Some Applied Mathematics: Series, Banking, and Loans

Matthew Zhang

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Interest Bearing Accounts

(a)
$S_0 = 10000$
$S_1 = S_0(1.05)$
$S_2 = S_1(1.05)$
$S_3 = S_2(1.05)$
$S_4 = S_3(1.05)$
$S_5 = S_4(1.05)$
$S_n = S_{n-1}(1.05)$

(b)
$S_n = S_{n-1}(1.05)$
$S_n = 10000(1.05)^n$
$S_n = 10000(1 + 0.05)^n$
Expand $(1 + 0.05)^n$ using Binomial Theorem

$$S_n = 10000 \sum_{k=0}^{n} \binom{n}{k} 1^{k-n} + 0.05^n$$

Remove 1^{k-n} term

$$S_n = 10000 \sum_{k=0}^{n} \binom{n}{k} 0.05^n$$

(c)
From part (a) $S_n = S_{n-1}(1.05)$

Plugging in S_{n-1}
$S_n = S_{n-2}(1.05)(1.05)$
$S_n = S_{n-2}(1.05)^2$

Continuing yields
$S_n = S_{n-3}(1.05)^3$
$S_n = S_{n-4}(1.05)^4$
$S_n = S_{n-n}(1.05)^n$
$S_n = S_0(1.05)^n$
$S_n = 10000(1.05)^n$

Loans

Fractional Reserve Banking