Mathematical Analyze

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Prove, that $\sqrt[n]{n!}$ is unlimited:
Let $\sqrt[n]{n!}$ is limited, then $\exists x : \forall n, \sqrt[n]{n!} \leq x$.

$$\sqrt[n]{n!} \leq x \mid \uparrow^n$$

$$n! \leq x^n$$

$$\underbrace{1 \times 2 \times \ldots \times n}_{n} \leq \underbrace{x \times x \times \ldots \times x}_{n}$$

We knows, that we can choose any value for n, then let $n = 2x^2$:

$$\underbrace{1 \times 2 \times \ldots \times x^2}_{x^2} \times \underbrace{(x+1)^2 \times \ldots \times 2x^2}_{x^2} \leq x^{2x^2}$$

So, we also know, that $\forall i,\ a_i + a_{x^2-i+1} = 2x^2 + 1$, for left side of inequality. We will group multipliers this way: $\forall i, a_i \times a_{x^2-i+1}$. After that, all multipliers will be more than $2x^2$. Then correct:

$$\underbrace{2x^2 \times 2x^2 \times \ldots \times 2x^2}_{x^2} \leq x^{2x^2}$$

$$(2x^2)^{x^2} \leq x^{2x^2}$$

$$2x^{2x^2} \leq x^{2x^2}$$

That's incorrect. The contradiction. Then $\sqrt[n]{n!}$ is unlimited.

Q.E.D.

$$\psi(n_1, n_2, \ldots, n_m) = \underbrace{\sum_{i_1=1}^{n_1} \sum_{i_2=1}^{n_2} \cdots \sum_{i_m=1}^{n_m}}_{m} \prod_{j=1}^{m} i_j \pmod p$$