(

)

. (x , y) - >
(x + a * y , y) , x
.

```
> points := [[30, 160], [30, 30], [60, 50]];
```

$$points := [[30, 160], [30, 30], [60, 50]] \quad (1.1)$$

,

x , a
.

```
> tracks := {op(map(p→p[1]+p[2]·a, points))};
  dists := { }:
  for i from 1 to nops(points) − 1 do
    p := points[i]:
    for j from i + 1 to nops(points) do
      q := points[j]:
      dists := {op(dists), abs(p[1]+p[2]·a − (q[1]+q[2]·a))}:
    end do:
  end do:
  dists;
```

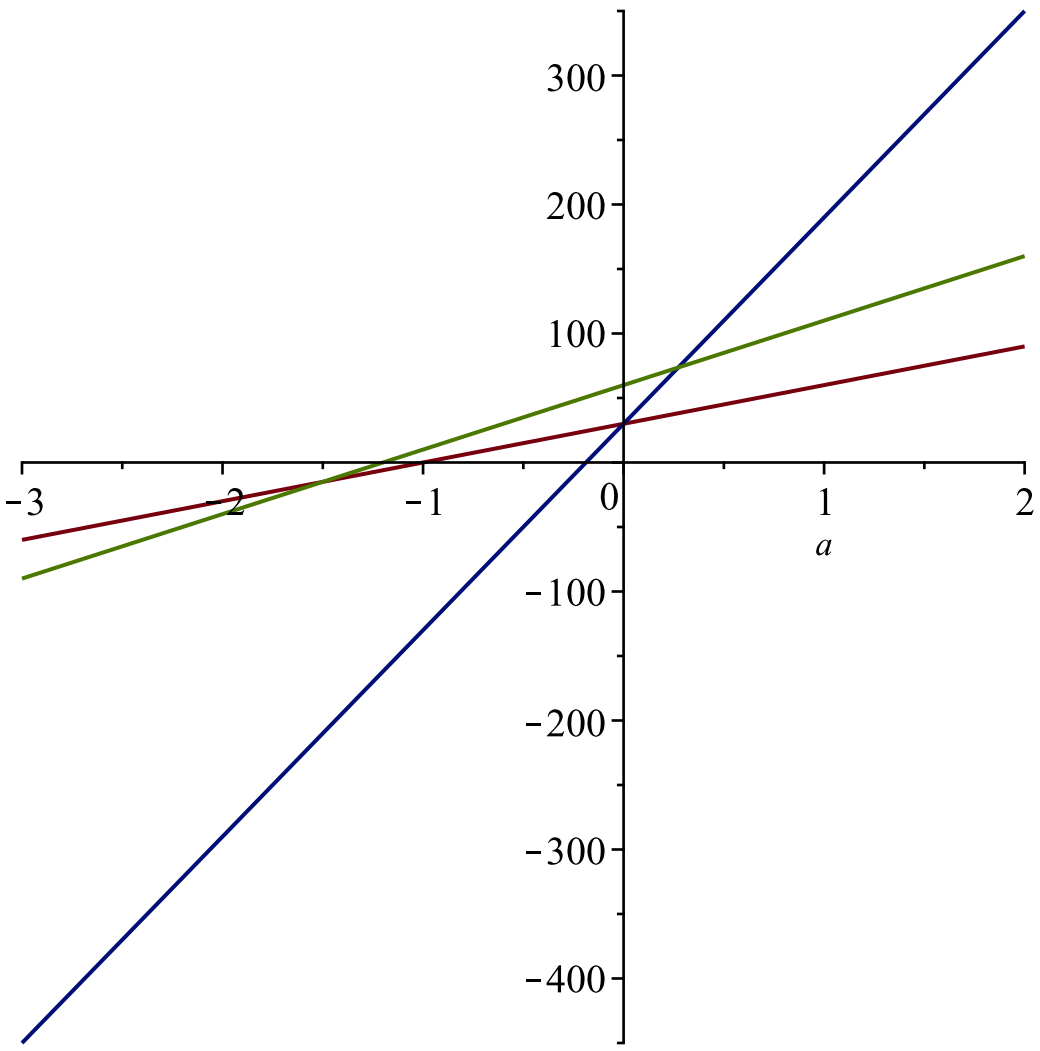
$$tracks := \{30 + 30\,a, 30 + 160\,a, 60 + 50\,a\}$$

$$\{130\,|a|, 10\,|-3 + 11\,a|, 10\,|3 + 2\,a|\} \quad (1.2)$$

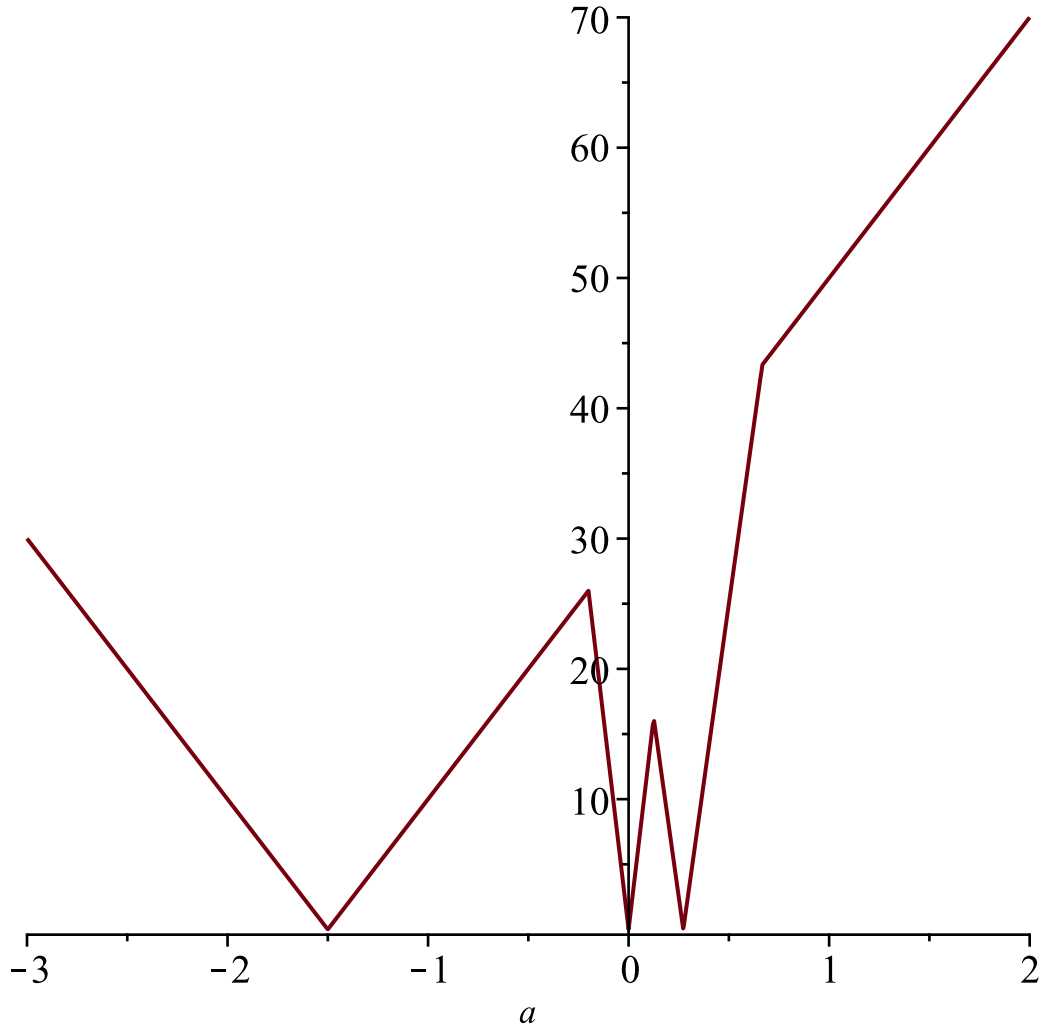
-

- x ,
(x , y) - > (x + a * y , y)

```
> plot(tracks, a =-3 ..2)
```




```
> plot(min(abs(130·a), abs(-30 + 110·a), abs(-30 - 20·a)), a = -3..2)
```



, -
p = (x , y) - > (p . x + a * p . y , p . y)
x ,
.

, ,
p=(x, y) -> (min(r.x)+(max(r.x)-min(r.x))*(p.x+a*p.y-min(r.x+a*r.y))/(max(r.x+a*r.y)
-min(r.x+a*r.y)), y)

m i n , m a x r
p ( ) .

" " - d (p , q) (a) x p , q

d(p,q)(a) = abs(p.x-q.x) = abs(
min(r.x)+(max(r.x)-min(r.x))*(p.x+a*p.y-min(r.x+a*r.y))/(max(r.x+a*r.y)-min(r.x+a*r.y))
-
min(r.x)+(max(r.x)-min(r.x))*(q.x+a*q.y-min(r.x+a*r.y))/(max(r.x+a*r.y)-min(r.x+a*r.y))
)

d(p,q)(a) = abs(p.x-q.x) = abs(
(max(r.x)-min(r.x))*(p.x+a*p.y-min(r.x+a*r.y))/(max(r.x+a*r.y)-min(r.x+a*r.y))
-
(max(r.x)-min(r.x))*(q.x+a*q.y-min(r.x+a*r.y))/(max(r.x+a*r.y)-min(r.x+a*r.y))
)
= (max(r.x)-min(r.x)) * abs(
(p.x+a*p.y-min(r.x+a*r.y))
-
(q.x+a*q.y-min(r.x+a*r.y))
)
/(max(r.x+a*r.y)-min(r.x+a*r.y))
= (max(r.x)-min(r.x)) * abs(
p.x+a*p.y
-
(q.x+a*q.y)
)
/(max(r.x+a*r.y)-min(r.x+a*r.y))
= (max(r.x)-min(r.x)) * abs(p.x+a*p.y-(q.x+a*q.y)) / (max(r.x+a*r.y)-min(r.x+a*r.y))

,
d(p,q)(a) = (max(r.x)-min(r.x)) * abs(p.x+a*p.y-(q.x+a*q.y)) / (max(r.x+a*r.y)-min(r.x+a*r.y))

p , q
.

d(a) = min(d(p,q)(a)) = min((max(r.x)-min(r.x)) * abs(p.x+a*p.y-(q.x+a*q.y)) / (max(r.x+a*r.y)-min(r.x+a*r.y)))
= (max(r.x)-min(r.x)) * min(abs(p.x+a*p.y-(q.x+a*q.y))) / (max(r.x+a*r.y)-min(r.x+a*r.y))

, . .
.

- ,
.

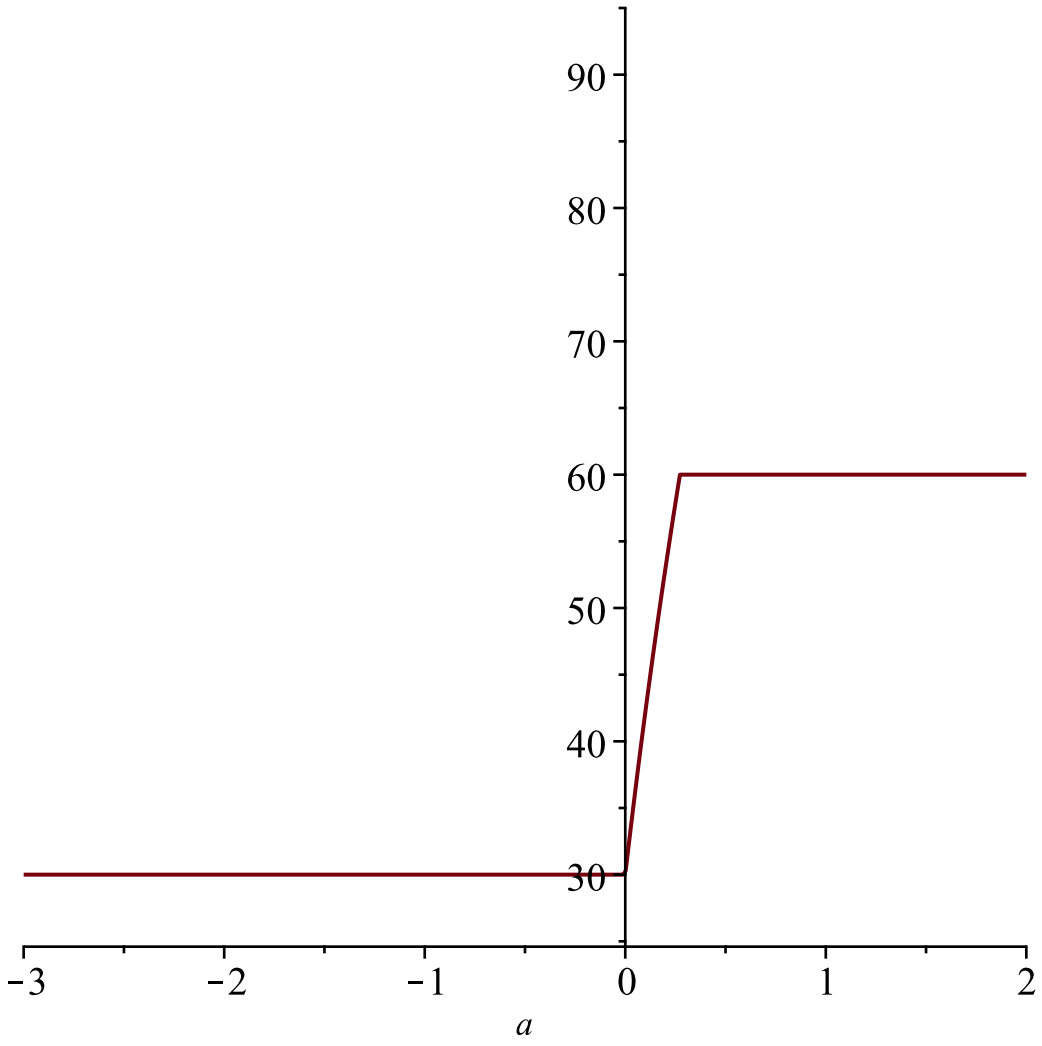
m i n
min(abs(p.x+a*p.y-(q.x+a*q.y)))
- ,

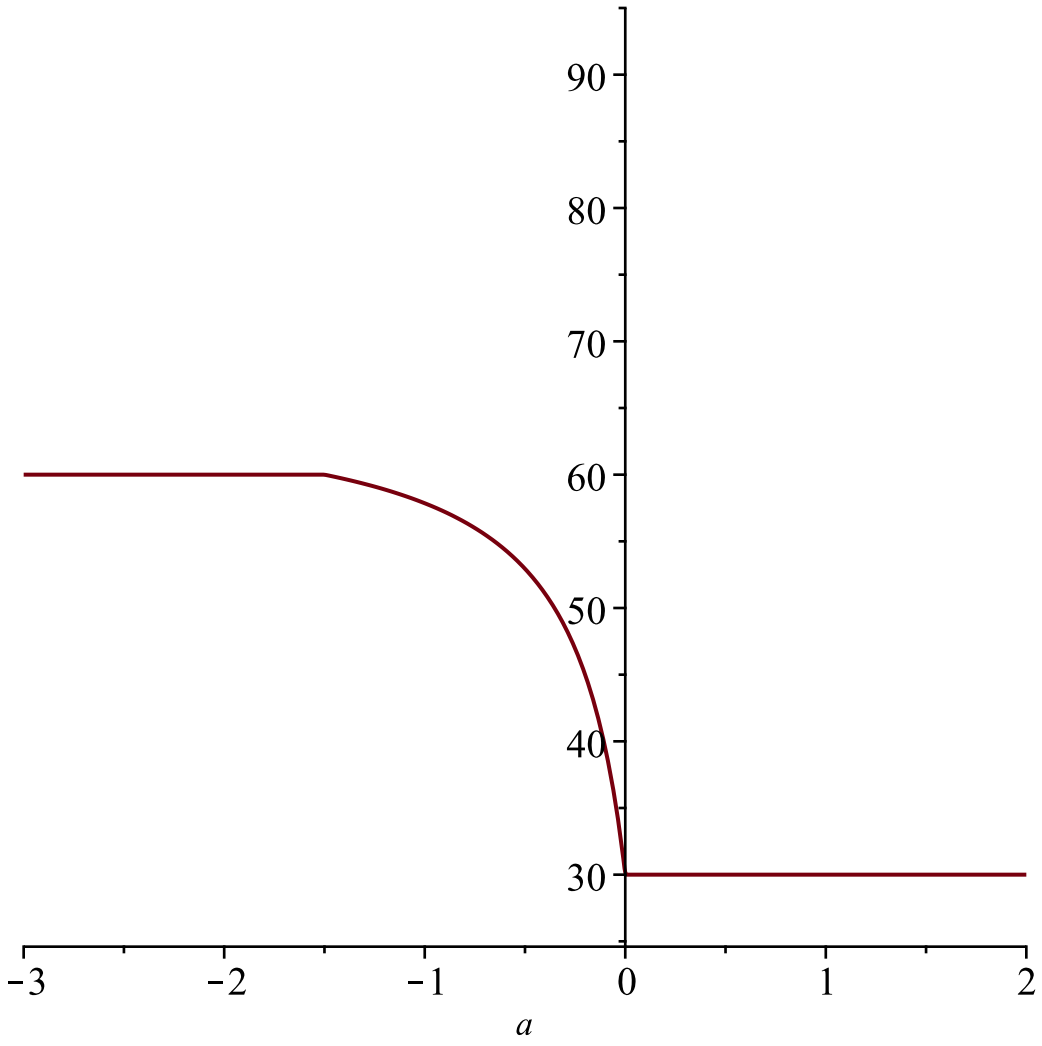
m a x (
r)
max(r.x+a*r.y)
- ,

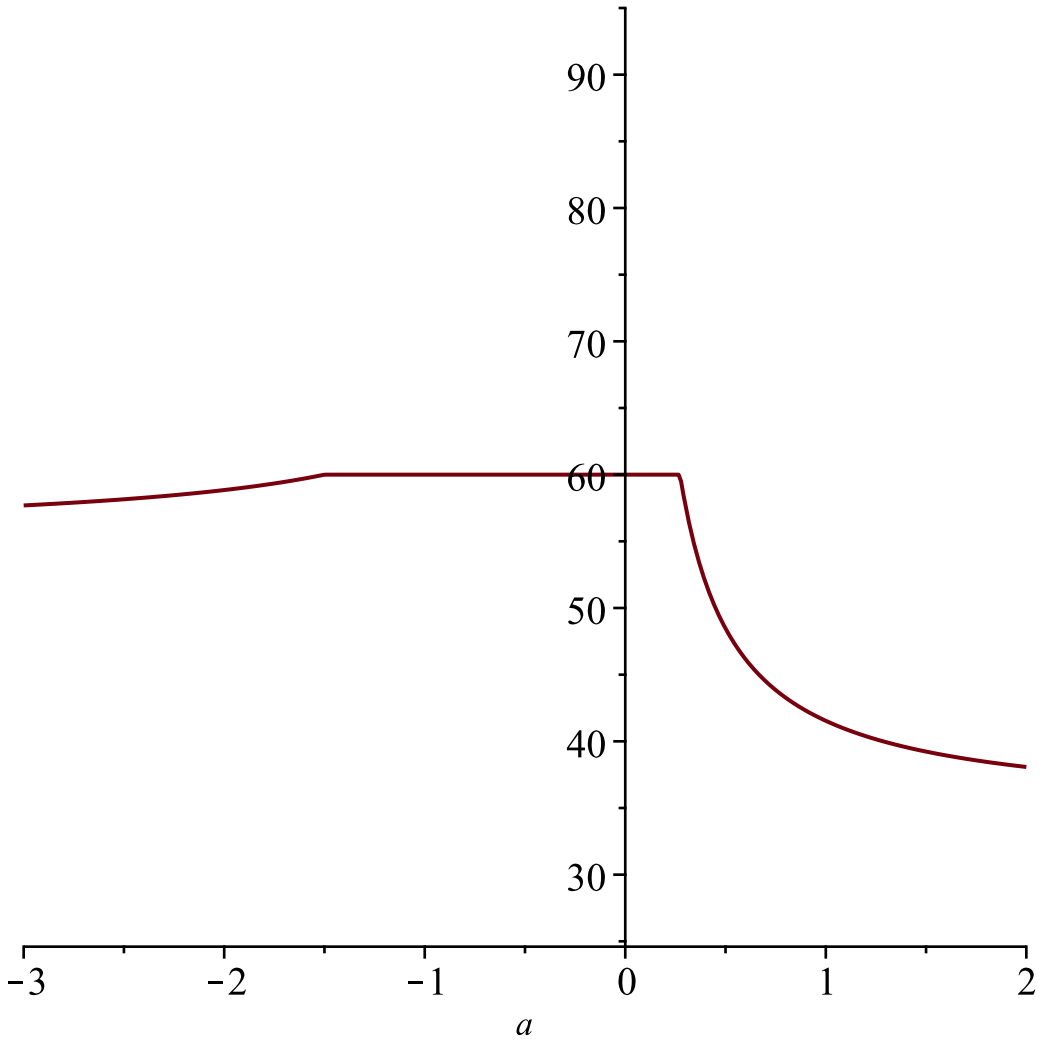
m i n (r)
max(r.x+a*r.y)
- ,

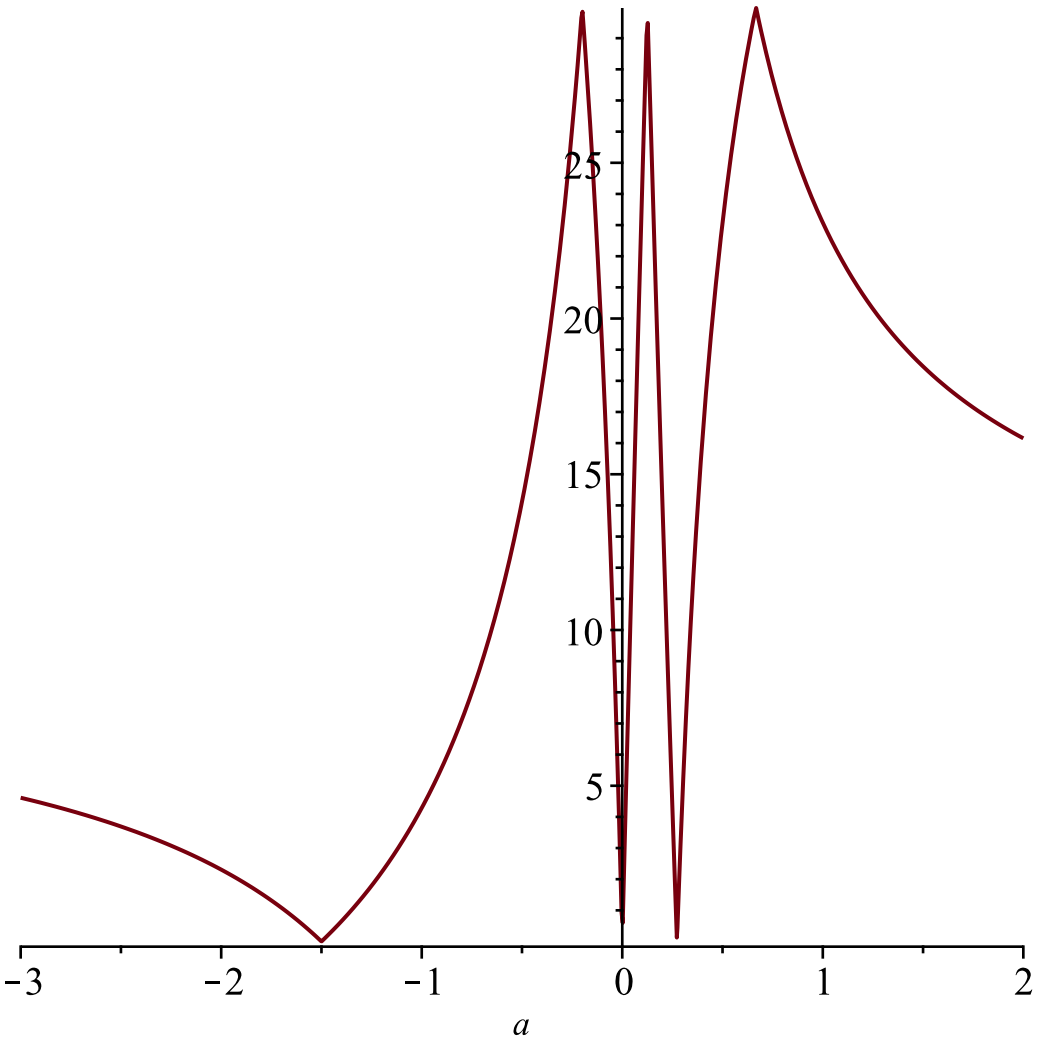
.

```
> plot(min(30, 30, 60) + ((max(30, 30, 60) - min(30, 30, 60))·(30 + 160·a - min(30
      + 160·a, 30 + 30·a, 60 + 50·a)))/(max(30 + 160·a, 30 + 30·a, 60 + 50·a)
      - min(30 + 160·a, 30 + 30·a, 60 + 50·a)), a = -3..2, 25..95);
  plot(min(30, 30, 60) + ((max(30, 30, 60) - min(30, 30, 60))·(30 + 30·a - min(30 + 160
      ·a, 30 + 30·a, 60 + 50·a)))/(max(30 + 160·a, 30 + 30·a, 60 + 50·a) - min(30
      + 160·a, 30 + 30·a, 60 + 50·a)), a = -3..2, 25..95);
  plot(min(30, 30, 60) + ((max(30, 30, 60) - min(30, 30, 60))·(60 + 50·a - min(30 + 160
      ·a, 30 + 30·a, 60 + 50·a)))/(max(30 + 160·a, 30 + 30·a, 60 + 50·a) - min(30
      + 160·a, 30 + 30·a, 60 + 50·a)), a = -3..2, 25..95);
  plot((max(30, 30, 60)·min(abs(30 + 160·a - (30 + 30·a)), abs(30 + 30·a - (60 + 50
      ·a)), abs(60 + 50·a - (30 + 160·a))))/(max(30 + 160·a, 30 + 30·a, 60 + 50·a)
      - min(30 + 160·a, 30 + 30·a, 60 + 50·a)), a = -3..2);
```

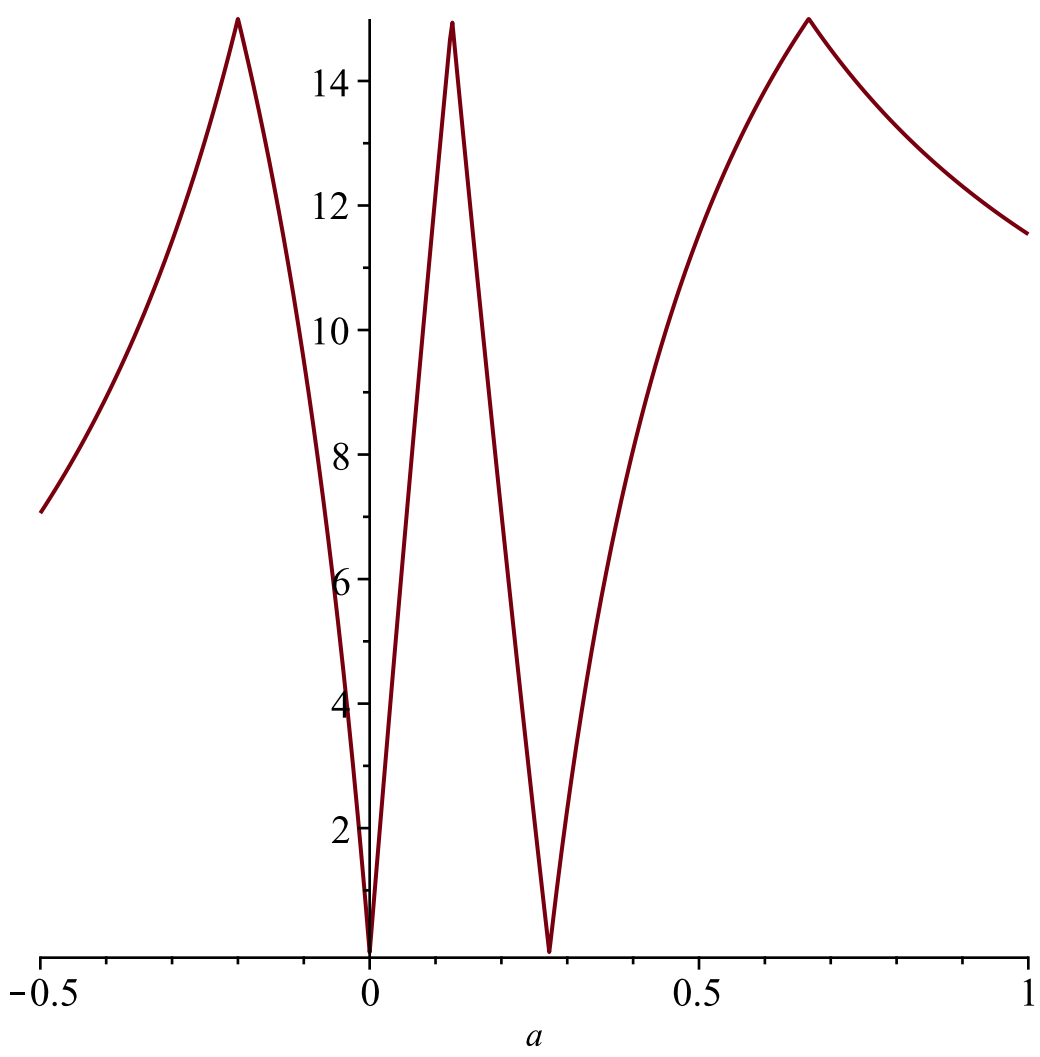

90
80
70
60
50
40
30
−3
−2
−1
0
1
2
a


90
80
70
60
50
40
30
−3
−2
−1
0
1
2
a


90
80
70
60
50
40
30
−3
−2
−1
0
1
2
a




```
> plot(((max(30, 30, 60) − min(30, 30, 60)) · min(abs(30 + 160·a − (30 + 30·a)), abs(30
  + 30·a − (60 + 50·a)), abs(60 + 50·a − (30 + 160·a)))) / (max(30 + 160·a, 30
  + 30·a, 60 + 50·a) − min(30 + 160·a, 30 + 30·a, 60 + 50·a)), a = -1/2 ..1)
```




```
> limit(((max(30, 30, 60) - min(30, 30, 60))·min(abs(30 + 160·a - (30 + 30·a)), abs(30
      + 30·a - (60 + 50·a)), abs(60 + 50·a - (30 + 160·a))))/(max(30 + 160·a, 30
      + 30·a, 60 + 50·a) - min(30 + 160·a, 30 + 30·a, 60 + 50·a)), a = -infinity)
```

$$\frac{60}{13} \tag{2.1}$$

```
> limit(((max(30, 30, 60) - min(30, 30, 60))·min(abs(30 + 160·a - (30 + 30·a)), abs(30
      + 30·a - (60 + 50·a)), abs(60 + 50·a - (30 + 160·a))))/(max(30 + 160·a, 30
      + 30·a, 60 + 50·a) - min(30 + 160·a, 30 + 30·a, 60 + 50·a)), a = infinity)
```

$$\frac{60}{13} \tag{2.2}$$

```
> extrema(((max(30, 30, 60) - min(30, 30, 60))·min(abs(30 + 160·a - (30 + 30·a)),
      abs(30 + 30·a - (60 + 50·a)), abs(60 + 50·a - (30 + 160·a))))/(max(30 + 160·a,
      30 + 30·a, 60 + 50·a) - min(30 + 160·a, 30 + 30·a, 60 + 50·a)), a)
```

$$\{0\} \tag{2.3}$$

```
> with(Optimization):
> Maximize(((max(30, 30, 60) - min(30, 30, 60))·min(abs(30 + 160·a - (30 + 30·a)),
      abs(30 + 30·a - (60 + 50·a)), abs(60 + 50·a - (30 + 160·a))))/(max(30 + 160·a,
      30 + 30·a, 60 + 50·a) - min(30 + 160·a, 30 + 30·a, 60 + 50·a)))
```

$$[14.9999999999999947, [a = 0.666666666666667]] \tag{2.4}$$

```
> enlarge := proc(points, minA, maxA)
  local a, d, m, i, p, j, q, f, minX, maxX;
  a := [ ]; d := [ ]; m := [ ];
  for i from 1 to nops(points)
  do
  p := points[i];
  a := [op(a), p[1]];
  m := [op(m), p[1] + p[2]·x];
  for j from i + 1 to nops(points)
  do
  q := points[j];
  d := [op(d), abs(p[1] + p[2]·x - q[1] - q[2]·x)];
  end do;
  end do;
  minX := min(op(a)); maxX := max(op(a));
  f := ((maxX - minX)·min(op(d)))/(max(op(m)) - min(op(m)));
  print(f);
  print(limit(f, x = -infinity));
  print(limit(f, x = infinity));
  print(Maximize(f));
  plot([f, (maxX - minX)/(nops(points) - 1)], x = minA..maxA);
  end proc;
```

```
enlarge := proc(points, minA, maxA)                                    (1)
  local a, d, m, i, p, j, q, f, minX, maxX;
  a := [ ];
  d := [ ];
  m := [ ];
  for i to nops(points) do
    p := points[i];
    a := [op(a), p[1]];
    m := [op(m), x*p[2] + p[1]];
    for j from i + 1 to nops(points) do
      q := points[j]; d := [op(d), abs(x*p[2] - x*q[2] + p[1] - q[1])]
    end do
  end do;
  minX := min(op(a));
  maxX := max(op(a));
  f := (maxX - minX)*min(op(d))/(max(op(m)) - min(op(m)));
  print(f);
  print(limit(f, x = -∞));
  print(limit(f, x = ∞));
  print(Maximize(f));
  plot([f, (maxX - minX)/(nops(points) - 1)], x = minA..maxA)
```

end proc

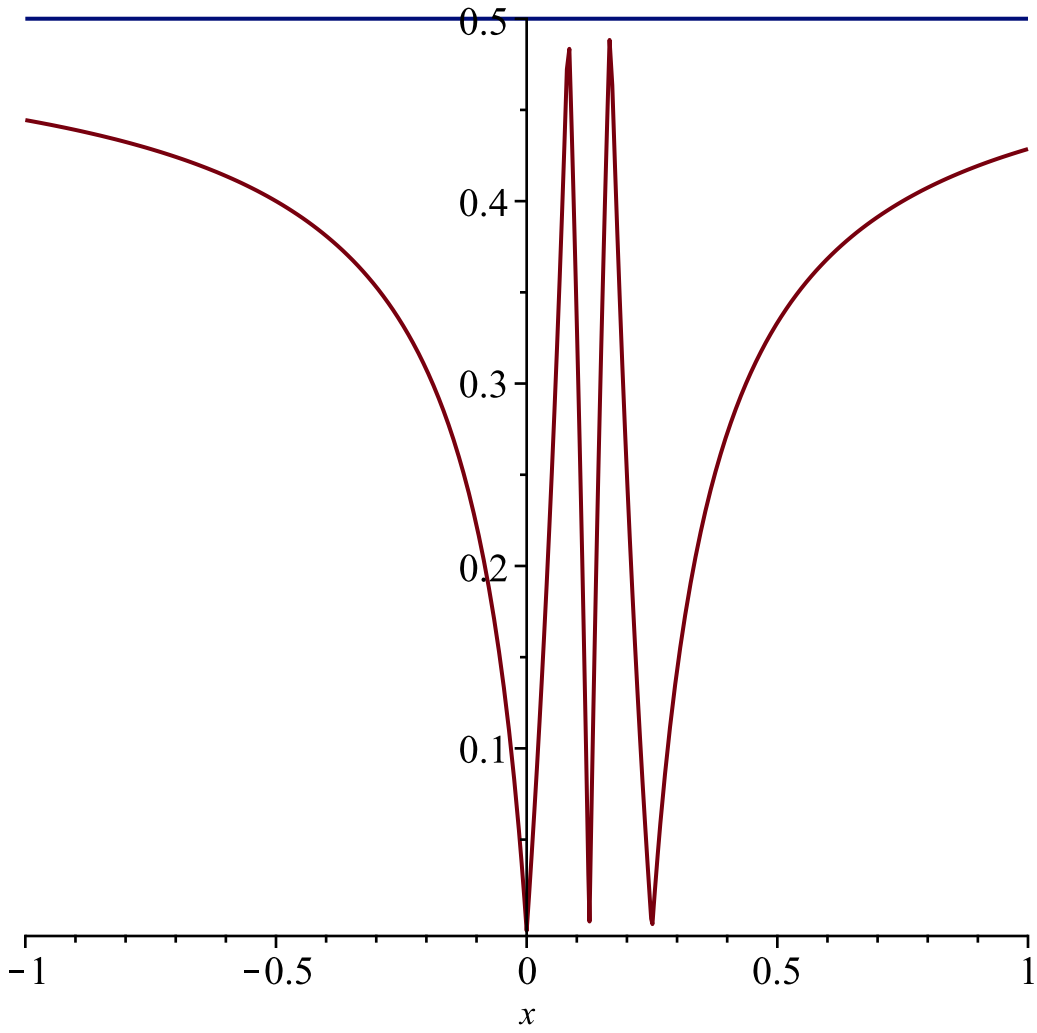
```
> with(Optimization) : enlarge([[2, 1], [1, 5], [2, -3]], -1, 1);
```

$$\frac{\min(4\,|x|,\ |4\,x-1|,\ |8\,x-1|)}{\max(-3\,x+2,\ x+2,\ 5\,x+1)-\min(-3\,x+2,\ x+2,\ 5\,x+1)}$$

$$\frac{1}{2}$$

$$\frac{1}{2}$$

$$\mathit{Maximize}\left(\frac{\min(4\,|x|,\ |4\,x-1|,\ |8\,x-1|)}{\max(-3\,x+2,\ x+2,\ 5\,x+1)-\min(-3\,x+2,\ x+2,\ 5\,x+1)}\right)$$



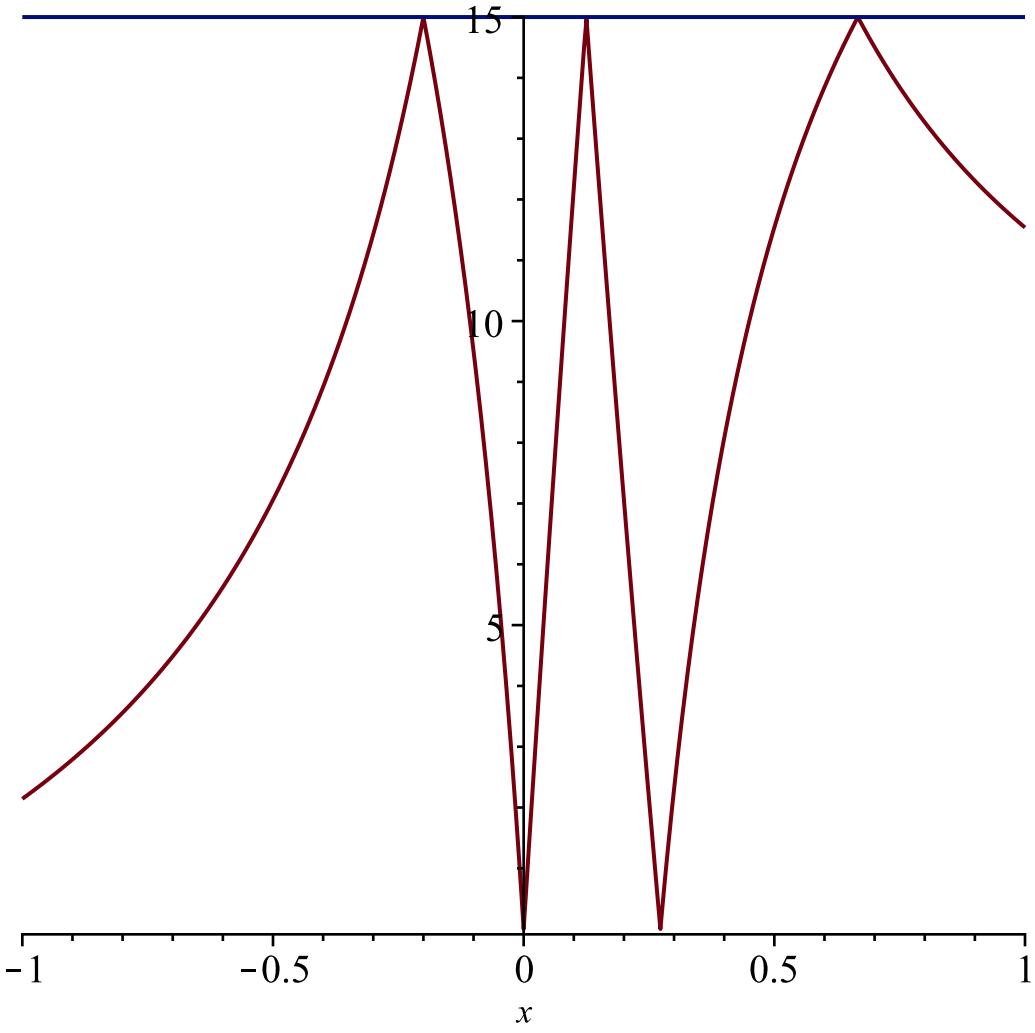

```
> enlarge([[30, 160], [30, 30], [60, 50]], -1, 1);
```

$$\frac{30\,\min(130\,|x|,\ 10\,|2\,x+3|,\ 10\,|11\,x-3|)}{\max(30\,x+30,\ 50\,x+60,\ 160\,x+30)-\min(30\,x+30,\ 50\,x+60,\ 160\,x+30)}$$

$$\frac{60}{13}$$

$$\frac{60}{13}$$

$$\mathit{Maximize}\left(\frac{30\min(130\,|x|,\ 10\,|2x+3|,\ 10\,|11x-3|)}{\max(30x+30,\ 50x+60,\ 160x+30)-\min(30x+30,\ 50x+60,\ 160x+30)}\right)$$



> $\mathit{Maximize}\left(\frac{30\min(130\,|x|,\ 10\,|2x+3|,\ 10\,|11x-3|)}{\max(30x+30,\ 50x+60,\ 160x+30)-\min(30x+30,\ 50x+60,\ 160x+30)}\right)$

$[14.9999999999999947,\ [x = 0.666666666666667]]$ **(2)**

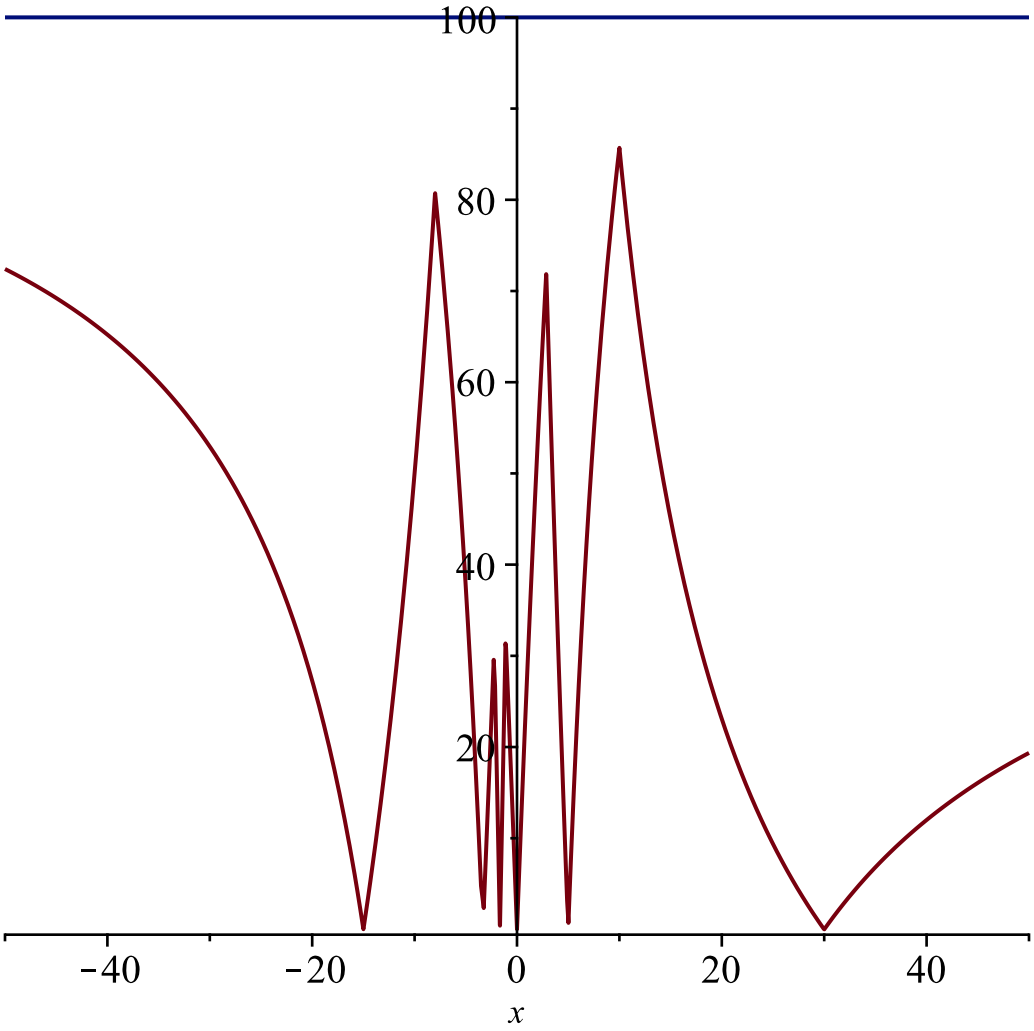
> $\mathit{enlarge}([[100, 60], [100, 30], [400, 50], [200, 90]], -50, 50)$

$(300\min(30\,|x|,\ 10\,|x-30|,\ 40\,|x-5|,\ 20\,|x+15|,\ 20\,|3x+5|,\ 10\,|3x+10|))/(\max(30x+100,\ 50x+400,\ 60x+100,\ 90x+200)-\min(30x+100,\ 50x+400,\ 60x+100,\ 90x+200))$

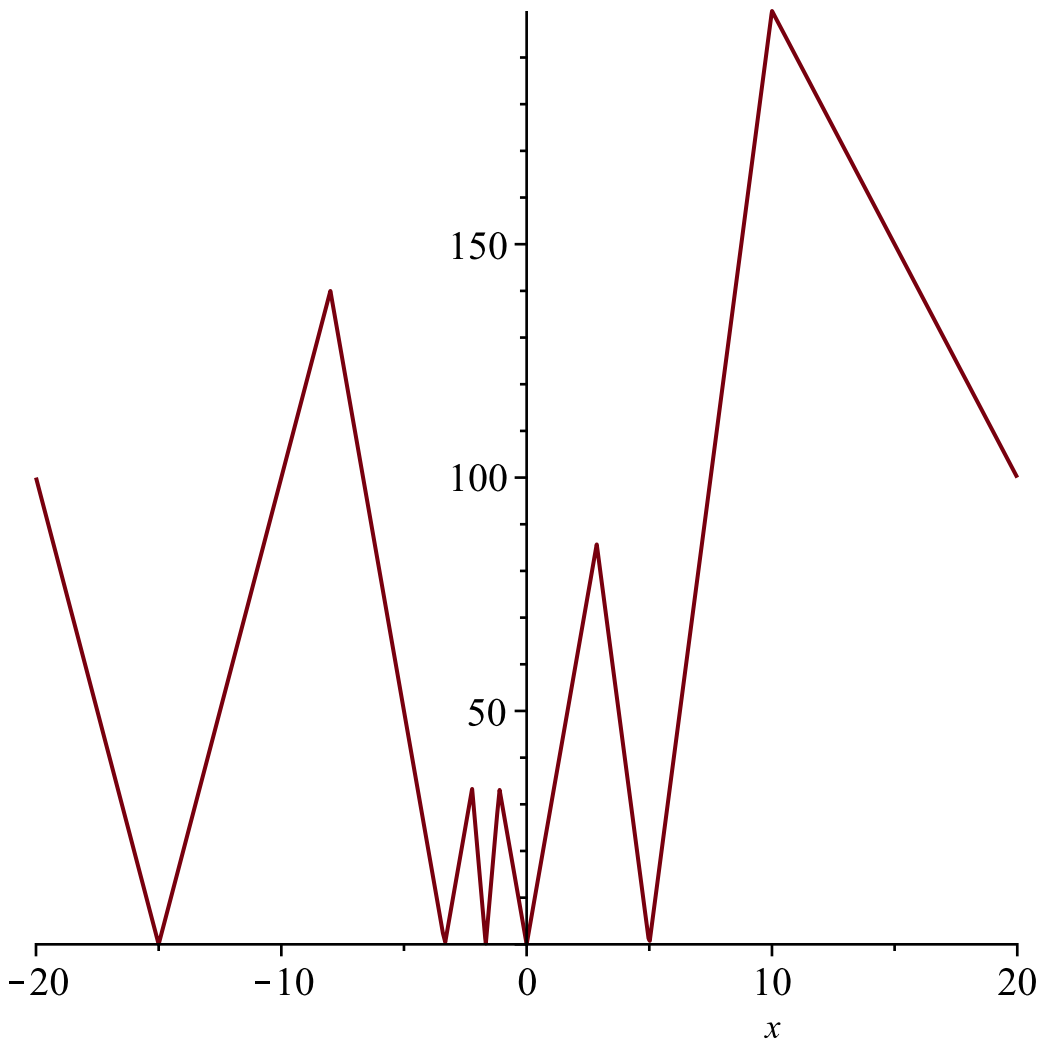
$$50$$

$$50$$

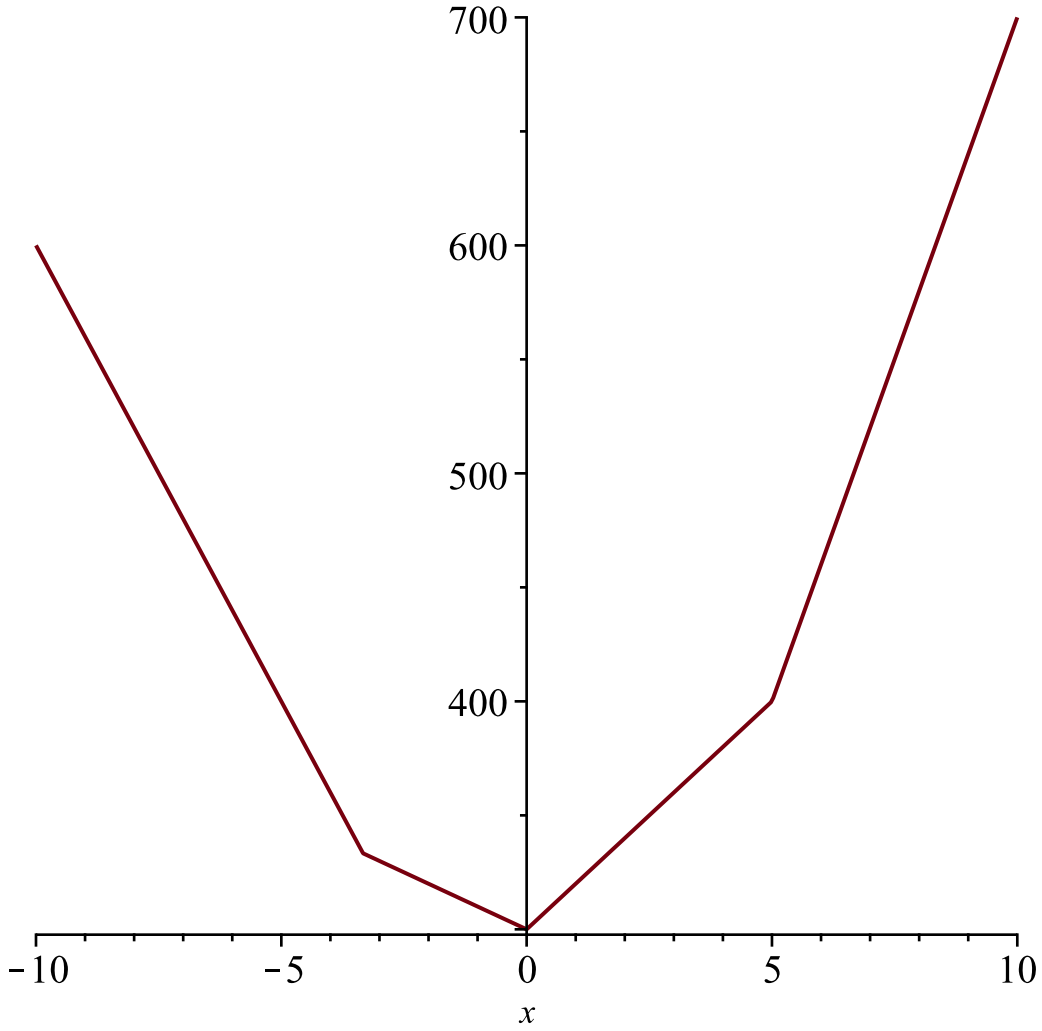
$$\mathit{Maximize}\left((300\min(30\,|x|,\ 10\,|x-30|,\ 40\,|x-5|,\ 20\,|x+15|,\ 20\,|3x+5|,\ 10\,|3x+10|))/(\max(30x+100,\ 50x+400,\ 60x+100,\ 90x+200)-\min(30x+100,\ 50x+400,\ 60x+100,\ 90x+200))\right)$$



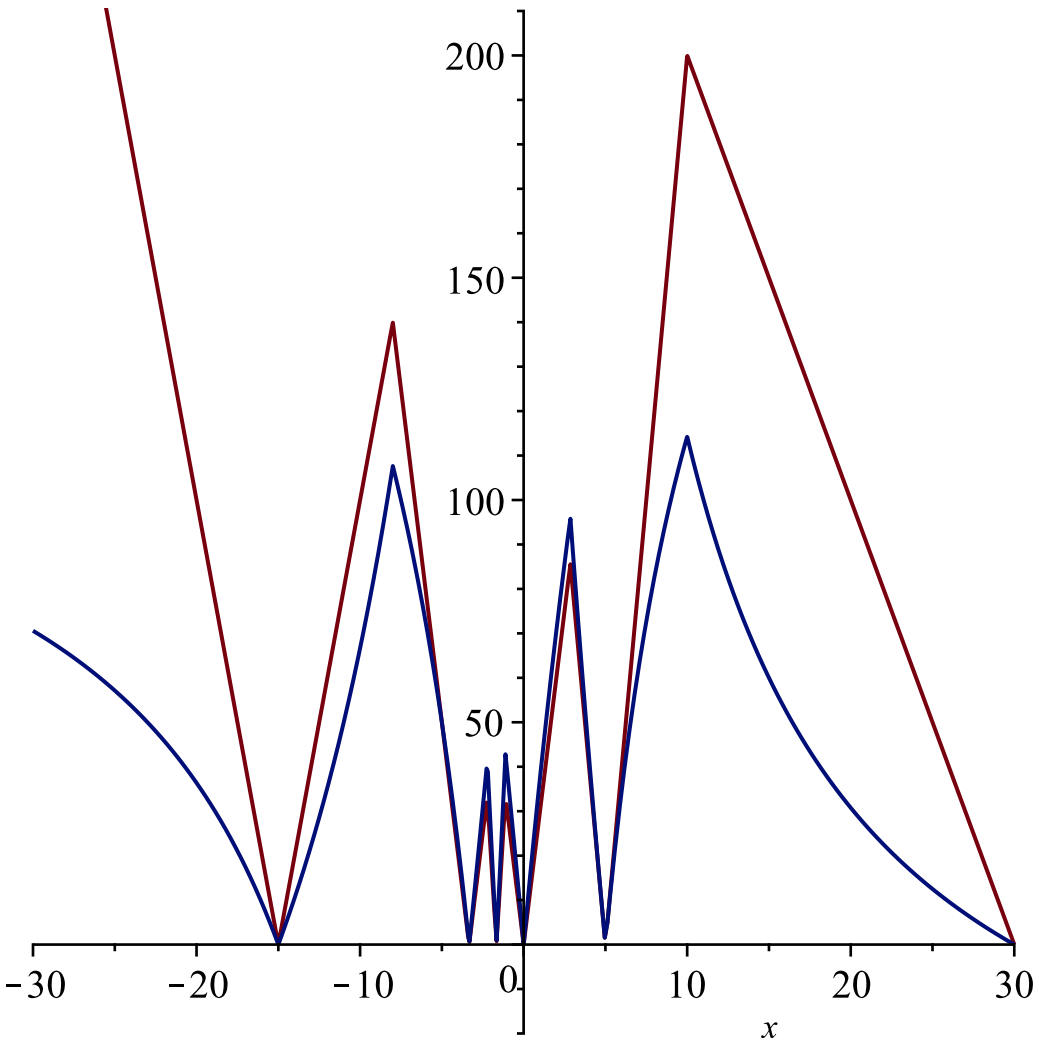

```
> plot(min(30 |x|, 10 |x − 30|, 40 |x − 5|, 20 |x + 15|, 20 |3 x + 5|, 10 |3 x + 10|), x = -20 ..20);
```



> $plot(\max(30\,x+100, 50\,x+400, 60\,x+100, 90\,x+200) - \min(30\,x+100, 50\,x+400, 60\,x+100, 90\,x+200))$




```
> plot([min(30 |x|, 10 |x − 30|, 40 |x − 5|, 20 |x + 15|, 20 |3 x + 5|, 10 |3 x + 10|),
  (400 min(30 |x|, 10 |x − 30|, 40 |x − 5|, 20 |x + 15|, 20 |3 x + 5|, 10 |3 x + 10|)) / (max(30 x
  + 100, 50 x + 400, 60 x + 100, 90 x + 200) − min(30 x + 100, 50 x + 400, 60 x + 100,
  90 x + 200))], x = −30 ..30, −20 ..210)
```


200
150
100
50
0
-30
-20
-10
10
20
30
x

>

>