

Test 1 Corrections

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1 Problem 4

Begin with an expression of the cosine angle addition identity

$$\cos(2x + 8x) = \cos(2x)\cos(8x) - \sin(2x)\sin(8x)$$

One could notice that through clever use of the cosine angle subtraction identity ($\cos(2x - 8x) = \cos(2x)\cos(8x) + \sin(2x)\sin(8x)$) the cosine terms on the right hand side could be eliminated, as follows:

$$\cos(2x+8x)-\cos(2x-8x) = \cos(2x)\cos(8x)-\sin(2x)\sin(8x)-[\cos(2x)\cos(8x)+\sin(2x)\sin(8x)]$$

Simplifying yields

$$\begin{aligned}\cos(2x + 8x) - \cos(2x - 8x) &= -2\sin(2x)\sin(8x) \\ \frac{-\cos(2x + 8x) + \cos(2x - 8x)}{2} &= \sin(2x)\sin(8x)\end{aligned}$$

2 Problem 5

$$\int \sin^{11} x \cos^6 x \, dx$$

$$\int \sin x (1 - \cos^2 x)^5 \cos^6 x \, dx$$

$$u = \cos x$$

$$du = -\sin x \, dx$$

$$-\int (1 - u^2)^5 u^6 \, du$$

$$-\int (1 - 5u^2 + 10u^4 - 10u^6 + 5u^8 - u^{10}) u^6 \, du$$

$$-\int (u^6 - 5u^8 + 10u^{10} - 10u^{12} + 5u^{14} - u^{16}) \, du$$

$$-\left(\frac{u^7}{7} - \frac{5u^9}{9} + \frac{10u^{11}}{11} - \frac{10u^{13}}{13} + \frac{u^{15}}{3} - \frac{u^{17}}{17}\right) + C$$

$$-\frac{u^7}{7} + \frac{5u^9}{9} - \frac{10u^{11}}{11} + \frac{10u^{13}}{13} - \frac{u^{15}}{3} + \frac{u^{17}}{17} + C$$

$$-\frac{\cos^7 x}{7} + \frac{5\cos^9 x}{9} - \frac{10\cos^{11} x}{11} + \frac{10\cos^{13} x}{13} - \frac{\cos^{15} x}{3} + \frac{\cos^{17} x}{17} + C$$

3 Problem 7

$$\frac{a}{2x-3} + \frac{b}{x+2} = \frac{3x-5}{(2x-3)(x+2)}$$

$$ax + 2a + 2bx - 3b = 3x - 5$$

Substitute -2 for x

$$-7b = -5 - 6$$

$$b = \frac{11}{7}$$

Substitute $\frac{3}{2}$ for x

$$\frac{7}{2}a = \frac{9}{2} - 5$$

$$\frac{7}{2}a = -\frac{1}{2}$$

$$a = -\frac{1}{7}$$

Plug in and integrate

$$\int \frac{-1}{7(2x-3)} dx + \int \frac{11}{7(x+2)} dx$$

$$\frac{-\ln|2x-3|}{14} + \frac{-11\ln|x+2|}{7} + C$$