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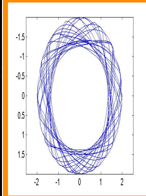
The Double Pendulum

LiHong Huang Herman

email: ipensive@yahoo.com



MATH

Assuming the there exists and admissible function $y(x)$ that minimizes the integral

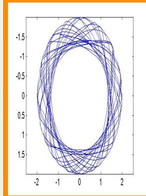
$$I = \int_{x_1}^{x_2} f(x, y, y')dx. \tag{1}$$

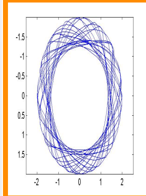
Let $\eta(x)$ be any function with the properties that $\eta''(x)$ is continuous and

$$\eta(x_1) = \eta(x_2) = 0. \tag{2}$$

If α is a small parameter, then

$$\bar{y}(x) = y(x) + \alpha\eta(x). \tag{3}$$





And if the well-defined real number I is in terms of α, then

$$\begin{aligned} I(\alpha) &= \int_{x_1}^{x_2} f(x, \bar{y}, \bar{y}')dx \\ &= \int_{x_1}^{x_2} f[x, y(x) + \alpha\eta(x), y'(x) + \alpha\eta'(x)]dx. \end{aligned}$$

If we differentiate function I with respect to α [1]**, we have**

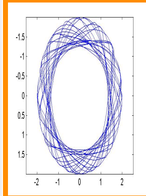
$$I'(\alpha) = \int_{x_1}^{x_2} \frac{\partial}{\partial\alpha} f(x, \bar{y}, \bar{y}')dx. \tag{4}$$

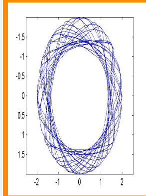
[1] We are trying to find the minimum point.

After series of calculation, we obtain

$$\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) - \frac{\partial f}{\partial y} = 0, \tag{5}$$

which is Euler Lagrangian Equation.





PHYSICS

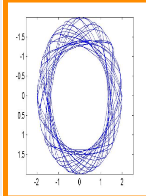
After some sophisticated argument in physics, we obtain the Lagrange's equations,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = 0 \tag{6}$$

where L is the Lagrange's equations describing the motion of a particle in a conservative force field. And

$$L = T - V. \tag{7}$$

T is the kinetic energy, V is the potential energy, q_k is the displacement in any direction.



The Combination of Physics and Math

This is a figure of double pendulum.

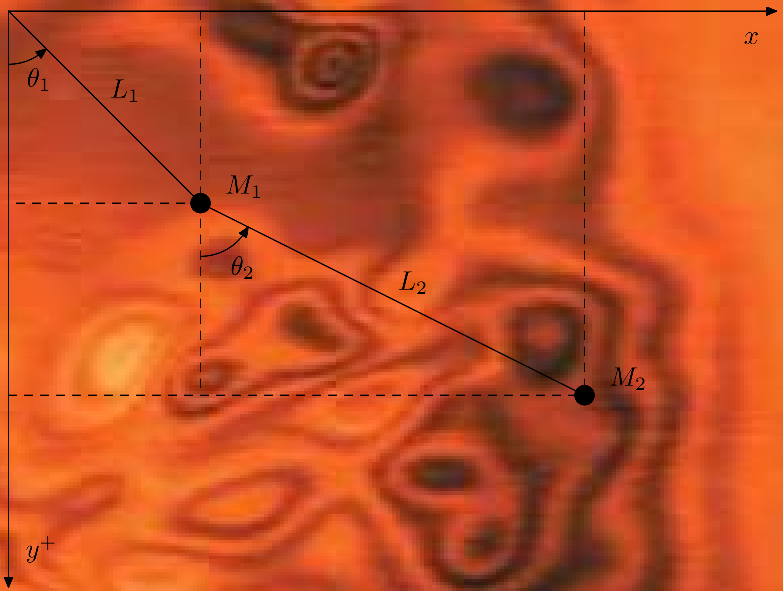
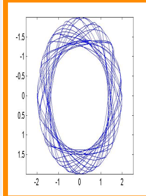


Figure 1:



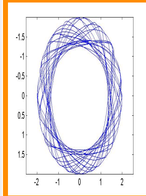
If we model the double pendulum, we obtain

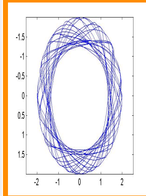
$$x_1 = l_1 \sin\theta_1 \tag{8}$$
$$x_2 = l_1 \sin\theta_1 + l_2 \sin\theta_2 \tag{9}$$
$$y_1 = l_1 \cos\theta_1 \tag{10}$$
$$y_2 = l_1 \cos\theta_1 + l_2 \cos\theta_2 \tag{11}$$

By using the Lagrange's equations, we have

$$\ddot{\theta}_1 = \frac{g(\sin\theta_2 \cos(\triangle\theta) - u \sin\theta_1) - (l_2 \dot{\theta}_2{}^2 + l_1 \dot{\theta}_1{}^2 \cos(\triangle\theta)) \sin(\triangle\theta)}{l_1(u - \cos^2(\triangle\theta)}$$
$$\ddot{\theta}_2 = \frac{gu(\sin\theta_1 \cos(\triangle\theta) - \sin\theta_2) + (u l_1 \dot{\theta}_1{}^2 + l_2 \dot{\theta}_2{}^2 \cos(\triangle\theta)) \sin(\triangle\theta)}{l_2(u - \cos^2(\triangle\theta)},$$

where $\triangle\theta = \theta_1 - \theta_2$ and $u = 1 + (m_1/m_2)$.

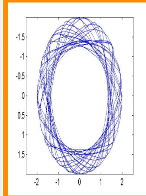




Motion of double pendulum

1. Periodic
2. Quasiperiodic
3. Chaotic

Periodic



Use this input, we will find the periodic motion:

$$M1 = 3$$

$$M2 = 3$$

$$L1 = 4$$

$$L2 = 3$$

$$AnVe1 = 0$$

$$AnVe2 = 0$$

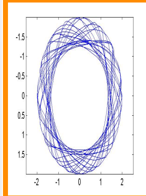
$$Angle1 = pi/4$$

$$Angle2 = pi/4$$

$$tolerance = 1e - 006$$

$$power = 1/3$$

Quasiperiodic



Use this input, we will find the quasiperiodic motion:

$$M1 = 3$$

$$M2 = 3$$

$$L1 = 4$$

$$L2 = 3$$

$$AnVe1 = 0$$

$$AnVe2 = 10$$

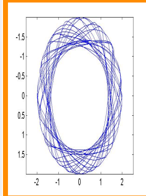
$$Angle1 = pi/2$$

$$Angle2 = pi/2$$

$$tolerance = 1e - 006$$

$$power = 1/4$$

Chaotic



Use this input, we will find the chaotic motion:

$$
\begin{aligned}
M1 &= 10 \\
M2 &= 1 \\
L1 &= 3 \\
L2 &= 3 \\
AnVe1 &= 2 \\
AnVe2 &= 10 \\
Angle1 &= pi \\
Angle2 &= pi \\
tolerance &= 1e-006 \\
power &= 1/3
\end{aligned}
$$

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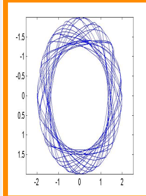

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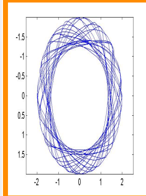

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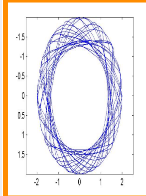

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IN SCIENCE, THERE IS ONLY PHYSICS; ALL THE REST IS STAMP COLLECTING

THE END