An Analytical Evaluation of the Integration Techniques Advanced Placement Calculus BC Assessment

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October 2018

1 Introduction

This paper aims to address and rectify various discrepancies found in the answers given on the recent Integration Techniques exam.
Following will be an itemized series of explanations and derivations to help resolve disparities which resulted in a non-optimal score yield.

2 Problem 4

The problem concerns the derivation of an expression equivalent to $sin(2x)cos(8x)$ from the trigonometric sum and difference formulas
We begin with an expression of the cosine angle addition identity

$$cos(2x + 8x) = cos(2x)cos(8x) - sin(2x)sin(8x)$$

One could notice that through clever use of the cosine angle subtraction identity ($cos(2x - 8x) = cos(2x)cos(8x) + sin(2x)sin(8x)$) the cosine terms on the right hand side could be eliminated, as follows:

$$cos(2x+8x)-cos(2x-8x) = cos(2x)cos(8x)-sin(2x)sin(8x)-[cos(2x)cos(8x)+sin(2x)sin(8x)]$$

Simplifying yields

$$cos(2x + 8x) - cos(2x - 8x) = -2sin(2x)sin(8x)$$

From here it is plain that one can reach the target expression by dividing both sides by -2

$$\frac{-cos(2x + 8x) + cos(2x - 8x)}{2} = sin(2x)sin(8x)$$

3 Problem 5

$$\int sin^{11}x\ cos^6x\ dx$$

$$\int sinx\ (1-cos^2x)^5\ cos^6x\ dx$$

$$u = cosx$$

$$du = -sinx\ dx$$

$$-\int (1-u^2)^5\ u^6\ du$$

$$-\int (1-5u^2+10u^4-10u^6+5u^8-u^{10})\ u^6\ du$$

$$-\int (u^6-5u^8+10u^{10}-10u^{12}+5u^{14}-u^{16})\ du$$

$$-(\frac{u^7}{7}-\frac{5u^9}{9}+\frac{10u^{11}}{11}-\frac{10u^{13}}{13}+\frac{u^{15}}{3}-\frac{u^{17}}{17})+C$$

$$-\frac{u^7}{7}+\frac{5u^9}{9}-\frac{10u^{11}}{11}+\frac{10u^{13}}{13}-\frac{u^{15}}{3}+\frac{u^{17}}{17}+C$$

$$-\frac{cos^7x}{7}+\frac{5cos^9x}{9}-\frac{10cos^{11}x}{11}+\frac{10cos^{13}x}{13}-\frac{cos^{15}x}{3}+\frac{cos^{17}x}{17}+C$$

4 Problem 7

$$\frac{a}{2x-3}+\frac{b}{x+2}=\frac{3x-5}{(2x-3)(x+2)}$$

$$ax+2a+2bx-3b=3x-5$$

Substitute -2 for x

$$-7b=-5-6$$

$$b=\frac{11}{7}$$

Substitute $\frac{3}{2}$ for x

$$\frac{7}{2}a=\frac{9}{2}-5$$

$$\frac{7}{2}a=-\frac{1}{2}$$

$$a=-\frac{1}{7}$$

Plug in and integrate

$$\int\frac{-1}{7(2x-3)}dx+\int\frac{11}{7(x+2)}dx$$

$$\frac{-ln|2x-3|}{14}+\frac{-11ln|x+2|}{7}+C$$