

Recall that

$$\overbrace{\vec{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} t + \begin{bmatrix} 0 \\ 2 \end{bmatrix}}^{\ell_1}, \quad \overbrace{\vec{x} = \begin{bmatrix} 1 \\ -2 \end{bmatrix} t + \begin{bmatrix} 1 \\ 3 \end{bmatrix}}^{\ell_2}.$$

1. Jack, you've made a mistake. The t 's can be different. You need to set them different when you solve, and then you'll see Naomi is correct.

2.

$$\begin{aligned} \ell_1 \cap \ell_2 &\neq \{\} \\ &\iff \\ \begin{bmatrix} 1 \\ 0 \end{bmatrix} t + \begin{bmatrix} 0 \\ 2 \end{bmatrix} &= \begin{bmatrix} 1 \\ -2 \end{bmatrix} s + \begin{bmatrix} 1 \\ 3 \end{bmatrix} \implies t = \frac{1}{2} \quad s = \frac{3}{2} \end{aligned}$$

So the lines intersect as Naomi said.

3. The ' t ' that shows up in the vector form of a line is a dummy variable. To determine if two lines written in vector form intersect, you need to replace t with a real variable. For example, you could solve

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} t + \begin{bmatrix} 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix} s + \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$$
