I. a) $E[XY] = \int_{-\infty}^{+\infty} E[XY \mid X=x] f_X(x)\,dx = \int_{-\infty}^{+\infty} E[xY] f_X(x)\,dx = \int_{-\infty}^{+\infty} E[Y]\,x f_X(x)\,dx$

$= \int_{-\infty}^{+\infty} x^2 f_X(x)\,dx = \int_0^1 2x^3\,dx = \frac{1}{2}$

b) $E[XZ] = \int_{-\infty}^{+\infty} E[XZ \mid X=x] f_X(x)\,dx = \int_{-\infty}^{+\infty} E[Z]\,x f_X(x)\,dx = \int_0^1 \frac{1}{x}\,x f_X(x)\,dx = 1$

II. a) Let $I_k = \begin{cases} 1, & \text{if k-th pile of 3 cards forms a match,} \\ 0, & \text{otherwise.} \end{cases}$ $E[I_k] = (\frac{1}{52})^2$

$E[M] = E[\sum_{k=1}^{52} I_k] = \sum_{k=1}^{52} E[I_k] = 52 \cdot (\frac{1}{52})^2 = \frac{1}{52}$

b) Let $J_k = \begin{cases} 1, & \text{if k-th pile of neiboring cards form a match,} \\ 0, & \text{otherwise.} \end{cases}$ $E[J_k] = P[J_k = 1] = \frac{103}{52^2}$

$E[N] = E[\sum_{k=1}^{52} J_k] = \sum_{k=1}^{52} E[J_k] = 52 \cdot \frac{103}{52^2} = \frac{103}{52}$

III. a) $E[X^2] = Var(X) + (E[X])^2 = \frac{2}{\lambda^2}$ $\quad E[Y^2] = E[a^2X^2] = a^2E[X^2] = \frac{2a^2}{\lambda^2}$

I, II

III. a) $E[X^2] = Var(X) + (E[X])^2 = \frac{2}{\lambda^2}$ $\quad E[Y^2] = E[a^2X^2] = a^2E[X^2] = \frac{2a^2}{\lambda^2}$

$E[XY] = E[aX^2] = aE[X^2] = \frac{2a}{\lambda^2}$ $\quad \sqrt{E[X^2]E[Y^2]} - |E[XY]| = 0$

b) $f_{XY}(x,y) = \begin{cases} 2, & (x,y) \text{ in triangle} \\ 0, & \text{otherwise} \end{cases}$ $\quad f_X(x) = \int_{-\infty}^{+\infty} f_{X,Y}(x,y)\,dy = \int_0^{1-x} 2\,dy = 2(1-x)$

$f_Y(y) = \int_{-\infty}^{+\infty} f_{X,Y}(x,y)\,dx = \int_0^{1-y} 2\,dx = 2(1-y)$

$E[X^2] = \int_{-\infty}^{+\infty} x^2 f_X(x)\,dx = \int_0^1 x^2 \cdot 2(1-x)\,dx = \frac{1}{6}$ $\quad E[Y^2] = \frac{1}{6}$

$E[XY] = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} xy\, f_{X,Y}(x,y)\,dx\,dy = \int_0^1 x \int_0^{1-x} y \cdot 2\,dy\,dx = \int_0^1 (x^3 - 2x^2 + x)\,dx = \frac{1}{12}$

$\sqrt{E[X^2]E[Y^2]} - |E[XY]| = \frac{1}{12}$

c) $P[X=0] = \frac{2}{3}$, $P[X=1] = \frac{1}{3}$, $P[Y=0] = \frac{2}{3}$, $P[Y=1] = \frac{1}{3}$

$E[X^2] = 0 \cdot P[X=0] + 1 \cdot P[X=1] = \frac{1}{3}$, $\quad E[Y^2] = \frac{1}{3}$ $\quad E[XY] = 0$

$\sqrt{E[X^2]E[Y^2]} - |E[XY]| = \frac{1}{3}$

III