

# HEURISTIC SEARCH TECHNIQUES:

## Search Algorithms

Many traditional search algorithms are used in AI applications. For complex problems, the traditional algorithms are unable to find the solutions within some practical time and space limits. Consequently, many special techniques are developed, using *heuristic functions*. The algorithms that use *heuristic functions* are called *heuristic algorithms*.

- Heuristic algorithms are not really intelligent; they appear to be intelligent because they achieve better performance.
- Heuristic algorithms are more efficient because they take advantage of feedback from the data to direct the search path.
- **Uninformed search algorithms** or *Brute-force algorithms*, search through the search space all possible candidates for the solution checking whether each candidate satisfies the problem's statement.
- **Informed search algorithms** use heuristic functions that are specific to the problem, apply them to guide the search through the search space to try to reduce the amount of time spent in searching.

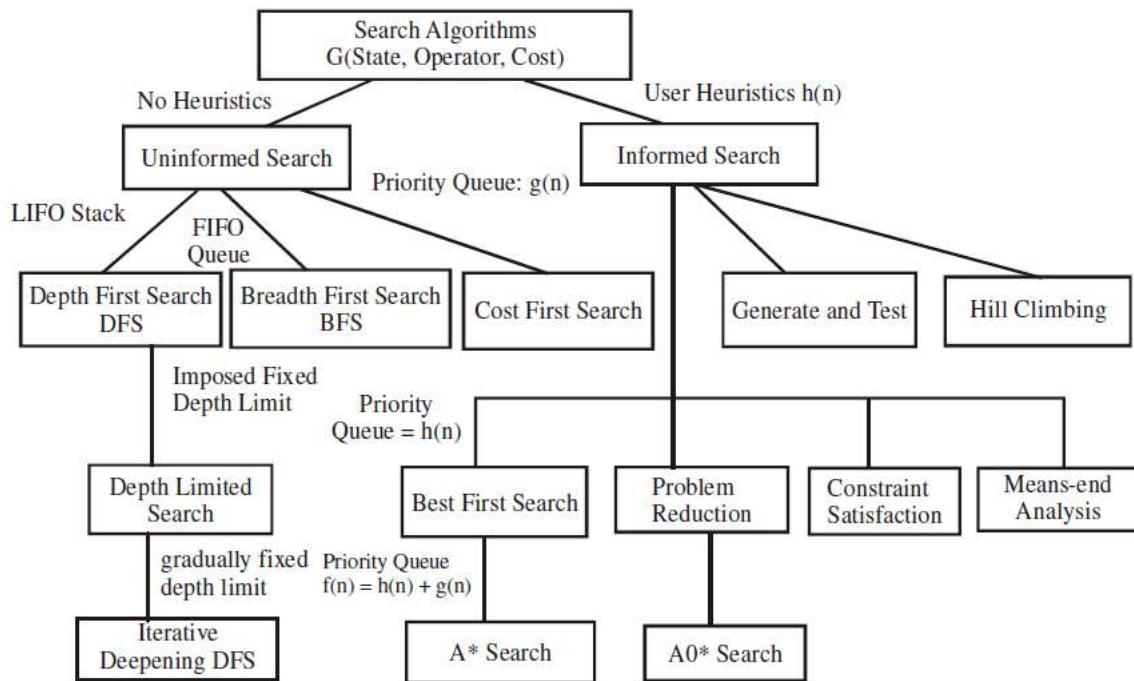
A good heuristic will make an informed search dramatically outperform any uninformed search: for example, the Traveling Salesman Problem (TSP), where the goal is to find a good solution instead of finding the best solution.

In such problems, the search proceeds using current information about the problem to predict which path is closer to the goal and follow it, although it does not always guarantee to find the best possible solution. Such techniques help in finding a solution within reasonable time and space (memory). Some prominent intelligent search algorithms are stated below:

1. **Generate and Test Search**
2. **Best-first Search**
3. **Greedy Search**
4. **A\* Search**
5. **Constraint Search**
6. **Means-ends analysis**

There are some more algorithms. They are either improvements or combinations of these.

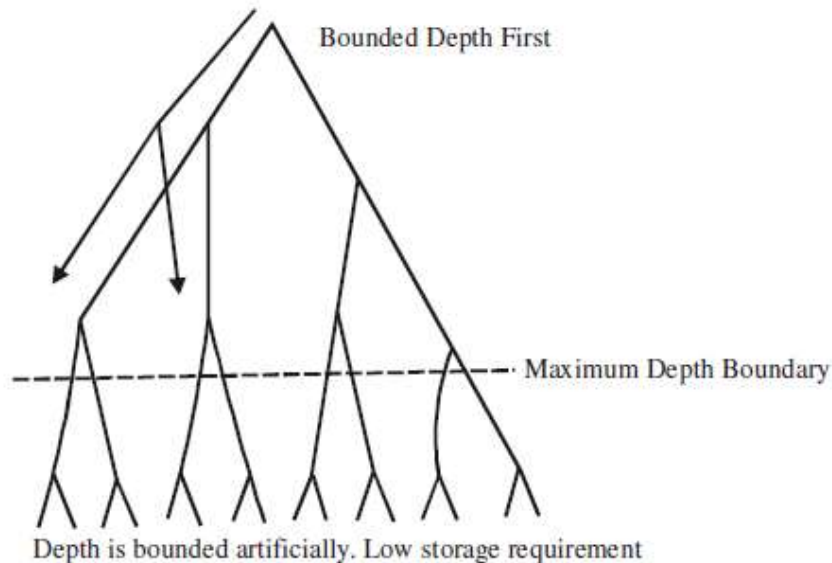
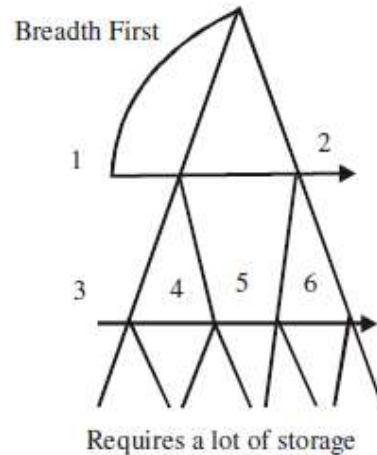
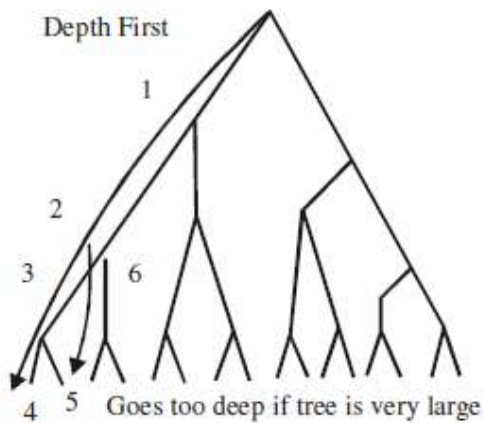
- **Hierarchical Representation of Search Algorithms:** A Hierarchical representation of most search algorithms is illustrated below. The representation begins with two types of search:
- **Uninformed Search:** Also called blind, exhaustive or brute-force search, it uses no information about the problem to guide the search and therefore may not be very efficient.
- **Informed Search:** Also called heuristic or intelligent search, this uses information about the problem to guide the search—usually guesses the distance to a goal state and is therefore efficient, but the search may not be always possible.



**Fig.** Different Search Algorithms

*The first requirement is that it causes motion*, in a game playing program, it moves on the board and in the water jug problem, filling water is used to fill jugs. It means the control strategies without the motion will never lead to the solution.

*The second requirement is that it is systematic*, that is, it corresponds to the need for global motion as well as for local motion. This is a clear condition that neither would it be rational to fill a jug and empty it repeatedly, nor it would be worthwhile to move a piece round and round on the board in a cyclic way in a game. We shall initially consider two systematic approaches for searching. Searches can be classified by the order in which operators are tried: depth-first, breadth-first, bounded depth-first.



### Breadth-first search

A Search strategy, in which the highest layer of a decision tree is searched completely before proceeding to the next layer is called *Breadth-first search (BFS)*.

- In this strategy, no viable solutions are omitted and therefore it is guaranteed that an optimal solution is found.
- This strategy is often not feasible when the search space is large.

### Algorithm

1. Create a variable called LIST and set it to be the starting state.
2. Loop until a goal state is found or LIST is empty, Do
  - a. Remove the first element from the LIST and call it E. If the LIST is empty, quit.
  - b. For every path each rule can match the state E, Do
    - (i) Apply the rule to generate a new state.
    - (ii) If the new state is a goal state, quit and return this state.
    - (iii) Otherwise, add the new state to the end of LIST.

### **Advantages**

1. Guaranteed to find an optimal solution (in terms of shortest number of steps to reach the goal).
2. Can always find a goal node if one exists (complete).

### **Disadvantages**

1. High storage requirement: *exponential* with tree depth.

### **Depth-first search**

A search strategy that extends the current path as far as possible before backtracking to the last choice point and trying the next alternative path is called *Depth-first search (DFS)*.

- This strategy does not guarantee that the optimal solution has been found.
- In this strategy, search reaches a satisfactory solution more rapidly than breadth first, an advantage when the search space is large.

### **Algorithm**

Depth-first search applies operators to each newly generated state, trying to drive directly toward the goal.

1. If the starting state is a goal state, quit and return success.
2. Otherwise, do the following until success or failure is signalled:
  - a. Generate a successor E to the starting state. If there are no more successors, then signal failure.
  - b. Call Depth-first Search with E as the starting state.
  - c. If success is returned signal success; otherwise, continue in the loop.

### **Advantages**

1. Low storage requirement: *linear* with tree depth.
2. Easily programmed: function call stack does most of the work of maintaining state of the search.

### **Disadvantages**

1. May find a sub-optimal solution (one that is deeper or more costly than the best solution).
2. Incomplete: without a depth bound, may not find a solution even if one exists.

#### **2.4.2.3 Bounded depth-first search**

Depth-first search can spend much time (perhaps infinite time) exploring a very deep path that does not contain a solution, when a shallow solution exists. An easy way to solve this problem is to put a maximum depth bound on the search. Beyond the depth bound, a failure is generated automatically without exploring any deeper.

Problems:

1. It's hard to guess how deep the solution lies.
2. If the estimated depth is too deep (even by 1) the computer time used is dramatically increased, by a factor of *b<sup>extra</sup>*.
3. If the estimated depth is too shallow, the search fails to find a solution; all that computer time is wasted.

### **Heuristics**

A heuristic is a method that improves the efficiency of the search process. These are like tour guides. There are good to the level that they may neglect the points in general interesting directions; they are bad to the level that they may neglect points of interest to particular individuals. Some heuristics help in the search process without sacrificing any claims to entirety that the process might previously had. Others may occasionally cause an excellent path to be overlooked. By sacrificing entirety it increases efficiency. Heuristics may not find the best

solution every time but guarantee that they find a good solution in a reasonable time. These are particularly useful in solving tough and complex problems, solutions of which would require infinite time, i.e. far longer than a lifetime for the problems which are not solved in any other way.

### Heuristic search

To find a solution in proper time rather than a complete solution in unlimited time we use heuristics. 'A heuristic function is a function that maps from problem state descriptions to measures of desirability, usually represented as numbers'. Heuristic search methods use knowledge about the problem domain and choose promising operators first. These heuristic search methods use heuristic functions to evaluate the next state towards the goal state. For finding a solution, by using the heuristic technique, one should carry out the following steps:

1. Add domain—specific information to select what is the best path to continue searching along.
2. Define a heuristic function  $h(n)$  that estimates the 'goodness' of a node  $n$ . Specifically,  $h(n)$  = estimated cost(or distance) of minimal cost path from  $n$  to a goal state.
3. The term, heuristic means 'serving to aid discovery' and is an estimate, based on domain specific information that is computable from the current state description of how close we are to a goal.

Finding a route from one city to another city is an example of a search problem in which different search orders and the use of heuristic knowledge are easily understood.

1. State: The current city in which the traveller is located.
2. Operators: Roads linking the current city to other cities.
3. Cost Metric: The cost of taking a given road between cities.
4. Heuristic information: The search could be guided by the direction of the goal city from the current city, or we could use airline distance as an estimate of the distance to the goal.

### Heuristic search techniques

For complex problems, the traditional algorithms, presented above, are unable to find the solution within some practical time and space limits. Consequently, many special techniques are developed, using *heuristic functions*.

- Blind search is not always possible, because it requires too much time or Space (memory).

Heuristics are *rules of thumb*; they do not guarantee a solution to a problem.

- Heuristic Search is a weak technique but can be effective if applied correctly; it requires domain specific information.

### Characteristics of heuristic search

- Heuristics are knowledge about domain, which help search and reasoning in its domain.
- Heuristic search incorporates domain knowledge to improve efficiency over blind search.
- Heuristic is a function that, when applied to a state, returns value as estimated merit of state, with respect to goal.
  - ✓ Heuristics might (for reasons) *underestimate* or *overestimate* the merit of a state with respect to goal.
  - ✓ Heuristics that underestimate are desirable and called admissible.
- Heuristic evaluation function estimates likelihood of given state leading to goal state.
- Heuristic search function estimates cost from current state to goal, presuming function is efficient.

## Heuristic search compared with other search

The Heuristic search is compared with Brute force or Blind search techniques below:

### Comparison of Algorithms

#### Brute force / Blind search

Can only search what it has knowledge about already

No knowledge about how far a node node from goal state

#### Heuristic search

Estimates 'distance' to goal state through explored nodes

Guides search process toward goal

Prefers states (nodes) that lead close to and not away from goal state

### Example: Travelling salesman

A salesman has to visit a list of cities and he must visit each city only once. There are different routes between the cities. The problem is to find the shortest route between the cities so that the salesman visits all the cities at once.

Suppose there are  $N$  cities, then a solution would be to take  $N!$  possible combinations to find the shortest distance to decide the required route. This is not efficient as with  $N=10$  there are 36,28,800 possible routes. This is an example of *combinatorial explosion*.

There are better methods for the solution of such problems: one is called *branch and bound*. First, generate all the complete paths and find the distance of the first complete path. If the next path is shorter, then save it and proceed this way avoiding the path when its length exceeds the saved shortest path length, although it is better than the previous method.

## Generate and Test Strategy

### Generate-And-Test Algorithm

Generate-and-test search algorithm is a very simple algorithm that guarantees to find a solution if done systematically and there exists a solution.

#### Algorithm: Generate-And-Test

1. Generate a possible solution.
2. Test to see if this is the expected solution.
3. If the solution has been found quit else go to step 1.

Potential solutions that need to be generated vary depending on the kinds of problems. For some problems the possible solutions may be particular points in the problem space and for some problems, paths from the start state.

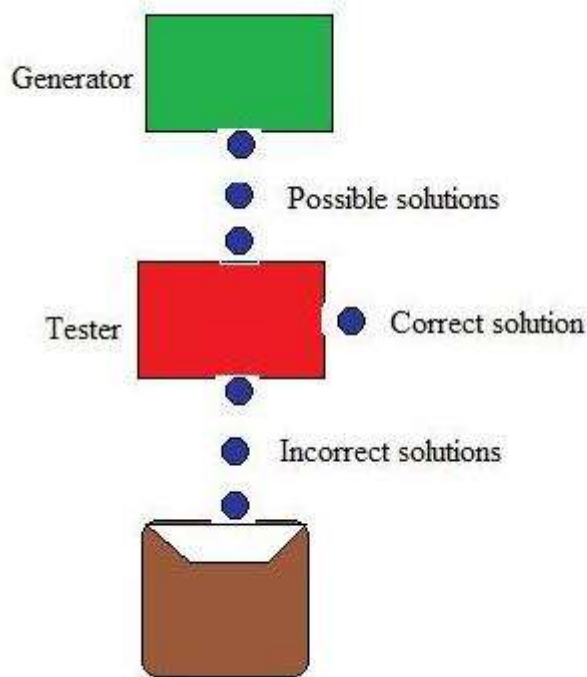


Figure: Generate And Test

Generate-and-test, like depth-first search, requires that complete solutions be generated for testing. In its most systematic form, it is only an exhaustive search of the problem space. Solutions can also be generated randomly but solution is not guaranteed. This approach is what is known as British Museum algorithm: finding an object in the British Museum by wandering randomly.

### **Systematic Generate-And-Test**

While generating complete solutions and generating random solutions are the two extremes there exists another approach that lies in between. The approach is that the search process proceeds systematically but some paths that unlikely to lead the solution are not considered. This evaluation is performed by a heuristic function.

Depth-first search tree with backtracking can be used to implement systematic generate-and-test procedure. As per this procedure, if some intermediate states are likely to appear often in the tree, it would be better to modify that procedure to traverse a graph rather than a tree.

### **Generate-And-Test And Planning**

Exhaustive generate-and-test is very useful for simple problems. But for complex problems even heuristic generate-and-test is not very effective technique. But this may be made effective by combining with other techniques in such a way that the space in which to search is restricted. An AI program DENDRAL, for example, uses plan-Generate-and-test technique. First, the planning process uses constraint-satisfaction techniques and creates lists of recommended and contraindicated substructures. Then the generate-and-test procedure uses the lists generated and required to explore only a limited set of structures. Constrained in this way, generate-and-test proved highly effective. A major weakness of planning is that it often produces inaccurate solutions as there is no feedback from the world. But if it is used to produce only pieces of solutions then lack of detailed accuracy becomes unimportant.



## Hill Climbing

Hill Climbing is heuristic search used for mathematical optimization problems in the field of Artificial Intelligence .

Given a large set of inputs and a good heuristic function, it tries to find a sufficiently good solution to the problem. This solution may not be the global optimal maximum.

- In the above definition, mathematical optimization problems implies that hill climbing solves the problems where we need to maximize or minimize a given real function by choosing values from the given inputs. Example-[Travelling salesman problem](#) where we need to minimize the distance traveled by salesman.
- 'Heuristic search' means that this search algorithm may not find the optimal solution to the problem. However, it will give a good solution in reasonable time.
- A heuristic function is a function that will rank all the possible alternatives at any branching step in search algorithm based on the available information. It helps the algorithm to select the best route out of possible routes.

### Features of Hill Climbing

1. Variant of generate and test algorithm : It is a variant of generate and test algorithm. The generate and test algorithm is as follows :

1. *Generate a possible solutions.*
2. *Test to see if this is the expected solution.*
3. *If the solution has been found quit else go to step 1.*

Hence we call Hill climbing as a variant of generate and test algorithm as it takes the feedback from test procedure. Then this feedback is utilized by the generator in deciding the next move in search space.

2. Uses the Greedy approach : At any point in state space, the search moves in that direction only which optimizes the cost of function with the hope of finding the optimal solution at the end.

### Types of Hill Climbing

1. Simple Hill climbing : It examines the neighboring nodes one by one and selects the first neighboring node which optimizes the current cost as next node.

Algorithm for Simple Hill climbing :

*Step 1 : Evaluate the initial state. If it is a goal state then stop and return success. Otherwise, make initial state as current state.*

*Step 2 : Loop until the solution state is found or there are no new operators present which can be applied to current state.*

*a) Select a state that has not been yet applied to the current state and apply it to produce a new state.*

*b) Perform these to evaluate new state*

- i. If the current state is a goal state, then stop and return success.*
- ii. If it is better than the current state, then make it current state and proceed further.*
- iii. If it is not better than the current state, then continue in the loop until a solution is found.*

*Step 3 : Exit.*



2. Steepest-Ascent Hill climbing : It first examines all the neighboring nodes and then selects the node closest to the solution state as next node.

*Step 1 : Evaluate the initial state. If it is goal state then exit else make the current state as initial state*

*Step 2 : Repeat these steps until a solution is found or current state does not change*

*i. Let 'target' be a state such that any successor of the current state will be better than it;*

*ii. for each operator that applies to the current state*

*a. apply the new operator and create a new state*

*b. evaluate the new state*

*c. if this state is goal state then quit else compare with 'target'*

*d. if this state is better than 'target', set this state as 'target'*

*e. if target is better than current state set current state to Target*

*Step 3 : Exit*

3. Stochastic hill climbing : It does not examine all the neighboring nodes before deciding which node to select .It just selects a neighboring node at random, and decides (based on the amount of improvement in that neighbor) whether to move to that neighbor or to examine another.

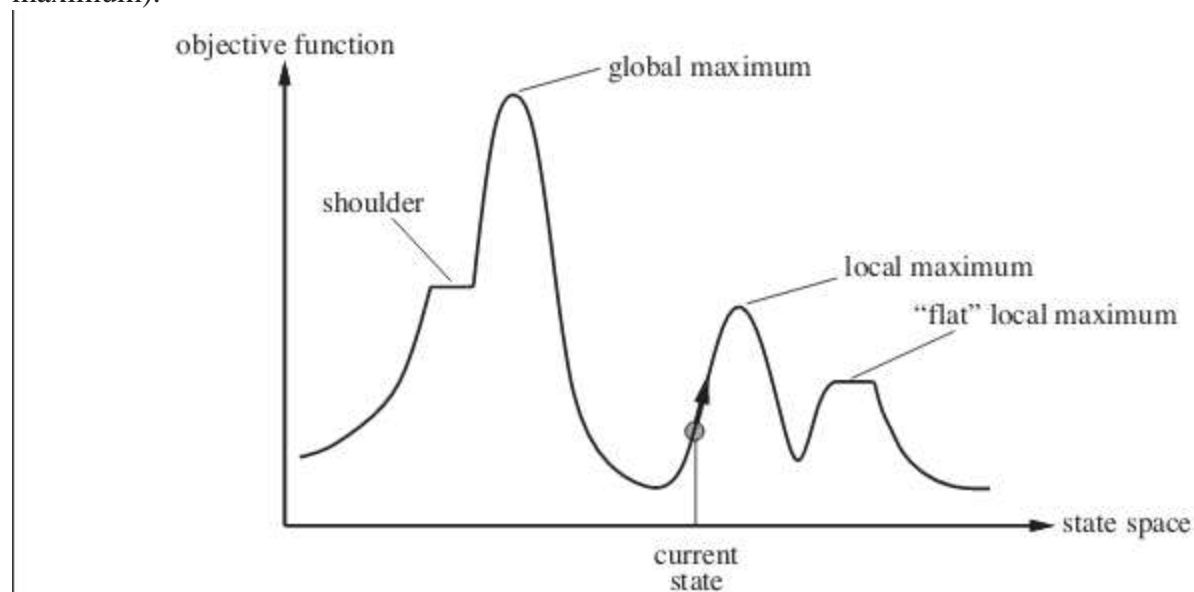
State Space diagram for Hill Climbing

State space diagram is a graphical representation of the set of states our search algorithm can reach vs the value of our objective function(the function which we wish to maximize).

X-axis : denotes the state space ie states or configuration our algorithm may reach.

Y-axis : denotes the values of objective function corresponding to a particular state.

The best solution will be that state space where objective function has maximum value(global maximum).



Different regions in the State Space Diagram

1. Local maximum : It is a state which is better than its neighboring state however there exists a state which is better than it(global maximum). This state is better because here value of objective function is higher than its neighbors.

2. Global maximum : It is the best possible state in the state space diagram. This because at this state, objective function has highest value.
3. Plateau/flat local maximum : It is a flat region of state space where neighboring states have the same value.
4. Ridge : It is region which is higher than its neighbours but itself has a slope. It is a special kind of local maximum.
5. Current state : The region of state space diagram where we are currently present during the search.
6. Shoulder : It is a plateau that has an uphill edge.

Problems in different regions in Hill climbing

Hill climbing cannot reach the optimal/best state(global maximum) if it enters any of the following regions :

1. Local maximum : At a local maximum all neighboring states have a values which is worse than than the current state. Since hill climbing uses greedy approach, it will not move to the worse state and terminate itself. The process will end even though a better solution may exist.  
To overcome local maximum problem : Utilize backtracking technique. Maintain a list of visited states. If the search reaches an undesirable state, it can backtrack to the previous configuration and explore a new path.
2. Plateau : On plateau all neighbors have same value . Hence, it is not possible to select the best direction.

To overcome plateaus : Make a big jump. Randomly select a state far away from current state.

Chances are that we will land at a non-plateau region

3. Ridge : Any point on a ridge can look like peak because movement in all possible directions is downward. Hence the algorithm stops when it reaches this state.  
To overcome Ridge : In this kind of obstacle, use two or more rules before testing. It implies moving in several directions at once.

## Best First Search (Informed Search)

In BFS and DFS, when we are at a node, we can consider any of the adjacent as next node. So both BFS and DFS blindly explore paths without considering any cost function. The idea of Best First Search is to use an evaluation function to decide which adjacent is most promising and then explore. Best First Search falls under the category of Heuristic Search or Informed Search.

We use a priority queue to store costs of nodes. So the implementation is a variation of BFS, we just need to change Queue to PriorityQueue.

Algorithm:

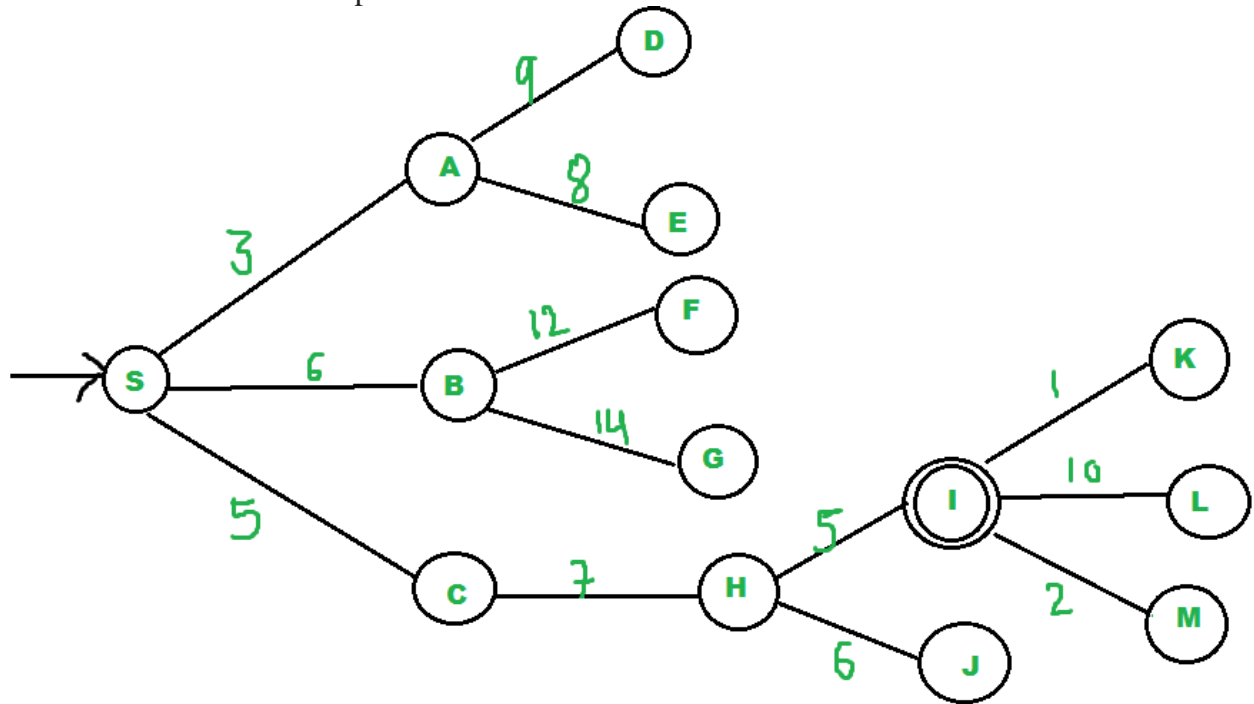
Best-First-Search(Graph g, Node start)

- 1) Create an empty PriorityQueue  
PriorityQueue pq;
- 2) Insert "start" in pq.  
pq.insert(start)
- 3) Until PriorityQueue is empty  
u = PriorityQueue.DeleteMin

```

    If u is the goal
      Exit
    Else
      Foreach neighbor v of u
        If v "Unvisited"
          Mark v "Visited"
          pq.insert(v)
          Mark v "Examined"
      End procedure
  
```

Let us consider below example.



We start from source "S" and search for goal "I" using given costs and Best First search.

pq initially contains S  
 We remove s from and process unvisited neighbors of S to pq.  
 pq now contains {A, C, B} (C is put before B because C has lesser cost)

We remove A from pq and process unvisited neighbors of A to pq.  
 pq now contains {C, B, E, D}

We remove C from pq and process unvisited neighbors of C to pq.  
pq now contains {B, H, E, D}

We remove B from pq and process unvisited neighbors of B to pq.  
pq now contains {H, E, D, F, G}

We remove H from pq. Since our goal "I" is a neighbor of H, we return.

#### **Analysis :**

- The worst case time complexity for Best First Search is  $O(n * \log n)$  where  $n$  is number of nodes. In worst case, we may have to visit all nodes before we reach goal. Note that priority queue is implemented using Min(or Max) Heap, and insert and remove operations take  $O(\log n)$  time.
- Performance of the algorithm depends on how well the cost or evaluation function is designed.

## **A\* Search Algorithm**

A\* is a type of search algorithm. Some problems can be solved by representing the world in the initial state, and then for each action we can perform on the world we generate states for what the world would be like if we did so. If you do this until the world is in the state that we specified as a solution, then the route from the start to this goal state is the solution to your problem.

In this tutorial I will look at the use of state space search to find the shortest path between two points (pathfinding), and also to solve a simple sliding tile puzzle (the 8-puzzle). Let's look at some of the terms used in Artificial Intelligence when describing this state space search.

### *Some terminology*

A *node* is a state that the problem's world can be in. In pathfinding a node would be just a 2d coordinate of where we are at the present time. In the 8-puzzle it is the positions of all the tiles. Next all the nodes are arranged in a *graph* where links between nodes represent valid steps in solving the problem. These links are known as *edges*. In the 8-puzzle diagram the edges are shown as blue lines. See figure 1 below.

*State space search*, then, is solving a problem by beginning with the start state, and then for each node we expand all the nodes beneath it in the graph by applying all the possible moves that can be made at each point.

### *Heuristics and Algorithms*

At this point we introduce an important concept, the *heuristic*. This is like an algorithm, but with a key difference. An algorithm is a set of steps which you can follow to solve a problem, which always works for valid input. For example you could probably write an algorithm yourself for

multiplying two numbers together on paper. A heuristic is not guaranteed to work but is useful in that it may solve a problem for which there is no algorithm.

We need a heuristic to help us cut down on this huge search problem. What we need is to use our heuristic at each node to make an estimate of how far we are from the goal. In pathfinding we know exactly how far we are, because we know how far we can move each step, and we can calculate the exact distance to the goal.

But the 8-puzzle is more difficult. There is no known algorithm for calculating from a given position how many moves it will take to get to the goal state. So various heuristics have been devised. The best one that I know of is known as the Nilsson score which leads fairly directly to the goal most of the time, as we shall see.

### Cost

When looking at each node in the graph, we now have an idea of a heuristic, which can estimate how close the state is to the goal. Another important consideration is the cost of getting to where we are. In the case of pathfinding we often assign a movement cost to each square. The cost is the same then the cost of each square is one. If we wanted to differentiate between terrain types we may give higher costs to grass and mud than to newly made road. When looking at a node we want to add up the cost of what it took to get here, and this is simply the sum of the cost of this node and all those that are above it in the graph.

### 8 Puzzle

Let's look at the 8 puzzle in more detail. This is a simple sliding tile puzzle on a 3\*3 grid where one tile is missing and you can move the other tiles into the gap until you get the puzzle into the goal position. See figure 1.

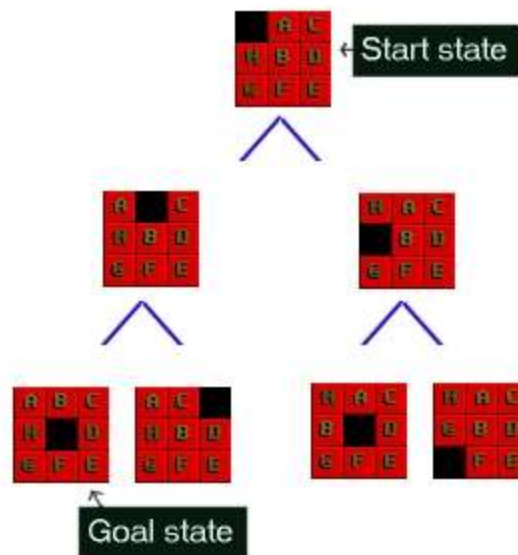


Figure 1 : The 8-Puzzle state space for a very simple example

There are 362,880 different states that the puzzle can be in, and to find a solution the search has to find a route through them. From most positions of the search the number of edges (that's the

blue lines) is two. That means that the number of nodes you have in each level of the search is  $2^d$  where  $d$  is the depth. If the number of steps to solve a particular state is 18, then that's 262,144 nodes just at that level.

The 8 puzzle game state is as simple as representing a list of the 9 squares and what's in them. Here are two states for example; the last one is the GOAL state, at which point we've found the solution. The first is a jumbled up example that you may start from.

Start state SPACE, A, C, H, B, D, G, F, E

Goal state A, B, C, H, SPACE, D, G, F, E

The rules that you can apply to the puzzle are also simple. If there is a blank tile above, below, to the left or to the right of a given tile, then you can move that tile into the space. To solve the puzzle you need to find the path from the start state, through the graph down to the goal state.

There is example code to to solve the 8-puzzle on the [github](#) site.

## Pathfinding

In a video game, or some other pathfinding scenario, you want to search a state space and find out how to get from somewhere you are to somewhere you want to be, without bumping into walls or going too far. For reasons we will see later, the A\* algorithm will not only find a path, if there is one, but it will find the shortest path. A state in pathfinding is simply a position in the world. In the example of a maze game like Pacman you can represent where everything is using a simple 2d grid. The start state for a ghost say, would be the 2d coordinate of where the ghost is at the start of the search. The goal state would be where pacman is so we can go and eat him.

There is also example code to do pathfinding on the [github](#) site.

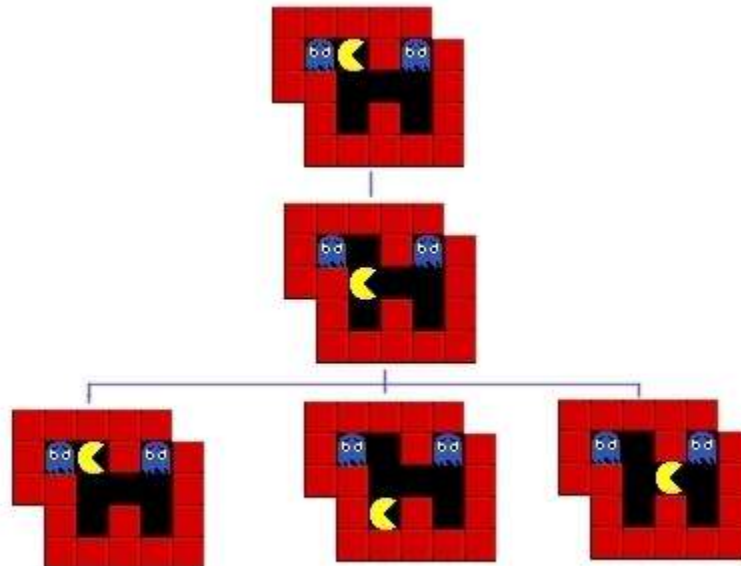


Figure 2 : The first three steps of a pathfinding state space

## Implementing A\*

We are now ready to look at the operation of the A\* algorithm. What we need to do is start with the goal state and then generate the graph downwards from there. Let's take the 8-puzzle in figure 1. We ask how many moves can we make from the start state? The answer is 2, there are two directions we can move the blank tile, and so our graph expands.

If we were just to continue blindly generating successors to each node, we could potentially fill the computer's memory before we found the goal node. Obviously we need to remember the best nodes and search those first. We also need to remember the nodes that we have expanded already, so that we don't expand the same state repeatedly.

Let's start with the OPEN list. This is where we will remember which nodes we haven't yet expanded. When the algorithm begins the start state is placed on the open list, it is the only state we know about and we have not expanded it. So we will expand the nodes from the start and put those on the OPEN list too. Now we are done with the start node and we will put that on the CLOSED list. The CLOSED list is a list of nodes that we have expanded.

$$f = g + h$$

Using the OPEN and CLOSED list lets us be more selective about what we look at next in the search. We want to look at the best nodes first. We will give each node a score on how good we think it is. This score should be thought of as the cost of getting from the node to the goal plus the cost of getting to where we are. Traditionally this has been represented by the letters f, g and h. 'g' is the sum of all the costs it took to get here, 'h' is our heuristic function, the estimate of what it will take to get to the goal. 'f' is the sum of these two. We will store each of these in our nodes.

Using the f, g and h values the A\* algorithm will be directed, subject to conditions we will look at further on, towards the goal and will find it in the shortest route possible.

So far we have looked at the components of the A\*, let's see how they all fit together to make the algorithm :

### Pseudocode

Hopefully the ideas we looked at in the preceding paragraphs will now click into place as we look at the A\* algorithm pseudocode. You may find it helpful to print this out or leave the window open while we discuss it.

To help make the operation of the algorithm clear we will look again at the 8-puzzle problem in figure 1 above. Figure 3 below shows the f,g and h scores for each of the tiles.



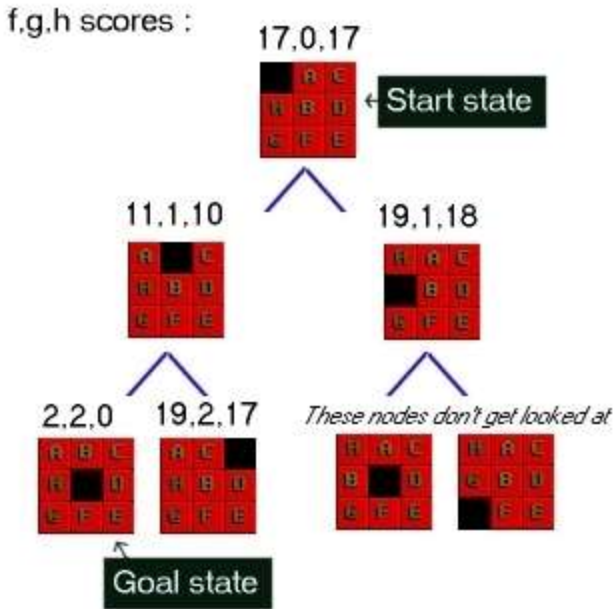


Figure 3 : 8-Puzzle state space showing f,g,h scores

First of all look at the g score for each node. This is the cost of what it took to get from the start to that node. So in the picture the center number is g. As you can see it increases by one at each level. In some problems the cost may vary for different state changes. For example in pathfinding there is sometimes a type of terrain that costs more than other types. Next look at the last number in each triple. This is h, the heuristic score. As I mentioned above I am using a heuristic known as Nilsson's Sequence, which converges quickly to a correct solution in many cases. Here is how you calculate this score for a given 8-puzzle state :

### Advantages:

It is complete and optimal.

It is the best one from other techniques. It is used to solve very complex problems.

It is optimally efficient, i.e. there is no other optimal algorithm guaranteed to expand fewer nodes than A\*.

### Disadvantages:

This algorithm is complete if the branching factor is finite and every action has fixed cost.

The speed execution of A\* search is highly dependant on the accuracy of the heuristic algorithm that is used to compute  $h(n)$ .

## **AO\* Search: (And-Or) Graph**

The Depth first search and Breadth first search given earlier for OR trees or graphs can be easily adopted by AND-OR graph. The main difference lies in the way termination conditions are determined, since all goals following an AND nodes must be realized; where as a single goal node following an OR node will do. So for this purpose we are using AO\* algorithm.

Like A\* algorithm here we will use two arrays and one heuristic function.

### **OPEN:**

It contains the nodes that has been traversed but yet not been marked solvable or unsolvable.

### **CLOSE:**

It contains the nodes that have already been processed.

**6 7:** The distance from current node to goal node.

### **Algorithm:**

**Step 1:** Place the starting node into OPEN.

**Step 2:** Compute the most promising solution tree say T0.

**Step 3:** Select a node n that is both on OPEN and a member of T0. Remove it from OPEN and place it in

CLOSE

**Step 4:** If n is the terminal goal node then leveled n as solved and leveled all the ancestors of n as solved. If the starting node is marked as solved then success and exit.

**Step 5:** If n is not a solvable node, then mark n as unsolvable. If starting node is marked as unsolvable, then return failure and exit.

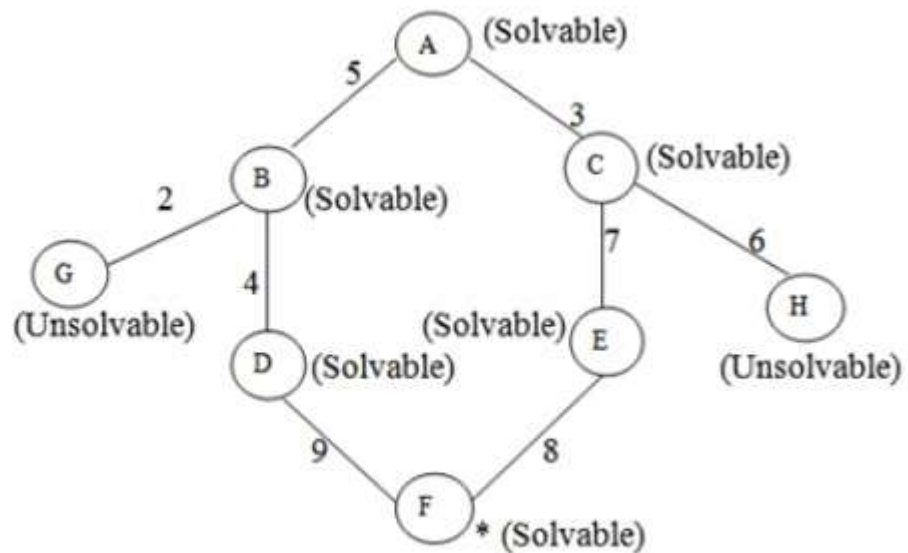
**Step 6:** Expand n. Find all its successors and find their h (n) value, push them into OPEN.

**Step 7:** Return to Step 2.

**Step 8:** Exit.

### Implementation:

Let us take the following example to implement the AO\* algorithm.



**Figure**

#### Step 1:

In the above graph, the solvable nodes are A, B, C, D, E, F and the unsolvable nodes are G, H. Take A as the starting node. So place A into OPEN.

i.e. OPEN = A CLOSE = (NULL)  $\phi$  A

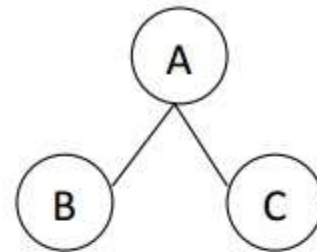
#### Step 2:

The children of A are B and C which are solvable. So place them into OPEN and place A into the CLOSE.

i.e. OPEN =



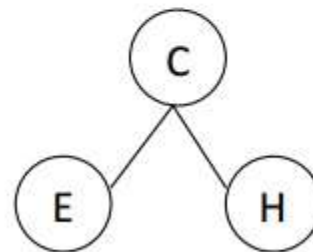
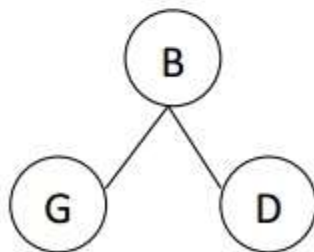
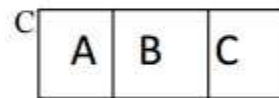
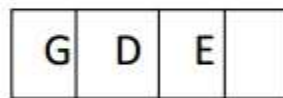
CLOSE =



### Step 3:

Now process the nodes B and C. The children of B and C are to be placed into OPEN. Also remove B and C from OPEN and place them into CLOSE.

So OPEN =



(O)

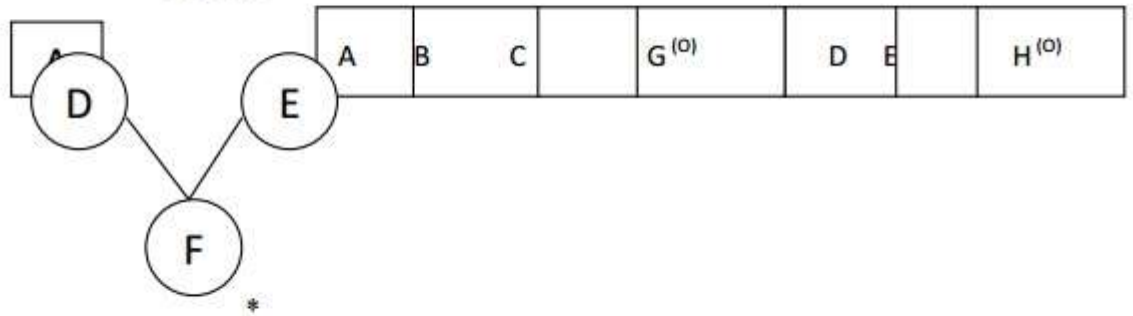
'O' indicated that the nodes G and H are unsolvable.

#### Step 4:

As the nodes G and H are unsolvable, so place them into CLOSE directly and process the nodes D and E.

i.e. OPEN =

CLOSE =



#### Step 5:

Now we have been reached at our goal state. So place F into CLOSE.

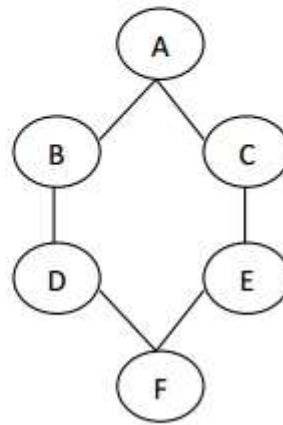
A	B	C		G <sup>(O)</sup>	D	E		H <sup>(O)</sup>	F
---	---	---	--	------------------	---	---	--	------------------	---

i.e. CLOSE =

**Step 6:**

Success and Exit

**AO\* Graph:**



**Figure**

**Advantages:**

It is an optimal algorithm.

If traverse according to the ordering of nodes. It can be used for both OR and AND graph.

**Disadvantages:**

Sometimes for unsolvable nodes, it can't find the optimal path. Its complexity is than other algorithms.

**PROBLEM REDUCTION**

**Problem Reduction with AO\* Algorithm.**

When a problem can be divided into a set of sub problems, where each sub problem can be solved separately and a combination of these will be a solution, AND-OR graphs or AND - OR trees are used for representing the solution. The decomposition of the problem or problem reduction generates AND arcs. One AND are may point to any number of successor nodes. All

these must be solved so that the arc will rise to many arcs, indicating several possible solutions. Hence the graph is known as AND - OR instead of AND. Figure shows an AND - OR graph.

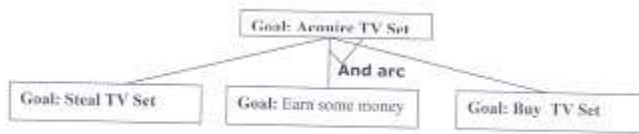


Figure shows AND - Or graph - an example.

An algorithm to find a solution in an AND - OR graph must handle AND area appropriately. A\* algorithm can not search AND - OR graphs efficiently. This can be understood from the given figure.

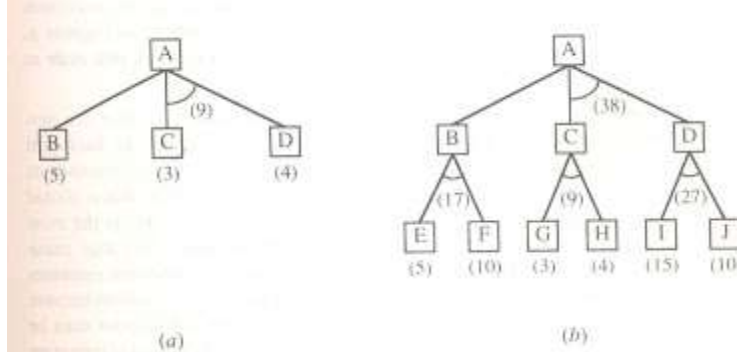


Figure 3.7: AND-OR Graphs

FIGURE : AND - OR graph

In figure (a) the top node A has been expanded producing two areas one leading to B and leading to C-D. The numbers at each node represent the value of  $f'$  at that node (cost of getting to the goal state from current state). For simplicity, it is assumed that every operation (i.e. applying a rule) has unit cost, i.e., each arc with single successor will have a cost of 1 and each of its components. With the available information till now, it appears that C is the most promising node to expand since its  $f' = 3$ , the lowest but going through B would be better since to use C we must also use D and the cost would be  $9(3+4+1+1)$ . Through B it would be  $6(5+1)$ .

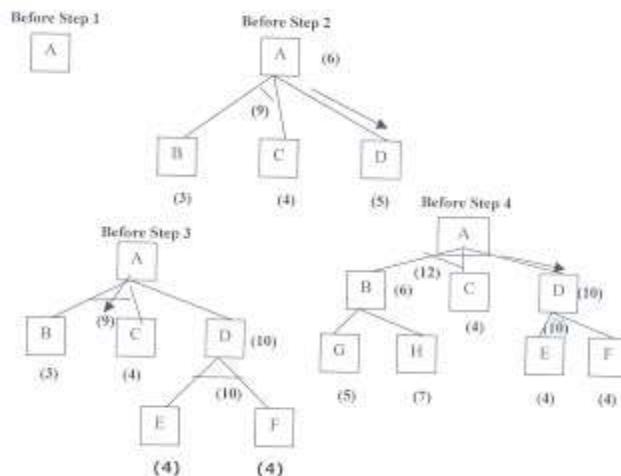
Thus the choice of the next node to expand depends not only on a value but also on whether that node is part of the current best path from the initial node. Figure (b) makes this clearer. In figure the node G appears to be the most promising node, with the least  $f'$  value. But G is not on the current best path, since to use G we must use GH with a cost of 9 and again this demands that arcs be used (with a cost of 27). The path from A through B, E-F is better with a total cost of  $(17+1=18)$ . Thus we can see that to search an AND-OR graph, the following three things must be done.

1. traverse the graph starting at the initial node and following the current best path, and accumulate the set of nodes that are on the path and have not yet been expanded.
2. Pick one of these unexpanded nodes and expand it. Add its successors to the graph and compute  $f'$  (cost of the remaining distance) for each of them.



3. Change the  $f'$  estimate of the newly expanded node to reflect the new information produced by its successors. Propagate this change backward through the graph. Decide which of the current best path.

The propagation of revised cost estimation backward in the tree is not necessary in A\* algorithm. This is because in AO\* algorithm expanded nodes are re-examined so that the current best path can be selected. The working of AO\* algorithm is illustrated in figure as follows:



Referring the figure. The initial node is expanded and D is Marked initially as promising node. D is expanded producing an AND arc E-F.  $f'$  value of D is updated to 10. Going backwards we can see that the AND arc B-C is better. It is now marked as current best path. B and C have to be expanded next. This process continues until a solution is found or all paths have led to dead ends, indicating that there is no solution. An A\* algorithm the path from one node to the other is always that of the lowest cost and it is independent of the paths through other nodes.

The algorithm for performing a heuristic search of an AND - OR graph is given below. Unlike A\* algorithm which used two lists OPEN and CLOSED, the AO\* algorithm uses a single structure G. G represents the part of the search graph generated so far. Each node in G points down to its immediate successors and up to its immediate predecessors, and also has with it the value of  $h'$  cost of a path from itself to a set of solution nodes. The cost of getting from the start nodes to the current node "g" is not stored as in the A\* algorithm. This is because it is not possible to compute a single such value since there may be many paths to the same state. In AO\* algorithm serves as the estimate of goodness of a node. Also a threshold value called FUTILITY is used. The estimated cost of a solution is greater than FUTILITY then the search is abandoned as too expensive to be practical.

For representing above graphs AO\* algorithm is as follows

#### AO\* ALGORITHM:

1. Let G consists only to the node representing the initial state call this node INTT. Compute  $h'$  (INIT).
2. Until INIT is labeled SOLVED or  $h_i$  (INIT) becomes greater than FUTILITY, repeat the following procedure.

- (I) Trace the marked arcs from INIT and select an unbounded node NODE.
- (II) Generate the successors of NODE . if there are no successors then assign FUTILITY as  $h'$  (NODE). This means that NODE is not solvable. If there are successors then for each one called SUCCESSOR, that is not also an ancestor of NODE do the following
- (a) add SUCCESSOR to graph G
  - (b) if successor is not a terminal node, mark it solved and assign zero to its  $h'$  value.
  - (c) If successor is not a terminal node, compute its  $h'$  value.
- (III) propagate the newly discovered information up the graph by doing the following . let S be a set of nodes that have been marked SOLVED. Initialize S to NODE. Until S is empty repeat the following procedure;
- (a) select a node from S call it CURRENT and remove it from S.
  - (b) compute  $h'$  of each of the arcs emerging from CURRENT , Assign minimum  $h'$  to CURRENT.
  - (c) Mark the minimum cost path as the best out of CURRENT.
  - (d) Mark CURRENT SOLVED if all of the nodes connected to it through the new marked are have been labeled SOLVED.
  - (e) If CURRENT has been marked SOLVED or its  $h'$  has just changed, its new status must be propagate backwards up the graph . hence all the ancestors of CURRENT are added to S.

(Referred From Artificial Intelligence TMH)

AO\* Search Procedure.

1. Place the start node on open.
2. Using the search tree, compute the most promising solution tree TP .
3. Select node n that is both on open and a part of tp, remove n from open and place it no closed.
4. If n is a goal node, label n as solved. If the start node is solved, exit with success where tp is the solution tree, remove all nodes from open with a solved ancestor.

5. If  $n$  is not solvable node, label  $n$  as unsolvable. If the start node is labeled as unsolvable, exit with failure. Remove all nodes from open, with unsolvable ancestors.

6. Otherwise, expand node  $n$  generating all of its successor compute the cost of for each newly generated node and place all such nodes on open.

7. Go back to step(2)

Note: AO\* will always find minimum cost solution.

### CONSTRAINT SATISFACTION:-

Many problems in AI can be considered as problems of constraint satisfaction, in which the goal state satisfies a given set of constraint. constraint satisfaction problems can be solved by using any of the search strategies. The general form of the constraint satisfaction procedure is as follows:

Until a complete solution is found or until all paths have led to lead ends, do

1. select an unexpanded node of the search graph.
2. Apply the constraint inference rules to the selected node to generate all possible new constraints.
3. If the set of constraints contains a contradiction, then report that this path is a dead end.
4. If the set of constraints describes a complete solution then report success.
5. If neither a constraint nor a complete solution has been found then apply the rules to generate new partial solutions. Insert these partial solutions into the search graph.

Example: consider the crypt arithmetic problems.

```
  SEND
+ MORE
-----
 MONEY
-----
```

Assign decimal digit to each of the letters in such a way that the answer to the problem is correct to the same letter occurs more than once, it must be assigned the same digit each time. no two different letters may be assigned the same digit. Consider the crypt arithmetic problem.

```

SEND
+ MORE
-----
MONEY
-----

```

CONSTRAINTS:-

1. no two digit can be assigned to same letter.
2. only single digit number can be assign to a letter.
1. no two letters can be assigned same digit.
2. Assumption can be made at various levels such that they do not contradict each other.
3. The problem can be decomposed into secured constraints. A constraint satisfaction approach may be used.
4. Any of search techniques may be used.
5. Backtracking may be performed as applicable us applied search techniques.
6. Rule of arithmetic may be followed.

Initial state of problem.

D=?

E=?

Y=?

N=?

R=?

O=?

S=?

M=?

C1=?

C2=?

C1 ,C 2, C3 stands for the carry variables respectively.

Goal State: the digits to the letters must be assigned in such a manner so that the sum is satisfied.

Solution Process:

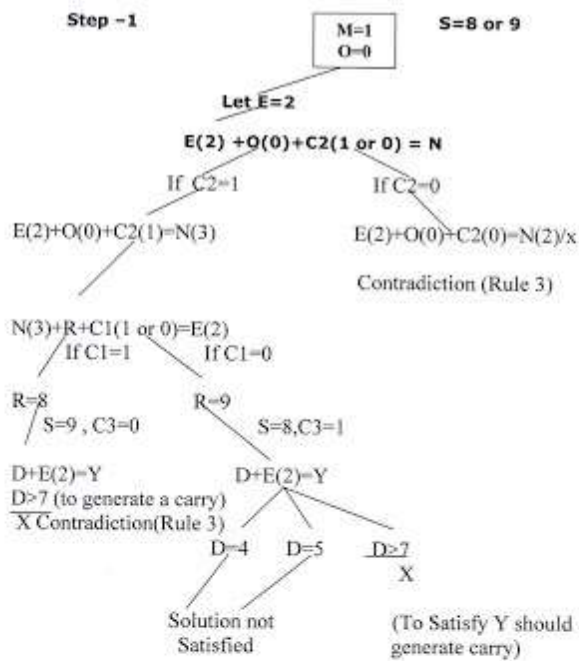
We are following the depth-first method to solve the problem.

1. initial guess m=1 because the sum of two single digits can generate at most a carry '1'.

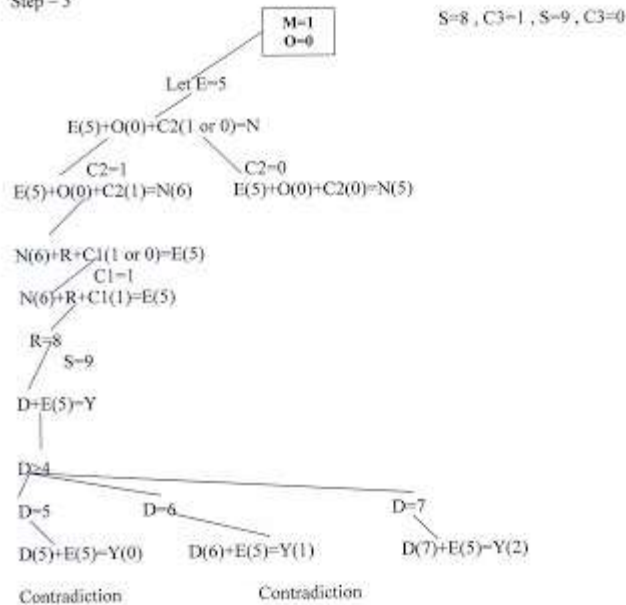
2. When  $n=1$   $o=0$  or  $1$  because the largest single digit number added to  $m=1$  can generate the sum of either  $0$  or  $1$  depend on the carry received from the carry sum. By this we conclude that  $o=0$  because  $m$  is already  $1$  hence we cannot assign same digit another letter(rule no.)

3. We have  $m=1$  and  $o=0$  to get  $o=0$  we have  $s=8$  or  $9$ , again depending on the carry received from the earlier sum.

The same process can be repeated further. The problem has to be composed into various constraints. And each constraints is to be satisfied by guessing the possible digits that the letters can be assumed that the initial guess has been already made . rest of the process is being shown in the form of a tree, using depth-first search for the clear understandability of the solution process.



After Step 2, we found that C1 cannot be Zero, Since Y has to generate a carry to satisfy goal state. From this step onwards, no need to branch for C1=0.  
Step - 3



At Step (4) we have assigned a single digit to every letter in accordance with the constraints & production rules.  
Now by backtracking, we find the different digits assigned to different letters and hence reach the solution state.

### Solution State:-

$$Y = 2$$

$$D = 7$$

$$S = 9$$

$$R = 8$$

$$N = 6$$

$$E = 5$$

$$O = 0$$

$$M = 1$$

$$C1 = 1$$

$$C2 = 0$$

$$C3 = 0$$

	C3(0)	C2(1)	C1(1)	
	S(9)	E(5)	N(6)	D(7)
+	M(1)	O(0)	R(8)	E(5)
<hr/>				
	M(1)	O(0)	N(6)	E(5)
				Y(2)
<hr/>				