Simulation of traffic flow

PHYS6017: COMPUTER TECHNIQUES IN PHYSICS

Physicists love trying to apply the techniques they know about to other people's prob-lems (often without bothering to learn what has already been done). Here we study the flow of traffic along a road. There are broadly two approaches to understanding traf-fic flow, one is 'hydrodynamic' model which ignores individual vehicles and models the traffic like a fluid. The other, which we use in this project, is analogous to kinetic theory in that it treats the motion of individual cars.

Consider a long road divided into L sites each of which may be empty or contain a car. There is a total of N cars. Each car has a velocity v which is 'quantised' in that it can take integer values 0, 1, $2 \cdots V_{max}$. Time t also advances in integer steps.

The motion of the cars is described by the following rules which describe how the state of the cars at time $t + 1$ is determined

1. Everyone tries to accelerate up to the speed limit. For each car, if $v < V_{max}$, v is increased to $v + 1$.

2. Drivers don't want to hit the car in front. If a car is at site i and the car in front is at site $i + d$ then if $v \geq d$, v is set equal to $d - 1$. This overrules the tendency to accelerate.

3. Unexpected events cause slowing down. This is a random component that is intended to model external influences, like what is happening on the other carriageway, or just irrational behaviour by drivers. If $v > 0$ then, with a probability p, v is reduced to $v - 1$. This happens after v has been updated by the first two rules.

4. Each vehicle is moved forward v places, using the new value of v.

Coding the simulation There are two ways to code this. The simplest uses two arrays $x\,[\,i\,]$ and $v\,[\,i\,]$ to store the position and velocity of car number i. The disadvantage of this approach is that it is difficult to generalise to more interesting problems. A more flexible approach is to use an array $x\,[\,i\,]$ of size L. If site i is occupied by a car with velocity v

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then $x\ [\ i\ ]$ is equal to v and if it is unoccupied then its value is some otherwise impossible number such as -1.

Start with the cars at rest in random positions. You will need L to be quite large, at least 200 and it is very convenient to pretend the road is a closed circle since this avoids deciding how to treat the ends. Use the built-in random number generator to choose where to put the cars; choose a site and if it is already occupied by a car choose another one until N cars have been inserted.

$V_{max} = 5$ is a good value to try. You will need to use graphical output to see what is happening, a good way is to show the positions of the cars along the road on one axis and successive time steps on the other. Colour coding the velocity makes the pictures easy to interpret. You need to follow the evolution for several hundred time steps.

What to do with the model With $p = 0$ then results are uninteresting; the flow becomes steady at the speed limit if the density permits this. The model with $p > 0$ is more interesting. Try $p = 0.25$. Slow moving traffic jams now develop that are fairly stable and slowly move backwards along the road.

The difficulty with this project is getting some solid conclusions. The model does show some features of real traffic flow but there is no point in investigating a model unless it has some predictive power. The result that jams develop only when $p > 0$ is an interesting but rather weak result. It is difficult to see how to get stronger results that are useful because the simplicity of the model is unlikely to produce quantitative results that could be applied to real roads. The best one can hope for is 'semi-quantitative' results that may be useful for road design and traffic management. An example of a question that can be investigated in the model and might also be relevant to the real world is 'Are journey times best reduced by increasing the speed limit or reducing extraneous diversions that cause random braking'. Note there is rough agreement between the model and the real world if we take one cell to be 7.5 metres, one time step equal to 1 second and V_{max} 120 km/hr.

Traffic flow versus traffic density One useful result you can get is to see how the traffic flow (cars passing a given point per time step) varies with the density (cars per site on the road). The former can be measured by counting for a number of time steps at, say, the end of the road, the number of cars that pass this point. The density is just N/L.

Before taking any measurements you should run the model until a statistically steady state is achieved. You ought also to average the flow measurements over many initial distributions.

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You will need to run for a *very* long time to get a reliable value of traffic flow since you need to count both free flowing vehicles and jams. Since all places on the (circular) road are equivalent you could measure the flow at lots of points along the road provided these are not too close together. This reduces the run time a lot.

You will find that the flow has a maximum; at low densities the flow is small because there are few cars, although these travel near the speed limit. At high densities the flow is small because the velocity drops rapidly. For a given V_{max} and p there is a density that produces the maximum traffic flow. This is quite difficult to find in the simulation because this is the region of the parameters where statistical fluctuations are largest. The changeover from free-flowing to jammed traffic has been regarded as analogous to a phase transition and it is well known that fluctations get large near phase transitions.

Traffic engineers obviously want to know how to get the maximum number of vehicles through a road in a given time, so you can investigate how the maximum flow depends on the parameters in the model. Getting reliable results will take a lot of computer time!

I suggest you aim to get at least one diagram of flow versus density. Beyond that I suggest you *either* investigate how this diagram changes with the parameters *or* do something else with the model. There are lots of things you could measure and lots of generalisations of the model; you are not expected to try more than one of the possibilities. Remember a few reliable results are much more valuable that a lot of uncertain ones.

Other measurements Maximum throughput is not the thing that one might want to optimise. What matters to an individual driver is the journey time, or the distribution of journey times; 'How often will I be late for work?'. Another criterion of driver satisfaction could be the probability of getting caught in any traffic jams.

Questions like 'If a traffic jam develops does it ever disperse without a reduction in density?' or 'What is the average time before it disperses?' 'How many jams are there on average in a given length of road?' can all be addressed with the model. The quantitative conclusions are unlikely to be correct but the way the numbers change with the parameters may give a useful insight for traffic management.

Generalisations There are several generalisations of the model. You can try any of them or invent your own.

- An easy one to model is to let different cars (or rather different drivers) have different values of p. You should see the development of queues behind the slowest driver!

- Use two different values for p depending on whether the driver is in a jam or moving freely. This is done at the beginning of each time step by setting a $flag$ to 1 if $v = 0$ or to 0 otherwise. The updates of v for acceleration and not hitting the car in front are carried out as before but when the random slowing is done p is much larger (say 0.75) if $flag = 1$ and smaller (say 0.02) if $flag = 0$. The interesting feature is the development of long-lived 'metastable' states of high flow that develop spontaneously into mega-jams and free flows. These states can occur when external conditions make drivers drive more carefully, in a tunnel for example.

- Model a finite section of road with a fixed rate of injection of cars at one end and a fixed rate of letting them off at the other.

- The obvious generalisation is to model two or three lane roads with overtaking.

- Without worrying about overtaking you could model two lanes (each with freely flowing traffic following the simple model) merging into a single one with some rule about who goes next.

Remember that you are trying to get results that are stronger than wooly phases like 'the model shows some resemblences to real traffic'. You need some useful predictions. There is also no point in over elaborate generalisations with lots of parameters. Keep it simple!

Further Reading

K Nagel and M Schreckenberg, Journal de Physique I, vol 2, pages 2221-2229, Dec 1992.