Application 5.2
Plotting Third-Order Solution Families

This application deals with the computer-plotting of solution families like those illustrated in Figs. 5.2.2 through 5.2.4 in the text. We know from Example 6 in Section 5.2 that the general solution of

$$y^{(3)} + 3y'' + 4y' + 12y = 0 \tag{1}$$

is

$$y(x) = c_1 e^{-3x} + c_2 \cos 2x + c_3 \sin 2x . \tag{2}$$

Then use the method of Example 6 (or a computer algebra system) to show that the particular solution of (1) satisfying the initial conditions $y(0) = a$, $y'(0) = b$ and $y''(0) = c$ is

$$y(x) = \frac{1}{26}\left[(8a + 2c)e^{-3x} + (18a - 2c)\cos 2x + (12a + 13b + 3c)\sin 2x\right]. \tag{3}$$

- For Fig. 5.2.2, substitution of $b = c = 0$ in (3) gives

$$y(x) = \frac{a}{13}\left(4e^{-3x} + 9\cos 2x + 6\sin 2x\right). \tag{4}$$

- For Fig. 5.2.3, substitution of $a = c = 0$ in (3) gives

$$y(x) = \frac{b}{2}\sin 2x . \tag{5}$$

- For Fig. 5.2.4, substitution of $a = b = 0$ in (3) gives

$$y(x) = \frac{c}{26}\left(2e^{-3x} - 2\cos 2x + 3\sin 2x\right). \tag{6}$$

In the following sections, we illustrate the use of computer systems like *Maple*, *Mathematica*, and MATLAB to plot simultaneously a family of solution curves like those defined by (4), (5), or (6). Start by reproducing Figs. 5.2.2 –5.2.4. Then plot similar families of solution curves for the differential equations in Problems 13–20 of Section 5.2 in the text.

Using *Maple*

Using Eq. (4), the particular solution of (1) with $y(0) = a$, $y'(0) = 0$ and $y''(0) = 0$ is defined by

```
partSoln := a*(4*exp(-3*x)+9*cos(2*x)+6*sin(2*x))/13;
```

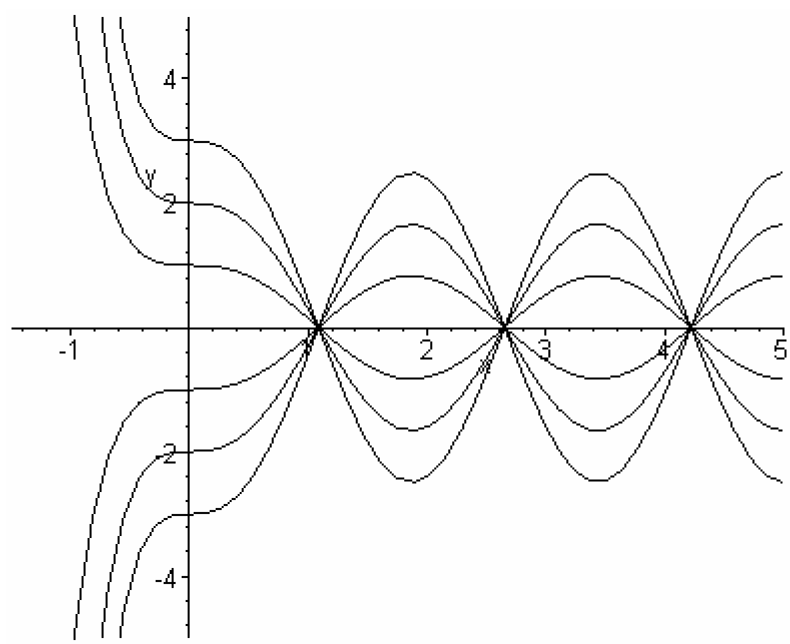
$$partSoln := \frac{1}{13}\,a\left(4\,e^{(-3x)} + 9\cos(2x) + 6\sin(2x)\right)$$

The set of such particular solutions with initial slopes $b = -3, -2, \ldots, 2, 3$ is then defined by

```
curves := {seq(partSoln, b =-3..3)}:
```

We plot these 7 curves simultaneously on the x-interval (–1.5, 5) with the single command

```
plot(curves, x =-1.5..5, y =-5..5);
```



Using *Mathematica*

Using Eq. (6), the particular solution of (1) with $y(0) = 0$, $y'(0) = 0$ and $y''(0) = c$ is defined by

```
partSoln = c(2 Exp[-3x] - 2 Cos[2x] + 3 Sin[2x])/26
```

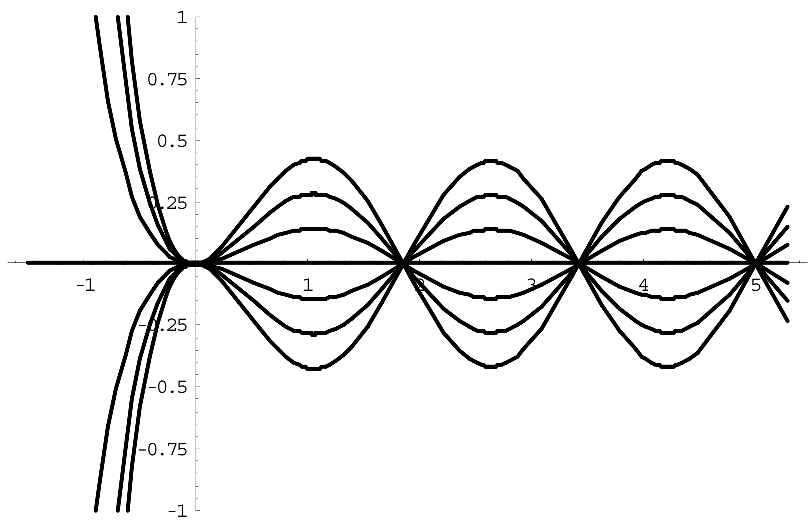
$$\frac{1}{26}\,c\left(2\,e^{-3x}-2\cos(2x)+3\sin(2x)\right)$$

The set of such particular solutions with initial slopes $c = -3, -2, \ldots, 2, 3$ is then defined by

```
curves = Table[partSoln, {c, -3, 3}];
```

We plot these 7 curves simultaneously on the x-interval $(-1.5, 5)$ with the single command

```
Plot[Evaluate[curves], {x,-1.5,5}, PlotRange->{-1,1}];
```



Using MATLAB

Using Eq. (3), the particular solution with $y(0) = y'(0) = y''(0) = a$ is defined by

$$y(x)=\frac{a}{13}\left(5e^{-3x}+8\cos 2x+14\sin 2x\right).$$

We can plot the 7 solution curves with $a = -6, -4, \ldots, 4, 6$ on the interval

```
x = -1.5 : 0.02 : 5;   % x-vector from x=-1.5 to x=5
```

with the single **for** loop

```
for a = -6 : 2 : 6
    c1 = 5*a/13;  c2 = 8*a/13;  c3 = 14*a/13;
    y = c1*exp(-3*x) + c2*cos(2*x) + c3*sin(2*x);
    plot(x,y,'k')
    axis([-1.5 5 -10 10]), hold on
    end
```