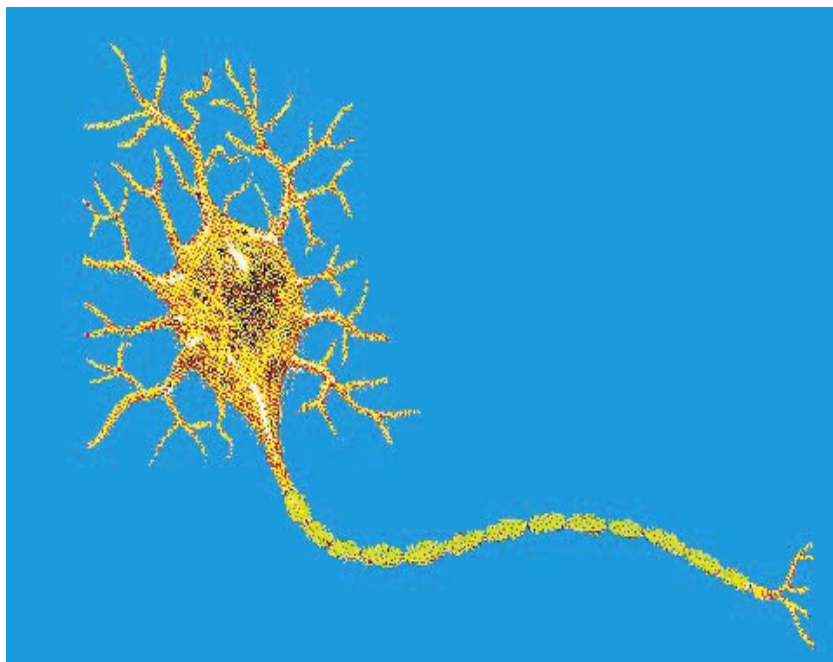


The Neuron Model

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5-21-99



Abstract

The model of the neuron is discussed as a set of differential equations that define whether a neuron is excited or bursting. This paper discusses the analysis of this system of differential equations.

1. Introduction

Neurons are the fundamental units of information processing in the human body and as such have been the subject of mathematical study. A.L. Hodgkin and A.F. Huxley in the early 1950's found that a neuron processes information by controlling the flow of charged ions through its cell membrane, which in turn generates an electric signal. Hodgkin and Huxley proposed that a neuron can be modeled by an equivalent electrical circuit and ultimately by a system of four differential equations. This work is the foundation of modern neuroscience and earned them the Nobel Prize. Since this pioneering work more and more detailed models of specific types of neurons have been developed through a combination of experiments and mathematics. Through the use of such software as Matlab and Scientific Notebook, new insights have been provided for the numerical study of these models about the function of neurons.

A neuron transmits information by generation "action potentials." In general, neurons exhibit two modes of operation, excitable and bursting. An excitable neuron is quiet (does not generate action potentials) except in response to an outside stimulus (for example from a second neuron through a synapse), A bursting neuron generates periodic trains of action potentials. This is often the case in neurons that help regulate rhythmic body functions such as heart contractions. Many neurons exhibit both of these behaviors depending on various factors. a common experiment is to apply different levels of current through an electrode to a neuron to determine whether and where a neuron transitions from excitable to bursting.

2. The Equation

The equation is called the Morris-Lecar equation, it is a system of two differential equations set to explore how a neuron responds to external stimulus. The Morris-Lecar equations were originally formulated to describe electrical activity in barnacle muscle fiber and are sometimes used as a simple caricature of the envelope of bursting neurons and only explicitly model the flow of electric neuron impulses. The variable v in the given equation denotes the voltage of the neuron while the variable w is known as a recovery variable and describes the percentage of open channels selectively permeable to the impulse.

$$v' = I + 2w(-0.7 - v) + 0.5(-0.5 - v) + 1.1m_{\infty}(v)(1 - v)$$

$$w' = \varepsilon \lambda(v)(w_\infty(v) - w)$$

These functions will be plotted in pplane5 from Matlab and analyzed. This will allow us to visualize how the equations will act. Is it a fast-mode (bursting) wave or is it a slow-mode (excitable) wave? These questions will be answered in the analysis of graphs, which will be discussed later.

The functions $m_\infty(v)$, $n_\infty(v)$, $\lambda(v)$ are given by

$$\begin{aligned} m_\infty(v) &= 1/2(1 + \tanh(\frac{v - v_1}{v_2})), \\ w_\infty(v) &= 1/2(1 + \tanh(\frac{v - v_3}{v_4})), \\ \lambda(v) &= \cosh(\frac{v - v_5}{2v_4}). \end{aligned}$$

The parameter I represents injected current into the model neuron. Both I and ε will be treated as parameters. The values of the other constants are given in the table below.

Constant	v_1	v_2	v_3	v_4	v_5	g_k	v_k	g_l	v_l	g_∞
Value	-0.01	0.15	-0.12	0.30	0.22	2.00	-0.70	0.50	-0.50	1.10

The variables are defined as follows:

V' = slow moving neuron system

W' = fast moving neuron system

v = potential difference of voltage

I = the current feeding through the artificial neuron network

ε = the change between slow and fast regions of the neuron

$$m_\infty(v) = 1/2(1 + \tanh((v - v_1)/v_2))$$

$$w_\infty(v) = 1/2(1 + \tanh((v - v_3)/v_4))$$

$$\lambda(v) = \cosh((v - v_5)/(2 * v_4))$$

3. Derivation

The basis of this equation branches from Ohm's Law. Ohm's Law deals with the relationship between voltage and current in an ideal conductor. This relationship states that the potential difference (voltage) across an ideal conductor is proportional to the current through it. The constant of proportionality is called the "resistance", and is denoted as R .

Ohm's Law is given as

$$V = IR,$$

where V is the potential difference between two points which include a resistance R . I is the current flowing through the resistance. For biological work, it is often preferable to use the conductance, $g = 1/R$; In this form Ohm's Law is

$$I = gV.$$

Ohm's Law can be used to solve simple circuits, such as a neuron. A complete circuit is one which is a closed loop. It contains at least one source of voltage (thus providing an increase of potential energy), and at least one potential drop (a place where potential energy decreases). The sum of the voltages around a complete circuit is zero. Because the neuron is in a closed loop Ohm's Law can be applied.

An increase of potential energy in a circuit causes a charge to move from a lower to a higher potential (voltage). Because the electrostatic force, which tries to move a positive charge from a higher to a lower potential, there must be another 'force' to move a charge from a lower potential to a higher inside the battery. This so-called force is called the electromotive force, or emf. The SI unit for the emf is a volt (and thus is not really a force, despite its name). We will use Epsilon to represent the equation in our model. So now the generated equation is

$$\varepsilon = IR.$$

A decrease of potential energy can occur by various means. For example, heat lost in a circuit due to come electrical resistance could be one source of energy drop. But because energy is conserved, the potential difference across a neuron must be equal to the potential difference across the rest of the circuit.

Therefore, applying Ohm's Law we are able to use Epsilon as the mediator of the system of differential equations. If Epsilon is a negative value then the

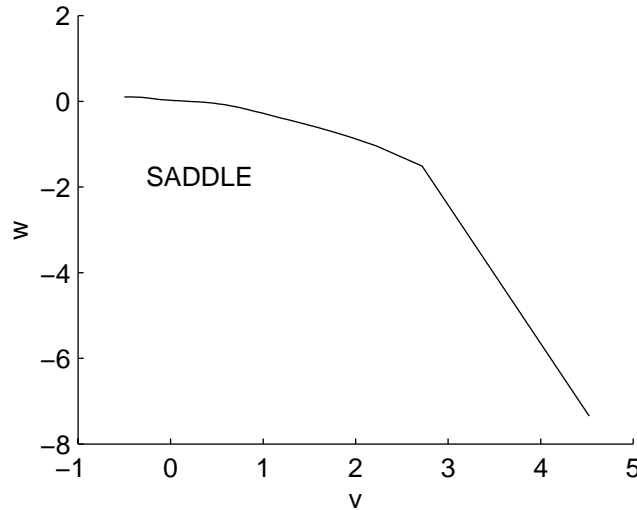


Figure 4.1: Ode45 file with $\epsilon = -0.1$

potential energy is lowered and the neuron will be a slow-mode wave (excited), but on the other hand if Epsilon is a positive value then the potential energy is raised and the neuron will be a fast-mode wave (bursting).

4. Analysis

One way to analyze the system was to study graphs produced by an ode45 program in Matlab. These files are not too difficult to create (for more information go to www.mathworks.com). In Figure 4.1, the graph appears to have both ends jolting out from one point of the solution. This calls for a central point to be conjoining the solution, which is called the equilibrium point. This graph produced a saddle, which is considered to be the slow-mode wave. In this case the equation plugged in has a negative ϵ value.

As the value of epsilon decreases more negatively, the slower the neuron will operate. This is due to the potential energy being lowered.

In Figure 4.2, ϵ has been changed to a positive value. It also has been graphed in an ode45 file from Matlab. This graph appears to show a circling towards the center of the graph, which hints towards an equilibrium point at the intersection. When ϵ is positive the potential energy is raised and the neuron is “bursting” causing a push for the impulse to be transmitted. The charge runs through the

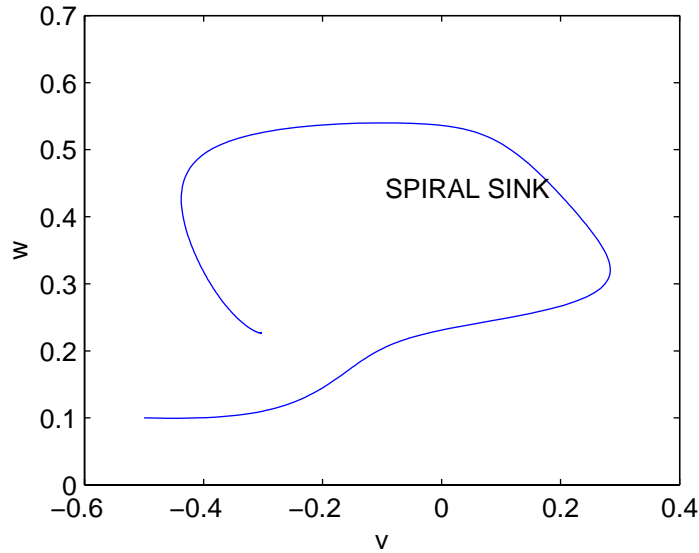


Figure 4.2: Ode45 file with $\epsilon = .1$

axon and over the synapse to the next neuron. The impulse sinks in then down the drain and on to the next stop, figuratively speaking. From the graph that may make a little more sense.

As you can see the ode45 graphs give a pretty good visual representation, but there needs to be more proof to let this assumption fly. This can be done using a different graphing method, which is called pplane5. It is also from Matlab, and allows manipulation and solutions to be plotted on the graph.

In pplane5 the solutions nullclines are plotted with respect to time (t). With the nullclines being plotted pplane5 shows arrows into the directions that the curve will go. These arrows can be followed to produce a sketch of the solutions.

In Figure 4.3, the value for ϵ is considered to be (-). From prior discussion this graph should produce a saddle, which is proof for a slow mode wave (excited) neuron. The impulse is not receiving messages to be transmitted throughout the system. Therefore the potential energy is lowered and the impulse is just passing through the neuron. Furthermore, when the value is negative and propelling away from the equilibrium point, this produces the graph of a saddle. For the system to be a saddle one eigen value must be (-) and one eigen value must be (+), and this is the case in this graph. As you can see the plots shown are veering towards the equilibrium point and then going away as predicted. The arrows on

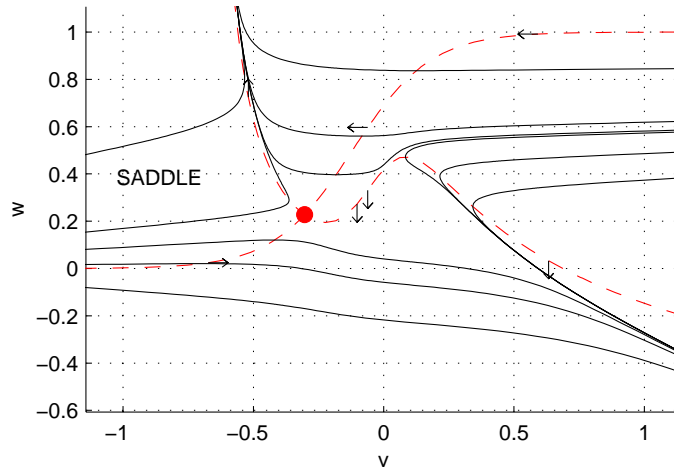


Figure 4.3: Pplane5 graph with $\epsilon = -.1$

the nullclines verify this assumption.

Next is the graph of ϵ as a (+) value, which is seen in Figure 4.4. This will generate a spiral source as discussed from the ode45 graph. This shows an increase in the potential energy within the system the neuron is circling to the center very quickly. The point of intersection of the two nullclines is the equilibrium point of the system. When this point is reached the neuron is dead, at which $\epsilon = 0$ and $I = 0$. When the value of ϵ is positive the spiral sink never reaches the exact center, it only appears to because of the programs error.

5. Conclusion

Modeling of the neuron is not a new idea. It has been researched for many years. It is just now though, where software is available to make the analysis of the equations easier. The technology to study neurons is amazing. Now there are artificial neural networks that are being produced to study the physiological understanding of the nervous systems and how it processes information. These artificial simulated networks incorporate other analysis tools such as: multivariable statistics, clustering algorithms, probabilistic modeling, fuzzy logic, expert or knowledge-based systems, signal processing, Fourier analysis, and wavelet analysis to name a few.

The key element of this paradigm is the novel structure of the information

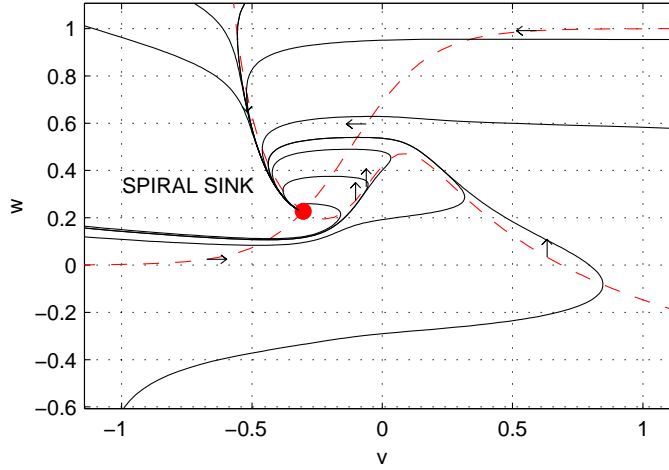


Figure 4.4: Pplane5 graph with $\epsilon = .1$

processing system. It is composed of a large number of highly interconnected processing elements that are analogous to neurons and are tied together with weighted connections that are analogous to synapses. Learning in biological systems involves adjustments to the synaptic connections that exist between the neurons. Learning often occurs by example through training, or exposure to a truthed set of input/output data where the training algorithm iteratively adjusts the connection weights (synapses).

Through the artificial simulation of the brain, the neuron is being studied at new heights. An electrode is injected into the system and studied as it flows through the network. This is a phenomenal idea to fathom. One day we may be able to understand how nerve impulses travel throughout our body by neurons. More research is still to be done on this topic. The discussion of this paper is only about how to interpret whether the neuron is in an “excited” or “bursting” state, which discussed means of slow or fast-mode waves, respectively. This is only the tip of the iceberg for what can be achieved by studying the neuron.

References

[1] Arnold, Dave. College of the Redwoods-Instructor.

[2] Neuron Models

`http://www.sci.wsu.edu/think/Neuron/model.html`

[3] Staley, Warren. College of the Redwoods-Math Instructor

[4] Blanchard, Devaney, and Hall, *Differential Equations*. Brooks/Cole Publishing Company, 1998