \in \{2,\dots,n-1\}, k \in [n] \setminus \{j\}}, pk_{n,\text{loc,rev}}, pk_{n,\text{rem,rev}})$
end if
else // we are host_funder
call VIRT.CIRCULATEKEYSCOINSTIMES($pk_{1,\text{fund,new}}, pk_{1,\text{right,fund,old}}, pk_{1,\text{out}}, c_{1,\text{right}}, t_1, pk_{\text{left,virt}}, pk_{\text{right,virt}}, (pk_{1,j,k})_{j \in \{2,\dots,n-1\}, k \in [n] \setminus \{j\}}, pk_{1,\text{loc,rev}}, pk_{1,\text{rem,rev}}$) of $\bar{P}$ and assign returned value to right_data
parse right_data as $((pk_{j,\text{fund,new}})_{j \in \{2,\dots,n\}}, (pk_{j,\text{left,fund,old}})_{j \in \{2,\dots,n\}}, (pk_{j,\text{right,fund,old}})_{j \in \{2,\dots,n-1\}}, (pk_{j,\text{out}})_{j \in \{2,\dots,n\}}, (c_{j,\text{left}})_{j \in \{2,\dots,n\}}, (c_{j,\text{right}})_{j \in \{2,\dots,n-1\}}, (t_j)_{j \in \{2,\dots,n\}}, (pk_{h,j,k})_{h \in \{2,\dots,n\}, j \in \{2,\dots,n-1\}, k \in [n] \setminus \{j\}}, (pk_{h,\text{loc,rev}})_{h \in \{2,\dots,n\}}, (pk_{h,\text{rem,rev}})_{h \in \{2,\dots,n\}})$
return (OK)
end if

Figure 50

Process VIRT

GETMIDTXS($i, n, c_{\text{virt}}, c_{\text{rem,left}}, c_{\text{loc,left}}, c_{\text{loc,right}}, c_{\text{rem,right}}, pk_{\text{rem,left,fund,old}}, pk_{\text{loc,left,fund,old}}, pk_{\text{loc,right,fund,old}}, pk_{\text{rem,right,fund,old}}, pk_{\text{rem,left,fund,new}}, pk_{\text{loc,left,fund,new}}, pk_{\text{loc,right,fund,new}}, pk_{\text{rem,right,fund,new}}, pk_{\text{left,virt}}, pk_{\text{right,virt}}, pk_{\text{loc,out}}, pk_{\text{funder,rev}}, pk_{\text{left,rev}}, pk_{\text{loc,rev}}, pk_{\text{right,rev}}, pk_{\text{fundee,rev}}, (pk_{h,j,k})_{h \in [n], j \in [n-1] \setminus \{1\}, k \in [n-1] \setminus \{1,j\}}, (pk_{h,2,1})_{h \in [n]}, (pk_{h,n-1,n})_{h \in [n]}, (t_j)_{j \in [n-1] \setminus \{1\}}$):
ensure $1 < i < n$
ensure $c_{\text{rem,left}} \geq c_{\text{virt}} \land c_{\text{loc,left}} \geq c_{\text{virt}}$ // left parties fund virtual channel
ensure $c_{\text{rem,left}} \geq c_{\text{loc,right}} \land c_{\text{rem,right}} \geq c_{\text{loc,left}}$ // avoid griefing attack
$c_{\text{left}} \leftarrow c_{\text{rem,left}} + c_{\text{loc,left}}$; $c_{\text{right}} \leftarrow c_{\text{loc,right}} + c_{\text{rem,right}}$
left_old_fund $\leftarrow 2/\{pk_{\text{rem,left,fund,old}}, pk_{\text{loc,left,fund,old}}\}$
right_old_fund $\leftarrow 2/\{pk_{\text{loc,right,fund,old}}, pk_{\text{rem,right,fund,old}}\}$
left_new_fund $\leftarrow 2/\{pk_{\text{rem,left,fund,new}}, pk_{\text{loc,left,fund,new}}\} \lor 2/\{pk_{\text{left,rev}}, pk_{\text{loc,rev}}\}$
right_new_fund $\leftarrow 2/\{pk_{\text{loc,right,fund,new}}, pk_{\text{rem,right,fund,new}}\} \lor 2/\{pk_{\text{loc,rev}}, pk_{\text{right,rev}}\}$
virt_fund $\leftarrow 2/\{pk_{\text{left,virt}}, pk_{\text{right,virt}}\}$
revocation $\leftarrow 4/\{pk_{\text{funder,rev}}, pk_{\text{loc,rev}}, pk_{\text{right,rev}}, pk_{\text{fundee,rev}}\}$
refund $\leftarrow (pk_{\text{loc,out}} + (p+s)) \lor 2/\{pk_{\text{left,rev}}, pk_{\text{loc,rev}}\}$
for all $j \in [n-1] \setminus \{1\}, k \in [n-1] \setminus \{1, j\}$ **do**
$all_{j,k} \leftarrow n/\{pk_{1,j,k}, \dots, pk_{n,j,k}\} \land$ "k"
end for
if $i = 2$ **then**
$all_{2,1} \leftarrow n/\{pk_{1,2,1}, \dots, pk_{n,2,1}\} \land$ "1"
end if
if $i = n-1$ **then**
$all_{n-1,n} \leftarrow n/\{pk_{1,n-1,n}, \dots, pk_{n,n-1,n}\} \land$ "n"
end if
// After funding is complete, A_j has the signature of all other parties for all $all_{j,k}$ inputs, but other parties do not have A_j's signature for this input, therefore only A_j can publish it.

```
    // TX_{i,j,k} := i-th move, j, k input interval start and end. j, k unneeded for i = 1, k unneeded for i = 2.
    TX_1 ← TX:
        inputs:
            (c_left, left_old_fund),
            (c_right, right_old_fund)
        outputs:
            (c_left − c_virt, left_new_fund),
            (c_right − c_virt, right_new_fund),
            (c_virt, refund),
            (c_virt,
                (if (i − 1 > 1) then all_{i−1,i} else False)
                ∨ (if (i + 1 < n) then all_{i+1,i} else False)
                ∨ revocation
                ∨ (
                    if (i − 1 = 1 ∧ i + 1 = n) then virt_fund
                    else if (i − 1 > 1 ∧ i + 1 = n) then virt_fund + t_{i−1}
                    else if (i − 1 = 1 ∧ i + 1 < n) then virt_fund + t_{i+1}
                    else /*i − 1 > 1 ∧ i + 1 < n*/ virt_fund + max (t_{i−1}, t_{i+1})
                )
            )
    if i = 2 then
        TX_{2,1} ← TX:
            inputs:
                (c_virt, all_{2,1}),
                (c_right, right_old_fund)
            outputs:
                (c_right − c_virt, right_new_fund),
                (c_virt, refund),
                (c_virt,
                    revocation ∨
                    (if (n > 3) then (all_{3,2} ∨ (virt_fund + t_3))
                    else virt_fund)
                )
    end if
    if i = n − 1 then
        TX_{2,n} ← TX:
            inputs:
                (c_left, left_old_fund),
                (c_virt, all_{n−1,n})
            outputs:
                (c_left − c_virt, left_new_fund),
                (c_virt, refund),
                (c_virt,
                    revocation ∨
                    (if (n − 2 > 1) then (all_{n−2,n−1} ∨ (virt_fund + t_{n−2}))
                    else virt_fund)
                )
    end if
    for all k ∈ {2, . . . , i − 1} do // i − 2 txs
        TX_{2,k} ← TX:
            inputs:
                (c_virt, all_{i,k}),
                (c_right, right_old_fund)
            outputs:
                (c_right − c_virt, right_new_fund),
                (c_virt, refund),
                (c_virt,
                    (if (k − 1 > 1) then all_{k−1,i} else False)
                    ∨ (if (i + 1 < n) then all_{i+1,k} else False)
                    ∨ revocation
                    ∨ (
                        if (k − 1 = 1 ∧ i + 1 = n) then virt_fund
                        else if (k − 1 > 1 ∧ i + 1 = n) then virt_fund + t_{k−1}
                        else if (k − 1 = 1 ∧ i + 1 < n) then virt_fund + t_{i+1}
                        else /*k − 1 > 1 ∧ i + 1 < n*/ virt_fund + max (t_{k−1}, t_{i+1})
                    )
                )
    end for
    for all k ∈ {i + 1, . . . , n − 1} do // n − i − 1 txs
        TX_{2,k} ← TX:
            inputs:
                (c_left, left_old_fund)
                (c_virt, all_{i,k}),
            outputs:
                (c_left − c_virt, left_new_fund),
                (c_virt, refund),
                (c_virt,
                    (if (i − 1 > 1) then all_{i−1,k} else False)
                    ∨ (if (k + 1 < n) then all_{k+1,i} else False)
                    ∨ revocation
                    ∨ (
                        if (i − 1 = 1 ∧ k + 1 = n) then virt_fund
                        else if (i − 1 > 1 ∧ k + 1 = n) then virt_fund + t_{i−1}
                        else if (i − 1 = 1 ∧ k + 1 < n) then virt_fund + t_{k+1}
                        else /*i − 1 > 1 ∧ k + 1 < n*/ virt_fund + max (t_{i−1}, t_{k+1})
                    )
                )
    end for
    if i = 2 then m ← 1 else m ← 2
    if i = n − 1 then l ← n else l ← n − 1
    for all (k_1, k_2) ∈ {m, . . . , i − 1} × {i + 1, . . . , l} do // (i − m) · (l − i) txs
        TX_{3,k_1,k_2} ← TX:
            inputs:
                (c_virt, all_{i,k_1}),
                (c_virt, all_{i,k_2})
            outputs:
                (c_virt, refund),
                (c_virt,
```

(if $(k_1 - 1 > 1)$ then $all_{k_1-1,\min(k_2,n-1)}$ else False)
$\lor$ (if $(k_2 + 1 < n)$ then $all_{k_2+1,\max(k_1,2)}$ else False)
$\lor$ `revocation`
$\lor$ (
if $(k_1 - 1 \leq 1 \land k_2 + 1 \geq n)$ then `virt_fund`
else if $(k_1 - 1 > 1 \land k_2 + 1 \geq n)$ then `virt_fund` $+ t_{k_1-1}$
else if $(k_1 - 1 \leq 1 \land k_2 + 1 < n)$ then `virt_fund` $+ t_{k_2+1}$
else /*$k_1 - 1 > 1 \land k_2 + 1 < n$*/
`virt_fund` $+ \max(t_{k_1-1}, t_{k_2+1})$
)
)
end for
return (
TX_1,
$(\mathrm{TX}_{2,k})_{k \in \{m,\dots,l\} \setminus \{i\}}$,
$(\mathrm{TX}_{3,k_1,k_2})_{(k_1,k_2) \in \{m,\dots,i-1\} \times \{i+1,\dots,l\}}$
)

Figure 51

Process VIRT

// left and right refer to the two counterparties, with left being the one closer to the funder. Note difference with left/right meaning in VIRT.GETMIDTXS.
GETENDPOINTTX(i, n, c_{virt}, c_{left}, c_{right}, $pk_{\mathrm{left,fund,old}}$, $pk_{\mathrm{right,fund,old}}$, $pk_{\mathrm{left,fund,new}}$, $pk_{\mathrm{right,fund,new}}$, $pk_{\mathrm{left,virt}}$, $pk_{\mathrm{right,virt}}$, $pk_{\mathrm{interm,rev}}$, $pk_{\mathrm{endpoint,rev}}$, $(pk_{\mathrm{all},j})_{j \in [n]}$, t):
ensure $i \in \{1, n\}$
ensure $c_{\mathrm{left}} \geq c_{\mathrm{virt}}$ // left party funds virtual channel
$c_{\mathrm{tot}} \leftarrow c_{\mathrm{left}} + c_{\mathrm{right}}$
`old_fund` $\leftarrow 2/\{pk_{\mathrm{left,fund,old}}, pk_{\mathrm{right,fund,old}}\}$
`new_fund` $\leftarrow 2/\{pk_{\mathrm{left,fund,new}}, pk_{\mathrm{right,fund,new}}\} \lor 2/\{pk_{\mathrm{left,rev}}, pk_{\mathrm{right,rev}}\}$
`virt_fund` $\leftarrow 2/\{pk_{\mathrm{left,virt}}, pk_{\mathrm{right,virt}}\}$
`revocation` $\leftarrow 2/\{pk_{\mathrm{interm,rev}}, pk_{\mathrm{endpoint,rev}}\}$
if $i = 1$ **then** // funder's tx
$all \leftarrow n/\{pk_{\mathrm{all},1}, \dots, pk_{\mathrm{all},n}\} \land$ "1"
else // fundee's tx
$all \leftarrow n/\{pk_{\mathrm{all},1}, \dots, pk_{\mathrm{all},n}\} \land$ "n"
end if
$\mathrm{TX}_1 \leftarrow \mathrm{TX}$: // endpoints only have an "initiator" tx
inputs:
(c_{tot}, `old_fund`)
outputs:
($c_{\mathrm{tot}} - c_{\mathrm{virt}}$, `new_fund`),
(c_{virt}, $all \lor$ `revocation` $\lor$ (`virt_fund` $+ t$))
return TX_1

Figure 52

Process VIRT.SIBLINGSIGS()

parse input as $\mathrm{sigs}_{\mathrm{byLeft}}$
if $i = 2$ **then** $m \leftarrow 1$ **else** $m \leftarrow 2$
if $i = n - 1$ **then** $l \leftarrow n$ **else** $l \leftarrow n - 1$
$(\mathrm{TX}_{i,1}$, $(\mathrm{TX}_{i,2,k})_{k \in \{m,\dots,l\} \setminus \{i\}}$, $(\mathrm{TX}_{i,3,k_1,k_2})_{(k_1,k_2) \in \{m,\dots,i-1\} \times \{i+1,\dots,l\}}) \leftarrow$ VIRT.GETMIDTXS(i, n, c_{virt}, $c_{i-1,\mathrm{right}}$, $c_{i,\mathrm{left}}$, $c_{i,\mathrm{right}}$, $c_{i+1,\mathrm{left}}$, $pk_{i-1,\mathrm{right,fund,old}}$, $pk_{i,\mathrm{left,fund,old}}$, $pk_{i,\mathrm{right,fund,old}}$, $pk_{i+1,\mathrm{left,fund,old}}$, $pk_{i-1,\mathrm{fund,new}}$, $pk_{i,\mathrm{fund,new}}$, $pk_{i,\mathrm{fund,new}}$, $pk_{i+1,\mathrm{fund,new}}$, $pk_{\mathrm{left,virt}}$, $pk_{\mathrm{right,virt}}$, $pk_{i,\mathrm{out}}$, $pk_{1,\mathrm{rev}}$, $pk_{i-1,\mathrm{rev}}$, $pk_{i,\mathrm{rev}}$, $pk_{i+1,\mathrm{rev}}$, $pk_{n,\mathrm{rev}}$, $(pk_{h,j,k})_{h \in [n], j \in [n-1] \setminus \{1\}, k \in [n-1] \setminus \{1,j\}}$, $(pk_{h,2,1})_{h \in [n]}$, $(pk_{h,n-1,n})_{h \in [n]}$, $(t_h)_{h \in [n-1] \setminus \{1\}}$)

// notation: sig(TX, pk) := sig with `ANYPREVOUT` flag such that VERIFY(TX, sig, pk) = True
ensure that the following signatures are present in $\mathrm{sigs}_{\mathrm{byLeft}}$ and store them:
- // $(l - m) \cdot (i - 1)$ signatures

$\forall k \in \{m, \dots, l\} \setminus \{i\}, \forall j \in [i - 1]$:
sig($\mathrm{TX}_{i,2,k}$, $pk_{j,i,k}$)
- // $2 \cdot (i - m) \cdot (l - i) \cdot (i - 1)$ signatures

$\forall k_1 \in \{m, \dots, i - 1\}, \forall k_2 \in \{i + 1, \dots, l\}, \forall j \in [i - 1]$:
sig($\mathrm{TX}_{i,3,k_1,k_2}$, pk_{j,i,k_1}), sig($\mathrm{TX}_{i,3,k_1,k_2}$, pk_{j,i,k_2})
$\mathrm{sigs}_{\mathrm{toRight}} \leftarrow \mathrm{sigs}_{\mathrm{byLeft}}$

for all $j \in \{2, \dots, n - 1\} \setminus \{i\}$ **do**
if $j = 2$ **then** $m' \leftarrow 1$ **else** $m' \leftarrow 2$
if $j = n - 1$ **then** $l' \leftarrow n$ **else** $l' \leftarrow n - 1$
$(\mathrm{TX}_{j,1}$, $(\mathrm{TX}_{j,2,k})_{k \in \{m',\dots,l'\} \setminus \{i\}}$, $(\mathrm{TX}_{j,3,k_1,k_2})_{(k_1,k_2) \in \{m',\dots,i-1\} \times \{i+1,\dots,l'\}}) \leftarrow$ GETMIDTXS(j, n, c_{virt}, $c_{j-1,\mathrm{right}}$, $c_{j,\mathrm{left}}$, $c_{j,\mathrm{right}}$, $c_{j+1,\mathrm{left}}$, $pk_{j-1,\mathrm{right,fund,old}}$, $pk_{j,\mathrm{left,fund,old}}$, $pk_{j,\mathrm{right,fund,old}}$, $pk_{j+1,\mathrm{left,fund,old}}$, $pk_{j-1,\mathrm{fund,new}}$, $pk_{j,\mathrm{fund,new}}$, $pk_{j,\mathrm{fund,new}}$, $pk_{j+1,\mathrm{fund,new}}$, $pk_{\mathrm{left,virt}}$, $pk_{\mathrm{right,virt}}$, $pk_{j,\mathrm{out}}$, $pk_{1,\mathrm{rev}}$, $pk_{j-1,\mathrm{rev}}$, $pk_{j,\mathrm{rev}}$, $pk_{j+1,\mathrm{rev}}$, $pk_{n,\mathrm{rev}}$, $(pk_{k,p,s})_{k \in [n], p \in [n-1] \setminus \{1\}, s \in [n-1] \setminus \{1,p\}}$, $(pk_{k,2,1})_{k \in [n]}$, $(pk_{k,n-1,n})_{k \in [n]}$, $(t_k)_{k \in [n-1] \setminus \{1\}}$)
if $j < i$ **then** sigs $\leftarrow \mathrm{sigs}_{\mathrm{toLeft}}$ **else** sigs $\leftarrow \mathrm{sigs}_{\mathrm{toRight}}$
for all $k \in \{m', \dots, l'\} \setminus \{j\}$ **do**
add SIGN($\mathrm{TX}_{j,2,k}$, $sk_{i,j,k}$, `ANYPREVOUT`) to sigs
end for
for all $k_1 \in \{m', \dots, j - 1\}, k_2 \in \{j + 1, \dots, l'\}$ **do**
add SIGN($\mathrm{TX}_{j,3,k_1,k_2}$, sk_{i,j,k_1}, `ANYPREVOUT`) to sigs
add SIGN($\mathrm{TX}_{j,3,k_1,k_2}$, sk_{i,j,k_2}, `ANYPREVOUT`) to sigs
end for
end for
if $i + 1 = n$ **then** // next hop is `host_fundee`
$\mathrm{TX}_{n,1} \leftarrow$ VIRT.GETENDPOINTTX(n, n, c_{virt}, $c_{n-1,\mathrm{right}}$, $c_{n,\mathrm{left}}$, $pk_{n-1,\mathrm{right,fund,old}}$, $pk_{n,\mathrm{left,fund,old}}$, $pk_{n-1,\mathrm{fund,new}}$, $pk_{n,\mathrm{fund,new}}$, $pk_{\mathrm{left,virt}}$, $pk_{\mathrm{right,virt}}$, $pk_{n-1,\mathrm{rev}}$, $pk_{n,\mathrm{rev}}$, $(pk_{j,n-1,n})_{j \in [n]}$, t_{n-1})
end if

call $\bar{P}$.CIRCULATEVIRTUALSIGS($\mathrm{sigs}_{\mathrm{toRight}}$) and assign returned value to $\mathrm{sigs}_{\mathrm{byRight}}$
ensure that the following signatures are present in $\mathrm{sigs}_{\mathrm{byRight}}$ and store them:
- // $(l - m) \cdot (n - i)$ signatures

$\forall k \in \{m, \ldots, l\} \setminus \{i\}, \forall j \in \{i+1, \ldots, n\}$:
    $\mathrm{sig}(\mathrm{TX}_{i,2,k}, pk_{j,i,k})$

- // $2 \cdot (i-m) \cdot (l-i) \cdot (n-i)$ signatures

$\forall k_1 \in \{m, \ldots, i-1\}, \forall k_2 \in \{i+1, \ldots, l\}, \forall j \in \{i+1, \ldots, n\}$:
    $\mathrm{sig}(\mathrm{TX}_{i,3,k_1,k_2}, pk_{j,i,k_1})$, $\mathrm{sig}(\mathrm{TX}_{i,3,k_1,k_2}, pk_{j,i,k_2})$
output (virtualSigsBack, $\mathrm{sigs}_{\mathrm{toLeft}}$, $\mathrm{sigs}_{\mathrm{byRight}}$)

Figure 53

Process virt.intermediarySigs()

if $i = 2$ **then** $m \leftarrow 1$ **else** $m \leftarrow 2$
if $i = n-1$ **then** $l \leftarrow n$ **else** $l \leftarrow n-1$
$(\mathrm{TX}_{i,1}, (\mathrm{TX}_{i,2,k})_{k \in \{m,\ldots,l\} \setminus \{i\}}, (\mathrm{TX}_{i,3,k_1,k_2})_{(k_1,k_2) \in \{m,\ldots,i-1\} \times \{i+1,\ldots,l\}}) \leftarrow$ virt.getMidTXs(i, n, c_{virt}, $c_{i-1,\mathrm{right}}$, $c_{i,\mathrm{left}}$, $c_{i,\mathrm{right}}$, $c_{i+1,\mathrm{left}}$, $pk_{i-1,\mathrm{right,fund,old}}$, $pk_{i,\mathrm{left,fund,old}}$, $pk_{i,\mathrm{right,fund,old}}$, $pk_{i+1,\mathrm{left,fund,old}}$, $pk_{i-1,\mathrm{fund,new}}$, $pk_{i,\mathrm{fund,new}}$, $pk_{i,\mathrm{fund,new}}$, $pk_{i+1,\mathrm{fund,new}}$, $pk_{\mathrm{left,virt}}$, $pk_{\mathrm{right,virt}}$, $pk_{i,\mathrm{out}}$, $pk_{1,\mathrm{rev}}$, $pk_{j-1,\mathrm{rev}}$, $pk_{j,\mathrm{rev}}$, $pk_{j+1,\mathrm{rev}}$, $pk_{n,\mathrm{rev}}$, $(pk_{h,j,k})_{h \in [n], j \in [n-1] \setminus \{1\}, k \in [n-1] \setminus \{1,j\}}$, $(pk_{h,2,1})_{h \in [n]}$, $(pk_{h,n-1,n})_{h \in [n]}$, $(t_h)_{h \in [n-1] \setminus \{1\}}$)
// not verifying our signatures in $\mathrm{sigs}_{\mathrm{byLeft}}$, our (trusted) `sibling` will do that
input (virtual sigs forward, $\mathrm{sigs}_{\mathrm{byLeft}}$) to `sibling`
virt.siblingSigs()
$\mathrm{sigs}_{\mathrm{toLeft}} \leftarrow \mathrm{sigs}_{\mathrm{byRight}} + \mathrm{sigs}_{\mathrm{toLeft}}$
if $i = 2$ **then** // previous hop is `host_funder`
    $\mathrm{TX}_{1,1} \leftarrow$ virt.getEndpointTX(1, n, c_{virt}, $c_{1,\mathrm{right}}$, $c_{2,\mathrm{left}}$, $pk_{1,\mathrm{right,fund,old}}$, $pk_{2,\mathrm{left,fund,old}}$, $pk_{1,\mathrm{fund,new}}$, $pk_{2,\mathrm{fund,new}}$, $pk_{\mathrm{left,virt}}$, $pk_{\mathrm{right,virt}}$, $pk_{2,\mathrm{rev}}$, $pk_{1,\mathrm{rev}}$, $(pk_{j,2,1})_{j \in [n]}$, t_2)
end if
return $\mathrm{sigs}_{\mathrm{toLeft}}$

Figure 54

Process virt.hostFundeeSigs()

$\mathrm{TX}_{n,1} \leftarrow$ virt.getEndpointTX(n, n, c_{virt}, $c_{n-1,\mathrm{right}}$, $c_{n,\mathrm{left}}$, $pk_{n-1,\mathrm{right,fund,old}}$, $pk_{n,\mathrm{right,fund,old}}$, $pk_{n-1,\mathrm{fund,new}}$, $pk_{n,\mathrm{fund,new}}$, $pk_{\mathrm{left,virt}}$, $pk_{\mathrm{right,virt}}$, $pk_{n-1,\mathrm{rev}}$, $pk_{n,\mathrm{rev}}$, $(pk_{j,n-1,n})_{j \in [n]}$, t_{n-1})
$\mathrm{sigs}_{\mathrm{toLeft}} \leftarrow \emptyset$
for all $j \in [n-1] \setminus \{1\}$ **do**
    if $j = 2$ **then** $m \leftarrow 1$ **else** $m \leftarrow 2$
    if $j = n-1$ **then** $l \leftarrow n$ **else** $l \leftarrow n-1$
    $(\mathrm{TX}_{j,1}, (\mathrm{TX}_{j,2,k})_{k \in \{m,\ldots,l\} \setminus \{j\}}, (\mathrm{TX}_{i,3,k_1,k_2})_{(k_1,k_2) \in \{m,\ldots,i-1\} \times \{i+1,\ldots,l\}}) \leftarrow$ virt.getMidTXs(j, n, c_{virt}, $c_{j-1,\mathrm{right}}$, $c_{j,\mathrm{left}}$, $c_{j,\mathrm{right}}$, $c_{j+1,\mathrm{left}}$, $pk_{j-1,\mathrm{right,fund,old}}$, $pk_{j,\mathrm{left,fund,old}}$, $pk_{j,\mathrm{right,fund,old}}$, $pk_{j+1,\mathrm{left,fund,old}}$, $pk_{j-1,\mathrm{fund,new}}$, $pk_{j,\mathrm{fund,new}}$, $pk_{j,\mathrm{fund,new}}$, $pk_{j+1,\mathrm{fund,new}}$, $pk_{\mathrm{left,virt}}$, $pk_{\mathrm{right,virt}}$, $pk_{j,\mathrm{out}}$, $pk_{1,\mathrm{rev}}$, $pk_{j-1,\mathrm{rev}}$, $pk_{j,\mathrm{rev}}$, $pk_{j+1,\mathrm{rev}}$, $pk_{n,\mathrm{rev}}$, $(pk_{h,s,k})_{h \in [n], s \in [n-1] \setminus \{1\}, k \in [n-1] \setminus \{1,s\}}$, $(pk_{h,2,1})_{h \in [n]}$, $(pk_{h,n-1,n})_{h \in [n]}$, $(t_h)_{h \in [n-1] \setminus \{1\}}$)
    for all $k \in \{m, \ldots, l\} \setminus \{j\}$ **do**
        add sign($\mathrm{TX}_{j,2,k}$, $sk_{n,j,k}$, ANYPREVOUT) to $\mathrm{sigs}_{\mathrm{toLeft}}$
    end for
    for all $k_1 \in \{m, \ldots, j-1\}, k_2 \in \{j+1, \ldots, l\}$ **do**
        add sign($\mathrm{TX}_{j,3,k_1,k_2}$, sk_{n,j,k_1}, ANYPREVOUT) to $\mathrm{sigs}_{\mathrm{toLeft}}$
        add sign($\mathrm{TX}_{j,3,k_1,k_2}$, sk_{n,j,k_2}, ANYPREVOUT) to $\mathrm{sigs}_{\mathrm{toLeft}}$
    end for
end for
return $\mathrm{sigs}_{\mathrm{toLeft}}$

Figure 55

Process virt.hostFunderSigs()

$\mathrm{sigs}_{\mathrm{toRight}} \leftarrow \emptyset$
for all $j \in [n-1] \setminus \{1\}$ **do**
    if $j = 2$ **then** $m \leftarrow 1$ **else** $m \leftarrow 2$
    if $j = n-1$ **then** $l \leftarrow n$ **else** $l \leftarrow n-1$
    $(\mathrm{TX}_{j,1}, (\mathrm{TX}_{j,2,k})_{k \in \{m,\ldots,l\} \setminus \{j\}}, (\mathrm{TX}_{i,3,k_1,k_2})_{(k_1,k_2) \in \{m,\ldots,i-1\} \times \{i+1,\ldots,l\}}) \leftarrow$ virt.getMidTXs(j, n, c_{virt}, $c_{j-1,\mathrm{right}}$, $c_{j,\mathrm{left}}$, $c_{j,\mathrm{right}}$, $c_{j+1,\mathrm{left}}$, $pk_{j-1,\mathrm{right,fund,old}}$, $pk_{j,\mathrm{left,fund,old}}$, $pk_{j,\mathrm{right,fund,old}}$, $pk_{j+1,\mathrm{left,fund,old}}$, $pk_{j-1,\mathrm{fund,new}}$, $pk_{j,\mathrm{fund,new}}$, $pk_{j,\mathrm{fund,new}}$, $pk_{j+1,\mathrm{fund,new}}$, $pk_{\mathrm{left,virt}}$, $pk_{\mathrm{right,virt}}$, $pk_{j,\mathrm{out}}$, $pk_{1,\mathrm{rev}}$, $pk_{j-1,\mathrm{rev}}$, $pk_{j,\mathrm{rev}}$, $pk_{j+1,\mathrm{rev}}$, $pk_{n,\mathrm{rev}}$, $(pk_{h,s,k})_{h \in [n], s \in [n-1] \setminus \{1\}, k \in [n-1] \setminus \{1,s\}}$, $(pk_{h,2,1})_{h \in [n]}$, $(pk_{h,n-1,n})_{h \in [n]}$, $(t_h)_{h \in [n-1] \setminus \{1\}}$)
    for all $k \in \{m, \ldots, l\} \setminus \{j\}$ **do**
        add sign($\mathrm{TX}_{j,2,k}$, $sk_{1,j,k}$, ANYPREVOUT) to $\mathrm{sigs}_{\mathrm{toRight}}$
    end for
    for all $k_1 \in \{m, \ldots, j-1\}, k_2 \in \{j+1, \ldots, l\}$ **do**
        add sign($\mathrm{TX}_{j,3,k_1,k_2}$, sk_{1,j,k_1}, ANYPREVOUT) to $\mathrm{sigs}_{\mathrm{toRight}}$
        add sign($\mathrm{TX}_{j,3,k_1,k_2}$, sk_{1,j,k_2}, ANYPREVOUT) to $\mathrm{sigs}_{\mathrm{toRight}}$
    end for
end for
call virt.circulateVirtualSigs($\mathrm{sigs}_{\mathrm{toRight}}$) of $\bar{P}$ and assign output to $\mathrm{sigs}_{\mathrm{byRight}}$
$\mathrm{TX}_{1,1} \leftarrow$ virt.getEndpointTX(1, n, c_{virt}, $c_{1,\mathrm{right}}$, $c_{2,\mathrm{left}}$, $pk_{1,\mathrm{right,fund,old}}$, $pk_{2,\mathrm{left,fund,old}}$, $pk_{1,\mathrm{fund,new}}$, $pk_{2,\mathrm{fund,new}}$, $pk_{\mathrm{left,virt}}$, $pk_{\mathrm{right,virt}}$, $pk_{2,\mathrm{rev}}$, $pk_{1,\mathrm{rev}}$, $(pk_{j,2,1})_{j \in [n]}$, t_2)
return (ok)

Figure 56

Process virt.circulateVirtualSigs($\mathrm{sigs}_{\mathrm{byLeft}}$)

if $1 < i < n$ **then** // we are not `host_funder` nor `host_fundee`
    return virt.intermediarySigs()
else if $i = 1$ **then** // we are `host_funder`
    return virt.hostFunderSigs()
else if $i = n$ **then** // we are `host_fundee`
    return virt.hostFundeeSigs()
end if // it is always $1 \leq i \leq n$ – c.f. Fig. 49, l. 12 and l. 37

Figure 57

Process $\textsc{virt.circulateFundingSigs}(\mathrm{sigs}_{\mathrm{byLeft}})$

if $1 < i < n$ **then** // we are not endpoint
  if $i = 2$ **then** $m \leftarrow 1$ **else** $m \leftarrow 2$
  if $i = n - 1$ **then** $l \leftarrow n$ **else** $l \leftarrow n - 1$
  ensure that the following signatures are present in $\mathrm{sigs}_{\mathrm{byLeft}}$ and store them:
  - // 1 signature
    $\mathrm{sig}(\mathrm{TX}_{i,1}, pk_{i-1,\mathrm{right,fund,old}})$
  - // $n - 3 + \chi_{i=2} + \chi_{i=n-1}$ signatures
    $\forall k \in \{m, \dots, l\} \setminus \{i\}$
      $\mathrm{sig}(\mathrm{TX}_{i,2,k}, pk_{i-1,\mathrm{right,fund,old}})$
  input ($\textsc{virtual base sig forward}$, $\mathrm{sigs}_{\mathrm{byLeft}}$) to `sibling`
  extract and store $\mathrm{sig}(\mathrm{TX}_{i,1}, pk_{i-1,\mathrm{right,fund,old}})$ and $\forall k \in \{m, \dots, l\} \setminus \{i\}$ $\mathrm{sig}(\mathrm{TX}_{i,2,k}, pk_{i-1,\mathrm{right,fund,old}})$ from $\mathrm{sigs}_{\mathrm{byLeft}}$ // same signatures as `sibling`
  $\mathrm{sigs}_{\mathrm{toRight}} \leftarrow \{\textsc{sign}(\mathrm{TX}_{i+1,1}, sk_{i,\mathrm{right,fund,old}}, \mathsf{ANYPREVOUT})\}$
  if $i + 1 < n$ **then**
    if $i + 1 = n - 1$ **then** $l' \leftarrow n$ **else** $l' \leftarrow n - 1$
    for all $k \in \{2, \dots, l'\}$ **do**
      add $\textsc{sign}(\mathrm{TX}_{i+1,2,k}, sk_{i,\mathrm{right,fund,old}}, \mathsf{ANYPREVOUT})$ to $\mathrm{sigs}_{\mathrm{toRight}}$
    end for
  else // $i + 1 = n$
    add $\textsc{sign}(\mathrm{TX}_{n,1}, sk_{i,\mathrm{right,fund,old}}, \mathsf{ANYPREVOUT})$ to $\mathrm{sigs}_{\mathrm{toRight}}$
  end if
  call $\textsc{virt.circulateFundingSigs}(\mathrm{sigs}_{\mathrm{toRight}})$ of $\bar{P}$ and assign returned values to $\mathrm{sigs}_{\mathrm{byRight}}$
  ensure that the following signatures are present in $\mathrm{sigs}_{\mathrm{byRight}}$ and store them:
  - // 1 signature
    $\mathrm{sig}(\mathrm{TX}_{i,1}, pk_{i+1,\mathrm{left,fund,old}})$
  - // $n - 3 + \chi_{i=2} + \chi_{i=n-1}$ signatures
    $\forall k \in \{m, \dots, l\} \setminus \{i\}$
      $\mathrm{sig}(\mathrm{TX}_{i,2,k}, pk_{i+1,\mathrm{right,fund,old}})$
  output ($\textsc{virtual base sig back}$, $\mathrm{sigs}_{\mathrm{byRight}}$)
  extract and store $\mathrm{sig}(\mathrm{TX}_{i,1}, pk_{i+1,\mathrm{right,fund,old}})$ and $\forall k \in \{m, \dots, l\} \setminus \{i\}$ $\mathrm{sig}(\mathrm{TX}_{i,2,k}, pk_{i+1,\mathrm{right,fund,old}})$ from $\mathrm{sigs}_{\mathrm{byRight}}$ // same signatures as `sibling`
  $\mathrm{sig}_{\mathrm{toLeft}} \leftarrow \{\textsc{sign}(\mathrm{TX}_{i-1,1}, sk_{i,\mathrm{left,fund,old}}, \mathsf{ANYPREVOUT})\}$
  if $i - 1 > 1$ **then**
    if $i - 1 = 2$ **then** $m' \leftarrow 1$ **else** $m' \leftarrow 2$
    for all $k \in \{m', \dots, n - 1\}$ **do**
      add $\textsc{sign}(\mathrm{TX}_{i-1,2,k}, sk_{i,\mathrm{left,fund,old}}, \mathsf{ANYPREVOUT})$ to $\mathrm{sigs}_{\mathrm{toLeft}}$
    end for
  else // $i - 1 = 1$
    add $\textsc{sign}(\mathrm{TX}_{1,1}, sk_{i,\mathrm{left,fund,old}}, \mathsf{ANYPREVOUT})$ to $\mathrm{sigs}_{\mathrm{toLeft}}$
  end if
  return $\mathrm{sigs}_{\mathrm{toLeft}}$
else if $i = 1$ **then** // we are `host_funder`
  $\mathrm{sigs}_{\mathrm{toRight}} \leftarrow \{\textsc{sign}(\mathrm{TX}_{2,1}, sk_{1,\mathrm{right,fund,old}}, \mathsf{ANYPREVOUT})\}$
  if $2 = n - 1$ **then** $l' \leftarrow n$ **else** $l' \leftarrow n - 1$
  for all $k \in \{3, \dots, l'\}$ **do**
    add $\textsc{sign}(\mathrm{TX}_{2,2,k}, sk_{1,\mathrm{right,fund,old}}, \mathsf{ANYPREVOUT})$ to $\mathrm{sigs}_{\mathrm{toRight}}$
  end for
  call $\textsc{virt.circulateFundingSigs}(\mathrm{sigs}_{\mathrm{toRight}})$ of $\bar{P}$ and assign returned value to $\mathrm{sigs}_{\mathrm{byRight}}$
  ensure that $\mathrm{sig}(\mathrm{TX}_{1,1}, pk_{2,\mathrm{left,fund,old}})$ is present in $\mathrm{sigs}_{\mathrm{byRight}}$ and store it
  return ($\textsc{ok}$)
else if $i = n$ **then** // we are `host_fundee`
  ensure $\mathrm{sig}(\mathrm{TX}_{n,1}, pk_{n-1,\mathrm{right,fund,old}})$ is present in $\mathrm{sigs}_{\mathrm{byLeft}}$ and store it
  $\mathrm{sigs}_{\mathrm{toLeft}} \leftarrow \{\textsc{sign}(\mathrm{TX}_{n-1,1}, sk_{n,\mathrm{left,fund,old}}, \mathsf{ANYPREVOUT})\}$
  if $n - 1 = 2$ **then** $m' \leftarrow 1$ **else** $m' \leftarrow 2$
  for all $k \in \{m', \dots, n - 2\}$ **do**
    add $\textsc{sign}(\mathrm{TX}_{n-1,2,k}, sk_{n,\mathrm{left,fund,old}}, \mathsf{ANYPREVOUT})$ to $\mathrm{sigs}_{\mathrm{toLeft}}$
  end for
  return $\mathrm{sigs}_{\mathrm{toLeft}}$
end if // it is always $1 \leq i \leq n$ – c.f. Fig. 49, l. 12 and l. 37

Figure 58

Process $\textsc{virt.circulateRevocations}$(`revoc_by_prev`)

if `revoc_by_prev` is given as argument **then** // we are not `host_funder`
  ensure `guest`.$\textsc{processRemoteRevocation}$(`revoc_by_prev`) returns ($\textsc{ok}$)
else // we are `host_funder`
  `revoc_for_next` $\leftarrow$ `guest`.$\textsc{revokePrevious}$()
  input ($\textsc{read}$) to $\mathcal{G}_{\mathrm{Ledger}}$ and assign ouput to Σ
  `last_poll` $\leftarrow |\Sigma|$
  call $\textsc{virt.circulateRevocations}$(`revoc_for_next`) of $\bar{P}$ and assign returned value to `revoc_by_next`
  ensure `guest`.$\textsc{processRemoteRevocation}$(`revoc_by_next`) returns ($\textsc{ok}$) // If the "ensure" fails, the opening process freezes, this is intentional. The channel can still close via ($\textsc{forceClose}$)
  return ($\textsc{ok}$)
end if
if we have a `sibling` **then** // we are not `host_fundee` nor `host_funder`
  input ($\textsc{virtual revocation forward}$) to `sibling`
  `revoc_for_next` $\leftarrow$ `guest`.$\textsc{revokePrevious}$()
  input ($\textsc{read}$) to $\mathcal{G}_{\mathrm{Ledger}}$ and assign ouput to Σ
  `last_poll` $\leftarrow |\Sigma|$
  call $\textsc{virt.circulateRevocations}$(`revoc_for_next`) of $\bar{P}$ and assign output to `revoc_by_next`
  ensure `guest`.$\textsc{processRemoteRevocation}$(`revoc_by_next`) returns ($\textsc{ok}$)
  output ($\textsc{hosts ready}$, t_i) to `guest` and expect reply ($\textsc{host ack}$)
  output ($\textsc{virtual revocation back}$)
end if
`revoc_for_prev` $\leftarrow$ `guest`.$\textsc{revokePrevious}$()
if $1 < i < n$ **then** // we are intermediary
  output ($\textsc{hosts ready}$, t_i) to `guest` and expect reply ($\textsc{host ack}$) // p is every how many blocks we have to check the chain
else // we are `host_fundee`, case of `host_funder` covered earlier
  output ($\textsc{hosts ready}$, $p + \sum_{j=2}^{n-1}(s - 1 + t_j)$) to `guest` and expect reply ($\textsc{host ack}$)
end if
return `revoc_for_prev`

Figure 59

Process VIRT – poll

On input (CHECK FOR LATERAL CLOSE) by $R \in \{$`guest`, `funder`, `fundee`$\}$:
input (READ) to $\mathcal{G}_{\text{Ledger}}$ and assign output to Σ
$k_1 \leftarrow 0$
if $\text{TX}_{i-1,1}$ is defined and $\text{TX}_{i-1,1} \in \Sigma$ **then**
$k_1 \leftarrow i - 1$
end if
for all $k \in [i-2]$ **do**
if $\text{TX}_{i-1,2,k}$ is defined and $\text{TX}_{i-1,2,k} \in \Sigma$ **then**
$k_1 \leftarrow k$
end if
end for
$k_2 \leftarrow 0$
if $\text{TX}_{i+1,1}$ is defined and $\text{TX}_{i+1,1} \in \Sigma$ **then**
$k_2 \leftarrow i + 1$
end if
for all $k \in \{i+2, \dots, n\}$ **do**
if $\text{TX}_{i+1,2,k}$ is defined and $\text{TX}_{i+1,2,k} \in \Sigma$ **then**
$k_2 \leftarrow k$
end if
end for
`last_poll` $\leftarrow |\Sigma|$
if $k_1 > 0 \lor k_2 > 0$ **then** // at least one neighbour has published its TX
ignore all messages except for (CHECK IF CLOSING) by R
State $\leftarrow$ CLOSING
sigs $\leftarrow \emptyset$
end if
if $k_1 > 0 \land k_2 > 0$ **then** // both neighbours have published their TXs
add $(\text{sig}(\text{TX}_{i,3,k_1,k_2}, pk_{p,i,k_1}))_{p \in [n] \setminus \{i\}}$ to sigs
add $(\text{sig}(\text{TX}_{i,3,k_1,k_2}, pk_{p,i,k_2}))_{p \in [n] \setminus \{i\}}$ to sigs
add SIGN($\text{TX}_{i,3,k_1,k_2}$, sk_{i,i,k_1}, ANYPREVOUT) to sigs
add SIGN($\text{TX}_{i,3,k_1,k_2}$, sk_{i,i,k_2}, ANYPREVOUT) to sigs
input (SUBMIT, $\text{TX}_{i,3,k_1,k_2}$, sigs) to $\mathcal{G}_{\text{Ledger}}$
else if $k_1 > 0$ **then** // only left neighbour has published its TX
add $(\text{sig}(\text{TX}_{i,2,k_1}, pk_{p,i,k_1}))_{p \in [n] \setminus \{i\}}$ to sigs
add SIGN($\text{TX}_{i,2,k_1}$, sk_{i,i,k_1}, ANYPREVOUT) to sigs
add SIGN($\text{TX}_{i,2,k_1}$, $sk_{i,\text{left,fund,old}}$, ANYPREVOUT) to sigs
input (SUBMIT, $\text{TX}_{i,2,k_1}$, sigs) to $\mathcal{G}_{\text{Ledger}}$
else if $k_2 > 0$ **then** // only right neighbour has published its TX
add $(\text{sig}(\text{TX}_{i,2,k_2}, pk_{p,i,k_2}))_{p \in [n] \setminus \{i\}}$ to sigs
add SIGN($\text{TX}_{i,2,k_2}$, sk_{i,i,k_2}, ANYPREVOUT) to sigs
add SIGN($\text{TX}_{i,2,k_2}$, $sk_{i,\text{right,fund,old}}$, ANYPREVOUT) to sigs
input (SUBMIT, $\text{TX}_{i,2,k_2}$, sigs) to $\mathcal{G}_{\text{Ledger}}$
end if

On input (CHECK FOR REVOKED) by $R \in \{$`guest`, `funder`, `fundee`$\}$:
input (READ) to $\mathcal{G}_{\text{Ledger}}$ and assign output to Σ
if $\text{TX}_{i-1,1} \in \Sigma \lor \exists k \in \mathbb{N} : \text{TX}_{i-1,2,k} \in \Sigma$ **then** // left counterparty maliciously published old virtual tx
input (SUBMIT, ($R_{\text{loc,left,virt}}$, $R_{\text{loc,left,fund}}$), (SIGN($R_{\text{loc,left,virt}}$, $sk_{i,\text{rev}}$), SIGN($R_{\text{loc,left,fund}}$, $sk_{i,\text{rev}}$))) to $\mathcal{G}_{\text{Ledger}}$
end if
if $\text{TX}_{i+1,1} \in \Sigma \lor \exists k \in \mathbb{N} : \text{TX}_{i+1,2,k} \in \Sigma$ **then** // right counterparty maliciously published old virtual tx
input (SUBMIT, ($R_{\text{loc,right,virt}}$, $R_{\text{loc,right,fund}}$), (SIGN($R_{\text{loc,right,virt}}$, $sk_{i,\text{rev}}$), SIGN($R_{\text{loc,right,fund}}$, $sk_{i,\text{rev}}$))) to $\mathcal{G}_{\text{Ledger}}$
end if
output (NOTHING REVOKED) to R

Figure 60

Process VIRT – On input (FORCECLOSE) by R:

// At most one of `funder`, `fundee` is defined
ensure $R \in \{$`guest`, `funder`, `fundee`$\}$
if *State* = CLOSED **then** output (CLOSED) to R
if *State* = GUEST PUNISHED **then** output (GUEST PUNISHED) to R
ensure *State* $\in$ {OPEN, CLOSING}
if $\texttt{host}_P \neq \mathcal{G}_{\text{Ledger}}$ **then** // $\texttt{host}_P$ is a VIRT
ignore all messages except for output (CLOSED) by $\texttt{host}_P$. Also relay to $\texttt{host}_P$ any (CHECK IF CLOSING) or (FORCECLOSE) input received
input (FORCECLOSE) to $\texttt{host}_P$
end if
// if we have a $\texttt{host}_P$, continue from here on output (CLOSED) by it
send (READ) to $\mathcal{G}_{\text{Ledger}}$ as R and assign reply to Σ
if $i \in \{1, n\} \land (\text{TX}_{(i-1)+\frac{2}{n-1}(n-i),1} \in \Sigma \lor \exists k \in [n] : \text{TX}_{(i-1)+\frac{2}{n-1}(n-i),2,k} \in \Sigma)$ **then** // we are an endpoint and our counterparty has closed – 1st subscript of TX is 2 if $i = 1$ and $n - 1$ if $i = n$
ignore all messages except for (CHECK IF CLOSING) and (FORCECLOSE) by R
State $\leftarrow$ CLOSING
give up execution token // control goes to $\mathcal{E}$
end if
let `tx` be the unique TX among $\text{TX}_{i,1}$, $(\text{TX}_{i,2,k})_{k \in [n]}$, $(\text{TX}_{i,3,k_1,k_2})_{k_1,k_2 \in [n]}$ that can be appended to Σ in a valid way // ignore invalid subscript combinations
let `sigs` be the set of stored signatures that sign `tx`
add SIGN(`tx`, $sk_{i,\text{left,fund,old}}$, ANYPREVOUT), SIGN(`tx`, $sk_{i,\text{right,fund,old}}$, ANYPREVOUT), $(\text{SIGN}(\texttt{tx}, sk_{i,j,k}, \text{ANYPREVOUT}))_{j,k \in [n]}$ to `sigs` // ignore invalid signatures
ignore all messages except for (CHECK IF CLOSING) by R
State $\leftarrow$ CLOSING
send (SUBMIT, `tx`, `sigs`) to $\mathcal{G}_{\text{Ledger}}$

Figure 61

Process VIRT – On input (CHECK IF CLOSING) by R:

ensure *State* = CLOSING
ensure $R \in \{$`guest`, `funder`, `fundee`$\}$
send (READ) to $\mathcal{G}_{\text{Ledger}}$ as R and assign reply to Σ
if $i = 1$ **then** // we are `host_funder`
ensure that there exists an output with $c_P + c_{\bar{P}} - c_{\text{virt}}$ coins and a $2/\{pk_{1,\text{fund,new}}, pk_{2,\text{fund,new}}\}$ spending method with expired/non-existent timelock in Σ // new base funding output

ensure that there exists an output with c_{virt} coins and a $2/\{pk_{\text{left,virt}}, pk_{\text{right,virt}}\}$ spending method with expired/non-existent timelock in Σ // virtual funding output
else if $i = n$ **then** // we are `host_fundee`
ensure that there exists an output with $c_P + c_{\bar{P}} - c_{\text{virt}}$ coins and a $2/\{pk_{n-1,\text{fund,new}}, pk_{n,\text{fund,new}}\}$ spending method with expired/non-existent timelock in Σ // new base funding output
ensure that there exists an output with c_{virt} coins and a $2/\{pk_{\text{left,virt}}, pk_{\text{right,virt}}\}$ spending method with expired/non-existent timelock in Σ // virtual funding output
else // we are intermediary
if `side` = "left" **then** $j \leftarrow i - 1$ **else** $j \leftarrow i + 1$ // `side` is defined for all intermediaries – c.f. Fig. 49, l. 11
ensure that there exists an output with $c_P + c_{\bar{P}} - c_{\text{virt}}$ coins and a $2/\{pk_{i,\text{fund,new}}, pk_{j,\text{fund,new}}\}$ spending method with expired/non-existent timelock and an output with c_{virt} coins and a $pk_{i,\text{out}}$ spending method with expired/non-existent timelock in Σ
end if
State ← CLOSED
output (CLOSED) to R

Figure 62

Process VIRT – On (COOP CLOSE, `sig_bal`, `left_comms_revkeys`) by $\bar{P}$:

// we are left intermediary or host of fundee
ensure *State* = OPEN
parse `sig_bal` as (c'_1, c'_2), sig_1, sig_2
ensure $c_{\text{virt}} = c'_1 + c'_2$
ensure VERIFY$((c'_1, c'_2), \text{sig}_1, pk_{\text{left,virt}})$ = True
ensure VERIFY$((c'_1, c'_2), \text{sig}_2, pk_{\text{right,virt}})$ = True
State ← COOP CLOSING
extract $\text{sig}_{i-1,\text{right},C}$, $pk_{i-1,\text{right},R}$ from `left_comms_revkeys`
if $i < n$ **then** M ← CHECK COOP CLOSE **else** M ← CHECK COOP CLOSE FUNDEE
output $(M, (c'_1, c'_2), \text{sig}_{i-1,\text{right},C}, pk_{i-1,\text{right},R})$ to `guest`
ensure *State* = OPEN // executed by `guest`
State ← COOP CLOSING
store received signature as $\text{sig}_{\bar{P},C,i+1}$ // in guests, i is the current state number
store received revocation key as $pk_{\bar{P},R,i+2}$
remove most recent keys from list of old funding keys and assign them to $sk'_{P,F}$, $pk'_{P,F}$ and $pk'_{\bar{P},F}$
$C_{P,i+1}$ ← TX {input: $(c_P + c_{\bar{P}} + c'_1 + c'_2, 2/\{pk'_{P,F}, pk'_{\bar{P},F}\})$, outputs: $(c_P + c'_2, (pk_{P,\text{out}} + (p+s)) \vee 2/\{pk_{\bar{P},R,i+1}, pk_{P,R,i+1}\})$, $(c_{\bar{P}} + c'_1, pk_{\bar{P},\text{out}})$}
ensure VERIFY$(C_{P,i+1}, \text{sig}_{\bar{P},C,i+1}, pk'_{\bar{P},F})$ = True
input (COOP CLOSE CHECK OK) to host_P
if $i < n$ **then** // we are intermediary
input (COOP CLOSE, `left_comms_keys`) to `sibling`
ensure *State* = OPEN // executed by `sibling`
State ← COOP CLOSING
output (COOP CLOSE SIGN COMM, (c'_1, c'_2)) to `guest`
ensure *State* = OPEN // executed by guest of `sibling`
State ← COOP CLOSING
remove most recent keys from list of old funding keys and assign them to $sk'_{P,F}$, $pk'_{P,F}$ and $pk'_{\bar{P},F}$
$C_{\bar{P},i+1}$ ← TX {input: $(c_P + c_{\bar{P}} + c'_1 + c'_2, 2/\{pk'_{P,F}, pk'_{\bar{P},F}\})$, outputs: $(c_P + c'_1, pk_{P,\text{out}})$, $(c_{\bar{P}} + c'_2, (pk_{\bar{P},\text{out}} + (p+s)) \vee 2/\{pk_{P,R,i+1}, pk_{\bar{P},R,i+1}\})$}
$\text{sig}_{P,C,i+1}$ ← SIGN$(C_{\bar{P},i+1}, sk'_{P,F})$
$(sk_{P,R,i+2}, pk_{P,R,i+2})$ ← KEYGEN()
input (NEW COMM TX, $\text{sig}_{P,C,i+1}$, $pk_{P,R,i+2}$) to host_P
rename received signature to $\text{sig}_{i,\text{right},C}$ // executed by `sibling`
rename received public key to $pk_{i,\text{right},R}$ // in hosts, i is our hop number
send (COOP CLOSE, `sig_bal`, (`left_comms_keys`, $\text{sig}_{i,\text{right},C}$, $pk_{i,\text{right},R}$) to $\bar{P}$ and expect reply (COOP CLOSE BACK, (`right_comms_revkeys`, `right_revocations`))
$R_{\text{loc,right,virt}}$ ← TX {input: $(c_{\text{virt}}, 2/\{pk_{i,\text{rev}}, pk_{i+1,\text{rev}}\})$, output: $(c_{\text{virt}}, pk_{i,\text{out}})$}
extract $\text{sig}_{i+1,\text{right,rev,virt}}$ from `right_revocations`
ensure VERIFY$(R_{\text{loc,right,virt}}, \text{sig}_{i+1,\text{right,rev,virt}}, pk_{i+1,\text{rev}})$ = True
$R_{\text{loc,right,fund}}$ ← TX {input: $(c_P + c_{\bar{P}}, 2/\{pk_{i,\text{rev}}, pk_{i+1,\text{rev}}\})$, output: $(c_P + c_{\bar{P}}, pk_{i,\text{out}})$}
extract $\text{sig}_{i+1,\text{right,rev,fund}}$ from `right_revocations`
ensure VERIFY$(R_{\text{loc,right,fund}}, \text{sig}_{i+1,\text{right,rev,fund}}, pk_{i+1,\text{rev}})$ = True
extract $\text{sig}_{i+1,\text{left},C}$ from `right_comms_revkeys`
extract $\text{sig}_{i+1,\text{left},R}$ from `right_revocations`
extract $pk_{i+1,\text{left},R}$ from `right_comms_revkeys`
output (VERIFY COMM REV, $\text{sig}_{i+1,\text{left},C}$, $\text{sig}_{i+1,\text{left},R}$, $pk_{i+1,\text{left},R}$) to `guest`
store received public key as $pk_{\bar{P},R,i+2}$ // executed by guest of `sibling`
store $\text{sig}_{i+1,\text{left},C}$ as $\text{sig}_{\bar{P},C,i+1}$, $pk_{\bar{P},R,i+2}$
$C_{P,i+1}$ ← TX {input: $(c_P + c_{\bar{P}} + c'_1 + c'_2, 2/\{pk'_{P,F}, pk'_{\bar{P},F}\})$, outputs: $(c_P + c_1, (pk_{P,\text{out}} + (p+s)) \vee 2/\{pk_{P,R,i+1}, pk_{\bar{P},R,i+1}\})$, $(c_{\bar{P}} + c'_2, pk_{\bar{P},\text{out}})$}
ensure VERIFY$(C_{P,i+1}, \text{sig}_{\bar{P},C,i+1}, pk'_{\bar{P},F})$ = True
store $\text{sig}_{i+1,\text{left},R}$ as $\text{sig}_{\bar{P},R,i}$
$R_{P,i}$ ← TX {input: $C_{\bar{P},i}$.outputs.$\bar{P}$, output: $(c_P + c_{\bar{P}}, pk_{P,\text{out}})$}
ensure VERIFY$(R_{P,i}, \text{sig}_{\bar{P},R,i}, pk_{\bar{P},R,i})$ = True
input (COMM REV VERIFIED) to host_P
output (COOP CLOSE BACK, `right_comms_revkeys`, `right_revocations`) to `sibling` // executed by `sibling`
$R_{\text{loc,left,virt}}$ ← TX {input: $(c_{\text{virt}}, 4/\{pk_{1,\text{rev}}, pk_{i-1,\text{rev}}, pk_{i,\text{rev}}, pk_{n,\text{rev}}\})$, output: $(c_{\text{virt}}, pk_{i,\text{out}})$}
// the input corresponds to the revocation path of the virtual output of all virtual txs owned by $\bar{P}$
extract $\text{sig}_{n,i,\text{left,rev,virt}}$ from `right_revocations`
ensure VERIFY$(R_{\text{loc,left}}, \text{sig}_{n,\text{left,rev}}, pk_{n,\text{rev}})$ = True
else // $i = n$, we are host of fundee
output (REVOKE) to `fundee`
$R_{\bar{P},i}$ ← TX {input: $C_{P,i}$.outputs.P, output: $(c_P, pk_{\bar{P},\text{out}})$} // executed by `fundee`
$\text{sig}_{P,R,i}$ ← SIGN$(R_{\bar{P},i}, sk_{P,R,i})$
input (REVOCATIONS, $\text{sig}_{P,R,i}$) to host_P
rename received signature to $\text{sig}_{n,\text{right},R}$
for all $j \in \{2, \ldots, n\}$ **do**
$R_{j,\text{left}}$ ← TX {input: $(c_{\text{virt}}, 4/\{pk_{1,\text{rev}}, pk_{j-1,\text{rev}}, pk_{j,\text{rev}}, pk_{n,\text{rev}}\})$, output: $(c_{\text{virt}}, pk_{j,\text{out}})$}
$\text{sig}_{n,j,\text{left,rev}}$ ← SIGN$(R_{j,\text{left}}, sk_{n,\text{rev}})$
end for
end if
output (NEW COMM REV) to `guest`

$C_{\bar{P},i+1} \leftarrow$ TX {input: $(c_P + c_{\bar{P}} + c'_1 + c'_2, 2/\{pk'_{\bar{P},F}, pk'_{P,F}\})$, outputs: $(c_{\bar{P}} + c'_1, (pk_{\bar{P},\text{out}} + (p+s)) \vee 2/\{pk_{\bar{P},R,i+1}, pk_{P,R,i+1}\})$, $(c_P + c'_2, pk_{P,\text{out}})$} // executed by guest
$\text{sig}_{P,C,i+1} \leftarrow \textsc{sign}(C_{\bar{P},i+1}, sk'_{P,F})$
$R_{\bar{P},i} \leftarrow$ TX {input: $C_{P,i}$.outputs.P, output: $(c_P + c_{\bar{P}}, pk_{\bar{P},\text{out}})$}
$\text{sig}_{P,R,i} \leftarrow \textsc{sign}(R_{\bar{P},i}, sk_{P,R,i})$
$(sk_{P,R,i+2}, pk_{P,R,i+2}) \leftarrow \textsc{keyGen}()$
input (NEW COMM REV, $\text{sig}_{P,C,i+1}$, $\text{sig}_{P,R,i}$, $pk_{P,R,i+2}$) to host$_P$
rename $\text{sig}_{P,C,i+1}$ to $\text{sig}_{i,\text{left},C}$
rename $\text{sig}_{P,R,i}$ to $\text{sig}_{i,\text{left},R}$
rename received public key to $pk_{i,\text{left},R}$
$R_{\text{rem,left,virt}} \leftarrow$ TX {input: $(c_{\text{virt}}, 2/\{pk_{i-1,\text{rev}}, pk_{i,\text{rev}}\})$, output: $(c_{\text{virt}}, pk_{i-1,\text{out}})$}
$\text{sig}_{i,\text{left,rev,virt}} \leftarrow \textsc{sign}(R_{\text{rem,left,virt}}, sk_{i,\text{rev}})$
$R_{\text{rem,left,fund}} \leftarrow$ TX {input: $(c_P + c_{\bar{P}}, 2/\{pk_{i-1,\text{rev}}, pk_{i,\text{rev}}\})$, output: $(c_P + c_{\bar{P}}, pk_{i-1,\text{out}})$}
$\text{sig}_{i,\text{left,rev,fund}} \leftarrow \textsc{sign}(R_{\text{rem,left,fund}}, sk_{i,\text{rev}})$
if $i < n$ **then** // we are intermediary
$M \leftarrow$ (COOP CLOSE BACK, ((right_comms_revkeys, $\text{sig}_{i,\text{left},C}$, $pk_{i,\text{left},R}$), (right_revocations, $\text{sig}_{i,\text{left,rev,virt}}$, $\text{sig}_{i,\text{left,rev,fund}}$, $\text{sig}_{i,\text{left},R}$)))
else // $i = n$, we are host of fundee
$M \leftarrow$ (COOP CLOSE BACK, ($\text{sig}_{i,\text{left},C}$, $pk_{i,\text{left},R}$, $\text{sig}_{n,\text{left},R}$), ($\text{sig}_{n,\text{left,rev,virt}}$, $\text{sig}_{n,\text{left,rev,fund}}$, $(\text{sig}_{n,j,\text{left,rev}})_{j \in \{2,\dots,n\}}$))
end if
send M to $\bar{P}$ and expect reply (COOP CLOSE REVOCATIONS, left_revocations)
extract $\text{sig}_{i-1,\text{right},R}$, $\text{sig}_{1,i,\text{right,rev}}$, $\text{sig}_{i-1,\text{right,rev}}$ from left_revocations
ensure VERIFY($R_{\text{loc,left,virt}}$, $\text{sig}_{1,\text{right,rev}}$, $pk_{1,\text{rev}}$) = True
ensure VERIFY($R_{\text{loc,left,virt}}$, $\text{sig}_{i-1,\text{right,rev}}$, $pk_{i-1,\text{rev}}$) = True
$R_{\text{loc,left,fund}} \leftarrow$ TX {input: $(c_P + c_{\bar{P}}, 2/\{pk_{i-1,\text{rev}}, pk_{i,\text{rev}}\})$, output: $(c_P + c_{\bar{P}}, pk_{i,\text{out}})$} // the input corresponds to the revocation path of the right funding output of all virtual txs owned by $\bar{P}$
extract $\text{sig}_{i-1,\text{left,rev,fund}}$ from left_revocations
ensure VERIFY($R_{\text{loc,left,fund}}$, $\text{sig}_{i-1,\text{left,rev,fund}}$, $pk_{i-1,\text{rev}}$) = True
output (VERIFY REV, $\text{sig}_{i-1,\text{right},R}$, host$_P$) to guest
store received signature as $\text{sig}_{barP,R,i}$ // executed by guest
$R_{P,i} \leftarrow$ TX {input: $C_{\bar{P},i}$.outputs.$\bar{P}$, output: $(c_P + c_{\bar{P}}, pk_{P,\text{out}})$}
ensure VERIFY($R_{P,i}$, $\text{sig}_{\bar{P},R,i}$, $pk_{\bar{P},R,i}$) = True
add host$_P$ to list of old hosts
assign received host to host$_P$
$i \leftarrow i+1$; $c_P \leftarrow c_P + c'_2$; $c_{\bar{P}} \leftarrow c_{\bar{P}} + c'_1$
add $sk_{P,F}$, $pk_{P,F}$, $pk_{\bar{P},F}$ to list of old enabler channel funding keys
$(sk_{P,F}, pk_{P,F}) \leftarrow (sk'_{P,F}, pk'_{P,F})$
layer $\leftarrow$ layer $- 1$
locked$_P$ $\leftarrow$ locked$_P$ $- c_{\text{virt}}$
State $\leftarrow$ OPEN
hosting $\leftarrow$ False
input (REV VERIFIED) to last old host
State $\leftarrow$ COOP CLOSED
if $i < n$ **then** // we are intermediary
send (COOP CLOSE REVOCATIONS, left_revocations) to sibling
output (COOP CLOSE REVOCATIONS, host$_P$) to guest // executed by sibling
$R_{\bar{P},i} \leftarrow$ TX {input: $C_{P,i}$.outputs.P, output: $(c_P, pk_{\bar{P},\text{out}})$} // executed by guest of sibling
$\text{sig}_{P,R,i} \leftarrow \textsc{sign}(R_{\bar{P},i}, sk_{P,R,i})$
add host$_P$ to list of old hosts
assign received host to host$_P$
$i \leftarrow i+1$; $c_P \leftarrow c_P + c'_1$; $c_{\bar{P}} \leftarrow c_{\bar{P}} + c'_2$
add $sk_{P,F}$, $pk_{P,F}$, $pk_{\bar{P},F}$ to list of old enabler channel funding keys
$(sk_{P,F}, pk_{P,F}) \leftarrow (sk'_{P,F}, pk'_{P,F})$
layer $\leftarrow$ layer $- 1$
locked$_P$ $\leftarrow$ locked$_P$ $- c_{\text{virt}}$
State $\leftarrow$ OPEN
hosting $\leftarrow$ False
input (REVOCATION, $\text{sig}_{P,R,i}$) to last old host
rename received signature to $\text{sig}_{i,\text{right},R}$ // executed by sibling
$R_{\text{rem,right,virt}} \leftarrow$ TX {input: $(c_{\text{virt}}, 4/\{pk_{1,\text{rev}}, pk_{i,\text{rev}}, pk_{i+1,\text{rev}}, pk_{n,\text{rev}}\})$, output: $(c_{\text{virt}}, pk_{i+1,\text{out}})$}
$\text{sig}_{i,\text{right,rev,virt}} \leftarrow \textsc{sign}(R_{\text{rem,right,virt}}, sk_{i,\text{rev}})$
$R_{\text{rem,right,fund}} \leftarrow$ TX {input: $(c_P + c_{\bar{P}}, 2/\{pk_{i,\text{rev}}, pk_{i+1,\text{rev}}\})$, output: $(c_P + c_{\bar{P}}, pk_{i+1,\text{out}})$}
$\text{sig}_{i,\text{right,rev,fund}} \leftarrow \textsc{sign}(R_{\text{rem,right,fund}}, sk_{i,\text{rev}})$
send (COOP CLOSE REVOCATIONS, (left_revocations, $\text{sig}_{i,\text{right},R}$, $\text{sig}_{i,\text{right,rev,virt}}$, $\text{sig}_{i,\text{right,rev,fund}}$) to $\bar{P}$)
else // $i = n$, we are host of fundee
extract $\text{sig}_{1,\text{right},R}$ from left_revocations
output (VERIFY REVOCATION, $\text{sig}_{1,\text{right},R}$) to fundee
store received signature as $\text{sig}_{\bar{P},R,i}$ // executed by fundee
$R_{P,i} \leftarrow$ TX {input: $C_{\bar{P},i}$.outputs.$\bar{P}$, output: $(c_{\bar{P}}, pk_{P,\text{out}})$}
ensure VERIFY($R_{P,i}$, $\text{sig}_{\bar{P},R,i}$, $pk_{\bar{P},R,i}$) = True
State $\leftarrow$ COOP CLOSED
if close_initiator = P **then** // $\mathcal{E}$ instructed us to close the channel
execute code of Fig. 45
else // $\mathcal{E}$ instructed another party to close the channel
send (COOPCLOSED) to close_initiator
end if
end if

Figure 63

Process VIRT – punishment handling

On input (USED REVOCATION) by guest: // (USED REVOCATION) by funder/fundee is ignored
State $\leftarrow$ GUEST PUNISHED
input (USED REVOCATION) to host$_P$, expect reply (USED REVOCATION OK)
if funder or fundee is defined **then**
output (ENABLER USED REVOCATION) to it
else // sibling is defined
output (ENABLER USED REVOCATION) to sibling
end if

On input (ENABLER USED REVOCATION) by sibling:
State $\leftarrow$ GUEST PUNISHED
output (ENABLER USED REVOCATION) to guest

On output (USED REVOCATION) by host$_P$:
State $\leftarrow$ GUEST PUNISHED

```
if funder or fundee is defined then
    output (ENABLER USED REVOCATION) to it
else // sibling is defined
    output (ENABLER USED REVOCATION) to sibling
end if
```

Figure 64

F LIVENESS

Proposition F.1. *Consider a synchronised honest party that submits a transaction* `tx` *to the ledger functionality [49] by the time the block indexed by h is added to* `state` *in its view. Then* `tx` *is guaranteed to be included in the block range $[h+1, h+s]$, where $s = (2 + q)\texttt{windowSize}$ and $q = \lceil(\texttt{maxTime}_{\texttt{window}} + \frac{\texttt{Delay}}{2})/\texttt{minTime}_{\texttt{window}}\rceil$.*

The proof can be found in [54].

G OMITTED PROOFS

Lemma G.1 (Real world balance security). *Consider a real world execution with $P \in \{Alice, Bob\}$ honest LN ITI and $\bar{P}$ the counterparty ITI. Assume that all of the following are true:*

- *the internal variable* `negligent` *of P has value "False",*
- *P has transitioned to the OPEN State for the first time after having received (OPEN, $c, \dots$) by either $\mathcal{E}$ or $\bar{P}$,*
- *P [has received (FUND ME, $f_i, \dots$) as input by another LN ITI while State was OPEN and subsequently P transitioned to OPEN State] n times,*
- *P [has received (CHECK COOP CLOSE FUNDEE, $(\_, r_i), \dots$) as output by* $\texttt{host}_P$ *while State was OPEN and subsequently P transitioned to OPEN State] j times,*
- *P [has received (COOP CLOSE SIGN COMM FUNDER, $(l_i, \_)$) as output by* $\texttt{host}_P$ *while State was OPEN and subsequently P transitioned to OPEN State] k times,*
- *P [has received (PAY, d_i) by $\mathcal{E}$ while State was OPEN and P subsequently transitioned to OPEN State] m times,*
- *P [has received (GET PAID, e_i) by $\mathcal{E}$ while State was OPEN and P subsequently transitioned to OPEN State] l times.*

Let $\phi = 1$ if $P = Alice$, or $\phi = 0$ if $P = Bob$.

- *If P receives (FORCECLOSE) by $\mathcal{E}$ and, if* $\texttt{host}_P \neq$ "`ledger`" *the output of* $\texttt{host}_P$ *is (CLOSED), then eventually the state obtained when P inputs (READ) to $\mathcal{G}_{\text{Ledger}}$ will contain h outputs each of value c_i and that has been spent or is exclusively spendable by $pk_{R,\text{out}}$ such that*

$$\sum_{i=1}^{h} c_i \geq \phi \cdot c - \sum_{i=1}^{n} f_i - \sum_{i=1}^{m} d_i + \sum_{i=1}^{l} e_i + \sum_{i=1}^{j} r_i + \sum_{i=1}^{k} l_i \qquad (3)$$

with overwhelming probability in the security parameter, where R is a local, kindred LN machine (i.e. either P, the `guest` *of* $\texttt{host}_P$*'s* `sibling`*, the party to which P sent FUND ME if such a message has been sent, or the* `guest` *of the* `sibling` *of one of the transitive closure of hosts of P).*
- *Assume that, at some particular instant during the execution,*
  (1) $\texttt{host}_P \neq$ "`ledger`",
  (2) P has State OPEN.
  Consider two alternative series of subsequent execution steps:
  (1) The `guest` *of* $\texttt{host}_P$ *(call them S) receives (FORCECLOSE) by $\mathcal{E}$. From that point onward, all protocol parties (even corrupted ones) honestly follow the protocol. Eventually a total of c_b coins is exclusively spendable by $pk_{R,\text{out}}$, where R is a machine kindred to S. Additionally, there is at least one funding output of P's channel $(c_P + c_{\bar{P}}, 2/\{pk_{P,F}, pk_{\bar{P},F}\})$ that is on-chain and unspent.*
  (2) P receives either (COOPCLOSE) by $\mathcal{E}$ or (COOP CLOSE,...) by some other ITI, and P's variable `hosting` *is False. Subsequently, P's State transitions to COOP CLOSED and then the State of S transitions to OPEN. The next time S is activated is via a (FORCECLOSE) input by $\mathcal{E}$ and eventually a total of c_t coins is exclusively spendable by $pk_{R,\text{out}}$.*
  It then holds that

$$c_t - c_b \geq \phi \cdot c - \sum_{i=1}^{n} f_i - \sum_{i=1}^{m} d_i + \sum_{i=1}^{l} e_i + \sum_{i=1}^{j} r_i + \sum_{i=1}^{k} l_i \qquad (4)$$

  with overwhelming probability in the security parameter.

Proof of Lemma G.1. We first note that, as signature forgeries only happen with negligible probability and only a polynomial number of signatures are verified by honest parties throughout an execution, the event in which any forged signature passes the verification of an honest party or of $\mathcal{G}_{\text{Ledger}}$ happens only with negligible probability. We can therefore ignore this event throughout this proof and simply add a computationally negligible distance between $\mathcal{E}$'s outputs in the real and the ideal world at the end.

We also note that $pk_{P,\text{out}}$ has been provided by $\mathcal{E}$, therefore it can freely use coins spendable by this key. This is why we allow for any of the $pk_{P,\text{out}}$ outputs to have been spent.

Define the *history* of a channel as $H = (F, C)$, where each of F, C is a list of lists of integers. A party P which satisfies the Lemma conditions has a unique, unambiguously and recursively defined history: If the value hops in the (OPEN, c, hops, ...) message was equal to "`ledger`", then F is the empty list, otherwise F is the concatenation of the F and C lists of the party that sent (FUNDED, ...) to P, as they were at the moment the latter message was sent. After initialised, F remains immutable. Observe that, if hops $\neq$ "`ledger`", both aforementioned messages must have been received before P transitions to the OPEN state.

The list C of party P is initialised to $[[g]]$ when P's *State* transitions for the first time to OPEN, where $g = c$ if $P = Alice$, or $g = 0$ if $P = Bob$; this represents the initial channel balance. The value x or $-x$ is appended to the last list in C when a payment is received (Fig. 36, l. 21) or sent (Fig. 36, l. 6) respectively by P. Moving on to the funding of new virtual channels, whenever P funds a new virtual channel (Fig. 33, l. 21), $[-c_{\text{virt}}]$ is appended to C and whenever P helps with the opening of a new virtual channel, but does not fund it (Fig. 33, l. 24), $[0]$ is appended to C. In case of cooperatively closing a channel (Figs. 44-47 & 63) to which P's channel is base, if this channel was initially funded by P, when the closing procedure completes (Fig. 47, l. 53) $[c_1']$ is appended to C. Likewise, if in the closed virtual channel P was the base of the `fundee` (Fig. 63, l. 128), then $[c_2']$ (Fig. 63, l. 9) is appended to C. In case P was a left intermediary for the closed virtual channel (Fig. 63, l. 10), then $[c_2']$ is appended to C. Lastly, in case P was a right intermediary for the closed virtual channel (Fig. 63, l. 23), then $[c_1' - c_{\text{virt}}]$ is appended to C. Therefore C consists of one list of integers for each sequence of inbound and outbound payments that have not been interrupted by a virtualisation step and a new list is added for every virtual layer that is created or torn down cooperatively. We also observe that a non-negligent party with history (F, C) satisfies the Lemma conditions and that the value of the right hand side of the inequality (3) is equal to $\sum\limits_{s \in C} \sum\limits_{x \in s} x$, as all inbound and outbound payment values, new channel funding values and cooperative closing refunds that appear in the Lemma conditions are recorded in C.

Let party P with a particular history. We will inductively prove that P satisfies the Lemma. The base case is when a channel is opened with `hops` = "`ledger`" and is closed right away, therefore $H = ([], [g])$, where $g = c$ if $P = Alice$ and $g = 0$ if $P = Bob$. P can transition to the OPEN *State* for the first time only if all of the following have taken place:

- It has received (OPEN, $c, \dots$) while in the INIT *State*. In case $P = Alice$, this message must have been received as input by $\mathcal{E}$ (Fig. 31, l. 1), or in case $P = Bob$, this message must have been received via the network by $\bar{P}$ (Fig. 26, l. 3).
- It has received $pk_{\bar{P},F}$. In case $P = Bob$, $pk_{\bar{P},F}$ must have been contained in the (OPEN, $\dots$) message by $\bar{P}$ (Fig. 26, l. 3), otherwise if $P = Alice$ $pk_{\bar{P},F}$ must have been contained in the (ACCEPT CHANNEL, $\dots$) message by $\bar{P}$ (Fig. 26, l. 16).
- It internally holds a signature on the commitment transaction $C_{P,0}$ that is valid when verified with public key $pk_{\bar{P},F}$ (Fig. 28, ll. 12 and 23).
- It has the transaction F in the $\mathcal{G}_{\text{Ledger}}$ state (Fig. 29, l. 3 or Fig. 30, l. 16).

We observe that P satisfies the Lemma conditions with $m = n = l = 0$. Before transitioning to the OPEN *State*, P has produced only one valid signature for the "funding" output $(c, 2/\{pk_{P,F}, pk_{\bar{P},F}\})$ of F with $sk_{P,F}$, namely for $C_{\bar{P},0}$ (Fig. 28, ll. 4 or 14), and sent it to $\bar{P}$ (Fig. 28, ll. 6 or 21), therefore the only two ways to spend $(c, 2/\{pk_{P,F}, pk_{\bar{P},F}\})$ are by either publishing $C_{P,0}$ or $C_{\bar{P},0}$. We observe that $C_{P,0}$ has a $(g, (pk_{P,\text{out}} + (t+s)) \vee 2/\{pk_{P,R}, pk_{\bar{P},R}\})$ output (Fig. 28, l. 2 or 3). The spending method $2/\{pk_{P,R}, pk_{\bar{P},R}\}$ cannot be used since P has not produced a signature for it with $sk_{P,R}$, therefore the alternative spending method, $pk_{P,\text{out}} + (t + s)$, is the only one that will be spendable if $C_{P,0}$ is included in $\mathcal{G}_{\text{Ledger}}$, thus contributing g to the sum of outputs that contribute to inequality (3). Likewise, if $C_{\bar{P},0}$ is included in $\mathcal{G}_{\text{Ledger}}$, it will contribute at least one $(g, pk_{P,\text{out}})$ output to this inequality, as $C_{\bar{P},0}$ has a $(g, pk_{P,\text{out}})$ output (Fig. 28, l. 2 or 3). Additionally, if P receives (FORCECLOSE) by $\mathcal{E}$ while $H = ([], [g])$, it attempts to publish $C_{P,0}$ (Fig. 42, l. 19), and will either succeed or $C_{\bar{P},0}$ will be published instead. We therefore conclude that in every case $\mathcal{G}_{\text{Ledger}}$ will eventually have a state Σ that contains at least one $(g, pk_{P,\text{out}})$ output, therefore satisfying the Lemma consequence.

Let P with history $H = (F, C)$. The induction hypothesis is that the Lemma holds for P. Let c_P the sum in the right hand side of inequality (3). In order to perform the induction step, assume that P is in the OPEN state. We will prove all the following (the facts to be proven are shown with emphasis for clarity):

- If P receives (FUND ME, $f, \dots$) by a (local, kindred) LN ITI R, subsequently transitions back to the OPEN state (therefore moving to history (F, C') where $C' = C + [-f]$) and finally receives (FORCECLOSE) by $\mathcal{E}$ and (CLOSED) by $\texttt{host}_P$ before any further change to its history, then *eventually P's $\mathcal{G}_{\text{Ledger}}$ state will contain h transaction outputs each of value c_i exclusively spendable or already spent by $pk_{P,\text{out}}$ that are descendants of an output with spending method $2/\{pk_{P,F}, pk_{\bar{P},F}\}$ such that* $\sum\limits_{i=1}^{h} c_i \geq \sum\limits_{s \in C'} \sum\limits_{x \in s} x$. Furthermore, given that P moves to the OPEN state after the (FUND ME, $\dots$) message, it also sends (FUNDED, $\dots$) to R (Fig. 33, l. 22). If subsequently the state of R transitions to OPEN (therefore obtaining history (F_R, C_R) where $F_R = F + C$ and $C_R = [[f]]$), and finally receives (FORCECLOSE) by $\mathcal{E}$ and (CLOSED) by $\texttt{host}_R$ ($\texttt{host}_R = \texttt{host}_P$ – Fig. 30, l. 3) before any further change to its history, then *eventually R's $\mathcal{G}_{\text{Ledger}}$ state will contain k transaction outputs each of value c_i^R exclusively spendable or already spent by $pk_{R,\text{out}}$ that are descendants of an output with spending method $2/\{pk_{R,F}, pk_{\bar{R},F}\}$ such that* $\sum\limits_{i=1}^{k} c_i^R \geq \sum\limits_{s \in C_R} \sum\limits_{x \in s} x$.
- If P receives (VIRTUALISING, $\dots$) by $\bar{P}$ or `sibling`, subsequently transitions back to OPEN (therefore moving to history (F, C') where $C' = C + [0]$) and finally receives (FORCECLOSE) by $\mathcal{E}$ and (CLOSED) by $\texttt{host}_P$ before any further change to its history, then *eventually P's $\mathcal{G}_{\text{Ledger}}$ state will contain h transaction outputs each of value c_i exclusively spendable or already spent by $pk_{P,\text{out}}$ that are descendants of an output with spending method $2/\{pk_{P,F}, pk_{\bar{P},F}\}$ such that* $\sum\limits_{i=1}^{h} c_i \geq \sum\limits_{s \in C} \sum\limits_{x \in s} x$. Furthermore, given that P moves to the OPEN state after the (VIRTUALISING, $\dots$) message and in case it sends (FUNDED, $\dots$) to some party R (Fig. 33, l. 19), the latter party is the (local, kindred) `fundee` of a new virtual channel. If subsequently the state of R transitions to OPEN (therefore obtaining history (F_R, C_R) where $F_R = F + C$ and $C_R = [[0]]$), and finally receives (FORCECLOSE) by $\mathcal{E}$ and (CLOSED) by $\texttt{host}_R$ ($\texttt{host}_R = \texttt{host}_P$ – Fig. 30, l. 3) before any further change to its history, then *eventually R's $\mathcal{G}_{\text{Ledger}}$ state will contain an output with a $2/\{pk_{R,F}, pk_{\bar{R},F}\}$ spending method.*
- If P receives (CHECK COOP CLOSE, $\dots$) by $\texttt{host}_P$, subsequently transitions back to OPEN (therefore moving to history (F, C') where $C' = C + [c_2']$), and finally receives (FORCECLOSE) by $\mathcal{E}$ and (CLOSED) by $\texttt{host}_P$ before any further change to its history, then *eventually P's $\mathcal{G}_{\text{Ledger}}$ state will contain h transaction outputs each of value c_i exclusively spendable or already spent by $pk_{P,\text{out}}$ that are descendants of an output with spending method $2/\{pk_{P,F}, pk_{\bar{P},F}\}$ such that* $\sum\limits_{i=1}^{h} c_i \geq \sum\limits_{s \in C} \sum\limits_{x \in s} x$.

- If P receives (COOP CLOSE SIGN COMM, ...) by $\texttt{host}_P$, subsequently transitions back to OPEN (therefore moving to history (F, C') where $C' = C + [c'_1 - c_{\text{virt}}]$), and finally receives (FORCECLOSE) by $\mathcal{E}$ and (CLOSED) by $\texttt{host}_P$ before any further change to its history, then *eventually P's $\mathcal{G}_{\text{Ledger}}$ state will contain h transaction outputs each of value c_i exclusively spendable or already spent by $pk_{P,\text{out}}$ that are descendants of an output with spending method $2/\{pk_{P,F}, pk_{\bar{P},F}\}$ such that $\sum\limits_{i=1}^{h} c_i \geq \sum\limits_{s \in C} \sum\limits_{x \in s} x$*. Furthermore, *there exists a local, kindred machine R that transitioned to the OPEN state after the last time control was obtained by one of P's kindred machines and before P transitioned to the OPEN state, such that R obtained $c'_2 = c_{\text{virt}} - c'_1$ coins during its last activation.* (In other words, P and R broke even on aggregate by first supporting the opening and then the cooperative closing of a virtual channel.)
- If P receives (COOP CLOSE SIG COMM FUNDER, ...) by $\texttt{host}_P$, subsequently transitions back to OPEN (therefore moving to history (F, C') where $C' = C + [c'_1]$) and finally receives (FORCECLOSE) by $\mathcal{E}$ and (CLOSED) by $\texttt{host}_P$ before any further change to its history, then *eventually P's $\mathcal{G}_{\text{Ledger}}$ state will contain h transaction outputs each of value c_i exclusively spendable or already spent by $pk_{P,\text{out}}$ that are descendants of an output with spending method $2/\{pk_{P,F}, pk_{\bar{P},F}\}$ such that $\sum\limits_{i=1}^{h} c_i \geq \sum\limits_{s \in C} \sum\limits_{x \in s} x$*.
- If P receives (CHECK COOP CLOSE FUNDEE, ...) by $\texttt{host}_P$, subsequently transitions back to OPEN (therefore moving to history (F, C') where $C' = C + [c'_2]$) and finally receives (FORCECLOSE) by $\mathcal{E}$ and (CLOSED) by $\texttt{host}_P$ before any further change to its history, then *eventually P's $\mathcal{G}_{\text{Ledger}}$ state will contain h transaction outputs each of value c_i exclusively spendable or already spent by $pk_{P,\text{out}}$ that are descendants of an output with spending method $2/\{pk_{P,F}, pk_{\bar{P},F}\}$ such that $\sum\limits_{i=1}^{h} c_i \geq \sum\limits_{s \in C} \sum\limits_{x \in s} x$*.
- If P receives (PAY, d) by $\mathcal{E}$, subsequently transitions back to OPEN (therefore moving to history (F, C') where C' is C with $-d$ appended to the last list of C) and finally receives (FORCECLOSE) by $\mathcal{E}$ and (CLOSED) by $\texttt{host}_P$ (the latter only if $\texttt{host}_P \neq$ "$\texttt{ledger}$" or equivalently $F \neq []$) before any further change to its history, then *eventually P's $\mathcal{G}_{\text{Ledger}}$ state will contain h transaction outputs each of value c_i exclusively spendable or already spent by $pk_{P,\text{out}}$ that are descendants of an output with a $2/\{pk_{P,F}, pk_{\bar{P},F}\}$ spending method such that $\sum\limits_{i=1}^{h} c_i \geq \sum\limits_{s \in C'} \sum\limits_{x \in s} x$*.
- If P receives (GET PAID, e) by $\mathcal{E}$, subsequently transitions back to OPEN (therefore moving to history (F, C') where C' is C with e appended to the last list of C) and finally receives (FORCECLOSE) by $\mathcal{E}$ and (CLOSED) by $\texttt{host}_P$ (the latter only if $\texttt{host}_P \neq$ "$\texttt{ledger}$" or equivalently $F = []$) before any further change to its history, then *eventually P's $\mathcal{G}_{\text{Ledger}}$ state will contain h transaction outputs each of value c_i exclusively spendable or already spent by $pk_{P,\text{out}}$ that are descendants of an output with a $2/\{pk_{P,F}, pk_{\bar{P},F}\}$ spending method such that $\sum\limits_{i=1}^{h} c_i \geq \sum\limits_{s \in C'} \sum\limits_{x \in s} x$*.

Consider the first bullet. By the induction hypothesis, before the funding procedure started P could close the channel and end up with on-chain transaction outputs exclusively spendable or already spent by $pk_{P,\text{out}}$ with a sum value of c_P. When P is in the OPEN state and receives (FUND ME, f, ...), it can only move again to the OPEN state after doing the following state transitions: OPEN $\rightarrow$ VIRTUALISING $\rightarrow$ WAITING FOR REVOCATION $\rightarrow$ WAITING FOR INBOUND REVOCATION $\rightarrow$ WAITING FOR HOSTS READY $\rightarrow$ OPEN. During this sequence of events, a new $\texttt{host}_P$ is defined (Fig. 33, l. 6), new commitment transactions are negotiated with $\bar{P}$ (Fig. 33, l. 9), control of the old funding output is handed over to $\texttt{host}_P$ (Fig. 33, l. 11), $\texttt{host}_P$ negotiates with its counterparty a new set of transactions and signatures that spend the aforementioned funding output and make available a new funding output with the keys $pk'_{P,F}, pk'_{\bar{P},F}$ as P instructed (Fig. 56 and 58) and the previous valid commitment transactions of both P and $\bar{P}$ are invalidated (Fig. 25, l. 1 and l. 14 respectively). We note that the use of the $\texttt{ANYPREVOUT}$ flag in all signatures that correspond to transaction inputs that may spend various different transaction outputs ensures that this is possible, as it avoids tying each input to a specific, predefined output. When P receives (FORCECLOSE) by $\mathcal{E}$, it inputs (FORCECLOSE) to $\texttt{host}_P$ (Fig. 42, l. 4). As per the Lemma conditions, $\texttt{host}_P$ will output (CLOSED). This can happen only when $\mathcal{G}_{\text{Ledger}}$ contains a suitable output for both P's and R's channel (Fig. 62, l. 5 and l. 6 respectively).

If the $\texttt{host}$ of $\texttt{host}_P$ is "$\texttt{ledger}$", then the funding output $o_{1,2} = (c_P + c_{\bar{P}}, 2/\{pk_{P,F}, pk_{\bar{P},F}\})$ for the $P, \bar{P}$ channel is already on-chain. Regarding the case in which $\texttt{host}_P \neq$ "$\texttt{ledger}$", after the funding procedure is complete, the new $\texttt{host}_P$ will have as its $\texttt{host}$ the old $\texttt{host}_P$ of P. If the (FORCECLOSE) sequence is initiated, the new $\texttt{host}_P$ will follow the same steps that will be described below once the old $\texttt{host}_P$ succeeds in closing the lower layer (Fig. 61, l. 6). The old $\texttt{host}_P$ however will see no difference in its interface compared to what would happen if P had received (FORCECLOSE) before the funding procedure, therefore it will successfully close by the induction hypothesis. Thereafter the process is identical to the one when the old $\texttt{host}_P =$ "$\texttt{ledger}$".

Moving on, $\texttt{host}_P$ is either able to publish its $\text{TX}_{1,1}$ (it has necessarily received a valid signature $\text{sig}(\text{TX}_{1,1}, pk_{\bar{P},F})$ (Fig. 58, l. 43) by its counterparty before it moved to the OPEN state for the first time), or the output $(c_P + c_{\bar{P}}, 2/\{pk_{P,F}, pk_{\bar{P},F}\})$ needed to spend $\text{TX}_{1,1}$ has already been spent. The only other transactions that can spend it are $\text{TX}_{2,1}$ and any of $(\text{TX}_{2,2,k})_{k>2}$, since these are the only transactions that spend the aforementioned output and that $\texttt{host}_P$ has signed with $sk_{P,F}$ (Fig. 58, ll. 37-41). The output can be also spent by old, revoked commitment transactions, but in that case $\texttt{host}_P$ would not have output (CLOSED); P would have instead detected this triggered by a (CHECK CHAIN FOR CLOSED) message by $\mathcal{E}$ (Fig. 40) and would have moved to the CLOSED state on its own accord (lack of such a message by $\mathcal{E}$ would lead P to become $\texttt{negligent}$, something that cannot happen according to the Lemma conditions). Every transaction among $\text{TX}_{1,1}$, $\text{TX}_{2,1}$, $(\text{TX}_{2,2,k})_{k>2}$ has a $(c_P + c_{\bar{P}} - f, 2/\{pk'_{P,F}, pk'_{\bar{P},F}\})$ output (Fig. 52, l. 19 and Fig. 51, ll. 29 and 97) which will end up in $\mathcal{G}_{\text{Ledger}}$ – call this output o_P. We will prove that at most $\sum_{i=2}^{n-1}(t_i+p+s-1)$ blocks after (forceClose) is received by P, an output o_R with c_{virt} coins and a $2/\{pk_{R,F}, pk_{\bar{R},F}\}$ spending condition without or with an expired timelock will be included in $\mathcal{G}_{\text{Ledger}}$. In case party $\bar{P}$ is idle, then $o_{1,2}$ is consumed by $\text{TX}_{1,1}$ and the timelock on its virtual output expires, therefore the required output o_R is on-chain. In case $\bar{P}$ is active, exactly one of $\text{TX}_{2,1}$, $(\text{TX}_{2,2,k})_{k>2}$ or $(\text{TX}_{2,3,1,k})_{k>2}$ is a descendant of $o_{1,2}$; if the transaction belongs to one of the two last transaction groups then necessarily $\text{TX}_{1,1}$ is on-chain in some block height h and given the timelock on the virtual output of $\text{TX}_{1,1}$, $\bar{P}$'s transaction can be at most at block height $h + t_2 + p + s - 1$. If $n = 3$ or $k = n - 1$, then $\bar{P}$'s unique transaction has the required output o_R (without a timelock). The rest of the cases are covered by the following sequence of events:

Closing sequence

1: `maxDel` $\leftarrow t_2 + p + s - 1$ // A_2 is active and the virtual output of $\text{TX}_{1,1}$ has a timelock of t_2
2: $i \leftarrow 3$
3: **loop**
4: **if** A_i is idle **then**
5: The timelock on the virtual output of the transaction published by A_{i-1} expires and therefore the required o_R is on-chain
6: **else** // A_i publishes a transaction that is a descendant of $o_{1,2}$
7: `maxDel` $\leftarrow$ `maxDel` $+ t_i + p + s - 1$
8: The published transaction can be of the form $\text{TX}_{i,2,2}$ or $(\text{TX}_{i,3,2,k})_{k>i}$ as it spends the virtual output which is encumbered with a public key controlled by R and R has only signed these transactions
9: **if** $i = n - 1$ or $k \geq n - 1$ **then** // The interval contains all intermediaries
10: The virtual output of the transaction is not timelocked and has only a $2/\{pk_{R,F}, pk_{\bar{R},F}\}$ spending method, therefore it is the required o_R
11: **else** // At least one intermediary is not in the interval
12: **if** the transaction is $\text{TX}_{i,3,2,k}$ **then** $i \leftarrow k$ **else** $i \leftarrow i + 1$
13: **end if**
14: **end if**
15: **end loop**
16: // `maxDel` $\leq \sum_{i=2}^{n-1}(t_i + p + s - 1)$

Figure 65

In every case o_P and o_R end up on-chain in at most s and $\sum_{i=2}^{n-1}(t_i + p + s - 1)$ blocks respectively from the moment (forceClose) is received. The output o_P an be spent either by $C_{P,i}$ or $C_{\bar{P},i}$. Both these transactions have a $(c_P - f, pk_{P,\text{out}})$ output. This output of $C_{P,i}$ is timelocked, but the alternative spending method cannot be used as P never signed a transaction that uses it (as it is reserved for revocation, which has not taken place yet in this virtualisation layer). We have now proven that if P completes the funding of a new channel then it can close its channel for a $(c_P - f, pk_{P,\text{out}})$ output that is a descendant of an output with spending method $2/\{pk_{P,F}, pk_{\bar{P},F}\}$ and that lower bound of value holds for the duration of the funding procedure, i.e. we have proven the first claim of the first bullet.

We will now prove that the newly funded party R can close its channel securely. After R receives (funded, $\texttt{host}_P$, . . .) by P and before moving to the open state, it receives $\text{sig}_{\bar{R},C,0} = \text{sig}(C_{R,0}, pk_{\bar{R},F})$ and sends $\text{sig}_{R,C,0} = \text{sig}(C_{\bar{R},0}, pk_{R,F})$. Both these transactions spend o_R. As we showed before, if R receives (forceClose) by $\mathcal{E}$ then o_R eventually ends up on-chain. After receiving (closed) from $\texttt{host}_P$, R attempts to add $C_{R,0}$ to $\mathcal{G}_{\text{Ledger}}$, which may only fail if $C_{\bar{R},0}$ ends up on-chain instead. Similar to the case of P, both these transactions have an $(f, pk_{R,\text{out}})$ output. This output of $C_{R,0}$ is timelocked, but the alternative spending method cannot be used as R never signed a transaction that uses it (as it is reserved for revocation, which has not taken place yet) so the timelock will expire and the desired spending method will be available. We have now proven that if R's channel is funded to completion (i.e. R moves to the open state for the first time) then it can close its channel for a $(f, pk_{R,\text{out}})$ output that is a descendant of o_R. We have therefore proven the first bullet.

We now move on to the second bullet. In case P is the `fundee` (i.e. $i = n$), then the same arguments as in the previous bullet hold here with "waiting for inbound revocation" replaced with "waiting for outbound revocation", $o_{1,2}$ with $o_{n-1,n}$, $\text{TX}_{1,1}$ with $\text{TX}_{n,1}$, $\text{TX}_{2,1}$ with $\text{TX}_{n-1,1}$, $(\text{TX}_{2,2,k})_{k>2}$ with $(\text{TX}_{n-1,2,k})_{k<n-1}$, $(\text{TX}_{2,3,1,k})_{k>2}$ with $(\text{TX}_{n-1,3,n,k})_{k<n-1}$, t_2 with t_{n-1}, $\text{TX}_{i,3,2,k}$ with $\text{TX}_{i,3,n-1,k}$, i is initialized to $n - 2$ in l. 2 of Fig. 65, i is decremented instead of incremented in l. 12 of the same Figure and f is replaced with 0. This is so because these two cases are symmetric.

In case P is not the `fundee` $(1 < i < n)$, then we only need to prove the first statement of the second bullet. By the induction hypothesis and since `sibling` is kindred, we know that both P's and `sibling`'s funding outputs either are or can be eventually put on-chain and that P's funding output has at least $c_P = \sum_{s \in C} \sum_{x \in s} x$ coins. If P is on the "left" of its `sibling` (i.e. there is an untrusted party that sent the (virtualising, . . .) message to P which triggered the latter to move to the virtualising state and to send a (virtualising, . . .) message to its own `sibling`), the "left" funding output o_{left} (the one held with the untrusted party to the left) can be spent by one of $\text{TX}_{i,1}$, $(\text{TX}_{i,2,k})_{k>i}$, $\text{TX}_{i-1,1}$, or $(\text{TX}_{i-1,2,k})_{k<i-1}$, as these are the only transactions that P has signed with $sk_{P,F}$. All these transactions have a $(c_P + c_{\bar{P}} - f, 2/\{pk_{P,F'}, pk_{\bar{P},F'}\})$ output that can in turn be spent by either $C_{P,0}$ or $C_{\bar{P},0}$, both of which have an output of value c_P and a $pk_{P,\text{out}}$ spending method and no other spending method can be used (as P has not signed the "revocation" spending method of $C_{P,0}$).

In the case that P is to the right of its `sibling` (i.e. P receives by `sibling` the (virtualising, . . .) message that causes P's transition to the virtualising state), the "right" funding output o_{right} (the one held with the untrusted party to the right) can be spent by one of $\text{TX}_{i,1}$, $(\text{TX}_{i,2,k})_{k<i}$, $\text{TX}_{i+1,1}$, or $(\text{TX}_{i+1,2,k})_{k>i+1}$, as these are the only transactions that P has signed with $sk_{P,F}$. All these transactions have a $(c_P + c_{\bar{P}} - f, 2/\{pk_{P,F'}, pk_{\bar{P},F'}\})$ output that can in turn be spent by either $C_{P,0}$ or $C_{\bar{P},0}$, both of which have an output of value $c_P - f$ and a $pk_{P,\text{out}}$ spending method and no other spending method can be used (as P has not signed the "revocation" spending method of $C_{P,0}$). P can get the remaining f coins as follows: $\mathrm{TX}_{i,1}$ and all of $(\mathrm{TX}_{i,2,k})_{k<i}$ already have an $(f, pk_{P,\mathrm{out}})$ output (Note that this output is also encumbered with a timelock, but the alternative spending method cannot be used as $\mathtt{host}_P$ has not signed the required revocation transaction). If instead $\mathrm{TX}_{i+1,1}$ or one of $(\mathrm{TX}_{i+1,2,k_2})_{k_2>i+1}$ spends o_{right}, then P will publish $\mathrm{TX}_{i,2,i+1}$ or $\mathrm{TX}_{i,2,k_2}$ respectively if o_{left} is unspent, otherwise o_{left} is spent by one of $\mathrm{TX}_{i-1,1}$ or $(\mathrm{TX}_{i-1,2,k_1})_{k_1<i-1}$ in which case P will publish one of $\mathrm{TX}_{i,3,k_1,i+1}$, $\mathrm{TX}_{i,3,i-1,k_2}$, $\mathrm{TX}_{i,3,i-1,i+1}$ or $\mathrm{TX}_{i,3,k_1,k_2}$. In particular, $\mathrm{TX}_{i,3,k_1,i+1}$ is published if $\mathrm{TX}_{i-1,2,k_1}$ and $\mathrm{TX}_{i+1,1}$ are on-chain, $\mathrm{TX}_{i,3,i-1,k_2}$ is published if $\mathrm{TX}_{i-1,1}$ and $\mathrm{TX}_{i+1,2,k_2}$ are on-chain, $\mathrm{TX}_{i,3,i-1,i+1}$ is published if $\mathrm{TX}_{i-1,1}$ and $\mathrm{TX}_{i+1,1}$ are on-chain, or $\mathrm{TX}_{i,3,k_1,k_2}$ is published if $\mathrm{TX}_{i-1,2,k_1}$ and $\mathrm{TX}_{i+1,2,k_2}$ are on-chain. All these transactions include an $(f, pk_{P,\mathrm{out}})$ output for which the revocation-based spending methd cannot be used since $\mathtt{host}_P$ has not produced the corresponding signature for the revocation transaction. We have therefore covered all cases and proven the second bullet.

We now focus on the third bullet. Once more the induction hypothesis guarantees that before (CHECK COOP CLOSE, ...) was received, P could close the channel resulting in on-chain outputs exclusively spendable or already spent by $pk_{P,\mathrm{out}}$ that are descendants of an output o_F with a $2/\{pk_{P,F}, pk_{\bar{P},F}\}$ spending method that have a sum value of $c_P = \sum\limits_{s \in C} \sum\limits_{x \in s} x$. When P receives (CHECK COOP CLOSE, ...), it moves to the COOP CLOSING state before returning to the OPEN state. It verifies the counterparty's signature on the new commitment transaction $C_{P,i+1}$, (Fig. 63, l. 16) which spends the latest old funding output (Fig. 63, l. 14), effectively removing one virtualisation layer. In $C_{P,i+1}$ P owns c'_2 more coins than before that moment (Fig. 63, l. 15). It then signs the corresponding commitment transaction for the counterparty (Fig. 63, l. 68) and expects a valid signature for the revocation transaction of the old commitment transaction of the counterparty (Fig. 63, l. 95). Once these are received, P transitions to the OPEN state. If the o_F output is spent while P is in the COOP CLOSING state, it can be spent by one of $C_{P,i+1}$ or some of $(C_{\bar{P},j})_{0 \le j \le i+1}$. If it is spent by $C_{P,i+1}$ or $C_{\bar{P},i+1}$, then these two transactions have a $(c_P + c'_2, pk_{P,\mathrm{out}})$ output. (Note that the former is encumbered with a timelock, but the alternative spending method cannot be used as P has not signed $R_{\bar{P},i+1}$.) If it is spent by $C_{\bar{P},i}$ then a $(c_P, pk_{P,\mathrm{out}})$ output becomes available instead, therefore P can still get the c_P coins that correspond to the previous state. If any of $(C_{\bar{P},j})_{0 \le j < i}$ spends this or another of our past funding outputs then it makes available a $pk_{P,\mathrm{out}}$ output with the coins that P had at state j and additionally P can publish $R_{P,j}$ that spends $\bar{P}$'s output of $C_{\bar{P},j}$ and obtain the entirety of $\bar{P}$'s coins at state j for a total of $c_P + c_{\bar{P}}$ coins. What is more, if o_F is spent by any virtual transaction, then $\mathtt{host}_P$ will punish the publisher of such transaction with the corresponding virtual revocation transaction (Fig. 63, l. 35, l. 38, l. 54, l. 87, l. 88 and l. 91) at the latest when P receives (CHECK CHAIN FOR CLOSED) (Fig. 40, l. 17) – note that the latter message is received periodically by P, since it is a non-negligent party. The virtual revocation transaction gives a sum equal to the entirety of the channel's funds to P. Therefore in every case P can claim at least c_P coins. In the case that P instead subsequently receives a valid signature to $R_{P,i}$ (Fig. 63, l. 95) and moves to the OPEN state, the above analysis of what can happen when o_F is spent holds similarly, with the difference that if $\bar{P}$ spends o_F with $C_{\bar{P},i}$ now P can publish $R_{P,i}$ which gives P the coins of $\bar{P}$. Therefore with this difference P is now guaranteed to gain at least $c_P + c'_2$ coins upon channel closure. We have therefore proven the third bullet.

We now focus on the fourth bullet. Once more the induction hypothesis guarantees that before (COOP CLOSE SIGN COMM, ...) was received, P could close the channel resulting in on-chain outputs exclusively spendable or already spent by $pk_{P,\mathrm{out}}$ that are descendants of an output o_F with a $2/\{pk_{P,F}, pk_{\bar{P},F}\}$ spending method that have a sum value of $c_P = \sum\limits_{s \in C} \sum\limits_{x \in s} x$. When P receives (COOP CLOSE SIGN COMM, ...), it moves to the COOP CLOSING state before returning to the OPEN state. It signs the new commitment transaction for the counterparty (Fig. 63, l. 27) which spends the latest old funding output (Fig. 63, l. 25), effectively removing one virtualisation layer. In $C_{P,i+1}$ P owns $c_{\mathrm{virt}} - c'_1$ less coins than before that moment (Fig. 63, l. 26) – note that P now lost access to c_{virt} coins from the refund output of its virtual transactions. It then verifies the counterparty's signatures on the corresponding new local commitment transaction $C_{P,i+1}$, (Fig. 63, l. 46) and on the revocation transaction of the old commitment transaction of the counterparty (Fig. 63, l. 49). Once these are received, P transitions to the OPEN state. If the o_F output is spent while P is in the COOP CLOSING state, it can be spent by one of $C_{P,i+1}$ or some of $(C_{\bar{P},j})_{0 \le j \le i+1}$. If it is spent by $C_{P,i+1}$ or $C_{\bar{P},i+1}$, then these two transactions have a $(c_P + c'_1, pk_{P,\mathrm{out}})$ output. (Note that the former is encumbered with a timelock, but the alternative spending method cannot be used as P has not signed $R_{\bar{P},i+1}$.) If it is spent by $C_{\bar{P},i}$ then a $(c_P, pk_{P,\mathrm{out}})$ output becomes available instead, therefore P can still get the c_P coins that correspond to the previous state. If any of $(C_{\bar{P},j})_{0 \le j < i}$ spends this or another of our past funding outputs then it makes available a $pk_{P,\mathrm{out}}$ output with the coins that P had at state j and additionally P can publish $R_{P,j}$ that spends $\bar{P}$'s output of $C_{\bar{P},j}$ and obtain the entirety of $\bar{P}$'s coins at state j for a total of $c_P + c_{\bar{P}}$ coins. Similarly to the previous bullet, if o_F is spent by any virtual transaction, then $\mathtt{host}_P$ will punish the publisher and P will obtain a sum equal to the entirety of the channel's funds. Therefore in every case P can claim at least c_P coins. In the case that P instead subsequently receives a valid signature to $R_{P,i}$ (Fig. 63, l. 95) and moves to the OPEN state, the above analysis of what can happen when o_F holds similarly, with the difference that if $\bar{P}$ spends o_F with $C_{\bar{P},i}$ now P can publish $R_{P,i}$ which gives P the coins of $\bar{P}$. Therefore with this difference P is now guaranteed to gain at least $c_P - c_{\mathrm{virt}} + c'_1$ coins upon channel closure. This proves the first claim of the fourth bullet.

Regarding the second claim, we observe that P can only move to the OPEN state if previously a local kindred LN ITI R moves to the OPEN state as well. Via direct application of the previous claim of the currently analysed bullet, R has gained c'_2 coins in the process, therefore guaranteeing that P and R have on aggregate access to the same number of coins as before the cooperative closing. What is more, throughout the cooperative closing process both parties had access to at least c_P and c_R coins respectively, thus ensuring that no loss of coins is possible. We have now proven the fourth bullet.

Moving on to the fifth bullet, the same reasoning as that of the treatment of the previous bullet holds, albeit with the `guest`'s signature verifications as they appear in Fig. 47.

The first claim of the sixth bullet holds due to an argument identical to that provided for the third bullet, since in both cases the relevant parts of the protocol execution are the same. Note that `funder`'s signature for the revocation of the lastt commitment transaction of the virtual channel has not been yet verified, but this is of no consequence for our balance as all other revocation signatures have been already verified and the connection with the `funder` has been severed due to the successful cooperative closing.

Regarding now the seventh bullet, once again the induction hypothesis guarantees that before (PAY, d) was received, P could close the channel resulting in on-chain outputs exclusively spendable or already spent by $pk_{P,\text{out}}$ that are descendants of an output with a $2/\{pk_{P,F}, pk_{\bar{P},F}\}$ spending method that have a sum value of $c_P = \sum\limits_{s \in C} \sum\limits_{x \in s} x$. (Note that $\sum\limits_{s \in C'} \sum\limits_{x \in s} x = d + \sum\limits_{s \in C} \sum\limits_{x \in s} x$.) When P receives (PAY, d) while in the OPEN state, it moves to the WAITING FOR COMMITMENT SIGNED state before returning to the OPEN state. It signs (Fig. 35, l. 2) the new commitment transaction $C_{\bar{P},i+1}$ in which the counterparty owns d more coins than before that moment (Fig. 35, l. 1), sends the signature to the counterparty (Fig. 35, l. 5) and expects valid signatures on its own updated commitment transaction (Fig. 36, l. 1) and the revocation transaction for the old commitment transaction of the counterparty (Fig. 36, l. 3). Upon verifying them, P transitions to the OPEN state. Note that if the counterparty does not respond or if it responds with missing/invalid signatures, either P can close the channel with the old commitment transaction $C_{P,i}$ exactly like before the update started (as it has not yet sent the signature for the old revocation transaction), or the counterparty will close the channel either with the new or with the old commitment transaction. In all cases in which validation fails and the channel closes, there is an output with a $pk_{P,\text{out}}$ spending method and no other useable spending method that carries at least $c_P - d$ coins. Only if the verification succeeds does P sign (Fig. 36, l. 5) and send (Fig. 36, l. 17) the counterparty's revocation transaction for P's previous commitment transaction.

Similarly to previous bullets, if $\texttt{host}_P \neq$ "`ledger`" the funding output can be put on-chain, otherwise the funding output is already on-chain. In both cases, since the closing procedure continues, one of $C_{P,i+1}$, $(C_{\bar{P},j})_{0 \leq j \leq i+1}$ will end up on-chain. If $C_{\bar{P},j}$ for some $j < i + 1$ is on-chain, then P submits $R_{P,j}$ (we discussed how P obtained $R_{P,i}$ and the rest of the cases are covered by induction) and takes the entire value of the channel which is at least $c_P - d$. If $C_{\bar{P},i+1}$ is on-chain, it has a $(c_P - d, pk_{P,\text{out}})$ output. If $C_{P,i+1}$ is on-chain, it has an output of value $c_P - d$, a timelocked $pk_{P,\text{out}}$ spending method and a non-timelocked spending method that needs the signature made with $sk_{P,R}$ on $R_{\bar{P},i+1}$. P however has not generated that signature, therefore this spending method cannot be used and the timelock will expire, therefore in all cases outputs that descend from the funding output, can be spent exclusively by $pk_{P,\text{out}}$ and carry at least $c_P - d$ coins are put on-chain. We have proven the seventh bullet.

For the eighth and last bullet, again by the induction hypothesis, before (GET PAID, e) was received P could close the channel resulting in on-chain outputs exclusively spendable or already spent by $pk_{P,\text{out}}$ that are descendants of an output o_F with a $2/\{pk_{P,F}, pk_{\bar{P},F}\}$ spending method and have a sum value of $c_P = \sum\limits_{s \in C} \sum\limits_{x \in s} x$. (Note that $e + \sum\limits_{s \in C'} \sum\limits_{x \in s} x = \sum\limits_{s \in C} \sum\limits_{x \in s} x$ and that o_F either is already on-chain or can be eventually put on-chain as we have argued in the previous bullets by the induction hypothesis.) When P receives (GET PAID, e) while in the OPEN state, if the balance of the counterparty is enough it moves to the WAITING TO GET PAID state (Fig. 38, l. 6). If subsequently it receives a valid signature for $C_{P,i+1}$ (Fig. 35, l. 9) which is a commitment transaction that can spend the o_F output and gives to P an additional e coins compared to $C_{P,i}$. Subsequently P's state transitions to WAITING FOR PAY REVOCATION and sends signatures for $C_{\bar{P},i+1}$ and $R_{\bar{P},i}$ to $\bar{P}$. If the o_F output is spent while P is in the latter state, it can be spent by one of $C_{P,i+1}$ or $(C_{\bar{P},j})_{0 \leq j \leq i+1}$. If it is spent by $C_{P,i+1}$ or $C_{\bar{P},i+1}$, then these two transactions have a $(c_P + e, pk_{P,\text{out}})$ output. (Note that the former is encumbered with a timelock, but the alternative spending method cannot be used as P has not signed $R_{\bar{P},i+1}$.) If it is spent by $C_{\bar{P},i}$ then a $(c_P, pk_{P,\text{out}})$ output becomes available instead, therefore P can still get the c_P coins that correspond to the previous state. If any of $(C_{\bar{P},j})_{0 \leq j < i}$ spends o_F then it makes available a $pk_{P,\text{out}}$ output with the coins that P had at state j and additionally P can publish $R_{P,j}$ that spends $\bar{P}$'s output of $C_{\bar{P},j}$ and obtain the entirety of $\bar{P}$'s coins at state j for a total of $c_P + c_{\bar{P}}$ coins. Therefore in every case P can claim at least c_P coins. In the case that P instead subsequently receives a valid signature to $R_{P,i}$ (Fig. 36, l. 20) it finally moves to the OPEN state once again. In this state the above analysis of what can happen when o_F holds similarly, with the difference that if $\bar{P}$ spends o_F with $C_{\bar{P},i}$ now P can publish $R_{P,i}$ which gives P the coins of $\bar{P}$. Therefore with this difference P is now guaranteed to gain at least $c_P + e$ coins upon channel closure. We have therefore proven the eighth bullet and with it the first bullet of the Lemma.

We now turn to proving the second bullet of the Lemma. We will take advantage of the results that have been derived earlier in this proof. If P is the `funder` of the virtual channel in process of cooperatively closing, it ensures that $c'_1 = c_P \wedge c'_2 = c_{\bar{P}}$ (Fig. 47, l. 4). If P is the `fundee`, it requests that the virtual channel be closed with the current honest coin balance (Fig. 46, l. 6), in which case it is $c'_1 = c_{\bar{P}} \wedge c'_2 = c_P$. Due to the arguments proving the first Lemma bullet, we know that

$$c_P = \sum_{s \in C} \sum_{x \in s} x \geq \phi \cdot c - \sum_{i=1}^{n} f_i - \sum_{i=1}^{m} d_i + \sum_{i=1}^{l} e_i + \sum_{i=1}^{j} r_i + \sum_{i=1}^{k} l_i \quad . \quad (5)$$

Just before the splitting of the two alternative scenarios, party S is entitled to c_b coins, since (i) in the first scenario all other parties honestly follow the protocol and thus they do not lose any coins to S and (ii) no action during the first scenario causes any transfer of coins. As we saw previously, if P transitions to the COOP CLOSED state, then S has also transitioned from the COOP CLOSING to the OPEN state and benefitted from an increase of the coins it can exclusively spend by c_P. It therefore holds that the difference of the coins $c_t - c_b$ that P owns at the end of the two scenarios is exactly c_P and due to (5) we can directly derive the required (4). The Lemma has now been proven. □

LEMMA G.2 (IDEAL WORLD BALANCE). *Consider an ideal world execution with functionality $\mathcal{G}_{\text{Chan}}$ and simulator $\mathcal{S}$. Let $P \in \{Alice, Bob\}$*

one of the two parties of $\mathcal{G}_{\text{Chan}}$. Assume that all of the following are true:

- *$State_P \neq$ IGNORED,*
- *P has transitioned to the OPEN State at least once. Additionally, if $P = Alice$, it has received $(\text{OPEN}, c, \dots)$ by $\mathcal{E}$ prior to transitioning to the OPEN State,*
- *P [has received $(\text{FUND ME}, f_i, \dots)$ as input by another $\mathcal{G}_{\text{Chan}}$/LN ITI while $State_P$ was OPEN and P subsequently transitioned to OPEN State] n times,*
- *$\mathcal{G}_{\text{Ledger}}$ [has received (COOP CLOSING, P, r_i) by $\mathcal{S}$ while $State_P$ was OPEN and subsequently P transitioned to OPEN State] k times,*
- *P [has received (PAY, d_i) by $\mathcal{E}$ while $State_P$ was OPEN and P subsequently transitioned to OPEN State] m times,*
- *P [has received $(\text{GET PAID}, e_i)$ by $\mathcal{E}$ while $State_P$ was OPEN and P subsequently transitioned to OPEN State] l times.*

Let $\phi = 1$ if $P = Alice$, or $\phi = 0$ if $P = Bob$. If $\mathcal{G}_{\text{Chan}}$ receives $(\text{FORCECLOSE}, P)$ by $\mathcal{S}$, then the following holds with overwhelming probability on the security parameter:

$$\texttt{balance}_P = \phi \cdot c - \sum_{i=1}^{n} f_i - \sum_{i=1}^{m} d_i + \sum_{i=1}^{l} e_i + \sum_{i=1}^{k} r_i \qquad (6)$$

PROOF OF LEMMA G.2. We will prove the Lemma by following the evolution of the $\texttt{balance}_P$ variable.

- When $\mathcal{G}_{\text{Chan}}$ is activated for the first time, it sets $\texttt{balance}_P \leftarrow 0$ (Fig. 9, l. 1).
- If $P = Alice$ and it receives $(\text{OPEN}, c, \dots)$ by $\mathcal{E}$, it stores c (Fig. 9, l. 11). If later $State_P$ becomes OPEN, $\mathcal{G}_{\text{Chan}}$ sets $\texttt{balance}_P \leftarrow c$ (Fig. 9, ll. 14 or 34). In contrast, if $P = Bob$, it is $\texttt{balance}_P = 0$ until at least the first transition of $State_P$ to OPEN (Fig. 9).
- Every time that P receives input $(\text{FUND ME}, f_i, \dots)$ by another party while $State_P =$ OPEN, P stores f_i (Fig. 11, l. 1). The next time $State_P$ transitions to OPEN (if such a transition happens), $\texttt{balance}_P$ is decremented by f_i (Fig. 11, l. 27). Therefore, if this cycle happens $n \geq 0$ times, $\texttt{balance}_P$ will be decremented by $\sum_{i=1}^{n} f_i$ in total.
- Every time $\mathcal{G}_{\text{Ledger}}$ receives (COOP CLOSING, P, r_i) by $\mathcal{S}$ while $State_P$ is OPEN, r_i is stored (Fig. 13, l. 1). The next time $State_P$ transitions to OPEN (if such a transition happens), $\texttt{balance}_P$ is incremented by r_i (Fig. 13, l. 9). Therefore, if this cycle happens $k \geq 0$ times, $\texttt{balance}_P$ will be incremented by $\sum_{i=1}^{k} r_i$ in total.
- Every time P receives input (PAY, d_i) by $\mathcal{E}$ while $State_P =$ OPEN, d_i is stored (Fig. 10, l. 2). The next time $State_P$ transitions to OPEN (if such a transition happens), $\texttt{balance}_P$ is decremented by d_i (Fig. 10, l. 13). Therefore, if this cycle happens $m \geq 0$ times, $\texttt{balance}_P$ will be decremented by $\sum_{i=1}^{m} d_i$ in total.
- Every time P receives input $(\text{GET PAID}, e_i)$ by $\mathcal{E}$ while $State_P =$ OPEN, e_i is stored (Fig. 10, l. 7). The next time $State_P$ transitions to OPEN (if such a transition happens) $\texttt{balance}_P$ is incremented by e_i (Fig. 10, l. 19). Therefore, if this cycle happens $l \geq 0$ times, $\texttt{balance}_P$ will be incremented by $\sum_{i=1}^{l} e_i$ in total.

On aggregate, after the above are completed and then $\mathcal{G}_{\text{Chan}}$ receives $(\text{FORCECLOSE}, P)$ by $\mathcal{S}$, it is $\texttt{balance}_P = c - \sum_{i=1}^{n} f_i - \sum_{i=1}^{m} d_i + \sum_{i=1}^{l} e_i + \sum_{i=1}^{k} r_i$ if $P = Alice$, or else if $P = Bob$, $\texttt{balance}_P = - \sum_{i=1}^{n} f_i - \sum_{i=1}^{m} d_i + \sum_{i=1}^{l} e_i + \sum_{i=1}^{k} r_i$. □

PROOF OF LEMMA 5.3. We prove the Lemma in two steps. We first show that if the conditions of Lemma G.2 hold, then the conditions of Lemma G.1 for the real world execution with protocol LN and the same $\mathcal{E}$ and $\mathcal{A}$ hold as well for the same k, m, n and l values.

For $State_P$ to become IGNORED, either $\mathcal{S}$ has to send (BECAME CORRUPTED OR NEGLIGENT, P) or $\texttt{host}_P$ must output (ENABLER USED REVOCATION) to $\mathcal{G}_{\text{Chan}}$ (Fig. 9, l. 5). The first case only happens when either P receives (CORRUPT) by $\mathcal{A}$ (Fig. 23, l. 1), which means that the simulated P is not honest anymore, or when P becomes $\texttt{negligent}$ (Fig. 23, l. 4), which means that the first condition of Lemma G.1 is violated. In the second case, it is $\texttt{host}_P \neq \mathcal{G}_{\text{Ledger}}$ and the state of $\texttt{host}_P$ is GUEST PUNISHED (Fig. 64, ll. 1 or 12), so in case P receives (FORCECLOSE) by $\mathcal{E}$ the output of $\texttt{host}_P$ will be (GUEST PUNISHED) (Fig. 61, l. 4). In all cases, some condition of Lemma G.1 is violated.

For $State_P$ to become OPEN at least once, the following sequence of events must take place (Fig. 9): If $P = Alice$, it must receive (INIT, pk) by $\mathcal{E}$ when $State_P =$ UNINIT, then either receive (OPEN, c, $\mathcal{G}_{\text{Ledger}}, \dots$) by $\mathcal{E}$ and (BASE OPEN) by $\mathcal{S}$ or (OPEN, c, $\texttt{hops}$ ($\neq \mathcal{G}_{\text{Ledger}}$), ...) by $\mathcal{E}$, (FUNDED, HOST, ...) by $\texttt{hops[0].left}$ and (VIRTUAL OPEN) by $\mathcal{S}$. In either case, $\mathcal{S}$ only sends its message only if all its simulated honest parties move to the OPEN state (Fig. 23, l. 10), therefore if the second condition of Lemma G.2 holds and $P = Alice$, then the second condition of Lemma G.1 holds as well. The same line of reasoning can be used to deduce that if $P = Bob$, then $State_P$ will become OPEN for the first time only if all honest simulated parties move to the OPEN *state*, therefore once more the second condition of Lemma G.2 holds only if the second condition of Lemma G.1 holds as well. We also observe that, if both parties are honest, they will transition to the OPEN state simultaneously.

Regarding the third Lemma G.2 condition, we assume (and will later show) that if both parties are honest and the state of one is OPEN, then the state of the other is also OPEN. Each time P receives input $(\text{FUND ME}, f, \dots)$ by $R \in \{\mathcal{G}_{\text{Chan}}, \text{LN}\}$, $State_P$ transitions to PENDING FUND, subsequently when a command to define a new VIRT ITI through P is intercepted by $\mathcal{G}_{\text{Chan}}$, $State_P$ transitions to TENTATIVE FUND and afterwards when $\mathcal{S}$ sends (FUND) to $\mathcal{G}_{\text{Chan}}$, $State_P$ transitions to SYNC FUND. In parallel, if $State_{\bar{P}} =$ IGNORED, then $State_P$ transitions directly back to OPEN. If on the other hand $State_{\bar{P}} =$ OPEN and $\mathcal{G}_{\text{Chan}}$ intercepts a similar VIRT ITI definition command through $\bar{P}$, $State_{\bar{P}}$ transitions to TENTATIVE HELP FUND. On receiving the aforementioned (FUND) message by $\mathcal{S}$ and given that $State_{\bar{P}} =$ TENTATIVE HELP FUND, $\mathcal{G}_{\text{Chan}}$ also sets $State_{\bar{P}}$ to SYNC HELP FUND. Then both $State_{\bar{P}}$ and $State_P$ transition simultaneously to OPEN (Fig. 11). This sequence of events may repeat any $n \geq 0$ times. We observe that throughout these steps, honest simulated P

has received (FUND ME, $f, \ldots$) and that $\mathcal{S}$ only sends (FUND) when all honest simulated parties have transitioned to the OPEN state (Fig. 23, l. 18 and Fig. 33, l. 12), so the third condition of Lemma G.1 holds with the same n as that of Lemma G.2.

Moving on to the fourth Lemma G.2 condition, we again assume that if both parties are honest and the state of one is OPEN, then the state of the other is also OPEN. Each time $\mathcal{G}_{\text{Chan}}$ receives (COOP CLOSING, P, r) by $\mathcal{S}$, $State_P$ transitions to COOP CLOSING and subsequently when $\mathcal{S}$ sends (COOP CLOSED, P) to $\mathcal{G}_{\text{Chan}}$, if $\texttt{layer}_P = 0$ then $State_P$ transitions to COOP CLOSED, else $State_P$ transitions to OPEN. This sequence of events may repeat any $k \geq 0$ times. We observe that throughout these steps, honest simulated P has transitioned to the COOP CLOSING state and that $\mathcal{S}$ only sends (COOP CLOSED, P) when honest simulated P transitions to either OPEN or COOP CLOSED state, so the sum of j (from the fourth condition of Lemma G.1) plus k (from the fifth condition of Lemma G.1) is equal to the k of Lemma G.2.

Regarding the sixth Lemma G.2 condition, we again assume that if both parties are honest and the state of one is OPEN, then the state of the other is also OPEN. Each time P receives input (PAY, d) by $\mathcal{E}$, $State_P$ tranisitions to TENTATIVE PAY and subsequently when $\mathcal{S}$ sends (PAY) to $\mathcal{G}_{\text{Chan}}$, $State_P$ transitions to (SYNC PAY, d). In parallel, if $State_{\bar{P}}$ = IGNORED, then $State_P$ transitions directly back to OPEN. If on the other hand $State_{\bar{P}}$ = OPEN and $\mathcal{G}_{\text{Chan}}$ receives (GET PAID, d) by $\mathcal{E}$ addressed to $\bar{P}$, $State_{\bar{P}}$ transitions to TENTATIVE GET PAID. On receiving the aforementioned (PAY) message by $\mathcal{S}$ and given that $State_{\bar{P}}$ = TENTATIVE GET PAID, $\mathcal{G}_{\text{Chan}}$ also sets $State_{\bar{P}}$ to SYNC GET PAID. Then both $State_P$ and $State_{\bar{P}}$ transition simultaneously to OPEN (Fig. 10). This sequence of events may repeat any $m \geq 0$ times. We observe that throughout these steps, honest simulated P has received (PAY, d) and that $\mathcal{S}$ only sends (PAY) when all honest simulated parties have completed sending or receiving the payment (Fig. 23, l. 16), so the sixth condition of Lemma G.1 holds with the same m as that of Lemma G.2. As far as the seventh condition of Lemma G.2 goes, we observe that this case is symmetric to the one discussed for its sixth condition above if we swap P and $\bar{P}$, therefore we deduce that if Lemma G.2 holds with some l, then Lemma G.1 holds with the same l.

As promised, we here argue that if both parties are honest and one party moves to the OPEN state, then the other party will move to the OPEN state as well. We already saw that the first time one party moves to the OPEN state, it will happen simultaneously with the same transition for the other party. We also saw that, when a party transitions from the SYNC HELP FUND or the SYNC FUND state to the OPEN state, then the other party will also transition to the OPEN state simultaneously. Additionally, we saw that if one party transitions from the COOP CLOSING state to the OPEN state, the other party will also transition to the OPEN state simultaneously. Furthermore, we saw that if one party transitions from the SYNC PAY or the SYNC GET PAID state to the OPEN state, the other party will also transition to the OPEN state simultaneously. Lastly we notice that we have exhausted all manners in which a party can transition to the OPEN state, therefore we have proven that transitions of honest parties to the OPEN state happen simultaneously.

Now, given that $\mathcal{S}$ internally simulates faithfully both LN parties and that $\mathcal{G}_{\text{Chan}}$ relinquishes to $\mathcal{S}$ complete control of the external communication of the parties as long as it does not halt, we deduce that $\mathcal{S}$ replicates the behaviour of the aforementioned real world. By combining these facts with the consequences of the two Lemmas and the check that leads $\mathcal{G}_{\text{Chan}}$ to halt if it fails (Fig. 12, l. 18), we deduce that if the conditions of Lemma G.2 hold for the honest parties of $\mathcal{G}_{\text{Chan}}$ and their kindred parties, then the functionality halts only with negligible probability.

In the second proof step, we show that if the conditions of Lemma G.2 do not hold, then the check of Fig. 12, l. 18 never takes place. We first discuss the $State_P$ = IGNORED case. We observe that the IGNORED $State$ is a sink state, as there is no way to leave it once in. Additionally, for the balance check to happen, $\mathcal{G}_{\text{Chan}}$ must receive (CLOSED, P) by $\mathcal{S}$ when $State_P \neq$ IGNORED (Fig. 12, l. 9). We deduce that, once $State_P$ = IGNORED, the balance check will not happen. Moving to the case where $State_P$ has never been OPEN, we observe that it is impossible to move to any of the states required by l. 9 of Fig. 12 without first having been in the OPEN state. Moreover if $P = Alice$, it is impossible to reach the OPEN state without receiving input (OPEN, $c, \ldots$) by $\mathcal{E}$. Lastly, as we have observed already, the three last conditions of Lemma G.2 are always satisfied. We conclude that if the conditions to Lemma G.2 do not hold, then the check of Fig. 12, l. 18 does not happen and therefore $\mathcal{G}_{\text{Chan}}$ does not halt.

On aggregate, $\mathcal{G}_{\text{Chan}}$ may only halt with negligible probability in the security parameter. ◻

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REFERENCES

[1] Nakamoto S.: Bitcoin: A Peer-to-Peer Electronic Cash System (2008)

[2] Croman K., Decker C., Eyal I., Gencer A. E., Juels A., Kosba A., Miller A., Saxena P., Shi E., Sirer E. G., et al.: On scaling decentralized blockchains. In International Conference on Financial Cryptography and Data Security: pp. 106–125: Springer (2016)

[3] Gudgeon L., Moreno-Sanchez P., Roos S., McCorry P., Gervais A.: SoK: Layer-Two Blockchain Protocols. In Financial Cryptography and Data Security - 24th International Conference, FC 2020, Kota Kinabalu, Malaysia, February 10-14, 2020 Revised Selected Papers: pp. 201–226: doi:10.1007/978-3-030-51280-4_12: URL https://doi.org/10.1007/978-3-030-51280-4_12 (2020)

[4] Decker C., Wattenhofer R.: A fast and scalable payment network with bitcoin duplex micropayment channels. In Symposium on Self-Stabilizing Systems: pp. 3–18: Springer (2015)

[5] Poon J., Dryja T.: The Bitcoin Lightning Network: Scalable Off-Chain Instant Payments. https://lightning.network/lightning-network-paper.pdf (2016)

[6] Dziembowski S., Eckey L., Faust S., Malinowski D.: Perun: Virtual Payment Hubs over Cryptocurrencies. In 2019 2019 IEEE Symposium on Security and Privacy (SP): pp. 344–361: IEEE Computer Society, Los Alamitos, CA, USA: ISSN 2375–1207: doi:10.1109/SP.2019.00020: URL https://doi.ieeecomputersociety.org/10.1109/SP.2019.00020 (2019)

[7] Dziembowski S., Faust S., Hostáková K.: General State Channel Networks. In Proceedings of the 2018 ACM SIGSAC Conference on Computer and Communications Security, CCS 2018, Toronto, ON, Canada, October 15-19, 2018: pp. 949–966: doi:10.1145/3243734.3243856: URL https://doi.org/10.1145/3243734.3243856 (2018)

[8] Aumayr L., Ersoy O., Erwig A., Faust S., Hostáková K., Maffei M., Moreno-Sanchez P., Riahi S.: Bitcoin-Compatible Virtual Channels. In IEEE Symposium on Security and Privacy, Oakland, USA; 2021-05-23 - 2021-05-27: https://eprint.iacr.org/2020/554.pdf (2021)

[9] Aumayr L., Ersoy O., Erwig A., Faust S., Hostakova K., Maffei M., Moreno-Sanchez P., Riahi S.: Generalized Bitcoin-Compatible Channels. Cryptology ePrint Archive, Report 2020/476: https://eprint.iacr.org/2020/476 (2020)

[10] Jourenko M., Larangeira M., Tanaka K.: Lightweight Virtual Payment Channels. In S. Krenn, H. Shulman, S. Vaudenay (editors), Cryptology and Network Security: pp. 365–384: Springer International Publishing, Cham: ISBN 978-3-030-65411-5 (2020)

[11] Wood G.: Ethereum: A secure decentralised generalised transaction ledger

[12] Canetti R.: Universally Composable Security: A New Paradigm for Cryptographic Protocols. In 42nd Annual Symposium on Foundations of Computer Science, FOCS 2001, 14-17 October 2001, Las Vegas, Nevada, USA: pp. 136–145: doi:10.1109/SFCS.2001.959888: URL https://eprint.iacr.org/2000/067.pdf (2001)
[13] Badertscher C., Canetti R., Hesse J., Tackmann B., Zikas V.: Universal Composition with Global Subroutines: Capturing Global Setup Within Plain UC. In Theory of Cryptography - 18th International Conference, TCC 2020, Durham, NC, USA, November 16-19, 2020, Proceedings, Part III: pp. 1–30: doi:10.1007/978-3-030-64381-2_1: URL https://doi.org/10.1007/978-3-030-64381-2_1 (2020)
[14] Spilman J.: Anti dos for tx replacement. https://lists.linuxfoundation.org/pipermail/bitcoin-dev/2013-April/002433.html (2013)
[15] Raiden Network. https://raiden.network/
[16] Malavolta G., Moreno-Sanchez P., Schneidewind C., Kate A., Maffei M.: Anonymous Multi-Hop Locks for Blockchain Scalability and Interoperability. In 26th Annual Network and Distributed System Security Symposium, NDSS 2019, San Diego, California, USA, February 24-27, 2019 (2019)
[17] Harris J., Zohar A.: Flood & Loot: A Systemic Attack on The Lightning Network. In Proceedings of the 2nd ACM Conference on Advances in Financial Technologies: AFT '20: pp. 202–213: Association for Computing Machinery, New York, NY, USA: ISBN 9781450381390: doi:10.1145/3419614.3423248: URL https://doi.org/10.1145/3419614.3423248 (2020)
[18] Sivaraman V., Venkatakrishnan S. B., Alizadeh M., Fanti G. C., Viswanath P.: Routing Cryptocurrency with the Spider Network. CoRR: vol. abs/1809.05088: URL http://arxiv.org/abs/1809.05088 (2018)
[19] Prihodko P., Zhigulin S., Sahno M., Ostrovskiy A., Osuntokun O.: Flare: An approach to routing in lightning network. White Paper (2016)
[20] Lee J., Kim S., Park S., Moon S. M.: RouTEE: A Secure Payment Network Routing Hub using Trusted Execution Environments (2020)
[21] Khalil R., Gervais A.: Revive: Rebalancing Off-Blockchain Payment Networks. In Proceedings of the 2017 ACM SIGSAC Conference on Computer and Communications Security, CCS 2017, Dallas, TX, USA, October 30 - November 03, 2017: pp. 439–453: doi:10.1145/3133956.3134033: URL https://doi.org/10.1145/3133956.3134033 (2017)
[22] Decker C., Russell R., Osuntokun O.: eltoo: A Simple Layer2 Protocol for Bitcoin. https://blockstream.com/eltoo.pdf
[23] Green M., Miers I.: Bolt: Anonymous Payment Channels for Decentralized Currencies. In Proceedings of the 2017 ACM SIGSAC Conference on Computer and Communications Security: CCS '17: p. 473–489: Association for Computing Machinery, New York, NY, USA: ISBN 9781450349468: doi:10.1145/3133956.3134093: URL https://doi.org/10.1145/3133956.3134093 (2017)
[24] Miller A., Bentov I., Kumaresan R., Cordi C., McCorry P.: Sprites and State Channels: Payment Networks that Go Faster than Lightning. ArXiv preprint arXiv:1702.05812 (2017)
[25] Dong M., Liang Q., Li X., Liu J.: Celer Network: Bring Internet Scale to Every Blockchain (2018)
[26] Chakravarty M. M. T., Coretti S., Fitzi M., Gazi P., Kant P., Kiayias A., Russell A.: Hydra: Fast Isomorphic State Channels. Cryptology ePrint Archive, Report 2020/299: https://eprint.iacr.org/2020/299 (2020)
[27] Chakravarty M. M. T., Kireev R., MacKenzie K., McHale V., Müller J., Nemish A., Nester C., Peyton Jones M., Thompson S., Valentine R., Wadler P.: Functional blockchain contracts. https://iohk.io/en/research/library/papers/functional-blockchain-contracts/ (2019)
[28] Burchert C., Decker C., Wattenhofer R.: Scalable funding of Bitcoin micropayment channel networks. In The Royal Society: doi:10.1098/rsos.180089 (2018)
[29] Egger C., Moreno-Sanchez P., Maffei M.: Atomic Multi-Channel Updates with Constant Collateral in Bitcoin-Compatible Payment-Channel Networks. In Proceedings of the 2019 ACM SIGSAC Conference on Computer and Communications Security: CCS '19: pp. 801–815: Association for Computing Machinery, New York, NY, USA: ISBN 9781450367479: doi:10.1145/3319535.3345666: URL https://doi.org/10.1145/3319535.3345666 (2019)
[30] Zhao L., Shuang H., Xu S., Huang W., Cui R., Bettadpur P., Lie D.: SoK: Hardware Security Support for Trustworthy Execution (2019)
[31] Lind J., Eyal I., Pietzuch P. R., Sirer E. G.: Teechan: Payment Channels Using Trusted Execution Environments. CoRR: vol. abs/1612.07766: URL http://arxiv.org/abs/1612.07766 (2016)
[32] Lind J., Naor O., Eyal I., Kelbert F., Sirer E. G., Pietzuch P.: Teechain: A Secure Payment Network with Asynchronous Blockchain Access. In Proceedings of the 27th ACM Symposium on Operating Systems Principles: SOSP '19: pp. 63–79: Association for Computing Machinery, New York, NY, USA: ISBN 9781450368735: doi:10.1145/3341301.3359627: URL https://doi.org/10.1145/3341301.3359627 (2019)
[33] Liao J., Zhang F., Sun W., Shi W.: Speedster: A TEE-assisted State Channel System (2021)
[34] Avarikioti G., Kogias E. K., Wattenhofer R., Zindros D.: Brick: Asynchronous Payment Channels (2020)
[35] Back A., Corallo M., Dashjr L., Friedenbach M., Maxwell G., Miller A., Poelstra A., Timón J., Wuille P.: Enabling blockchain innovations with pegged sidechains. URL http://web.archive.org/save/https://pdfs.semanticscholar.org/1b23/cd2050d5000c05e1da3c9997b308ad5b7903.pdf (2014)
[36] Gaži P., Kiayias A., Zindros D.: Proof-of-Stake Sidechains. In 2019 2019 IEEE Symposium on Security and Privacy (SP): pp. 677–694: IEEE Computer Society, Los Alamitos, CA, USA: ISSN 2375-1207: doi:10.1109/SP.2019.00040: URL https://doi.ieeecomputersociety.org/10.1109/SP.2019.00040 (2019)
[37] Kiayias A., Zindros D.: Proof-of-Work Sidechains. In Financial Cryptography and Data Security - FC 2019 International Workshops, VOTING and WTSC, St. Kitts, St. Kitts and Nevis, February 18-22, 2019, Revised Selected Papers: pp. 21–34: doi:10.1007/978-3-030-43725-1_3: URL https://doi.org/10.1007/978-3-030-43725-1_3 (2019)
[38] Poon J., Buterin V.: Plasma: Scalable Autonomous Smart Contracts. http://plasma.io/plasma.pdf
[39] Konstantopoulos G.: Plasma Cash: Towards more efficient Plasma constructions (2019)
[40] Dziembowski S., Fabiański G., Faust S., Riahi S.: Lower Bounds for Off-Chain Protocols: Exploring the Limits of Plasma. Cryptology ePrint Archive, Report 2020/175: https://eprint.iacr.org/2020/175 (2020)
[41] Adler J.: The Why's of Optimistic Rollup. https://medium.com/@adlerjohn/the-why-s-of-optimistic-rollup-7c6a22cbb61a (2019)
[42] Armknecht F., Karame G. O., Mandal A., Youssef F., Zenner E.: Ripple: Overview and Outlook. In Trust and Trustworthy Computing - 8th International Conference, TRUST 2015, Heraklion, Greece, August 24-26, 2015, Proceedings: pp. 163–180: doi:10.1007/978-3-319-22846-4_10: URL https://doi.org/10.1007/978-3-319-22846-4_10 (2015)
[43] Mazieres D.: The stellar consensus protocol: A federated model for internet-level consensus. Stellar Development Foundation (2015)
[44] Malavolta G., Moreno-Sanchez P., Kate A., Maffei M.: SilentWhispers: Enforcing Security and Privacy in Decentralized Credit Networks (2016)
[45] Roos S., Moreno-Sanchez P., Kate A., Goldberg I.: Settling Payments Fast and Private: Efficient Decentralized Routing for Path-Based Transactions. In 25th Annual Network and Distributed System Security Symposium, NDSS 2018, San Diego, California, USA, February 18-21, 2018: URL http://wp.internetsociety.org/ndss/wp-content/uploads/sites/25/2018/02/ndss2018_09-3_Roos_paper.pdf (2018)
[46] Aumayr L., Moreno-Sanchez P., Kate A., Maffei M.: Donner: UTXO-Based Virtual Channels Across Multiple Hops. Cryptology ePrint Archive, Report 2021/855: https://eprint.iacr.org/2021/855 (2021)
[47] Decker C., Towns A.: SIGHASH_ANYPREVOUT for Taproot Scripts. https://github.com/bitcoin/bips/blob/master/bip-0118.mediawiki
[48] Badertscher C., Maurer U., Tschudi D., Zikas V.: Bitcoin as a transaction ledger: A composable treatment. In Annual International Cryptology Conference: pp. 324–356: Springer (2017)
[49] Badertscher C., Gaži P., Kiayias A., Russell A., Zikas V.: Ouroboros genesis: Composable proof-of-stake blockchains with dynamic availability. In Proceedings of the 2018 ACM SIGSAC Conference on Computer and Communications Security: pp. 913–930: ACM (2018)
[50] Katz J., Maurer U., Tackmann B., Zikas V.: Universally Composable Synchronous Computation. In Theory of Cryptography - 10th Theory of Cryptography Conference, TCC 2013, Tokyo, Japan, March 3-6, 2013. Proceedings: pp. 477–498: doi:10.1007/978-3-642-36594-2_27: URL https://doi.org/10.1007/978-3-642-36594-2_27 (2013)
[51] Wuille P., Nick J., Towns A.: Taproot: SegWit version 1 spending rules. https://github.com/bitcoin/bips/blob/master/bip-0341.mediawiki
[52] Danezis G., Goldberg I.: Sphinx: A compact and provably secure mix format. In Security and Privacy, 2009 30th IEEE Symposium on: pp. 269–282: IEEE (2009)
[53] Lindell Y.: How to Simulate It - A Tutorial on the Simulation Proof Technique. In Tutorials on the Foundations of Cryptography: pp. 277–346: doi:10.1007/978-3-319-57048-8_6: URL https://doi.org/10.1007/978-3-319-57048-8_6 (2017)
[54] Kiayias A., Litos O. S. T.: A Composable Security Treatment of the Lightning Network. In 33rd IEEE Computer Security Foundations Symposium, CSF 2020, Boston, MA, USA, June 22-26, 2020: pp. 334–349: doi:10.1109/CSF49147.2020.00031: URL https://doi.org/10.1109/CSF49147.2020.00031 (2020)

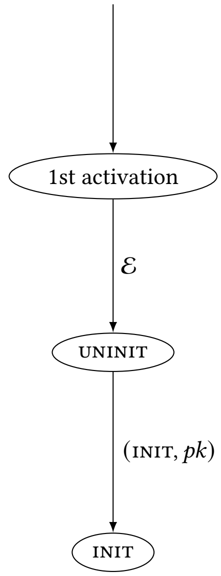

Figure 14: $\mathcal{G}_{\text{Chan}}$ state machine up to INIT (both parties)

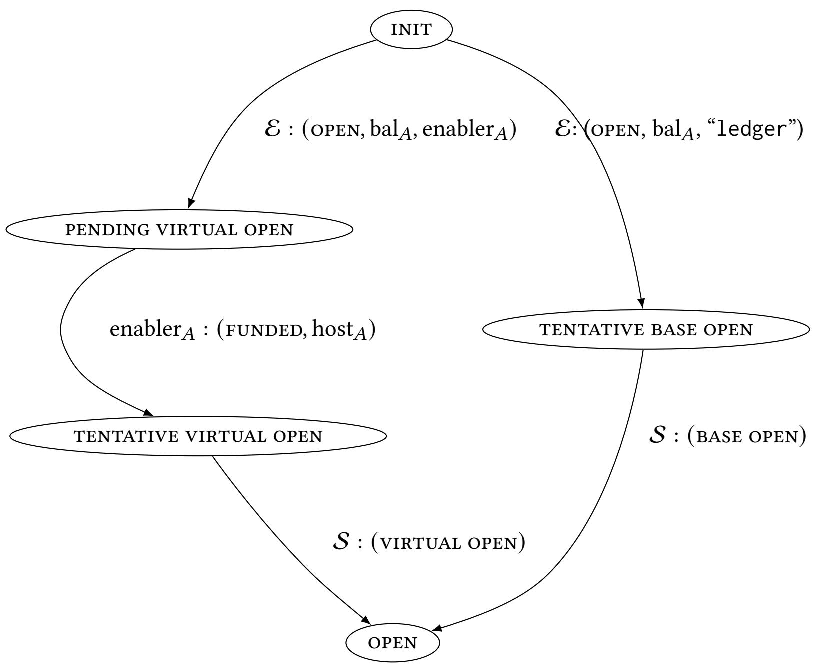

Figure 15: $\mathcal{G}_{\text{Chan}}$ state machine from INIT up to OPEN (funder)

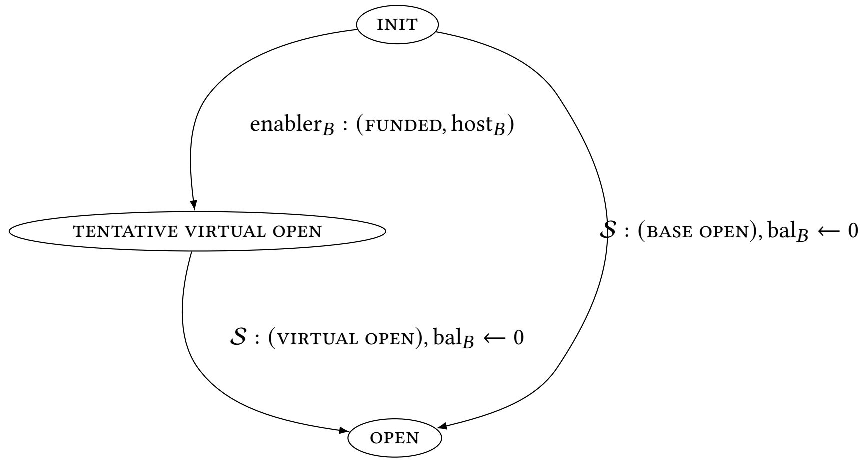

Figure 16: $\mathcal{G}_{\text{Chan}}$ state machine from INIT up to OPEN (fundee)

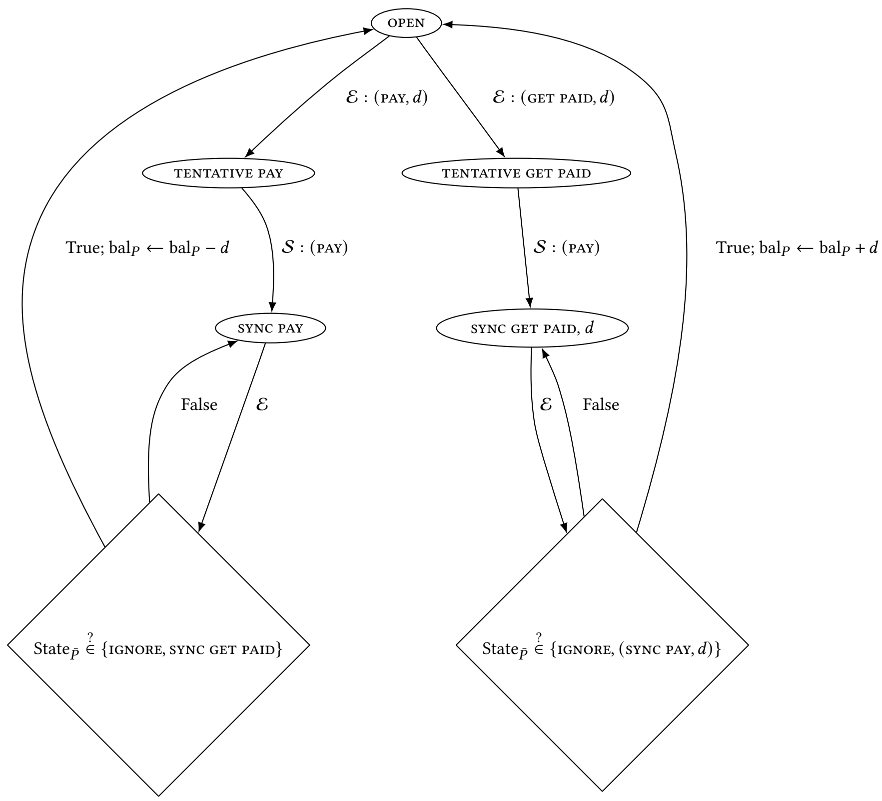

Figure 17: $\mathcal{G}_{\text{Chan}}$ state machine for payments (both parties)

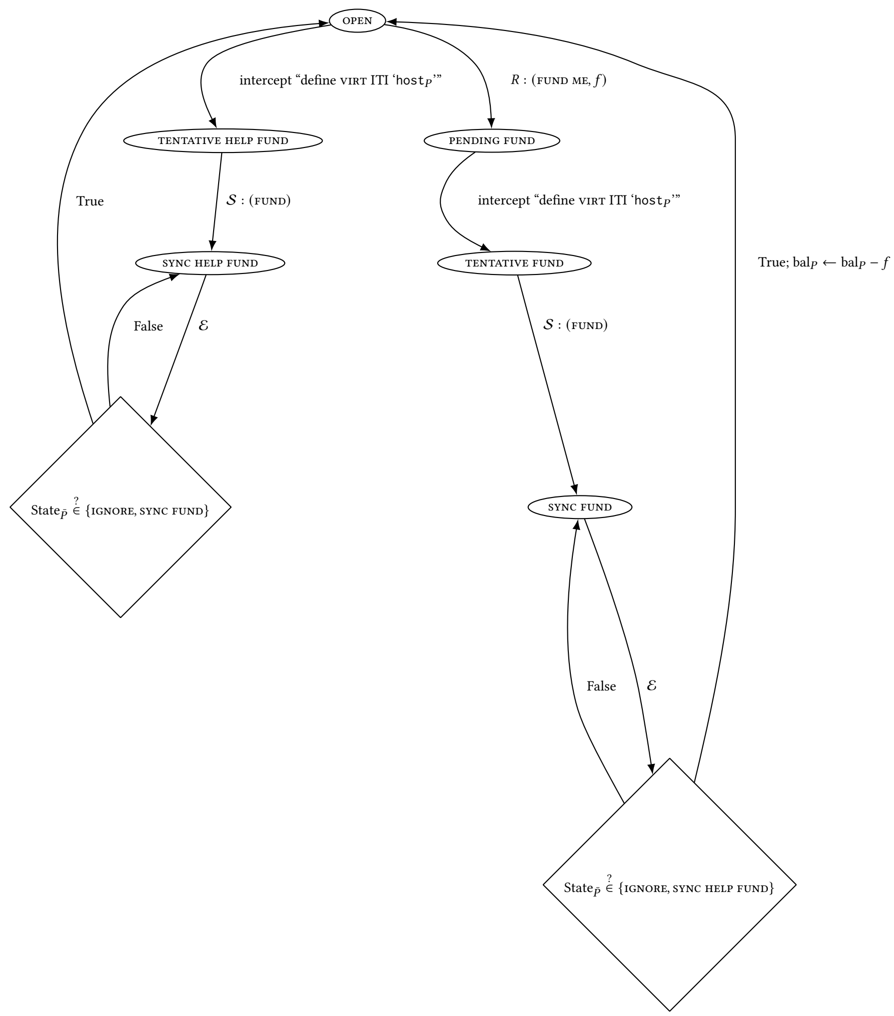

Figure 18: $\mathcal{G}_{\text{Chan}}$ state machine for funding new virtuals (both parties)

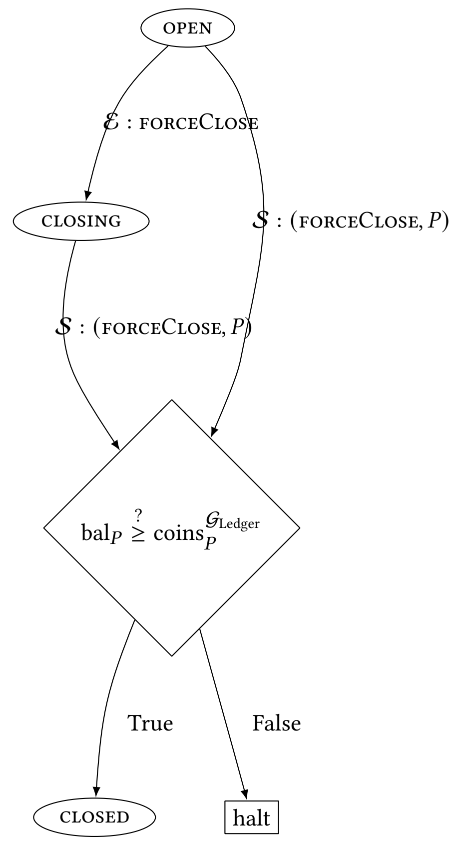

Figure 19: $\mathcal{G}_{\text{Chan}}$ state machine for channel closure (both parties)

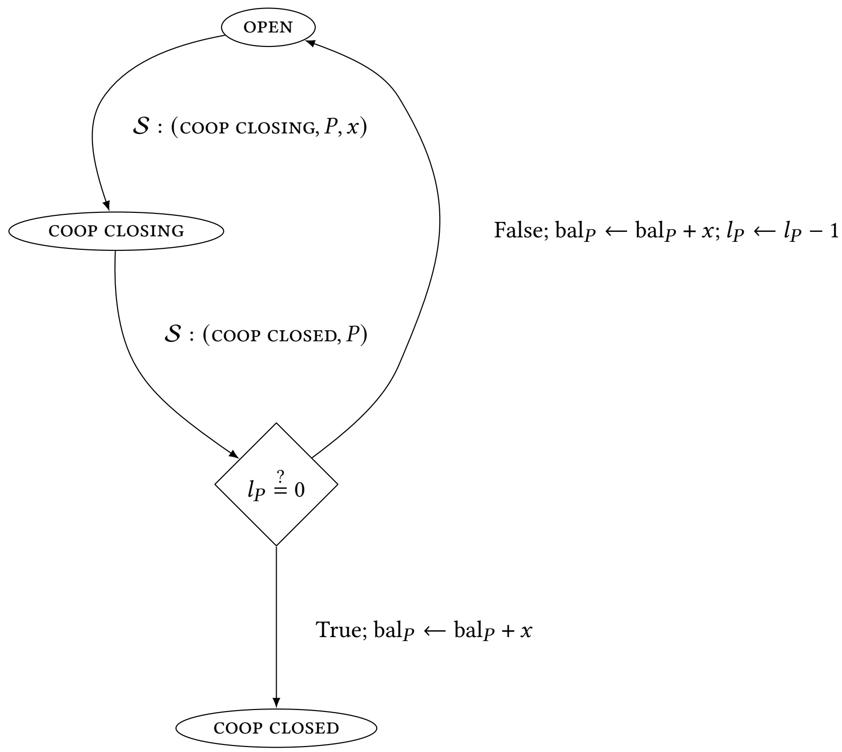

Figure 20: $\mathcal{G}_{\text{Chan}}$ state machine for cooperative channel closure (all parties)

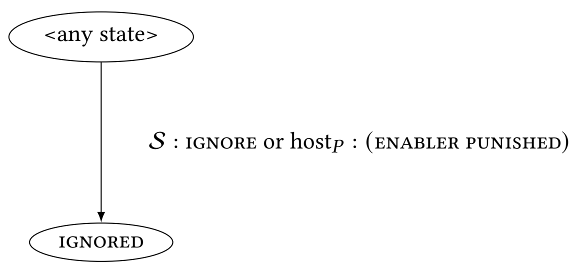

Figure 21: $\mathcal{G}_{\text{Chan}}$ state machine for corruption, negligence or punishment of the counterparty of a lower layer (both parties)