```
Protocol \Pi_{\mathrm{Chan}}
 1: Initialisation:
         State \leftarrow \text{INIT}
 3: On top up, check top up by \mathcal{E}, act as \mathcal{F}_{\mathrm{Chan}} (Fig. 3, lines 4-8 and 9-15
     respectively)
 4: On (OPEN, c_F, pk_{A,out}, pk_{B,out}) by \mathcal{E}:
         ensure State = \text{TOPPED UP}
         State \leftarrow \text{Opening base channel}
 6:
         do LN (other box)
 7:
 8: On (CHECK FUNDING) by \mathcal{E}:
 9:
         ensure State = WAITING FOR LEDGER
         send (READ) to \mathcal{G}_{\text{Ledger}} and assign reply to \Sigma
10:
         ensure F\in \varSigma
11:
12:
         c_A \leftarrow c; c_B \leftarrow 0 // c received in OPEN
13:
         State \leftarrow \text{OPEN BASE}
14:
         output (OPEN SUCCESS) to {\mathcal E}
15: On (PAY, x) by \mathcal{E}:
16:
         ensure State \in \{\text{OPEN BASE}, \text{OPEN VIRTUAL}\}
17:
         ensure c_A \geq x
         do LN payment (these channels won't be async) (balance change here)
18:
19:
         output (OK) to \mathcal{E}
20: On (BALANCE) by \mathcal{E}:
         ensure State \in \{\text{OPEN BASE}, \text{OPEN VIRTUAL}\}
22:
         output (BALANCE, (c_A, c_B, locked_A, locked_B)) to \mathcal{E}
23: On (CLOSE) by \mathcal{E}:
         \mathbf{if} \ \mathit{State} = \mathtt{OPEN} \ \mathtt{BASE} \ \mathbf{then}
24:
25:
              prepare C TODO
26:
              send (SUBMIT, C) to \mathcal{G}_{Ledger}
27:
          else if State = OPEN VIRTUAL then
28:
              TODO
29:
         end if
```

Fig. 1.

```
Protocol \Pi_{\mathrm{Chan}} – virtual
 1: // notification to funder
 2: // trust that Alice has c in her channel
3: On (FUND YOU, c, Bob) by Charlie as input:
        ensure State = INIT
 5:
        State \leftarrow \text{Opening virtual channel}
 6:
        do LN with Bob – TODO
 7:
        State \leftarrow \text{OPEN VIRTUAL}
        output (OK) to Charlie
9: On (FUND, c, hops, sub_parties = (fundee, counterparty), outer_parties =
    (Alice, Dave), pk_{VA,out}, pk_{VB,out}) by \mathcal{E}:
        do the same as in \mathcal{F}_{Chan}, Fig. 8, lines 1-11, skipping line 6 // "as Alice"
10:
    sender labels are applied anyway, since we are Alice
        do VChan() with hops – TODO //P_{i-1}P_i, P_iP_{i+1} and all P_1P_n held by
    BOTH R_{i-1} and L_i. P_{i-1}P_i held only by R_{i-1}, P_iP_{i+1} held only by L_i. This
    (probably) ensures that only relevant parties can close their channels (with the
    exception of honest R_{i-1} wanting to leave channels virtual but corrupted L_i
    demoting them to base, which however doesn't cost funds to anyone), but that
    they have minimal impact to the decisions of ajdacent channels. All P_{i-1}P_i
    inputs must be signed by R_{i-1} and all P_i P_{i+1} inputs by L_i.
12:
        do the same as in \mathcal{F}_{Chan}, Fig. 8, lines 19-26
13:
        output (OK) to \mathcal{E}
14: // notification to fundee
15: On (ALLOW FUND, ...) by Charlie, act as \mathcal{F}_{Chan} (Fig 8, line 9):
```

Fig. 2.

```
Functionality \mathcal{F}_{\operatorname{Chan}} - \operatorname{init} \, \& \, \operatorname{top} \, \operatorname{up}
 1: Initialisation: // runs on first activation
 2:
           State \leftarrow \text{init}
           (\operatorname{locked}_A, \operatorname{locked}_B) \leftarrow (0, 0)
3:
4: On (top up, c_{\min}) by Alice:
          ensure State = INIT
 6:
           State \leftarrow \text{sent key}
 7:
           (sk, pk) \leftarrow \text{keyGen}()
          output (Public Key, pk) to Alice
9: On (CHECK TOP UP) by Alice:
10:
           ensure State = SENT KEY
11:
           input (READ) to \mathcal{G}_{\mathrm{Ledger}} as \mathit{Alice} and assign ouput to \varSigma
12:
           ensure \exists tx \in \Sigma, c_{on} : c_{on} \ge c_{min} \land (c_{on}, pk) \in tx.outputs
13:
           \texttt{base\_output} \leftarrow (c_{\text{on}}, pk) \text{ of } tx
14:
           State \leftarrow (\text{TOPPED UP}, Alice)
15:
           output (TOPPED UP) to Alice
```

Fig. 3.

```
Functionality \mathcal{F}_{\operatorname{Chan}} - \operatorname{base}
 1: On (OPEN, c_F, pk_{A,out}, pk_{B,out}) by Alice:
 2:
         ensure State = (TOPPED UP, Alice)
 3:
         ensure c_F = c_{\rm on}
 4:
         (sk_{A,F}, pk_{A,F}) \leftarrow \text{KEYGEN}(); (sk_{B,F}, pk_{B,F}) \leftarrow \text{KEYGEN}()
 5:
         F \leftarrow TX \{\text{input: base\_output, output: } (c_F, 2/\{pk_{A,F}, pk_{B,F}\})\}
         \operatorname{sig}_F \leftarrow \operatorname{SIGN}(F, sk)
 6:
         State \leftarrow \texttt{WAITING FOR LEDGER}
 7:
         send (OPEN, c_F, pk_{A,\text{out}}, pk_{B,\text{out}}, F, \text{sig}_F, Alice) to A
9: On (CHECK FUNDING) by Alice:
         ensure State = WAITING FOR LEDGER
10:
         input (READ) to \mathcal{G}_{Ledger} as Alice and assign output to \Sigma
11:
         ensure F \in \Sigma
12:
13:
         c_A \leftarrow c; c_B \leftarrow 0
14:
          State \leftarrow \text{OPEN BASE}
15:
         output (OPEN SUCCESS) to Alice
16: On (PAY, x) by Dave \in \{Alice, Bob\}:
17:
         ensure State \in \{\text{OPEN BASE}, \text{OPEN VIRTUAL}\}
         ensure c_D - \operatorname{locked}_D \ge x
18:
         send (PAY, x, Dave) to A and expect reply (OK)
19:
20:
         c_D \leftarrow c_D - x; c_{\bar{D}} \leftarrow c_{\bar{D}} + x //\bar{D} is Alice if D is Bob and vice-versa
21:
         output (PAY SUCCESS) to Dave
22: On (BALANCE) by Dave \in \{Alice, Bob\}:
         ensure State \in \{\text{OPEN BASE}, \text{OPEN VIRTUAL}\}
23:
24:
         output (BALANCE, (c_A, c_B, locked_A, locked_B)) to Dave
```

Fig. 4.

```
Functionality \mathcal{F}_{Chan} – close Pt. 1
 1: On (CLOSE) by Alice:
 2:
          if State = OPEN base then
 3:
               if both channel parties are honest then
                    C \leftarrow \text{TX } \{\text{input: } F.\text{output, outputs: } (c_A, pk_{A,\text{out}} \land t), (c_B, pk_{B,\text{out}})\}
 4:
 5:
                    \operatorname{sig}_{B,C} \leftarrow \operatorname{SIGN}(C, sk_{B,F})
 6:
               end if // note that rest is run only if Alice is honest
 7:
               \operatorname{sig}_{A,C} \leftarrow \operatorname{SIGN}(C, sk_{A,F})
 8:
               State \leftarrow \text{CLOSED}
               input (Submit, (C,\operatorname{sig}_{A,C},\operatorname{sig}_{B,C})) to \mathcal{G}_{\operatorname{Ledger}}
 9:
          else if State = OPEN VIRTUAL then TODO: think more about closing w/
10:
     one corrupted party
11:
               State \leftarrow \text{CLOSED}
12:
               output (CLOSING, c_A, c_B) to opener
13:
          end if
14: On (CLOSE) by A:
          ensure that exactly one party in {Alice, Bob} is corrupted
15:
           wlog, let Alice corrupted, Bob honest
16:
17:
          C' \leftarrow \text{TX {input: } } F.\text{output, outputs: } (c_B, pk_{B,\text{out}}), (c_A, pk_{A,\text{out}} \land t) \}
18:
          \operatorname{sig}_{B,C'} \leftarrow \operatorname{SIGN}(C',sk_{B,F})
19:
           State \leftarrow \text{CLOSED}
20:
          send (C', \mathrm{sig}_{B,C'}) to \mathcal A
21:
          TODO: asynchronously, if a lot of time passes and C' not in \mathcal{G}_{Ledger},
     submit C to \mathcal{G}_{Ledger} for good measure
```

Fig. 5.

```
Functionality \mathcal{F}_{\mathrm{Chan}} - \mathrm{close}\ \mathrm{Pt.}\ 2
 1: On (CLOSING, c_{\text{left}}, c_{\text{right}}) by \mathcal{F}_{\text{Chan}}:
         ensure State \in \{\text{OPEN BASE}, \text{OPEN VIRTUAL}\}
 2:
3:
         ensure ((c_L, c_R), hops, (Charlie, Dave), (Frank, George), id) \in funded with
     Frank \in \{Alice, Bob\}
         ensure c_{\text{left}} \leq c_L + c_R
 5:
         remove\ entry\ from\ {\tt funded}
         output (CLOSED VIRTUAL, c_{\text{right}}, id) to Frank
 7: On (CLOSED VIRTUAL, c_{\text{right}}, id) by \mathcal{F}_{\text{Chan}}:
         ensure State \in \{\text{OPEN BASE}, \text{OPEN VIRTUAL}\}
 8:
9:
         ensure (virtual, c, \mathcal{F}_{Chan}, Dave, id) \in funded
         ensure c_{\text{right}} \leq c
10:
11:
         send (CLOSED) to virtual and expect reply (YES)
12:
         c_D \leftarrow c_D + c_{\text{right}}
13:
         remove\ entry\ from\ {\tt funded}
14: On (CLOSED) by P:
         if State = CLOSED then
16:
              send (YES) to P
17:
          \mathbf{else}
18:
              send (NO) to P
19:
          end if
```

Fig. 6.

```
Functionality \mathcal{F}_{Chan} – fund virtual
 1: On (FUND, c, hops, sub_parties = (fundee, counterparty), outer_parties =
    (Alice, Dave), pk_{VA,out}, pk_{VB,out}) by Alice: // we fund another channel
    TODO: use pk_{VA} in virtual LN()
         ensure State \in \{\text{OPEN BASE}, \text{OPEN VIRTUAL}\}
         ensure c_A - \operatorname{locked}_A > c
         input (FUND YOU, c, counterparty) to fundee as Alice, ensure output is
    (ok)
         generate random id
 5:
         \texttt{send} \ (\texttt{FUND} \ c, \ \texttt{hops}, \ \texttt{sub\_parties} = (\texttt{fundee}, \ \texttt{counterparty}), \ \texttt{outer\_parties}
    = (Alice, Dave), funder = Alice, id) to A and ensure reply is (OK)
         (L_0, R_0) \leftarrow (Alice, Bob)
         for all (L_i, R_i) \in \text{hops do } // i \in \{1, \dots, |\text{hops}|\}
 8:
             ensure R_{i-1} = L_i
 9:
10:
             send (ALLOW FUND, c, sub_parties, local_funder \leftarrow L_i, id,
    i \stackrel{!}{=} |\text{hops}|) to L_i as Alice and ensure reply is (OK)
         end for
11:
12:
         if both channel parties are honest then
13:
             send (IS OPEN SUCCESSFUL, id) to A and ensure reply is (OK)
14:
         else if only Alice is honest then
15:
              (sk_{A,V}, pk_{A,V}) \leftarrow \text{KEYGEN}()
             send (UPDATE TO VIRTUAL, pk_{A,V}) to \mathcal{A} and assign reply to (V = \mathrm{TX}
16:
     {input: F.output, outputs: (c_A + c_B - c, 2/\{pk_{A,V}, pk_{B,V}\}), (c, c, c, c)
    2/\{pk_{G,V}, pk_{A,V}\}\), (0, |hops|/\{hops_i.pk\}_i)\}, sig_{B,V}, C' = TX \{input:
    V.outputs.0, outputs: (c_A - \text{locked}_A - c, pk_{A,\text{out}} \land t), (c_B - \text{locked}_B, pk_{B,\text{out}}),
    \operatorname{sig}_{B,C'}) TODO: think about locked coins
17:
             ensure VERIFY(V, sig_{B,V}, pk_{B,F}) = VERIFY(C', sig_{B,C'}, pk_{A,V}) = True
18:
         for all (L_i, R_i) \in \text{hops do } // i \in \{1, \dots, |\text{hops}|\}
19:
20:
             send (FUND DONE, id) to L_i as Alice and ensure reply is (OK)
         end for
21:
22:
         c_A \leftarrow c_A - c
23:
         if only Alice is honest then
             C \leftarrow C'; \, \operatorname{sig}_{B,C} \leftarrow \operatorname{sig}_{B,C'}
24:
25:
         end if
26:
         add ((c, 0), hops, sub_parties, outer_parties, id) to funded
27:
         output (OK) to Alice
```

Fig. 7.

```
Functionality \mathcal{F}_{\operatorname{Chan}} - \operatorname{virtual}
 1: On (FUND YOU, c, Dave) by Charlie as input to Alice: // Alice is funded by
     Charlie
 2:
        ensure State = INIT
 3:
        Bob \leftarrow Dave
 4:
        send (FUND YOU, c, Bob, Charlie, Alice) to A and ensure reply is (OK)
 5:
        c_A \leftarrow c; c_B \leftarrow 0
        \mathtt{opener} \leftarrow \mathit{Charlie}
 6:
 7:
        State \leftarrow \texttt{OPEN VIRTUAL}
 8:
        output (OK) to Charlie
9: On (ALLOW FUND, c, sub_parties, Dave, id, is_last) by Charlie:
        ensure State \in \{\text{OPEN BASE}, \text{OPEN VIRTUAL}\}
11:
        ensure Dave \in \{Alice, Bob\}
12:
        ensure c_D - \operatorname{locked}_D \ge c
        output received message to Dave and ensure reply is (OK)
13:
14:
        \mathrm{locked}_D \leftarrow \mathrm{locked}_D + c
        add (id, is_last, sub_parties, c, Dave) to pending
15:
16:
        send (OK) to Charlie
17: On (FUND DONE, id) by Charlie:
        ensure State \in \{\text{OPEN BASE}, \text{OPEN VIRTUAL}\}
19:
        ensure (id, is_last, sub_parties, c, Dave) \in pending
20:
        remove (id, is_last, sub_parties, c, Dave) from pending
21:
        if is_last then
22:
            add ((0, c), \perp, sub\_parties.reverse(), (Dave, \perp), id) to funded
23:
         end if
        send (OK) to Charlie
24:
```

Fig. 8.

```
Functionality \mathcal{F}_{\operatorname{Chan}} – corruption
 1: On (CORRUPT) by P, addressed to Alice:
 2:
         ensure P \in \{\text{opener}, \mathcal{A}\}
         for all (\_, \_, (fundee, \_), (Alice, \_)) \in funded do
 3:
 4:
             send (CORRUPT) to fundee and ensure reply is (OK)
 5:
         from now on, allow A to handle all Alice's messages, i.e. act as a relay
 6:
 7:
         if Bob is not corrupted then
             from now on, override reactions to messages (OPEN) and (PAY)
     addressed to Bob with those defined in the current Figure:
 9:
         end if
10:
         output (OK) to P
11: On (OPEN, c_F, pk_{A,\text{out}}, pk_{B,\text{out}}) by Bob:
         ensure State = (TOPPED UP, Bob) TODO: decide what happens when
     channel funded by corrupted party
13:
         ensure c_F = c_{\text{on}}
          (sk_{B,F}, pk_{B,F}) \leftarrow \text{KEYGEN}()
14:
15:
         send (OPEN, c_F, pk_{A,out}, pk_{B,out}, pk_{B,F}, Bob) to A, assign reply to (pk_{A,F},
     C = TX \{\text{input: } F.\text{output, outputs: } (c_F, pk_{B,\text{out}} \land t), (0, pk_{A,\text{out}})\}, \text{sig}_{A,C} \}
16:
         ensure VERIFY(C, sig_{A,C}, pk_{A,F}) = True
17:
         F \leftarrow TX \{\text{input: base\_output, output: } (c_F, 2/\{pk_{A,F}, pk_{B,F}\})\}
         \operatorname{sig}_F \leftarrow \operatorname{SIGN}(F, sk)
18:
          State \leftarrow \text{Waiting for Ledger}
19:
         input (SUBMIT, (F, \operatorname{sig}_F)) to \mathcal{G}_{\text{Ledger}}
20:
21: On (PAY, x) by Bob:
22:
         ensure State \in \{\text{OPEN BASE}, \text{OPEN VIRTUAL}\}
23:
         ensure c_B - \operatorname{locked}_B \ge x
         send (PAY, x, Bob) to A and assign reply to (C' = TX {input: F.output,
24:
     outputs: (c_A + x, pk_{A,\text{out}}), (c_B - \text{locked}_B - x, pk_{B,\text{out}} \wedge t)\}, \text{ sig}'_{A,C}) TODO:
     think about locked coins again
25:
         ensure VERIFY(C', sig'_{A,C}, pk_{A,F}) = True
         C \leftarrow C'; \operatorname{sig}_{A,C} \leftarrow \operatorname{sig}_{A,C}'
26:
27:
         c_B \leftarrow c_B - x; c_A \leftarrow c_A + x
28:
         output (PAY SUCCESS) to Bob
     TODO: receive payment from corrupted counterparty
```

Fig. 9.

```
Simulator S-Pt. 1
 1: On (OPEN, c_F, pk_{A,out}, pk_{B,out}, F, sig_FAlice) by \mathcal{F}_{Chan}: // both honest
 2:
        simulate Alice receiving input (OPEN, c_F, pk_{A,out}, pk_{B,out}) by \mathcal{E}
        ensure simulated Alice inputs (SUBMIT, (F', \operatorname{sig}_{F'})) to \mathcal{G}_{Ledger}
 3:
        input (SUBMIT, (F, \operatorname{sig}_F)) to \mathcal{G}_{\text{Ledger}}
 5: On (OPEN, c_F, pk_{A,\text{out}}, pk_{B,\text{out}}, pk_{B,F}, Bob) by \mathcal{F}_{\text{Chan}}: // Alice corrupted
        send LN message (OPEN, pk_{B,F}) to Alice and relay reply to \mathcal{F}_{Chan} TODO:
    change msg to fit LN, ensure Alice doesn't see a difference from real world
 7: On (PAY, x, Dave) by \mathcal{F}_{Chan}:
        if both channel parties are honest then
9:
             simulate Dave receiving input (PAY, x) by \mathcal{E}
10:
             ensure simulated Dave outputs (OK)
             send (OK) to \mathcal{F}_{Chan}
11:
12:
         else if only Dave's counterparty is corrupted then // else just relay to A
             simulate Dave receiving input (PAY, x) by \mathcal{E}
13:
14:
             ensure simulated Dave outputs (OK)
15:
             extract the latest commitment transaction C and its signature by
    Dave's counterparty \operatorname{sig}_{\bar{D},C} from simulated Dave's state
16:
             send (C, \operatorname{sig}_{\bar{D},C}) to \mathcal{F}_{\operatorname{Chan}}
17:
         end if
18: On (FUND YOU, c, Bob, Charlie, Alice) by \mathcal{F}_{Chan}:
        simulate Alice receiving input (FUND YOU, c, Bob) by Charlie
19:
20:
        ensure simulated Alice outputs (OK) to Charlie
21:
        send (OK) to \mathcal{F}_{Chan}
22: On (FUND c, hops, sub_parties = (fundee, counterparty), outer_parties =
    (Charlie, Dave), funder = Alice, id) by \mathcal{F}_{Chan}:
23:
        add the message data to virtual_opening
        simulate execution of line 7 of Fig. 8 with Alice//\mathcal{S} knows Bob (Alice's
24:
    counterparty) through opening procedure
25:
        send (OK) to \mathcal{F}_{Chan}
26: On (ALLOW FUND, c, sub_parties, local_funder = L_i, id, i \stackrel{?}{=} |\text{hops}|) by
    \mathcal{F}_{Chan}'s Alice to Charlie:
27:
        simulate receiving message with Charlie by Alice and all subsequent
    communication
28:
        ensure the simulated Charlie sends (OK) to the simulated Alice
        intercept this message and send it to \mathcal{F}_{Chan}'s Alice
29:
```

Fig. 10.

Simulator S- Pt. 2

- 1: On (IS OPEN SUCCESSFUL, id) by \mathcal{F}_{Chan} :
- 2: retrieve and remove from virtual_opening the data marked with id
- 3: simulate line 11 of Fig. 2 with Alice using this data
- 4: ensure Alice completes execution of VChan() successfully
- 5: send (OK) to \mathcal{F}_{Chan}
- 6: On (update to virtual) by \mathcal{F}_{Chan} :
- 7: retrieve and remove from virtual_opening the data marked with id
- 8: simulate line 11 of Fig. 2 with Alice using this data
- 9: ensure Alice completes execution of VChan() successfully
- 10: extract from Alice's state the new virtual funding TX V for pre-existing channel
- 11: extract from Alice's state the new commitment TX C that spends the on-chain funding TX
- 12: send (V, C) to \mathcal{F}_{Chan}
- 13: On (FUND DONE, id) by \mathcal{F}_{Chan} 's Alice to Charlie:
- 14: simulate receiving message with *Charlie* by *Alice* and all subsequent communication
- 15: ensure the simulated Charlie sends (OK) to the simulated Alice
- 16: intercept this message and send it to \mathcal{F}_{Chan} 's Alice

Fig. 11.

1 Security Proof

When \mathcal{E} sends (FUND, c, hops, (fundee, counterparty), (*Charlie*, *Dave*), $pk_{VA,out}$, $pk_{VB,out}$) to *Alice* in the real world, lines 1-4 of Fig. 8 are executed and then control is handed over to the "fundee" ITI, which executes lines 3-8 of Fig. 2. This ITI will output (OK) if and only if line 6 of Fig. 2 succeeds.

When \mathcal{E} sends (FUND, c, hops, (fundee, counterparty), (*Charlie*, *Dave*)) to *Alice* in the ideal world, lines 1-4 of Fig. 8 are executed and then control is handed over to the functionality that controls the "fundee", which executes lines 1-4 of Fig. 8 and then hands control over to \mathcal{E} . The latter in turn simulates lines 3-8 of Fig. 2, thus following the exact same steps as in the real world, therefore it will send (OK) to $\mathcal{F}_{\text{Chan}}$ if and only if the simulated line 6 of Fig. 2 succeeds. From this and the previous paragraph, we see that, up to this point, the two worlds are perfectly indistinguishable.

Moving on, in the ideal world subsequently lines 5-6 of Fig. 8 are executed, which results in S executing lines 22-25 of Fig. 10. During the latter steps, S simulates executing line 7 of Fig. 8 with *Alice*.

Similarly in the real world, *Alice* executes lines 5 and 7 of Fig. 8, therefore the two worlds still are perfectly indistinguishable.

The "for" loop of lines 8-11 of Fig. 8 is then executed in both the real and the ideal worlds. The message of line 10 results in the execution of lines 3-8 of Fig. 2 by L_i in both worlds: in the real world directly, in the ideal world simulated by S.

In the ideal world, line 18 in Fig. 8 prompts S to simulate line 11 of Fig. 2 with Alice, which is exactly the code that would be directly run by Alice in the real world. Therefore the two worlds remain perfectly indistinguishable.

The "for" loop of lines 19-26 of Fig. 8 is also perfectly indistinguishable in the two worlds. With argumentation similar to that of the previous "for" loop, we conclude that the FUND message does not induce any chance of distinguishability between the two worlds.

Theorem 1. Assume that at the end of the execution, \mathcal{G}_{Ledger} contains exactly one "groups" transaction that precedes all "funding" transactions and contains as payload a partition \mathcal{G} into groups of all VChan parties, with each group containing the parties that belong to the same (human) owner. Then the following holds:

 $\forall G \in \mathcal{G} \text{ such that all parties in } G \text{ are honest},$

$$\sum_{P \in G} \operatorname{logged-coins}(P) = \sum_{P \in G} \operatorname{ledger-coins}(P) =$$

$$= \sum_{P \in G} (\operatorname{top-up}(P) + \sum_{m \in \mathcal{T}} \operatorname{pay-in}(m, P) - \sum_{m \in \mathcal{T}} \operatorname{pay-out}(m, P)) \ ,$$

where \mathcal{T} is the execution transcript and:

logged-coins(P) = c_P , as recorded in $\mathcal{F}_{Chan}/\Pi_{Chan}$

 $\label{eq:coins} \mbox{ledger-coins}(P) = coins \ spendable \ with \ the \ secret \ key \ sk \ of \ P \ if \ the \ closing$ $\ transactions \ of \ all \ open \ channels \ are \ submitted \ to \ \mathcal{G}_{\rm Ledger}$ and added to the state of all parties and then t new blocks enter the state of all honest parties

$$\operatorname{top-up}(P) = \begin{cases} c_{\operatorname{on}}, \ as \ determined \ on \ message \ (\operatorname{CHECK \ TOP \ UP}), \\ if \ such \ a \ message \ was \ handled \\ 0, & otherwise \\ \end{cases}$$

$$\operatorname{pay-in}(m,P) = \begin{cases} x, & \text{if } message \ m \ updated \ the \ channel \ to \\ a \ state \ in \ which \ P \ had \ x \ more \ coins \ TODO: \ improve \ prev \\ 0, & otherwise \\ \end{cases}$$

$$\operatorname{pay-out}(m,P) = \begin{cases} x, \ if \ m = (\operatorname{PAY},x) \ was \ received \ by \ P \ and \\ P \ output \ (\operatorname{PAY \ SUCCESS}) \ as \ a \ result \\ 0, & otherwise \end{cases}$$

References