Mobile Beacon-Based 3D-Localization with Multidimensional Scaling in Large Sensor Networks

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***Abstract*—Localization is essential in wireless sensor networks to handle the reporting of events from sensor nodes. For 3-D applications, we propose a mobile beacon-based localization using classical multidimensional scaling (MBL-MDS) by taking full advantage of MDS with connectivity and measurements. To further improve location performance, MBL-MDS adopts a selection rule to choose useful reference points, and a decision rule to prevent a failure case due to reference points placed on the same plane. Simulation results show improved performance of MBL-MDS in terms of location accuracy and computation complexity.**

***Index Terms*—Sensor networks, localization, mobile beacon, multidimensional scaling.**

I. Introduction

RECENTLY, research on localization has been actively investigated in large-scale wireless sensor networks, with many schemes introduced in various simulation environments. Most of them target locating nodes in 2-D space, e.g., probabilistic method for relative location estimation [1]. In practice, sensor nodes are deployed in 3-D space. Locating nodes in 3-D space does not simply reflect just the addition of one extra dimension to the locating problem, and thus it is necessary to develop 3-D localization [2].

One effective approach is range-based location using mobile beacons because each mobile beacon can directly provide its current position to sensor nodes and sensor nodes can measure distances to the mobile beacon [3], [4]. Zhang *et al*. [3] encourages a sensor node to improve location accuracy with an unscented Kalman filter (UKF). While their scheme gives a converged node position under continuous measurements, it often fails in the improvement due to discontinuous measurements when a mobile node flies out of the listenable range of a sensor node. Hu *et al.* [4] uses a least-square method for initial state of the same UKF of Zhang's scheme but a least-square method is sensitive to deployment of reference positions. As a good approach for discontinuous measurements, multidimensional scaling (MDS) has been widely used to locate nodes with static beacons in 2-D space [5]. In 3-D localization with mobile beacons, MDS is still appropriate because all required distances are obtainable, but there is scarcely any effort to use MDS. Besides, there are open problems that MDS requires expensive computation for large number of reference positions and relocation on the coordinates of reference positions.

In this letter, we propose a mobile beacon-based localization using classical multidimensional scaling (MBL-MDS). MBL-MDS applies MDS to 3-D localization using a mobile beacon and adopts two rules to improve location performance by handling the problems: 1) a selection rule to choose sufficient sets of reference positions among all received ones, and 2) a decision rule to determine which of the two candidates is in the correct node position, in the case that given reference positions are placed on the same plane.

II. Principles for the Proposed Scheme

This section describes multidimensional scaling (MDS) and linear transformation to locate a sensor node $\boldsymbol{s}$ that has the M number of beacon points as $\mathbf{P} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_M]$.

A. Classical Multidimensional Scaling

For points $\mathbf{X} = [\boldsymbol{s}, \boldsymbol{x}_1, \ldots, \boldsymbol{x}_M]$, distances between all pairs of points are given as $[\mathbf{D}]_{ij} = d_{ij} = ||\boldsymbol{x}_i - \boldsymbol{x}_j||$ for $i, j = 0, 1, \cdots, M$, where $\boldsymbol{x}_0 = \boldsymbol{s}$. Then, MDS computes relative points $\mathbf{X}' = [\boldsymbol{s}', \boldsymbol{x}'_1, \ldots, \boldsymbol{x}'_M]$ from $\mathbf{D}$ as follows [5]:

For given distance matrix $\mathbf{D}$, symmetric matrix $\mathcal{D}_2$ has the square of the distance for each component, such that $[\mathcal{D}_2]_{ij} = d_{ij}^2$. Then, symmetric matrix $\mathbf{B}$, defined as $\mathbf{B} \triangleq \mathbf{X'}^\top \mathbf{X}'$, can be obtained from $\mathcal{D}_2$ as

$$\mathbf{B} = -\frac{1}{2}\mathbf{J}\mathcal{D}_2\mathbf{J}, \quad \mathbf{J} = \mathbf{I}_{M+1} - \frac{1}{M+1}\mathbf{1}_{M+1}\mathbf{1}_{M+1}^\top, \quad (1)$$

where $\mathbf{I}_{M+1}$ is an $(M+1 \times M+1)$ identity matrix and $\mathbf{1}_{M+1} = [1, 1, \cdots, 1]^\top$ is an $(M+1 \times 1)$ unit vector. Decomposing $\mathbf{B}$ using a singular value decomposition (SVD) gives

$$\mathbf{B} = \mathbf{U}\mathbf{V}\mathbf{U}^\top, \quad (2)$$

where $\mathbf{V} = \mathrm{diag}(\lambda_1, \lambda_2, \cdots, \lambda_{M+1}),\ \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{M+1}$ is a matrix of which diagonals are eigenvalues of $\mathbf{B}$ and $\mathbf{U} = [\boldsymbol{u}_1, \boldsymbol{u}_2, \cdots, \boldsymbol{u}_{M+1}]$ is a matrix composed of the eigenvectors $\boldsymbol{u}_i$ for λ_i. Thus, the relative points $\mathbf{X}'$ can be obtained as

$$\mathbf{X}' = \mathbf{B}^{1/2} = \mathbf{U}\mathbf{V}^{1/2}. \quad (3)$$

Finally, because the $M+1$ points are placed in a p-dimensional space $(M+1 > p)$, rank($\mathbf{B}$) = p, the relative points $\mathbf{X}'$ for $\mathbf{X}$ are obtained by selecting the dominant eignevalues and corresponding eigenvectors as

$$\mathbf{X}' = \mathbf{U}_p\mathbf{V}_p^{1/2}, \quad (4)$$

where $\mathbf{V}_p^{1/2} = \mathrm{diag}(\lambda_1^{1/2}, \lambda_2^{1/2}, \cdots, \lambda_p^{1/2})$ and $\mathbf{U}_p = [\boldsymbol{u}_1, \boldsymbol{u}_2, \cdots, \boldsymbol{u}_p]$.

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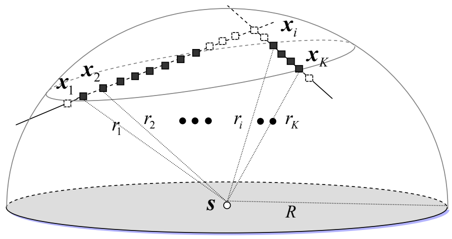


Fig. 1. Gathered beacon points x_i and measured distances r_i, for $i = 1, 2, \cdots, K$, inside the listenable range R of the n-th sensor node s.

B. Linear Transformation to Adjust $\mathbf{P}'$ Close to $\mathbf{P}$

Given two matrices $\mathbf{P}' = [\boldsymbol{x}'_1, \boldsymbol{x}'_2, \ldots, \boldsymbol{x}'_M]$ and $\mathbf{P} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_M]$, there are some known solutions for determining the parameters to linearly transform $\mathbf{P}'$ most close to $\mathbf{P}$, such that

$$\arg\min_{c,\mathbf{R},\boldsymbol{t}} \sum_{i=1}^{M} ||c\mathbf{R}\boldsymbol{x}'_i + \boldsymbol{t} - \boldsymbol{x}_i||^2. \tag{5}$$

Of these solutions, Arun's method [6] is used in our proposed scheme, because: 1) the matrix $\mathbf{P}'$ is based on the distances between all pairs of points in $\mathbf{P}$; hence, the scale between $\mathbf{P}'$ and $\mathbf{P}$ does not change, and 2) Arun's method provides low complexity by using the unit scale $c = 1$. In Arun's method, a rotation matrix $\mathbf{R}$ and a translation vector $\boldsymbol{t}$ are given by

$$\mathbf{R} = \mathbf{U}\mathbf{V}^\top, \quad \boldsymbol{t} = \boldsymbol{\mu}_x - \mathbf{R}\boldsymbol{\mu}_{x'}, \tag{6}$$

where $\boldsymbol{\mu}_x = \frac{1}{M}\sum_{i=1}^{M} \boldsymbol{x}_i$ and $\boldsymbol{\mu}_{x'} = \frac{1}{M}\sum_{i=1}^{M} \boldsymbol{x}'_i$. Also, $\mathbf{U}$ and $\mathbf{V}$ are the SVD results of the centralized matrices as $\bar{\mathbf{P}}\bar{\mathbf{P}}'^\top = \mathbf{U}\mathbf{S}\mathbf{V}^\top$, where the centralized matrices are

$$\bar{\mathbf{P}} = \mathbf{P} - \boldsymbol{\mu}_x \cdot \mathbf{1}_M^\top, \; \bar{\mathbf{P}}' = \mathbf{P}' - \boldsymbol{\mu}_{x'} \cdot \mathbf{1}_M^\top. \tag{7}$$

III. Proposed Location Scheme: MBL-MDS

This section presents a system model and the proposed MBL-MDS with respect to a certain sensor node. The other nodes can locate themselves in the same manner.

A. System Model

As shown in Fig. 1, a mobile beacon flies over sensor nodes. While traveling every beacon distance, the mobile beacon broadcasts a beacon packet that contains its current position at the sending point (beacon point). To this end, we assume that the beacon has mobility, self-localizability, and sufficient energy to travel over the sensing area. Every node has the same limited listenable range R to receive beacon packets. Receiving a beacon packet, a sensor node obtains the beacon point $\boldsymbol{x}_i$ and measures the distance r_i to $\boldsymbol{x}_i$, $i = 1, 2, \cdots, K$. Based on the received signal-strength technique, the measurement error ϵ_i in a measured distance r_i is usually proportional to the actual distance d_i, modeled as the Gaussian random variable [3]

$$r_i = d_i + \epsilon_i, \quad \epsilon_i \sim N(0, \sigma_i^2), \tag{8}$$

with variance of the measurement error $\sigma_i^2 = (\gamma \cdot d_i)^2$. The γ is a ratio of standard deviation of the measurement error to the actual distance.

B. Decision Rule for Resolving Ambiguous Reflection

Let $\mathbf{P} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_M]$ and $\boldsymbol{r} = [r_1, r_2, \cdots, r_M]^\top$ denote M beacon points and their corresponding measured distances, respectively. The beacon points are chosen out of total K number of beacon points by a selection rule to be mentioned in the next subsection. Based on $\mathbf{P}$ and $\boldsymbol{r}$, a sensor node makes a distance matrix $\mathbf{D}$ as

$$\mathbf{D} = \begin{bmatrix} 0 & \boldsymbol{r}^\top \\ \boldsymbol{r} & \mathbf{D}_P \end{bmatrix}, \tag{9}$$

where $[\mathbf{D}_P]_{ij} = d_{ij} = ||\boldsymbol{x}_i - \boldsymbol{x}_j||$ for $i, j = 1, 2, \cdots, M$. Using MDS, a sensor node computes $\mathbf{X}' = [\boldsymbol{s}', \mathbf{P}']$ from $\mathbf{D}$, where $\boldsymbol{s}'$ and $\mathbf{P}'$ are relative positions of the sensor node and the selected beacon points, respectively.

In relocating $\mathbf{X}'$ in the coordinates of beacon points, we have to consider the ambiguous reflection case when all beacon points are almost placed on the same plane in Fig. 1. In this case, there exist two possible candidates for node position; one is the correct position and the other is a reflected position with respect to the coplanar. For the correct position, a decision rule of MBL-MDS first computes both candidates as

$$\boldsymbol{s}^+ = \mathbf{R}^+\boldsymbol{s}' + \boldsymbol{t}^+, \quad \boldsymbol{s}^- = \mathbf{R}^-\boldsymbol{s}' + \boldsymbol{t}^- \tag{10}$$

where $\mathbf{R}^+ = \mathbf{U}\mathbf{V}^\top$, and $\boldsymbol{t}^+ = \boldsymbol{\mu}_x - \mathbf{R}^+\boldsymbol{\mu}_{x'}$ in (6). The reflected rotation matrix $\mathbf{R}^-$ and the corresponding translation vector are given as

$$\mathbf{R}^- = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^\top, \quad \boldsymbol{t}^- = \boldsymbol{\mu}_x - \mathbf{R}^-\boldsymbol{\mu}_{x'}, \tag{11}$$

where $\boldsymbol{\Sigma} = \text{diag}(1, 1, -1)$. Then the correct position from the two possible candidates is determined as

$$\tilde{\boldsymbol{s}} = \begin{cases} \boldsymbol{s}^+ & \text{if } \boldsymbol{s}^+(z) \le \boldsymbol{\mu}_x(z) \\ \boldsymbol{s}^- & \text{otherwise} \end{cases} \tag{12}$$

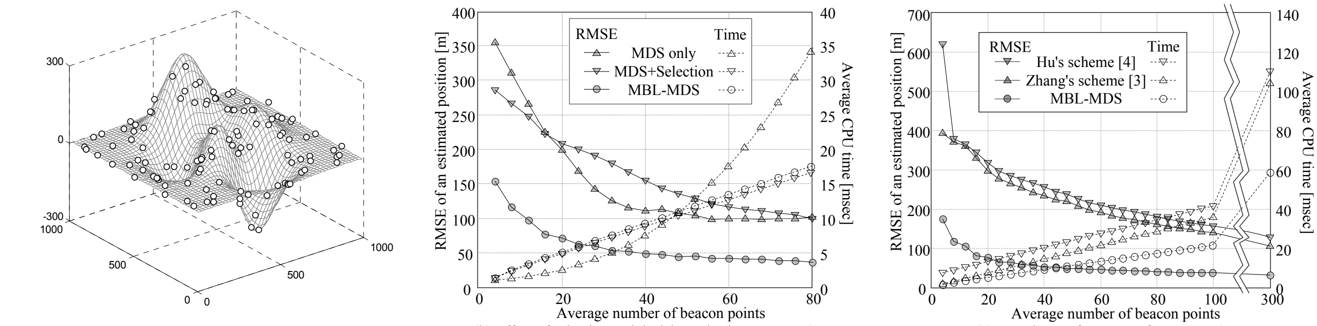
where $\boldsymbol{s}^+(z)$ and $\boldsymbol{\mu}_x(z)$ are z-values of $\boldsymbol{s}^+$ and $\boldsymbol{\mu}_x$, because the sensor node is always located below the beacon points.

C. Locating Node using Selection Rule

MDS and the linear transformation technique result in exponential growth in complexity, such as $O(K^3)$ for the K number of beacon points due to SVD computation. To reduce complexity, the selection rule of MBL-MDS collects κ sets composed of M beacon points ($M \ll K$), and then MDS and linear transformation are applied to each set, independently. Hence, the complexity of MBL-MDS is denoted by $O(\kappa \cdot M^3)$ and linearly increases with κ, $1 \le \kappa \le \lfloor K/M \rfloor$, where a real number inside $\lfloor \cdot \rfloor$ is mapped to the next smallest integer.

Selecting well-distributed beacon points is also important to improve location accuracy. To that end, a selection rule chooses M beacon points in a set as follows:

1) Compute the mean of the given beacon points $\boldsymbol{\mu}_x$ and dividing the x-y plane of beacon points into 4 parts (quadrants) with the center of $\boldsymbol{\mu}_x$.
2) Gather M beacon points by randomly selecting the $\lfloor M/4 \rfloor$-tuple beacon points from each quadrant.
3) Make a matrix $\mathbf{P}$ with the selected beacon points and the corresponding distance vector $\boldsymbol{r}$.



(a) Node deployment (b) Effect of selection and decision rules in MBL-MDS (c) Location performance of MBL-MDS

Fig. 2. Simulation results: (a) Node deployment, (b) Effect of selection and decision rules in MBL-MDS, and (c) Location performance of MBL-MDS.

4) Estimate a node position $\tilde{\boldsymbol{s}}_i$ based on $\mathbf{P}$ and $\boldsymbol{r}$, which is computed with aforementioned MDS and linear transformation.
5) Remove $\mathbf{P}$ from the given beacon points.

Repeating steps 1) to 5) for κ times, MBL-MDS obtains a set $W = \{\tilde{\boldsymbol{s}}_i : i = 1, 2, \ldots, \kappa\}$. Then, the final node position $\hat{\boldsymbol{s}}$ is determined as the median of $\tilde{\boldsymbol{s}}_j$ such that

$$\hat{\boldsymbol{s}} = \arg\min_{\tilde{\boldsymbol{s}}_j \in W} \sum_i |\tilde{\boldsymbol{s}}_i - \tilde{\boldsymbol{s}}_j|, \tag{13}$$

because the median is much better than the average when the size of W is not large enough and there are some estimated positions with large errors.

IV. Performance Evaluation

MBL-MDS was evaluated in the complex 3-D terrain in Fig. 2(a); 1000×1000 in size and altitude of -300 to 300. The linear track of a mobile beacon starts from $[0, 0, 300]^\top$ to $[x, y, z]^\top$, where x, y, and z are randomly selected with uniform distribution in the ranges of $0 \leq x, y \leq 1000$ and $300 \leq z \leq z_{\max}$. After arriving, the mobile beacon heads to another randomly selected destination point. Beacon points are arranged at every 50 distance along the linear track. The number of sensor nodes is 200, the listenable range is 600, and γ in (8) is given as 10% of the actual distances. The number of selected beacon points in a set is fixed to four ($M = 4$). Simulation results were obtained using MATLAB v7.1, 2.0 GHz CPU, and 1 Gbytes memory.

Fig. 2(b) presents the effect of selection and decision rules of MBL-MDS on location performance. All beacon points are placed on the same plane, $z_{\max} = 300$. The performance is shown in terms of location accuracy with dashed lines and average CPU time with dotted lines. MBL-MDS uses both rules, *MDS+Selection* uses the only selection rule, and *MDS only* uses none of two rules. MDS is applied to all cases. Decision rule of MBL-MDS shows the improvement in location accuracy by comparing MBL-MDS to the others; it reduces RMSE to 40% of the others with 80 beacon points. Comparing RMSE of *MDS only* and *MDS+Selection*, *MDS+Selection* is superior to *MDS only* in small beacon number due to relatively well-distributed beacon points by its selection rule. Conversely, *MDS only* is superior in large beacon number because *MDS only* has well-distributed beacon points and improved location accuracy due to many beacon points in computing MDS. Selection rule of MBL-MDS and *MDS+Selection* depicts linear growth in computation complexity.

Fig. 2(c) presents the location performance of MBL-MDS compared to the conventional location schemes mentioned in Introduction under $z_{\max} = 400$. This figure shows that MBL-MDS outperforms the others with respect to RMSE of location errors, e.g., MBL-MDS reduces RMSE to about 26% of Zhang's scheme [3] and 24% of Hu's schemes [4], with 20 beacon points. Also, MBL-MDS takes lower average time to locate a node than others, about 1.5 times faster than Zhang's scheme and 2.5 times faster than Hu's scheme. Note that the UKF mechanism in Zhang's and Hu's schemes should run for every beacon point, while MDS and linear transformation in MBL-MDS run once every M beacon points.

V. Conclusions

In this letter, we proposed a mobile beacon-based localization with multidimensional scaling (MBL-MDS) for 3-D applications of large-scale sensor networks. To further improve location performance in applying MDS, MBL-MDS adopts a selection rule to choose sufficient sets of beacon points, and a decision rule to resolve ambiguous reflection for a node position. Simulation results confirmed the improvement of MBL-MDS, e.g., MBL-MDS reduces RMSE to 26% of Zhang's scheme [3] with 1.5 times faster time.

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