Machine Learning from Data – IDC

HW5 – Theory + SVM

Note: Solutions are encouraged to be submitted in a typed format. You may write in hand writing, but unclear handwriting will be deducted points with no option for appeal.

1.
   a. Let K, L be two kernels (operating on the same space) and let α, β be two positive scalars.

      Prove that $\alpha K + \beta L$ is a kernel.

   b. Let K be a non-zero kernel. Is $-K$ a kernel? Prove your answer.

2. Let $K = 200$ be the number of samples from some d-dimensional space.
   a. Using Cover's Counting Theorem, state the required dimension d, such that the probability that a dichotomy is linearly separable is bigger or equal to the following probabilities: *all probabilities from 0.001 in steps of 0.001 up to 0.9*, (meaning {0.001,0.002,…,0.899,0.9})

   b. Draw the graph of d on the x-axis and the probability that a dichotomy is linearly separable on the y-axis and include it in your answer (Do not include your code).

3. Let N be any be any positive integer. For every $x, x' \in \{1,2, \dots, N\}$ define

$$K(x, x') = \min\,(x, x')$$

   Prove the K is a valid kernel. Namely, find a mapping ψ: $\{1,2, \dots, N\} \longrightarrow E$, where E is some Euclidean space, such that

$$\forall\, x, x', K(x, x') = \langle\, \psi(x), \psi(x')\,\rangle$$

4. Use Lagrange Multipliers to find the maximum and minimum values of the function subject to the given constraints:

   Function: $f(x, y, z) = x^2 + y^2 + z^2$. Constraint: $g(x, y, z) = \frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} + \frac{z^2}{\gamma^2} = 1$, where $\alpha > \beta > \gamma > 0$.

5. Let $X = \mathbb{R}^3$. Let

$$C = H = \{h(l,u,r) = \{(x,y,z)\ s.t\ x^2 + y^2 \leq r,\ \ l \leq z \leq u\}\ s.t.\ \ l,u \in \mathbb{R}, r \in \mathbb{R}_+\}$$

the set of all cylinders threaded on the z axis the cylinder can be both on the positive and negative part of z (see picture). Describe a polynomial sample complexity algorithm L that learns C using H. State the time complexity and the sample complexity of your suggested algorithm. Prove all your steps.