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Synthesis of noise-free code

Laboratory Work №2

Variant 63

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1. Determine the variant of work.
2. Determine four messages depending on the variant.
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5. Determine one decoded message as a sequence of 11-characters code.
6. Build a scheme of decoding a classical hamming code (15;11).
7. Analyze and comment on errors in messages (part №2).
8. Add the numbers of all 5 variants of tasks and multiply by 4. Take this number as the number of information bits in transmitted message. Calculate minimum for a given number the number of test bits and the redundancy factor.
9. Side Quest №1: Write program on any programming language, which take the sequence of 7 digits "0" and "1", analyze message as a hamming code (7;4) and give correct message, and indicate bits with errors, if it exists.

2 Solutions and calculations

2.1 Variant

My ISU is 466537 ⇒ number of variant is **63**.

2.2 Determine messages №1

№	001 1 r_1	010 2 r_2	011 3 i_1	100 4 r_3	101 5 i_2	110 6 i_3	111 7 i_4
45	0	0	1	0	0	1	1
82	1	1	0	1	1	0	1
7	0	1	1	1	0	0	0
46	0	0	1	1	0	1	1

Table 1: Sequences according to the variant

2.3 Scheme of decoding Hamming's code (7;4)

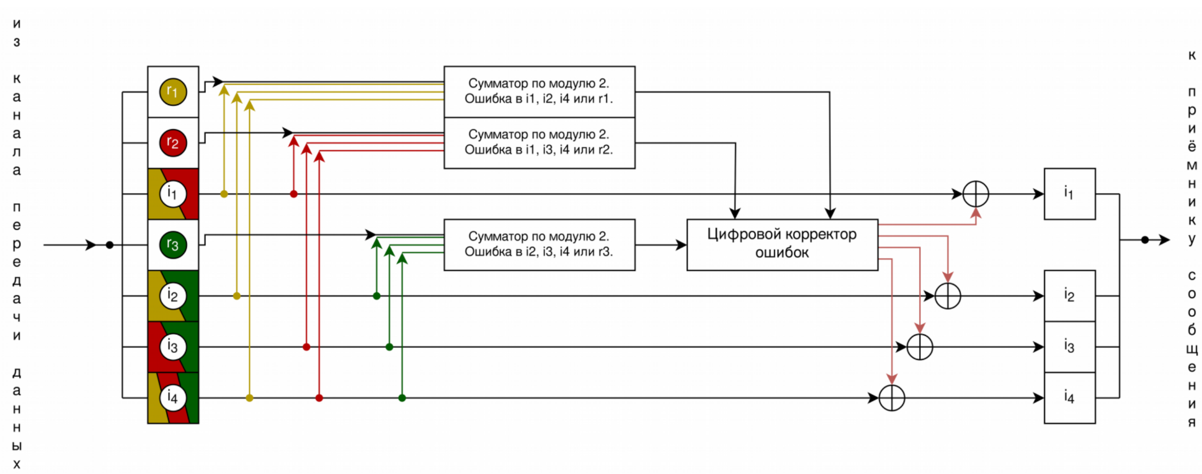

Fig. 1: This scheme shows algorithm of decoding Hamming's code (7;4)

2.4 Decoding messages

2.4.1 №45

- Count control sums:

$$s_0 = r_1 \oplus i_1 \oplus i_2 \oplus i_4 = 0 \oplus 1 \oplus 0 \oplus 1 = 0$$
$$s_1 = r_2 \oplus i_1 \oplus i_3 \oplus i_4 = 0 \oplus 1 \oplus 1 \oplus 1 = 1$$
$$s_2 = r_3 \oplus i_2 \oplus i_3 \oplus i_4 = 0 \oplus 0 \oplus 1 \oplus 1 = 0$$

- Then syndrome of sequence is: $S = (s_0,\ s_1,\ s_2)$
- Find errors in the transmitted message:

$$\overline{s_2 s_1 s_0}_{(2)} = 010_{(2)} = 2_{(10)} \quad \Rightarrow \quad r_2$$

- Correct code is:

№45	001 1 r_1	010 2 r_2	011 3 i_1	100 4 r_3	101 5 i_2	110 6 i_3	111 7 i_4
Transmitted	0	0	1	0	0	1	1
Correct	0	1	1	0	0	1	1

2.4.2 №82

- Count control sums:

$$s_0 = r_1 \oplus i_1 \oplus i_2 \oplus i_4 = 1 \oplus 0 \oplus 1 \oplus 1 = 1$$
$$s_1 = r_2 \oplus i_1 \oplus i_3 \oplus i_4 = 1 \oplus 0 \oplus 0 \oplus 1 = 0$$
$$s_2 = r_3 \oplus i_2 \oplus i_3 \oplus i_4 = 1 \oplus 1 \oplus 0 \oplus 1 = 1$$

- Then syndrome of sequence is: $S = (s_0,\ s_1,\ s_2)$
- Find errors in the transmitted message:

$$\overline{s_2 s_1 s_0}_{(2)} = 101_{(2)} = 5_{(10)} \quad \Rightarrow \quad i_2$$

- Correct code is:

№82	001 1 r_1	010 2 r_2	011 3 i_1	100 4 r_3	101 5 i_2	110 6 i_3	111 7 i_4
Transmitted	1	1	0	1	1	0	1
Correct	1	1	0	1	0	0	1

2.4.3 №7

- Count control sums:

$$s_0 = r_1 \oplus i_1 \oplus i_2 \oplus i_4 = 0 \oplus 1 \oplus 0 \oplus 0 = 1$$
$$s_1 = r_2 \oplus i_1 \oplus i_3 \oplus i_4 = 1 \oplus 1 \oplus 0 \oplus 0 = 0$$
$$s_2 = r_3 \oplus i_2 \oplus i_3 \oplus i_4 = 1 \oplus 0 \oplus 0 \oplus 0 = 1$$

- Then syndrome of sequence is: $S = (s_0, s_1, s_2)$
- Find errors in the transmitted message:

$$\overline{s_2 s_1 s_0}_{(2)} = 101_{(2)} = 5_{(10)} \quad \Rightarrow \quad i_2$$

- Correct code is:

№7	001 1 r_1	010 2 r_2	011 3 i_1	100 4 r_3	101 5 i_2	110 6 i_3	111 7 i_4
Transmitted	0	1	1	1	0	0	0
Correct	0	1	1	1	1	0	0

2.4.4 №46

- Count control sums:

$$s_0 = r_1 \oplus i_1 \oplus i_2 \oplus i_4 = 0 \oplus 1 \oplus 0 \oplus 1 = 0$$
$$s_1 = r_2 \oplus i_1 \oplus i_3 \oplus i_4 = 0 \oplus 1 \oplus 1 \oplus 1 = 1$$
$$s_2 = r_3 \oplus i_2 \oplus i_3 \oplus i_4 = 1 \oplus 0 \oplus 1 \oplus 1 = 1$$

- Then syndrome of sequence is: $S = (s_0, s_1, s_2)$

- Find errors in the transmitted message:

$$\overline{s_2 s_1 s_0}_{(2)} = 110_{(2)} = 6_{(10)} \quad \Rightarrow \quad i_3$$

- Correct code is:

№46	001 1 r_1	010 2 r_2	011 3 i_1	100 4 r_3	101 5 i_2	110 6 i_3	111 7 i_4
Transmitted	0	0	1	1	0	1	1
Correct	0	0	1	1	0	0	1

2.5 Determine messages №2

№	0001 1 r_1	0010 2 r_2	0011 3 i_1	0100 4 r_3	0101 5 i_2	0110 6 i_3	0111 7 i_4	1000 8 r_4	1001 9 i_5	1010 10 i_6	1011 11 i_7	1100 12 i_8	1101 13 i_9	1110 14 i_{10}	1111 15 i_{11}
63	0	1	0	0	0	1	1	1	1	1	1	0	0	1	1

Table 2: Sequences according to the variant

2.6 Scheme of decoding Hamming's code (15;11)

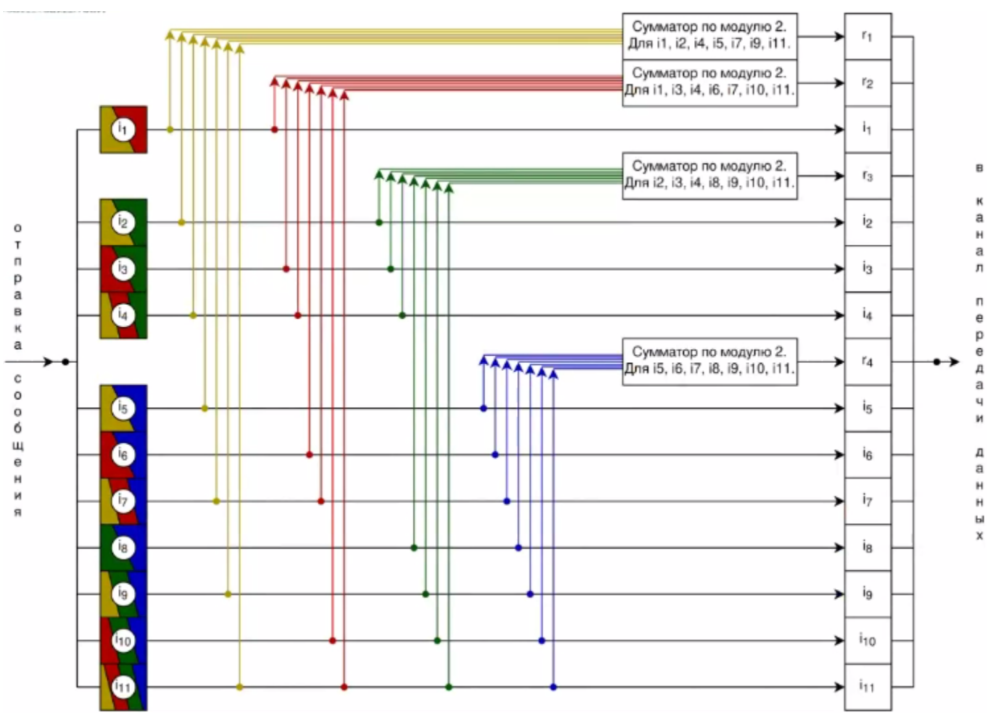

Fig. 2: This scheme shows algorithm of decoding Hamming's code (15;11)

2.7 Decoding messages №2

2.7.1 №63

- Count control sums:

$$s_0 = r_1 \oplus i_1 \oplus i_2 \oplus i_4 \oplus i_5 \oplus i_7 \oplus i_9 \oplus i_{11} = \\ = 0 \oplus 0 \oplus 0 \oplus 1 \oplus 1 \oplus 1 \oplus 0 \oplus 1 = 0$$

$$s_1 = r_2 \oplus i_1 \oplus i_3 \oplus i_4 \oplus i_6 \oplus i_7 \oplus i_{10} \oplus i_{11} = \\ = 1 \oplus 0 \oplus 1 \oplus 1 \oplus 1 \oplus 1 \oplus 1 \oplus 1 = 1$$

$$s_2 = r_3 \oplus i_2 \oplus i_3 \oplus i_4 \oplus i_8 \oplus i_9 \oplus i_{10} \oplus i_{11} = \\ = 0 \oplus 0 \oplus 1 \oplus 1 \oplus 0 \oplus 0 \oplus 1 \oplus 1 = 0$$

$$s_3 = r_4 \oplus i_5 \oplus i_6 \oplus i_7 \oplus i_8 \oplus i_9 \oplus i_{10} \oplus i_{11} = \\ = 1 \oplus 1 \oplus 1 \oplus 1 \oplus 0 \oplus 0 \oplus 1 \oplus 1 = 0$$

- Then syndrome of sequence is: $S = (s_0,\ s_1,\ s_2,\ s_3)$
- Find errors in the transmitted message:

$$\overline{s_3 s_2 s_1 s_0}_{(2)} = 0010_{(2)} = 2_{(10)} \quad \Rightarrow \quad r_2$$

- Correct code is:

№63	0001 1 r_1	0010 2 r_2	0011 3 i_1	0100 4 r_3	0101 5 i_2	0110 6 i_3	0111 7 i_4	1000 8 r_4	1001 9 i_5	1010 10 i_6	1011 11 i_7	1100 12 i_8	1101 13 i_9	1110 14 i_{10}	1111 15 i_{11}
T.[1]	0	1	0	0	0	1	1	1	1	1	1	0	0	1	1
C.[2]	0	0	0	0	0	1	1	1	1	1	1	0	0	1	1

Look to subscript for T.[1] and C.[2].

[1] T. — Transmitted message №63.
[2] C. — Correct form of message.

2.8 Calculations test bits and redundancy factor

2.8.1 Calculate number

$$4(45 + 82 + 7 + 46 + 63) = 972_{(10)} = 1111001100_{(2)}$$

2.8.2 Calculate minimum number of test bits

r — number of test bits, i — number of information bits

$$2^r \geq r + i + 1$$

We know, that $i = 10$(number of information bits), then we need:

$$r \geq 4 \quad \Longrightarrow \quad r_{\min} = 4$$

2.8.3 Calculate redundancy factor

$$\frac{r}{i + r} = \frac{4}{10 + 4} = \frac{2}{7} = 0.\overline{285714}$$

2.9 <u>Side Quest №1</u>

```cpp
#include <iostream>
#include <string>

int log2(const int n) {
  int ans = 0;
  for (int i = 0; i < 31; i++) {
    if ((1 << i) & n) ans = i;
  }
  return ans;
}

std::string end(const int n) {
  int r = n % 10;
  std::string ans;
  switch (r) {
    case 1:ans = "st";
    break;
    case 2:ans = "nd";
    break;
    case 3:ans = "rd";
    break;
    default:ans = "th";
  }
  if (n / 10 % 10 == 1) {
    ans = "th";
  }
  return ans;
}

int main() {
  std::string code;
  std::cin >> code;
  int n = (int) code.length();
  int pow = log2(n);
  int err = 0;
  for (int i = pow; i >= 0; i--) {
    int cnt = 0;
    int s = 0;
    for (int j = (1 << i) - 1; j < n; j++) {
      s ^= code[j] - '0';
      if (++cnt == (1 << i)) j += cnt, cnt = 0;
    }
    err <<= 1;
    err += s;
  }
  if (err) {
    code[err - 1] ^= 1;
    std::cout << code << "\nTransmitted code is incorrect. Error in " << err << end(err) << " bit.";
  } else {
    std::cout << code << "\nTransmitted code is correct.";
  }
}
```

3 Answering questions

1. Classic Hamming's code has a structure $(2^r - 1; 2^r - 1 - r)$, where r — number of test bits. Non-classic Hamming's code has a structure $(2^r; 2^r - 1 - r)$. The difference is in one more bit at the beginning non-classic Hamming's code, which can provide detection error with 2 bits, but also like a classic version can't recover it. This extra bit serves even parity for the message being sent.

2. We need to use $(31; 26)$ classic Hamming's code. Extra bits will be nulled.

3. This means that ratio between initial code length and resulted code length. In other words:

$$\frac{|s_{\text{initial}}|}{|s_{\text{final}}|} \text{ — } \textit{compression ratio.}$$

4. Control sum is a more general concept that refers to a value calculated by a specific algorithm. Parity bit is a special case of control sum.

5. Different processing methods are needed to compare the results and reduce the probability of error.

6. Forbidden combinations is all combinations, where number of test bits doesn't allowed determine the correctness of the information bits.

7. Redundancy factor is a ratio length between initial input code and resulted output code. Compression ratio is a ratio between number of test bits and number of all bits.

4 Conclusion

I learned what the Hamming code is, learned how to work with it, wrote an algorithm that works on its basis, and also learned about the internal structure of storing integer numbers in the computer's memory.

5 References

- Balakshin's lecture as a video.
- Hamming's code: [site].
  URL:https://en.wikipedia.org/wiki/Hamming_code