Machine learning Exercise 3

This exercise is composed of 2 parts:

1. Probability theory part

2. Coding part

Submission guidelines: zip only the completed jupyter notebook and the PDF with your solution for the theory part. Do not include the data or any directories. Name the file ID1_ID2.zip and submit **only one copy of the assignment**.

Probability Theory Questions

1. MLE for Poisson. Given a random sample $\{x_1, x_2, \dots, x_n\}$, derive the maximum likelihood estimator λ of the Poisson distribution.

$$P(x, \lambda) = \frac{e^{-\lambda}\lambda^x}{x!}$$

2. A radar at the beach is used to detect ships. Ships are located in one of four zones: A, B, C and D. The probability of detection per zone is $0.75, 0.5, 0.3, 0.4$ for A, B, C, D respectively. The probability of being at a specific zone is $0.4, 0.2, 0.3, 0.1$ for A, B, C, D, respectively.
    a. What is the probability that a ship will be detected?
    b. Given that a ship is detected, what is the probability that it was in zone C?
    c. Given that a ship is detected, what is the probability that it was in zone B?

3. Find 3 random variables X, Y, C such that:
    a. $X \perp Y \mid C$ – (X and Y are independent given C.)
    b. X and Y are not independent.
    c. X, Y take integer values such that $1 \leq X, Y \leq 10$ and C is binary.
    d. The following conditions hold:
        i. $P(1 \leq X \leq 5) = 0.3$
        ii. $P(1 \leq Y \leq 5) = 0.3$
        iii. $P(C = 0) = 0.5$

    You need to specify the value of $P(X = x, Y = y, C = c)$ for all relevant x, y, c. How many such relevant values exist?

4. The probability of having a decent meal in Karnaf is 0.65.
    a. What is the probability of having 3 descent meals in a week (5 days)?
    b. What is the probability of having at least 2 descent meals in a week?
    c. A class of 300 students recorded the number of descent meals they had during a specific week. They averaged their results. What do you expect the value of that average to be?

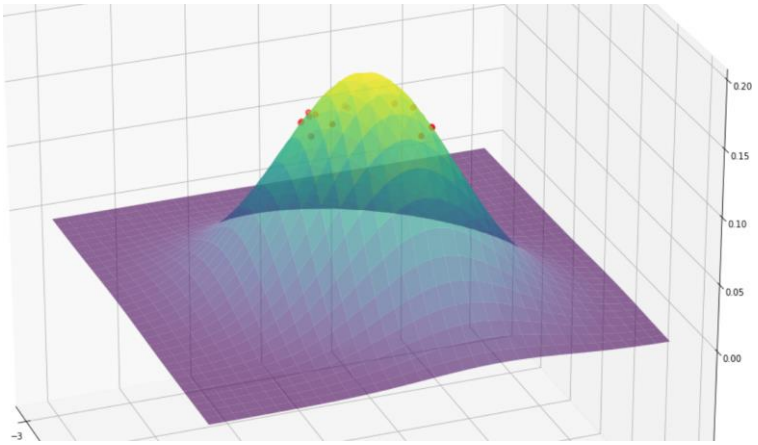
5. Bivariate Normal Distribution: you are given a dataset of 1,000 (x_1, x_2) points drawn from a Bivariate Normal distribution with unknown parameters (data/bivariate_normal_data.csv).
    a. Estimate the distribution parameters using the following (these are the MLE parameters):

$$\mu_i = \frac{1}{N} \cdot \sum_{k=1}^{N} x_i^{(k)}$$

$$\sigma_i = \frac{1}{N} \sum_{k=1}^{N} \left(x_i^{(k)} - \mu_i \right)^2$$

$$\rho = \frac{1}{N} \sum_{k=1}^{N} \left(x_1^{(k)} - \mu_1 \right) \cdot \left(x_2^{(k)} - \mu_2 \right)$$

    b. Using the parameters you found, plot the distribution in 3d and mark on the plot the points from the dataset we provided you that correspond to some narrow equiprobable range. For example, given the range $P(0.14 \leq X \leq 0.15)$:



Coding Exercise

Follow the instructions in the MAP classifier Jupyter notebook.

Good luck!