Model of the Vegetation-Erosion Relationship using Differential Equations

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Vegetation-Erosion Dynamics

- Vegetation-Erosion Dynamics study the relationship between the Vegetation Coverage Density (VCD) of an area to its rate of Erosion
- There is an inverse relationship between VCD and erosion in that the higher the VCD the less erosion, while the higher the erosion the harder for vegetation to take hold.
- Vegetation-Erosion Dynamics also integrate the effects of natural forces and human interactions on the change in VCD and Erosion of an area
- Erosion is a serious problem that affects the farmlands and natural habitats.
- Models predict the long-term nature of watersheds
- Models show the negative and positive influences humans have on VE dynamics and provide analysis to devise a plan of action to improve the ecosystem.

Vegetation-Erosion Dynamics Model

The basic equation for Vegetation-Erosion dynamics is given by

$$\begin{aligned} \frac{dV}{dt} &= aV - cE \\ \frac{dE}{dt} &= dE - fV \end{aligned} \tag{1}$$

where V and E represent the percentage of vegetation-coverage and the rate of erosion in ton/km^2 respectively.

The coefficients a,c,d, and f are the natural rates of VCD, erosion, erosion effects that further erosion, and vegetation coverage that reduces erosion, which are specific to the area.

Vegetation-Erosion Dynamics Model

Natural and human stresses on vegetation and erosion must also be evaluated to create a complete model giving us the linear differential equation model with forcing terms of V_τ and E_τ

$$\begin{aligned} \frac{dV}{dt} &= aV - cE + V_\tau \\ \frac{dE}{dt} &= dE - fV + E_\tau \end{aligned} \tag{2}$$

where E_τ represents human affects on erosion V_τ represents natural and human ecological stresses.

Solving the Equation

The solution to the linear differential equations (2) is given by the homogeneous solution plus the nonhomogeneous solution

$$\begin{aligned} V &= c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} + V_1(t) \\ E &= c_1 \frac{a-\lambda_1}{c} e^{\lambda_1 t} + c_2 \frac{a-\lambda_2}{c} e^{\lambda_2 t} + E_1(t) \end{aligned} \tag{3}$$

where c_1 and c_2 are constants determined from the initial conditions and λ_1 and λ_2 are eigenvalues in terms of the coefficients $a, c, d,$ and f given by

$$\begin{aligned} \lambda_1 &= \frac{(a+d) - \sqrt{(a+d)^2 - 4(ad-cf)}}{2} \\ \lambda_2 &= \frac{(a+d) + \sqrt{(a+d)^2 - 4(ad-cf)}}{2}. \end{aligned} \tag{4}$$

Qualitative Analysis

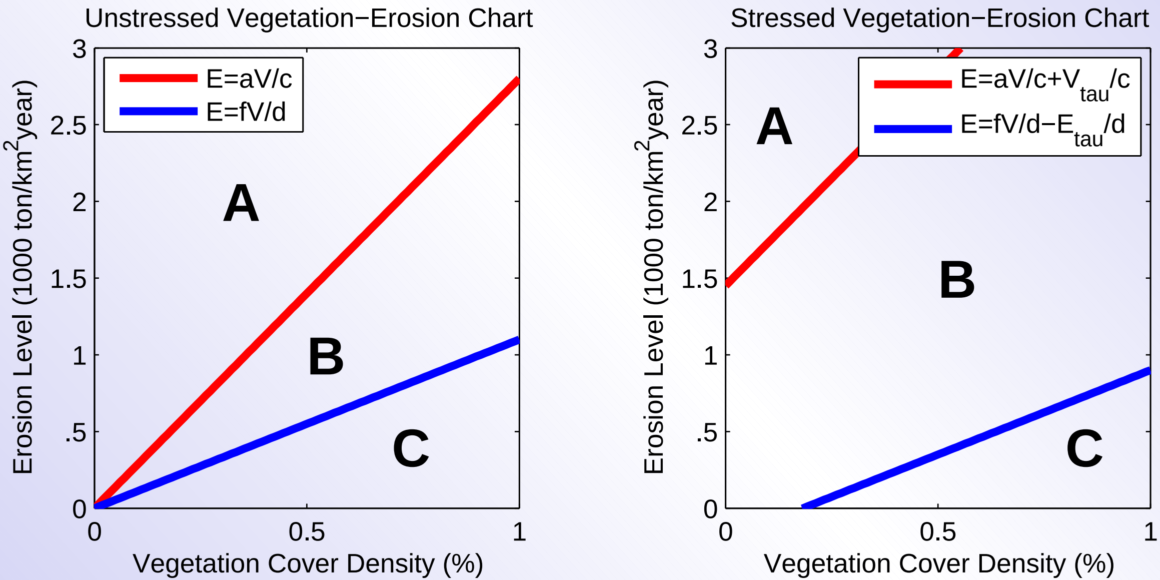
Now that we have solved our equations we can use qualitative analysis to determine the current condition of the system and the likelihood of the ecological system to recover or deteriorate.

Using the equations (2) from our original system, we can find the nullclines of the system by setting dV/dt and dE/dt equal to zero and solving for E.

$$
\begin{aligned}
E &= \frac{a}{c}V + \frac{1}{c}V_\tau \\
E &= \frac{f}{d}V - \frac{1}{d}E_\tau
\end{aligned}
\tag{5}
$$

Vegetation-Erosion Chart

Evaluating the vegetation density versus the erosion produces a Vegetation-Erosion Chart.



Heishui River Basin

The Heishui River Basin is a small tributary of the Yangtze river that was targeted as a study for erosion control due to its high erosion rates of 7,243 ton/km^2 per year and low vegetation coverage of 7.6%.

Beginning in 1978 the government reforested the river basin at a rate of 4% per year(V_{τ_0}), while also installing erosion control dams that reduced erosion by 650 ton/km^2 per year(E_{τ_0}).

The ecological stresses in this case are constant where

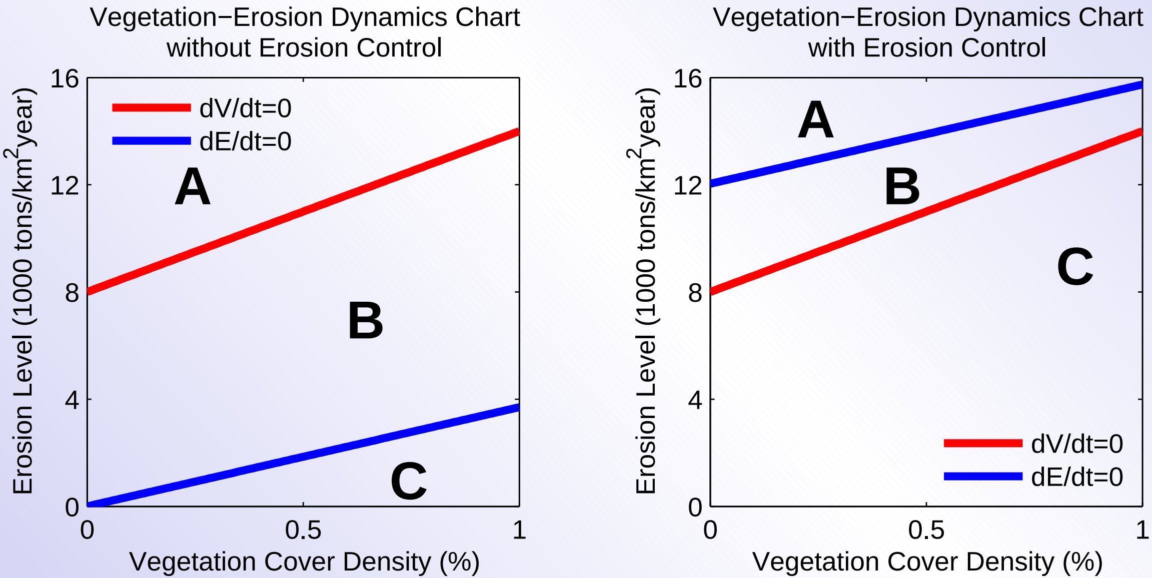
$$V_\tau(t) = V_{\tau_0} \text{ and } E_\tau(t) = E_{\tau_0}, \tag{6}$$

Evaluating the solution from (3) gives the equations

$$\begin{aligned} V(t) &= c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} - \frac{dV_{\tau_0} + cE_{\tau_0}}{ad - cf} \\ E(t) &= c_1 \frac{a - \lambda_1}{c} e^{\lambda_1 t} + c_2 \frac{a - \lambda_2}{c} e^{\lambda_2 t} + - \frac{fV_{\tau_0} + aE_{\tau_0}}{ad - cf} \end{aligned} \tag{7}$$

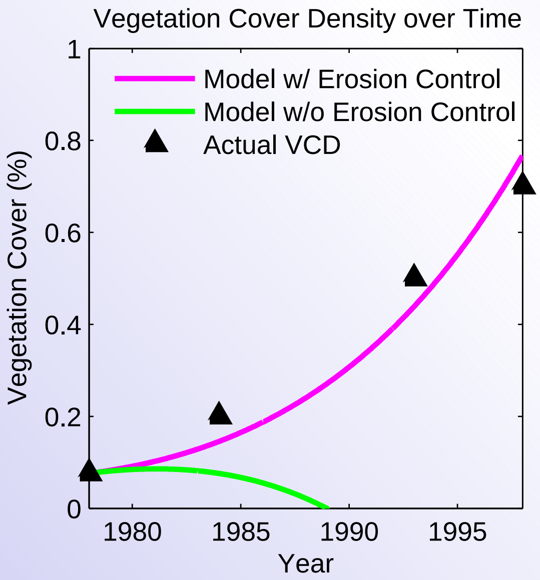
Heishui: Qualitative Analysis

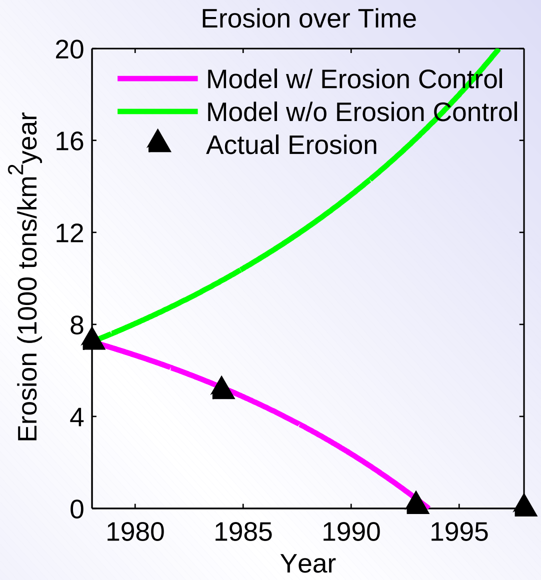
The Vegetation-Erosion Chart of the Heishui model demonstrates how improving vegetation and erosion conditions decrease the high risk (A) and the medium risk (B) sections.



Heishui Graphs

Using the initial conditions of $V(1978) = .076$ $E(1978) = 7,243$ we can simulate the activity of the Vegetation Coverage and the Erosion in the Heishui Basin.





We simulated the model with and without erosion control along with the measured to show the importance of erosion control.

Conclusion

- Vegetation-Erosion Dynamics is a relatively new area in science that has shown the important interconnectedness of vegetation to erosion.
- Human activities have a great impact on the environment and should take responsibility to sustain a vibrant ecosystem.
- Vegetation-Erosion models can help determine the current conditions and help develop effective means to reduce erosion.
- Vegetation-Erosion analysis will eventually be incorporated into civil engineering projects to help minimize impacts to the environment.