Name: Haorui Chen   ID: 4454226030

I. repeatable: 15^4; non-repeatable: $\binom{15}{4}$.

II. a) $\binom{k}{m}\binom{100-k}{M-m} / \binom{100}{M}$ b) $\left[\binom{100-k}{M}+\binom{100-k}{M-1}\cdot k\right] / \binom{100}{M}$

III. a) $P[0]=1/3$; $P[1]=1/3$; $P[2]=1/3$ b) $P(i=0)=\varepsilon$, $P(i=1)=1-\varepsilon$, $P(i=2)=0$

IV. When the amount of black & white balls is the same in both urn A and urn B.

V. $P[A\cup B]P[C]=(P[A]+P[B]-P[AB])P[C]=P[AC]+P[BC]-P[ABC]=P[AC]+P[BC]-P[ACBC]$
$=P[AC\cup BC]=P[(A\cup B)C]$ $\therefore (A\cup B)$ and C are independent.

VI. a) $P[\text{Win}]=P[\text{We play team 1, Win}]+P[\text{We play team 2, Win}]+P[\text{We play team 3, Win}]=7/24$

b) $P[\text{We play team 3} \mid \text{Win}]=P[\text{We play team 3, Win}]/P[\text{Win}]=1/7$.

c) $P[\text{We play either team 1 or 2} \mid \text{Win}]=1-P[\text{We play team 3} \mid \text{Win}]=6/7$.

d) $P[\text{We play either team 1 or 2} \mid \text{Loose}]=\left(P[\text{We play team 1, Loose}]+P[\text{We play team 2, Loose}]\right)/P[\text{Loose}]$
$=10/17$

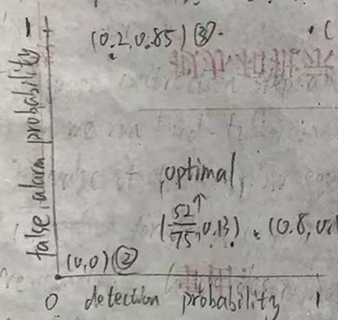
e) $P[\text{We play team 3} \mid \text{Win} \cup \{\text{We play team 2}\}]=\dfrac{P[\text{We play team 3, Win}]+P[\text{We play team 3, We play team 2}]}{P[\text{Win} \cup \{\text{We play team 2}\}]}$
$=(1/24+0)/(P[\text{Win}]+P[\text{We play team 2, Loose}])=$
$=\frac{1}{24}/(7/24+1/4)=1/13$.

I, II, III, IV, V, VI

IX. a). $P[A\cup B\cup C]=P[A]+P[B]+P[C]-P[AB]-P[AC]-P[BC]+P[ABC]$.
$=\frac{1}{4}+\frac{1}{8}+\frac{1}{16}-\frac{1}{32}-\frac{1}{64}-\frac{1}{128}+P[ABC]$
$\because 0\le P[ABC]\le \max\{P[AB], P[AC], P[BC]\}=\frac{1}{32}$ $\therefore \frac{49}{128}\le P[A\cup B\cup C]\le\frac{53}{128}$.

b). Let $P[C\mid AB]=1$, then $P[ABC]=P[C\mid AB]\cdot P[AB]=\frac{1}{32}$.

XI. a). 4 functions. ① $Y=1$; ② $Y=0$; ③ $X=1\to Y=1$; $X=0\to Y=0$; ④ $X=0\to Y=1$, $X=1\to Y=0$.



b) linear programming:

$$\begin{cases} P_1+P_2+P_3+P_4=1; \\ P_1+0.85P_3+0.15P_4\le 0.13; \\ \text{maximize } P_1+0.2P_3+0.8P_4 \end{cases} \Rightarrow \begin{cases} P_1=0, \\ P_2=\frac{2}{15}, \\ P_3=0, \\ P_4=\frac{13}{15} \end{cases}$$

$\therefore$ Optimal $P_{detect}^{new}=\frac{52}{75}$.

IX, XI

VII. a) $P[C] = P[MD^c, C] + P[MD, C] = 15/32$.

b) $P[R] = P[MD, R] + P[M^cD, R] = 5/32$, $P[O] = P[M^cD, O] + P[MD, O] + P[MD^c, O] + P[M^cD^c, O] = 12/32$

c) $P[D|C] = \dfrac{P[D, C]}{P[C]} = \dfrac{P[MD, C] + P[M^cD, C]}{P[C]} = \dfrac{3/16 + 0}{15/32} = \dfrac{2}{5}$.

d) $P[M, C] = P[C] - P[M^c, C] = P[C] - P[M^cD, C] - P[M^cD^c, C] = 15/32 - 0 - 0 = 15/32$

e) $P[O | M^cD] = P[M^cD, O] / P[M^cD] = 1/2$.

VIII. a) $P[L = 100] = P[L_A = 100] + P[L_B = 100] + P[L_C = 100] = \left(1/200 + \frac{1}{100} \cdot \left(\frac{99}{100}\right)^{99}\right) \times \frac{1}{3}$

b) $P[L \geq 100] = P[L_A \geq 100] + (1 - P[L_B < 100]) + P[L_C \geq 100] = \left[101/200 + (99/100)^{99} + 1/2\right] \times 1/3$

c) $P[\text{Type A} \mid L \geq 100] = 101/200 \Big/ \left[201/200 + (99/100)^{99}\right]$

d) $P[L \geq 100 \mid (A \cup B)] = (P[\text{Type A}, L \geq 100] + P[\text{Type B}, L \geq 100]) / P[A \cup B] = \left[101/200 + (99/100)^{99}\right] \times \frac{1}{2}$

e) $P[\text{Type A} \mid L = 50] = P[\text{Type A}, L = 50] / P[L = 50] = 1/200 \times 1/3 \Big/ \left(1/200 + \frac{1}{100}\left(\frac{99}{100}\right)^{49} + 1/2\right) \times 1/3$

X. Suppose: $P(A) = 0.34$, $P(B) = 0.50$, $P(A \cup B) = 0.70$,

What is $P(A|B)$?

$$P(A|B) = \frac{P(AB)}{P(B)} = \frac{P(A) + P(B) - P(A \cup B)}{P(B)} = \frac{14}{50}.$$

VII, VIII, X