COGA
Combinatorial Optimization & Graph Algorithms

Applied Network Optimization (for Traffic and Transportation)

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April 14, 2015

DFG Research Center MATHEON
mathematics for key technologies

Content

- Part 1: Traffic guidances policies in rush hour traffic
  - Multicommodity flows, traffic flows, user equilibrium and system optimum, price of anarchy, traffic guidance policies with fairness constraints, Kuhn Tucker conditions and Frank-Wolfe algorithm for computing such policies
- Part 2: Traffic guidance with tolls
  - Traffic flows and tolls, characterizing traffic flows that can be enforced by tolls, computing such tolls with linear programming, what can be achieved by restricting tolls to subnetworks
- Part 3: Periodic time tabling
  - Periodic time tabling in train traffic, characterizing periodic time tables via the cycle space of the underlying graph, computing optimal periodic timetables with integer programming

References

- Part 1: Traffic guidances policies in rush hour traffic

  - O. Jahn, R. H. Möhring, A. S. Schulz, and N. E. Stier Moses. *System-optimal routing of traffic flows with user constraints in networks with congestion*. Oper. Res., 53(4):600–616, 2005.
  - E. Köhler, R. H. Möhring, and M. Skutella. *Traffic networks and flows over time*. LNCS 5515, pages 166–196, 2009.

- Part 2: Traffic guidance with tolls

  - L. K. Fleischer, K. Jain, and M. Mahdian. *Tolls for heterogeneous selfish users in multicommodity networks and generalized congestion games*. In Proceedings of the 45th Annual Symposium on Foundations of Computer Science (FOCS), pages 277–285, 2004.
  - T. Harks, I. Kleinert, M. Klimm, and R. H. Möhring. *Computing network tolls with support constraints*. Technical Report, to appear in Networks

- Part 3: Periodic time tabling

  - C. Liebchen and R. H. Möhring. *The modeling power of the periodic event scheduling problem: Railway timetables - and beyond*. LNCS 4359, pages 3–40.

Some of my applied projects

- Adaptive Traffic Control

  PTV GROUP the mind of movement

  Bundesministerium für Bildung und Forschung

- Routing of AGVs in the Hamburg harbor

  HHLA

  Bundesministerium für Bildung und Forschung

- Constructing periodic timetables in public transport

  PTV GROUP the mind of movement

  Die Bahn DB

  BVG

  S Bahn Berlin Deutsche Bahn Gruppe

- Coordinated traffic light control in networks

  PTV GROUP the mind of movement

  Bundesministerium für Bildung und Forschung

- Ship Traffic Optimization for the Kiel Canal

  WSV.de Wasser- und Schifffahrtsverwaltung des Bundes

Adaptive Traffic Control

- Decentralized methods for optimizing traffic flows, *Sándor Fekete, TU Braunschweig*
- An integrated model for traffic assignment and traffic light optimization in traffic networks, *Ekkehard Köhler, BTU Cottbus*
- Optimized traffic guidance in large scale micro-simulation, *Rolf Möhring, TU Berlin*
- Planning and guiding traffic in megacities, *Kai Nagel, TU Berlin*
- Traffic guidance at great events and evacuation planning, *Martin Skutella, TU Berlin*

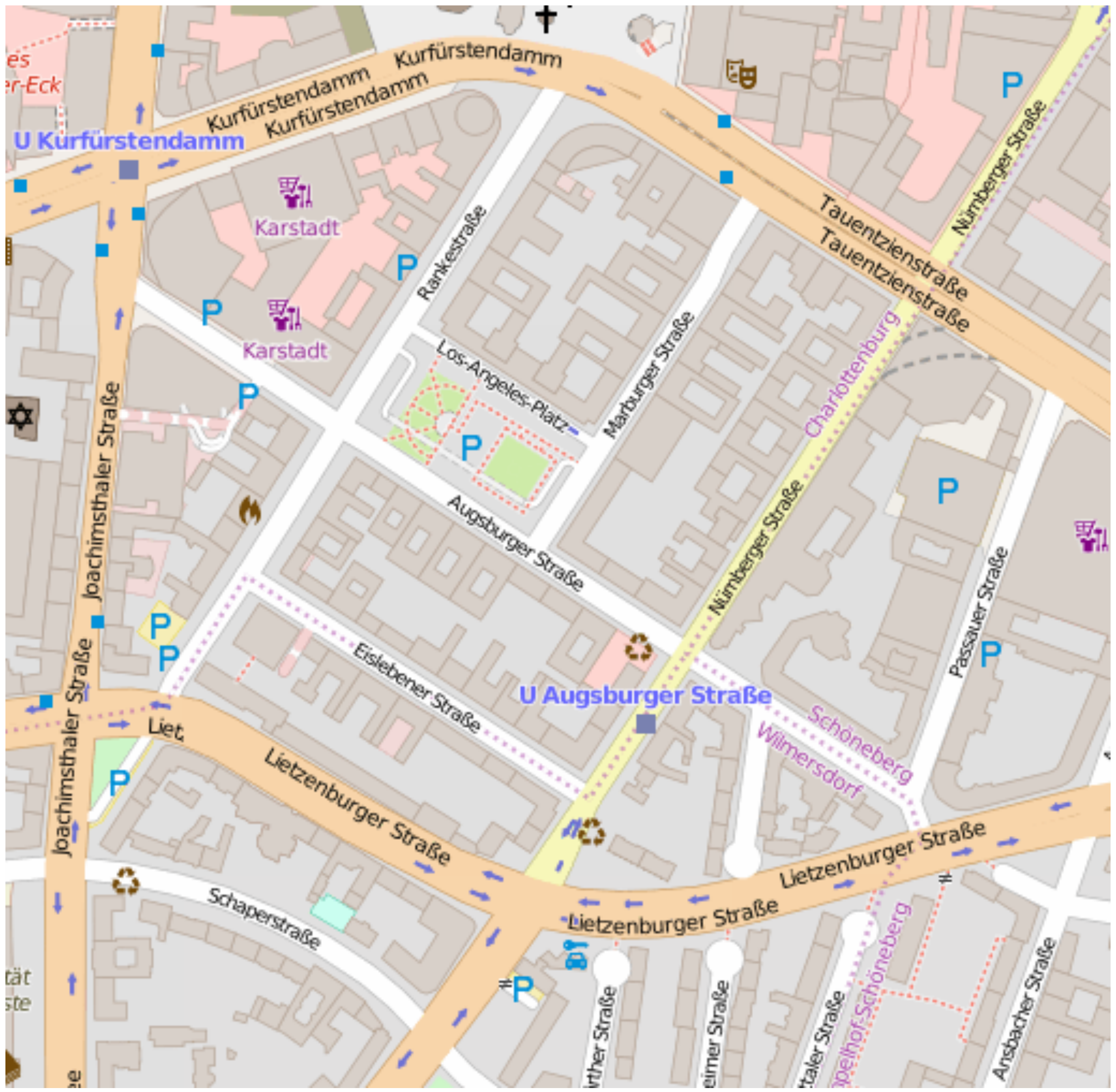
Bundesministerium für Bildung und Forschung

Technische Universität Braunschweig

Brandenburgische Technische Universität Cottbus

http://www3.math.tu-berlin.de/coga/projects/traffic/advest/

Modeling of street maps as graphs



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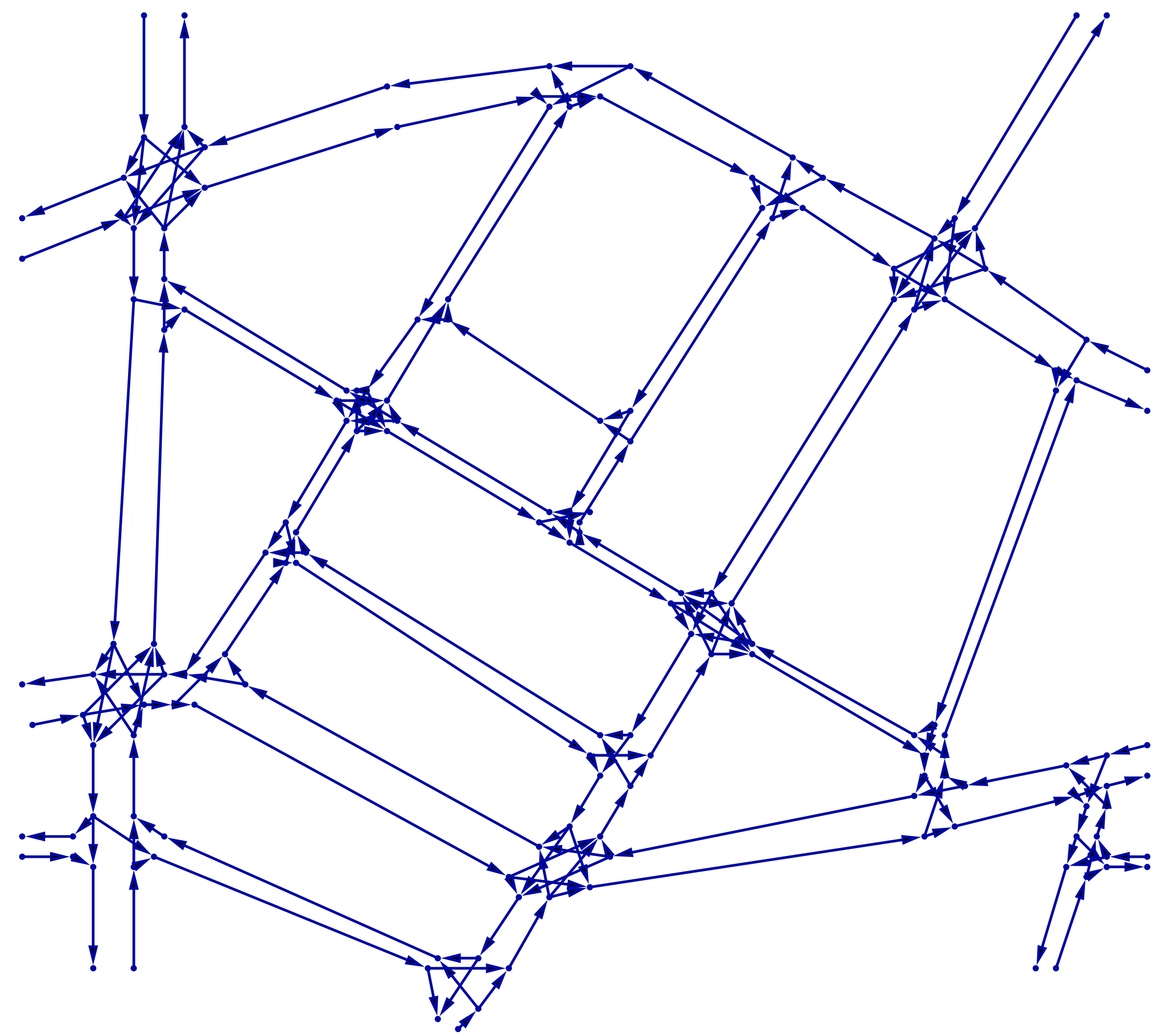


Berlin graph: 10,000 nodes 30,000 arcs

Germany graph: 4.8 mio nodes 11.9 mio arcs

Central theme

[with Tobias Harks, Max Klimm, Ingo Kleinert]

Microscopic ⟷ **Macroscopic**

Microscopic

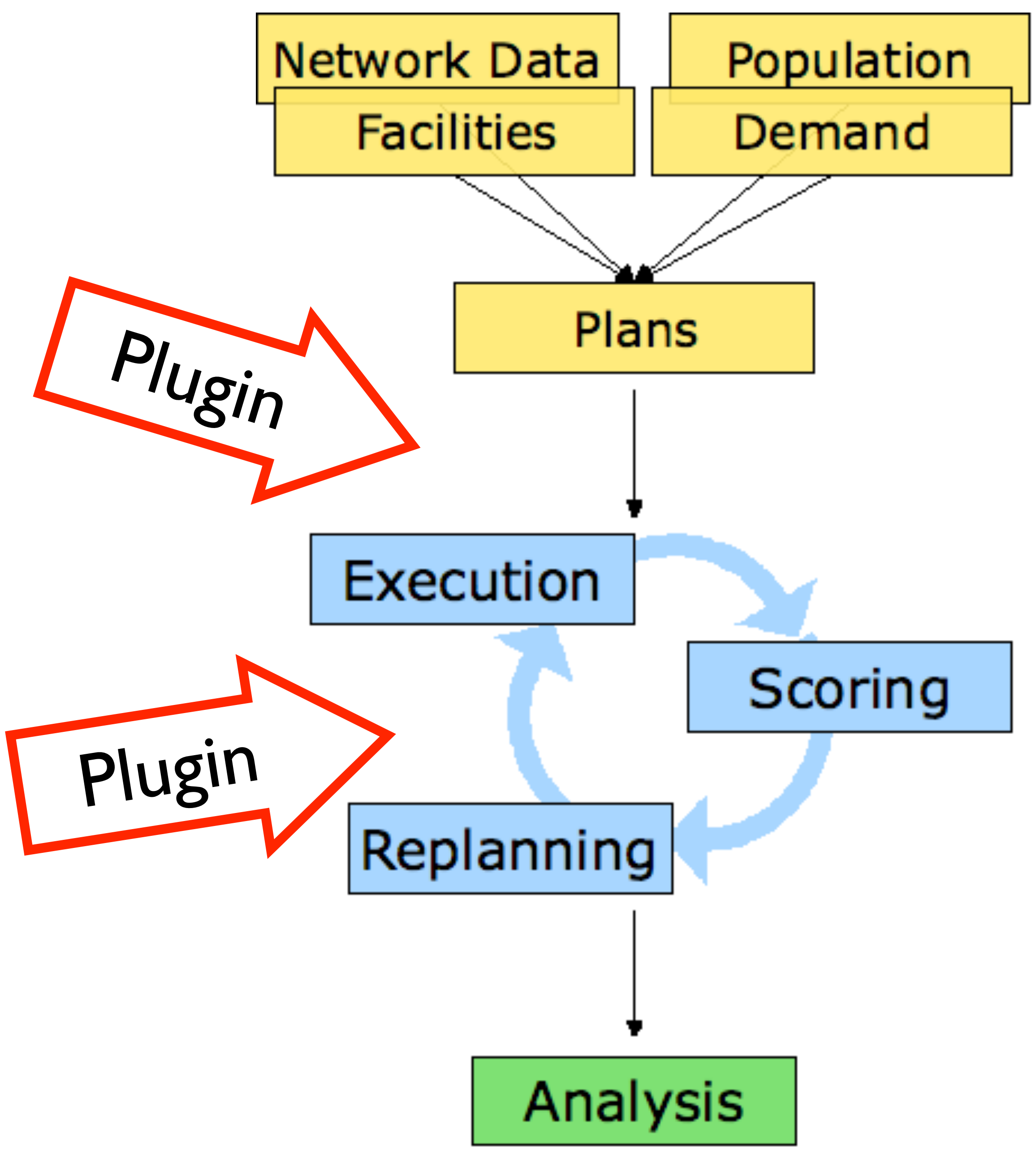
- Detailed simulation
- Study of many real life effects
- No optimization in the mathematical sense

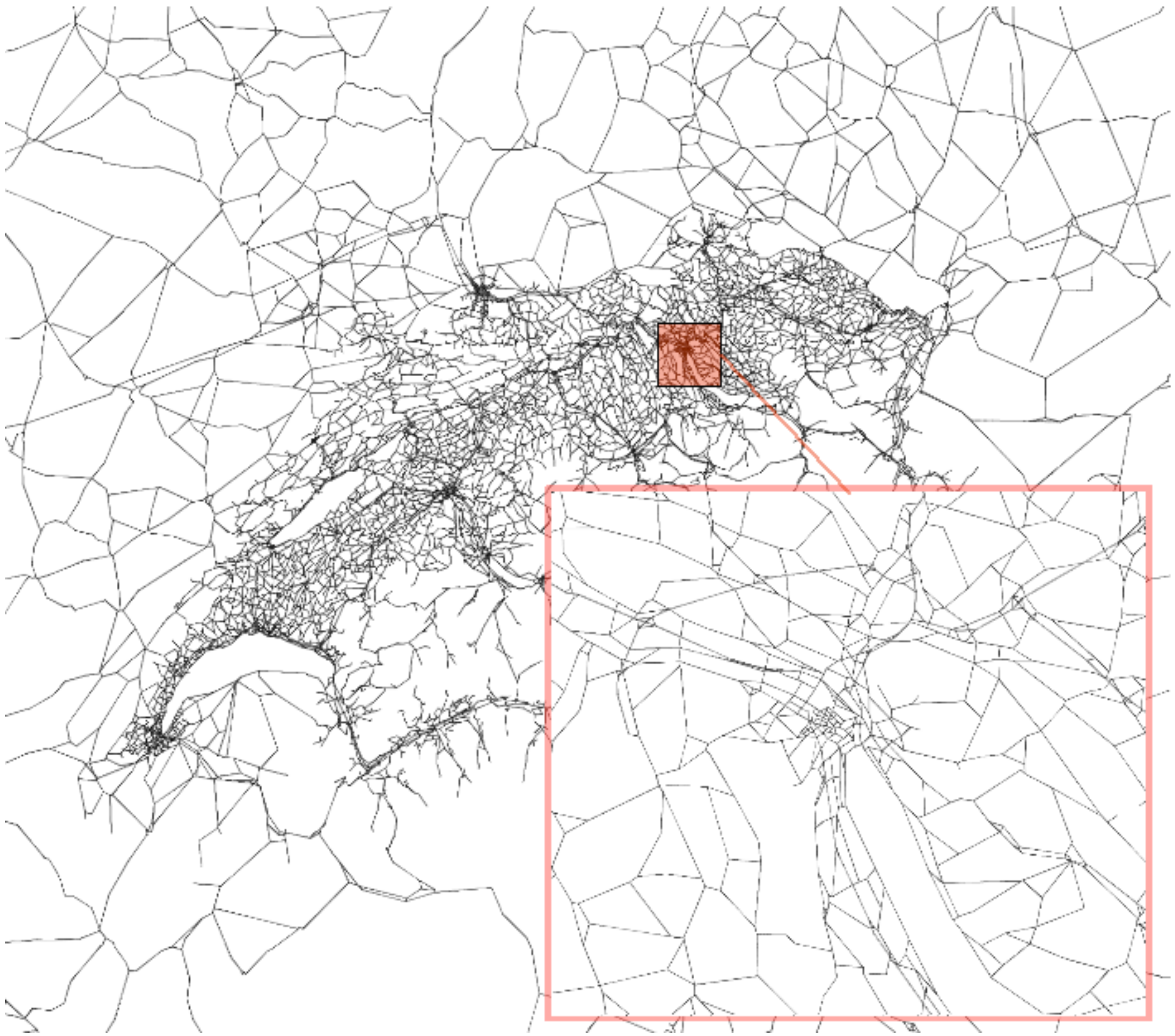
Macroscopic

- Coarse models
- Miss many real life aspects
- Optimization methods available or under development

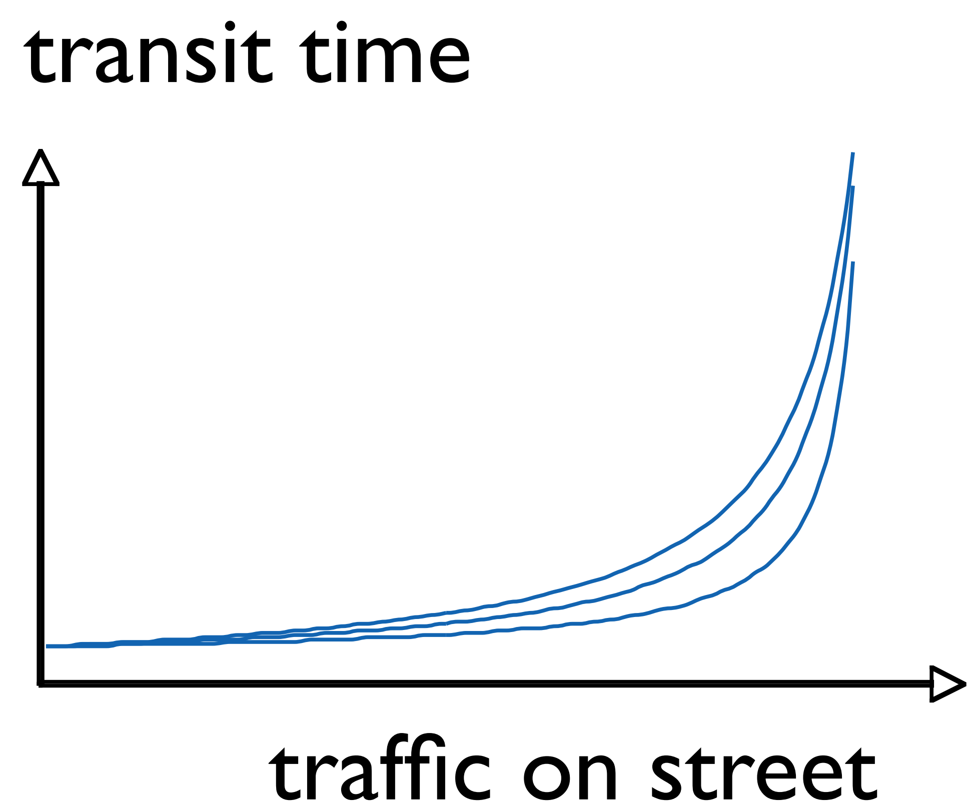
The MATSim microsimulation

www.matsim.org [Kai Nagel et al.]



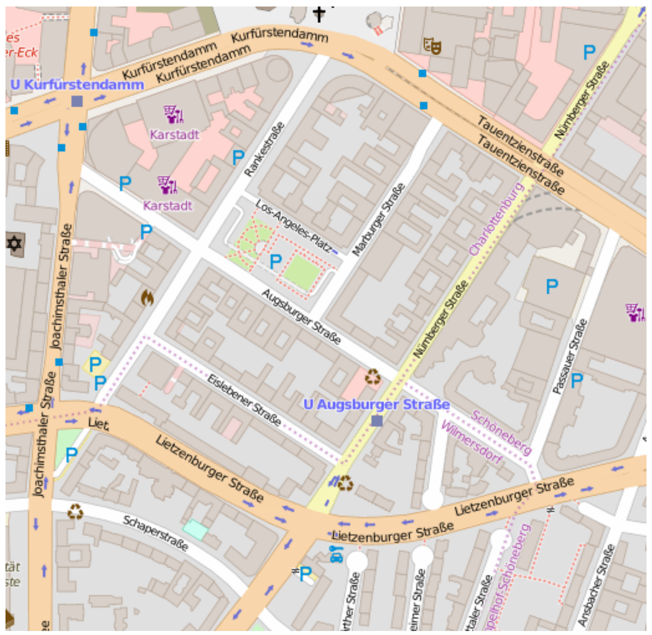


Modeling congestion



Streets have capacities ... and traffic-dependent transit times

An optimization model for the rush hour



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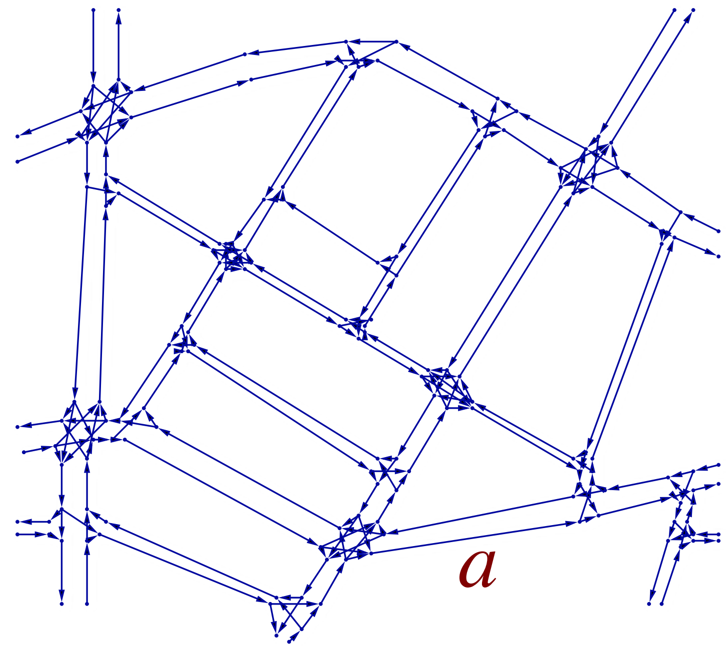
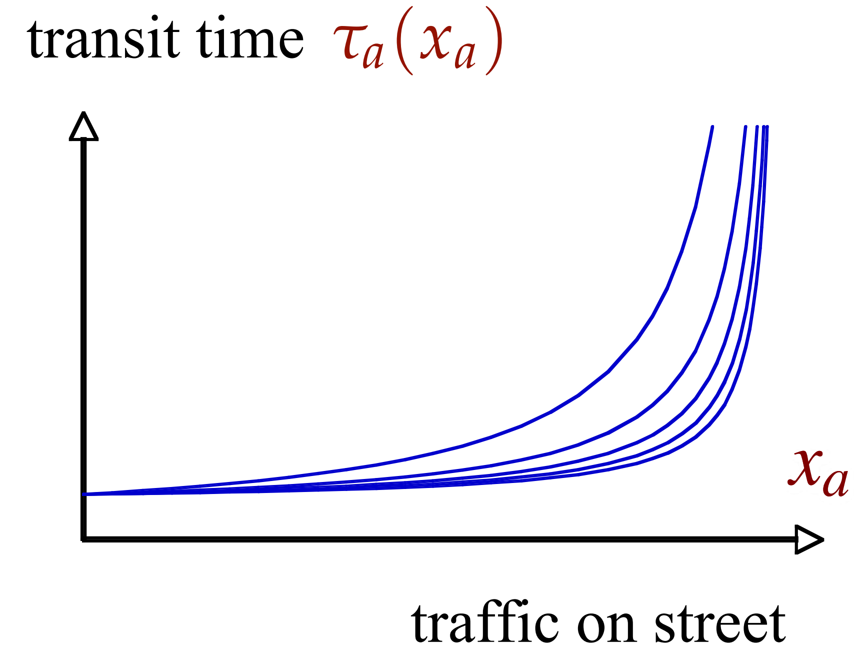




- Street graph with capacities and transit time functions
- Origin-destination demands for the rush hour
- Route the demand subject to the capacities such that the total travel time is "small" (system optimum)

$\sum_{a \in A} x_a \cdot \tau_a(x_a)$

Selfish routing leads to “user equilibrium”

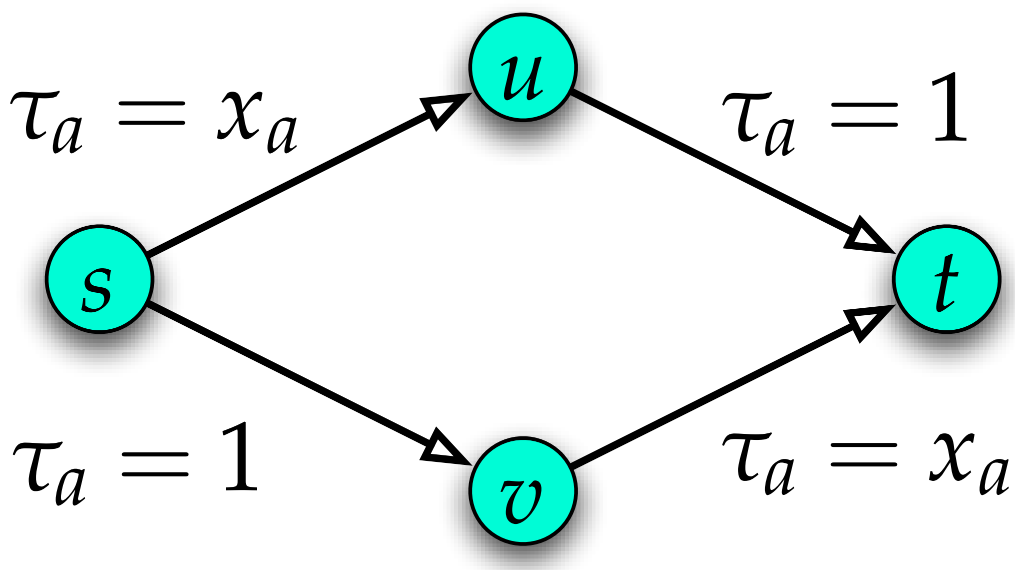
Nash equilibrium

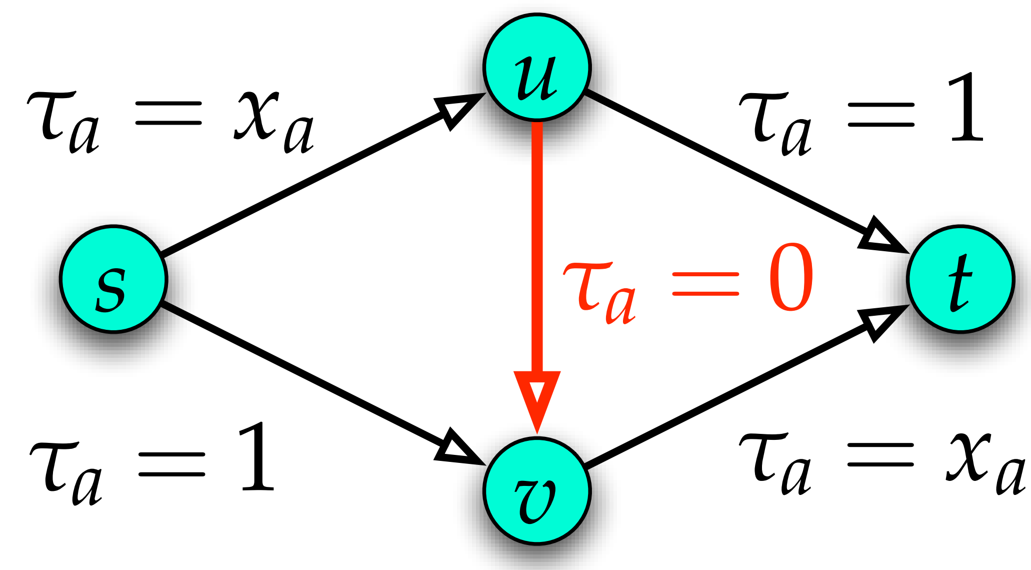
- Nobody can improve his route just by himself
- All alternatives take at least as long as the current route

User equilibrium is paradox ridden

User equilibrium has higher network load than necessary
Price of Anarchy

The Braess paradox of the user equilibrium





Send $d = 1$ from s to t

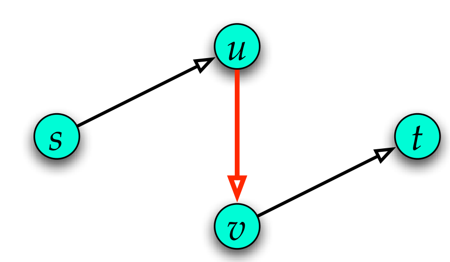
Send $1/2$ along each path

$\Rightarrow \tau_P(f) = 3/2$ on each path

$$\Rightarrow \sum_{P \in \mathcal{P}} x_P \cdot \tau_P(f) = 3/2$$

Build new fast road

Send 1 along

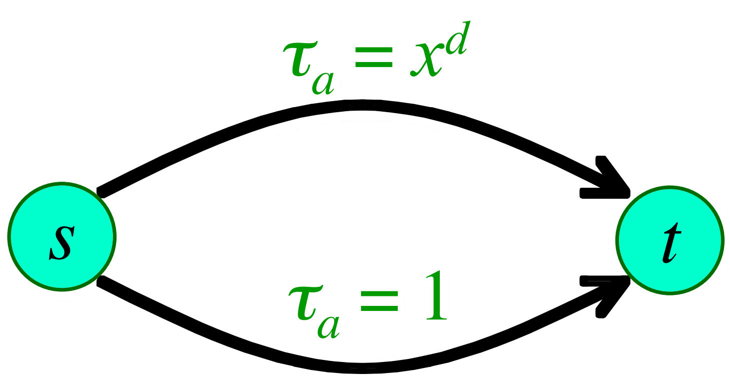


$\Rightarrow \tau_P(f) = 2$ on that path

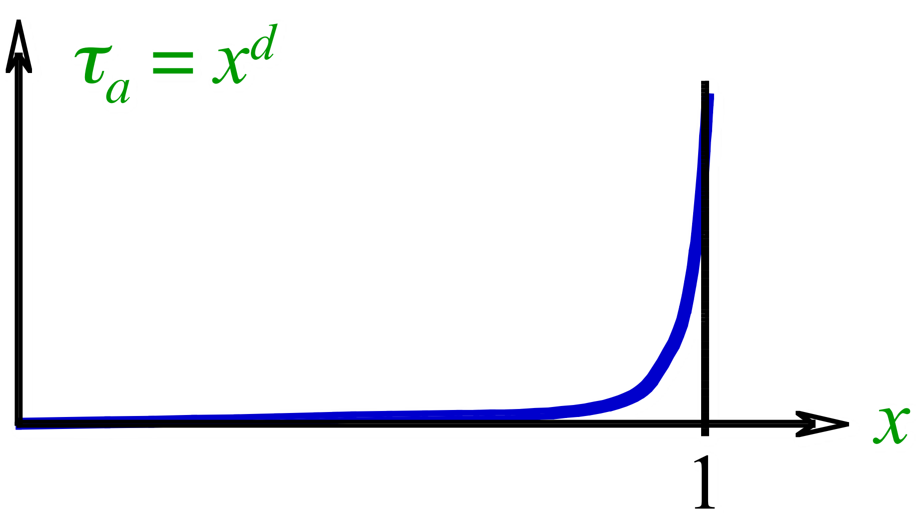
The price of anarchy

$$\text{Price of anarchy} := \frac{\text{Network load in user equilibrium}}{\text{Network load in system optimum}} = \frac{\text{UE}}{\text{SO}}$$

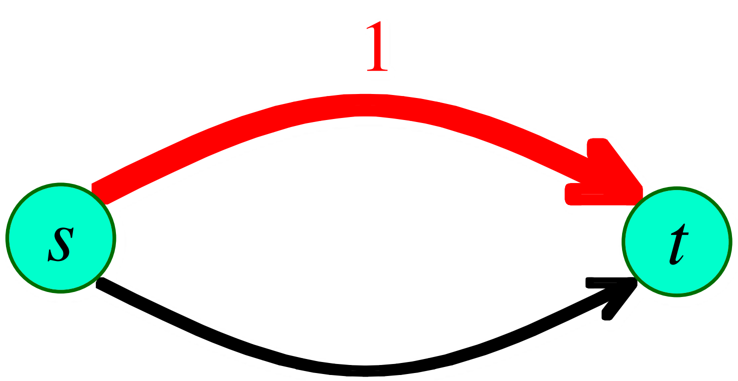
Pigou's example [1920]



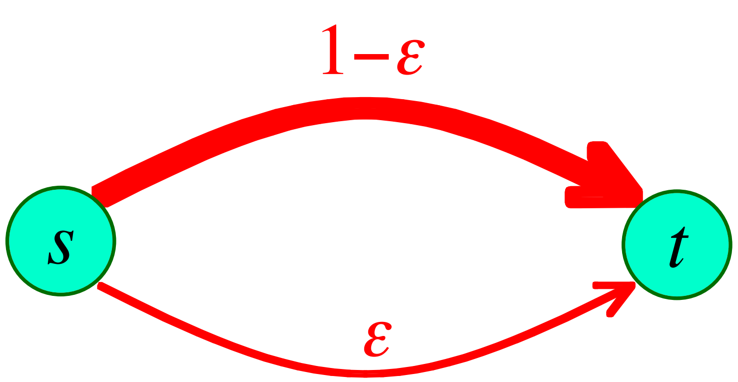
Demand b = 1



User equilibrium has value 1

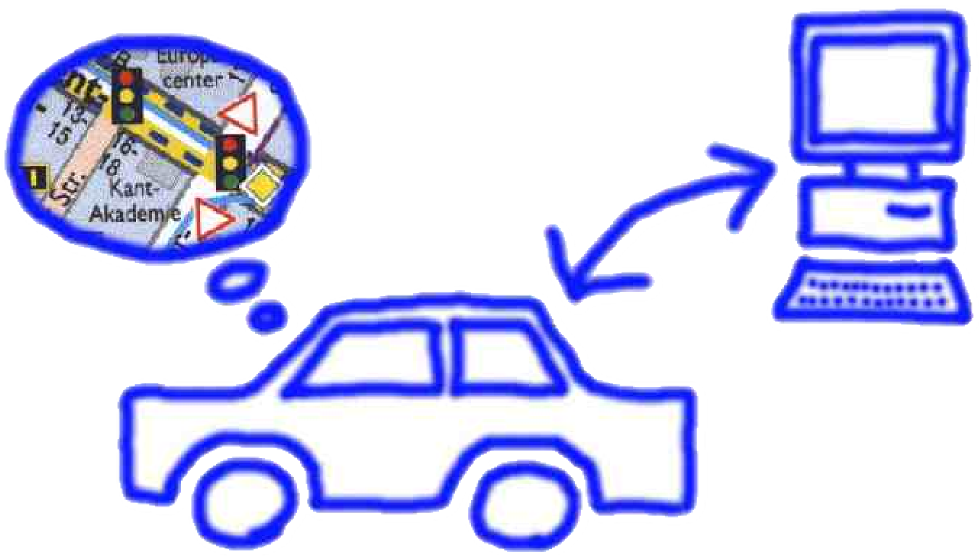


System optimum has value ~ 0



Centralized traffic management

Technological requirements



- exact knowledge of current position
- 2-way communication to a main server
- server has "complete" knowledge" of current state

- Can achieve the "system optimum"

But

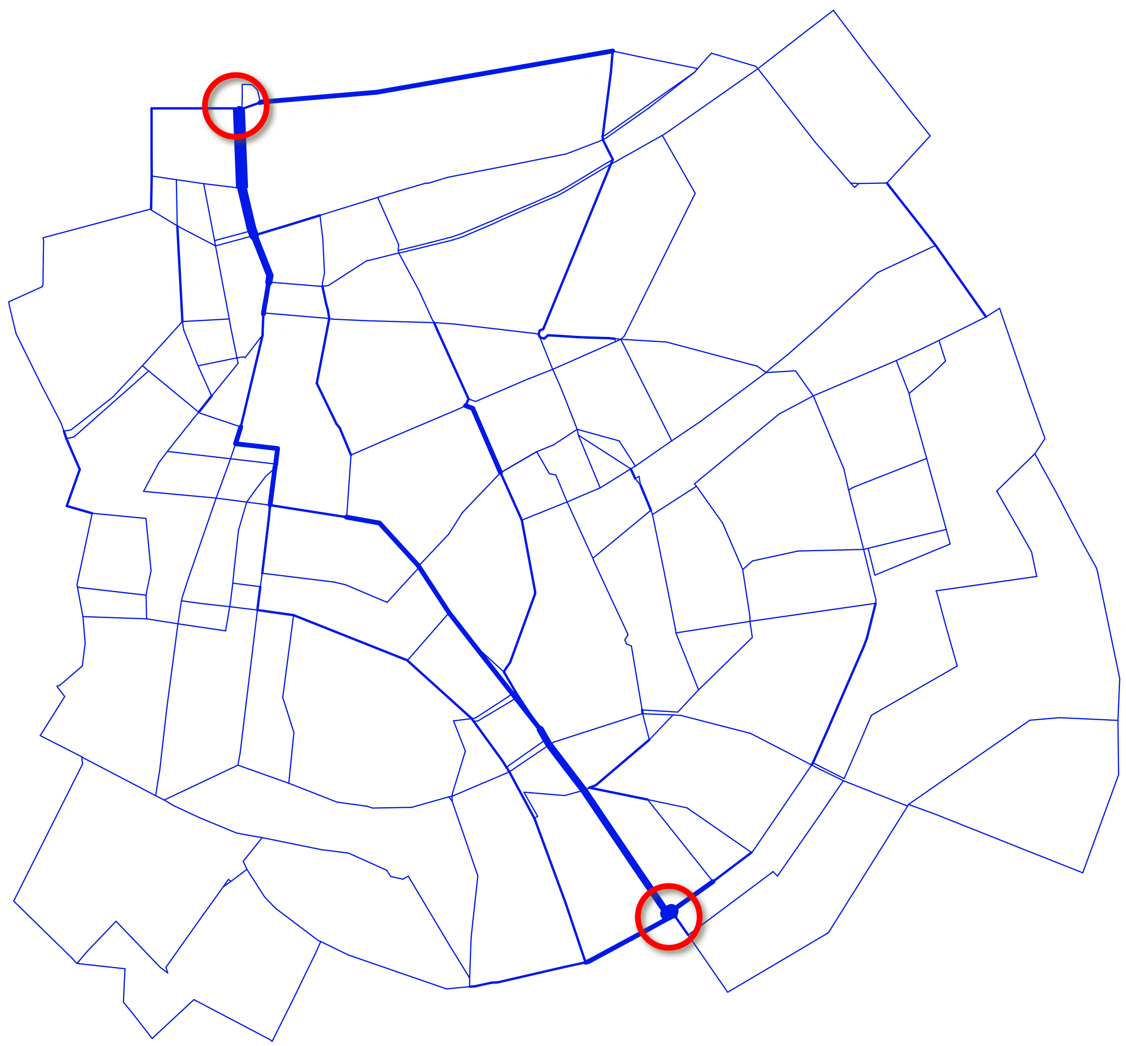
- Individual routes may be far too long!

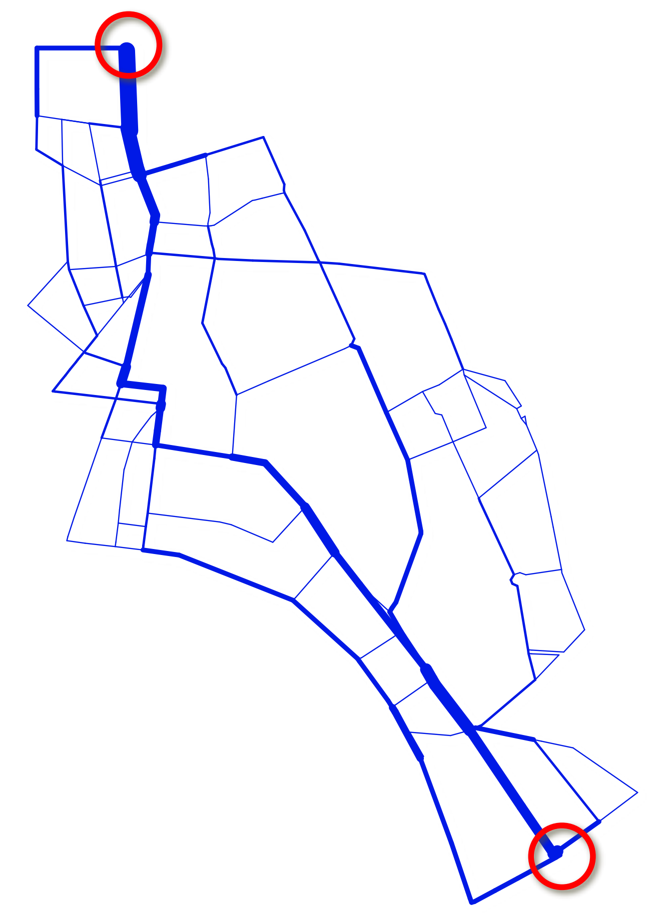
Central assignment of routes

System optimum without ...

... and with fairness conditions

Central assignment of routes





System optimum without ... ... and with fairness conditions

Definition of fairness

Unfairness of a route guidance policy $:=$

$$\max_{\text{all OD pairs } i} \frac{\text{maximum travel time on a route for OD Paar } i}{\text{travel time in user equilibrium for OD Paar } i}$$

Unfairness (user equilibrium) $= 1$

Unfairness (system optimum) may be arbitrarily large

Our "fairness" algorithm CSO

[Jahn, M., Schulz, Stier 2005]

- Fractional multicommodity flow problem
  - convex separable objectice
  - side constraints on routes

Berlin in 20 minutes

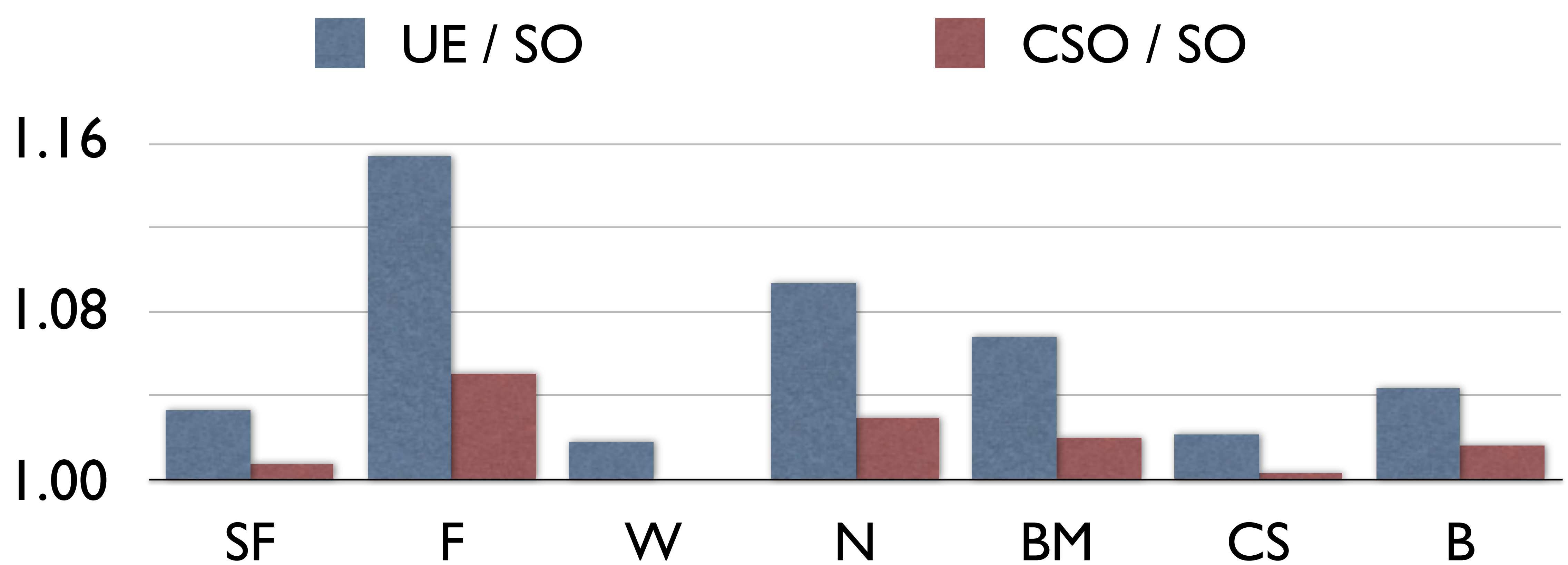
Gradient method (variant of Frank-Wolfe)

Simplex algorithm with column generation

Computing constrained shortest paths

Shortest paths (e.g. Dijkstra)

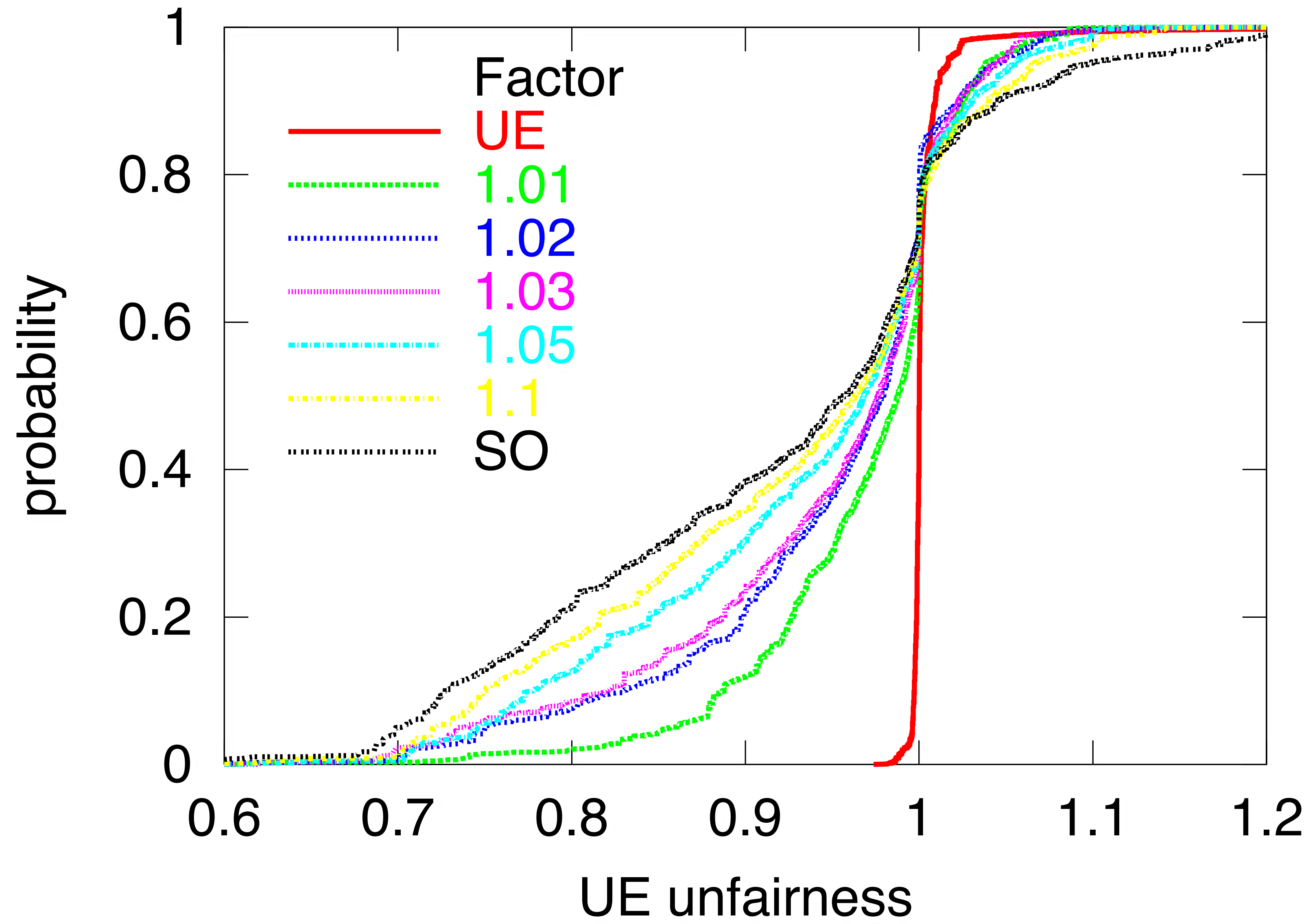
Results for some cities



SF Sioux Falls	F Friedrichshain	W Winnipeg	N Neukölln
BM Berlin Mitte	CS Chicago Sketch	B Berlin	

- initial theory [Roughgarden 03] $\frac{UE}{SO} \leq 2.151$
- improved analysis [Correa, Schulz, Stier 05] $\frac{UE}{SO} \leq 1.365$

Analysis of fairness



- 75% of the users travel less than in equilibrium
- Only 0.4% of the users travels 10% more than in equilibrium
- For the system optimum, these are more than 5%

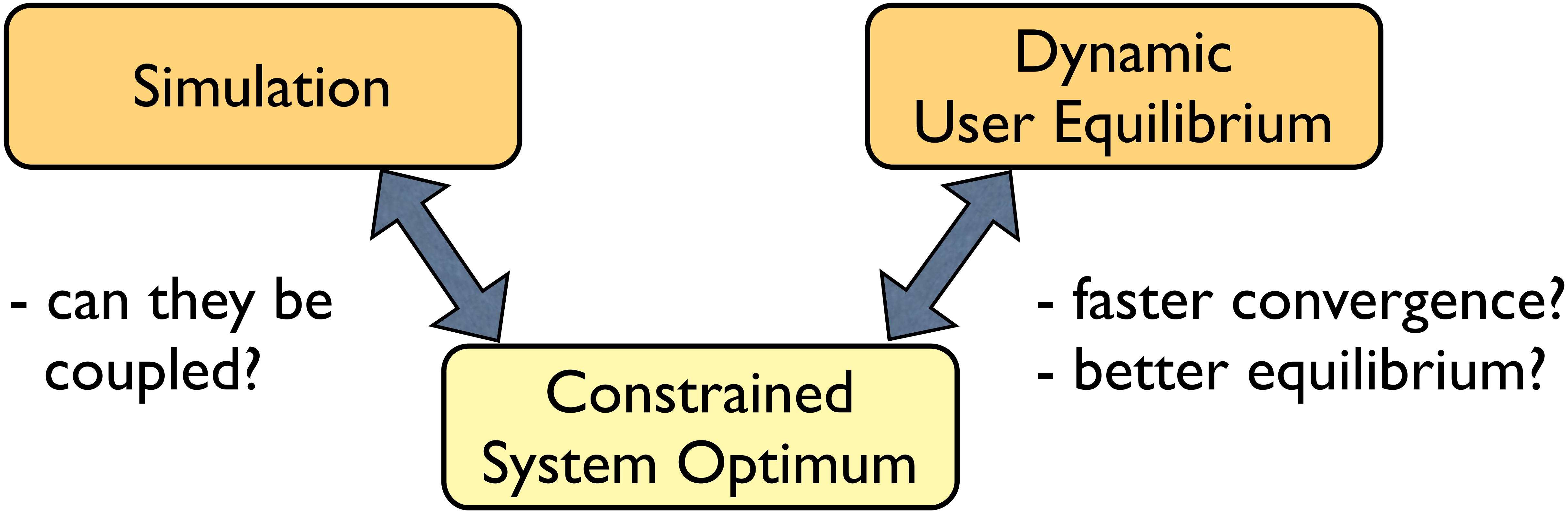
Dynamic flows difficult to analyze

- Only simulation works in large networks
- Problems become NP hard
- Ekkehard Köhler, Martin Skutella: Theory of such flows, approximation algorithms for idealized problems

transit time determined by inflow rate into arc	transit time determined by total flow on arc
there is an approximation scheme	only 2-approximation APX-hard
FIFO not satisfied	closer to real life, used in simulation models

Questions and results

[Tobias Harks, Max Klimm, Ingo Kleinert, M. 2011]



Stackelberg routing

„TomTom view“

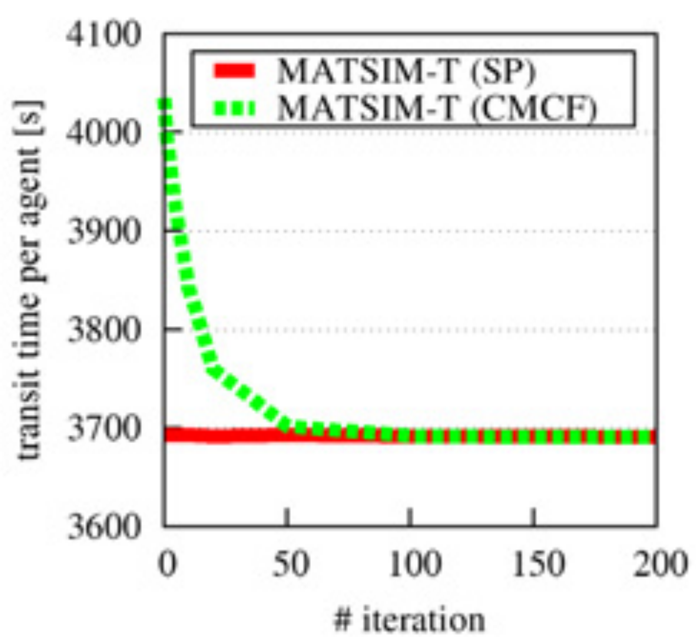
Congestion pricing

„Government view“

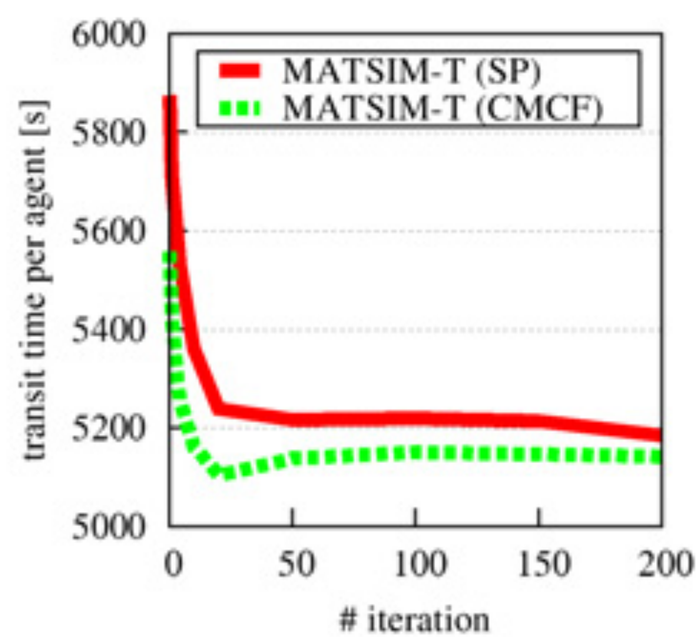
Optimization helps simulation

- Use optimal routes from constrained system optimum as start routes in simulation
- Tests with MATSim

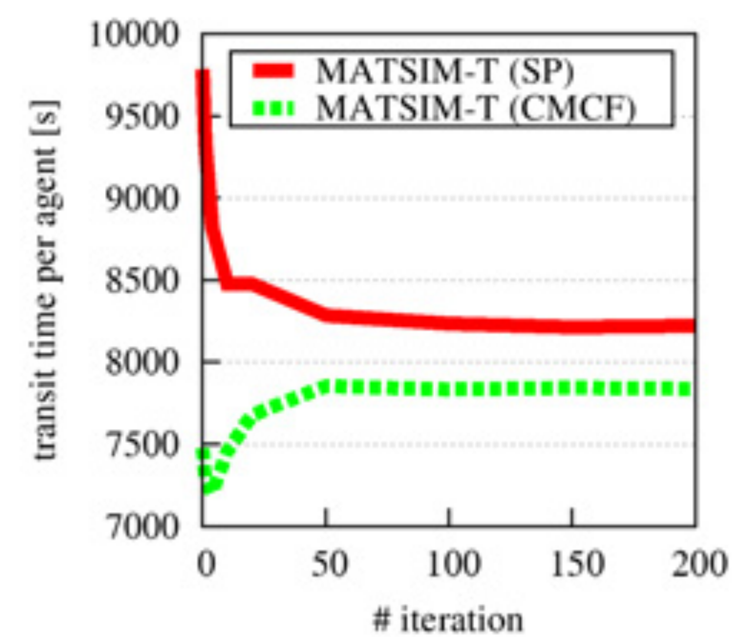
Results for Zurich



2,000 OD pairs

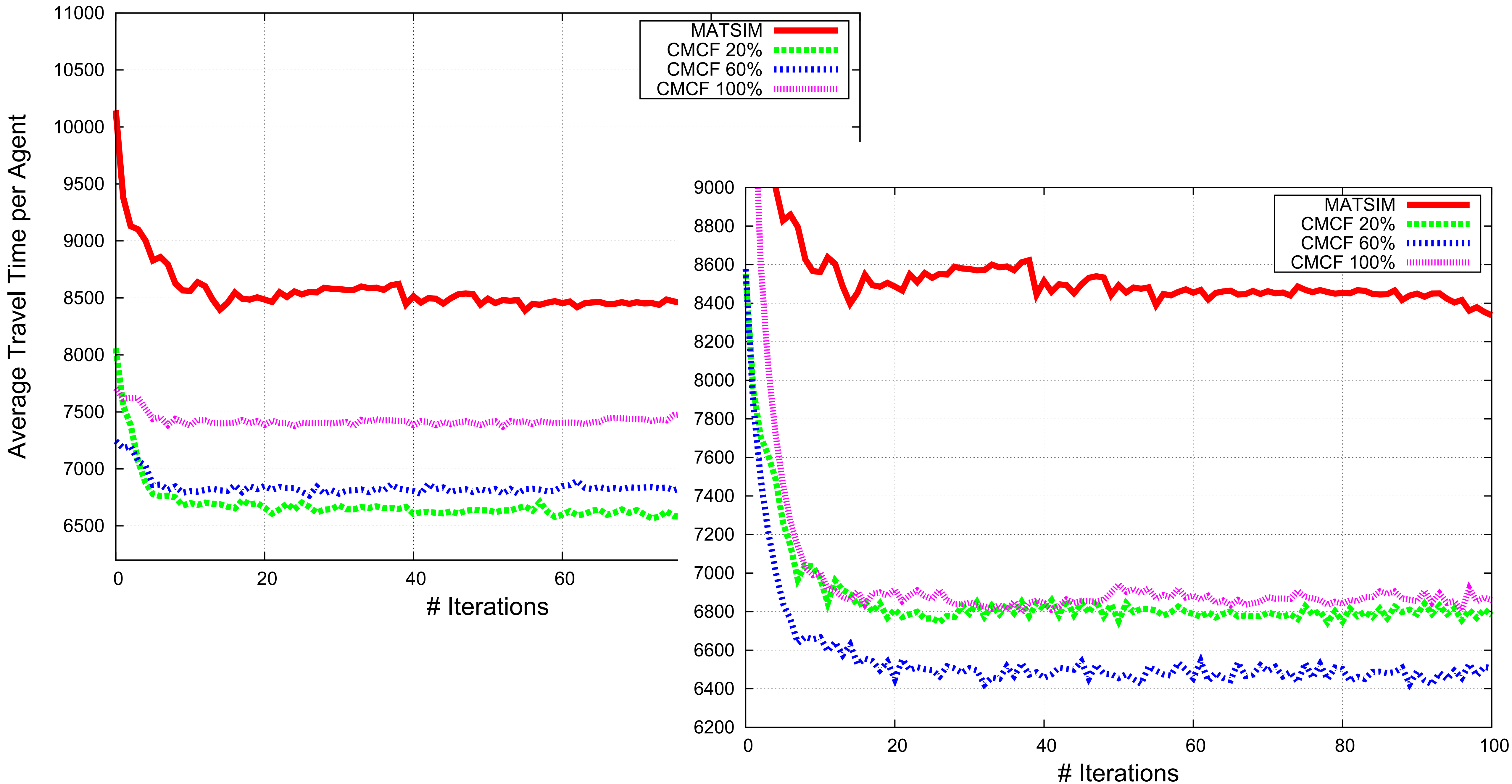


20,000 OD pairs



50,000 OD pairs

Looking more closely



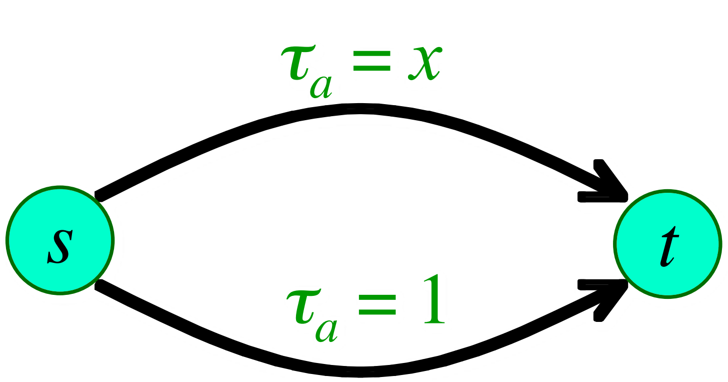
multiple dynamic equilibria for fixed departure times

Congestion pricing

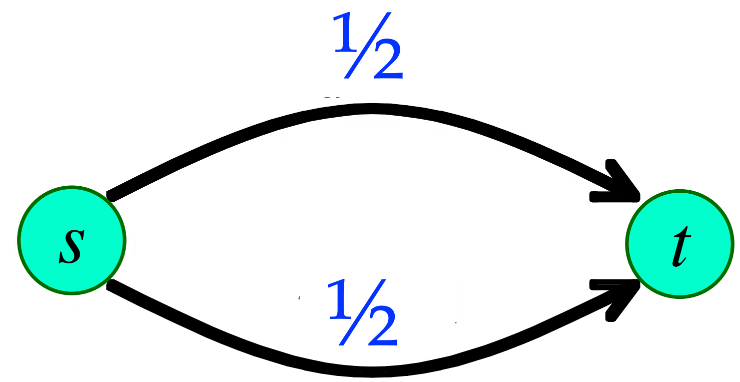
- Marginal cost pricing on all edges leads to the system optimum [Beckmann et al. `56]
  - f is SO w.r.t. $\tau_a(x_a)$ $\Leftrightarrow$ f is UE w.r.t. $\tau_a(x_a) + x_a\tau_a'(x_a)$

- tolls can „in principle“ achieve SO
  - need to choose arc cost as $\underbrace{\tau_a(x_a)}_{\text{time}} + \underbrace{x_a\tau_a'(x_a)}_{\text{toll, depends on flow } x_a}$

Tolls achieve system optimum on Pigou's example



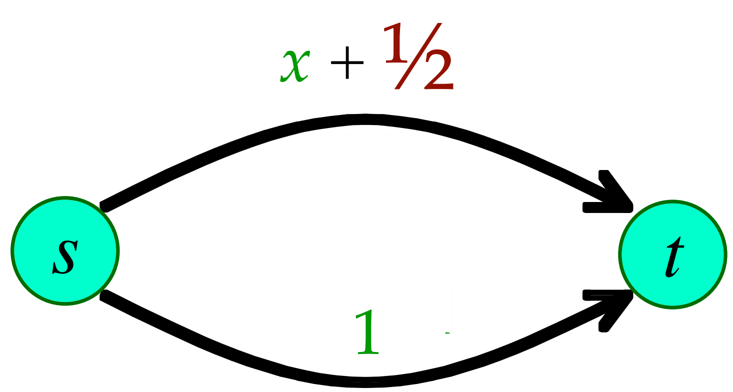
demand $b = 1$



system optimum

Tolls compute as $x_a \tau_a'(x_a)$ for the system optimal flow

$\Rightarrow$ $½ * 1$ on the top and 0 on the bottom



tolled travel times,

achieve system optimum at equilibrium

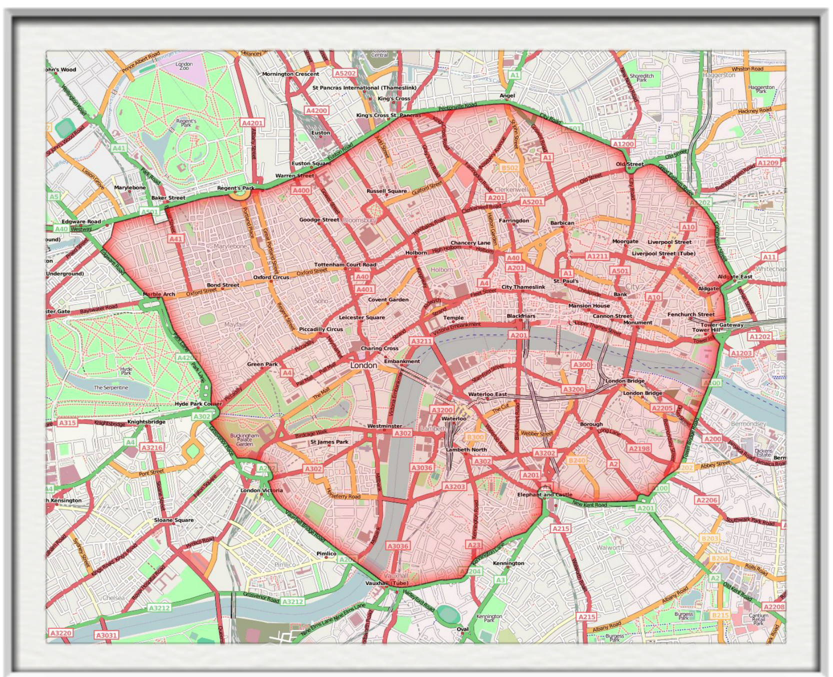
Known results on congestion pricing

- Polyhedral description of opt-inducing tolls (path-based and arc-based via node potentials)
  [Bergendorff et al. (1997)] [Hearn and Ramana (1998)] [Larsson and Patriksson (1999)]
- Optimize toll-dependent objective function over opt-inducing tolls; in particular: min revenue, min toll-booth problem
  [Hearn and Ramana (1998)] [Dial (1999)]
- Heterogeneous vs. homogeneous users
  [Cole et al. (2003)] [Fleischer et al. (2004)] [Swamy (2007)]
- Characterizing (arc-based) flows that are enforcable by tolls
  [Fleischer et al. (2004)]

Congestion pricing in practice

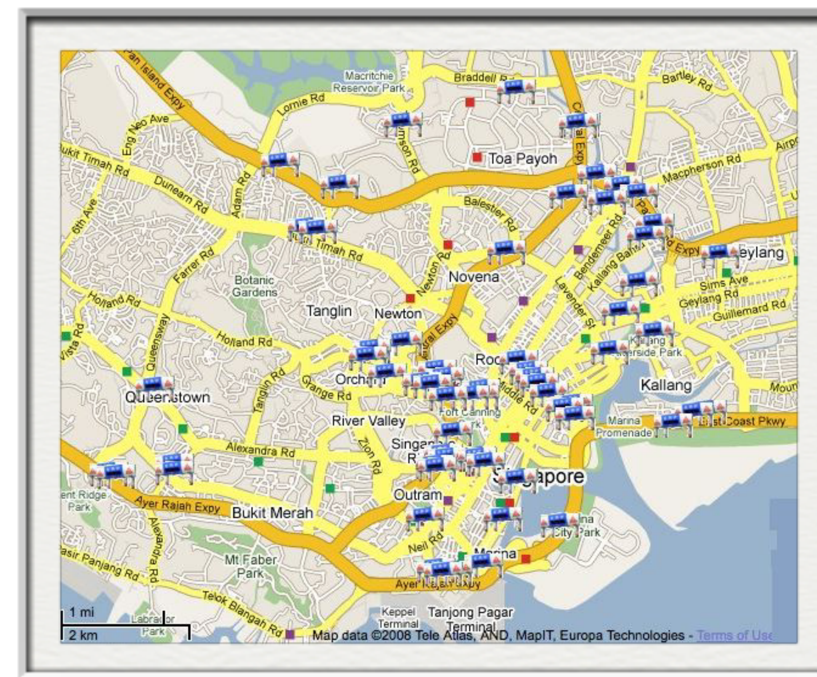
- Marginal cost pricing on all edges not feasible in practice

London



Stockholm

Singapore



- Only a subset of edges has tolls
- Need to study network toll problem with support constraints

Best tolls with limited number of toll booths

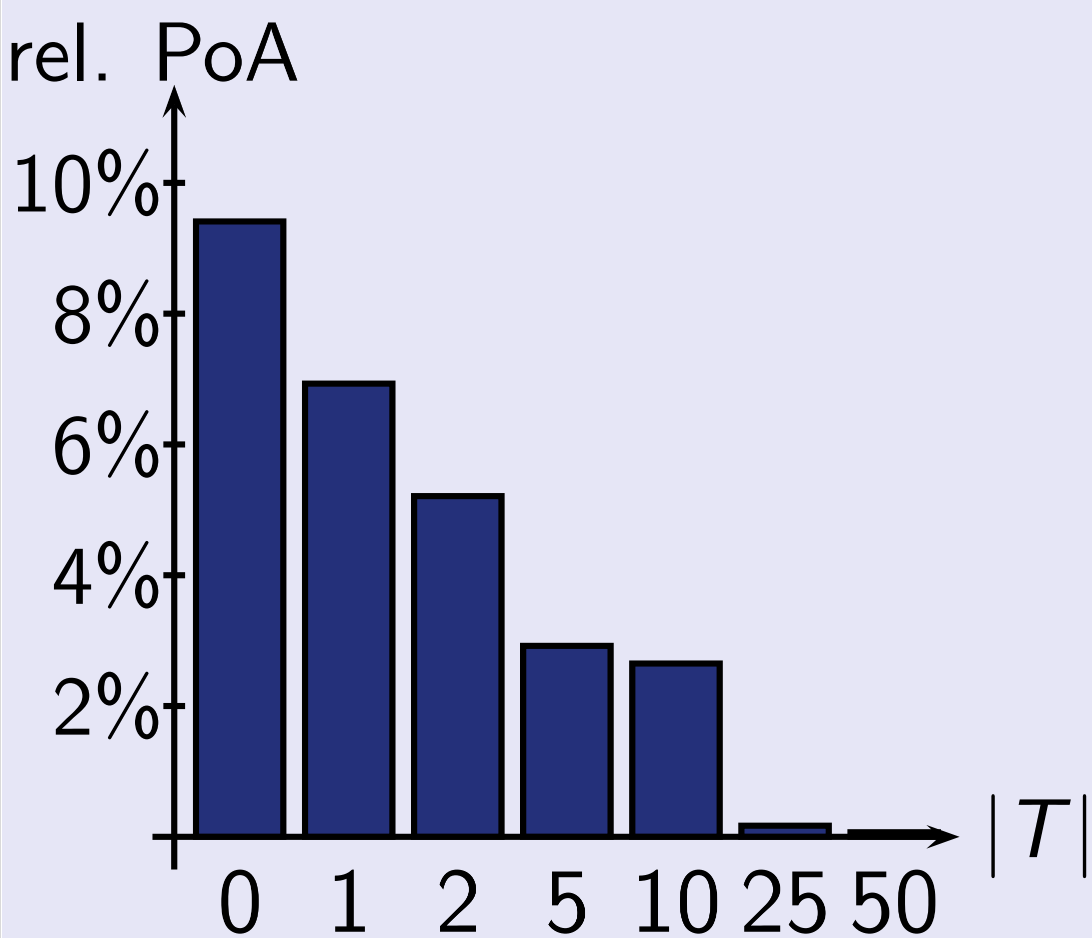
[Tobias Harks, Max Klimm, Ingo Kleinert, Rolf Möhring 2011]

- Is NP-hard, even for two commodities and linear travel times
  [Hoefer et al. 2008]
- No hope for exact solution
- Implemented and tested several algorithms
  - motivated by steepest decent approaches
- Tested on real-world networks

Few tollable edges suffice to reduce congestion

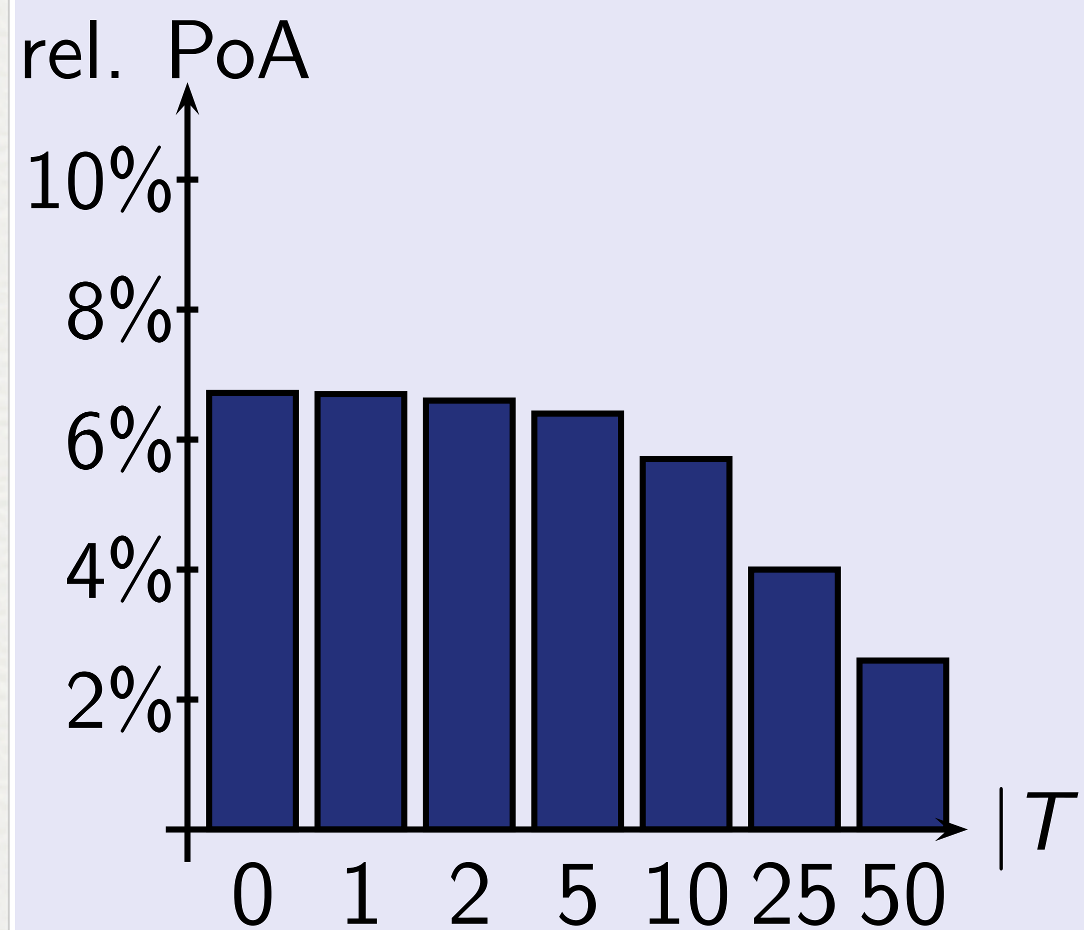
B.-F'hain

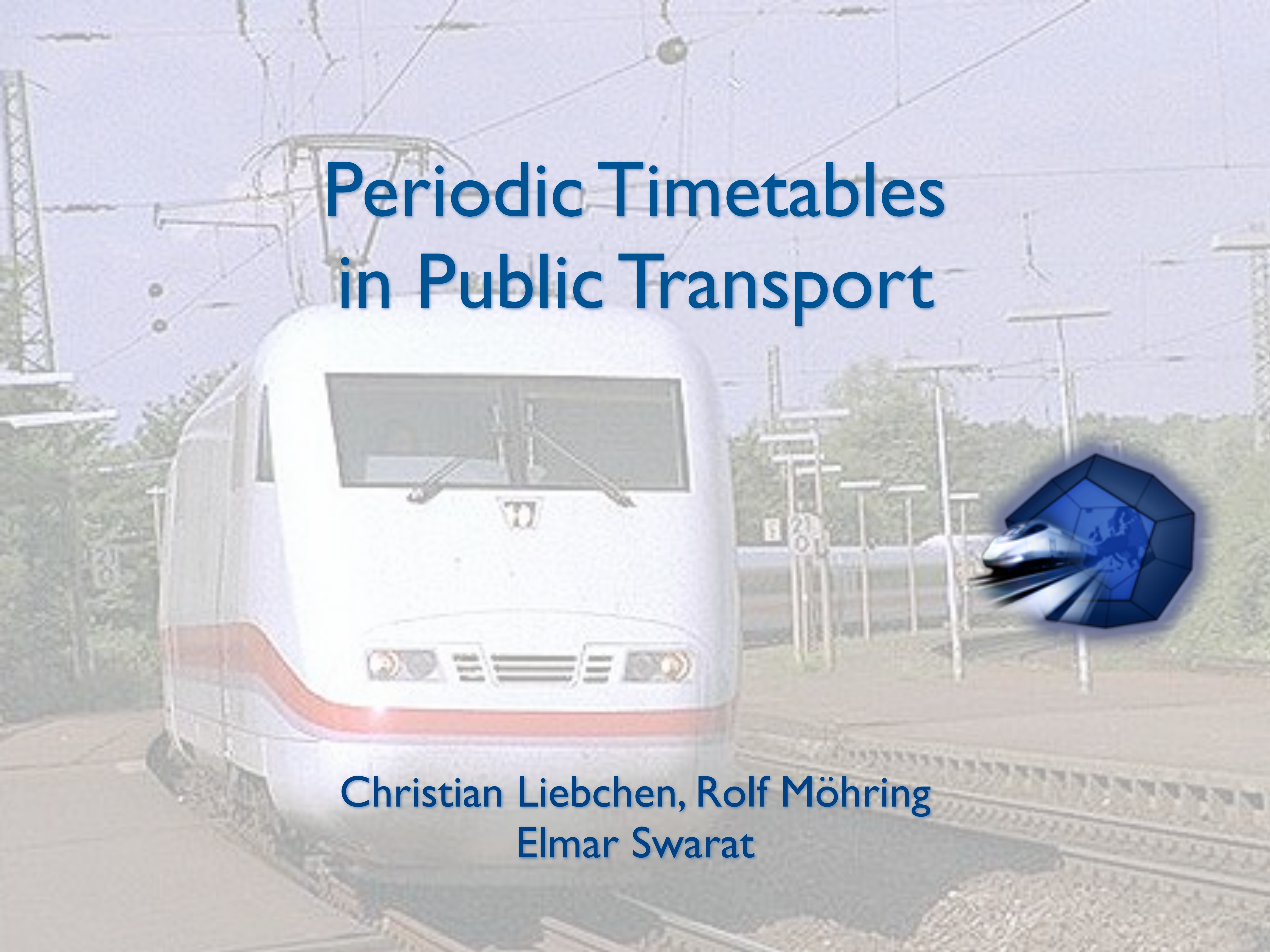
$n = 224,\ m = 523,\ k = 506$



B.-Mitte

$n = 1782,\ m = 2935,\ k = 29$

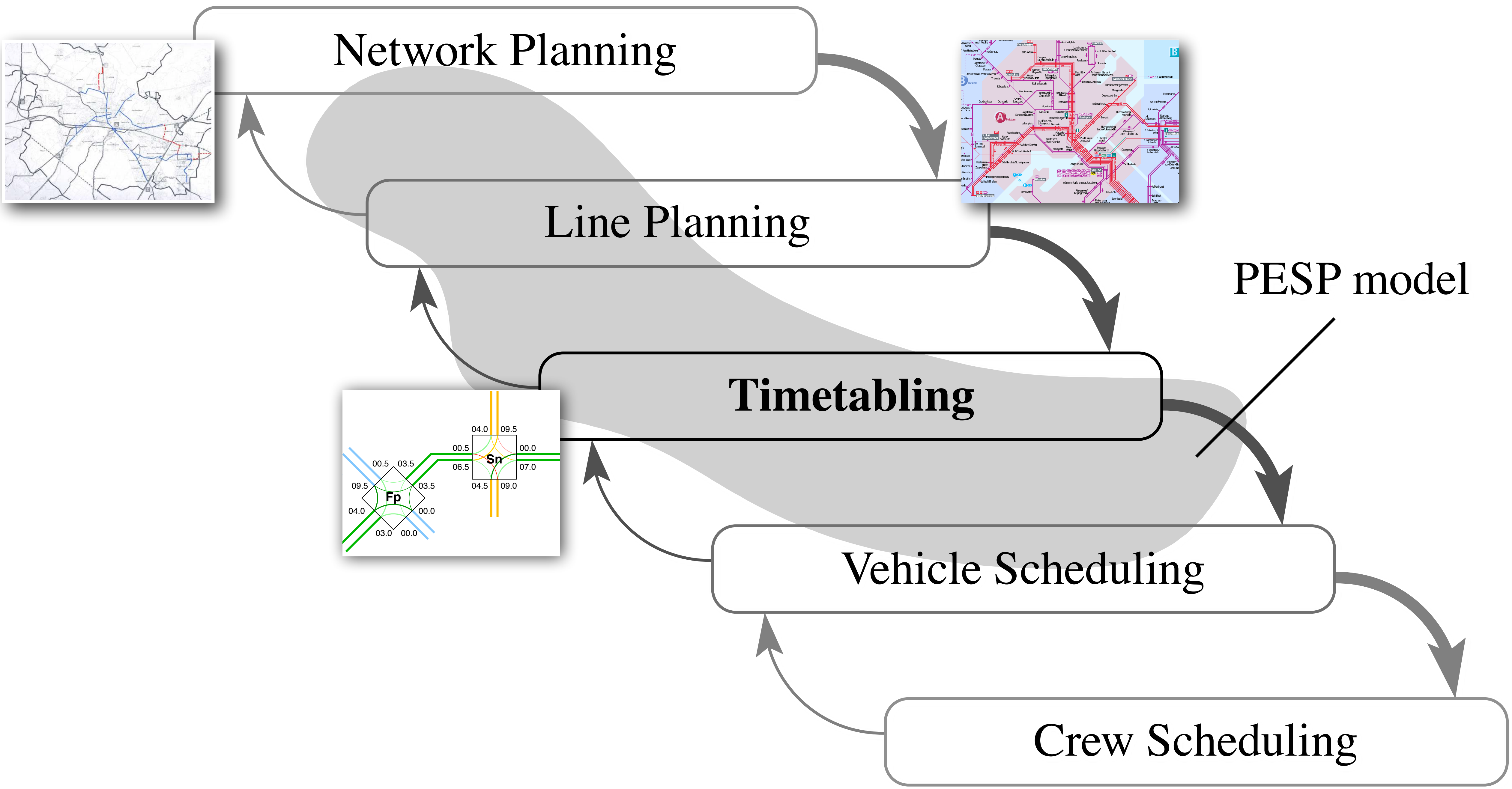




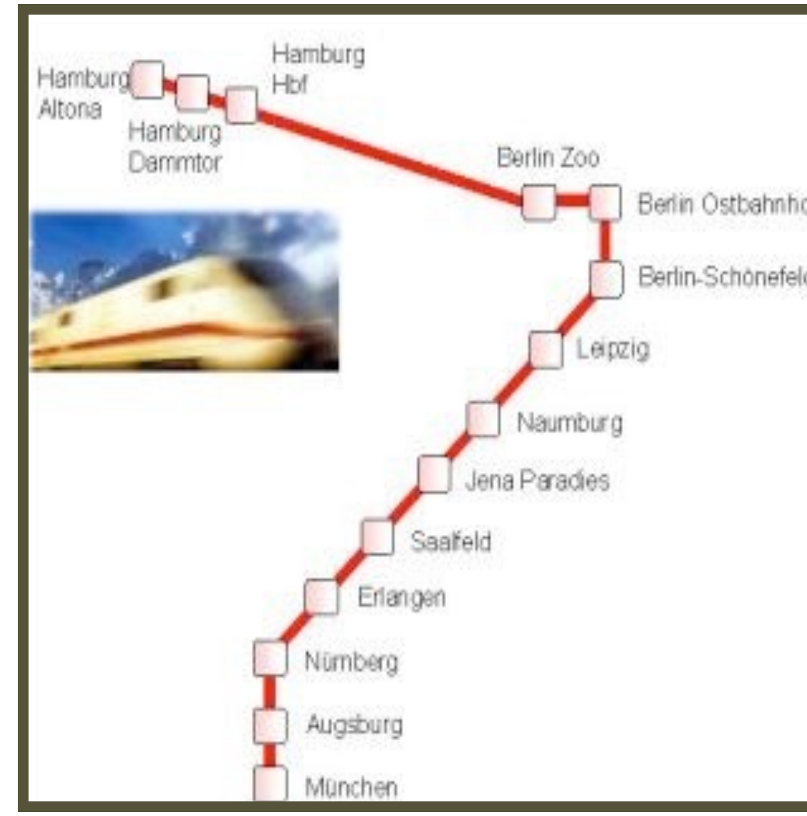
Periodic Timetables in Public Transport

Christian Liebchen, Rolf Möhring
Elmar Swarat

Planning steps in railbound traffic



An informal problem formulation

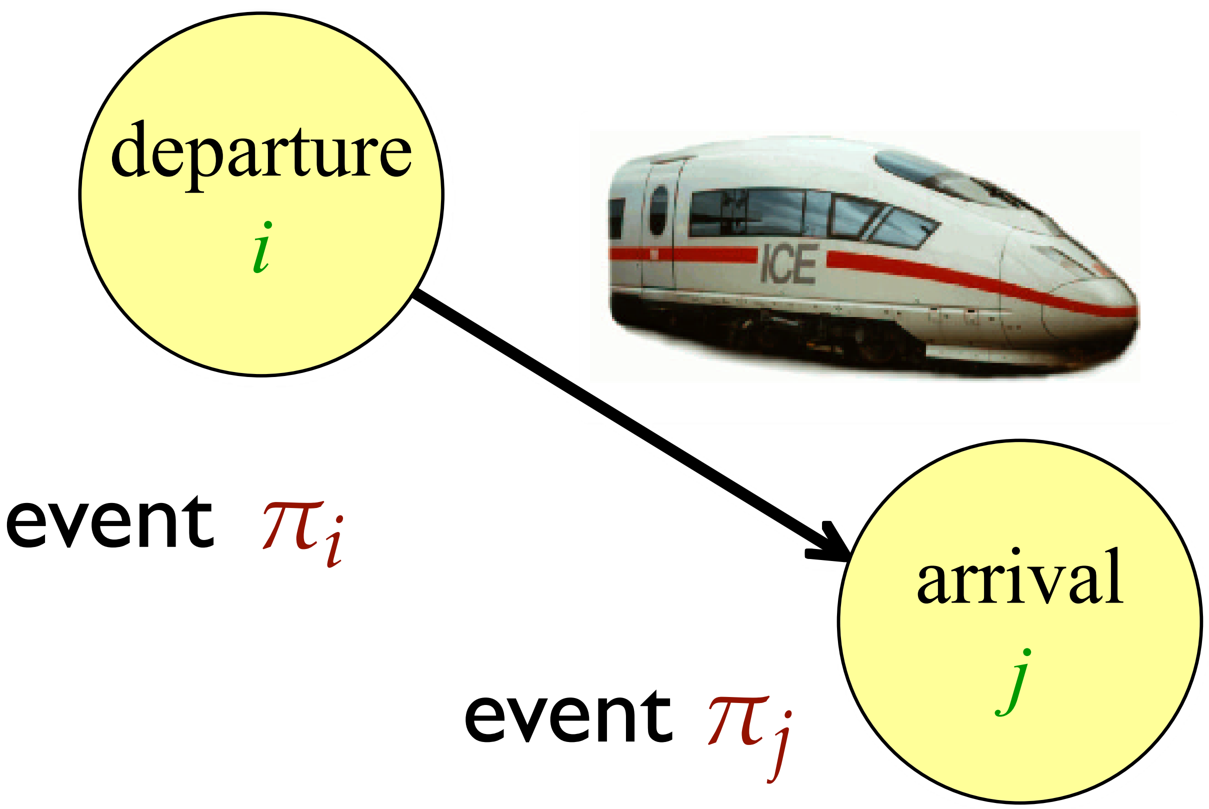


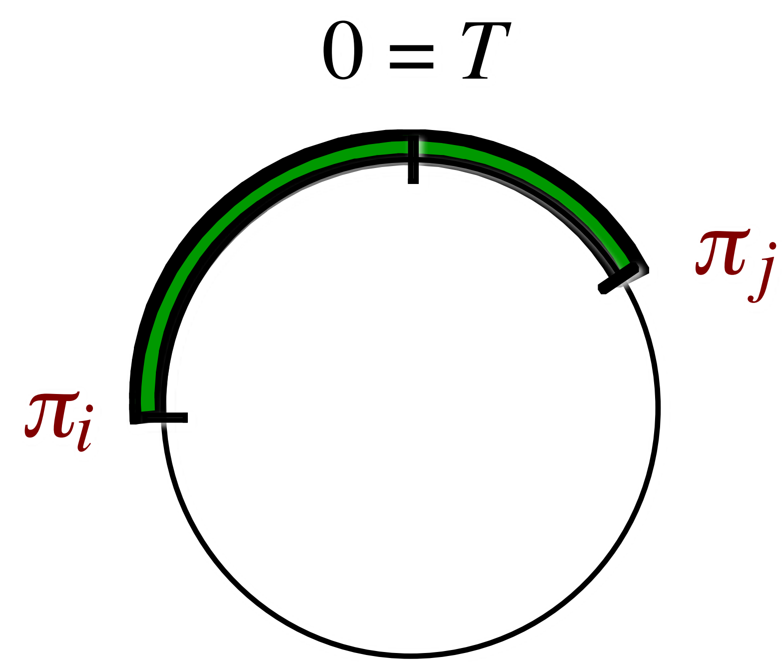
- Lines are already determined
- Line = sequence of stations
- Wanted: timetable
  - arrival and departure times at stations
  - and technically important points
- subject to temporal constraints of various kinds
  - timetable must be periodic
- Objectives to be minimized
  - total (weighted) changeover time
  - number of used vehicles/trains (rolling stock)

PESP: The core of the model

[Serafini & Ukovich '89]

Consider geographic points i, j along a line



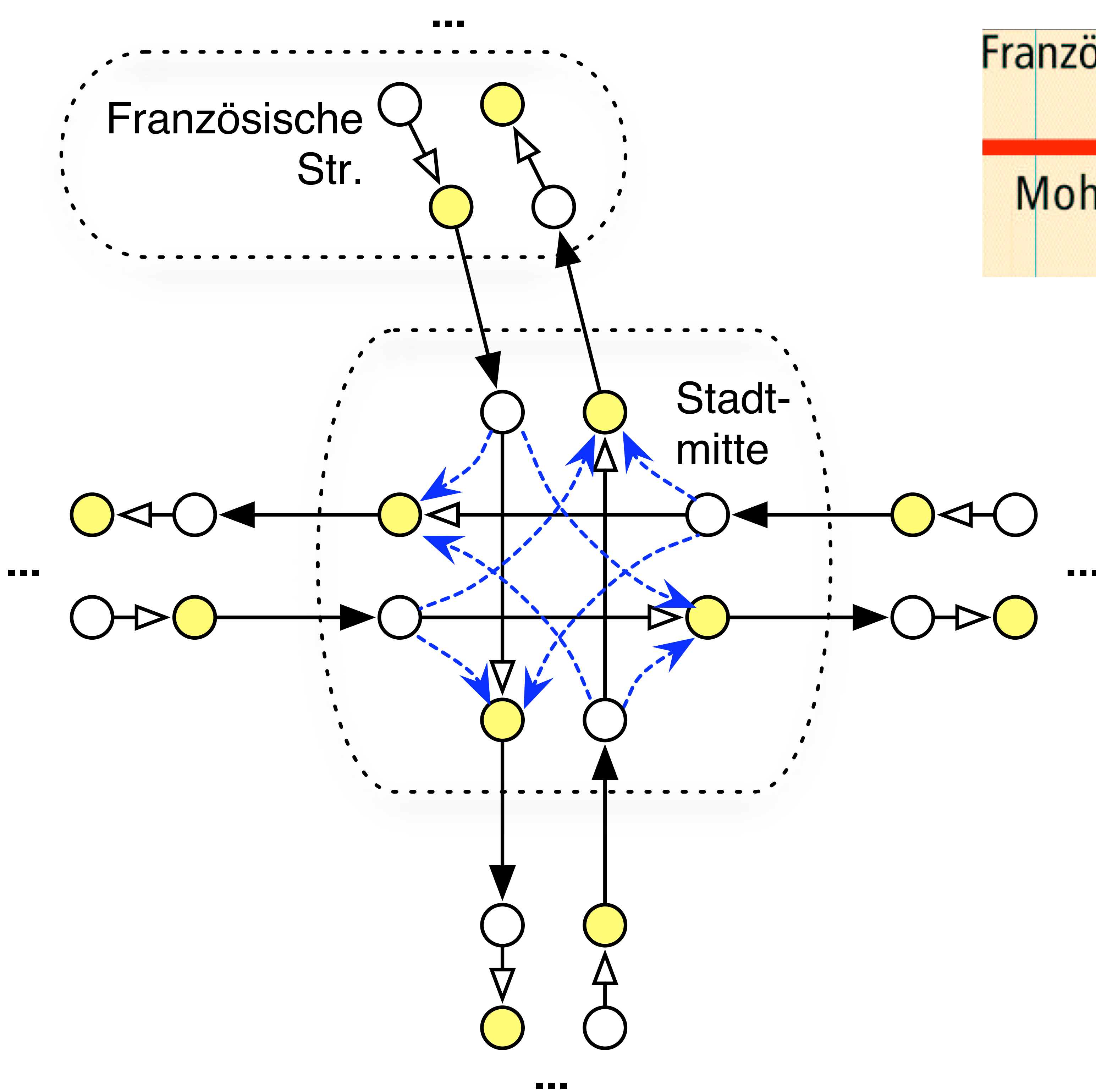


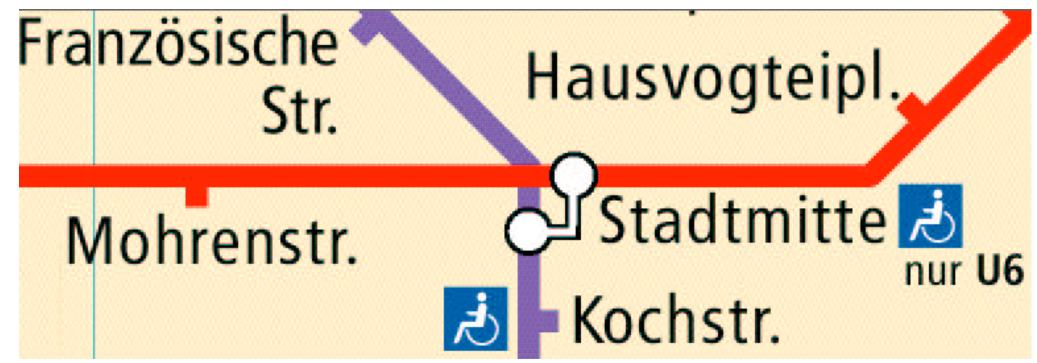
$$\ell_{ij} \le \underbrace{\pi_j - \pi_i}_{x_{ij}} + p_{ij} \cdot T \le u_{ij}$$

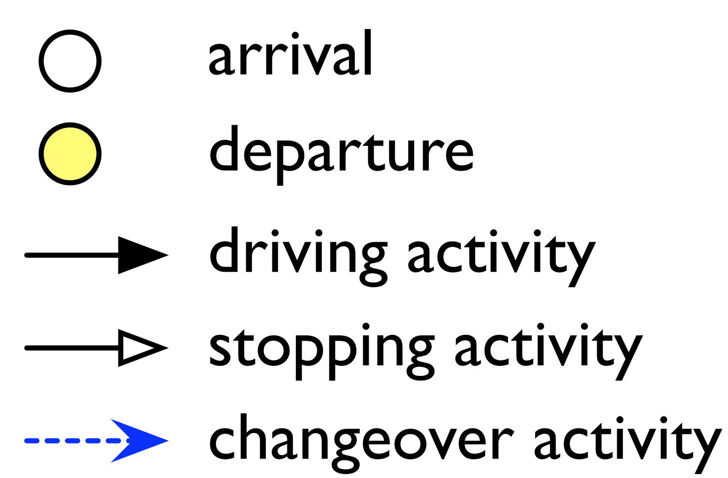
p_{ij} integer

notation $x_{ij} \in [\ell_{ij}, u_{ij}]_T$

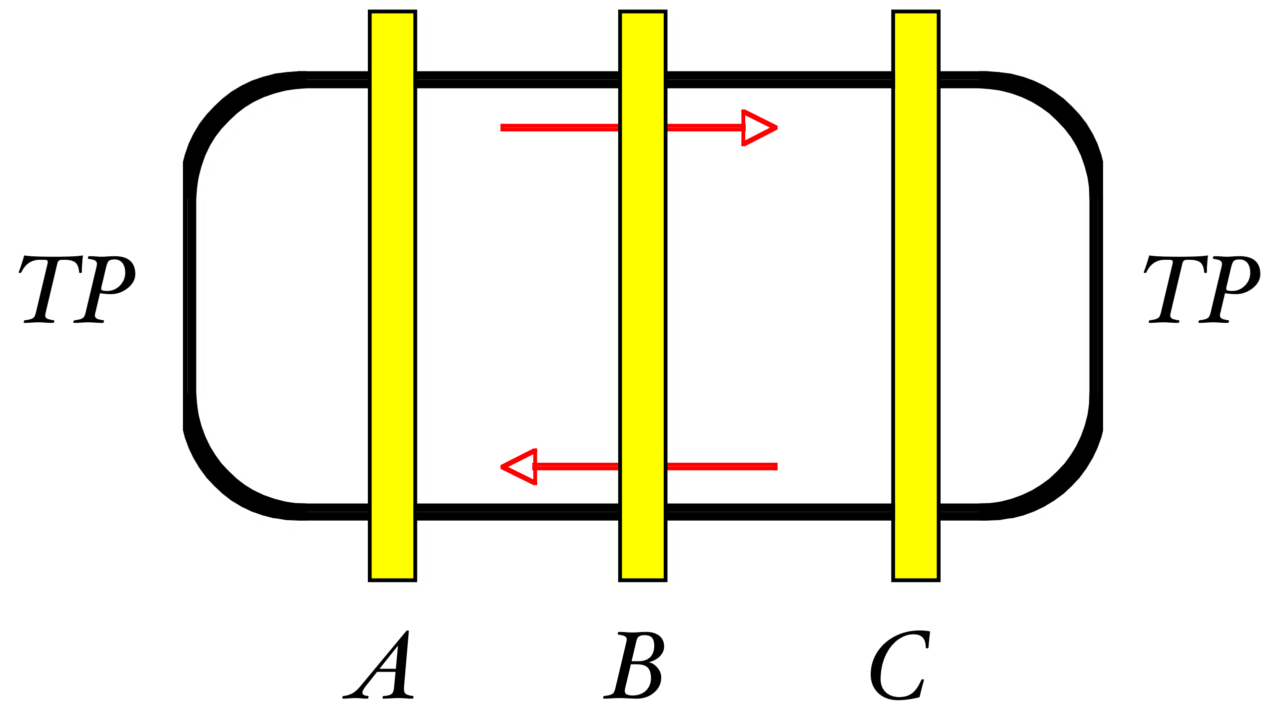
From the line plan to the graph model







Example: a single line



stopping condition in B

$\pi_{\text{dep in } B} - \pi_{\text{arr in } B} \in [2,4]_T$

xx:59 arr in B

xx:02 dep in B

travel condition

$\pi_{\text{arr in } A} - \pi_{\text{dep in } B} \in [11,11]_T$

xx:48 dep in B

xx:59 arr in A

turning condition at *TP*

$\pi_{\text{arr in } A} - \pi_{\text{dep in } A} \in [5,10]_T$

xx:02 dep in A

xx:10 arr in A

More complex conditions

- Single tracks used in both directions
- Safety distance between successive trains (headway)
- Coupling to other traffic systems
- Coordinated servicing of central transfer points
- ...

all can be modeled by a digraph with conditions

$x_{ij} \in [\ell_{ij}, u_{ij}]_T$ at the arcs (i, j)

Modeling the PESP as MIP

Linear objective $\sum w_{ij} x_{ij}$

- Sum of changeover times
- Time spent at turning points (rolling stock)
- Routing of rolling stock (line changes at endpoints)
- Penalties for weak side constraints

Have side constraints for every arc $a = (i, j)$

$$
\begin{aligned}
x_a &= \pi_j - \pi_i \\
\ell_a &\leq x_a + p_a \cdot T \leq u_a \\
0 &\leq x_a \leq T - 1 \\
p_a & \text{ integer}
\end{aligned}
$$

Integer LP in variables x_a and p_a

Complexity of the PESP

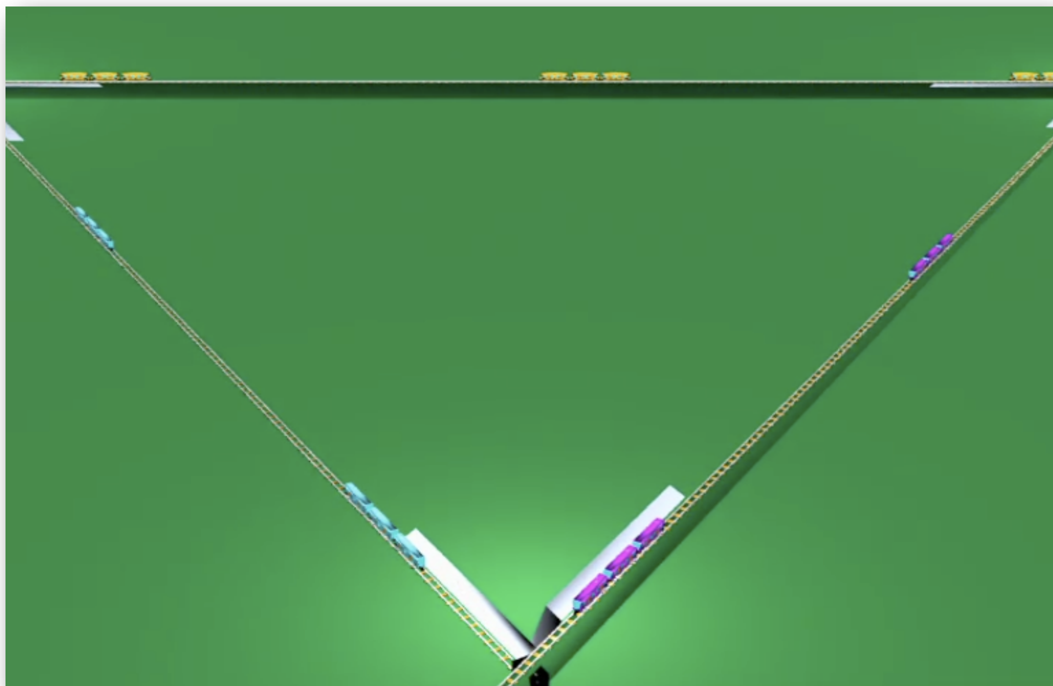
- Decision problem is NP-complete [Odijk '94]
- Generalizes MAXIMUM k-COLORABLE SUBGRAPH,
  so MAX-PESP is MAXSNP-complete [Liebchen '05]
- Natural Mixed Integer Programming (MIP) formulations are extremely hard to solve
- Small instances have entered MIPLIB
  - `timetab1`: 56 nodes, 226 arcs (after contraction)
  - `timetab2`: 88 nodes, 381 arcs (after contraction)

Solution methods

- Integer linear optimization
  - Reduction of the solution space via "good" cycle bases
  - Cutting planes
  - Problems are extremely difficult to solve
- Genetic algorithms
  - work well in practice
  - use integer linear optimization for quality checks
- Software integrated into
  - PTV Software Visum
  - DB Software Prosim Express

Characterizing periodic tension

[Serafini & Ukovich 89]



Explains the role of (undirected) cycles C by looking at the sum of potential differences $\sum_C x_{ij}$ along C

x is a periodic tension for period length T

$\Leftrightarrow$ $\sum_C x_{ij}$ is an integer multiple of T for *every cycle* C of G

$\Leftrightarrow$ $\sum_C x_{ij}$ is an integer multiple of T for *every fundamental cycle* C of G w.r.t. to an arbitrary spanning tree of G

Better MIP formulations based on cycle bases

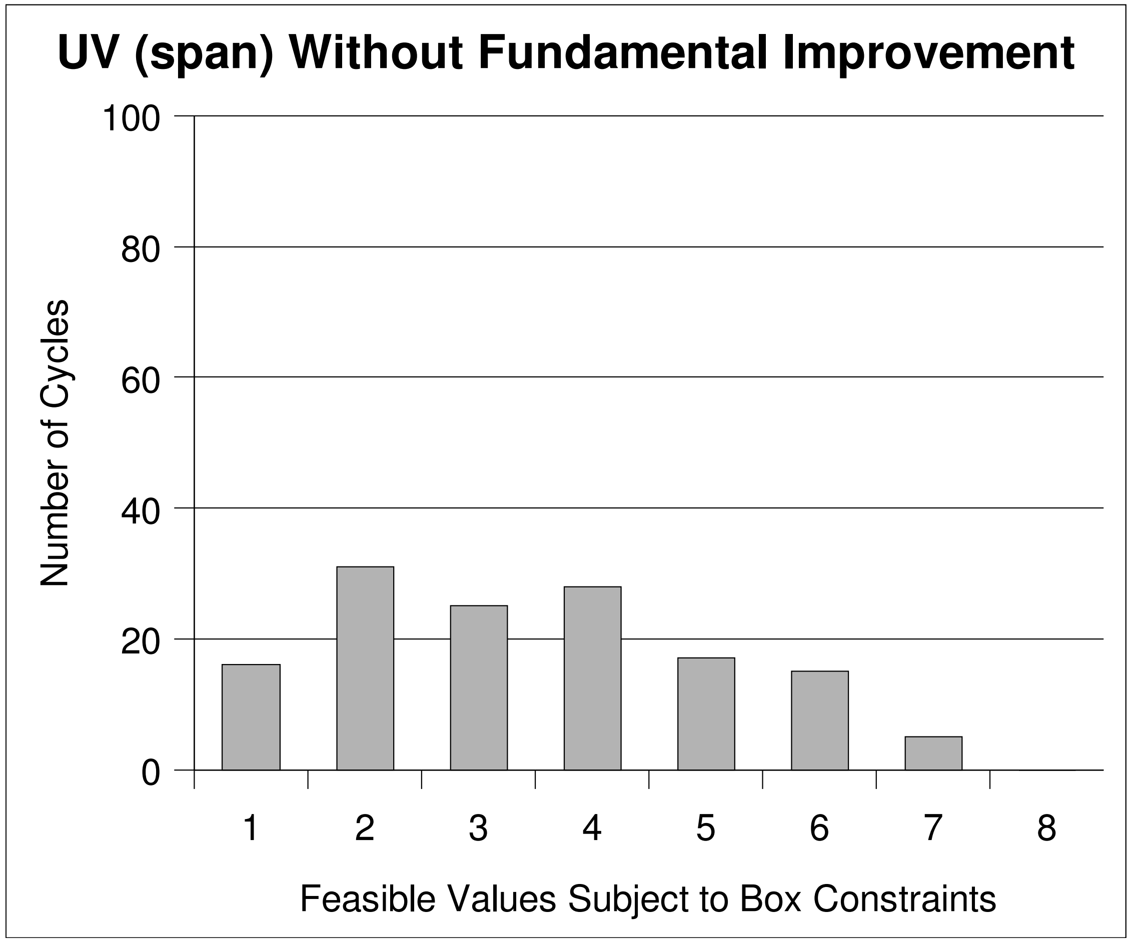
- Along every cycle C, the period offsets p_a must sum up to an integer multiple q_C of the period length T
- Suffices to require this property for the cycles of an integer cycle basis of the underlying graph
- Gives a different IP formulation in $m-n+1$ cycle variables q_C

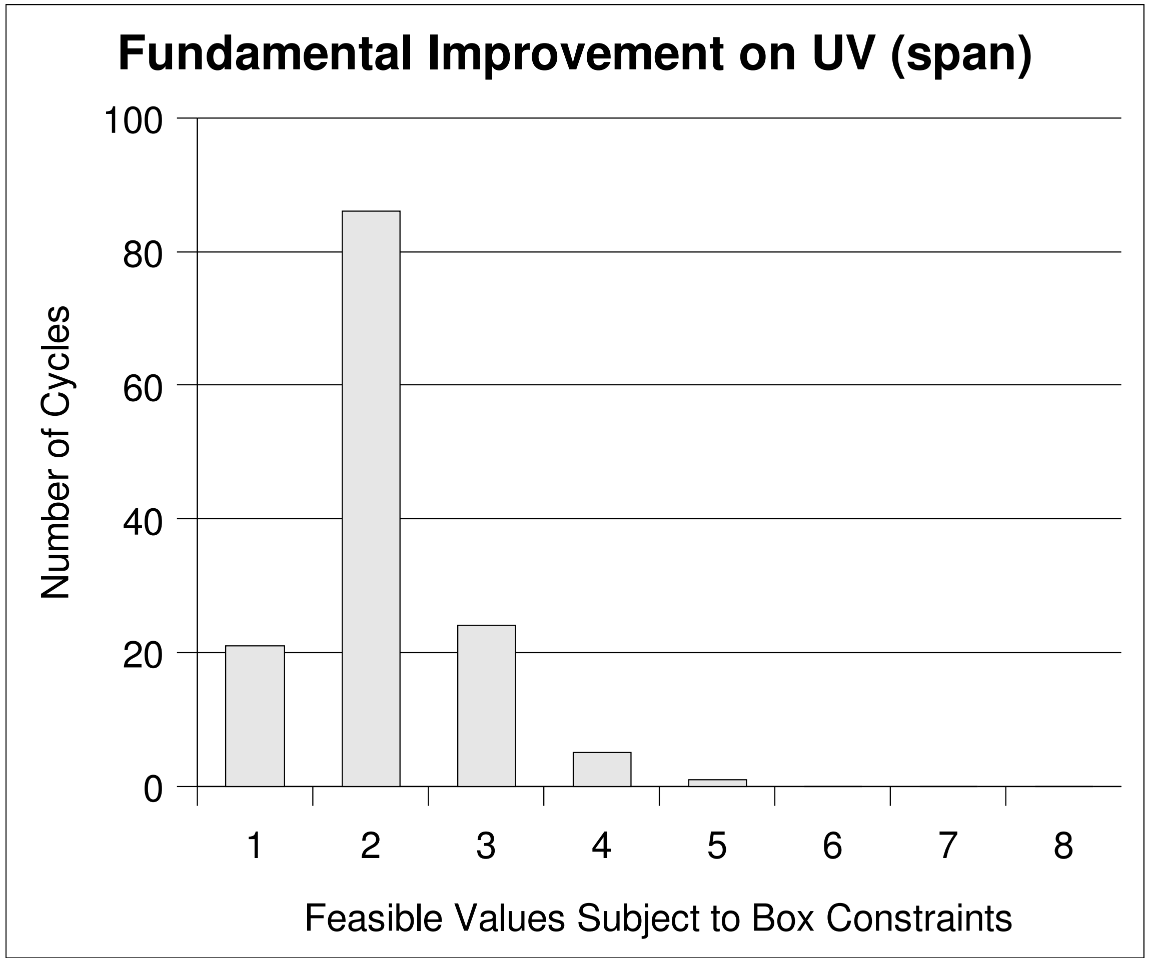
$$\Gamma^{\mathsf{T}} \cdot x = q \cdot T$$

network matrix of basis	$\cdot$	vector x	$=$	vector of cycle multiples of basis cycles	$\cdot$	period length T

- The formulation is "good" if the cycle base is "short"

Numerical evidence for good cycle bases



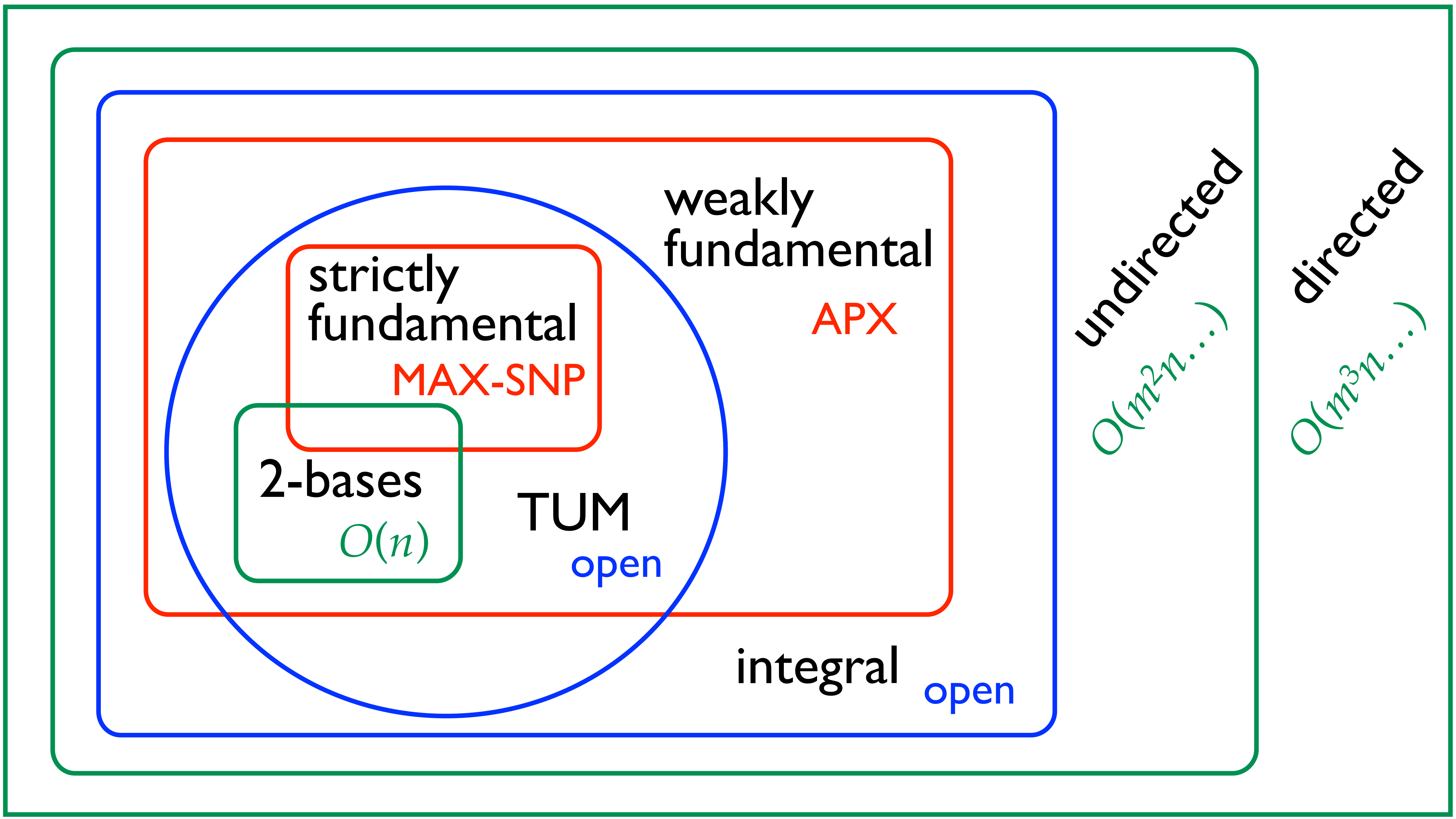


Runtime and memory drop by a factor of 2 to 10

How to compute a suitable short bases

- Undirected graphs
  - $O(m^3 n)$ [Horton ‘87]
  - $O(m^2 n + mn^2 log\ n)$ [Kavitha, Mehlhorn, Michal & Paluch ‘04]
- Directed graphs
  - $\tilde{O}(m^{\omega+1} n)$[Liebchen & Rizzi ‘04]
  - $O(m^3 n + m^2 n^2 log\ n)$ [Hariharan, Kavitha, Mehlhorn ’06,07]
- Hardness
  - NP-hard for tree bases [Deo, Prabhu, Krishnamoorthy ‘82]
  - MAX-SNP hard for tree bases [Amaldi & Galbiati ’03]
- Complexity open for integral bases
- Cycle exchange approaches [Berger & Gritzmann ’04, others]

The complexity of computing a short cycle basis



Main criteria of improvement

criterion	before	after optimization
stop time of trains	up to 3.5 min	at most 2.5 min
changeover times at 50 top connections	up to 5.5 min	at most 5 min
changeover times at 120 other connections	average 4.84 min	average 4.26 min
number of trains	n	n–1

-30%

-12%

1 train