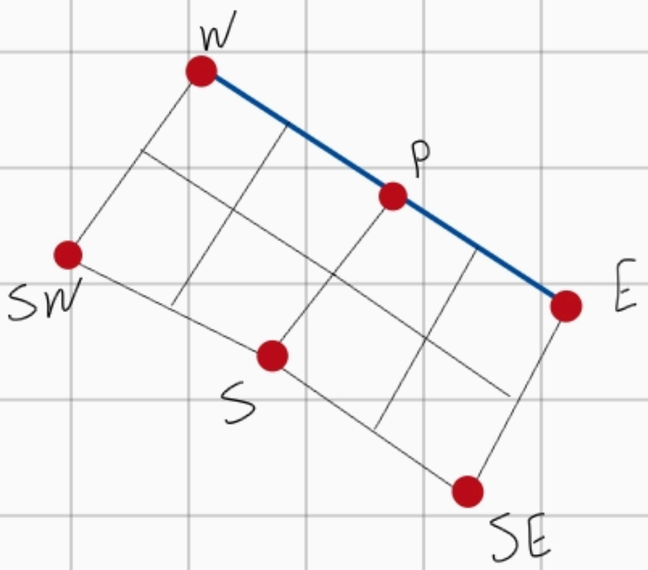
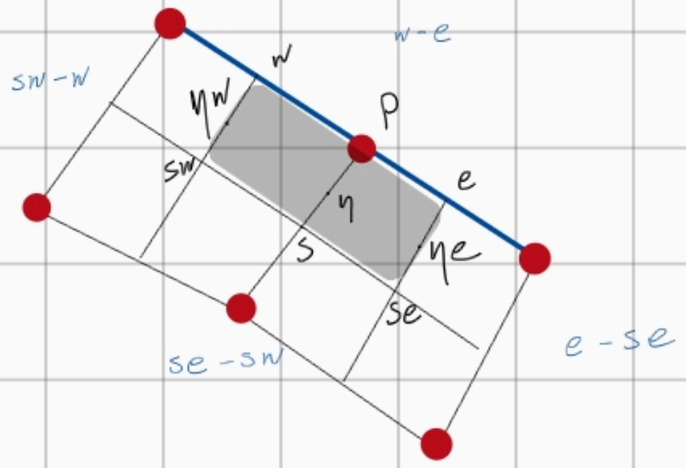
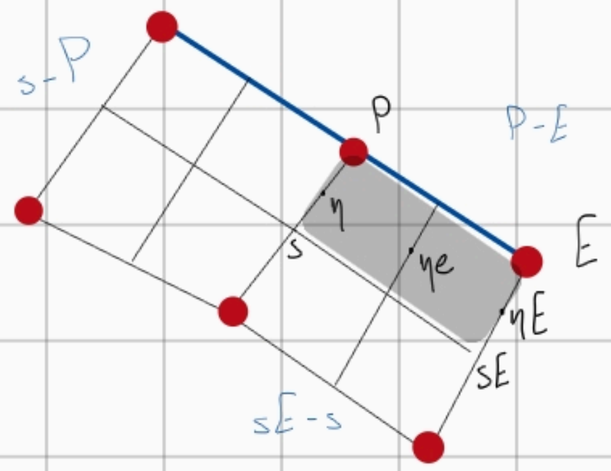
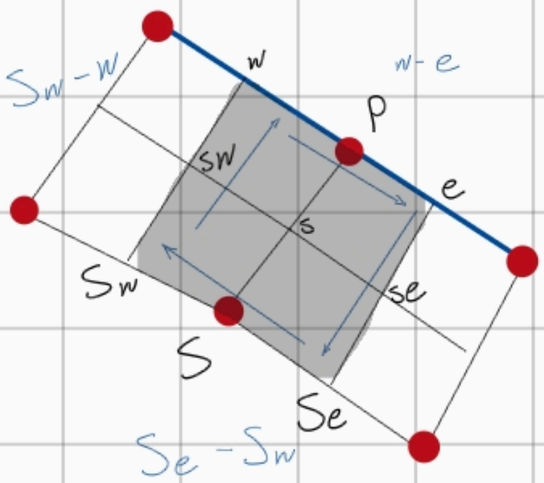
North

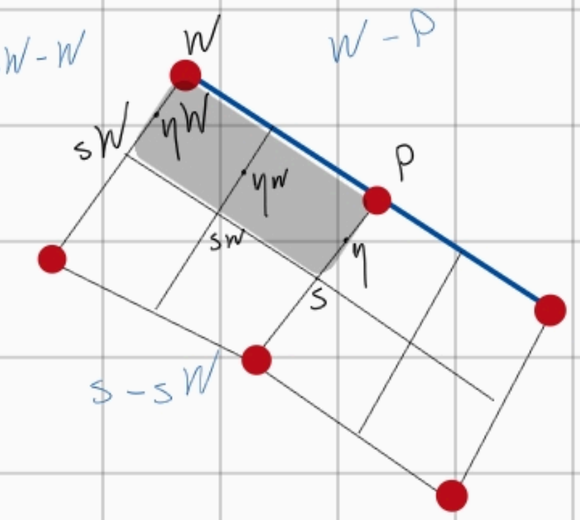








e-Se



P-s

$$\nabla^2 T|_P \approx \frac{1}{S^\eta}\oint_{\partial S^\eta}\nabla T\cdot n\,dl = \frac{1}{S^\eta}\left[\int_{|sw}^{|se}(\nabla T)\,n\,dl + \int_{|se}^{|e}(\nabla T)\,n\,dl + \int_{|e}^{|w}(\nabla T)\,n\,dl + \int_{|w}^{|sw}(\nabla T)\,n\,dl\right] \equiv 3.4 \quad (A.13)$$

$$\nabla^2 T|_P \approx \frac{1}{S^\eta}\oint_{\partial S^\eta}\nabla T\cdot n\,dS \approx \frac{1}{S^\eta}\left[\nabla T|_P\cdot n\,\Delta l_e^w + \frac{\partial T}{\partial x}\Big|_{\eta e}\Delta y_{se}^{e} - \frac{\partial T}{\partial y}\Big|_{\eta e}\Delta x_{se}^{e} + \frac{\partial T}{\partial x}\Big|_{s}\Delta y_{sw}^{se} - \frac{\partial T}{\partial y}\Big|_{s}\Delta x_{sw}^{se} + \frac{\partial T}{\partial x}\Big|_{\eta w}\Delta y_{w}^{sw} - \frac{\partial T}{\partial y}\Big|_{\eta w}\Delta x_{w}^{sw}\right] \equiv 3.16$$

$$\frac{\partial T}{\partial x}\Big|_{\eta e} \approx \frac{1}{S^{\eta e}}\left(\Delta y_{s}^{sE}\,T|_{se} + \Delta y_{sE}^{E}\,T|_{\eta E} + \Delta y_{E}^{p}\,T|_{e} + \Delta y_{p}^{s}\,T|_{\eta}\right) \quad (A.14)$$

$$\frac{\partial T}{\partial y}\Big|_{\eta e} \approx \frac{-1}{S^{\eta e}}\left(\Delta x_{s}^{sE}\,T|_{se} + \Delta x_{sE}^{E}\,T|_{\eta E} + \Delta x_{E}^{p}\,T|_{e} + \Delta x_{p}^{s}\,T|_{\eta}\right)$$

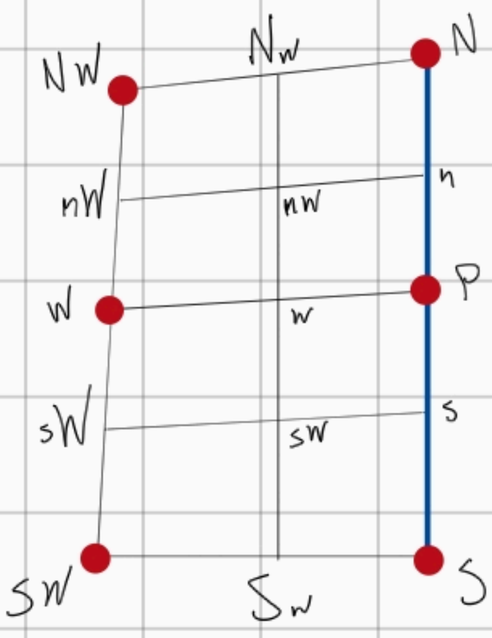
$$\frac{\partial T}{\partial x}\Big|_{s} \approx \frac{1}{S^{s}}\left(\Delta y_{Sw}^{Se}\,T|_{S} + \Delta y_{Se}^{e}\,T|_{se} + \Delta y_{e}^{w}\,T|_{p} + \Delta y_{w}^{Sw}\,T|_{sw}\right) \equiv 3.19 - 3.26 \quad (A.15 - A.20)$$

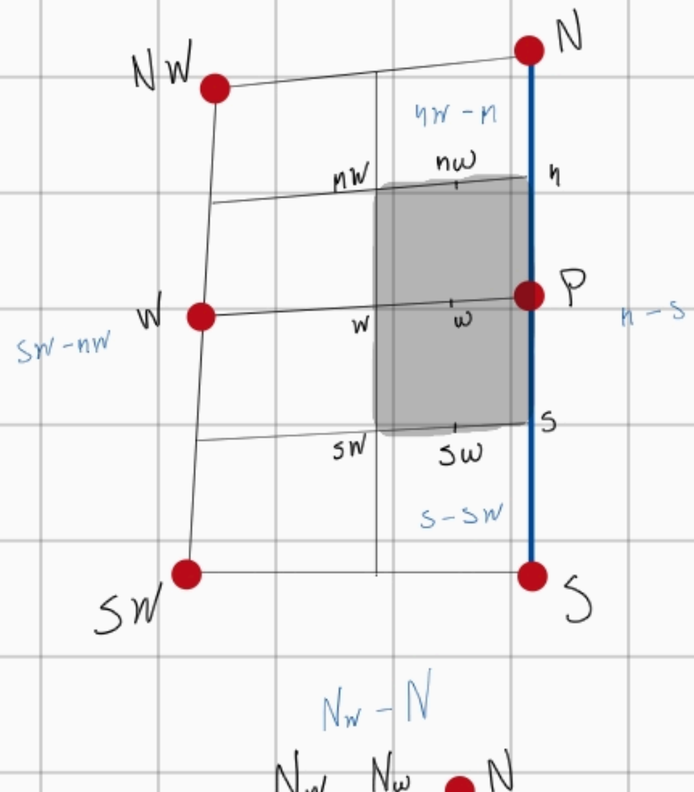
$$\frac{\partial T}{\partial y}\Big|_{s} \approx \frac{-1}{S^{s}}\left(\Delta x_{Sw}^{Se}\,T|_{S} + \Delta x_{Se}^{e}\,T|_{se} + \Delta x_{e}^{w}\,T|_{p} + \Delta x_{w}^{Sw}\,T|_{sw}\right)$$

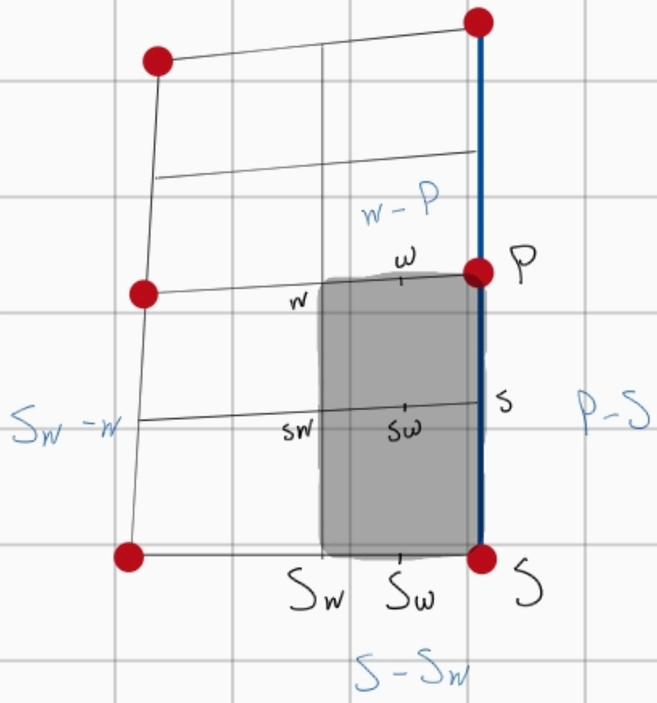
$$\frac{\partial T}{\partial x}\Big|_{\eta w} \approx \frac{1}{S^{\eta w}}\left(\Delta y_{sw}^{s}\,T|_{sw} + \Delta y_{s}^{p}\,T|_{\eta} + \Delta y_{p}^{w}\,T|_{w} + \Delta y_{w}^{sw}\,T|_{\eta w}\right)$$

$$\frac{\partial T}{\partial y}\Big|_{\eta w} \approx \frac{-1}{S^{\eta w}}\left(\Delta x_{sw}^{s}\,T|_{sw} + \Delta x_{s}^{p}\,T|_{\eta} + \Delta x_{p}^{w}\,T|_{w} + \Delta x_{w}^{sw}\,T|_{\eta w}\right)$$

East







$$\nabla^2 T|_P \approx \frac{1}{S^\omega}\oint_{\partial S^\omega}\nabla T\cdot n\,dl = \frac{1}{S^\omega}\left[\int_{l_{SW}^{S}}(\nabla T)_n\,dl + \int_{S}^{l_{nw}^{n}}(\nabla T)_n\,dl + \int_{l_{nw}^{nw}}^{n}(\nabla T)_n\,dl + \int_{l_{NW}^{sw}}^{nw}(\nabla T)_n\,dl\right] \equiv 3.4 \quad (A.13)$$

$$\nabla^2 T|_P \approx \frac{1}{S^\omega}\oint_{\partial S^\omega}\nabla T\cdot n\,dS \approx \frac{1}{S^\omega}\left[\nabla T|_P\cdot n\,\Delta l_S^n + \frac{\partial T}{\partial x}\Big|_{SW}\Delta y_{SW}^{S} - \frac{\partial T}{\partial y}\Big|_{SW}\Delta x_{SW}^{S} + \frac{\partial T}{\partial x}\Big|_{W}\Delta y_{nW}^{SW} - \frac{\partial T}{\partial y}\Big|_{W}\Delta x_{nW}^{SW} + \frac{\partial T}{\partial x}\Big|_{nw}\Delta y_{n}^{nw} - \frac{\partial T}{\partial y}\Big|_{nw}\Delta x_{n}^{nw}\right] \equiv 3.16 \quad (A.14)$$

$$\frac{\partial T}{\partial x}\Big|_{SW} \approx \frac{1}{S^{SW}}\left(\Delta y_{W}^{S_W}\,T|_{SW} + \Delta y_{S_W}^{S}\,T|_{S_W} + \Delta y_{S}^{P}\,T|_{S} + \Delta y_{P}^{W}\,T|_{W}\right)$$

$$\frac{\partial T}{\partial y}\Big|_{SW} \approx \frac{-1}{S^{SW}}\left(\Delta x_{W}^{S_W}\,T|_{SW} + \Delta x_{S_W}^{S}\,T|_{S_W} + \Delta x_{S}^{P}\,T|_{S} + \Delta x_{P}^{W}\,T|_{W}\right)$$

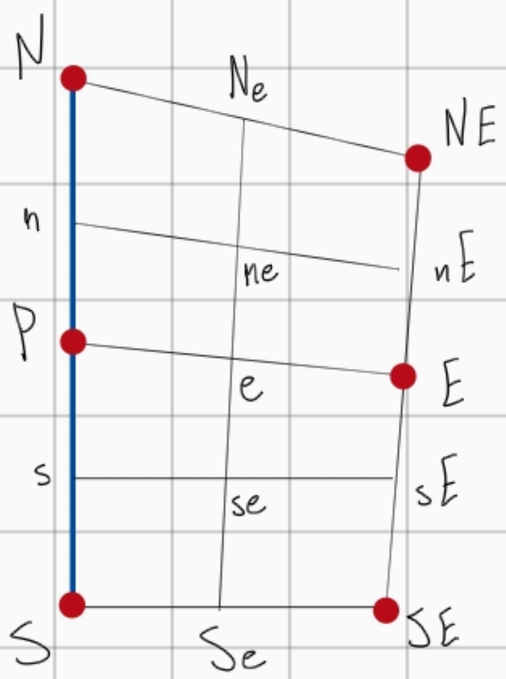
$$\frac{\partial T}{\partial x}\Big|_{W} \approx \frac{1}{S^{W}}\left(\Delta y_{nW}^{sW}\,T|_{W} + \Delta y_{sW}^{s}\,T|_{sW} + \Delta y_{s}^{n}\,T|_{P} + \Delta y_{n}^{nW}\,T|_{nW}\right) \equiv 3.19 - 3.26 \quad (A.15 - A.20)$$

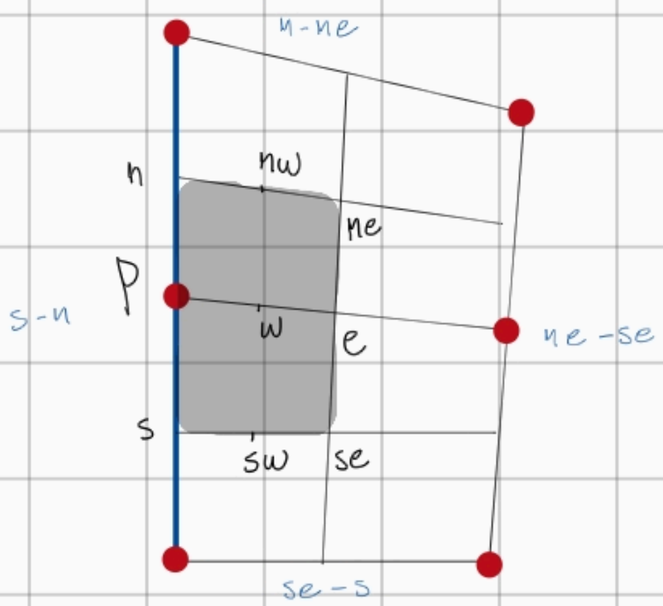
$$\frac{\partial T}{\partial y}\Big|_{W} \approx \frac{-1}{S^{W}}\left(\Delta x_{nW}^{sW}\,T|_{W} + \Delta x_{sW}^{s}\,T|_{sW} + \Delta x_{s}^{n}\,T|_{P} + \Delta x_{n}^{nW}\,T|_{nW}\right)$$

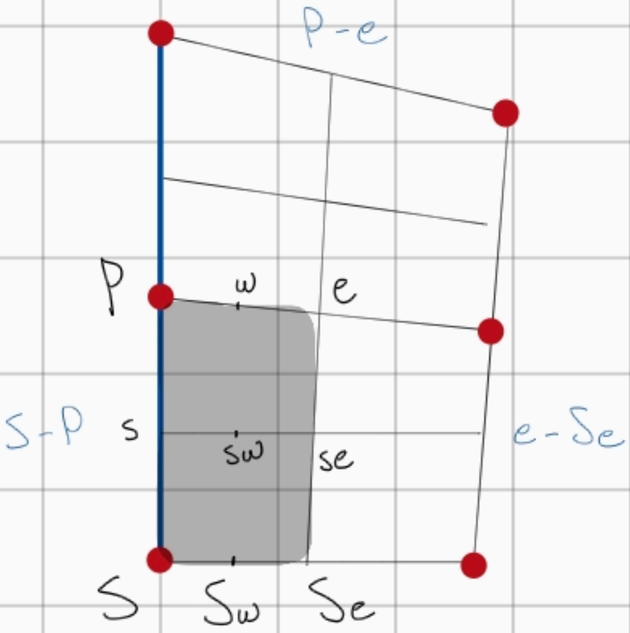
$$\frac{\partial T}{\partial x}\Big|_{nw} \approx \frac{1}{S^{nw}}\left(\Delta y_{N_W}^{W}\,T|_{nW} + \Delta y_{W}^{P}\,T|_{W} + \Delta y_{P}^{N}\,T|_{n} + \Delta y_{N}^{N_W}\,T|_{N_W}\right)$$

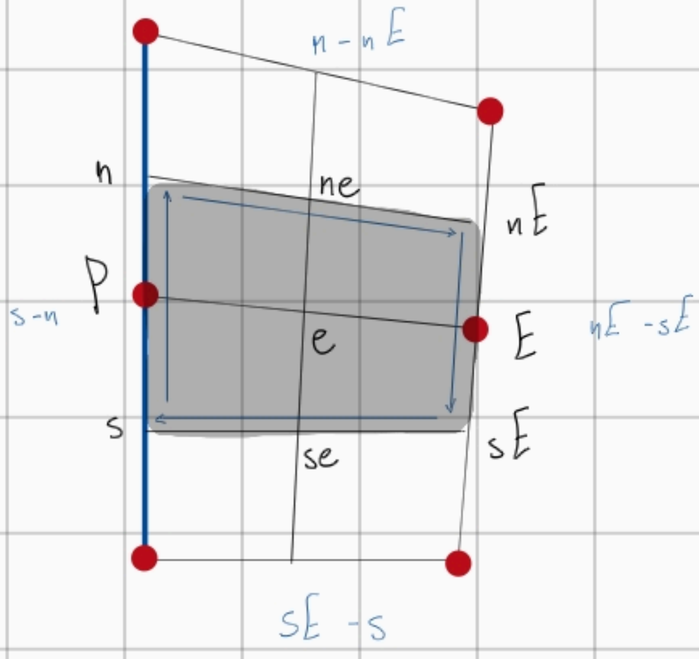
$$\frac{\partial T}{\partial y}\Big|_{nw} \approx \frac{-1}{S^{nw}}\left(\Delta x_{N_W}^{W}\,T|_{nW} + \Delta x_{W}^{P}\,T|_{W} + \Delta x_{P}^{N}\,T|_{n} + \Delta x_{N}^{N_W}\,T|_{N_W}\right)$$

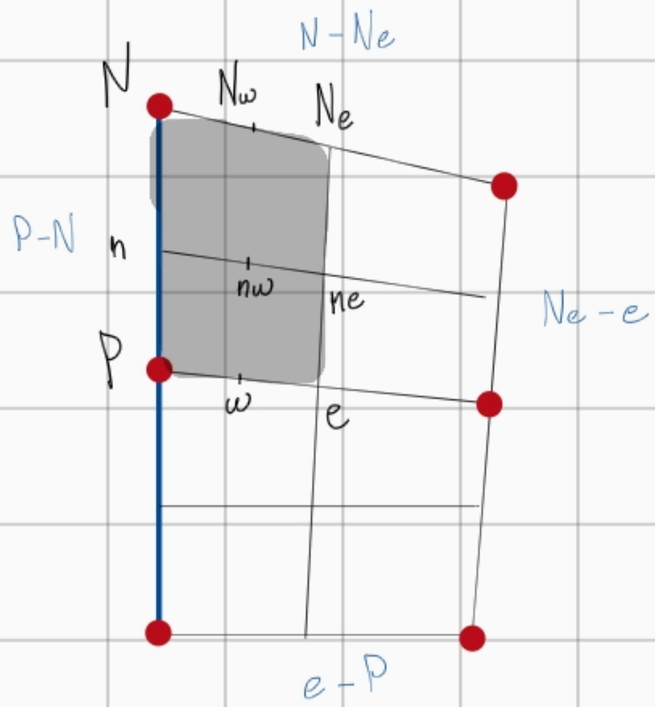
West











$$\nabla^2 T|_P \approx \frac{1}{S^w}\oint_{\partial S^w} \nabla T \cdot n\, dl = \frac{1}{S^w}\left[\int_{l_S^{se}} (\nabla T)\, n\, dl + \int_{l_{se}^{ne}} (\nabla T)\, n\, dl + \int_{l_{ne}^{n}} (\nabla T)\, n\, dl + \int_{l_n^{s}} (\nabla T)\, n\, dl\right] \equiv 3.4 \quad (A.13)$$

$$\nabla^2 T|_P \approx \frac{1}{S^w}\oint_{\partial S^w} \nabla T \cdot n\, dS \approx \frac{1}{S^w}\left[\nabla T|_P \cdot n\, \Delta l_n^s + \frac{\partial T}{\partial x}\Big|_{sw} \Delta y_s^{se} - \frac{\partial T}{\partial y}\Big|_{sw} \Delta x_s^{se} + \frac{\partial T}{\partial x}\Big|_{e} \Delta y_{se}^{ne} - \frac{\partial T}{\partial y}\Big|_{e} \Delta x_{se}^{ne} + \frac{\partial T}{\partial x}\Big|_{nw} \Delta y_{ne}^{n} - \frac{\partial T}{\partial y}\Big|_{nw} \Delta x_{ne}^{n}\right] \equiv 3.16 \quad (A.14)$$

$$\frac{\partial T}{\partial x}\Big|_{sw} \approx \frac{1}{S^{sw}}\left(\Delta y_P^S\, T|_S + \Delta y_S^{Se}\, T|_{Sw} + \Delta y_{Se}^{e}\, T|_{se} + \Delta y_e^P\, T|_w\right)$$

$$\frac{\partial T}{\partial y}\Big|_{sw} \approx \frac{-1}{S^{sw}}\left(\Delta x_P^S\, T|_S + \Delta x_S^{Se}\, T|_{Sw} + \Delta x_{Se}^{e}\, T|_{se} + \Delta x_e^P\, T|_w\right)$$

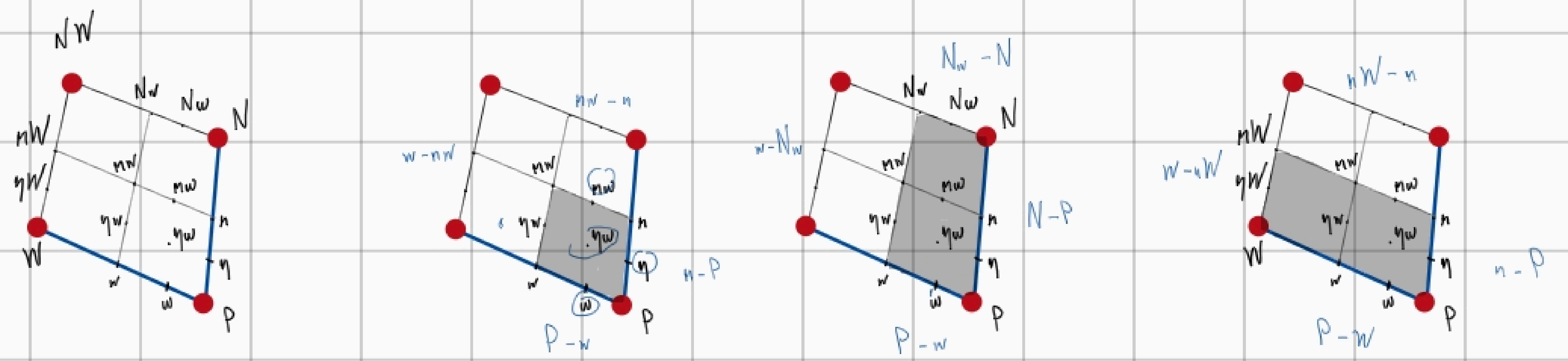
$$\frac{\partial T}{\partial x}\Big|_{e} \approx \frac{1}{S^{e}}\left(\Delta y_n^s\, T|_P + \Delta y_s^{sE}\, T|_{se} + \Delta y_{sE}^{nE}\, T|_E + \Delta y_{nE}^{n}\, T|_{ne}\right) \equiv 3.19 - 3.26 \quad (A.15 - A.20)$$

$$\frac{\partial T}{\partial y}\Big|_{e} \approx \frac{-1}{S^{e}}\left(\Delta x_n^s\, T|_P + \Delta x_s^{sE}\, T|_{se} + \Delta x_{sE}^{nE}\, T|_E + \Delta x_{nE}^{n}\, T|_{ne}\right)$$

$$\frac{\partial T}{\partial x}\Big|_{nw} \approx \frac{1}{S^{nw}}\left(\Delta y_N^P\, T|_n + \Delta y_P^e\, T|_w + \Delta y_e^{Ne}\, T|_{ne} + \Delta y_{Ne}^{N}\, T|_{Nw}\right)$$

$$\frac{\partial T}{\partial y}\Big|_{nw} \approx \frac{-1}{S^{nw}}\left(\Delta x_N^P\, T|_n + \Delta x_P^e\, T|_w + \Delta x_e^{Ne}\, T|_{ne} + \Delta x_{Ne}^{N}\, T|_{Nw}\right)$$

Corners SE



$$\nabla^2 T|_P \approx \frac{1}{S^{\eta w}} \oint_{\partial S^{\eta w}} \nabla T \cdot n\, dl = \frac{1}{S^{\eta w}} \left[\int_{l_w}^{P} (\nabla T) n\, dl + \int_{l_P}^{n} (\nabla T) n\, dl + \int_{l_n}^{nw} (\nabla T) n\, dl + \int_{l_{nw}}^{w} (\nabla T) n\, dl \right] \equiv 3.4 \ (A.13)$$

Mid-Point Rule: $\nabla^2 T|_P = \frac{1}{S^{\eta w}} \left[\frac{\partial T}{\partial x}\Big|_{\eta w} \Delta y_{nw}^{w} - \frac{\partial T}{\partial y}\Big|_{\eta w} \Delta x_{nw}^{w} + \nabla T|_w\, n \Delta l_w^P + \nabla T|_\eta\, n \Delta l_P^n + \frac{\partial T}{\partial x}\Big|_{nw} \Delta y_n^{nw} - \frac{\partial T}{\partial y}\Big|_{nw} \Delta y_n^{nw} \right]$

$$\frac{\partial T}{\partial x}\Big|_{\eta w} \approx \frac{1}{S^{\eta w}} \left(\Delta y_{nW}^{W}\, T|_{\eta W} + \Delta y_{W}^{P}\, T|_{w} + \Delta y_{P}^{n}\, T|_{\eta} + \Delta y_{n}^{nW}\, T|_{nw} \right)$$

$$\frac{\partial T}{\partial y}\Big|_{\eta w} \approx \frac{1}{S^{\eta w}} \left(\Delta x_{nW}^{W}\, T|_{\eta W} + \Delta x_{W}^{P}\, T|_{w} + \Delta x_{P}^{n}\, T_{\eta} + \Delta x_{n}^{nW}\, T|_{nw} \right.$$

$$\frac{\partial T}{\partial x}\Big|_{nw} \approx \frac{1}{S^{nw}} \left(\Delta y_{N_W}^{w}\, T|_{nw} + \Delta y_{w}^{P}\, T|_{w} + \Delta y_{P}^{N}\, T|_{n} + \Delta y_{N}^{N_w}\, T|_{N_w} \right.$$

$$\frac{\partial T}{\partial y}\Big|_{nw} \approx \frac{-1}{S^{nw}} \left(\Delta x_{N_w}^{w}\, T|_{nw} + \Delta x_{w}^{P}\, T|_{w} + \Delta x_{P}^{N}\, T|_{n} + \Delta x_{N}^{N_w}\, T|_{N_w} \right.$$

$\nabla T|_w$ — Neumann: $= q = 0$; Robin: $= \frac{a}{\lambda}(T_w - T_{\infty_S})$

$\nabla T|_\eta$ — Neumann: $= q = 0$; Robin: $= \frac{a}{\lambda}(T_\eta - T_{\infty_E})$