Congestion Management Important Points

- *GOAL*
  - Optimization of routing autonomous vehicles (both passenger vehicles and empty rebalancing vehicles) in congested / capacitated transportation networks.
  - Decoupling the two problems:
    - Routing customers
    - Rebalancing vehicles

- *CONTRIBUTIONS*
  - Customer-carrying and empty rebalancing vehicles are represented as **flows over a capacitated road network**. When the flow of vehicles along a road reaches a critical capacity value, congestion effects occur.
  - A cut condition for the road graph needs to be satisfied for congestion-free customer and rebalancing flows to exist.
  - Under the assumption of a symmetric road network:
    - rebalancing does not increase congestion
    - For certain cost functions, the problems of finding customer and rebalancing flows can be decoupled.

- *CONGESTION MODEL*
  - Free flow phase of traffic: In classical traffic flow theory, at low vehicle densities on a road link, vehicles travel at the free flow speed (approximately constant) of the road (imposed by the speed limit).
  - Flow rate: Number of vehicles passing through the link per unit time.
    - Product of the speed and density of vehicles.
  - Capacity of road link: Maximum stationary flow rate.
    - Beyond the critical flow rate, vehicle speeds are dramatically reduced and the flow decreases, signaling the beginning of traffic congestion.
  - Road capacities are constraints on the flow of vehicles. The model captures the behavior of vehicles up to the onset of congestion.
  - Did not model:
    - delays at intersections
    - spillback behavior due to congestion
    - bottleneck behavior due to the reduction of the number of lanes on a road link.

- *Network Flow Model of AMoD system*

  Directed graph $G = (V, E)$

  V ---> node set and $E \subseteq V \times V$ ---> the edge set

Each constraint represents the capacity of the road upon the onset of congestion.
c(u, v) : ε → N >0: Capacity of link, for each road link (u, v) ∈ E
t(u, v) : ε → R ≥0: Free flow time required to traverse road link (u, v)

Flow rate (u,v) > c(u,v),   t(u,v) = ∞
Therefore, avoid the onset of congestion.

Overall capacity entering each node = Capacity exiting each node

$$\sum_{(u,v)\in\mathcal{E}:\ u\in\mathcal{S},\, v\in\bar{\mathcal{S}}} c(u,v) = \sum_{(v,u)\in\mathcal{E}:\ u\in\mathcal{S},\, v\in\bar{\mathcal{S}}} c(v,u)$$

$$\sum_{u\in\mathcal{V}:(u,v)\in\mathcal{E}} c(u,v) = \sum_{w\in\mathcal{V}:(v,w)\in\mathcal{E}} c(v,w)$$

$\mathcal{M} = \{(s_m, t_m, \lambda_m)\}_m$: set of transportation requests
s ∈ V ---> origin of the requests
t ∈ V ----> destination
λ ∈ R >0 ---> rate of requests (customers per unit time)

f m (u, v) : ε→ R ≥0 , m = {1, . . . , M }, the amount of flow from origin s m to destination t m that uses link (u, v).
f R (u, v) : ε→ R ≥0, the amount of rebalancing flow traversing edge (u, v) needed to realign the vehicles with the asymmetric distribution of transportation requests.

- *The Routing Problem*
  - **Cost Function:** Weighted sum (with weight ρ) of the overall duration of all passenger trips and the duration of rebalancing trips.
  - **Constraints:**
    - Continuity of each trip, i.e., flow conservation across nodes.
    - Ensure every outbound customer flow is matched by an inbound flow of rebalancing vehicles and vice-a-versa.
    - Capacity constraint on each link
  - $$V_{\min} = \left\lceil \sum_{m\in\mathcal{M}} \sum_{(u,v)\in\mathcal{E}} t(u,v)\left(f_m^*(u,v) + f_R^*(u,v)\right) \right\rceil$$

- Minimize the number of vehicles needed to operate an AMoD system.

- ***Existence of Congestion-free flows***
  - Assume there exists a set of feasible customer flows {f m (u, v)} (u,v),m ,but there does not exist a set of feasible rebalancing flows {f R (u, v)} (u,v) . Then, there exists a partial rebalancing flow { f ˆ R (u, v)} (u,v) that induces a graph cut (S, S¯) with the following properties: (i) all defective destinations are in S, (ii)all defective origins are in S¯, and (iii) all edges in (S, S¯) are saturated.
  - For symmetric road networks it is always possible to rebalance the autonomous vehicles without increasing congestion – in other words, the rebalancing of autonomous vehicles in a symmetric road network does not lead to an increase in congestion.
  - If the cost function in the CRRP only depends on the customer flows (that is, ρ = 0 and the goal is to minimize the customers' travel times), then the CRRP problem can be decoupled and the customers and rebalancing flows can be solved separately without loss of optimality.
  - The set of (integral) rebalancing flows {f R (u, v)} (u,v) is then decomposed into a set of rebalancing paths via a flow decomposition algorithm. Each rebalancing path connects one origin region with one destination region: thus, rebalancing paths represent the set of routes that excess vehicles should follow to rebalance to regions with a deficit of vehicles.