

★ 第二次习题课 (week 6)

定解条件  $\begin{cases} \text{初始条件: 关于 } t \text{ 个数由 } t \text{ 的偏导阶数} \\ \text{边界条件: 关于坐标变量} \end{cases}$

eg. for 齐次化原理.

§1 Exercise 10.

$$\text{利求解} \begin{cases} \frac{\partial u}{\partial t} + a \frac{\partial u}{\partial x} = f(t, x) & (t > 0, -\infty < x < +\infty) \\ u(0, x) = \varphi(x) & (a \neq 0, a \text{ 为 const}) \end{cases}$$

由叠加原理, 原定解问题的解  $u = u_1 + u_2$ , 其中  $u_1, u_2$  分别为定解问题 ①, ② 的解

$$\textcircled{1} \begin{cases} \frac{\partial u_1}{\partial t} + a \frac{\partial u_1}{\partial x} = 0 \\ u_1|_{t=0} = \varphi(x) \end{cases} \quad \textcircled{2} \begin{cases} \frac{\partial u_2}{\partial t} + a \frac{\partial u_2}{\partial x} = f(t, x) \\ u_2|_{t=0} = 0 \end{cases}$$

① 代换:  $\xi = x - at, \eta = t$

$$\begin{cases} \frac{\partial u_1}{\partial \eta} = 0 \\ u_1|_{\eta=0} = \varphi(\xi) \end{cases} \quad \therefore u_1 = \varphi(x - at)$$

② 令  $t_1 = t - \tau$

波的平移, 起点归零: 好算. (齐次化原理平移出去再代换平移回来)

$$\frac{\partial u_2}{\partial t_1} + a \frac{\partial u_2}{\partial x} = f(t_1, x) \quad \text{先用齐次化原理.}$$

$$\text{先求解} \begin{cases} \frac{\partial w}{\partial t_1} + a \frac{\partial w}{\partial x} = 0 \\ w|_{t_1=0} = f(t_1, x) \end{cases}$$

$$\text{代换: } \begin{cases} \frac{\partial w}{\partial t_1} + a \frac{\partial w}{\partial x} = 0 \\ w|_{t_1=0} = f(t_1, x) \end{cases} \quad \leftarrow \tau \text{ 和 } t_1 \text{ 为无关, 所有代换只是形式上的.}$$

利用①的解形式,  $w = f(\tau, x - a t_1) = f(\tau, x - a(t - \tau))$

代入齐次化原理的形式解.  $u_2 = \int_0^t f(\tau, x - a(t - \tau)) d\tau$

$\therefore u = \varphi(x - at) + \int_0^t f(\tau, x - a(t - \tau)) d\tau$