

Abstract. Blockchains are slow. Layer-2 largely solves this problem. PCNs constitute the most prominent layer-2/off-chain protocols. LN is the most widely used PCN and works on Bitcoin. Opening a channel requires 1 on-chain transaction, which can at times be avoided by performing a multi-hop payment. Then however fees to the intermediaries must be paid, routing becomes an issue, payment delay is proportional to the number of intermediaries and per-payment privacy suffers. We propose Recursive Channels, which allow for new channels to be opened on top of an arbitrarily long path of existing channels in a recursive manner (i.e. the preexisting channels may themselves be virtual), answering the question of feasibility in the affirmative. Our construction relies on the proposed **ANYPREVOUT** signature type.

1 Introduction

The popularity of blockchains in recent years has stretched their performance to its limits. Due to their need for synchronisation their latency is large (e.g. Bitcoin has a latency of 1h [1]) and due to the need for massive redundancy their throughput is low (Bitcoin can handle at most 7 transactions per second [2]). To circumvent these inherent limitations of blockchains, a prominent solution is to optimistically handle payments off-chain via a Payment Channel Network (PCN) **TODO: cite PCN SoK/many papers** and only use the blockchain as an arbiter in case of dispute.

The most popular PCN is the Lightning Network (LN) [3], which works on top of Bitcoin. With this, parties can open a pairwise channel with a single on-chain transaction and subsequently pay each other an unlimited number of times, only limited by the speed of their internet connection. What is more, a party can pay another even if they do not have a direct channel. They can instead leverage a path of channels for a fee and perform a so-called multi-hop payment in an atomic manner. Unfortunately a multi-hop payment needs active cooperation by all intermediaries, therefore increasing the latency and the probability of failure of the payment.

To mitigate this issue, virtual payment channels have been proposed **TODO: cite**. These enable two parties, say Alice and Bob, to open a payment channel over two preexisting channels, one between Alice and Charlie and another between Charlie and Bob. **TODO: check if recursive channels exist**

However, due to the limited scripting language of Bitcoin, it has proved challenging to build a secure protocol that allows virtual channels to be opened over more than two underlying channels, **TODO: delete following phrase if the previous's TODO answer is affirmative** as well as to make this construction recursive in the sense that further virtual channels can be opened on top of other virtual channels.

This work fills this gap by providing a concrete protocol that allows for arbitrarily many channels to be opened on top of arbitrarily long channel paths, where the underlying channels may themselves be virtual. This is achieved using standard Bitcoin script and an elaborate transaction configuration. We formally

prove the security of the protocol in the UC [4] setting. The construction relies on the ANYPREVOUT signature type, which does not sign the hash of the transaction it spends, therefore allowing for a single pre-signed transaction to spend any output with a suitable script. We conjecture that this primitive cannot be achieved without ANYPREVOUT.

2 Related Work

3 High Level Explanation

4 Preliminaries & Notation

5 Construction

6 Future work

- Add support for cooperative adding multiple virtuals to single channel (needs cooperation by all hops of all existing virtuals of current channel)
- Add support for cooperative closing
- Use eltoo instead of lightning to avoid balance restriction that prevents the revoked-griefing attack
- Allow for user-defined “leeway” timeout and timeout renegotiation
- Incorporate fees
- Prevent DoS attacks

Functionality $\mathcal{F}_{\text{Chan}}$ – general message handling rules

- On receiving (msg) by party R to $P \in \{Alice, Bob\}$ by means of $\text{mode} \in \{\text{input}, \text{output}, \text{network}\}$, handle it according to the corresponding rule in Fig. 2, 3, 5, or 4 (if any) and subsequently send $(\text{RELAY}, \text{msg}, P, \mathcal{E}, \text{input})$ \mathcal{A} .
// all messages are relayed to \mathcal{A}
- On receiving $(\text{RELAY}, \text{msg}, P, R, \text{mode})$ by \mathcal{A} ($\text{mode} \in \{\text{input}, \text{output}, \text{network}\}$, $P \in \{Alice, Bob\}$), relay msg to R as P by means of mode . // \mathcal{A} fully controls outgoing messages by $\mathcal{F}_{\text{Chan}}$
- On receiving $(\text{INFO}, \text{msg})$ by \mathcal{A} , handle (msg) according to the corresponding rule in Fig. 2, 3, 5, or 4 (if any). After handling the message or after an “ensure” fails, send $(\text{HANDLED}, \text{msg})$ to \mathcal{A} . // $(\text{INFO}, \text{msg})$ messages by \mathcal{S} always return control to \mathcal{S} without any side-effect to any other ITI, except if $\mathcal{F}_{\text{Chan}}$ halts
- $\mathcal{F}_{\text{Chan}}$ keeps track of two state machines, one for each of *Alice*, *Bob*. If there are more than one suitable rules for a particular message, or if a rule matches the message for both parties, then both rule versions are executed. // the two rules act on different state machines, so the order of execution does not matter

Fig. 1.

Functionality $\mathcal{F}_{\text{Chan}}$ – state machine up to OPEN for $P \in \{\text{Alice}, \text{Bob}\}$

- 1: On first activation: // before handing the message
- 2: $pk_P \leftarrow \perp$; $\text{host}_P \leftarrow \perp$; $\text{enabler}_P \leftarrow \perp$; $\text{balance}_P \leftarrow 0$;
- 3: $\text{State}_P \leftarrow \text{UNINIT}$
- 4: On (BECAME CORRUPTED OR NEGLIGENT, P) by \mathcal{A} or on output (ENABLER USED REVOCATION) by host_P when in any state:
- 5: $\text{State}_P \leftarrow \text{IGNORED}$
- 6: On (INIT, pk) to P by \mathcal{E} when $\text{State}_P = \text{UNINIT}$:
- 7: $pk_P \leftarrow pk$
- 8: $\text{State}_P \leftarrow \text{INIT}$
- 9: On (OPEN, x , $\mathcal{G}_{\text{Ledger}}, \dots$) to *Alice* by \mathcal{E} when $\text{State}_A = \text{INIT}$:
- 10: store x
- 11: $\text{State}_A \leftarrow \text{TENTATIVE BASE OPEN}$
- 12: On (BASE OPEN) by \mathcal{A} when $\text{State}_A = \text{TENTATIVE BASE OPEN}$:
- 13: $\text{balance}_A \leftarrow x$
- 14: $\text{State}_A \leftarrow \text{OPEN}$
- 15: On (BASE OPEN) by \mathcal{A} when $\text{State}_B = \text{INIT}$:
- 16: $\text{State}_B \leftarrow \text{OPEN}$
- 17: On (OPEN, x , $\text{hops} \neq \mathcal{G}_{\text{Ledger}}, \dots$) to *Alice* by \mathcal{E} when $\text{State}_A = \text{INIT}$:
- 18: store x
- 19: $\text{enabler}_A \leftarrow \text{hops}[0].\text{left}$
- 20: add enabler_A to *Alice's* trusted parties
- 21: $\text{State}_A \leftarrow \text{PENDING VIRTUAL OPEN}$
- 22: On output (FUNDED, host , \dots) to *Alice* by enabler_A when $\text{State}_A = \text{PENDING VIRTUAL OPEN}$:
- 23: $\text{host}_A \leftarrow \text{host}[0].\text{left}$
- 24: $\text{State}_A \leftarrow \text{TENTATIVE VIRTUAL OPEN}$
- 25: On output (FUNDED, host , \dots) to *Bob* by ITI $R \in \{\mathcal{F}_{\text{Chan}}, \text{LN}\}$ when $\text{State}_B = \text{INIT}$:
- 26: $\text{enabler}_B \leftarrow R$
- 27: add enabler_B to *Bob's* trusted parties
- 28: $\text{host}_B \leftarrow \text{host}$
- 29: $\text{State}_B \leftarrow \text{TENTATIVE VIRTUAL OPEN}$
- 30: On (VIRTUAL OPEN) by \mathcal{A} when $\text{State}_P = \text{TENTATIVE VIRTUAL OPEN}$:
- 31: if $P = \text{Alice}$ then $\text{balance}_P \leftarrow x$
- 32: $\text{State}_P \leftarrow \text{OPEN}$

Fig. 2.
3

Functionality $\mathcal{F}_{\text{Chan}}$ – payments state machine for $P \in \{\text{Alice}, \text{Bob}\}$

```

1: On (PAY,  $x$ ) by  $\mathcal{E}$  when  $\text{State}_P = \text{OPEN}$ : //  $P$  pays  $\bar{P}$ 
2:   store  $x$ 
3:    $\text{State}_P \leftarrow \text{TENTATIVE PAY}$ 

4: On (PAY) by  $\mathcal{A}$  when  $\text{State}_P = \text{TENTATIVE PAY}$ : //  $P$  pays  $\bar{P}$ 
5:    $\text{State}_P \leftarrow (\text{SYNC PAY}, x)$ 

6: On (GET PAID,  $y$ ) by  $\mathcal{E}$  when  $\text{State}_P = \text{OPEN}$ : //  $\bar{P}$  pays  $P$ 
7:   store  $y$ 
8:    $\text{State}_P \leftarrow \text{TENTATIVE GET PAID}$ 

9: On (PAY) by  $\mathcal{A}$  when  $\text{State}_P = \text{TENTATIVE GET PAID}$ : //  $\bar{P}$  pays  $P$ 
10:   $\text{State}_P \leftarrow (\text{SYNC GET PAID}, x)$ 

11: When  $\text{State}_P = (\text{SYNC PAY}, x)$ :
12:   if  $\text{State}_{\bar{P}} \in \{\text{IGNORED}, (\text{SYNC GET PAID}, x)\}$  then
13:     balance $_P \leftarrow \text{balance}_P - x$ 
14:     // if  $\bar{P}$  honest, this state transition happens simultaneously with l. 21
15:      $\text{State}_P \leftarrow \text{OPEN}$ 
16:   end if

17: When  $\text{State}_P = (\text{SYNC GET PAID}, x)$ :
18:   if  $\text{State}_{\bar{P}} \in \{\text{IGNORED}, (\text{SYNC PAY}, x)\}$  then
19:     balance $_P \leftarrow \text{balance}_P + x$ 
20:     // if  $\bar{P}$  honest, this state transition happens simultaneously with l. 15
21:      $\text{State}_P \leftarrow \text{OPEN}$ 
22:   end if

```

Fig. 3.

Functionality $\mathcal{F}_{\text{Chan}}$ – fundings state machine for $P \in \{\text{Alice}, \text{Bob}\}$

```

1: On input (FUND ME,  $x, \dots$ ) by ITI  $R \in \{\mathcal{F}_{\text{Chan}}, \text{LN}\}$  when  $\text{State}_P = \text{OPEN}$ :
2:   store  $x$ 
3:   add  $R$  to  $P$ 's trusted parties
4:    $\text{State}_P \leftarrow \text{PENDING FUND}$ 

5: When  $\text{State}_P = \text{PENDING FUND}$ :
6:   if we intercept the command “define new VIRT ITI host” by  $\mathcal{A}$ , routed
   through  $P$  then
7:     store host
8:      $\text{State}_P \leftarrow \text{TENTATIVE FUND}$ 
9:     continue executing  $\mathcal{A}$ 's command
10:  end if

11: On (FUND) by  $\mathcal{A}$  when  $\text{State}_P = \text{TENTATIVE FUND}$ :
12:    $\text{State}_P \leftarrow \text{SYNC FUND}$ 

13: When  $\text{State}_P = \text{OPEN}$ :
14:   if we intercept the command “define new VIRT ITI host” by  $\mathcal{A}$ , routed
   through  $P$  then
15:     store host
16:      $\text{State}_P \leftarrow \text{TENTATIVE HELP FUND}$ 
17:     continue executing  $\mathcal{A}$ 's command
18:   end if
19:   if we receive a RELAY message with  $\text{msg} = (\text{INIT}, \dots, \text{fundee})$  addressed
   from  $P$  by  $\mathcal{A}$  then
20:     add fundee to  $P$ 's trusted parties
21:     continue executing  $\mathcal{A}$ 's command
22:   end if

23: On (FUND) by  $\mathcal{A}$  when  $\text{State}_P = \text{TENTATIVE HELP FUND}$ :
24:    $\text{State}_P \leftarrow \text{SYNC HELP FUND}$ 

25: When  $\text{State}_P = \text{SYNC FUND}$ :
26:   if  $\text{State}_{\bar{P}} \in \{\text{IGNORED}, \text{SYNC HELP FUND}\}$  then
27:      $\text{balance}_P \leftarrow \text{balance}_P - x$ 
28:      $\text{host}_P \leftarrow \text{host}$ 
29:     // if  $\bar{P}$  honest, this state transition happens simultaneously with l. 36
30:      $\text{State}_P \leftarrow \text{OPEN}$ 
31:   end if

32: When  $\text{State}_P = \text{SYNC HELP FUND}$ :
33:   if  $\text{State}_{\bar{P}} \in \{\text{IGNORED}, \text{SYNC FUND}\}$  then
34:      $\text{host}_P \leftarrow \text{host}$ 
35:     // if  $\bar{P}$  honest, this state transition happens simultaneously with l. 30
36:      $\text{State}_P \leftarrow \text{OPEN}$ 
37:   end if

```

Fig. 4.

Functionality $\mathcal{F}_{\text{Chan}}$ – closure state machine for $P \in \{\text{Alice}, \text{Bob}\}$

```

1: On (CLOSE) by  $\mathcal{E}$  when  $\text{State}_P = \text{OPEN}$ :
2:    $\text{State}_P \leftarrow \text{CLOSING}$ 

3: On input (BALANCE) to  $P$  by  $R$  where  $R$  is trusted by  $P$ :
4:   if  $\text{State}_P \notin \{\text{UNINIT}, \text{INIT}, \text{PENDING VIRTUAL OPEN}, \text{TENTATIVE VIRTUAL OPEN}, \text{TENTATIVE BASE OPEN}, \text{IGNORED}, \text{CLOSED}\}$  then
5:     reply (MY BALANCE,  $\text{balance}_P$ ,  $pk_P$ ,  $\text{balance}_{\bar{P}}$ ,  $pk_{\bar{P}}$ )
6:   else
7:     reply (MY BALANCE, 0,  $pk_P$ , 0,  $pk_{\bar{P}}$ )
8:   end if

9: On (CLOSE,  $P$ ) by  $\mathcal{A}$  when  $\text{State}_P \notin \{\text{UNINIT}, \text{INIT}, \text{PENDING VIRTUAL OPEN}, \text{TENTATIVE VIRTUAL OPEN}, \text{TENTATIVE BASE OPEN}, \text{IGNORED}\}$ :
10:  input (READ) to  $\mathcal{G}_{\text{Ledger}}$  as  $P$  and assign output to  $\Sigma$ 
11:   $\text{coins} \leftarrow$  sum of values of outputs exclusively spendable or spent by  $pk_P$  in  $\Sigma$ 
12:   $\text{balance} \leftarrow \text{balance}_P$ 
13:  for all  $P$ 's trusted parties  $R$  do
14:    input (BALANCE) to  $R$  as  $P$  and extract  $\text{balance}_R$ ,  $pk_R$  from response
15:     $\text{balance} \leftarrow \text{balance} + \text{balance}_R$ 
16:     $\text{coins} \leftarrow \text{coins} +$  sum of values of outputs exclusively spendable or
    spent by  $pk_R$  in  $\Sigma$ 
17:  end for
18:  if  $\text{coins} \geq \text{balance}$  then
19:     $\text{State}_P \leftarrow \text{CLOSED}$ 
20:  else // balance security is broken
21:    halt
22:  end if

```

Fig. 5.

Simulator \mathcal{S} – general message handling rules

- On receiving (RELAY, in_msg , P , R , in_mode) by $\mathcal{F}_{\text{Chan}}$ ($\text{in_mode} \in \{\text{input}, \text{output}, \text{network}\}$, $P \in \{\text{Alice}, \text{Bob}\}$), handle (in_msg) with the simulated party P as if it was received from R by means of in_mode . In case simulated P does not exist yet, initialise it as an LN ITI. If there is a resulting message out_msg that is to be sent by simulated P to R' by means of $\text{out_mode} \in \{\text{input}, \text{output}, \text{network}\}$, send (RELAY, out_msg , P , R' , out_mode) to $\mathcal{F}_{\text{Chan}}$.
- On receiving by $\mathcal{F}_{\text{Chan}}$ a message to be sent by P to R via the network, carry on with this action (i.e. send this message via the internal \mathcal{A}).
- Relay any other incoming message to the internal \mathcal{A} unmodified.
- On receiving a message (msg) by the internal \mathcal{A} , if it is addressed to one of the parties that correspond to $\mathcal{F}_{\text{Chan}}$, handle the message internally with the corresponding simulated party. Otherwise relay the message to its intended recipient unmodified. // Other recipients are \mathcal{E} , $\mathcal{G}_{\text{Ledger}}$ or parties unrelated to $\mathcal{F}_{\text{Chan}}$.

Given that $\mathcal{F}_{\text{Chan}}$ relays all messages and that we simulate the real-world machines that correspond to $\mathcal{F}_{\text{Chan}}$, the simulation is perfectly indistinguishable from the real world.

Fig. 6.

Simulator \mathcal{S} – notifications to $\mathcal{F}_{\text{Chan}}$

- “ P ” refers one of the parties that correspond to $\mathcal{F}_{\text{Chan}}$.
 - When an action in this Figure interrupts an ITI simulation, continue simulating from the interruption location once action is over/ $\mathcal{F}_{\text{Chan}}$ hands control back.
- 1: On (CORRUPT) by \mathcal{A} , addressed to P :
 - 2: // After executing this code and getting control back from $\mathcal{F}_{\text{Chan}}$ (which always happens, c.f. Fig. 1), deliver (CORRUPT) to simulated P (c.f. Fig. 6.
 - 3: send (INFO, BECAME CORRUPTED OR NEGLIGENT, P) to $\mathcal{F}_{\text{Chan}}$
 - 4: When simulated P sets variable **negligent** to True (Fig. 8, l. 7/Fig. 9, l. 26):
 - 5: send (INFO, BECAME CORRUPTED OR NEGLIGENT, P) to $\mathcal{F}_{\text{Chan}}$
 - 6: When simulated honest *Alice* receives (OPEN, x , **hops**, ...) by \mathcal{E} :
 - 7: store **hops** // will be used to inform $\mathcal{F}_{\text{Chan}}$ once the channel is open
 - 8: When simulated honest *Bob* receives (OPEN, x , **hops**, ...) by *Alice*:
 - 9: **if** *Alice* is corrupted **then** store **hops** // if *Alice* is honest, we already have **hops**. If *Alice* became corrupted after receiving (OPEN, ...), overwrite **hops**
 - 10: When the last of the honest simulated $\mathcal{F}_{\text{Chan}}$ ’s parties moves to the OPEN *State* for the first time (Fig. 12, l. 19/Fig. 14, l. 5/Fig. 15, l. 18):
 - 11: **if** **hops** = $\mathcal{G}_{\text{Ledger}}$ **then**
 - 12: send (INFO, BASE OPEN) to $\mathcal{F}_{\text{Chan}}$
 - 13: **else**
 - 14: send (INFO, VIRTUAL OPEN) to $\mathcal{F}_{\text{Chan}}$
 - 15: **end if**
 - 16: When (both $\mathcal{F}_{\text{Chan}}$ ’s simulated parties are honest and complete sending and receiving a payment (Fig. 20, ll. 6 and 21 respectively), or (when only one party is honest and (completes either receiving or sending a payment)): // also send this message if both parties are honest when Fig. 20, l. 6 is executed by one party, but its counterparty is corrupted before executing Fig. 20, l. 21
 - 17: send (INFO, PAY) to $\mathcal{F}_{\text{Chan}}$
 - 18: When honest P executes Fig. 17, l. 20 or (when honest P executes Fig. 17, l. 18 and \bar{P} is corrupted): // in the first case if \bar{P} is honest, it has already moved to the new host, (Fig 38, ll. 7, 23): lifting to next layer is done
 - 19: send (INFO, FUND) to $\mathcal{F}_{\text{Chan}}$
 - 20: When one of the honest simulated $\mathcal{F}_{\text{Chan}}$ ’s parties P moves to the CLOSED state (Fig. 24, l. 8 or l. 11):
 - 21: send (INFO, CLOSE, P) to $\mathcal{F}_{\text{Chan}}$

Fig. 7.

Process LN – init

```

1: // When not specified, input comes from and output goes to  $\mathcal{E}$ .
2: // The ITI knows whether it is Alice (funder) or Bob (fundee). The activated
   party is  $P$  and the counterparty is  $\bar{P}$ .
3: On every activation, before handling the message:
4:   if  $\text{last\_poll} \neq \perp$  then // channel has opened
5:     input (READ) to  $\mathcal{G}_{\text{Ledger}}$  and assign output to  $\Sigma$ 
6:     if  $\text{last\_poll} + p < |\Sigma|$  then //  $p$  is a global parameter
7:        $\text{negligent} \leftarrow \text{True}$ 
8:     end if
9:   end if

10: On (INIT,  $pk_{P,\text{out}}$ ):
11:   ensure  $\text{State} = \perp$ 
12:    $\text{State} \leftarrow \text{INIT}$ 
13:   store  $pk_{P,\text{out}}$ 
14:    $(c_A, c_B, \text{locked}_A, \text{locked}_B) \leftarrow (0, 0, 0, 0)$ 
15:    $(\text{paid\_out}, \text{paid\_in}) \leftarrow (\emptyset, \emptyset)$ 
16:    $\text{negligent} \leftarrow \text{False}$ 
17:    $\text{last\_poll} \leftarrow \perp$ 
18:   output (INIT OK)

19: On (TOP UP):
20:   ensure  $P = \text{Alice}$  // activated party is the funder
21:   ensure  $\text{State} = \text{INIT}$ 
22:    $(sk_{P,\text{chain}}, pk_{P,\text{chain}}) \leftarrow \text{KEYGEN}()$ 
23:   input (READ) to  $\mathcal{G}_{\text{Ledger}}$  and assign output to  $\Sigma$ 
24:   output (TOP UP TO,  $pk_{P,\text{chain}}$ )
25:   while  $\nexists \text{tx} \in \Sigma, c_{P,\text{chain}} : (c_{P,\text{chain}}, pk_{P,\text{chain}}) \in \text{tx.outputs}$  do
26:     // while waiting, all other messages by  $P$  are ignored
27:     wait for input (CHECK TOP UP)
28:     input (READ) to  $\mathcal{G}_{\text{Ledger}}$  and assign output to  $\Sigma$ 
29:   end while
30:    $\text{State} \leftarrow \text{TOPPED UP}$ 
31:   output (TOP UP OK,  $c_{P,\text{chain}}$ )

32: On (BALANCE):
33:   ensure  $\text{State}^P \in \{\text{OPEN}, \text{CLOSED}\}$ 
34:   output (BALANCE,  $c_A, pk_{A,\text{out}}, c_B, pk_{B,\text{out}}, \text{locked}_A, \text{locked}_B$ )

```

Fig. 8.

Process LN – methods used by VIRT

```

1: REVOKEPREVIOUS():
2:   ensure  $State \in \text{WAITING FOR (OUTBOUND) REVOCATION}$ 
3:    $R_{\bar{P},i} \leftarrow \text{TX}$  {input:  $C_{P,i}.\text{outputs}.P$ , output:  $(C_{P,i}.\text{outputs}.P.\text{value},$ 
    $pk_{\bar{P},\text{out}})$ }
4:    $\text{sig}_{A,R,i} \leftarrow \text{SIGN}(R_{\bar{P},i}, sk_{P,R})$ 
5:   if  $State = \text{WAITING FOR REVOCATION}$  then
6:      $State \leftarrow \text{WAITING FOR INBOUND REVOCATION}$ 
7:   else //  $State = \text{WAITING FOR OUTBOUND REVOCATION}$ 
8:      $i \leftarrow i + 1$ 
9:      $State \leftarrow \text{WAITING FOR HOSTS READY}$ 
10:  end if
11:   $\text{host}_P \leftarrow \text{host}'_P$  // forget old host, use new host instead
12:   $\text{layer} \leftarrow \text{layer} + 1$ 
13:  return  $\text{sig}_{P,R,i}$ 

14: PROCESSREMOTEREVOCATION( $\text{sig}_{\bar{P},R,i}$ ):
15:   ensure  $State = \text{WAITING FOR (INBOUND) REVOCATION}$ 
16:    $R_{P,i} \leftarrow \text{TX}$  {input:  $C_{\bar{P},i}.\text{outputs}.P$ , output:  $(C_{\bar{P},i}.\text{outputs}.\bar{P}.\text{value},$ 
    $pk_{P,\text{out}})$ }
17:   ensure  $\text{VERIFY}(R_{P,i}, \text{sig}_{\bar{P},R,i}, pk_{\bar{P},R}) = \text{True}$ 
18:   if  $State = \text{WAITING FOR REVOCATION}$  then
19:      $State \leftarrow \text{WAITING FOR OUTBOUND REVOCATION}$ 
20:   else //  $State = \text{WAITING FOR INBOUND REVOCATION}$ 
21:      $i \leftarrow i + 1$ 
22:      $State \leftarrow \text{WAITING FOR HOSTS READY}$ 
23:   end if
24:   return (OK)

25: NEGLIGENT():
26:    $\text{negligent} \leftarrow \text{True}$ 
27:   return (OK)

```

Fig. 9.

Process LN.EXCHANGEOPENKEYS()

```

1:  $(sk_{A,F}, pk_{A,F}) \leftarrow \text{KEYGEN}(); (sk_{A,R}, pk_{A,R}) \leftarrow \text{KEYGEN}()$ 
2:  $State \leftarrow \text{WAITING FOR OPENING KEYS}$ 
3: send (OPEN,  $c$ , hops,  $pk_{A,F}$ ,  $pk_{A,R}$ ,  $pk_{A,out}$ ) to fundee
4: // colored code is run by honest fundee. Validation is implicit
5: ensure we run the code of Bob
6: ensure  $State = \text{INIT}$ 
7: store  $pk_{A,F}$ ,  $pk_{A,R}$ ,  $pk_{A,out}$ 
8:  $(sk_{B,F}, pk_{B,F}) \leftarrow \text{KEYGEN}(); (sk_{B,R}, pk_{B,R}) \leftarrow \text{KEYGEN}()$ 
9: if hops =  $\mathcal{G}_{\text{Ledger}}$  then // opening base channel
10:   layer  $\leftarrow 0$ 
11:    $t_P \leftarrow s + p$  //  $s$  is the upper bound of  $\eta$  from Lemma 7.19 of [5]
12:    $State \leftarrow \text{WAITING FOR COMM SIG}$ 
13: else // opening virtual channel
14:    $State \leftarrow \text{WAITING FOR CHECK KEYS}$ 
15: end if
16: reply (ACCEPT CHANNEL,  $pk_{B,F}$ ,  $pk_{B,R}$ ,  $pk_{B,out}$ )
17: ensure  $State = \text{WAITING FOR OPENING KEYS}$ 
18: store  $pk_{B,F}$ ,  $pk_{B,R}$ ,  $pk_{B,out}$ 
19:  $State \leftarrow \text{OPENING KEYS OK}$ 

```

Fig. 10.

Process LN.PREPAREBASE()

```

1: if hops =  $\mathcal{G}_{\text{Ledger}}$  then // opening base channel
2:    $F \leftarrow \text{TX}$  {input:  $(c, pk_{A,chain})$ , output:  $(c, 2/\{pk_{A,F}, pk_{B,F}\})$ }
3:    $host_P \leftarrow \mathcal{G}_{\text{Ledger}}$ 
4:   layer  $\leftarrow 0$ 
5:    $t_P \leftarrow s + p$ 
6: else // opening virtual channel
7:   input (FUND ME, Alice, Bob, hops,  $c$ ,  $pk_{A,F}$ ,  $pk_{B,F}$ ) to hops[0].left and
   expect output (FUNDED,  $host_P$ , funder_layer,  $t_P$ ) // ignore any other
   message
8:   layer  $\leftarrow$  funder_layer
9: end if

```

Fig. 11.

Process LN.EXCHANGEOPENSIGS()

```

1: //  $s = (2 + \lceil \text{maxTime}_{\text{window}} + \frac{\text{Delay}}{2} / \text{minTime}_{\text{window}} \rceil) \text{windowSize}$ , where
    $\text{maxTime}_{\text{window}}$ ,  $\text{Delay}$ ,  $\text{minTime}_{\text{window}}$  and  $\text{windowSize}$  are defined in
   Proposition ?? TODO: recheck and include proposition
2:  $C_{A,0} \leftarrow \text{TX}$  {input:  $(c, 2/\{pk_{A,F}, pk_{B,F}\})$ , outputs:  $(c, (pk_{A,\text{out}} + (t + s)) \vee$ 
    $2/\{pk_{A,R}, pk_{B,R}\})$ ,  $(0, pk_{B,\text{out}})\}$ 
3:  $C_{B,0} \leftarrow \text{TX}$  {input:  $(c, 2/\{pk_{A,F}, pk_{B,F}\})$ , outputs:  $(c, pk_{A,\text{out}})$ ,  $(0,$ 
    $(pk_{B,\text{out}} + (t + s)) \vee 2/\{pk_{A,R}, pk_{B,R}\})\}$ 
4:  $\text{sig}_{A,C,0} \leftarrow \text{SIGN}(C_{B,0}, sk_{A,F})$ 
5:  $\text{State} \leftarrow \text{WAITING FOR COMM SIG}$ 
6: send (FUNDING CREATED,  $(c, pk_{A,\text{chain}})$ ,  $\text{sig}_{A,C,0}$ ) to fundee
7: ensure  $\text{State} = \text{WAITING FOR COMM SIG}$  // if opening virtual channel, we have
   received (FUNDED, host_fundee) by hops[-1].right (Fig 14, l. 10)
8: if hops =  $\mathcal{G}_{\text{Ledger}}$  then // opening base channel
9:    $F \leftarrow \text{TX}$  {input:  $(c, pk_{A,\text{chain}})$ , output:  $(c, 2/\{pk_{A,F}, pk_{B,F}\})\}$ 
10: end if
11:  $C_{B,0} \leftarrow \text{TX}$  {input:  $(c, 2/\{pk_{A,F}, pk_{B,F}\})$ , outputs:  $(c, pk_{A,\text{out}})$ ,  $(0,$ 
    $(pk_{B,\text{out}} + (t + s)) \vee 2/\{pk_{A,R}, pk_{B,R}\})\}$ 
12: ensure  $\text{VERIFY}(C_{B,0}, \text{sig}_{A,C,0}, pk_{A,F}) = \text{True}$ 
13:  $C_{A,0} \leftarrow \text{TX}$  {input:  $(c, 2/\{pk_{A,F}, pk_{B,F}\})$ , outputs:  $(c, (pk_{A,\text{out}} + (t + s)) \vee$ 
    $2/\{pk_{A,R}, pk_{B,R}\})$ ,  $(0, pk_{B,\text{out}})\}$ 
14:  $\text{sig}_{B,C,0} \leftarrow \text{SIGN}(C_{A,0}, sk_{B,F})$ 
15: if hops =  $\mathcal{G}_{\text{Ledger}}$  then // opening base channel
16:    $\text{State} \leftarrow \text{WAITING TO CHECK FUNDING}$ 
17: else // opening virtual channel
18:    $c_A \leftarrow c$ ;  $c_B \leftarrow 0$ ;  $i \leftarrow 0$ 
19:    $\text{State} \leftarrow \text{OPEN}$ 
20: end if
21: reply (FUNDING SIGNED,  $\text{sig}_{B,C,0}$ )
22: ensure  $\text{State} = \text{WAITING FOR COMM SIG}$ 
23: ensure  $\text{VERIFY}(C_{A,0}, \text{sig}_{B,C,0}, pk_{B,F}) = \text{True}$ 

```

Fig. 12.

Process LN.COMMITBASE()

```

1:  $\text{sig}_F \leftarrow \text{SIGN}(F, sk_{A,\text{chain}})$ 
2: input (SUBMIT,  $(F, \text{sig}_F)$ ) to  $\mathcal{G}_{\text{Ledger}}$  // enter “while” below before sending
3: while  $F \notin \Sigma$  do
4:   wait for input (CHECK FUNDING) // ignore all other messages
5:   input (READ) to  $\mathcal{G}_{\text{Ledger}}$  and assign output to  $\Sigma$ 
6: end while

```

Fig. 13.

Process LN – external open messages for *Bob*

```

1: On input (CHECK FUNDING):
2:   ensure  $State = \text{WAITING TO CHECK FUNDING}$ 
3:   input (READ) to  $\mathcal{G}_{\text{Ledger}}$  and assign output to  $\Sigma$ 
4:   if  $F \in \Sigma$  then
5:      $State \leftarrow \text{OPEN}$ 
6:     reply (OPEN OK)
7:   end if

8: On output (FUNDED,  $host_P$ ,  $funder\_layer$ ,  $t_P$ ) by  $hops[-1].right$ :
9:   ensure  $State = \text{WAITING FOR FUNDED}$ 
10:  store  $host_P$  // we will talk directly to  $host_P$ 
11:   $layer \leftarrow funder\_layer$ 
12:   $State \leftarrow \text{WAITING FOR COMM SIG}$ 
13:  reply (FUND ACK)

14: On output (CHECK KEYS,  $(pk_1, pk_2)$ ) by  $hops[-1].right$ :
15:  ensure  $State = \text{WAITING FOR CHECK KEYS}$ 
16:  ensure  $pk_1 = pk_{A,F} \wedge pk_2 = pk_{B,F}$ 
17:   $State \leftarrow \text{WAITING FOR FUNDED}$ 
18:  reply (KEYS OK)

```

Fig. 14.

Process LN – On (OPEN, c , hops, fundee):

```

1: // fundee is Bob
2: ensure we run the code of Alice // activated party is the funder
3: if hops =  $\mathcal{G}_{\text{Ledger}}$  then // opening base channel
4:   ensure State = TOPPED UP
5:   ensure  $c = c_{A,\text{chain}}$ 
6: else // opening virtual channel
7:   ensure len(hops)  $\geq 2$  // cannot open a virtual over 1 channel
8: end if
9: LN.EXCHANGEOPENKEYS()
10: LN.PREPAREBASE()
11: LN.EXCHANGEOPENSIGS()
12: if hops =  $\mathcal{G}_{\text{Ledger}}$  then
13:   LN.COMMITBASE()
14: end if
15: input (READ) to  $\mathcal{G}_{\text{Ledger}}$  and assign output to  $\Sigma$ 
16: last_poll  $\leftarrow |\Sigma|$ 
17:  $c_A \leftarrow c$ ;  $c_B \leftarrow 0$ ;  $i \leftarrow 0$ 
18: State  $\leftarrow$  OPEN
19: output (OPEN OK,  $c$ , fundee, hops)

```

Fig. 15.

Process LN.UPDATEFORVIRTUAL()

```

1:  $C_{\bar{P},i+1} \leftarrow C_{\bar{P},i}$  with  $pk'_{P,F}$  and  $pk'_{\bar{P},F}$  instead of  $pk_{P,F}$  and  $pk_{\bar{P},F}$  respectively,
   reducing the input and  $\bar{P}$ 's output by  $c_{\text{virt}}$ 
2:  $\text{sig}_{P,C,i+1} \leftarrow \text{SIGN}(C_{\bar{P},i+1})$  // kept by  $\bar{P}$ 
3: send (UPDATE FORWARD,  $\text{sig}_{P,C,i+1}$ ) to  $\bar{P}$ 
4: //  $P$  refers to payer and  $\bar{P}$  to payee both in local and remote code
5:  $C_{\bar{P},i+1} \leftarrow C_{\bar{P},i}$  with  $pk'_{P,F}$  and  $pk'_{\bar{P},F}$  instead of  $pk_{P,F}$  and  $pk_{\bar{P},F}$  respectively,
   reducing the input and  $\bar{P}$ 's output by  $c_{\text{virt}}$ 
6: ensure  $\text{VERIFY}(C_{\bar{P},i+1}, \text{sig}_{P,C,i+1}, pk'_{P,F}) = \text{True}$ 
7:  $C_{P,i+1} \leftarrow C_{P,i}$  with  $pk'_{\bar{P},F}$  and  $pk'_{P,F}$  instead of  $pk_{\bar{P},F}$  and  $pk_{P,F}$  respectively,
   reducing the input and  $P$ 's output by  $c_{\text{virt}}$ 
8:  $\text{sig}_{\bar{P},C,i+1} \leftarrow \text{SIGN}(C_{P,i+1}, sk'_{\bar{P},F})$  // kept by  $P$ 
9: reply (UPDATE BACK,  $\text{sig}_{\bar{P},C,i+1}$ )
10:  $C_{P,i+1} \leftarrow C_{P,i}$  with  $pk'_{\bar{P},F}$  and  $pk'_{P,F}$  instead of  $pk_{\bar{P},F}$  and  $pk_{P,F}$  respectively,
   reducing the input and  $P$ 's output by  $c_{\text{virt}}$ 
11: ensure  $\text{VERIFY}(C_{P,i+1}, \text{sig}_{\bar{P},C,i+1}, pk'_{\bar{P},F}) = \text{True}$ 

```

Fig. 16.

Process LN – virtualise start and end

- 1: On input (FUND ME, c_{virt} , **fundee**, **hops**, $pk_{A,V}$, $pk_{B,V}$) by **funder**:
- 2: ensure $State = \text{OPEN}$
- 3: ensure $c_P - \text{locked}_P \geq c_{\text{virt}}$
- 4: $State \leftarrow \text{VIRTUALISING}$
- 5: $(sk'_{P,F}, pk'_{P,F}) \leftarrow \text{KEYGEN}()$
- 6: define new VIRT ITI host'_P
- 7: send (VIRTUALISING, host'_P , $pk'_{P,F}$, **hops**, **fundee**, c_{virt} , 2, $\text{len}(\text{hops})$) to \bar{P}
and expect reply (VIRTUALISING ACK, $\text{host}'_{\bar{P}}$, $pk'_{\bar{P},F}$)
- 8: ensure $pk'_{\bar{P},F}$ is different from $pk_{\bar{P},F}$ and all older \bar{P} 's funding public keys
- 9: LN.UPDATEFORVIRTUAL()
- 10: $State \leftarrow \text{WAITING FOR REVOCATION}$
- 11: input (HOST ME, **funder**, **fundee**, $\text{host}'_{\bar{P}}$, host_P , c_P , $c_{\bar{P}}$, c_{virt} , $pk_{A,V}$, $pk_{B,V}$,
 $(sk'_{P,F}, pk'_{P,F})$, $(sk_{P,F}, pk_{P,F})$, $pk_{\bar{P},F}$, $pk'_{\bar{P},F}$, $pk_{P,\text{out}}$, $\text{len}(\text{hops})$) to host'_P
- 12: On output (HOSTS READY, t_P) by host_P : // host_P is the new host, renamed
in Fig. 9, l. 12
- 13: ensure $State = \text{WAITING FOR HOSTS READY}$
- 14: $State \leftarrow \text{OPEN}$
- 15: move $pk_{P,F}$, $pk_{\bar{P},F}$ to list of old funding keys
- 16: $(sk_{P,F}, pk_{P,F}) \leftarrow (sk'_{P,F}, pk'_{P,F})$; $pk_{\bar{P},F} \leftarrow pk'_{\bar{P},F}$
- 17: **if** $\text{len}(\text{hops}) = 1$ **then** // we are the last hop
- 18: output (FUNDED, host_P , **layer**, t_P) to **fundee** and expect reply (FUND
ACK)
- 19: **else if** we have received input FUND ME just before we moved to the
VIRTUALISING state **then** // we are the first hop
- 20: $c_P \leftarrow c_P - c_{\text{virt}}$
- 21: output (FUNDED, host_P , **layer**, t_P) to **funder** // do not expect reply
by **funder**
- 22: **end if**
- 23: reply (HOST ACK)

Fig. 17.

Process LN – virtualise hops

```

1: On (VIRTUALISING,  $\text{host}'_{\bar{P}}$ ,  $pk'_{\bar{P},F}$ ,  $\text{hops}$ ,  $\text{fundee}$ ,  $c_{\text{virt}}$ ,  $i$ ,  $n$ ) by  $\bar{P}$ :
2:   ensure  $\text{State} = \text{OPEN}$ 
3:   ensure  $c_{\bar{P}} - \text{locked}_{\bar{P}} \geq c$ ;  $1 \leq i \leq n$ 
4:   ensure  $pk'_{\bar{P},F}$  is different from  $pk_{\bar{P},F}$  and all older  $\bar{P}$ 's funding public keys
5:    $\text{State} \leftarrow \text{VIRTUALISING}$ 
6:    $\text{locked}_{\bar{P}} \leftarrow \text{locked}_{\bar{P}} + c$  // if  $\bar{P}$  is hosting the funder,  $\bar{P}$  will transfer  $c_{\text{virt}}$ 
   coins instead of locking them, but the end result is the same
7:    $(sk'_{P,F}, pk'_{P,F}) \leftarrow \text{KEYGEN}()$ 
8:   if  $\text{len}(\text{hops}) > 1$  then // we are not the last hop
9:     define new VIRT ITI  $\text{host}'_P$ 
10:    input (VIRTUALISING,  $\text{host}'_P$ ,  $(sk'_{P,F}, pk'_{P,F})$ ,  $pk'_{\bar{P},F}$ ,  $pk_{P,\text{out}}$ ,  $\text{hops}[1:]$ ,
     $\text{fundee}$ ,  $c_{\text{virt}}$ ,  $c_{\bar{P}}$ ,  $c_P$ ,  $i$ ,  $n$ ) to  $\text{hops}[1].\text{left}$  and expect reply (VIRTUALISING
    ACK,  $\text{host\_sibling}$ ,  $pk_{\text{sib},\bar{P},F}$ )
11:    input (INIT,  $\text{host}_P$ ,  $\text{host}'_{\bar{P}}$ ,  $\text{host\_sibling}$ ,  $(sk'_{P,F}, pk'_{P,F})$ ,  $pk'_{\bar{P},F}$ ,
     $pk_{\text{sib},\bar{P},F}$ ,  $(sk_{P,F}, pk_{P,F})$ ,  $pk_{\bar{P},F}$ ,  $pk_{P,\text{out}}$ ,  $c_P$ ,  $c_{\bar{P}}$ ,  $c_{\text{virt}}$ ,  $i$ ,  $t_P$ , "left",  $n$ ) to  $\text{host}'_P$ 
    and expect reply (HOST INIT OK)
12:   else // we are the last hop
13:     input (INIT,  $\text{host}_P$ ,  $\text{host}'_{\bar{P}}$ ,  $\text{fundee}=\text{fundee}$ ,  $(sk'_{P,F}, pk'_{P,F})$ ,  $pk'_{\bar{P},F}$ ,
     $(sk_{P,F}, pk_{P,F})$ ,  $pk_{\bar{P},F}$ ,  $pk_{P,\text{out}}$ ,  $c_P$ ,  $c_{\bar{P}}$ ,  $c_{\text{virt}}$ ,  $t_P$ ,  $i$ , "left",  $n$ ) to new VIRT ITI
     $\text{host}'_P$  and expect reply (HOST INIT OK)
14:   end if
15:    $\text{State} \leftarrow \text{WAITING FOR REVOCATION}$ 
16:   send (VIRTUALISING ACK,  $\text{host}'_P$ ,  $pk'_{P,F}$ ) to  $\bar{P}$ 

17: On input (VIRTUALISING,  $\text{host\_sibling}$ ,  $(sk'_{P,F}, pk'_{P,F})$ ,  $pk_{\text{sib},\bar{P},F}$ ,  $pk_{\text{sib},\text{out}}$ ,
     $\text{hops}$ ,  $\text{fundee}$ ,  $c_{\text{virt}}$ ,  $c_{\text{sib},\text{rem}}$ ,  $c_{\text{sib}}$ ,  $i$ ,  $n$ ) by  $\text{sibling}$ :
18:   ensure  $\text{State} = \text{OPEN}$ 
19:   ensure  $c_P - \text{locked}_P \geq c$ 
20:   ensure  $c_{\text{sib},\text{rem}} \geq c_P \wedge c_{\bar{P}} \geq c_{\text{sib}}$  // avoid value loss by griefing attack: one
   counterparty closes with old version, the other stays idle forever
21:    $\text{State} \leftarrow \text{VIRTUALISING}$ 
22:    $\text{locked}_P \leftarrow \text{locked}_P + c$ 
23:   define new VIRT ITI  $\text{host}'_P$ 
24:   send (VIRTUALISING,  $\text{host}'_P$ ,  $pk'_{P,F}$ ,  $\text{hops}$ ,  $\text{fundee}$ ,  $c_{\text{virt}}$ ,  $i+1$ ,  $n$ ) to
     $\text{hops}[0].\text{right}$  and expect reply (VIRTUALISING ACK,  $\text{host}'_{\bar{P}}$ ,  $pk'_{\bar{P},F}$ )
25:   ensure  $pk'_{\bar{P},F}$  is different from  $pk_{\bar{P},F}$  and all older  $\bar{P}$ 's funding public keys
26:   LN.UPDATEFORVIRTUAL()
27:   input (INIT,  $\text{host}_P$ ,  $\text{host}'_{\bar{P}}$ ,  $\text{host\_sibling}$ ,  $(sk'_{P,F}, pk'_{P,F})$ ,  $pk'_{\bar{P},F}$ ,  $pk_{\text{sib},\bar{P},F}$ ,
     $(sk_{P,F}, pk_{P,F})$ ,  $pk_{\bar{P},F}$ ,  $pk_{\text{sib},\text{out}}$ ,  $c_P$ ,  $c_{\bar{P}}$ ,  $c_{\text{virt}}$ ,  $i$ , "right",  $n$ ) to  $\text{host}'_P$  and expect
    reply (HOST INIT OK)
28:    $\text{State} \leftarrow \text{WAITING FOR REVOCATION}$ 
29:   output (VIRTUALISING ACK,  $\text{host}'_P$ ,  $pk'_{P,F}$ ) to  $\text{sibling}$ 

```

Fig. 18.

Process LN.SIGNATURESROUNDTRIP()

- 1: $C_{\bar{P},i+1} \leftarrow C_{\bar{P},i}$ with x coins moved from P 's to \bar{P} 's output
- 2: $\text{sig}_{P,C,i+1} \leftarrow \text{SIGN}(C_{\bar{P},i+1}, sk_{P,F})$ // kept by \bar{P}
- 3: $State \leftarrow \text{WAITING FOR COMMITMENT SIGNED}$
- 4: send (PAY, x , $\text{sig}_{P,C,i+1}$) to \bar{P}
- 5: // P refers to payer and \bar{P} to payee both in local and remote code
- 6: ensure $State = \text{WAITING TO GET PAID} \wedge x = y$
- 7: $C_{\bar{P},i+1} \leftarrow C_{\bar{P},i}$ with x coins moved from P 's to \bar{P} 's output
- 8: ensure $\text{VERIFY}(C_{\bar{P},i+1}, \text{sig}_{P,C,i+1}, pk_{P,F}) = \text{True}$
- 9: $C_{P,i+1} \leftarrow C_{P,i}$ with x coins moved from P 's to \bar{P} 's output
- 10: $\text{sig}_{\bar{P},C,i+1} \leftarrow \text{SIGN}(C_{P,i+1}, sk_{\bar{P},F})$ // kept by P
- 11: $R_{P,i} \leftarrow \text{TX}$ {input: $C_{\bar{P},i}.\text{outputs}.P$, output: $(c_{\bar{P}}, pk_{P,\text{out}})$ }
- 12: $\text{sig}_{\bar{P},R,i} \leftarrow \text{SIGN}(R_{P,i}, sk_{\bar{P},R})$
- 13: $State \leftarrow \text{WAITING FOR PAY REVOCATION}$
- 14: reply (COMMITMENT SIGNED, $\text{sig}_{\bar{P},C,i+1}$, $\text{sig}_{\bar{P},R,i}$)
- 15: ensure $State = \text{WAITING FOR COMMITMENT SIGNED}$
- 16: $C_{P,i+1} \leftarrow C_{P,i}$ with x coins moved from P 's to \bar{P} 's output

Fig. 19.

Process LN.REVOCATIONSTRIP()

```

1: ensure VERIFY( $C_{P,i+1}$ ,  $\text{sig}_{\bar{P},C,i+1}$ ,  $pk_{\bar{P},F}$ ) = True
2:  $R_{P,i} \leftarrow \text{TX}$  {input:  $C_{\bar{P},i}.\text{outputs}.\bar{P}$ , output: ( $c_{\bar{P}}$ ,  $pk_{P,\text{out}}$ )}
3: ensure VERIFY( $R_{P,i}$ ,  $\text{sig}_{\bar{P},R,i}$ ,  $pk_{\bar{P},R}$ ) = True
4:  $R_{\bar{P},i} \leftarrow \text{TX}$  {input:  $C_{P,i}.\text{outputs}.P$ , output: ( $c_P$ ,  $pk_{\bar{P},\text{out}}$ )}
5:  $\text{sig}_{P,R,i} \leftarrow \text{SIGN}(R_{\bar{P},i}, sk_{P,R})$ 
6: add  $x$  to paid_out
7:  $c_P \leftarrow c_P - x$ ;  $c_{\bar{P}} \leftarrow c_{\bar{P}} + x$ ;  $i \leftarrow i + 1$ 
8:  $State \leftarrow \text{OPEN}$ 
9: if  $\text{host}_P \neq \mathcal{G}_{\text{Ledger}} \wedge$  we have a host_sibling then // we are intermediary
   channel
10:   input (NEW BALANCE,  $c_P$ ,  $c_{\bar{P}}$ ) to host_P
11:   relay message as input to sibling // run by VIRT
12:   relay message as output to guest // run by VIRT
13:   store new sibling balance and reply (NEW BALANCE OK)
14:   output (NEW BALANCE OK) to sibling // run by VIRT
15:   output (NEW BALANCE OK) to guest // run by VIRT
16: end if
17: send (REVOKE AND ACK,  $\text{sig}_{P,R,i}$ ) to  $\bar{P}$ 
18: ensure  $State = \text{WAITING FOR PAY REVOCATION}$ 
19:  $R_{\bar{P},i} \leftarrow \text{TX}$  {input:  $C_{P,i}.\text{outputs}.\bar{P}$ , output: ( $c_P$ ,  $pk_{\bar{P},\text{out}}$ )}
20: ensure VERIFY( $R_{\bar{P},i}$ ,  $\text{sig}_{P,R,i}$ ,  $pk_{P,R}$ ) = True
21: add  $x$  to paid_in
22:  $c_P \leftarrow c_P - x$ ;  $c_{\bar{P}} \leftarrow c_{\bar{P}} + x$ ;  $i \leftarrow i + 1$ 
23:  $State \leftarrow \text{OPEN}$ 
24: if  $\text{host}_P \neq \mathcal{G}_{\text{Ledger}} \wedge \bar{P}$  has a host_sibling then // we are intermediary
   channel
25:   input (NEW BALANCE,  $c_{\bar{P}}$ ,  $c_P$ ) to host_P
26:   relay message as input to sibling // run by VIRT
27:   relay message as output to guest // run by VIRT
28:   store new sibling balance and reply (NEW BALANCE OK)
29:   output (NEW BALANCE OK) to sibling // run by VIRT
30:   output (NEW BALANCE OK) to guest // run by VIRT
31: end if

```

Fig. 20.

Process LN – On (PAY, x):

- 1: ensure $State = OPEN \wedge c_P \geq x$
- 2: **if** $host_P \neq \mathcal{G}_{Ledger} \wedge P$ has a **host_sibling** **then** // we are intermediary channel
- 3: **ensure** $c_{sib,rem} \geq c_P - x \wedge c_{\bar{P}} + x \geq c_{sib}$ // avoid value loss by grieving attack: one counterparty closes with old version, the other stays idle forever
- 4: **end if**
- 5: LN.SIGNATURESROUNDTRIP()
- 6: LN.REVOCATIONSROUNDTRIP()
- 7: // No output is given to the caller, this is intentional

Fig. 21.

Process LN – On (GET PAID, y):

- 1: ensure $State = OPEN \wedge c_{\bar{P}} \geq y$
- 2: **if** $host_P \neq \mathcal{G}_{Ledger} \wedge P$ has a **host_sibling** **then** // we are intermediary channel
- 3: **ensure** $c_P + y \leq c_{sib,rem} \wedge c_{sib} \leq c_{\bar{P}} - y$ // avoid value loss by grieving attack
- 4: **end if**
- 5: store y
- 6: $State \leftarrow WAITING\ TO\ GET\ PAID$

Fig. 22.

Process LN – On (CHECK FOR LATERAL CLOSE):

- 1: **if** $host_P \neq \mathcal{G}_{Ledger}$ **then**
- 2: input (CHECK FOR LATERAL CLOSE) to $host_P$
- 3: **end if**

Fig. 23.

Process LN – On (CHECK CHAIN FOR CLOSED):

```

1: ensure  $State \notin \{\perp, \text{INIT}, \text{TOPPED UP}\}$  // channel open
2: // even virtual channels check  $\mathcal{G}_{\text{Ledger}}$  directly. This is intentional
3: input (READ) to  $\mathcal{G}_{\text{Ledger}}$  and assign reply to  $\Sigma$ 
4:  $\text{last\_poll} \leftarrow |\Sigma|$ 
5: if  $\exists 0 \leq j < i : C_{\bar{P},j} \in \Sigma$  then // counterparty has closed maliciously
6:    $State \leftarrow \text{CLOSING}$ 
7:   LN.SUBMITANDCHECKREVOCATION( $j$ )
8:    $State \leftarrow \text{CLOSED}$ 
9:   output (CLOSED)
10: else if  $C_{P,j} \in \Sigma \wedge C_{\bar{P},j} \in \Sigma$  then
11:    $State \leftarrow \text{CLOSED}$ 
12:   output (CLOSED)
13: end if

```

Fig. 24.

Process LN.SUBMITANDCHECKREVOCATION(j)

```

1:  $\text{sig}_{P,R,j} \leftarrow \text{SIGN}(R_{P,j}, sk_{P,R})$ 
2: input (SUBMIT, ( $R_{P,j}$ ,  $\text{sig}_{P,R,j}$ ,  $\text{sig}_{\bar{P},R,j}$ )) to  $\mathcal{G}_{\text{Ledger}}$ 
3: while  $\nexists R_{P,j} \in \Sigma$  do
4:   wait for input (CHECK REVOCATION) // ignore other messages
5:   input (READ) to  $\mathcal{G}_{\text{Ledger}}$  and assign output to  $\Sigma$ 
6: end while
7:  $c_P \leftarrow c_P + c_{\bar{P}}$ 
8: if  $\text{host}_P \neq \mathcal{G}_{\text{Ledger}}$  then
9:   input (USED REVOCATION) to  $\text{host}_P$ 
10: end if

```

Fig. 25.

Process LN – On (CLOSE):

```

1: ensure  $State \notin \{\perp, \text{INIT}, \text{TOPPED UP}, \text{CLOSED}, \text{BASE PUNISHED}\}$  // channel open
2: if  $\text{host}_P \neq \mathcal{G}_{\text{Ledger}}$  then // we have a virtual channel
3:    $State \leftarrow \text{HOST CLOSING}$ 
4:   input (CLOSE) to  $\text{host}_P$  and keep relaying inputs (CHECK IF CLOSING) to
      $\text{host}_P$  until receiving output (CLOSED) by  $\text{host}_P$ 
5:    $\text{host}_P \leftarrow \mathcal{G}_{\text{Ledger}}$ 
6: end if
7:  $State \leftarrow \text{CLOSING}$ 
8: input (READ) to  $\mathcal{G}_{\text{Ledger}}$  and assign output to  $\Sigma$ 
9: if  $C_{\bar{P},i} \in \Sigma$  then // counterparty has closed honestly
10:  no-op // do nothing
11: else if  $\exists 0 \leq j < i : C_{\bar{P},j} \in \Sigma$  then // counterparty has closed maliciously
12:  LN.SUBMITANDCHECKREVOCATION( $j$ )
13: else // counterparty is idle
14:  while  $\nexists$  unspent output  $\in \Sigma$  that  $C_{P,i}$  can spend do // possibly due to an
    active timelock
15:    wait for input (CHECK VIRTUAL) // ignore other messages
16:    input (READ) to  $\mathcal{G}_{\text{Ledger}}$  and assign output to  $\Sigma$ 
17:  end while
18:   $\text{sig}'_{P,C,i} \leftarrow \text{SIGN}(C_{P,i}, sk_{P,F})$ 
19:  input (SUBMIT,  $(C_{P,i}, \text{sig}_{P,C,i}, \text{sig}'_{P,C,i})$ ) to  $\mathcal{G}_{\text{Ledger}}$ 
20: end if

```

Fig. 26.

Process LN – On output (ENABLER USED REVOCATION) by host_P :

```

1:  $State \leftarrow \text{BASE PUNISHED}$ 

```

Fig. 27.

Process VIRT

```

1: On every activation, before handling the message:
2:   if  $\text{last\_poll} \neq \perp$  then // virtual layer is ready
3:     input (READ) to  $\mathcal{G}_{\text{Ledger}}$  and assign output to  $\Sigma$ 
4:     if  $\text{last\_poll} + p < |\Sigma|$  then
5:       for  $P \in \{\text{guest}, \text{funder}, \text{fundee}\}$  do // at most 1 of funder, fundee
        is defined
7:         ensure  $P.\text{NEGLIGENT}()$  returns (OK)
8:       end for
9:     end if

```

```

9:   end if

10: // guest is trusted to give sane inputs, therefore a state machine and input
    verification are redundant
11: On input (INIT, hostP,  $\bar{P}$ , sibling, fundee, ( $sk_{loc,fund,new}$ ,  $pk_{loc,fund,new}$ ),
     $pk_{rem,fund,new}$ ,  $pk_{sib,rem,fund,new}$ , ( $sk_{loc,fund,old}$ ,  $pk_{loc,fund,old}$ ),  $pk_{rem,fund,old}$ ,
     $pk_{loc,out}$ ,  $c_P$ ,  $c_{\bar{P}}$ ,  $c_{virt}$ ,  $t_P$ ,  $i$ , side,  $n$ ) by guest:
12:   ensure  $1 < i \leq n$  // host_funder ( $i = 1$ ) is initialised with HOST ME
13:   ensure side  $\in \{\text{"left"}, \text{"right"}\}$ 
14:   store message contents and guest // sibling,  $pk_{sib,\bar{P},F}$  are missing for
    endpoints, fundee is present only in last node
15:   ( $sk_{i,fund,new}$ ,  $pk_{i,fund,new}$ )  $\leftarrow$  ( $sk_{loc,fund,new}$ ,  $pk_{loc,fund,new}$ )
16:    $pk_{myRem,fund,new} \leftarrow pk_{rem,fund,new}$ 
17:   if  $i < n$  then // we are not last hop
18:      $pk_{sibRem,fund,new} \leftarrow pk_{sib,rem,fund,new}$ 
19:   end if
20:   if side = "left" then
21:     side'  $\leftarrow$  "right"; myRem  $\leftarrow i - 1$ ; sibRem  $\leftarrow i + 1$ 
22:   else // side = "right"
23:     side'  $\leftarrow$  "left"; myRem  $\leftarrow i + 1$ ; sibRem  $\leftarrow i - 1$ 
24:   end if
25:   ( $sk_{i,side,fund,old}$ ,  $pk_{i,side,fund,old}$ )  $\leftarrow$  ( $sk_{loc,fund,old}$ ,  $pk_{loc,fund,old}$ )
26:    $pk_{myRem,side',fund,old} \leftarrow pk_{rem,fund,old}$ 
27:   if side = "left" then
28:      $pk_{i,out} \leftarrow pk_{loc,out}$ 
29:   end if // otherwise sibling will send  $pk_{i,out}$  in KEYS AND COINS FORWARD
30:   ( $c_{i,side}$ ,  $c_{myRem,side'}$ ,  $t_{i,side}$ )  $\leftarrow$  ( $c_P$ ,  $c_{\bar{P}}$ ,  $t_P$ )
31:   last_poll  $\leftarrow \perp$ 
32:   if side = "left"  $\wedge i \neq n$  then
33:     ( $sk_{i,j,k}$ ,  $pk_{i,j,k}$ ) $j \in \{2, \dots, n-1\}, k \in [n] \setminus \{j\}$   $\leftarrow$  KEYGEN() $(n-2)(n-1)$ 
34:   end if
35:   output (HOST INIT OK) to guest

36: On input (HOST ME, funder, fundee,  $\bar{P}$ , hostP,  $c_P$ ,  $c_{\bar{P}}$ ,  $c_{virt}$ ,  $pk_{left,virt}$ ,
     $pk_{right,virt}$ , ( $sk_{1,fund,new}$ ,  $pk_{1,fund,new}$ ), ( $sk_{1,right,fund,old}$ ,  $pk_{1,right,fund,old}$ ),
     $pk_{2,left,fund,old}$ ,  $pk_{2,left,fund,new}$ ,  $pk_{1,out}$ ,  $n$ ) by guest:
37:   last_poll  $\leftarrow \perp$ 
38:    $i \leftarrow 1$ 
39:    $c_{1,right} \leftarrow c_P$ ;  $c_{2,left} \leftarrow c_{\bar{P}}$ 
40:   ( $sk_{1,j,k}$ ,  $pk_{1,j,k}$ ) $j \in \{2, \dots, n-1\}, k \in [n] \setminus \{j\}$   $\leftarrow$  KEYGEN() $(n-2)(n-1)$ 
41:   ensure VIRT.CIRCULATEKEYSCOINSTIMES() returns (OK)
42:   ensure VIRT.CIRCULATEVIRTUALSIGS() returns (OK)
43:   ensure VIRT.CIRCULATEFUNDINGSIGS() returns (OK)
44:   ensure VIRT.CIRCULATEREVOCATIONS() returns (OK)
45:   output (HOSTS READY,  $p + \sum_{j=2}^{n-1} (s - 1 + t_j)$ ) to guest //  $p$  is every how
    many blocks we have to check the chain

```

Fig. 28.

```

Process VIRT.CIRCULATEKEYSCOINSTIMES(left_data):
1: if left_data is given as argument then // we are not host_funder
2:   parse left_data as  $((pk_{j,\text{fund,new}})_{j \in [i-1]}, (pk_{j,\text{left,fund,old}})_{j \in \{2, \dots, i-1\}},$ 
    $(pk_{j,\text{right,fund,old}})_{j \in [i-1]}, (pk_{j,\text{out}})_{j \in [i-1]}, (c_{j,\text{left}})_{j \in \{2, \dots, i-1\}}, (c_{j,\text{right}})_{j \in [i-1]},$ 
    $(t_j)_{j \in [i-1]}, pk_{\text{left,virt}}, pk_{\text{right,virt}}, (pk_{h,j,k})_{h \in [i-1], j \in \{2, \dots, n-1\}, k \in [n] \setminus \{j\}})$ 
3:   if we have a sibling then // we are not host_fundee
4:     input (KEYS AND COINS FORWARD, (left_data,  $(sk_{i,\text{left,fund,old}},$ 
    $pk_{i,\text{left,fund,old}}), pk_{i,\text{out}}, c_{i,\text{left}}, t_{i,\text{left}}, (sk_{i,j,k}, pk_{i,j,k})_{j \in \{2, \dots, n-1\}, k \in [n] \setminus \{j\}}$ ) to
   sibling
5:     store input as left_data and parse it as  $((pk_{j,\text{fund,new}})_{j \in [i-1]},$ 
    $(pk_{j,\text{left,fund,old}})_{j \in \{2, \dots, i\}}, (pk_{j,\text{right,fund,old}})_{j \in [i-1]}, (pk_{j,\text{out}})_{j \in [i]}, (c_{j,\text{left}})_{j \in \{2, \dots, i\}},$ 
    $(c_{j,\text{right}})_{j \in [i-1]}, (t_j)_{j \in [i-1]}, sk_{i,\text{left,fund,old}}, t_{i,\text{left}}, pk_{\text{left,virt}}, pk_{\text{right,virt}},$ 
    $(pk_{h,j,k})_{h \in [i], j \in \{2, \dots, n-1\}, k \in [n] \setminus \{j\}}, (sk_{i,j,k})_{j \in \{2, \dots, n-1\}, k \in [n] \setminus \{j\}})$ 
6:      $t_i \leftarrow \max(t_{i,\text{left}}, t_{i,\text{right}})$ 
7:     replace  $t_{i,\text{left}}$  in left_data with  $t_i$ 
8:     remove  $sk_{i,\text{left,fund,old}}$  and  $(sk_{i,j,k})_{j \in \{2, \dots, n-1\}, k \in [n] \setminus \{j\}}$  from left_data
9:     call VIRT.CIRCULATEKEYSCOINSTIMES(left_data) of  $\bar{P}$  and assign
   returned value to right_data
10:    parse right_data as  $((pk_{j,\text{fund,new}})_{j \in \{i+1, \dots, n\}},$ 
    $(pk_{j,\text{left,fund,old}})_{j \in \{i+1, \dots, n\}}, (pk_{j,\text{right,fund,old}})_{j \in \{i+1, \dots, n-1\}}, (pk_{j,\text{out}})_{j \in \{i+1, \dots, n\}},$ 
    $(c_{j,\text{left}})_{j \in \{i+1, \dots, n\}}, (c_{j,\text{right}})_{j \in \{i+1, \dots, n-1\}}, (t_j)_{j \in \{i+1, \dots, n\}},$ 
    $(pk_{h,j,k})_{h \in \{i+1, \dots, n\}, j \in \{2, \dots, n-1\}, k \in [n] \setminus \{j\}})$ 
11:    output (KEYS AND COINS BACK, right_data,  $(sk_{i,\text{right,fund,old}},$ 
    $pk_{i,\text{right,fund,old}}), c_{i,\text{right}}, t_i)$ 
12:    store output as right_data and parse it as  $((pk_{j,\text{fund,new}})_{j \in \{i+1, \dots, n\}},$ 
    $(pk_{j,\text{left,fund,old}})_{j \in \{i+1, \dots, n\}}, (pk_{j,\text{right,fund,old}})_{j \in \{i, \dots, n-1\}}, (pk_{j,\text{out}})_{j \in \{i+1, \dots, n\}},$ 
    $(c_{j,\text{left}})_{j \in \{i+1, \dots, n\}}, (c_{j,\text{right}})_{j \in \{i, \dots, n-1\}}, (t_j)_{j \in \{i, \dots, n\}},$ 
    $(pk_{h,j,k})_{h \in \{i+1, \dots, n\}, j \in \{2, \dots, n-1\}, k \in [n] \setminus \{j\}}, sk_{i,\text{right,fund,old}})$ 
13:    remove  $sk_{i,\text{right,fund,old}}$  from right_data
14:    return (right_data,  $pk_{i,\text{fund,new}}, pk_{i,\text{left,fund,old}}, pk_{i,\text{out}}, c_{i,\text{left}})$ 
15:  else // we are host_fundee
16:    output (CHECK KEYS,  $(pk_{\text{left,virt}}, pk_{\text{right,virt}})$ ) to fundee and expect
    reply (KEYS OK)
17:    return  $(pk_{n,\text{fund,new}}, pk_{n,\text{left,fund,old}}, pk_{n,\text{out}}, c_{n,\text{left}}, t_n)$ 
18:  end if
19: else // we are host_funder
20:   call VIRT.CIRCULATEKEYSCOINSTIMES( $pk_{1,\text{fund,new}}, pk_{1,\text{right,fund,old}}, pk_{1,\text{out}},$ 
    $c_{1,\text{right}}, t_1, pk_{\text{left,virt}}, pk_{\text{right,virt}}, (pk_{1,j,k})_{j \in \{2, \dots, n-1\}, k \in [n] \setminus \{j\}}$ ) of  $\bar{P}$  and assign
   returned value to right_data
21:   parse right_data as  $((pk_{j,\text{fund,new}})_{j \in \{2, \dots, n\}}, (pk_{j,\text{left,fund,old}})_{j \in \{2, \dots, n\}},$ 
    $(pk_{j,\text{right,fund,old}})_{j \in \{2, \dots, n-1\}}, (pk_{j,\text{out}})_{j \in \{2, \dots, n\}}, (c_{j,\text{left}})_{j \in \{2, \dots, n\}},$ 
    $(c_{j,\text{right}})_{j \in \{2, \dots, n-1\}}, (t_j)_{j \in \{2, \dots, n\}}, (pk_{h,j,k})_{h \in \{2, \dots, n\}, j \in \{2, \dots, n-1\}, k \in [n] \setminus \{j\}})$ 
22:   return (OK)
23: end if

```

Fig. 29.

Process VIRT

```

1: GETMIDTXS( $i, n, c_{\text{virt}}, c_{\text{rem},\text{left}}, c_{\text{loc},\text{left}}, c_{\text{loc},\text{right}}, c_{\text{rem},\text{right}}, pk_{\text{rem},\text{left},\text{fund},\text{old}},$ 
 $pk_{\text{loc},\text{left},\text{fund},\text{old}}, pk_{\text{loc},\text{right},\text{fund},\text{old}}, pk_{\text{rem},\text{right},\text{fund},\text{old}}, pk_{\text{rem},\text{left},\text{fund},\text{new}},$ 
 $pk_{\text{loc},\text{left},\text{fund},\text{new}}, pk_{\text{loc},\text{right},\text{fund},\text{new}}, pk_{\text{rem},\text{right},\text{fund},\text{new}}, pk_{\text{left},\text{virt}}, pk_{\text{right},\text{virt}},$ 
 $pk_{\text{loc},\text{out}}, (pk_{p,j,k})_{p \in [n], j \in [n-1] \setminus \{1\}, k \in [n-1] \setminus \{1,j\}}, (pk_{p,2,1})_{p \in [n]}, (pk_{p,n-1,n})_{p \in [n]},$ 
 $(t_j)_{j \in [n-1] \setminus \{1\}})$ :
2:   ensure  $1 < i < n$ 
3:   ensure  $c_{\text{rem},\text{left}} \geq c_{\text{virt}} \wedge c_{\text{loc},\text{left}} \geq c_{\text{virt}}$  // left parties fund virtual channel
4:   ensure  $c_{\text{rem},\text{left}} \geq c_{\text{loc},\text{right}} \wedge c_{\text{rem},\text{right}} \geq c_{\text{loc},\text{left}}$  // avoid griefing attack
5:    $c_{\text{left}} \leftarrow c_{\text{rem},\text{left}} + c_{\text{loc},\text{left}}; c_{\text{right}} \leftarrow c_{\text{loc},\text{right}} + c_{\text{rem},\text{right}}$ 
6:    $\text{left\_old\_fund} \leftarrow 2/\{pk_{\text{rem},\text{left},\text{fund},\text{old}}, pk_{\text{loc},\text{left},\text{fund},\text{old}}\}$ 
7:    $\text{right\_old\_fund} \leftarrow 2/\{pk_{\text{loc},\text{right},\text{fund},\text{old}}, pk_{\text{rem},\text{right},\text{fund},\text{old}}\}$ 
8:    $\text{left\_new\_fund} \leftarrow 2/\{pk_{\text{rem},\text{left},\text{fund},\text{new}}, pk_{\text{loc},\text{left},\text{fund},\text{new}}\}$ 
9:    $\text{right\_new\_fund} \leftarrow 2/\{pk_{\text{loc},\text{right},\text{fund},\text{new}}, pk_{\text{rem},\text{right},\text{fund},\text{new}}\}$ 
10:   $\text{virt\_fund} \leftarrow 2/\{pk_{\text{left},\text{virt}}, pk_{\text{right},\text{virt}}\}$ 
11:  for all  $j \in [n-1] \setminus \{1\}, k \in [n-1] \setminus \{1,j\}$  do
12:     $all_{j,k} \leftarrow n/\{pk_{1,j,k}, \dots, pk_{n,j,k}\} \wedge "k"$ 
13:  end for
14:  if  $i = 2$  then
15:     $all_{2,1} \leftarrow n/\{pk_{1,2,1}, \dots, pk_{n,2,1}\} \wedge "1"$ 
16:  end if
17:  if  $i = n-1$  then
18:     $all_{n-1,n} \leftarrow n/\{pk_{1,n-1,n}, \dots, pk_{n,n-1,n}\} \wedge "n"$ 
19:  end if
20:  // After funding is complete,  $A_j$  has the signature of all other parties for
  all  $all_{j,k}$  inputs, but other parties do not have  $A_j$ 's signature for this input,
  therefore only  $A_j$  can publish it.
21:  //  $TX_{i,j,k} := i$ -th move,  $j, k$  input interval start and end.  $j, k$  unneeded for
 $i = 1, k$  unneeded for  $i = 2$ .
22:   $TX_1 \leftarrow TX$ :
23:    inputs:
24:      ( $c_{\text{left}}, \text{left\_old\_fund}$ ),
25:      ( $c_{\text{right}}, \text{right\_old\_fund}$ )
26:    outputs:
27:      ( $c_{\text{left}} - c_{\text{virt}}, \text{left\_new\_fund}$ ),
28:      ( $c_{\text{right}} - c_{\text{virt}}, \text{right\_new\_fund}$ ),
29:      ( $c_{\text{virt}}, pk_{\text{loc},\text{out}}$ ),
30:      ( $c_{\text{virt}},$ 
31:        (if  $(i-1 > 1)$  then  $all_{i-1,i}$  else False)
32:         $\vee$  (if  $(i+1 < n)$  then  $all_{i+1,i}$  else False)
33:         $\vee$  (
34:          if  $(i-1 = 1 \wedge i+1 = n)$  then  $\text{virt\_fund}$ 

```



```

35:         else if  $(i - 1 > 1 \wedge i + 1 = n)$  then virt_fund +  $t_{i-1}$ 
36:         else if  $(i - 1 = 1 \wedge i + 1 < n)$  then virt_fund +  $t_{i+1}$ 
37:         else  $/^*i - 1 > 1 \wedge i + 1 < n^*/$  virt_fund +  $\max(t_{i-1}, t_{i+1})$ 
38:     )
39: )
40: if  $i = 2$  then
41:     TX2,1  $\leftarrow$  TX:
42:     inputs:
43:         (cvirt, all2,1),
44:         (cright, right_old_fund)
45:     outputs:
46:         (cright - cvirt, right_new_fund),
47:         (cvirt, pkloc,out),
48:         (cvirt,
49:             if  $(n > 3)$  then (all3,2  $\vee$  (virt_fund +  $t_3$ ))
50:             else virt_fund
51:         )
52: end if
53: if  $i = n - 1$  then
54:     TX2,n  $\leftarrow$  TX:
55:     inputs:
56:         (cleft, left_old_fund),
57:         (cvirt, all $n-1,n$ )
58:     outputs:
59:         (cleft - cvirt, left_new_fund),
60:         (cvirt, pkloc,out),
61:         (cvirt,
62:             if  $(n - 2 > 1)$  then (all $n-2,n-1$   $\vee$  (virt_fund +  $t_{n-2}$ ))
63:             else virt_fund
64:         )
65: end if
66: for all  $k \in \{2, \dots, i - 1\}$  do //  $i - 2$  txs
67:     TX2,k  $\leftarrow$  TX:
68:     inputs:
69:         (cvirt, all $i,k$ ),
70:         (cright, right_old_fund)
71:     outputs:
72:         (cright - cvirt, right_new_fund),
73:         (cvirt, pkloc,out),
74:         (cvirt,
75:             if  $(k - 1 > 1)$  then all $k-1,i$  else False)
76:              $\vee$  (if  $(i + 1 < n)$  then all $i+1,k$  else False)
77:              $\vee$  (

```

```

78:         if  $(k - 1 = 1 \wedge i + 1 = n)$  then virt_fund
79:         else if  $(k - 1 > 1 \wedge i + 1 = n)$  then virt_fund +  $t_{k-1}$ 
80:         else if  $(k - 1 = 1 \wedge i + 1 < n)$  then virt_fund +  $t_{i+1}$ 
81:         else /*  $k - 1 > 1 \wedge i + 1 < n$  */ virt_fund +  $\max(t_{k-1}, t_{i+1})$ 
82:     )
83: )
84: end for
85: for all  $k \in \{i + 1, \dots, n - 1\}$  do //  $n - i - 1$  txs
86:     TX2,k  $\leftarrow$  TX:
87:     inputs:
88:         (cleft, left_old_fund)
89:         (cvirt, alli,k),
90:     outputs:
91:         (cleft - cvirt, left_new_fund),
92:         (cvirt, pkloc,out),
93:         (cvirt,
94:             (if  $(i - 1 > 1)$  then alli-1,k else False)
95:              $\vee$  (if  $(k + 1 < n)$  then allk+1,i else False)
96:              $\vee$  (
97:                 if  $(i - 1 = 1 \wedge k + 1 = n)$  then virt_fund
98:                 else if  $(i - 1 > 1 \wedge k + 1 = n)$  then virt_fund +  $t_{i-1}$ 
99:                 else if  $(i - 1 = 1 \wedge k + 1 < n)$  then virt_fund +  $t_{k+1}$ 
100:                 else /*  $i - 1 > 1 \wedge k + 1 < n$  */
101:                 virt_fund +  $\max(t_{i-1}, t_{k+1})$ 
102:             )
103:         )
104:     )
105: end for
106: if  $i = 2$  then  $m \leftarrow 1$  else  $m \leftarrow 2$ 
107: if  $i = n - 1$  then  $l \leftarrow n$  else  $l \leftarrow n - 1$ 
108: for all  $(k_1, k_2) \in \{m, \dots, i - 1\} \times \{i + 1, \dots, l\}$  do //  $(i - m) \cdot (l - i)$  txs
109:     TX3,k1,k2  $\leftarrow$  TX:
110:     inputs:
111:         (cvirt, alli,k1),
112:         (cvirt, alli,k2)
113:     outputs:
114:         (cvirt, pkloc,out),
115:         (cvirt,
116:             (if  $(k_1 - 1 > 1)$  then allk1-1,min(k2,n-1) else False)
117:              $\vee$  (if  $(k_2 + 1 < n)$  then allk2+1,max(k1,2) else False)
118:              $\vee$  (
119:                 if  $(k_1 - 1 \leq 1 \wedge k_2 + 1 \geq n)$  then virt_fund
120:                 else if  $(k_1 - 1 > 1 \wedge k_2 + 1 \geq n)$  then virt_fund +  $t_{k_1-1}$ 
121:                 else if  $(k_1 - 1 \leq 1 \wedge k_2 + 1 < n)$  then virt_fund +  $t_{k_2+1}$ 
122:                 else /*  $k_1 - 1 > 1 \wedge k_2 + 1 < n$  */

```

```

121:         virt_fund + max( $t_{k_1-1}, t_{k_2+1}$ )
122:     )
123: )
124: end for
125: return (
126:     TX1,
127:     (TX2,k)k ∈ {m,...,l} \ {i},
128:     (TX3,k1,k2)(k1,k2) ∈ {m,...,i-1} × {i+1,...,l}
129: )

```

Fig. 30.

Process VIRT

```

1: // left and right refer to the two counterparties, with left being the one closer
   to the funder. Note difference with left/right meaning in VIRT.GETMIDTXS.
2: GETENDPOINTTX( $i, n, c_{\text{virt}}, c_{\text{left}}, c_{\text{right}}, pk_{\text{left,fund,old}}, pk_{\text{right,fund,old}},$ 
    $pk_{\text{left,fund,new}}, pk_{\text{right,fund,new}}, pk_{\text{left,virt}}, pk_{\text{right,virt}}, (pk_{\text{all},j})_{j \in [n]}, t$ ):
3:   ensure  $i \in \{1, n\}$ 
4:   ensure  $c_{\text{left}} \geq c_{\text{virt}}$  // left party funds virtual channel
5:    $c_{\text{tot}} \leftarrow c_{\text{left}} + c_{\text{right}}$ 
6:    $\text{old\_fund} \leftarrow 2 / \{pk_{\text{left,fund,old}}, pk_{\text{right,fund,old}}\}$ 
7:    $\text{new\_fund} \leftarrow 2 / \{pk_{\text{left,fund,new}}, pk_{\text{right,fund,new}}\}$ 
8:    $\text{virt\_fund} \leftarrow 2 / \{pk_{\text{left,virt}}, pk_{\text{right,virt}}\}$ 
9:   if  $i = 1$  then // funder's tx
10:      $\text{all} \leftarrow n / \{pk_{\text{all},1}, \dots, pk_{\text{all},n}\} \wedge "1"$ 
11:   else // fundee's tx
12:      $\text{all} \leftarrow n / \{pk_{\text{all},1}, \dots, pk_{\text{all},n}\} \wedge "n"$ 
13:   end if
14:   TX1  $\leftarrow$  TX: // endpoints only have an "initiator" tx
15:   inputs:
16:     ( $c_{\text{tot}}, \text{old\_fund}$ )
17:   outputs:
18:     ( $c_{\text{tot}} - c_{\text{virt}}, \text{new\_fund}$ ),
19:     ( $c_{\text{virt}}, \text{all} \vee (\text{virt\_fund} + t)$ )
20:   return TX1

```

Fig. 31.

Process VIRT.SIBLINGSIGS()

```

1: parse input as sigsbyLeft
2: if  $i = 2$  then  $m \leftarrow 1$  else  $m \leftarrow 2$ 
3: if  $i = n - 1$  then  $l \leftarrow n$  else  $l \leftarrow n - 1$ 
4:  $(TX_{i,1}, (TX_{i,2,k})_{k \in \{m, \dots, l\} \setminus \{i\}}, (TX_{i,3,k_1,k_2})_{(k_1,k_2) \in \{m, \dots, i-1\} \setminus \{i+1, \dots, l\}}) \leftarrow$ 
   VIRT.GETMIDTXS( $i, n, c_{virt}, c_{i-1, \text{right}}, c_{i, \text{left}}, c_{i, \text{right}}, c_{i+1, \text{left}},$ 
    $pk_{i-1, \text{right}, \text{fund}, \text{old}}, pk_{i, \text{left}, \text{fund}, \text{old}}, pk_{i, \text{right}, \text{fund}, \text{old}}, pk_{i+1, \text{left}, \text{fund}, \text{old}},$ 
    $pk_{i-1, \text{fund}, \text{new}}, pk_{i, \text{fund}, \text{new}}, pk_{i, \text{fund}, \text{new}}, pk_{i+1, \text{fund}, \text{new}}, pk_{\text{left}, \text{virt}}, pk_{\text{right}, \text{virt}},$ 
    $pk_{i, \text{out}}, (pk_{i,j,k})_{i \in [n], j \in [n-1] \setminus \{1\}, k \in [n-1] \setminus \{1, j\}}, (pk_{i,2,1})_{i \in [n]}, (pk_{i,n-1,n})_{i \in [n]},$ 
    $(t_i)_{i \in [n-1] \setminus \{1\}})$ 
5: // notation:  $\text{sig}(TX, pk) := \text{sig}$  with ANYPREVOUT flag such that
    $\text{VERIFY}(TX, \text{sig}, pk) = \text{True}$ 
6: ensure that the following signatures are present in sigsbyLeft and store them:
   - //  $(l - m) \cdot (i - 1)$  signatures
7:    $\forall k \in \{m, \dots, l\} \setminus \{i\}, \forall j \in [i - 1] :$ 
8:      $\text{sig}(TX_{i,2,k}, pk_{j,i,k})$ 
   - //  $2 \cdot (i - m) \cdot (l - i) \cdot (i - 1)$  signatures
9:    $\forall k_1 \in \{m, \dots, i - 1\}, \forall k_2 \in \{i + 1, \dots, l\}, \forall j \in [i - 1] :$ 
10:     $\text{sig}(TX_{i,3,k_1,k_2}, pk_{j,i,k_1}), \text{sig}(TX_{i,3,k_1,k_2}, pk_{j,i,k_2})$ 
11: sigstoRight  $\leftarrow$  sigsbyLeft

12: for all  $j \in \{2, \dots, n - 1\} \setminus \{i\}$  do
13:   if  $j = 2$  then  $m' \leftarrow 1$  else  $m' \leftarrow 2$ 
14:   if  $j = n - 1$  then  $l' \leftarrow n$  else  $l' \leftarrow n - 1$ 
15:    $(TX_{j,1}, (TX_{j,2,k})_{k \in \{m', \dots, l'\} \setminus \{j\}}, (TX_{j,3,k_1,k_2})_{(k_1,k_2) \in \{m', \dots, i-1\} \setminus \{i+1, \dots, l'\}}) \leftarrow$ 
     GETMIDTXS( $j, n, c_{virt}, c_{j-1, \text{right}}, c_{j, \text{left}}, c_{j, \text{right}}, c_{j+1, \text{left}}, pk_{j-1, \text{right}, \text{fund}, \text{old}},$ 
      $pk_{j, \text{left}, \text{fund}, \text{old}}, pk_{j, \text{right}, \text{fund}, \text{old}}, pk_{j+1, \text{left}, \text{fund}, \text{old}}, pk_{j-1, \text{fund}, \text{new}}, pk_{j, \text{fund}, \text{new}},$ 
      $pk_{j, \text{fund}, \text{new}}, pk_{j+1, \text{fund}, \text{new}}, pk_{\text{left}, \text{virt}}, pk_{\text{right}, \text{virt}}, pk_{j, \text{out}},$ 
      $(pk_{k,p,s})_{k \in [n], p \in [n-1] \setminus \{1\}, s \in [n-1] \setminus \{1, p\}}, (pk_{k,2,1})_{k \in [n]}, (pk_{k,n-1,n})_{k \in [n]},$ 
      $(t_k)_{k \in [n-1] \setminus \{1\}})$ 
16:   if  $j < i$  then sigs  $\leftarrow$  sigstoLeft else sigs  $\leftarrow$  sigstoRight
17:   for all  $k \in \{m', \dots, l'\} \setminus \{j\}$  do
18:     add SIGN( $TX_{j,2,k}, sk_{i,j,k}, \text{ANYPREVOUT}$ ) to sigs
19:   end for
20:   for all  $k_1 \in \{m', \dots, j - 1\}, k_2 \in \{j + 1, \dots, l'\}$  do
21:     add SIGN( $TX_{j,3,k_1,k_2}, sk_{i,j,k_1}, \text{ANYPREVOUT}$ ) to sigs
22:     add SIGN( $TX_{j,3,k_1,k_2}, sk_{i,j,k_2}, \text{ANYPREVOUT}$ ) to sigs
23:   end for
24: end for
25: if  $i + 1 = n$  then // next hop is host_fundee
26:    $TX_{n,1} \leftarrow \text{VIRT.GETENDPOINTTX}(n, n, c_{virt}, c_{n-1, \text{right}}, c_{n, \text{left}},$ 
      $pk_{n-1, \text{right}, \text{fund}, \text{old}}, pk_{n, \text{left}, \text{fund}, \text{old}}, pk_{n-1, \text{fund}, \text{new}}, pk_{n, \text{fund}, \text{new}}, pk_{\text{left}, \text{virt}},$ 
      $pk_{\text{right}, \text{virt}}, (pk_{j,n-1,n})_{j \in [n]}, t_{n-1})$ 
27: end if

```

```

28: call  $\bar{P}.$ CIRCULATEVIRTUALSIGS(sigstoRight) and assign returned value to
    sigsbyRight
29: ensure that the following signatures are present in sigsbyRight and store them:
    - //  $(l - m) \cdot (n - i)$  signatures
30:    $\forall k \in \{m, \dots, l\} \setminus \{i\}, \forall j \in \{i + 1, \dots, n\} :$ 
31:     sig(TX $i,2,k$ ,  $pk_{j,i,k}$ )
    - //  $2 \cdot (i - m) \cdot (l - i) \cdot (n - i)$  signatures
32:    $\forall k_1 \in \{m, \dots, i - 1\}, \forall k_2 \in \{i + 1, \dots, l\}, \forall j \in \{i + 1, \dots, n\} :$ 
33:     sig(TX $i,3,k_1,k_2$ ,  $pk_{j,i,k_1}$ ), sig(TX $i,3,k_1,k_2$ ,  $pk_{j,i,k_2}$ )
34: output (VIRTUALSIGSBACK, sigstoLeft, sigsbyRight)

```

Fig. 32.

Process VIRT.INTERMEDIARYSIGS()

```

1: if  $i = 2$  then  $m \leftarrow 1$  else  $m \leftarrow 2$ 
2: if  $i = n - 1$  then  $l \leftarrow n$  else  $l \leftarrow n - 1$ 
3: (TX $i,1$ , (TX $i,2,k$ ) $k \in \{m, \dots, l\} \setminus \{i\}$ , (TX $i,3,k_1,k_2$ ) $(k_1,k_2) \in \{m, \dots, i-1\} \times \{i+1, \dots, l\}$ )  $\leftarrow$ 
    VIRT.GETMIDTXS( $i, n, c_{\text{virt}}, c_{i-1,\text{right}}, c_{i,\text{left}}, c_{i,\text{right}}, c_{i+1,\text{left}},$ 
     $pk_{i-1,\text{right},\text{fund},\text{old}}, pk_{i,\text{left},\text{fund},\text{old}}, pk_{i,\text{right},\text{fund},\text{old}}, pk_{i+1,\text{left},\text{fund},\text{old}},$ 
     $pk_{i-1,\text{fund},\text{new}}, pk_{i,\text{fund},\text{new}}, pk_{i,\text{fund},\text{new}}, pk_{i+1,\text{fund},\text{new}}, pk_{i,\text{left},\text{virt}}, pk_{i,\text{right},\text{virt}},$ 
     $pk_{i,\text{out}}, (pk_{i,j,k})_{i \in [n], j \in [n-1] \setminus \{1\}, k \in [n-1] \setminus \{1,j\}}, (pk_{i,2,1})_{i \in [n]}, (pk_{i,n-1,n})_{i \in [n]},$ 
     $(t_i)_{i \in [n-1] \setminus \{1\}})$ 
4: // not verifying our signatures in sigsbyLeft, our (trusted) sibling will do that
5: input (VIRTUAL SIGS FORWARD, sigsbyLeft) to sibling
6: VIRT.SIBLINGSIGS()
7: sigstoLeft  $\leftarrow$  sigsbyRight + sigstoLeft
8: if  $i = 2$  then // previous hop is host_funder
9:   TX $1,1$   $\leftarrow$  VIRT.GETENDPOINTTX( $1, n, c_{\text{virt}}, c_{1,\text{right}}, c_{2,\text{left}}, pk_{1,\text{right},\text{fund},\text{old}},$ 
     $pk_{2,\text{left},\text{fund},\text{old}}, pk_{1,\text{fund},\text{new}}, pk_{2,\text{fund},\text{new}}, pk_{i,\text{left},\text{virt}}, pk_{i,\text{right},\text{virt}}, (pk_{j,2,1})_{j \in [n]}, t_2)$ 
10: end if
11: return sigstoLeft

```

Fig. 33.

Process VIRT.HOSTFUNDEESIGS()

```

1: TX $n,1$   $\leftarrow$  VIRT.GETENDPOINTTX( $n, n, c_{\text{virt}}, c_{n-1,\text{right}}, c_{n,\text{left}},$ 
     $pk_{n-1,\text{right},\text{fund},\text{old}}, pk_{n,\text{right},\text{fund},\text{old}}, pk_{n-1,\text{fund},\text{new}}, pk_{n,\text{fund},\text{new}}, pk_{i,\text{left},\text{virt}},$ 
     $pk_{i,\text{right},\text{virt}}, (pk_{j,n-1,n})_{j \in [n]}, t_{n-1})$ 
2: for all  $j \in [n - 1] \setminus \{1\}$  do

```

```

3:   if  $j = 2$  then  $m \leftarrow 1$  else  $m \leftarrow 2$ 
4:   if  $j = n - 1$  then  $l \leftarrow n$  else  $l \leftarrow n - 1$ 
5:    $(TX_{j,1}, (TX_{j,2,k})_{k \in \{m, \dots, l\} \setminus \{j\}}, (TX_{i,3,k_1,k_2})_{(k_1,k_2) \in \{m, \dots, i-1\} \setminus \{i+1, \dots, l\}}) \leftarrow$ 
   VIRT.GETMIDTXS( $j, n, c_{\text{virt}}, c_{j-1, \text{right}}, c_{j, \text{left}}, c_{j, \text{right}}, c_{j+1, \text{left}},$ 
    $pk_{j-1, \text{right}, \text{fund}, \text{old}}, pk_{j, \text{left}, \text{fund}, \text{old}}, pk_{j, \text{right}, \text{fund}, \text{old}}, pk_{j+1, \text{left}, \text{fund}, \text{old}},$ 
    $pk_{j-1, \text{fund}, \text{new}}, pk_{j, \text{fund}, \text{new}}, pk_{j, \text{fund}, \text{new}}, pk_{j+1, \text{fund}, \text{new}}, pk_{\text{left}, \text{virt}}, pk_{\text{right}, \text{virt}},$ 
    $pk_{j, \text{out}}, (pk_{j,s,k})_{j \in [n], s \in [n-1] \setminus \{1\}, k \in [n-1] \setminus \{1, s\}}, (pk_{j,2,1})_{j \in [n]}, (pk_{j,n-1,n})_{j \in [n]},$ 
    $(t_j)_{j \in [n-1] \setminus \{1\}})$ 
6:    $\text{sigs}_{\text{toLeft}} \leftarrow \emptyset$ 
7:   for all  $k \in \{m, \dots, l\} \setminus \{j\}$  do
8:     add SIGN( $TX_{j,2,k}, sk_{n,j,k}, \text{ANYPREVOUT}$ ) to  $\text{sigs}_{\text{toLeft}}$ 
9:   end for
10:  for all  $k_1 \in \{m, \dots, j-1\}, k_2 \in \{j+1, \dots, l\}$  do
11:    add SIGN( $TX_{j,3,k_1,k_2}, sk_{n,j,k_1}, \text{ANYPREVOUT}$ ) to  $\text{sigs}_{\text{toLeft}}$ 
12:    add SIGN( $TX_{j,3,k_1,k_2}, sk_{n,j,k_2}, \text{ANYPREVOUT}$ ) to  $\text{sigs}_{\text{toLeft}}$ 
13:  end for
14: end for
15: return  $\text{sigs}_{\text{toLeft}}$ 

```

Fig. 34.

Process VIRT.HOSTFUNDERSIGS()

```

1: for all  $j \in [n-1] \setminus \{1\}$  do
2:   if  $j = 2$  then  $m \leftarrow 1$  else  $m \leftarrow 2$ 
3:   if  $j = n - 1$  then  $l \leftarrow 1$  else  $l \leftarrow 2$ 
4:    $(TX_{j,1}, (TX_{j,2,k})_{k \in \{m, \dots, l\} \setminus \{j\}}, (TX_{i,3,k_1,k_2})_{(k_1,k_2) \in \{m, \dots, i-1\} \setminus \{i+1, \dots, l\}}) \leftarrow$ 
   VIRT.GETMIDTXS( $j, n, c_{\text{virt}}, c_{j-1, \text{right}}, c_{j, \text{left}}, c_{j, \text{right}}, c_{j+1, \text{left}},$ 
    $pk_{j-1, \text{right}, \text{fund}, \text{old}}, pk_{j, \text{left}, \text{fund}, \text{old}}, pk_{j, \text{right}, \text{fund}, \text{old}}, pk_{j+1, \text{left}, \text{fund}, \text{old}},$ 
    $pk_{j-1, \text{fund}, \text{new}}, pk_{j, \text{fund}, \text{new}}, pk_{j, \text{fund}, \text{new}}, pk_{j+1, \text{fund}, \text{new}}, pk_{\text{left}, \text{virt}}, pk_{\text{right}, \text{virt}},$ 
    $pk_{j, \text{out}}, (pk_{j,s,k})_{j \in [n], s \in [n-1] \setminus \{1\}, k \in [n-1] \setminus \{1, s\}}, (pk_{j,2,1})_{j \in [n]}, (pk_{j,n-1,n})_{j \in [n]},$ 
    $(t_j)_{j \in [n-1] \setminus \{1\}})$ 
5:    $\text{sigs}_{\text{toRight}} \leftarrow \emptyset$ 
6:   for all  $k \in \{m, \dots, l\} \setminus \{j\}$  do
7:     add SIGN( $TX_{j,2,k}, sk_{1,j,k}, \text{ANYPREVOUT}$ ) to  $\text{sigs}_{\text{toRight}}$ 
8:   end for
9:   for all  $k_1 \in \{m, \dots, j-1\}, k_2 \in \{j+1, \dots, l\}$  do
10:    add SIGN( $TX_{j,3,k_1,k_2}, sk_{1,j,k_1}, \text{ANYPREVOUT}$ ) to  $\text{sigs}_{\text{toRight}}$ 
11:    add SIGN( $TX_{j,3,k_1,k_2}, sk_{1,j,k_2}, \text{ANYPREVOUT}$ ) to  $\text{sigs}_{\text{toRight}}$ 
12:   end for
13: end for
14: call VIRT.CIRCULATEVIRTUALSIGS( $\text{sigs}_{\text{toRight}}$ ) of  $\bar{P}$  and assign output to
    $\text{sigs}_{\text{byRight}}$ 
15:  $TX_{1,1} \leftarrow \text{VIRT.GETENDPOINTTX}(1, n, c_{\text{virt}}, c_{1, \text{right}}, c_{2, \text{left}}, pk_{1, \text{right}, \text{fund}, \text{old}},$ 
    $pk_{2, \text{left}, \text{fund}, \text{old}}, pk_{1, \text{fund}, \text{new}}, pk_{2, \text{fund}, \text{new}}, pk_{\text{left}, \text{virt}}, pk_{\text{right}, \text{virt}}, (pk_{j,2,1})_{j \in [n]}, t_2)$ 

```

```
16: return (OK)
```

Fig. 35.

Process VIRT.CIRCULATEVIRTUALSIGS(sigs_{byLeft})

```
1: if 1 < i < n then // we are not host_funder nor host_fundee
2:   return VIRT.INTERMEDIARYSIGS()
3: else if i = 1 then // we are host_funder
4:   return VIRT.HOSTFUNDERSIGS()
5: else if i = n then // we are host_fundee
6:   return VIRT.HOSTFUNDEESIGS()
7: end if // it is always 1 ≤ i ≤ n - c.f. Fig. 28, l. 12 and l. 39
```

Fig. 36.

Process VIRT.CIRCULATEFUNDINGSIGS(sigs_{byLeft})

```
1: if 1 < i < n then // we are not endpoint
2:   if i = 2 then m ← 1 else m ← 2
3:   if i = n - 1 then l ← n else l ← n - 1
4:   ensure that the following signatures are present in sigsbyLeft and store them:
   - // 1 signature
5:     sig(TXi,1, pki-1,right,fund,old)
   - // n - 3 + χi=2 + χi=n-1 signatures
6:     ∀k ∈ {m, ..., l} \ {i}
7:       sig(TXi,2,k, pki-1,right,fund,old)
8:   input (VIRTUAL BASE SIG FORWARD, sigsbyLeft) to sibling
9:   extract and store sig(TXi,1, pki-1,right,fund,old) and ∀k ∈ {m, ..., l} \ {i}
   sig(TXi,2,k, pki-1,right,fund,old) from sigsbyLeft // same signatures as sibling
10:  sigstoRight ← {SIGN(TXi+1,1, ski,right,fund,old, ANYPREVOUT)}
11:  if i + 1 < n then
12:    if i + 1 = n - 1 then l' ← n else l' ← n - 1
13:    for all k ∈ {2, ..., l'} do
14:      add SIGN(TXi+1,2,k, ski,right,fund,old, ANYPREVOUT) to sigstoRight
15:    end for
16:  end if
17:  call VIRT.CIRCULATEFUNDINGSIGS(sigstoRight) of P̄ and assign returned
  values to sigsbyRight
18:  ensure that the following signatures are present in sigsbyRight and store
  them:
```

```

- // 1 signature
19:   sig(TXi,1, pki+1,left,fund,old)
- // n - 3 + χi=2 + χi=n-1 signatures
20:   ∀k ∈ {m, ..., l} \ {i}
21:   sig(TXi,2,k, pki+1,right,fund,old)
22:   output (VIRTUAL BASE SIG BACK, sigsbyRight)
23:   extract and store sig(TXi,1, pki+1,right,fund,old) and ∀k ∈ {m, ..., l} \ {i}
sig(TXi,2,k, pki+1,right,fund,old) from sigsbyRight // same signatures as sibling
24:   sigtoLeft ← {SIGN(TXi-1,1, ski,left,fund,old, ANYPREVOUT)}
25:   if i - 1 > 1 then
26:     if i - 1 = 2 then m' ← 1 else m' ← 2
27:     for all k ∈ {m', ..., n - 1} do
28:       add SIGN(TXi-1,2,k, ski,left,fund,old, ANYPREVOUT) to sigstoLeft
29:     end for
30:   end if
31:   return sigstoLeft
32: else if i = 1 then // we are host_funder
33:   sigstoRight ← {SIGN(TX2,1, sk1,right,fund,old, ANYPREVOUT)}
34:   if 2 = n - 1 then l' ← n else l' ← n - 1
35:   for all k ∈ {3, ..., l'} do
36:     add SIGN(TX2,2,k, sk1,right,fund,old, ANYPREVOUT) to sigstoRight
37:   end for
38:   call VIRT.CIRCULATEFUNDINGSIGS(sigstoRight) of  $\bar{P}$  and assign returned
value to sigsbyRight
39:   ensure that sig(TX1,1, pk2,left,fund,old) is present in sigsbyRight and store it
40:   return (OK)
41: else if i = n then // we are host_fundee
42:   ensure sig(TXn,1, pkn-1,right,fund,old) is present in sigsbyLeft and store it
43:   sigstoLeft ← {SIGN(TXn-1,1, skn,left,fund,old, ANYPREVOUT)}
44:   if n - 1 = 2 then m' ← 1 else m' ← 2
45:   for all k ∈ {m', ..., n - 2} do
46:     add SIGN(TXn-1,2,k, skn,left,fund,old, ANYPREVOUT) to sigstoLeft
47:   end for
48:   return sigstoLeft
49: end if // it is always 1 ≤ i ≤ n - c.f. Fig. 28, l. 12 and l. 39

```

Fig. 37.

Process VIRT.CIRCULATEREVOCATIONS(revoc_by_prev)

```

1: if revoc_by_prev is given as argument then // we are not host_funder
2:   ensure guest.PROCESSREMOTEREVOCATION(revoc_by_prev) returns (OK)
3: else // we are host_funder
4:   revoc_for_next ← guest.REVOKEPREVIOUS()

```



```

5:   input (READ) to  $\mathcal{G}_{\text{Ledger}}$  and assign output to  $\Sigma$ 
6:    $\text{last\_poll} \leftarrow |\Sigma|$ 
7:   call  $\text{VIRT.CIRCULATEREVOCATIONS}(\text{revoc\_for\_next})$  of  $\bar{P}$  and assign
   returned value to  $\text{revoc\_by\_next}$ 
8:   ensure  $\text{guest.PROCESSREMOTEREVOCATION}(\text{revoc\_by\_next})$  returns (OK)
   // If the “ensure” fails, the opening process freezes, this is intentional. The
   channel can still close via (CLOSE)
9:   return (OK)
10: end if
11: if we have a sibling then // we are not host_fundee nor host_funder
12:   input (VIRTUAL REVOCATION FORWARD) to sibling
13:    $\text{revoc\_for\_next} \leftarrow \text{guest.REVOKEPREVIOUS}()$ 
14:   input (READ) to  $\mathcal{G}_{\text{Ledger}}$  and assign output to  $\Sigma$ 
15:    $\text{last\_poll} \leftarrow |\Sigma|$ 
16:   call  $\text{VIRT.CIRCULATEREVOCATIONS}(\text{revoc\_for\_next})$  of  $\bar{P}$  and assign
   output to  $\text{revoc\_by\_next}$ 
17:   ensure  $\text{guest.PROCESSREMOTEREVOCATION}(\text{revoc\_by\_next})$  returns (OK)
18:   output (HOSTS READY,  $t_i$ ) to guest and expect reply (HOST ACK)
19:   output (VIRTUAL REVOCATION BACK)
20: end if
21:  $\text{revoc\_for\_prev} \leftarrow \text{guest.REVOKEPREVIOUS}()$ 
22: if  $1 < i < n$  then // we are intermediary
23:   output (HOSTS READY,  $t_i$ ) to guest and expect reply (HOST ACK) //  $p$  is
   every how many blocks we have to check the chain
24: else // we are host_fundee, case of host_funder covered earlier
25:   output (HOSTS READY,  $p + \sum_{j=2}^{n-1} (s - 1 + t_j)$ ) to guest and expect reply
   (HOST ACK)
26: end if
27: return  $\text{revoc\_for\_prev}$ 

```

Fig. 38.

Process VIRT – poll

```

1: On input (CHECK FOR LATERAL CLOSE) by  $R \in \{\text{guest, funder, fundee}\}$ :
2:   input (READ) to  $\mathcal{G}_{\text{Ledger}}$  and assign output to  $\Sigma$ 
3:    $k_1 \leftarrow 0$ 
4:   if  $\text{TX}_{i-1,1}$  is defined and  $\text{TX}_{i-1,1} \in \Sigma$  then
5:      $k_1 \leftarrow i - 1$ 
6:   end if
7:   for all  $k \in [i - 2]$  do
8:     if  $\text{TX}_{i-1,2,k}$  is defined and  $\text{TX}_{i-1,2,k} \in \Sigma$  then
9:        $k_1 \leftarrow k$ 
10:    end if

```

```

11:   end for
12:    $k_2 \leftarrow 0$ 
13:   if  $\text{TX}_{i+1,1}$  is defined and  $\text{TX}_{i+1,1} \in \Sigma$  then
14:      $k_2 \leftarrow i + 1$ 
15:   end if
16:   for all  $k \in \{i + 2, \dots, n\}$  do
17:     if  $\text{TX}_{i+1,2,k}$  is defined and  $\text{TX}_{i+1,2,k} \in \Sigma$  then
18:        $k_2 \leftarrow k$ 
19:     end if
20:   end for
21:   last_poll  $\leftarrow |\Sigma|$ 
22:   if  $k_1 > 0 \vee k_2 > 0$  then // at least one neighbour has published its TX
23:     ignore all messages except for (CHECK IF CLOSING) by  $R$ 
24:     State  $\leftarrow$  CLOSING
25:     sigs  $\leftarrow \emptyset$ 
26:   end if
27:   if  $k_1 > 0 \wedge k_2 > 0$  then // both neighbours have published their TXs
28:     add (sig( $\text{TX}_{i,3,k_1,k_2}, pk_{p,i,k_1}$ )) $_{p \in [n] \setminus \{i\}}$  to sigs
29:     add (sig( $\text{TX}_{i,3,k_1,k_2}, pk_{p,i,k_2}$ )) $_{p \in [n] \setminus \{i\}}$  to sigs
30:     add SIGN( $\text{TX}_{i,3,k_1,k_2}, sk_{i,i,k_1}$ , ANYPREVOUT) to sigs
31:     add SIGN( $\text{TX}_{i,3,k_1,k_2}, sk_{i,i,k_2}$ , ANYPREVOUT) to sigs
32:     input (SUBMIT,  $\text{TX}_{i,3,k_1,k_2}$ , sigs) to  $\mathcal{G}_{\text{Ledger}}$ 
33:   else if  $k_1 > 0$  then // only left neighbour has published its TX
34:     add (sig( $\text{TX}_{i,2,k_1}, pk_{p,i,k_1}$ )) $_{p \in [n] \setminus \{i\}}$  to sigs
35:     add SIGN( $\text{TX}_{i,2,k_1}, sk_{i,i,k_1}$ , ANYPREVOUT) to sigs
36:     add SIGN( $\text{TX}_{i,2,k_1}, sk_{i,\text{left},\text{fund},\text{old}}$ , ANYPREVOUT) to sigs
37:     input (SUBMIT,  $\text{TX}_{i,2,k_1}$ , sigs) to  $\mathcal{G}_{\text{Ledger}}$ 
38:   else if  $k_2 > 0$  then // only right neighbour has published its TX
39:     add (sig( $\text{TX}_{i,2,k_2}, pk_{p,i,k_2}$ )) $_{p \in [n] \setminus \{i\}}$  to sigs
40:     add SIGN( $\text{TX}_{i,2,k_2}, sk_{i,i,k_2}$ , ANYPREVOUT) to sigs
41:     add SIGN( $\text{TX}_{i,2,k_2}, sk_{i,\text{right},\text{fund},\text{old}}$ , ANYPREVOUT) to sigs
42:     input (SUBMIT,  $\text{TX}_{i,2,k_2}$ , sigs) to  $\mathcal{G}_{\text{Ledger}}$ 
43:   end if

```

Fig. 39.

Process VIRT – On input (CLOSE) by $R \in \{\text{guest}, \text{funder}, \text{fundee}\}$:

```

1: // At most one of funder, fundee is defined
2: if State = CLOSED then output (CLOSED) to  $R$ 
3: if State = GUEST PUNISHED then output (GUEST PUNISHED) to  $R$ 
4: ensure State = OPEN
5: if  $\text{host}_P \neq \mathcal{G}_{\text{Ledger}}$  then //  $\text{host}_P$  is a VIRT
6:   ignore all messages except for output (CLOSED) by  $\text{host}_P$ . Also relay to
    $\text{host}_P$  any (CHECK IF CLOSING) input received

```

```

7:   input (CLOSE) to  $\text{host}_P$ 
8: end if
9: // if we have a  $\text{host}_P$ , continue from here on output (CLOSED) by it
10: send (READ) to  $\mathcal{G}_{\text{Ledger}}$  as  $R$  and assign reply to  $\Sigma$ 
11: if  $i \in \{1, n\} \wedge (\text{TX}_{(i-1)+\frac{2}{n-1}(n-i),1} \in \Sigma \vee \exists k \in [n] : \text{TX}_{(i-1)+\frac{2}{n-1}(n-i),2,k} \in \Sigma)$ 
    then // we are an endpoint and our counterparty has closed – 1st subscript of
        TX is 2 if  $i = 1$  and  $n - 1$  if  $i = n$ 
12:   ignore all messages except for (CHECK IF CLOSING) by  $R$ 
13:    $\text{State} \leftarrow \text{CLOSING}$ 
14:   give up execution token // control goes to  $\mathcal{E}$ 
15: end if
16: let  $\text{tx}$  be the unique TX among  $\text{TX}_{i,1}, (\text{TX}_{i,2,k})_{k \in [n]}, (\text{TX}_{i,3,k_1,k_2})_{k_1,k_2 \in [n]}$ 
    that can be appended to  $\Sigma$  in a valid way // ignore invalid subscript
    combinations
17: let  $\text{sigs}$  be the set of stored signatures that sign  $\text{tx}$ 
18: add  $\text{SIGN}(\text{tx}, sk_{i,\text{left},\text{fund},\text{old}}, \text{ANYPREVOUT}), \text{SIGN}(\text{tx}, sk_{i,\text{right},\text{fund},\text{old}},$ 
     $\text{ANYPREVOUT}), (\text{SIGN}(\text{tx}, sk_{i,j,k}, \text{ANYPREVOUT}))_{j,k \in [n]}$  to  $\text{sigs}$  // ignore invalid
    signatures
19: ignore all messages except for (CHECK IF CLOSING) by  $R$ 
20:  $\text{State} \leftarrow \text{CLOSING}$ 
21: send (SUBMIT,  $\text{tx}, \text{sigs}$ ) to  $\mathcal{G}_{\text{Ledger}}$ 

```

Fig. 40.

Process VIRT – On input (CHECK IF CLOSING) by $R \in \{\text{guest}, \text{funder}, \text{funder}\}$:

```

1: ensure  $\text{State} = \text{CLOSING}$ 
2: send (READ) to  $\mathcal{G}_{\text{Ledger}}$  as  $R$  and assign reply to  $\Sigma$ 
3: if  $i = 1$  then // we are  $\text{host\_funder}$ 
4:   ensure that there exists an output with  $c_P + c_{\bar{P}} - c_{\text{virt}}$  coins and a
     $2/\{pk_{1,\text{fund},\text{new}}, pk_{2,\text{fund},\text{new}}\}$  spending method with expired/non-existent
    timelock in  $\Sigma$  // new base funding output
5:   ensure that there exists an output with  $c_{\text{virt}}$  coins and a
     $2/\{pk_{\text{left},\text{virt}}, pk_{\text{right},\text{virt}}\}$  spending method with expired/non-existent timelock
    in  $\Sigma$  // virtual funding output
6: else if  $i = n$  then // we are  $\text{host\_funder}$ 
7:   ensure that there exists an output with  $c_P + c_{\bar{P}} - c_{\text{virt}}$  coins and a
     $2/\{pk_{n-1,\text{fund},\text{new}}, pk_{n,\text{fund},\text{new}}\}$  spending method with expired/non-existent
    timelock in  $\Sigma$  // new base funding output
8:   ensure that there exists an output with  $c_{\text{virt}}$  coins and a
     $2/\{pk_{\text{left},\text{virt}}, pk_{\text{right},\text{virt}}\}$  spending method with expired/non-existent timelock
    in  $\Sigma$  // virtual funding output
9: else // we are intermediary
10:  if  $\text{side} = \text{“left”}$  then  $j \leftarrow i - 1$  else  $j \leftarrow i + 1$  // side is defined for all
    intermediaries – c.f. Fig. 28, l. 11

```

```

11:   ensure that there exists an output with  $c_P + c_{\bar{P}} - c_{\text{virt}}$  coins and a
       $2/\{pk_{i,\text{fund,new}}, pk_{j,\text{fund,new}}\}$  spending method with expired/non-existent
      timelock and an output with  $c_{\text{virt}}$  coins and a  $pk_{i,\text{out}}$  spending method with
      expired/non-existent timelock in  $\Sigma$ 
12: end if
13:  $State \leftarrow \text{CLOSED}$ 
14: output (CLOSED) to  $R$ 

```

Fig. 41.

Process VIRT – punishment handling

```

1: On input (USED REVOCATION) by guest: // (USED REVOCATION) by
   funder/fundee is ignored
2:    $State \leftarrow \text{GUEST PUNISHED}$ 
3:   input (USED REVOCATION) to hostP, expect reply (USED REVOCATION OK)
4:   if funder or fundee is defined then
5:     output (ENABLER USED REVOCATION) to it
6:   else // sibling is defined
7:     output (ENABLER USED REVOCATION) to sibling
8:   end if

9: On input (ENABLER USED REVOCATION) by sibling:
10:    $State \leftarrow \text{GUEST PUNISHED}$ 
11:   output (ENABLER USED REVOCATION) to guest

12: On output (USED REVOCATION) by hostP:
13:    $State \leftarrow \text{GUEST PUNISHED}$ 
14:   if funder or fundee is defined then
15:     output (ENABLER USED REVOCATION) to it
16:   else // sibling is defined
17:     output (ENABLER USED REVOCATION) to sibling
18:   end if

```

Fig. 42.

Lemma 1 (Real world balance security). *Consider a real world execution with $P \in \{\text{Alice}, \text{Bob}\}$ honest LN ITI and \bar{P} the counterparty ITI. Assume that all of the following are true:*

- the internal variable *negligent* of P has value “False”,
- P has transitioned to the OPEN State for the first time after having received (OPEN, c, \dots) by either \mathcal{E} or \bar{P} ,
- P [has received $(\text{FUND ME}, f_i, \dots)$ as input by another LN ITI while State was OPEN and subsequently P transitioned to OPEN State] n times,

- P [has received (PAY, d_i) by \mathcal{E} while State was OPEN and P subsequently transitioned to OPEN State] m times,
- P [has received $(\text{GET PAID}, e_i)$ by \mathcal{E} while State was OPEN and P subsequently transitioned to OPEN State] l times.

Let $\phi = 1$ if $P = \text{Alice}$, or $\phi = 0$ if $P = \text{Bob}$. If P receives (CLOSE) by \mathcal{E} and, if $\text{host}_P \neq \mathcal{G}_{\text{Ledger}}$ the output of host_P is (CLOSED) , then eventually the state obtained when P inputs (READ) to $\mathcal{G}_{\text{Ledger}}$ will contain h outputs each of value c_i and that has been spent or is exclusively spendable by $pk_{R,\text{out}}$ such that

$$\sum_{i=1}^h c_i \geq \phi \cdot c - \sum_{i=1}^n f_i - \sum_{i=1}^m d_i + \sum_{i=1}^l e_i \quad (1)$$

with overwhelming probability in the security parameter, where R is a local, trusted machine (i.e. either P , P 's *sibling*, the party to which P sent FUND ME if such a message has been sent, or the *sibling* of one of the transitive closure of hosts of P).

Proof. We first note that, as signature forgeries only happen with negligible probability and only a polynomial number of signatures are verified by honest parties throughout an execution, the event in which any forged signature passes the verification of an honest party or of $\mathcal{G}_{\text{Ledger}}$ happens only with negligible probability. We can therefore ignore this event throughout this proof and simply add a computationally negligible distance between \mathcal{E} 's outputs in the real and the ideal world at the end.

We also note that $pk_{P,\text{out}}$ has been provided by \mathcal{E} , therefore it can freely use coins spendable by this key. This is why we allow for any of the $pk_{P,\text{out}}$ outputs to have been spent.

Define the *history* of a channel as $H = (F, C)$, where each of F, C is a list of lists of integers. A party P which satisfies the Lemma conditions has a unique, unambiguously and recursively defined history: If the value **hops** in the $(\text{OPEN}, c, \text{hops}, \dots)$ message was equal to $\mathcal{G}_{\text{Ledger}}$, then F is the empty list, otherwise F is the concatenation of the F and C lists of the party that sent (FUNDED, \dots) to P , as they were at the moment the latter message was sent. After initialised, F remains immutable. Observe that, if **hops** $\neq \mathcal{G}_{\text{Ledger}}$, both aforementioned messages must have been received before P transitions to the OPEN state.

The list C of party P is initialised to $[[g]]$ when P 's *State* transitions for the first time to OPEN, where $g = c$ if $P = \text{Alice}$, or $g = 0$ if $P = \text{Bob}$; this represents the initial channel balance. The value x or $-x$ is appended to the last list in C when a payment is received (Fig. 20, l. 21) or sent (Fig. 20, l. 6) respectively by P . Moving on to the funding of new virtual channels, whenever P funds a new virtual channel (Fig. 17, l. 20), $[-c_{\text{virt}}]$ is appended to C and whenever P helps with the opening of a new virtual channel, but does not fund it (Fig. 17, l. 23), $[0]$ is appended to C . Therefore C consists of one list of integers for each sequence of inbound and outbound payments that have not been interrupted by a virtualisation step and a new list is added for every new virtual layer. We

also observe that a non-negligent party with history (F, C) satisfies the Lemma conditions and that the value of the right hand side of the inequality (1) is equal to $\sum_{s \in C} \sum_{x \in s} x$, as all inbound and outbound payment values and new channel funding values that appear in the Lemma conditions are recorded in C .

Let party P with a particular history. We will inductively prove that P satisfies the Lemma. The base case is when a channel is opened with $\text{hops} = \mathcal{G}_{\text{Ledger}}$ and is closed right away, therefore $H = ([], [g])$, where $g = c$ if $P = \text{Alice}$ and $g = 0$ if $P = \text{Bob}$. P can transition to the OPEN State for the first time only if all of the following have taken place:

- It has received (OPEN, c, \dots) while in the INIT State . In case $P = \text{Alice}$, this message must have been received as input by \mathcal{E} (Fig. 15, l. 1), or in case $P = \text{Bob}$, this message must have been received via the network by \bar{P} (Fig. 10, l. 3).
- It has received $pk_{\bar{P}, F}$. In case $P = \text{Bob}$, $pk_{\bar{P}, F}$ must have been contained in the (OPEN, \dots) message by \bar{P} (Fig. 10, l. 3), otherwise if $P = \text{Alice}$ $pk_{\bar{P}, F}$ must have been contained in the $(\text{ACCEPT CHANNEL}, \dots)$ message by \bar{P} (Fig. 10, l. 16).
- It internally holds a signature on the commitment transaction $C_{P,0}$ that is valid when verified with public key $pk_{\bar{P}, F}$ (Fig. 12, ll. 12 and 23).
- It has the transaction F in the $\mathcal{G}_{\text{Ledger}}$ state (Fig. 13, l. 3 or Fig. 14, l. 5).

We observe that P satisfies the Lemma conditions with $m = n = l = 0$. Before transitioning to the OPEN State , P has produced only one valid signature for the “funding” output $(c, 2/\{pk_{P,F}, pk_{\bar{P},F}\})$ of F with $sk_{P,F}$, namely for $C_{P,0}$ (Fig. 12, ll. 4 or 14), and sent it to \bar{P} (Fig. 12, ll. 6 or 21), therefore the only two ways to spend $(c, 2/\{pk_{P,F}, pk_{\bar{P},F}\})$ are by either publishing $C_{P,0}$ or $C_{\bar{P},0}$. We observe that $C_{P,0}$ has a $(g, (pk_{P,\text{out}} + (t+s)) \vee 2/\{pk_{P,R}, pk_{\bar{P},R}\})$ output (Fig. 12, l. 2 or 3). The spending method $2/\{pk_{P,R}, pk_{\bar{P},R}\}$ cannot be used since P has not produced a signature for it with $sk_{P,R}$, therefore the alternative spending method, $pk_{P,\text{out}} + (t+s)$, is the only one that will be spendable if $C_{P,0}$ is included in $\mathcal{G}_{\text{Ledger}}$, thus contributing g to the sum of outputs that contribute to inequality (1). Likewise, if $C_{\bar{P},0}$ is included in $\mathcal{G}_{\text{Ledger}}$, it will contribute at least one $(g, pk_{P,\text{out}})$ output to this inequality, as $C_{\bar{P},0}$ has a $(g, pk_{P,\text{out}})$ output (Fig. 12, l. 2 or 3). Additionally, if P receives (CLOSE) by \mathcal{E} while $H = ([], [g])$, it attempts to publish $C_{P,0}$ (Fig. 26, l. 19), and will either succeed or $C_{\bar{P},0}$ will be published instead. We therefore conclude that in every case $\mathcal{G}_{\text{Ledger}}$ will eventually have a state Σ that contains at least one $(g, pk_{P,\text{out}})$ output, therefore satisfying the Lemma consequence.

Let P with history $H = (F, C)$. The induction hypothesis is that the Lemma holds for P . Let c_P the sum in the right hand side of inequality (1). In order to perform the induction step, assume that P is in the OPEN state . We will prove all the following (the facts to be proven are shown with emphasis for clarity):

- If P receives $(\text{FUND ME}, f, \dots)$ by a (local, trusted) LN ITI R , subsequently transitions back to the OPEN state (therefore moving to history (F, C') where $C' = C + [-f]$) and finally receives (CLOSE) by \mathcal{E} and (CLOSED) by host_P

before any further change to its history, then *eventually* P 's $\mathcal{G}_{\text{Ledger}}$ state will contain h transaction outputs each of value c_i exclusively spendable or already spent by $pk_{P,\text{out}}$ that are descendants of an output with spending

method $2/\{pk_{P,F}, pk_{\bar{P},F}\}$ such that $\sum_{i=1}^h c_i \geq \sum_{s \in C'} \sum_{x \in s} x$. Furthermore, given

that P moves to the OPEN state after the (FUND ME, ...) message, it also sends (FUNDED, ...) to R (Fig. 17, l. 21). If subsequently the state of R transitions to OPEN (therefore obtaining history (F_R, C_R) where $F_R = F + C$ and $C_R = [[f]]$), and finally receives (CLOSE) by \mathcal{E} and (CLOSED) by host_R ($\text{host}_R = \text{host}_P$ – Fig. 14, l. 10) before any further change to its history, then *eventually* R 's $\mathcal{G}_{\text{Ledger}}$ state will contain k transaction outputs each of value c_i^R exclusively spendable or already spent by $pk_{R,\text{out}}$ that are descendants of an output with spending method $2/\{pk_{R,F}, pk_{\bar{R},F}\}$ such that

$$\sum_{i=1}^k c_i^R \geq \sum_{s \in C_R} \sum_{x \in s} x.$$

- If P receives (VIRTUALISING, ...) by \bar{P} , subsequently transitions back to OPEN (therefore moving to history (F, C') where $C' = C + [0]$) and finally receives CLOSE by \mathcal{E} and (CLOSED) by host_P before any further change to its history, then *eventually* P 's $\mathcal{G}_{\text{Ledger}}$ state will contain h transaction outputs each of value c_i exclusively spendable or already spent by $pk_{P,\text{out}}$ that are descendants of an output with spending method $2/\{pk_{P,F}, pk_{\bar{P},F}\}$ such that

$$\sum_{i=1}^h c_i \geq \sum_{s \in C} \sum_{x \in s} x.$$

Furthermore, given that P moves to the OPEN state after the (VIRTUALISING, ...) message and in case it sends (FUNDED, ...) to some party R (Fig. 17, l. 18), the latter party is the (local, trusted) **fundee** of a new virtual channel. If subsequently the state of R transitions to OPEN (therefore obtaining history (F_R, C_R) where $F_R = F + C$ and $C_R = [[0]]$), and finally receives (CLOSE) by \mathcal{E} and (CLOSED) by host_R ($\text{host}_R = \text{host}_P$ – Fig. 14, l. 10) before any further change to its history, then *eventually* R 's $\mathcal{G}_{\text{Ledger}}$ state will contain an output with a $2/\{pk_{R,F}, pk_{\bar{R},F}\}$ spending method.

- If P receives (PAY, d) by \mathcal{E} , subsequently transitions back to OPEN (therefore moving to history (F, C') where C' is C with $-d$ appended to the last list of C) and finally receives CLOSE by \mathcal{E} and (CLOSED) by host_P (the latter only if $\text{host}_P \neq \mathcal{G}_{\text{Ledger}}$ or equivalently $F \neq []$) before any further change to its history, then *eventually* P 's $\mathcal{G}_{\text{Ledger}}$ state will contain h transaction outputs each of value c_i exclusively spendable or already spent by $pk_{P,\text{out}}$ that are descendants of an output with a $2/\{pk_{P,F}, pk_{\bar{P},F}\}$ spending method such that

$$\sum_{i=1}^h c_i \geq \sum_{s \in C'} \sum_{x \in s} x.$$

- If P receives (GET PAID, e) by \mathcal{E} , subsequently transitions back to OPEN (therefore moving to history (F, C') where C' is C with e appended to the last list of C) and finally receives CLOSE by \mathcal{E} and (CLOSED) by host_P (the latter only if $\text{host}_P \neq \mathcal{G}_{\text{Ledger}}$ or equivalently $F = []$) before any further change

to its history, then *eventually* P 's $\mathcal{G}_{\text{Ledger}}$ state will contain h transaction outputs each of value c_i exclusively spendable or already spent by $pk_{P,\text{out}}$ that are descendants of an output with a $2/\{pk_{P,F}, pk_{\bar{P},F}\}$ spending method

$$\text{such that } \sum_{i=1}^h c_i \geq \sum_{s \in C'} \sum_{x \in s} x.$$

By the induction hypothesis, before the funding procedure started P could close the channel and end up with on-chain transaction outputs exclusively spendable or already spent by $pk_{P,\text{out}}$ with a sum value of c_P . When P is in the OPEN state and receives (FUND ME, f , ...), it can only move again to the OPEN state after doing the following state transitions: OPEN \rightarrow VIRTUALISING \rightarrow WAITING FOR REVOCATION \rightarrow WAITING FOR INBOUND REVOCATION \rightarrow WAITING FOR HOSTS READY \rightarrow OPEN. During this sequence of events, a new $\text{host}_{\bar{P}}$ is defined (Fig. 17, l. 6), new commitment transactions are negotiated with \bar{P} (Fig. 17, l. 9), control of the old funding output is handed over to host_P (Fig. 17, l. 11), host_P negotiates with its counterparty a new set of transactions and signatures that spend the aforementioned funding output and make available a new funding output with the keys $pk'_{P,F}, pk'_{\bar{P},F}$ as P instructed (Fig. 35 and 37) and the previous valid commitment transactions of both P and \bar{P} are invalidated (Fig. 9, l. 1 and l. 14 respectively). We note that the use of the ANYPREVOUT flag in all signatures that correspond to transaction inputs that may spend various different transaction outputs ensures that this is possible, as it avoids tying each input to a specific, predefined output. When P receives (CLOSE) by \mathcal{E} , it inputs (CLOSE) to host_P (Fig. 26, l. 4). As per the Lemma conditions, host_P will output (CLOSED). This can happen only when $\mathcal{G}_{\text{Ledger}}$ contains a suitable output for both P 's and R 's channel (Fig. 41, and 4 ll. 5 respectively).

If the host of host_P is $\mathcal{G}_{\text{Ledger}}$, then the funding output $o_{1,2} = (c_P + c_{\bar{P}}, 2/\{pk_{P,F}, pk_{\bar{P},F}\})$ for the P, \bar{P} channel is already on-chain. Regarding the case in which $\text{host}_P \neq \mathcal{G}_{\text{Ledger}}$, after the funding procedure is complete, the new host_P will have as its host the old host_P of P . If the (CLOSE) sequence is initiated, the new host_P will follow the same steps that will be described below once the old host_P succeeds in closing the lower layer (Fig. 40, l. 5). The old host_P however will see no difference in its interface compared to what would happen if P had received (CLOSE) before the funding procedure, therefore it will successfully close by the induction hypothesis. Thereafter the process is identical to the one when the old $\text{host}_P = \mathcal{G}_{\text{Ledger}}$.

Moving on, host_P is either able to publish its $\text{TX}_{1,1}$ (it has necessarily received a valid signature $\text{sig}(\text{TX}_{1,1}, pk_{\bar{P},F})$ (Fig. 37, l. 39) by its counterparty before it moved to the OPEN state for the first time), or the output $(c_P + c_{\bar{P}}, 2/\{pk_{P,F}, pk_{\bar{P},F}\})$ needed to spend $\text{TX}_{1,1}$ has already been spent. The only other transactions that can spend it are $\text{TX}_{2,1}$ and any of $(\text{TX}_{2,2,k})_{k>2}$, since these are the only transactions that spend the aforementioned output and that host_P has signed with $sk_{P,F}$ (Fig. 37, ll. 33-37). The output can be also spent by old, revoked commitment transactions, but in that case host_P would not have output (CLOSED); P would have instead detected this triggered by a (CHECK CHAIN FOR CLOSED) message by \mathcal{E} (Fig. 24) and would have moved to the

CLOSED state on its own accord (lack of such a message by \mathcal{E} would lead P to become **negligent**, something that cannot happen according to the Lemma conditions). Every transaction among $\text{TX}_{1,1}$, $\text{TX}_{2,1}$, $(\text{TX}_{2,2,k})_{k>2}$ has a $(c_P + c_{\bar{P}} - f, 2/\{pk'_{P,F}, pk'_{\bar{P},F}\})$ output (Fig. 31, l. 18 and Fig. 30, ll. 27 and 91) which will end up in $\mathcal{G}_{\text{Ledger}}$ – call this output o_P . We will prove that at most $\sum_{i=2}^{n-1} (t_i + p + s - 1)$ blocks after (CLOSE) is received by P , an output o_R with c_{virt} coins and a $2/\{pk_{R,F}, pk_{\bar{R},F}\}$ spending condition without or with an expired timelock will be included in $\mathcal{G}_{\text{Ledger}}$. In case party \bar{P} is idle, then $o_{1,2}$ is consumed by $\text{TX}_{1,1}$ and the timelock on its virtual output expires, therefore the required output o_R is on-chain. In case \bar{P} is active, exactly one of $\text{TX}_{2,1}$, $(\text{TX}_{2,2,k})_{k>2}$ or $(\text{TX}_{2,3,1,k})_{k>2}$ is a descendant of $o_{1,2}$; if the transaction belongs to one of the two last transaction groups then necessarily $\text{TX}_{1,1}$ is on-chain in some block height h and given the timelock on the virtual output of $\text{TX}_{1,1}$, \bar{P} 's transaction can be at most at block height $h + t_2 + p + s - 1$. If $n = 3$ or $k = n - 1$, then \bar{P} 's unique transaction has the required output o_R (without a timelock). The rest of the cases are covered by the following sequence of events:

Closing sequence

- 1: $\text{maxDel} \leftarrow t_2 + p + s - 1$ // A_2 is active and the virtual output of $\text{TX}_{1,1}$ has a timelock of t_2
- 2: $i \leftarrow 3$
- 3: **loop**
- 4: **if** A_i is idle **then**
- 5: The timelock on the virtual output of the transaction published by A_{i-1} expires and therefore the required o_R is on-chain
- 6: **else** // A_i publishes a transaction that is a descendant of $o_{1,2}$
- 7: $\text{maxDel} \leftarrow \text{maxDel} + t_i + p + s - 1$
- 8: The published transaction can be of the form $\text{TX}_{i,2,2}$ or $(\text{TX}_{i,3,2,k})_{k>i}$ as it spends the virtual output which is encumbered with a public key controlled by R and R has only signed these transactions
- 9: **if** $i = n - 1$ or $k \geq n - 1$ **then** // The interval contains all intermediaries
- 10: The virtual output of the transaction is not timelocked and has only a $2/\{pk_{R,F}, pk_{\bar{R},F}\}$ spending method, therefore it is the required o_R
- 11: **else** // At least one intermediary is not in the interval
- 12: **if** the transaction is $\text{TX}_{i,3,2,k}$ **then** $i \leftarrow k$ **else** $i \leftarrow i + 1$
- 13: **end if**
- 14: **end if**
- 15: **end loop**
- 16: // $\text{maxDel} \leq \sum_{i=2}^{n-1} (t_i + p + s - 1)$

Fig. 43.

In every case o_P and o_R end up on-chain in at most s and $\sum_{i=2}^{n-1} (t_i + p + s - 1)$ blocks respectively from the moment (CLOSE) is received. The output o_P can be spent either by $C_{P,i}$ or $C_{\bar{P},i}$. Both these transactions have a $(c_P - f, pk_{P,\text{out}})$ output. This output of $C_{P,i}$ is timelocked, but the alternative spending method cannot be used as P never signed a transaction that uses it (as it is reserved for revocation, which has not taken place yet in this virtualisation layer). We have now proven that if P completes the funding of a new channel then it can close its channel for a $(c_P - f, pk_{P,\text{out}})$ output that is a descendant of an output with spending method $2/\{pk_{P,F}, pk_{\bar{P},F}\}$ and that lower bound of value holds for the duration of the funding procedure, i.e. we have proven the first claim of the first bullet.

We will now prove that the newly funded party R can close its channel securely. After R receives (FUNDED, host_P, \dots) by P and before moving to the OPEN state, it receives $\text{sig}_{\bar{R},C,0} = \text{sig}(C_{R,0}, pk_{\bar{R},F})$ and sends $\text{sig}_{R,C,0} = \text{sig}(C_{\bar{R},0}, pk_{R,F})$. Both these transactions spend o_R . As we showed before, if R receives (CLOSE) by \mathcal{E} then o_R eventually ends up on-chain. After receiving (CLOSED) from host_P , R attempts to add $C_{R,0}$ to $\mathcal{G}_{\text{Ledger}}$, which may only fail if $C_{\bar{R},0}$ ends up on-chain instead. Similar to the case of P , both these transactions have an $(f, pk_{R,\text{out}})$ output. This output of $C_{R,0}$ is timelocked, but the alternative spending method cannot be used as R never signed a transaction that uses it (as it is reserved for revocation, which has not taken place yet) so the timelock will expire and the desired spending method will be available. We have now proven that if R 's channel is funded to completion (i.e. R moves to the OPEN state for the first time) then it can close its channel for a $(f, pk_{R,\text{out}})$ output that is a descendant of o_R . We have therefore proven the first bullet.

We now move on to the second bullet. In case P is the **funder** (i.e. $i = n$), then the same arguments as in the previous bullet hold here with “WAITING FOR INBOUND REVOCATION” replaced with “WAITING FOR OUTBOUND REVOCATION”, $o_{1,2}$ with $o_{n-1,n}$, $\text{TX}_{1,1}$ with $\text{TX}_{n,1}$, $\text{TX}_{2,1}$ with $\text{TX}_{n-1,1}$, $(\text{TX}_{2,2,k})_{k>2}$ with $(\text{TX}_{n-1,2,k})_{k<n-1}$, $(\text{TX}_{2,3,1,k})_{k>2}$ with $(\text{TX}_{n-1,3,n,k})_{k<n-1}$, t_2 with t_{n-1} , $\text{TX}_{i,3,2,k}$ with $\text{TX}_{i,3,n-1,k}$, i is initialized to $n - 2$ in l. 2 of Fig. 43, i is decremented instead of incremented in l. 12 of the same Figure and f is replaced with 0. This is so because these two cases are symmetric.

In case P is not the **funder** ($1 < i < n$), then we only need to prove the first statement of the second bullet. By the induction hypothesis and since **sibling** is trusted, we know that both P 's and **sibling**'s funding outputs either are or can be eventually put on-chain and that P 's funding output has at least $c_P = \sum_{s \in C} \sum_{x \in s} x$ coins. If P is on the “left” of its **sibling** (i.e. there is an untrusted party that sent the (VIRTUALISING, ...) message to P which triggered the latter to move to the VIRTUALISING state and to send a (VIRTUALISING, ...) message to its own **sibling**), the “left” funding output o_{left} (the one held with the untrusted party to the left) can be spent by one of $\text{TX}_{i,1}$, $(\text{TX}_{i,2,k})_{k>i}$, $\text{TX}_{i-1,1}$,

or $(\text{TX}_{i-1,2,k})_{k < i-1}$, as these are the only transactions that P has signed with $sk_{P,F}$. All these transactions have a $(c_P + c_{\bar{P}} - f, 2/\{pk_{P,F'}, pk_{\bar{P},F'}\})$ output that can in turn be spent by either $C_{P,0}$ or $C_{\bar{P},0}$, both of which have an output of value c_P and a $pk_{P,\text{out}}$ spending method and no other spending method can be used (as P has not signed the “revocation” spending method of $C_{P,0}$).

In the case that P is to the right of its **sibling** (i.e. P receives by **sibling** the (VIRTUALISING, ...) message that causes P 's transition to the VIRTUALISING state), the “right” funding output o_{right} (the one held with the untrusted party to the right) can be spent by one of $\text{TX}_{i,1}$, $(\text{TX}_{i,2,k})_{k < i}$, $\text{TX}_{i+1,1}$, or $(\text{TX}_{i+1,2,k})_{k > i+1}$, as these are the only transactions that P has signed with $sk_{P,F}$. All these transactions have a $(c_P + c_{\bar{P}} - f, 2/\{pk_{P,F'}, pk_{\bar{P},F'}\})$ output that can in turn be spent by either $C_{P,0}$ or $C_{\bar{P},0}$, both of which have an output of value $c_P - f$ and a $pk_{P,\text{out}}$ spending method and no other spending method can be used (as P has not signed the “revocation” spending method of $C_{P,0}$). P can get the remaining f coins as follows: $\text{TX}_{i,1}$ and all of $(\text{TX}_{i,2,k})_{k < i}$ already have an $(f, pk_{P,\text{out}})$ output. If instead $\text{TX}_{i+1,1}$ or one of $(\text{TX}_{i+1,2,k_2})_{k_2 > i+1}$ spends o_{right} , then P will publish $\text{TX}_{i,2,i+1}$ or $\text{TX}_{i,2,k_2}$ respectively if o_{left} is unspent, otherwise o_{left} is spent by one of $\text{TX}_{i-1,1}$ or $(\text{TX}_{i-1,2,k_1})_{k_1 < i-1}$ in which case P will publish one of $\text{TX}_{i,3,k_1,i+1}$, $\text{TX}_{i,3,i-1,k_2}$, $\text{TX}_{i,3,i-1,i+1}$ or $\text{TX}_{i,3,k_1,k_2}$. In particular, $\text{TX}_{i,3,k_1,i+1}$ is published if $\text{TX}_{i-1,2,k_1}$ and $\text{TX}_{i+1,1}$ are on-chain, $\text{TX}_{i,3,i-1,k_2}$ is published if $\text{TX}_{i-1,1}$ and $\text{TX}_{i+1,2,k_2}$ are on-chain, $\text{TX}_{i,3,i-1,i+1}$ is published if $\text{TX}_{i-1,1}$ and $\text{TX}_{i+1,1}$ are on-chain, or $\text{TX}_{i,3,k_1,k_2}$ is published if $\text{TX}_{i-1,2,k_1}$ and $\text{TX}_{i+1,2,k_2}$ are on-chain. All these transactions include an $(f, pk_{P,\text{out}})$ output. We have therefore covered all cases and proven the second bullet.

Regarding now the third bullet, once again the induction hypothesis guarantees that before (PAY, d) was received, P could close the channel resulting in on-chain outputs exclusively spendable or already spent by $pk_{P,\text{out}}$ that are descendants of an output with a $2/\{pk_{P,F}, pk_{\bar{P},F}\}$ spending method that have a sum value of $c_P = \sum_{s \in C} \sum_{x \in s} x$. (Note that $\sum_{s \in C'} \sum_{x \in s} x = d + \sum_{s \in C} \sum_{x \in s} x$.) When P receives (PAY, d) while in the OPEN state, it moves to the WAITING FOR COMMITMENT SIGNED state before returning to the OPEN state. It signs (Fig. 19, l. 2) the new commitment transaction $C_{\bar{P},i+1}$ in which the counterparty owns d more coins than before that moment (Fig. 19, l. 1), sends the signature to the counterparty (Fig. 19, l. 4) and expects valid signatures on its own updated commitment transaction (Fig. 20, l. 1) and the revocation transaction for the old commitment transaction of the counterparty (Fig. 20, l. 3). Note that if the counterparty does not respond or if it responds with missing/invalid signatures, either P can close the channel with the old commitment transaction $C_{P,i}$ exactly like before the update started (as it has not yet sent the signature for the old revocation transaction), or the counterparty will close the channel either with the new or with the old commitment transaction. In all cases in which validation fails and the channel closes, there is an output with a $pk_{P,\text{out}}$ spending method and no other useable spending method that carries at least $c_P - d$ coins. Only if the verification succeeds does P sign (Fig. 20, l. 5) and send (Fig. 20, l. 17) the counterparty's revocation transaction for P 's previous commitment transaction.

Similarly to previous bullets, if $\text{host}_P \neq \mathcal{G}_{\text{Ledger}}$ the funding output can be put on-chain, otherwise the funding output is already on-chain. In both cases, since the closing procedure continues, one of $C_{P,i+1}$ ($C_{\bar{P},j}$) $_{0 \leq j \leq i+1}$ will end up on-chain. If $C_{\bar{P},j}$ for some $j < i+1$ is on-chain, then P submits $R_{P,j}$ (we discussed how P obtained $R_{P,i}$ and the rest of the cases are covered by induction) and takes the entire value of the channel which is at least $c_P - d$. If $C_{\bar{P},i+1}$ is on-chain, it has a $(c_P - d, pk_{P,\text{out}})$ output. If $C_{P,i+1}$ is on-chain, it has an output of value $c_P - d$, a timelocked $pk_{P,\text{out}}$ spending method and a non-timelocked spending method that needs the signature made with $sk_{P,R}$ on $R_{\bar{P},i+1}$. P however has not generated that signature, therefore this spending method cannot be used and the timelock will expire, therefore in all cases outputs that descend from the funding output, can be spent exclusively by $pk_{P,\text{out}}$ and carry at least $c_P - d$ coins are put on-chain. We have proven the third bullet.

For the fourth and last bullet, again by the induction hypothesis, before (GET PAID, e) was received P could close the channel resulting in on-chain outputs exclusively spendable or already spent by $pk_{P,\text{out}}$ that are descendants of an output o_F with a $2/\{pk_{P,F}, pk_{\bar{P},F}\}$ spending method and have a sum value of $c_P = \sum_{s \in C} \sum_{x \in s} x$. (Note that $e + \sum_{s \in C'} \sum_{x \in s} x = \sum_{s \in C} \sum_{x \in s} x$ and that o_F either is already on-chain or can be eventually put on-chain as we have argued in the previous bullets by the induction hypothesis.) When P receives (GET PAID, e) while in the OPEN state, if the balance of the counterparty is enough it moves to the WAITING TO GET PAID state (Fig. 22, l. 6). If subsequently it receives a valid signature for $C_{P,i+1}$ (Fig. 19, l. 8) which is a commitment transaction that can spend the o_F output and gives to P an additional e coins compared to $C_{P,i}$. Subsequently P 's state transitions to WAITING FOR PAY REVOCATION and sends signatures for $C_{\bar{P},i+1}$ and $R_{\bar{P},i}$ to \bar{P} . If the o_F output is spent while P is in the latter state, it can be spent by one of $C_{P,i+1}$ or $(C_{\bar{P},j})_{0 \leq j \leq i+1}$. If it is spent by $C_{P,i+1}$ or $C_{\bar{P},i+1}$, then these two transactions have a $(c_P + e, pk_{P,\text{out}})$ output. (Note that the former is encumbered with a timelock, but the alternative spending method cannot be used as P has not signed $R_{\bar{P},i+1}$.) If it is spent by $C_{\bar{P},i}$ then a $(c_P, pk_{P,\text{out}})$ output becomes available instead, therefore P can still get the c_P coins that correspond to the previous state. If any of $(C_{\bar{P},j})_{0 \leq j < i}$ spends o_F then it makes available a $pk_{P,\text{out}}$ output with the coins that P had at state j and additionally P can publish $R_{P,j}$ that spends \bar{P} 's output of $C_{\bar{P},j}$ and obtain the entirety of \bar{P} 's coins at state j for a total of $c_P + c_{\bar{P}}$ coins. Therefore in every case P can claim at least c_P coins. In the case that P instead subsequently receives a valid signature to $R_{P,i}$ (Fig. 20, l. 20) it finally moves to the OPEN state once again. In this state the above analysis of what can happen when o_F holds similarly, with the difference that if \bar{P} spends o_F with $C_{\bar{P},i}$ now P can publish $R_{P,i}$ which gives P the coins of \bar{P} . Therefore with this difference P is now guaranteed to gain at least $c_P + e$ coins upon channel closure. We have therefore proven the fourth bullet. \square

Lemma 2 (Ideal world balance). Consider an ideal world execution with functionality $\mathcal{F}_{\text{Chan}}$ and simulator \mathcal{S} . Let $P \in \{\text{Alice}, \text{Bob}\}$ one of the two parties of $\mathcal{F}_{\text{Chan}}$. Assume that all of the following are true:

- $\text{State}_P \neq \text{IGNORED}$,
- P has transitioned to the OPEN State at least once. Additionally, if $P = \text{Alice}$, it has received (OPEN, c, \dots) by \mathcal{E} prior to transitioning to the OPEN State,
- P [has received $(\text{FUND ME}, f_i, \dots)$ as input by another $\mathcal{F}_{\text{Chan}}/\text{LN ITI}$ while $\text{State}_P = \text{OPEN}$ and P subsequently transitioned to OPEN State] $n \geq 0$ times,
- P [has received (PAY, d_i) by \mathcal{E} while $\text{State}_P = \text{OPEN}$ and P subsequently transitioned to OPEN State] $m \geq 0$ times,
- P [has received $(\text{GET PAID}, e_i)$ by \mathcal{E} while $\text{State}_P = \text{OPEN}$ and P subsequently transitioned to OPEN State] $l \geq 0$ times.

Let $\phi = 1$ if $P = \text{Alice}$, or $\phi = 0$ if $P = \text{Bob}$. If $\mathcal{F}_{\text{Chan}}$ receives (CLOSE, P) by \mathcal{S} , then the following holds with overwhelming probability on the security parameter:

$$\text{balance}_P = \phi \cdot c - \sum_{i=1}^n f_i - \sum_{i=1}^m d_i + \sum_{i=1}^l e_i \quad (2)$$

Proof. We will prove the Lemma by following the evolution of the balance_P variable.

- When $\mathcal{F}_{\text{Chan}}$ is activated for the first time, it sets $\text{balance}_P \leftarrow 0$ (Fig. 2, l. 1).
- If $P = \text{Alice}$ and it receives (OPEN, c, \dots) by \mathcal{E} , it stores c (Fig. 2, l. 10). If later State_P becomes OPEN, $\mathcal{F}_{\text{Chan}}$ sets $\text{balance}_P \leftarrow c$ (Fig. 2, ll. 13 or 31). In contrast, if $P = \text{Bob}$, it is $\text{balance}_P = 0$ until at least the first transition of State_P to OPEN (Fig. 2).
- Every time P receives input $(\text{FUND ME}, f_i, \dots)$ by another party while $\text{State}_P = \text{OPEN}$, P stores f_i (Fig. 4, l. 1). The next time State_P transitions to OPEN (if such a transition happens), balance_P is decremented by f_i (Fig. 4, l. 27). Therefore, if this cycle happens $n \geq 0$ times, balance_P will be decremented by $\sum_{i=1}^n f_i$ in total.
- Every time P receives input (PAY, d_i) by \mathcal{E} while $\text{State}_P = \text{OPEN}$, d_i is stored (Fig. 3, l. 2). The next time State_P transitions to OPEN (if such a transition happens), balance_P is decremented by d_i (Fig. 3, l. 13). Therefore, if this cycle happens $m \geq 0$ times, balance_P will be decremented by $\sum_{i=1}^m d_i$ in total.
- Every time P receives input $(\text{GET PAID}, e_i)$ by \mathcal{E} while $\text{State}_P = \text{OPEN}$, e_i is stored (Fig. 3, l. 7). The next time State_P transitions to OPEN (if such a transition happens) balance_P is incremented by e_i (Fig. 3, l. 19). Therefore, if this cycle happens $l \geq 0$ times, balance_P will be incremented by $\sum_{i=1}^l e_i$ in total.

On aggregate, after the above are completed and then $\mathcal{F}_{\text{Chan}}$ receives (CLOSE, P) by \mathcal{S} , it is $\text{balance}_P = c - \sum_{i=1}^n f_i - \sum_{i=1}^m d_i + \sum_{i=1}^l e_i$ if $P = \text{Alice}$, or else if $P = \text{Bob}$, $\text{balance}_P = - \sum_{i=1}^n f_i - \sum_{i=1}^m d_i + \sum_{i=1}^l e_i$. \square

Lemma 3 (No halt). *In an ideal execution with $\mathcal{F}_{\text{Chan}}$ and \mathcal{S} , if the trusted parties of the honest parties of $\mathcal{F}_{\text{Chan}}$ are themselves honest, then the functionality halts with negligible probability in the security parameter (i.e. l. 21 of Fig. 5 is executed negligibly often).*

Proof. We prove the Lemma in two steps. We first show that if the conditions of Lemma 2 hold, then the conditions of Lemma 1 for the real world execution with protocol LN and the same \mathcal{E} and \mathcal{A} hold as well for the same m, n and l values.

For State_P to become IGNORED, either \mathcal{S} has to send (BECAME CORRUPTED OR NEGLIGENT, P) or host_P must output (ENABLER USED REVOCATION) to $\mathcal{F}_{\text{Chan}}$ (Fig. 2, l. 4). The first case only happens when either P receives (CORRUPT) by \mathcal{A} (Fig. 7, l. 1), which means that the simulated P is not honest anymore, or when P becomes negligent (Fig. 7, l. 4), which means that the first condition of Lemma 1 is violated. In the second case, it is $\text{host}_P \neq \mathcal{G}_{\text{Ledger}}$ and the state of host_P is GUEST PUNISHED (Fig. 42, ll. 1 or 12), so in case P receives (CLOSE) by \mathcal{E} the output of host_P will be (GUEST PUNISHED) (Fig. 40, l. 3). In all cases, some condition of Lemma 1 is violated.

For State_P to become OPEN at least once, the following sequence of events must take place (Fig. 2): If $P = \text{Alice}$, it must receive (INIT, pk) by \mathcal{E} when $\text{State}_P = \text{UNINIT}$, then either receive (OPEN, c , $\mathcal{G}_{\text{Ledger}}$, ...) by \mathcal{E} and (BASE OPEN) by \mathcal{S} or (OPEN, c , hops ($\neq \mathcal{G}_{\text{Ledger}}$), ...) by \mathcal{E} , (FUNDED, HOST, ...) by hops[0].left and (VIRTUAL OPEN) by \mathcal{S} . In either case, \mathcal{S} only sends its message only if all its simulated honest parties move to the OPEN state (Fig. 7, l. 10), therefore if the second condition of Lemma 2 holds and $P = \text{Alice}$, then the second condition of Lemma 1 holds as well. The same line of reasoning can be used to deduce that if $P = \text{Bob}$, then State_P will become OPEN for the first time only if all honest simulated parties move to the OPEN state, therefore once more the second condition of Lemma 2 holds only if the second condition of Lemma 1 holds as well. We also observe that, if both parties are honest, they will transition to the OPEN state simultaneously.

Regarding the third Lemma 2 condition, we assume (and will later show) that if both parties are honest and the state of one is OPEN, then the state of the other is also OPEN. Each time P receives input (FUND ME, f , ...) by $R \in \{\mathcal{F}_{\text{Chan}}, \text{LN}\}$, State_P transitions to PENDING FUND, subsequently when a command to define a new VIRT ITI through P is intercepted by $\mathcal{F}_{\text{Chan}}$, State_P transitions to TENTATIVE FUND and afterwards when \mathcal{S} sends (FUND) to $\mathcal{F}_{\text{Chan}}$, State_P transitions to SYNC FUND. In parallel, if $\text{State}_{\bar{P}} = \text{IGNORED}$, then $\text{State}_{\bar{P}}$ transitions directly back to OPEN. If on the other hand $\text{State}_{\bar{P}} = \text{OPEN}$ and $\mathcal{F}_{\text{Chan}}$ intercepts a similar VIRT ITI definition command through \bar{P} , $\text{State}_{\bar{P}}$ transitions

to TENTATIVE HELP FUND. On receiving the aforementioned (FUND) message by \mathcal{S} and given that $State_{\bar{P}} = \text{TENTATIVE HELP FUND}$, $\mathcal{F}_{\text{Chan}}$ also sets $State_{\bar{P}}$ to SYNC HELP FUND. Then both $State_{\bar{P}}$ and $State_P$ transition simultaneously to OPEN (Fig. 4). This sequence of events may repeat any $n \geq 0$ times. We observe that throughout these steps, honest simulated P has received (FUND ME, f , ...) and that \mathcal{S} only sends (FUND) when all honest simulated parties have transitioned to the OPEN state (Fig. 7, l. 18 and Fig. 17, l. 12), so the third condition of Lemma 1 holds with the same n as that of Lemma 2.

Regarding the fourth Lemma 2 condition, we again assume that if both parties are honest and the state of one is OPEN, then the state of the other is also OPEN. Each time P receives input (PAY, d) by \mathcal{E} , $State_P$ transitions to TENTATIVE PAY and subsequently when \mathcal{S} sends (PAY) to $\mathcal{F}_{\text{Chan}}$, $State_P$ transitions to (SYNC PAY, d). In parallel, if $State_{\bar{P}} = \text{IGNORED}$, then $State_{\bar{P}}$ transitions directly back to OPEN. If on the other hand $State_{\bar{P}} = \text{OPEN}$ and $\mathcal{F}_{\text{Chan}}$ receives (GET PAID, d) by \mathcal{E} addressed to \bar{P} , $State_{\bar{P}}$ transitions to TENTATIVE GET PAID. On receiving the aforementioned (PAY) message by \mathcal{S} and given that $State_{\bar{P}} = \text{TENTATIVE GET PAID}$, $\mathcal{F}_{\text{Chan}}$ also sets $State_{\bar{P}}$ to SYNC GET PAID. Then both $State_P$ and $State_{\bar{P}}$ transition simultaneously to OPEN (Fig. 3). This sequence of events may repeat any $m \geq 0$ times. We observe that throughout these steps, honest simulated P has received (PAY, d) and that \mathcal{S} only sends (PAY) when all honest simulated parties have completed sending or receiving the payment (Fig. 7, l. 16), so the fourth condition of Lemma 1 holds with the same m as that of Lemma 2. As far as the fifth condition of Lemma 2 goes, we observe that this case is symmetric to the one discussed for its fourth condition above if we swap P and \bar{P} , therefore we deduce that if Lemma 2 holds with some l , then Lemma 1 holds with the same l .

As promised, we here argue that if both parties are honest and one party moves to the OPEN state, then the other party will move to the OPEN state as well. We already saw that the first time one party moves to the OPEN state, it will happen simultaneously with the same transition for the other party. We also saw that, when a party transitions from the SYNC HELP FUND or the SYNC FUND state to the OPEN state, then the other party will also transition to the OPEN state simultaneously. Furthermore, we saw that if one party transitions from the SYNC PAY or the SYNC GET PAID state to the OPEN state, the other party will also transition to the OPEN state simultaneously. Lastly we notice that we have exhausted all manners in which a party can transition to the OPEN state, therefore we have proven that transitions of honest parties to the OPEN state happen simultaneously.

Now, given that \mathcal{S} internally simulates faithfully both LN parties and that $\mathcal{F}_{\text{Chan}}$ relinquishes to \mathcal{S} complete control of the external communication of the parties as long as it does not halt, we deduce that \mathcal{S} replicates the behaviour of the aforementioned real world. By combining these facts with the consequences of the two Lemmas and the check that leads $\mathcal{F}_{\text{Chan}}$ to halt if it fails (Fig. 5, l. 18), we deduce that if the conditions of Lemma 2 hold for the honest parties of

$\mathcal{F}_{\text{Chan}}$ and their trusted parties, then the functionality halts only with negligible probability.

In the second proof step, we show that if the conditions of Lemma 2 do not hold, then the check of Fig. 5, l. 18 never takes place. We first discuss the $\text{State}_P = \text{IGNORED}$ case. We observe that the IGNORED State is a sink state, as there is no way to leave it once in. Additionally, for the balance check to happen, $\mathcal{F}_{\text{Chan}}$ must receive (CLOSED, P) by \mathcal{S} when $\text{State}_P \neq \text{IGNORED}$ (Fig. 5, l. 9). We deduce that, once $\text{State}_P = \text{IGNORED}$, the balance check will not happen. Moving to the case where State_P has never been OPEN, we observe that it is impossible to move to any of the states required by l. 9 of Fig. 5 without first having been in the OPEN state. Moreover if $P = \text{Alice}$, it is impossible to reach the OPEN state without receiving input (OPEN, c, \dots) by \mathcal{E} . Lastly, as we have observed already, the three last conditions of Lemma 2 are always satisfied. We conclude that if the conditions to Lemma 2 do not hold, then the check of Fig. 5, l. 18 does not happen and therefore $\mathcal{F}_{\text{Chan}}$ does not halt.

On aggregate, $\mathcal{F}_{\text{Chan}}$ may only halt with negligible probability in the security parameter. \square

Theorem 1 (Recursive Virtual Payment Channel Security). *The protocol Π_{Chan} realises $\mathcal{F}_{\text{Chan}}$ given a global functionality $\mathcal{G}_{\text{Ledger}}$ and assuming the security of the underlying digital signature. Specifically,*

$$\forall \text{PPT } \mathcal{A}, \exists \text{PPT } \mathcal{S} : \forall \text{PPT } \mathcal{E} \text{ it is } \text{EXEC}_{\Pi_{\text{Chan}}, \mathcal{A}, \mathcal{E}}^{\mathcal{G}_{\text{Ledger}}} \approx \text{EXEC}_{\mathcal{S}, \mathcal{E}}^{\mathcal{F}_{\text{Chan}}, \mathcal{G}_{\text{Ledger}}}$$

Proof. By inspection of Figs. 1 and 6 we can deduce that for a particular \mathcal{E} , in the ideal world execution $\text{EXEC}_{\mathcal{S}, \mathcal{A}, \mathcal{E}}^{\mathcal{F}_{\text{Chan}}, \mathcal{G}_{\text{Ledger}}}$, $\mathcal{S}_{\mathcal{A}}$ simulates internally the two Π_{Chan} parties exactly as they would execute in the real world execution, $\text{EXEC}_{\Pi_{\text{Chan}}, \mathcal{A}, \mathcal{E}}^{\mathcal{G}_{\text{Ledger}}}$ in case $\mathcal{F}_{\text{Chan}}$ does not halt. Indeed, $\mathcal{F}_{\text{Chan}}$ only halts with negligible probability according to Lemma 3, therefore the two executions are computationally indistinguishable. \square

We now generalise Theorem 1 to prove the indistinguishability of multiple $\mathcal{F}_{\text{Chan}}$ instances from multiple Π_{Chan} instances, leveraging the definition of the multi-session extension of an ideal functionality [6].

Definition 1 (Multi-Session Extension of a Protocol). *Let protocol π . Its multi-session extension $\hat{\pi}$ has the same code as π and has 2 session ids: the “sub-session id” ssid which replaces the session id of π and the usual session id sid which has no further function apart from what is prescribed by the UC framework.*

Theorem 2 (Indistinguishability of multiple sessions). *Let $\hat{\mathcal{F}}_{\text{Chan}}$ the multi-session extension of $\mathcal{F}_{\text{Chan}}$ and $\hat{\Pi}_{\text{Chan}}$ the protocol-multi-session extension of Π_{Chan} .*

$$\forall \text{PPT } \mathcal{A}, \exists \text{PPT } \mathcal{S} : \forall \text{PPT } \mathcal{E} \text{ it is } \text{EXEC}_{\hat{\Pi}_{\text{Chan}}, \mathcal{A}, \mathcal{E}}^{\mathcal{G}_{\text{Ledger}}} \approx \text{EXEC}_{\mathcal{S}, \mathcal{E}}^{\hat{\mathcal{F}}_{\text{Chan}}, \mathcal{G}_{\text{Ledger}}}$$

Proof. We observe that $\hat{\mathcal{F}}_{\text{Chan}}$ uses $\mathcal{F}_{\text{Chan}}$ internally. According to the UC theorem [4] and given that Π_{Chan} UC-realises $\mathcal{F}_{\text{Chan}}$ (Theorem 1), $\hat{\mathcal{F}}_{\text{Chan}}^{\mathcal{F}_{\text{Chan}} \rightarrow \Pi_{\text{Chan}}}$ UC-emulates $\hat{\mathcal{F}}_{\text{Chan}}$. We now observe that $\hat{\mathcal{F}}_{\text{Chan}}^{\mathcal{F}_{\text{Chan}} \rightarrow \Pi_{\text{Chan}}}$ behaves identically to a session with $\hat{\Pi}_{\text{Chan}}$ protocols, as the former routes each message to the same internal Π_{Chan} instance that would handle the same message in the latter case, therefore $\hat{\mathcal{F}}_{\text{Chan}}^{\mathcal{F}_{\text{Chan}} \rightarrow \Pi_{\text{Chan}}}$ UC-emulates $\hat{\Pi}_{\text{Chan}}$. By the transitivity of UC-emulation, we deduce that $\hat{\mathcal{F}}_{\text{Chan}}$ UC-emulates $\hat{\Pi}_{\text{Chan}}$. \square

References

1. Nakamoto S.: Bitcoin: A Peer-to-Peer Electronic Cash System (2008)
2. Croman K., Decker C., Eyal I., Gencer A. E., Juels A., Kosba A., Miller A., Saxena P., Shi E., Sirer E. G., et al.: On scaling decentralized blockchains. In International Conference on Financial Cryptography and Data Security: pp. 106–125: Springer (2016)
3. Poon J., Dryja T.: The Bitcoin Lightning Network: Scalable Off-Chain Instant Payments. <https://lightning.network/lightning-network-paper.pdf> (2016)
4. Canetti R.: Universally Composable Security: A New Paradigm for Cryptographic Protocols. In 42nd Annual Symposium on Foundations of Computer Science, FOCS 2001, 14-17 October 2001, Las Vegas, Nevada, USA: pp. 136–145: doi:10.1109/SFCS.2001.959888: URL <https://eprint.iacr.org/2000/067.pdf> (2001)
5. Badertscher C., Maurer U., Tschudi D., Zikas V.: Bitcoin as a transaction ledger: A composable treatment. In Annual International Cryptology Conference: pp. 324–356: Springer (2017)
6. Canetti R., Rabin T.: Universal Composition with Joint State. In Advances in Cryptology - CRYPTO 2003, 23rd Annual International Cryptology Conference, Santa Barbara, California, USA, August 17-21, 2003, Proceedings: pp. 265–281: doi:10.1007/978-3-540-45146-4_16: URL https://doi.org/10.1007/978-3-540-45146-4_16 (2003)