Statistics and probability. First block Academic year 2017/18

The study of the muon decay ($\mu^- \rightarrow e^- \nu_\mu \nu_e$) provided significant information about the weak interaction but also about the proceses that generated the muons. One of the possible measurements is the muon polarisation, defined as the fraction of muons with positive helicity. The polarisation of a muon beam contains information about the muon creation mechanisms. The angular distribution of the electrons produced in muon decays is described by the probability:

$$\frac{d\Gamma}{d\cos\theta} = \frac{1}{2}(1 - \frac{1}{3}P_\mu \cos\theta) \quad P_\mu \in [-1., 1.]$$

where P_μ is the muon polarisation that carries information about the muon production phenomena and the cos θ is the angle between the electron and the muon polarisation vector in the muon rest frame.

1. Demonstrate that the normalised differential cross-section $d\Gamma/d\cos\Theta$ is a probability density function.

2. Build a Monte Carlo able to generate this probability density function the try-reject:
   - Calculate the mean, variance, skewness and kurtosis of the distribution based on the Monte Carlo.

3. The $d\Gamma/d\cos\Theta$ mean depends on the polarisation P_μ in a simple manner: mean=$P_\mu/9$.
   - Show that the Monte Carlo predicts this dependency by changing the value of P_μ
   - What is the variance of the parameter P_μ?: compute it numerically using Monte Carlo techniques for a given P_μ value.

4. Generate a continuous series of N events using the Monte Carlo and compute the mean of the distribution and the estimated P_μ as 9*mean.
   - Show that the P_μ tends to the true value as predicted by the law of large numbers.

5. Generate several Monte Carlo experiments, each with N events,
   - Build for each experiment the student's t variable for P_μ and show that it follows the Students't distribution.
   - Show the validity of the Central Limit Theorem.

6. Generate N random events according to the probability density function $d\Gamma/d\cos\Theta$ and fill the obtained $\cos\Theta$ values in a histogram
   - What is the probability density function associated to the number of entries per bin?.
   - What is the expected p.d.f. for the the number of entries per bin when the number N is very large?
   - Show that the χ^2 of the obtained numbers per bin follows a χ^2 distribution. To simplify, you shall use the nominal value obtained from the p.d.f. formula per bin as the central value in the bin.
   - How does the χ^2 computed above change when you change the number of bins?

7. Take one of the histograms generated in the previous step and construct the conditional probability, P(histogram I P_μ), of obtaining the entries in the obtained histogram for a given value of ρ
   - Find the most probable value of ρ used to generate the histogram using the conditional probability P(histogramIP_μ).