

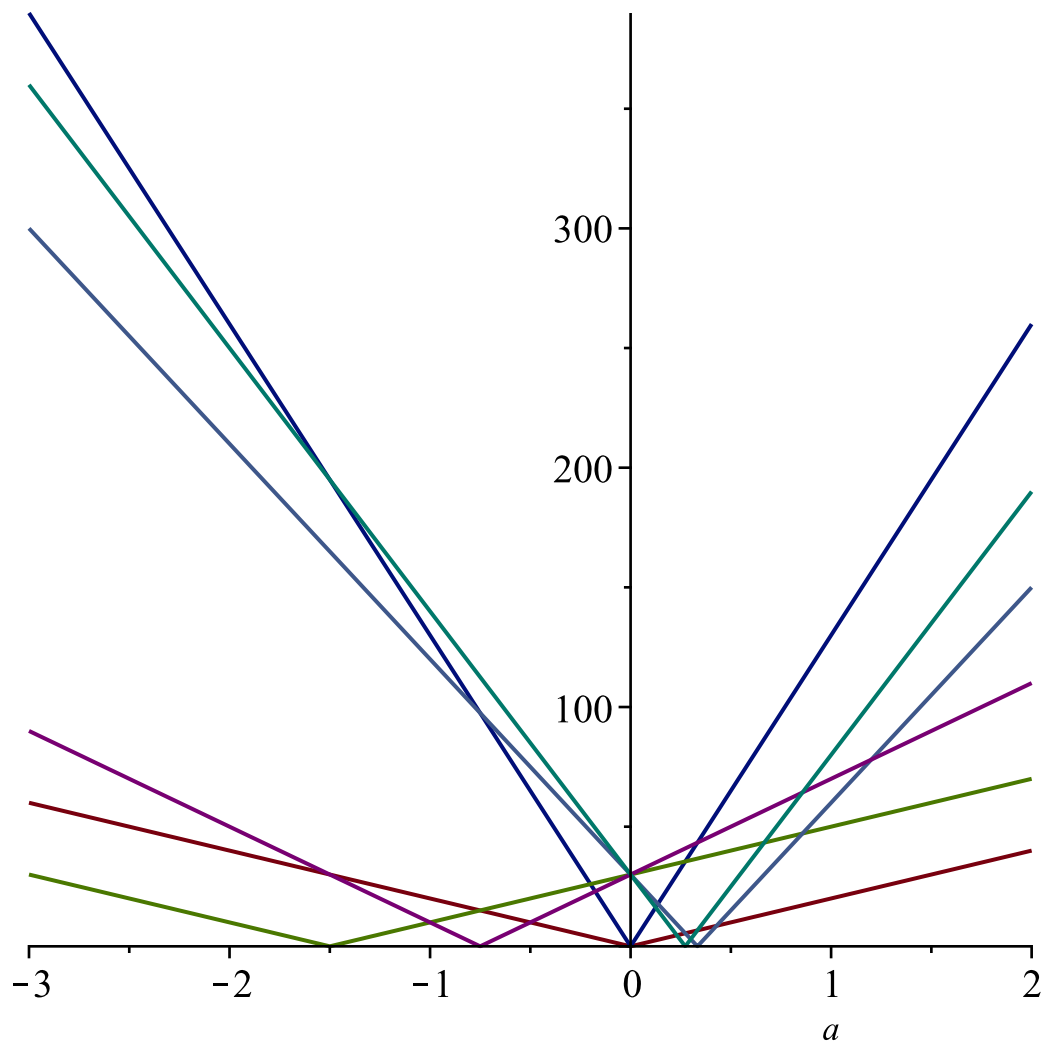
$$\begin{aligned} & \cdot \quad (x, y) \rightarrow \\ & (x + a \cdot y, y), \quad x \\ & \cdot \end{aligned}$$

$$\left[\begin{aligned} & \text{points} := [[30, 160], [30, 30], [60, 50], [60, 70]]; \\ & \text{points} := [[30, 160], [30, 30], [60, 50], [60, 70]] \end{aligned} \right. \quad (1.1)$$

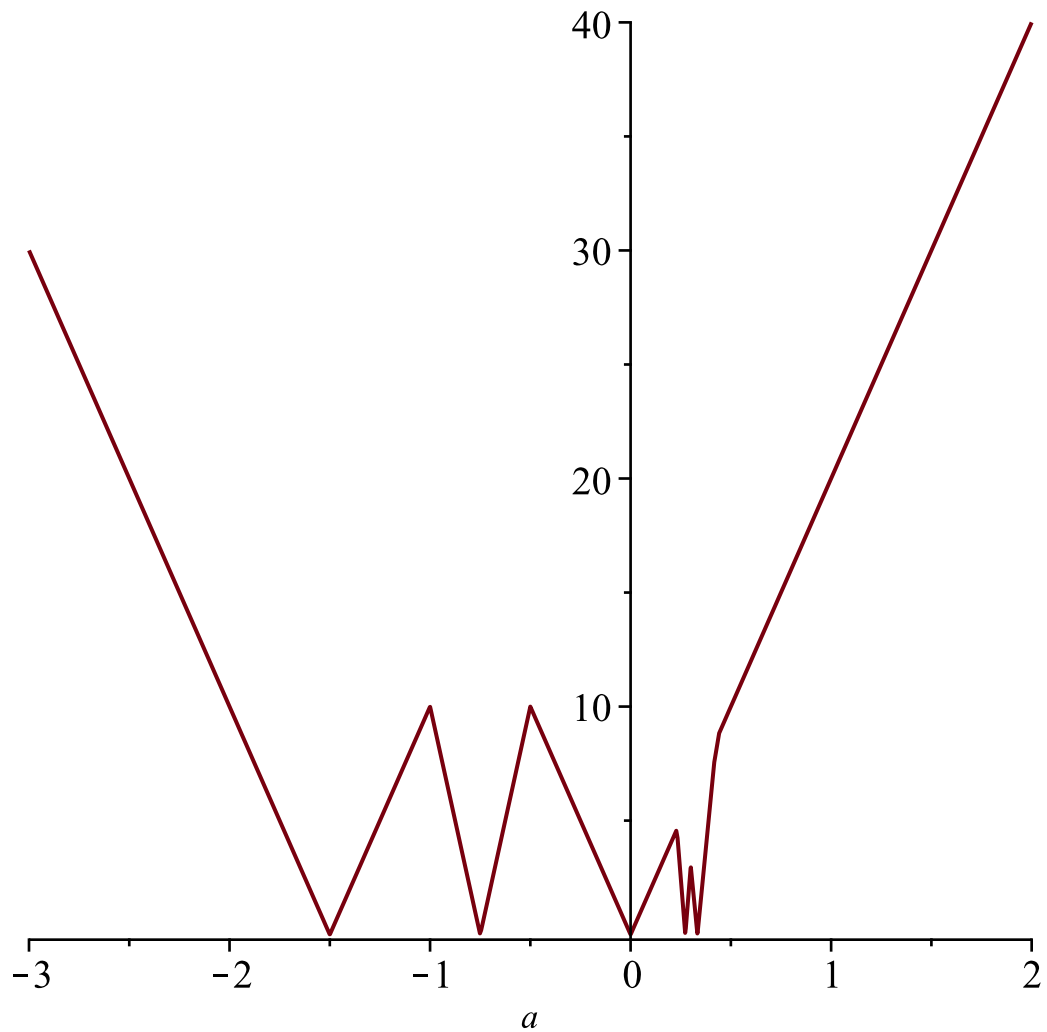
$$\begin{aligned} & \cdot \quad x, \quad a \\ & \cdot \end{aligned}$$

$$\left[\begin{aligned} & \text{tracks} := \{op(map(p \rightarrow p[1] + p[2] \cdot a, points))\}; \\ & distances := \{\}; \\ & \text{for } i \text{ from } 1 \text{ to } nops(points) - 1 \text{ do} \\ & \quad p := points[i]; \\ & \quad \text{for } j \text{ from } i + 1 \text{ to } nops(points) \text{ do} \\ & \quad \quad q := points[j]; \\ & \quad \quad distances := \{op(distances), abs(p[1] + p[2] \cdot a - (q[1] + q[2] \cdot a))\}; \\ & \quad \text{end do;} \\ & \text{end do;} \\ & distances; \\ & \quad \text{tracks} := \{30 \cdot a + 30, 50 \cdot a + 60, 70 \cdot a + 60, 160 \cdot a + 30\} \\ & \quad \{20 |a|, 130 |a|, 10 |2 \cdot a + 3|, 30 |3 \cdot a - 1|, 10 |4 \cdot a + 3|, 10 |11 \cdot a - 3|\} \end{aligned} \right. \quad (1.2)$$

$$\begin{aligned} & - \\ & \quad - \quad x \\ & \quad (x, y) \rightarrow (x + a \cdot y, y) \\ & \left[\begin{aligned} & \text{plot}(distances, a = -3..2) \end{aligned} \right. \end{aligned}$$



```
> plot(min(op(distances)), a=-3..2)
```



```

> ac := { } :
  for i from 1 to nops(distances) - 1 do
    di := distances[i];
    for j from i + 1 to nops(distances) do
      dj := distances[j];
      s := {solve(di = dj) };
      ac := {op(ac), op(s) };
    end do;
  end do;
  ac := sort([op(ac)]);
ac := [ -3/2, -1, -3/4, -1/2, -1/5, -3/17, 0, 1/8, 3/22, 3/13, 3/11, 3/10, 1/3, 3/7, 2/3, 6/7,
        6/5 ]
  (1.3.1)
  > map(t→eval(subs(a = t, distances)), ac);

```

$$\left[\{0, 30, 165, 195\}, \{10, 20, 120, 130, 140\}, \left\{0, 15, \frac{195}{2}, \frac{225}{2}\right\}, \{10, 20, 65, 75, 85\}, \right. \quad (1.3.2)$$

$$\left. \begin{aligned} &\{4, 22, 26, 48, 52\}, \left\{\frac{60}{17}, \frac{390}{17}, \frac{450}{17}, \frac{780}{17}, \frac{840}{17}\right\}, \{0, 30\}, \left\{35, \frac{5}{2}, \frac{65}{2}, \frac{65}{4}, \right. \\ &\left. \frac{75}{4}\right\}, \left\{15, \frac{30}{11}, \frac{195}{11}, \frac{360}{11}, \frac{390}{11}\right\}, \left\{30, \frac{60}{13}, \frac{120}{13}, \frac{450}{13}, \frac{510}{13}\right\}, \left\{0, \frac{60}{11}, \right. \\ &\left. \frac{390}{11}, \frac{450}{11}\right\}, \{3, 6, 36, 39, 42\}, \left\{0, \frac{20}{3}, \frac{110}{3}, \frac{130}{3}\right\}, \left\{\frac{60}{7}, \frac{120}{7}, \frac{270}{7}, \frac{330}{7}, \right. \\ &\left. \frac{390}{7}\right\}, \left\{30, \frac{40}{3}, \frac{130}{3}, \frac{170}{3}, \frac{260}{3}\right\}, \left\{\frac{120}{7}, \frac{330}{7}, \frac{450}{7}, \frac{780}{7}\right\}, \{24, 54, 78, \\ &102, 156\} \end{aligned} \right]$$

> *map*(*t*→*eval*(*subs*(*a*=*t*, *distances*)), *ac*); *max*(%);

$$\left[\{0, 30, 165, 195\}, \{10, 20, 120, 130, 140\}, \left\{0, 15, \frac{195}{2}, \frac{225}{2}\right\}, \{10, 20, 65, 75, 85\}, \right. \\ \left. \{4, 22, 26, 48, 52\}, \left\{\frac{60}{17}, \frac{390}{17}, \frac{450}{17}, \frac{780}{17}, \frac{840}{17}\right\}, \{0, 30\}, \left\{35, \frac{5}{2}, \frac{65}{2}, \frac{65}{4}, \right. \right. \\ \left. \frac{75}{4}\right\}, \left\{15, \frac{30}{11}, \frac{195}{11}, \frac{360}{11}, \frac{390}{11}\right\}, \left\{30, \frac{60}{13}, \frac{120}{13}, \frac{450}{13}, \frac{510}{13}\right\}, \left\{0, \frac{60}{11}, \right. \\ \left. \frac{390}{11}, \frac{450}{11}\right\}, \{3, 6, 36, 39, 42\}, \left\{0, \frac{20}{3}, \frac{110}{3}, \frac{130}{3}\right\}, \left\{\frac{60}{7}, \frac{120}{7}, \frac{270}{7}, \frac{330}{7}, \right. \\ \left. \frac{390}{7}\right\}, \left\{30, \frac{40}{3}, \frac{130}{3}, \frac{170}{3}, \frac{260}{3}\right\}, \left\{\frac{120}{7}, \frac{330}{7}, \frac{450}{7}, \frac{780}{7}\right\}, \{24, 54, 78, \\ 102, 156\} \right]$$

$$195 \quad (1.3.3)$$

> *with*((*Optimization*)) :

$$\text{Maximize}(\min(\text{abs}(130 \cdot a), \text{abs}(-30 + 110 \cdot a), \text{abs}(-30 - 20 \cdot a)), a = -2 .. 1);$$

$$[26.00000000000000, [a = -0.2000000000000000]] \quad (1.3.4)$$

$$\begin{matrix} , \\ \text{p} = (\text{x} , \text{y}) \end{matrix} \rightarrow \begin{matrix} - \\ (\text{p} . \text{x} + \text{a} * \text{p} . \text{y} , \text{p} . \text{y}) \end{matrix}$$

$$\begin{matrix} , \\ \text{p} = (\text{x} , \text{y}) \end{matrix} \rightarrow (\min(\text{r.x}) + (\max(\text{r.x}) - \min(\text{r.x})) * (\text{p.x} + \text{a} * \text{p.y} - \min(\text{r.x} + \text{a} * \text{r.y})) / (\max(\text{r.x} + \text{a} * \text{r.y}) - \min(\text{r.x} + \text{a} * \text{r.y})), \text{y})$$

min , max r
p () .

" " - d (p , q) (a) x p , q

d(p,q)(a) = abs(p.x-q.x) = abs(
min(r.x)+(max(r.x)-min(r.x))*(p.x+a*p.y-min(r.x+a*r.y))/(max(r.x+a*r.y)-min(r.x+a*
r.y))
-
min(r.x)+(max(r.x)-min(r.x))*(q.x+a*q.y-min(r.x+a*r.y))/(max(r.x+a*r.y)-min(r.x+a*
r.y))
)

d(p,q)(a) = abs(p.x-q.x) = abs(
(max(r.x)-min(r.x))*(p.x+a*p.y-min(r.x+a*r.y))/(max(r.x+a*r.y)-min(r.x+a*r.y))
-
(max(r.x)-min(r.x))*(q.x+a*q.y-min(r.x+a*r.y))/(max(r.x+a*r.y)-min(r.x+a*r.y))
)
= (max(r.x)-min(r.x)) * abs(
(p.x+a*p.y-min(r.x+a*r.y))
-
(q.x+a*q.y-min(r.x+a*r.y))
)
/(max(r.x+a*r.y)-min(r.x+a*r.y))
= (max(r.x)-min(r.x)) * abs(
p.x+a*p.y
-
(q.x+a*q.y)
)
/(max(r.x+a*r.y)-min(r.x+a*r.y))
= (max(r.x)-min(r.x)) * abs(p.x+a*p.y-(q.x+a*q.y)) / (max(r.x+a*r.y)-min(r.x+a*r.y))

,
d(p,q)(a) = (max(r.x)-min(r.x)) * abs(p.x+a*p.y-(q.x+a*q.y)) / (max(r.x+a*r.y)-min(r.
x+a*r.y))

p , q

.

d(a) = min(d(p,q)(a)) = min((max(r.x)-min(r.x)) * abs(p.x+a*p.y-(q.x+a*q.y)) / (max
(r.x+a*r.y)-min(r.x+a*r.y)))

$$= (\max(r.x) - \min(r.x)) * \min(\text{abs}(p.x + a * p.y - (q.x + a * q.y))) / (\max(r.x + a * r.y) - \min(r.x + a * r.y))$$

, . .

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- ,

.

m i n

$$\min(\text{abs}(p.x + a * p.y - (q.x + a * q.y)))$$

- ,

m a x (

r)

$$\max(r.x + a * r.y)$$

- ,

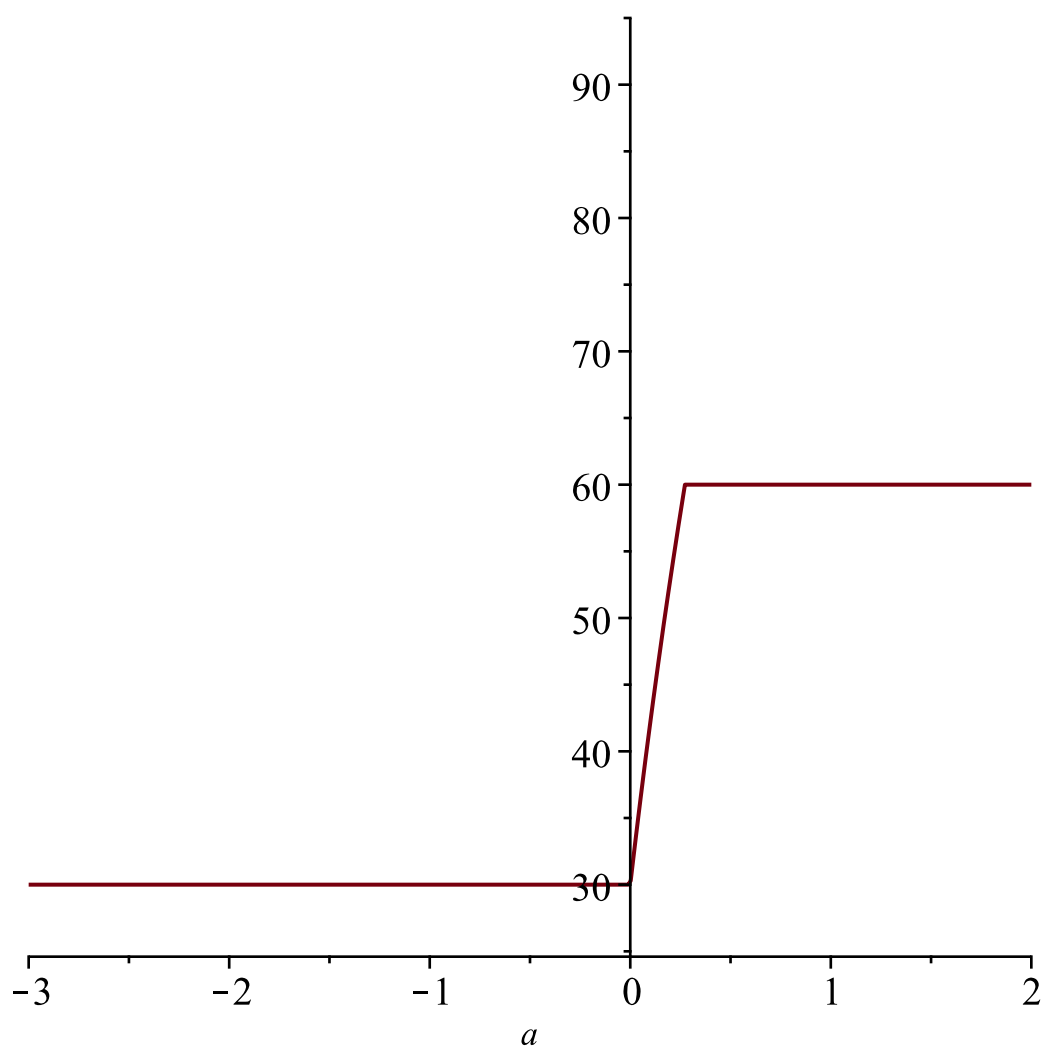
m i n (r)

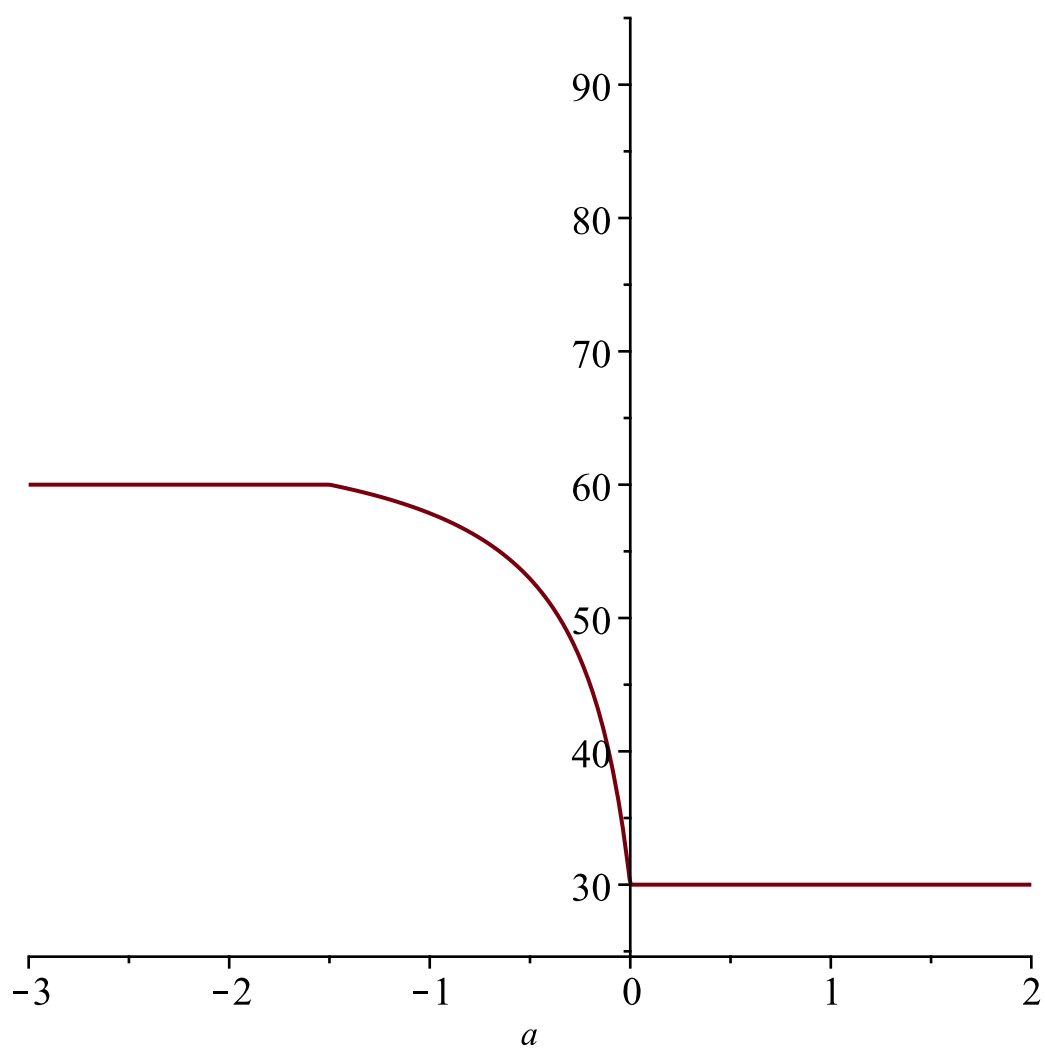
$$\max(r.x + a * r.y)$$

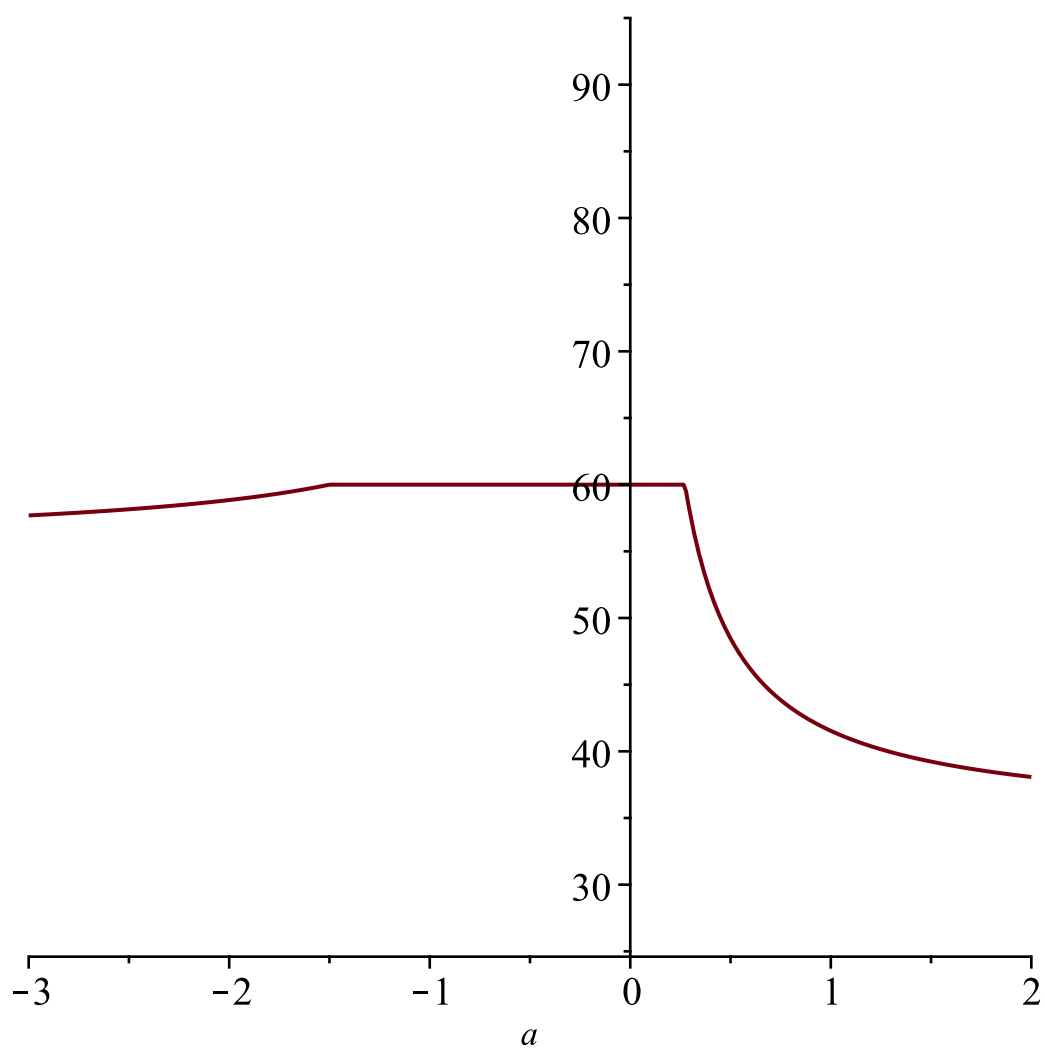
- ,

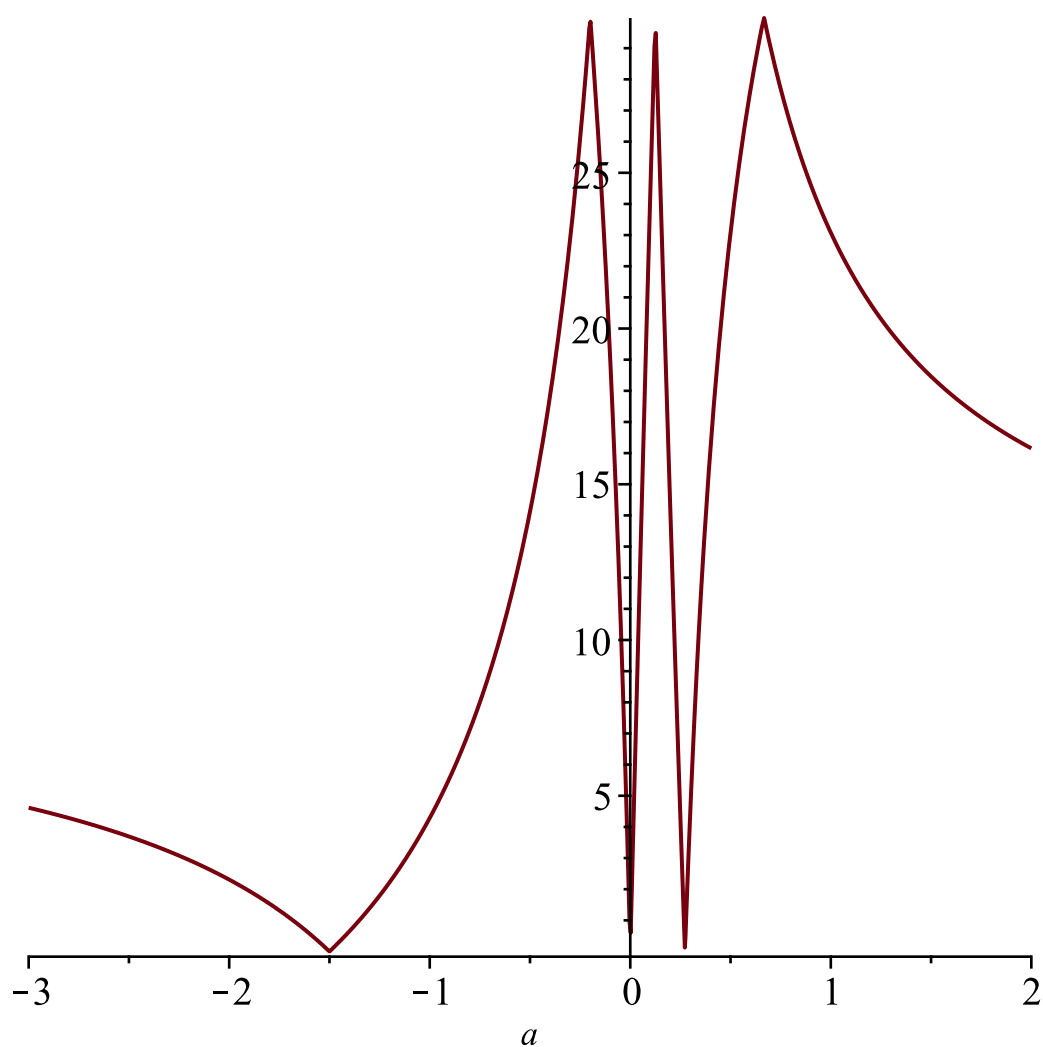
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```
> plot(min(30, 30, 60) + ((max(30, 30, 60) - min(30, 30, 60)) * (30 + 160 * a - min(30 + 160 * a, 30 + 30 * a, 60 + 50 * a))) / (max(30 + 160 * a, 30 + 30 * a, 60 + 50 * a) - min(30 + 160 * a, 30 + 30 * a, 60 + 50 * a)), a = -3 .. 2, 25 .. 95);
plot(min(30, 30, 60) + ((max(30, 30, 60) - min(30, 30, 60)) * (30 + 30 * a - min(30 + 160 * a, 30 + 30 * a, 60 + 50 * a))) / (max(30 + 160 * a, 30 + 30 * a, 60 + 50 * a) - min(30 + 160 * a, 30 + 30 * a, 60 + 50 * a)), a = -3 .. 2, 25 .. 95);
plot(min(30, 30, 60) + ((max(30, 30, 60) - min(30, 30, 60)) * (60 + 50 * a - min(30 + 160 * a, 30 + 30 * a, 60 + 50 * a))) / (max(30 + 160 * a, 30 + 30 * a, 60 + 50 * a) - min(30 + 160 * a, 30 + 30 * a, 60 + 50 * a)), a = -3 .. 2, 25 .. 95);
plot((max(30, 30, 60) * min(abs(30 + 160 * a - (30 + 30 * a)), abs(30 + 30 * a - (60 + 50 * a)), abs(60 + 50 * a - (30 + 160 * a)))) / (max(30 + 160 * a, 30 + 30 * a, 60 + 50 * a) - min(30 + 160 * a, 30 + 30 * a, 60 + 50 * a)), a = -3 .. 2);
```

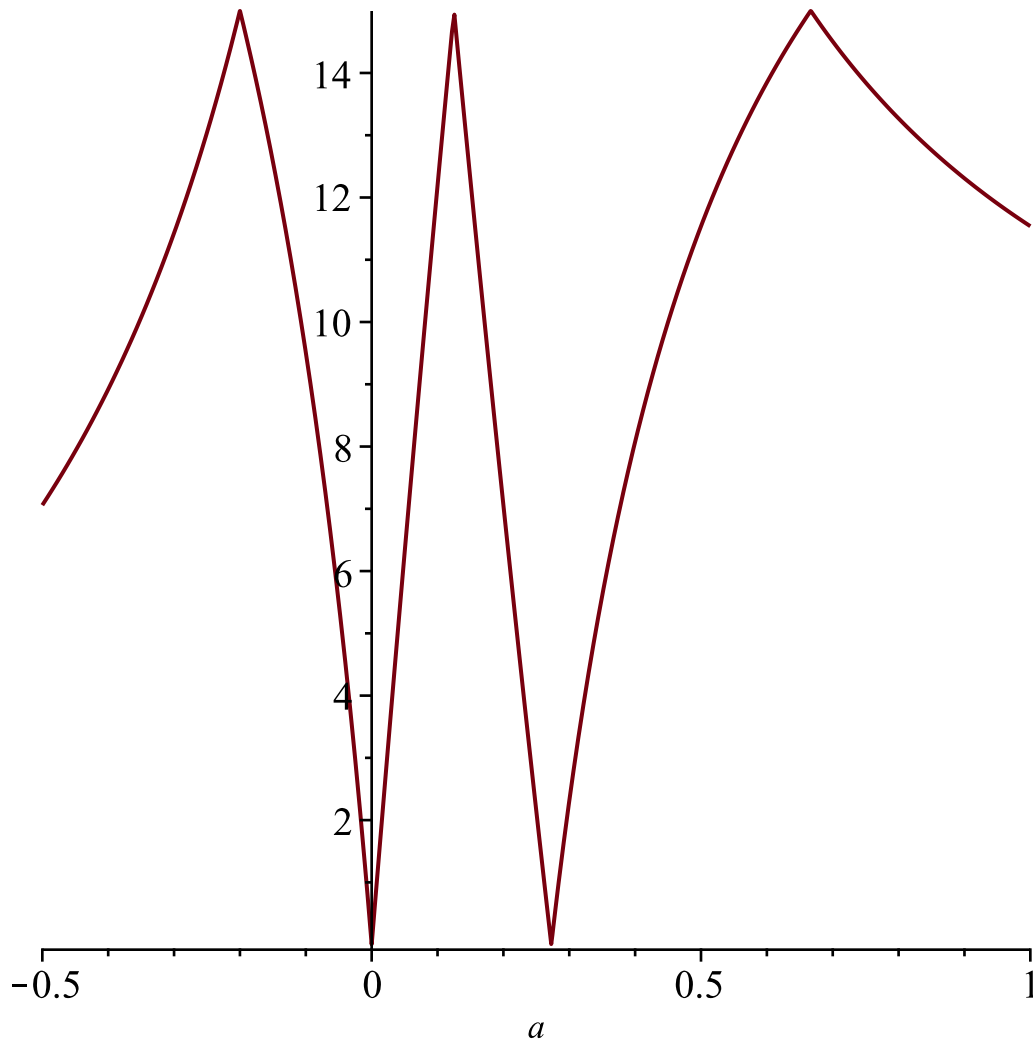








```
> plot( ( (max(30, 30, 60) - min(30, 30, 60)) * min(abs(30 + 160*a - (30 + 30*a)), abs(30
+ 30*a - (60 + 50*a)), abs(60 + 50*a - (30 + 160*a))) ) / (max(30 + 160*a, 30
+ 30*a, 60 + 50*a) - min(30 + 160*a, 30 + 30*a, 60 + 50*a)), a = -1/2 .. 1 )
```



$$\begin{aligned} &> \text{limit}(((\max(30, 30, 60) - \min(30, 30, 60)) \cdot \min(\text{abs}(30 + 160 \cdot a - (30 + 30 \cdot a)), \text{abs}(30 \\ &\quad + 30 \cdot a - (60 + 50 \cdot a)), \text{abs}(60 + 50 \cdot a - (30 + 160 \cdot a))) / (\max(30 + 160 \cdot a, 30 \\ &\quad + 30 \cdot a, 60 + 50 \cdot a) - \min(30 + 160 \cdot a, 30 + 30 \cdot a, 60 + 50 \cdot a)), a = -\text{infinity}) \\ &\quad \frac{60}{13} \end{aligned} \quad (2.1)$$

$$\begin{aligned} &> \text{limit}(((\max(30, 30, 60) - \min(30, 30, 60)) \cdot \min(\text{abs}(30 + 160 \cdot a - (30 + 30 \cdot a)), \text{abs}(30 \\ &\quad + 30 \cdot a - (60 + 50 \cdot a)), \text{abs}(60 + 50 \cdot a - (30 + 160 \cdot a))) / (\max(30 + 160 \cdot a, 30 \\ &\quad + 30 \cdot a, 60 + 50 \cdot a) - \min(30 + 160 \cdot a, 30 + 30 \cdot a, 60 + 50 \cdot a)), a = \text{infinity}) \\ &\quad \frac{60}{13} \end{aligned} \quad (2.2)$$

$$\begin{aligned} &> \text{extrema}(((\max(30, 30, 60) - \min(30, 30, 60)) \cdot \min(\text{abs}(30 + 160 \cdot a - (30 + 30 \cdot a)), \\ &\quad \text{abs}(30 + 30 \cdot a - (60 + 50 \cdot a)), \text{abs}(60 + 50 \cdot a - (30 + 160 \cdot a))) / (\max(30 + 160 \cdot a, \\ &\quad 30 + 30 \cdot a, 60 + 50 \cdot a) - \min(30 + 160 \cdot a, 30 + 30 \cdot a, 60 + 50 \cdot a)), a) \\ &\quad \{0\} \end{aligned} \quad (2.3)$$

> with(Optimization) :

$$\begin{aligned} &> \text{Maximize}(((\max(30, 30, 60) - \min(30, 30, 60)) \cdot \min(\text{abs}(30 + 160 \cdot a - (30 + 30 \cdot a)), \\ &\quad \text{abs}(30 + 30 \cdot a - (60 + 50 \cdot a)), \text{abs}(60 + 50 \cdot a - (30 + 160 \cdot a))) / (\max(30 + 160 \cdot a, \\ &\quad 30 + 30 \cdot a, 60 + 50 \cdot a) - \min(30 + 160 \cdot a, 30 + 30 \cdot a, 60 + 50 \cdot a))) \\ &\quad [14.9999999999999947, [a = 0.6666666666666667]] \end{aligned} \quad (2.4)$$

LL>

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```
> enlarge := proc( points, minA, maxA)
  local a, d, m, i, p, j, q, f, minX, maxX;
  a := [ ]; d := [ ]; m := [ ];
  for i from 1 to nops(points)
  do
    p := points[i];
    a := [op(a), p[1]];
    m := [op(m), p[1] + p[2]·x];
    for j from i + 1 to nops(points)
    do
      q := points[j];
      d := [op(d), abs(p[1] + p[2]·x - q[1] - q[2]·x)];
    end do;
  end do;
  minX := min(op(a)); maxX := max(op(a));
  f := 
$$\frac{(maxX - minX) \cdot \min(op(d))}{\max(op(m)) - \min(op(m))}$$
;
  print(f);
  print(limit(f, x = -infinity));
  print(limit(f, x = infinity));
  print(Maximize(f));
  plot(
$$\left[ f, \frac{maxX - minX}{nops(points) - 1} \right], x = minA .. maxA$$
);
end proc;
```

enlarge := proc(points, minA, maxA)

local a, d, m, i, p, j, q, f, minX, maxX;

a := [];

d := [];

m := [];

for i to nops(points) do

p := points[i];

a := [op(a), p[1]];

m := [op(m), x*p[2] + p[1]];

for j from i + 1 to nops(points) do

q := points[j]; d := [op(d), abs(x*p[2] - x*q[2] + p[1] - q[1])]

end do

end do;

minX := min(op(a));

maxX := max(op(a));

f :=
$$(maxX - minX) * \min(op(d)) / (\max(op(m)) - \min(op(m)))$$
;

print(f);

print(limit(f, x = -∞));

print(limit(f, x = ∞));

print(Maximize(f));

plot([f, (maxX - minX) / (nops(points) - 1)], x = minA .. maxA)

(1)

end proc

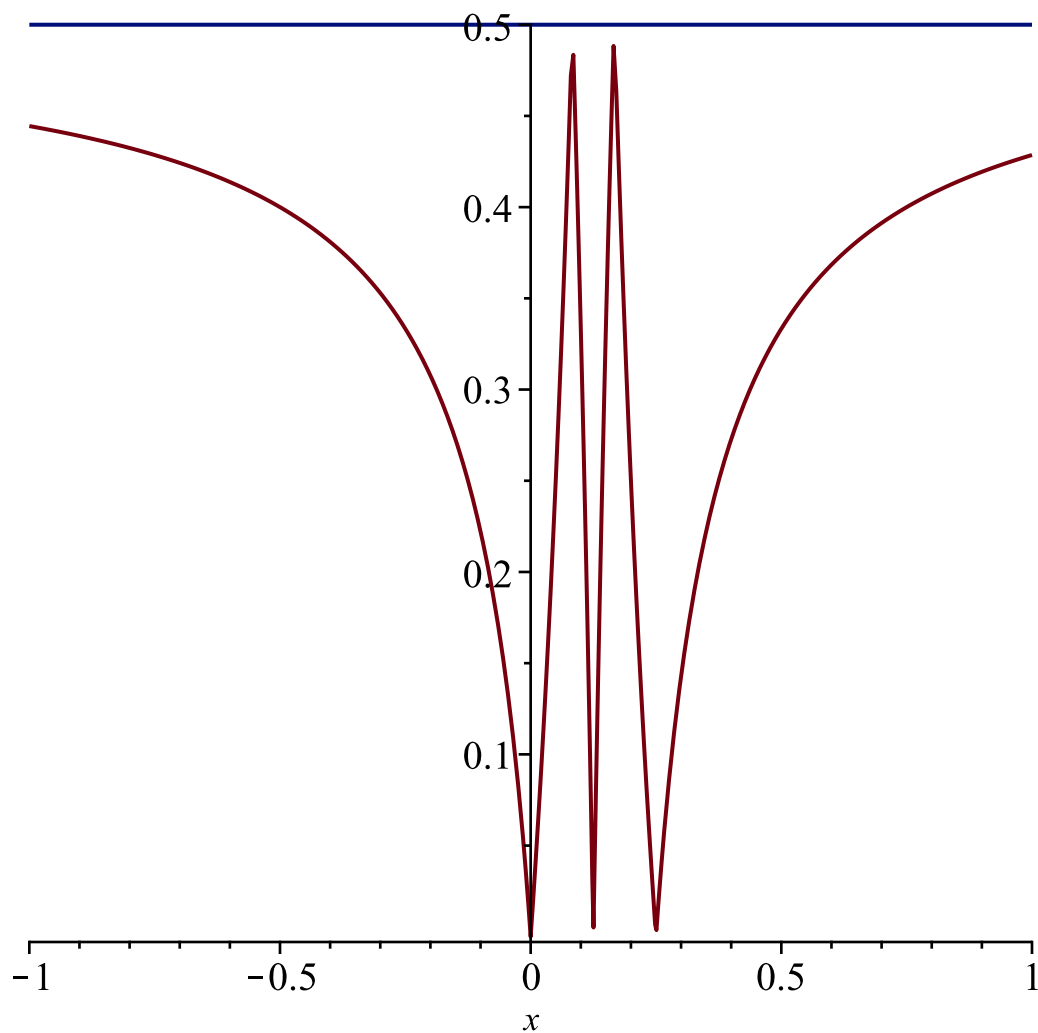
> with(Optimization) : enlarge([[2, 1], [1, 5], [2, -3]], -1, 1);

$$\frac{\min(4|x|, |4x-1|, |8x-1|)}{\max(-3x+2, x+2, 5x+1) - \min(-3x+2, x+2, 5x+1)}$$

$$\frac{1}{2}$$

$$\frac{1}{2}$$

$$\text{Maximize}\left(\frac{\min(4|x|, |4x-1|, |8x-1|)}{\max(-3x+2, x+2, 5x+1) - \min(-3x+2, x+2, 5x+1)}\right)$$



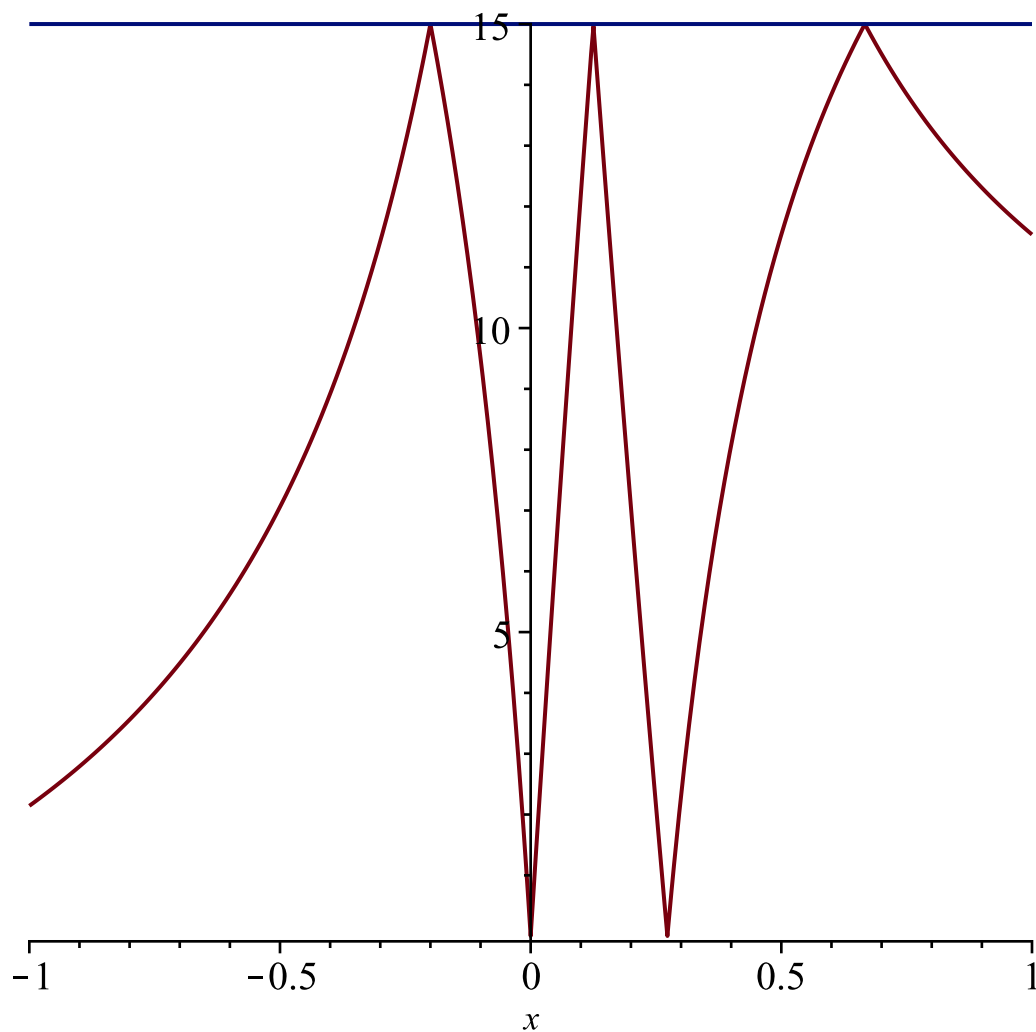
> enlarge([[30, 160], [30, 30], [60, 50]], -1, 1);

$$\frac{30 \min(130|x|, 10|2x+3|, 10|11x-3|)}{\max(30x+30, 50x+60, 160x+30) - \min(30x+30, 50x+60, 160x+30)}$$

$$\frac{60}{13}$$

$$\frac{60}{13}$$

$$\text{Maximize} \left(\frac{30 \min(130 |x|, 10 |2x + 3|, 10 |11x - 3|)}{\max(30x + 30, 50x + 60, 160x + 30) - \min(30x + 30, 50x + 60, 160x + 30)} \right)$$



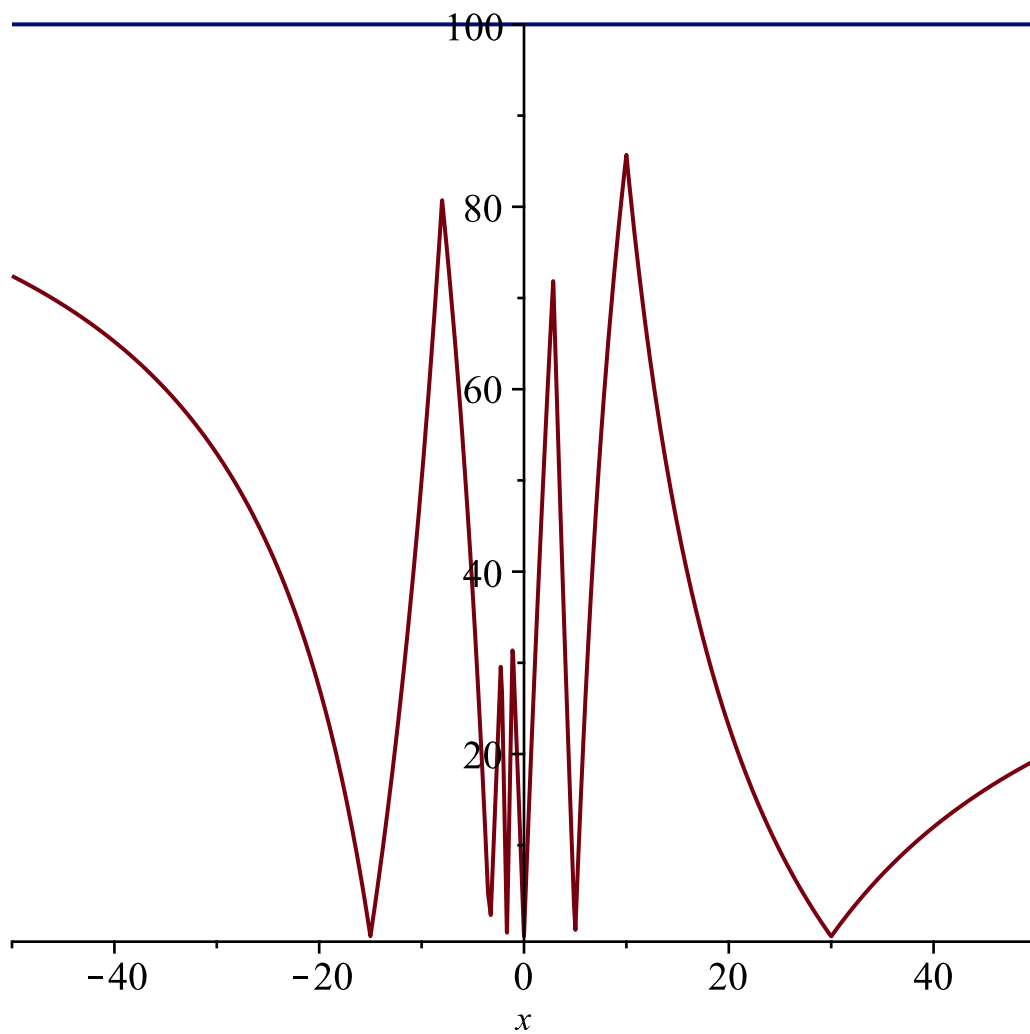
$$\begin{aligned} &> \text{Maximize} \left(\frac{30 \min(130 |x|, 10 |2x + 3|, 10 |11x - 3|)}{\max(30x + 30, 50x + 60, 160x + 30) - \min(30x + 30, 50x + 60, 160x + 30)} \right) \\ &\quad [14.9999999999999947, [x = 0.6666666666666667]] \end{aligned} \quad (2)$$

$$\begin{aligned} &> \text{enlarge}([[100, 60], [100, 30], [400, 50], [200, 90]], -50, 50) \\ &\quad (300 \min(30 |x|, 10 |x - 30|, 40 |x - 5|, 20 |x + 15|, 20 |3x + 5|, 10 |3x + 10|)) / (\max(30x \\ &\quad + 100, 50x + 400, 60x + 100, 90x + 200) - \min(30x + 100, 50x + 400, 60x + 100, \\ &\quad 90x + 200)) \end{aligned}$$

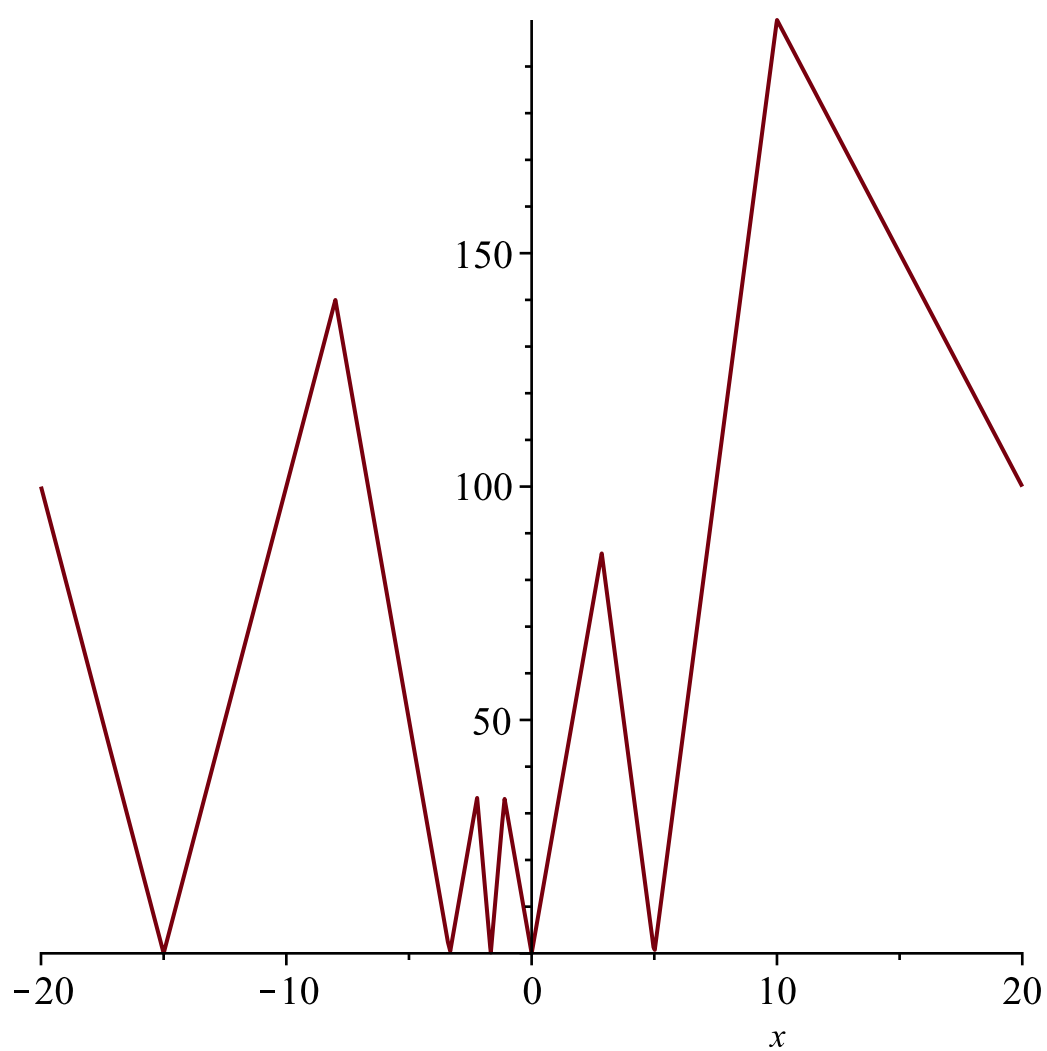
50

50

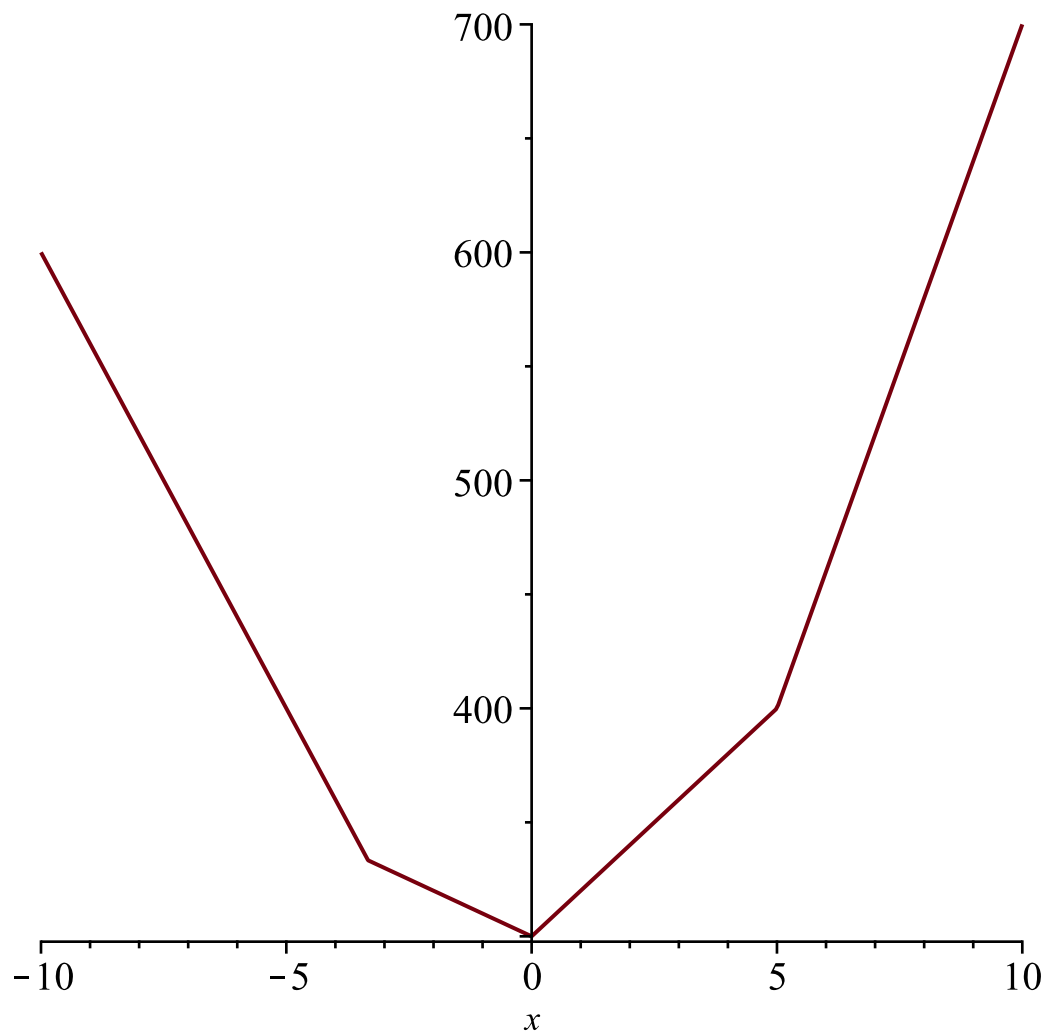
$$\begin{aligned} &\text{Maximize} ((300 \min(30 |x|, 10 |x - 30|, 40 |x - 5|, 20 |x + 15|, 20 |3x + 5|, 10 |3x + 10|)) / \\ &\quad (\max(30x + 100, 50x + 400, 60x + 100, 90x + 200) - \min(30x + 100, 50x + 400, 60x \\ &\quad + 100, 90x + 200))) \end{aligned}$$



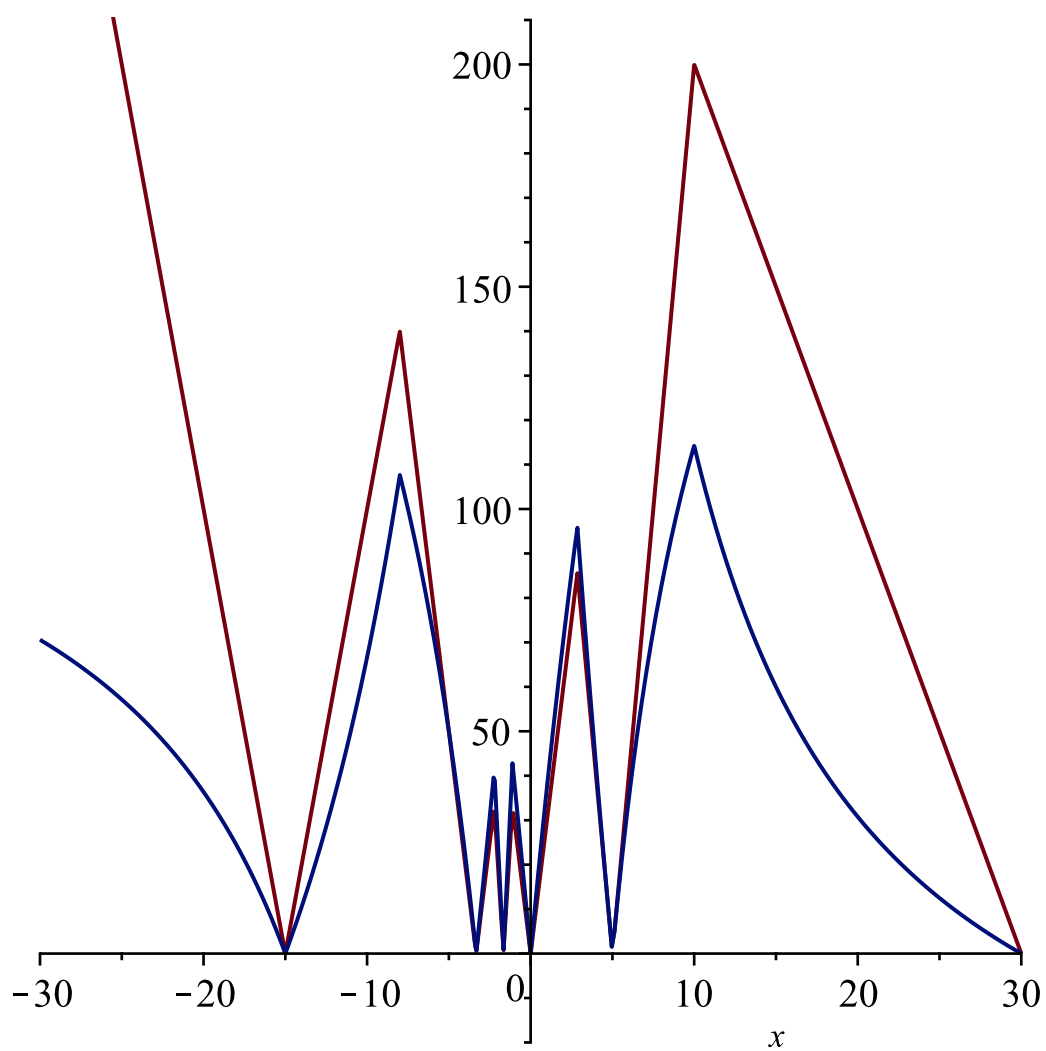
`> plot(min(30 |x|, 10 |x - 30|, 40 |x - 5|, 20 |x + 15|, 20 |3 x + 5|, 10 |3 x + 10|), x=-20..20);`



```
> plot(max(30 x + 100, 50 x + 400, 60 x + 100, 90 x + 200) - min(30 x + 100, 50 x + 400, 60 x + 100, 90 x + 200))
```

```
> plot( [ min(30 |x|, 10 |x - 30|, 40 |x - 5|, 20 |x + 15|, 20 |3 x + 5|, 10 |3 x + 10|),
(400 min(30 |x|, 10 |x - 30|, 40 |x - 5|, 20 |x + 15|, 20 |3 x + 5|, 10 |3 x + 10|) ) / (max(30 x
+ 100, 50 x + 400, 60 x + 100, 90 x + 200) - min(30 x + 100, 50 x + 400, 60 x + 100,
90 x + 200) ) ], x=-30..30,-20..210)
```



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