MAE 5730 Final Project

Dynamic modelling of a spy ziplining into a objective

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Contents

Project Specification

My project is to model the dynamics of a human taking a zipline through a arbitrarily defined course. The human, or "spy" in the context of this document consists of a 3-link body connected to a point mass. The point mass is representative of the zipline trolley, and the 3 links represent the head-neck connection, neck-waist connection and then waist to toe connection.

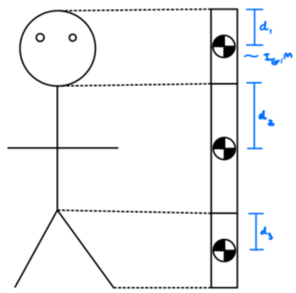


Figure 1 Description of anatomical model

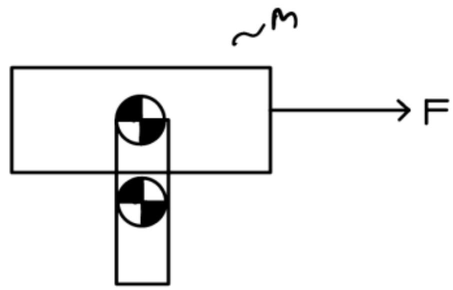


Figure 2 Point Mass Description

A description of the 3 link connection is established visually above in Figure 1 where each of the links are described as rigid bodies which have their own lengths, masses, moments of inertia as well as their rotation as defined from the perpendicular of the previous link. For some more anatomical accuracy, the body's joints will also act as torque dampers.

This body's head is then connected to a point mass which serves as the trolley and the forcing of the system that drives the spy through the course.

Figure 2 describes the point mass setup with the first link (head connection) also being shown. As mentioned, the point mass has a forcing which will drive it through the course.

The course itself will be described by 3 distinct functions (subject to change) and is described visually in figure 3 (I will make this prettier for real report).

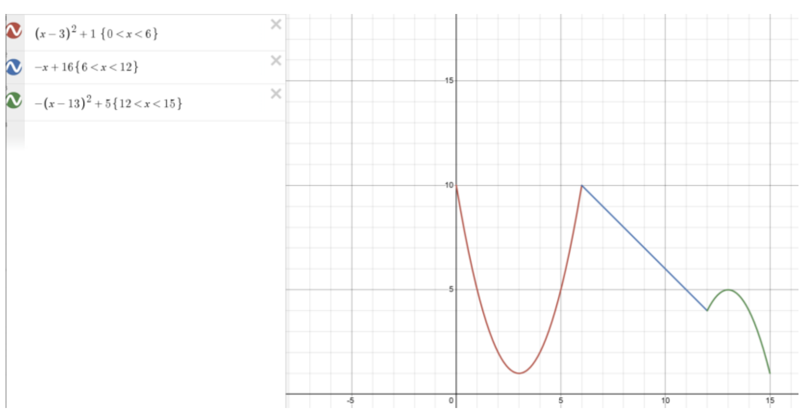


Figure 3 Course Description

The spy's trolley will be attached to the lines shown and follow the course. Where the start is at X = 0, and the end at X = 15. (Subject to Change)

The goal of the project is to develop the model and a visual representation of the system that can be plotted. An additional goal would be to find out how much forcing can be applied to the spy's trolley to get them to the end of the course as quick as possible before the links loop upon themselves which will metaphorically mean that the spy has died and unsuccessfully made it to the end.

Analytical Setup

Global Free Body Diagram

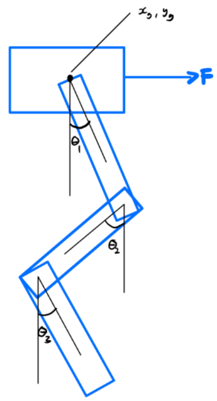


Figure 4 General Free Body Diagram

Figure 4 depicts the general free body diagram. In minimal coordinates the trolley has 2 degrees of freedom in x and y, while the links are free to rotate with one degree of freedom each with their own teeth.

Equations of Motion

Euler-Lagrange Method

Constraints

The setup consists of one force leading to a non-conservative force, one holonomic constraint coming from the course, and one non-holonomic constraint coming from the torque dampers. I will deal with the holonomic constraint by using a Lagrange multiplier. An example of how I deal with the trivial case of in the course where $y = -x + 16$, the holonomic constraint becomes $f(x, y) = y + x - 16 = 0$, with the differential form being $f dt = dx + dy = 0$, which leads to the lagrange multipliers of $f_x = 1, f_y = 1$. These can then be incorporated into the equations of motion calculations. For the torque damper, I'll use the Raleigh dissipation equation to reduce the torque on the joints. Different sections of the course will be piece wised together.

Lagrange Equations

The generalized coordinates are chosen as the minimal set of coordinates $q = x, y, \theta_1, \theta_2, \theta_3$.

The Lagrange equations come from the kinetic and potential energy. The kinetic energy can be found by:

$$E\_k \ = 1/2 * \ (m1 * v_1^2 + m2 * v_2^2 + I_{g1} * th\dot{e}ta_1{}^2 + m3 * v_3^2 + I_{g2} * th\dot{e}ta_2{}^2 + m4 * v_4^2 + I_{g3} * th\dot{e}ta3^2$$

Where:

$$v_1 = \sqrt{x_g'^2 + y_g'^2}$$

$$v_2 = \sqrt{\left(x_g' + d_1\theta_1' \cos(\theta_1)\right)^2 + \left(y_g' - d_1\theta_1' \sin(\theta_1)\right)^2}$$

$$v_3 = \sqrt{\left(x_g' + d_1\theta_1' \cos(\theta_1) + d_2\theta_2' \cos(\theta_2)\right)^2 + \left(y_g' - d_1\theta_1' \sin(\theta_1) - d_2\theta_2' \sin(\theta_2)\right)^2}$$

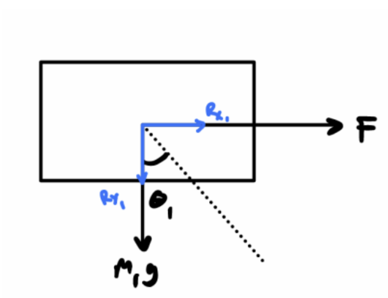
$$v_4 = \sqrt{\left(x_g' + d_1\theta_1' \cos(\theta_1) + d_2\theta_2' \cos(\theta_2) + d_3\theta_3' \cos(\theta_3)\right)^2 + \left(y_g' - d_1\theta_1' \sin(\theta_1) - d_2\theta_2' \sin(\theta_2) - d_3\theta_3' \sin(\theta_3)\right)^2}$$

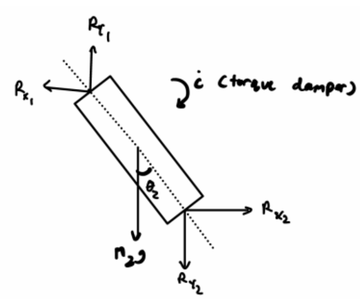
The Potential energy is similarly defined by:

$$PE = g\left(m_1 y_g + m_2\left(y_g - d_1\cos(\theta_1)\right) + m_3\left(y_g - d_1\cos(\theta_1) - d_2\cos(\theta_2)\right) + m_4\left(y_g - d_1\cos(\theta_1) - d_2\cos(\theta_2) - d_3\cos(\theta_3)\right)\right)$$

Newton-Euler DAE Method

Individual Free Body Diagrams





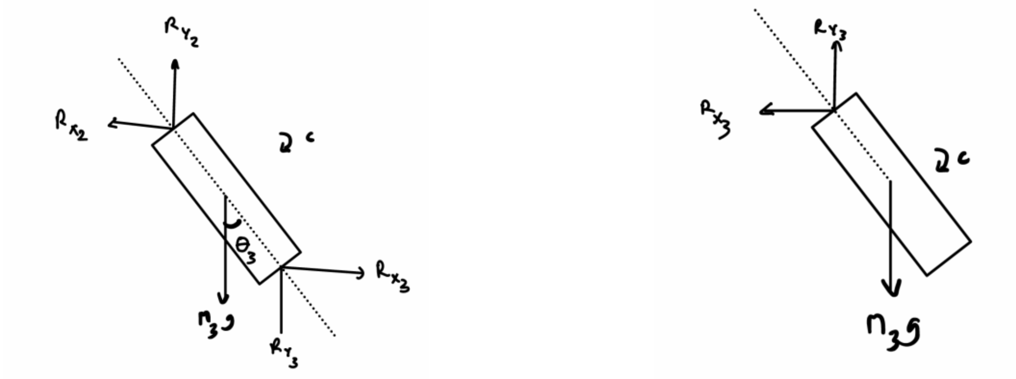
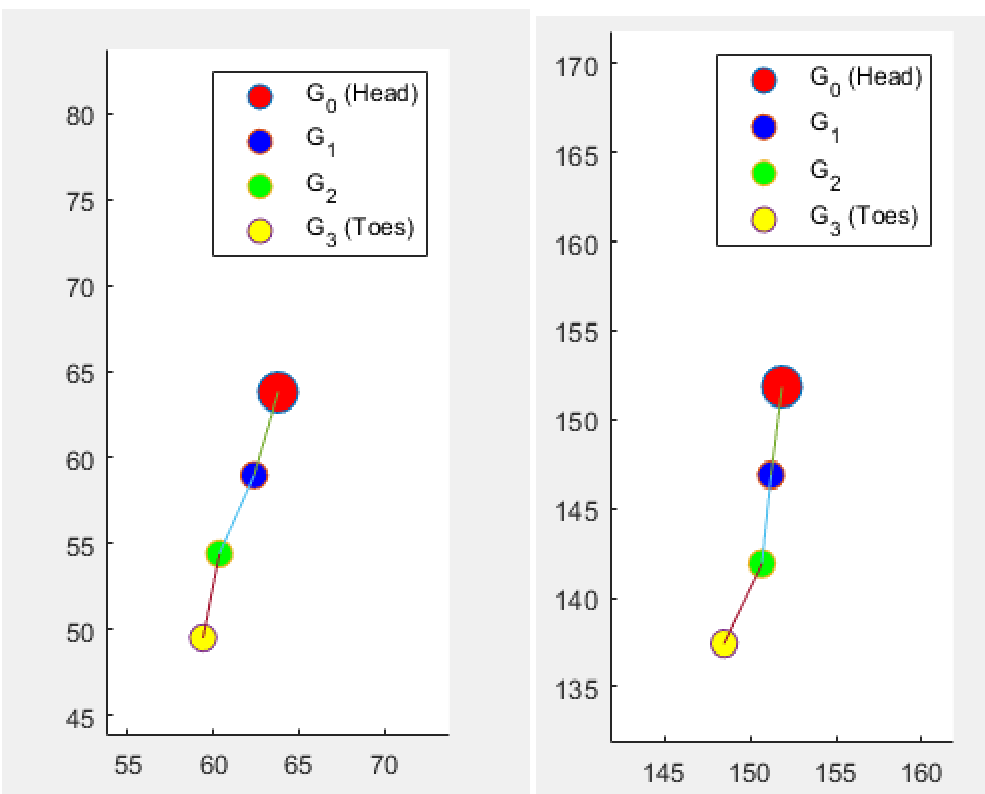


Figure 5 Body Specific Free Body Diagrams

Figure 5 depicts each of the free body diagrams for the trolley and the 3 links comprising each of the limbs.

Animation Still Frames

Currently the Euler Lagrange approach works, but I'm currently working on the DAE method. I'm struggling with incorporating constraints.



Discussion

WHATS LEFT TO DO:

- Finish the Euler Lagrange approach to complete the full course
    - Piecewise connect the equations of motion together for the different holonomic constraints
    - Use the end of the state space (X) as the initial conditions for the next equations of motion
- Continue and finish the DAE method for the equations of motion
- Compare the two methods and try to find the maximum force before the human ends up no longer making into the end alive.
- Finish writing the report

Appendix

Equation's of Motion:

I'm not certain how to display my equations of motion as they are absurdly long.