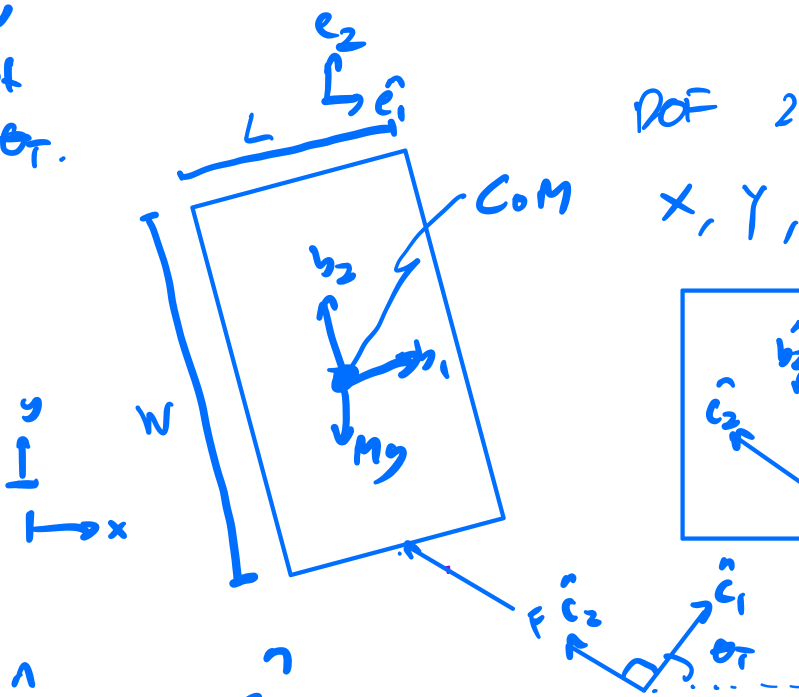
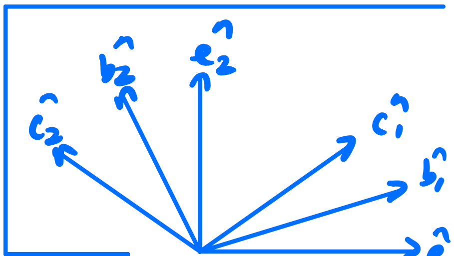
Rigid bar of L, W with input Force of Mag F and angle θ_T.

MIMO system.



DOF $2+1+\cancel{1}$

$x, y, \cancel{\theta_T}, \theta_b$



e_3, b_3, c_3 oop.

LMB

$$M(\ddot{x}\hat{e}_1+\ddot{y}\hat{e}_2) = -Mg\hat{e}_2 + F\hat{c}_2$$

$$= -Mg\hat{e}_2 + F(-\sin\theta_T\hat{e}_1 + \cos\theta_T\hat{e}_2)$$

	$\hat{e}_1$	$\hat{e}_2$
$\hat{c}_1$	$\cos\theta_T$	$\sin\theta_T$
$\hat{c}_2$	$-\sin\theta_T$	$\cos\theta_T$

$$M\ddot{x}\hat{e}_1 + M\ddot{y}\hat{e}_2 = -F\sin\theta_T\hat{e}_1 + (F\cos\theta_T - Mg)\hat{e}_2$$

$\hat{e}_1/$ $\ddot{x} = -\frac{F}{M}\sin\theta_T\hat{e}_1$ $\quad$ $\hat{e}_2/$ $\ddot{y} = \frac{F\cos\theta_T}{M} - g$

$+$ e_1 e_2 e_3

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$$I_G\ddot{\theta}_B = \sum M = -\frac{L}{2}\hat{b}_2 \times F\hat{c}_2 \qquad \left(= \sum_i^{n\text{ forces}} r_i \times F_i\right)$$

	$\hat{e}_1$	$\hat{e}_2$
$\hat{b}_1$	$\cos\theta_b$	$\sin\theta_b$
$\hat{b}_2$	$-\sin\theta_b$	$\cos\theta_b$

$$= -\frac{L}{2}(-\sin\theta_b\hat{e}_1 + \cos\theta_b\hat{e}_2) \times F(-\sin\theta_T\hat{e}_1 + \cos\theta_T\hat{e}_2)$$

$$= \left(\frac{L}{2}\sin\theta_b\hat{e}_1 - \frac{L}{2}\cos\theta_b\hat{e}_2\right) \times \left(-F\sin\theta_T\hat{e}_1 + F\cos\theta_T\hat{e}_2\right)$$

$$= \frac{FL}{2}\sin\theta_b\cos\theta_T\hat{e}_3 - \frac{FL}{2}\cos\theta_b\sin\theta_T\hat{e}_3$$

$\hat{e}_3/$ $$I_G\ddot{\theta}_B = \frac{FL}{2}(\sin\theta_b\cos\theta_T - \cos\theta_b\sin\theta_T)$$

EOM /

(1) $$\ddot{\theta}_B = \frac{FL}{2I_G}(\sin\theta_b \cos\theta_T - \cos\theta_b \sin\theta_T)$$

(2) $$\ddot{x} = -\frac{F}{M}\sin\theta_T$$

(3) $$\ddot{y} = \frac{F\cos\theta_T - g}{M}$$

MIMO:

Do control via state space.

1. write in SS ($Ax + Bu$)
2. Linearize
3. Place.

$$x = \begin{bmatrix} x \\ y \\ \theta_B \\ \dot{x} \\ \dot{y} \\ \dot{\theta}_B \end{bmatrix} \quad SS = 6\times 1 \quad \dot{x} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta}_B \\ \ddot{x} \\ \ddot{y} \\ \ddot{\theta}_B \end{bmatrix} \quad u = \begin{bmatrix} \theta_B \\ F \end{bmatrix}$$

$mg = \cos\theta_T$

$$\dot{x} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta}_B \\ \ddot{x} \\ \ddot{y} \\ \ddot{\theta}_B \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta}_B \\ -\frac{F}{M}\sin\theta_T \\ \frac{F\cos\theta_T - g}{M} \\ \frac{FL}{2I_G}(\sin\theta_b \cos\theta_T - \cos\theta_b \sin\theta_T) \end{bmatrix}$$

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} x$$