Model of the Vegetation-Erosion Relationship using Differential Equations

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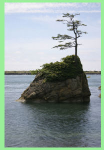
May 6, 2005

Abstract

Erosion and vegetation are intimately connected. The dynamics of vegetation and erosion can be evaluated using a system of differential equations. By examining the natural and human influences on these systems we can create an accurate model that simulates the long-term progression of a watershed system. Qualitative analysis can help determine whether an ecosystem is in danger of eroding away. This data can help people develop effective methods to stop erosion and promote vegetation growth.



1. Introduction

In this community and in many communities in this world, the effects of erosion on our rivers and ecosystems are often devastating. Recently, however, mathematicians and scientists have been investigating the dynamic relationship between vegetation and erosion processes and the effects of human stresses on a particular watershed system. Through the use of a model derived using first order differential equations, we are able to illustrate the constant relationship between vegetation and erosion. Vegetation-erosion charts produced from the original differential model can predict the long-term effects that certain conditions have on specific watersheds and can be used to devise a plan of action to improve the current ecosystem's chances of survival.

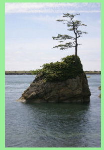
It is not surprising that increases in erosion will have an inverse relationship with the vegetation-cover density (VCD) in any given area. We may also find it easy to understand that human activities such as logging, hunting, building roads, and so on will affect vegetation development. Beyond human causes, however, certain ecological events such as wind storms, pests, drought, air pollution, and severe temperatures have been targeted as causes for stress on a watershed's vegetation.

Unfortunately, the lower the vegetation-cover density in an area the more susceptible the earth will be to erosion from human and environmental stresses. Without help, some borderline watersheds are destined for destruction. Because of the urgency involved in the protection of our earth and its fragile ecosystems, the simulation of the vegetation-erosion dynamic is an important tool that has proven to be an applicable and clear method for predicting the long-term nature of watersheds. The relationship shared between soil erosion and ground cover can actually reach an equilibrium when exposed to and kept under the correct conditions. This is where the conceptualization of the VCD and the soil's stresses can be exceptionally useful.

2. The Basic Model

2.1. The Parameters

Before we can begin to piece together the model, there are a few important terms that must be explained. Primarily, the ability of the soil to bind to the roots of plants growing on the surface is an important aspect of the model. The term vegetation-cover density or VCD has been mentioned previously and is defined as the percentage of ground area being covered by plants in a specific region. Naturally, a greater VCD will be desirable in the reduction of erosion. Since grasslands to do not provide a sufficient root base to effectively reduce erosion, we are only considering vegetation density of trees and shrubs. The



VCD, which we will define simply as V for the model, will hence become a very important indicator of the success of a watershed because it can actually be used to represent the level of vegetation development.

The VCD can be affected drastically by many types of environmental and human occurrences and this fact makes its calculation rather complicated. For example, air pollution and drought can negatively affect an area's abilities in vegetation development. Though air pollution poses a much more enduring affect, a drought can cause a substantially greater, yet less lengthy, stress on the vegetation. If we define A_T as being the stress caused by air pollution, A as the amount of pollutant over a certain time interval, and A_a as the long term average of that same pollutant, we can identify the relationship

$$A_\tau = \frac{A - A_a}{A_a} \tag{1}$$

Similarly, the precipitation in a given area can be defined as

$$P_\tau = \frac{P - P_e}{P_e}, P < P_e \tag{2}$$

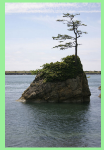
where P =the annual precipitation levels, P_e =the estimated water demand of a certain vegetated system, and P_τ =the stress caused by a lack of precipitation in a watershed.

Along with air pollution and precipitation, certain instantaneous events affect vegetation. Consider the meteorite that struck a Siberian forest in 1908 that flattened trees up to ten miles around the impact site. This instantaneous stress will be defined as $K_{inst}(t_0)$ or the instant stress at some time t_0.

Though humans can cause great stress on the environment through mining, road constructing, and farming activities, one human practice actually acts as one of the main deterrents of erosion and bolster to vegetation: the human reforestation effort. With continuity, the replanting of trees and shrubs in watersheds creates a strong and effective system that controls erosion. This continuous function will be defined as V_R. With all this, we now have the ability to construct a fairly thorough model for simulating vegetation development:

$$\frac{dV}{dt} = aV - cE + K_p P_t - K_a A_t - K_{inst}(t_0) + V_R \tag{3}$$

where E =erosion rate, a and c are rate coefficients for vegetation cover density and erosion respectively, K_p and K_a represent the affects of air pollution and precipitation on a system, and V_R is a continuous function representing reforestation. In many case studies we find that the model can be reduced to a simpler form because there are only a few significant stresses on the system.



A similar equation can be obtained which represents the erosion rate in watersheds.

$$\frac{dE}{dt} = dE - fV + E_R \tag{4}$$

where d and f are rate coefficients representing erosion and vegetation and E_R is a continuous function representing the influence of human activities on erosion.

2.2. The Dynamics

Many factors determine the relationship between vegetation and erosion in watersheds. Originally, the equations were slightly altered and included other coefficients such as b which denoted the effect of wildlife on vegetation. Because it is unlikely that deer or herbivorous animals greatly effect the ability of a watershed to vegetate, this coefficient was presumed to be zero and eliminated.

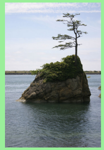
In equation (3) we see several other variables that have been previously defined as rate coefficients. It may be helpful to understand the actual meaning of each of these parameters. Coefficient a, which is the rate coefficient for vegetation, represents the effect that existing plant-life has on promoting the growth of new trees and shrubs. The greater the value of a in the model, the more quickly an area will be able to produce new vegetation. Of the others, c denotes the percentage of the erosion that is actually hindering the ability of the system to reproduce plant-life, d is the level of erosion that is seen to induce excess erosion, and f is the proportion of vegetation that stunts erosion.

The magnitude of these parameters can give us valuable insight into the topographical and climatic behavior of an area. For example, in tropical regions that have high levels of rainfall and mountainous terrain, we would see greater values of a, c, d, and f because of the significant relationship between vegetation and erosion.

2.3. Solving the Model

The differential equations (3) and (4) are nonhomogeneous and linear and can therefore be solved analytically. By redefining the stresses imposed on vegetation due to environmental and human interference and the influences of humans on the erosion rate as we can produce a simpler differential set of differentials that will be easier to manipulate.

$$\begin{gathered} K_p P_t - K_a A_t - K_{inst}\delta(t_0) + V_R = V_\tau \\ E_R = E\tau \end{gathered} \tag{5}$$



Substituting these new definitions into equations 3 and 4 we produce the following set of equations:

$$\begin{aligned} \frac{dV}{dt} &= aV + cE + V_\tau \\ \frac{dE}{dt} &= dE - fV + E_\tau \end{aligned} \tag{6}$$

which can also be written in the linear form

$$\begin{aligned} V' - aV + cE &= V_\tau \\ E' - dE + fV &= E_\tau \end{aligned} \tag{7}$$

The general solution to any linear system must be comprised of the linear combination of the homogeneous solution and a particular solution which is associated to the original linear system.

The homogeneous solution is the solution to the system

$$\begin{aligned} V' - aV + cE &= 0 \\ E' - dE + fV &= 0 \end{aligned} \tag{8}$$

The method for solving such a system begins when the system is rewritten using matrix notation and the eigenvalues along with their eigenvectors are found. When written in matrix notation the system takes the form $\mathbf{x}' = \mathbf{A}\mathbf{x}$ where $\mathbf{x}$ is a vector and $\mathbf{A}$ is a matrix of coefficients. In our system,

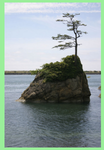
$$\begin{bmatrix} V \\ E \end{bmatrix}' = \begin{bmatrix} a & -c \\ -f & d \end{bmatrix} \begin{bmatrix} V \\ E \end{bmatrix}. \tag{9}$$

The eigenvalues are determined by deriving the characteristic equation associated with the system which includes certain values of λ. The characteristic polynomial is

$$\begin{aligned} p(\lambda) &= \left| A - \lambda I \right| \\ &= \left| \begin{bmatrix} a & -c \\ -f & d \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| \\ &= \begin{vmatrix} a - \lambda & -c \\ -f & d - \lambda \end{vmatrix} \end{aligned} \tag{10}$$

Finally, in order to solve for the eigenvalues we must take the determinant of equation (10)

$$\begin{aligned} p(\lambda) &= (a - \lambda)(d - \lambda) - (-c)(-f) \\ &= \lambda^2 - (a + d)\lambda + (ad - cf) \end{aligned} \tag{11}$$



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and solve for λ

$$\begin{aligned}\lambda_1 &= \frac{\left[(a+d)+\sqrt{(a+d)^2-4(ad-cf)}\right]}{2}\\ \lambda_2 &= \frac{\left[(a+d)-\sqrt{(a+d)^2-4(ad-cf)}\right]}{2}\end{aligned} \tag{12}$$

These eigenvalues determine a certain set of eigenvectors which happen to be the basis of the null space of the system. Therefore, we can calculate the eigenvectors by solving the equation

$$\left[A-\lambda I\right]\mathbf{v}=\mathbf{0} \tag{13}$$

for $\mathbf{v}$. Upon determining the eigenvectors, we can calculate the solution of the homogeneous equation in the form $\mathbf{x}=e^{\lambda t}\mathbf{v}$ where $\mathbf{x}$ is a vector containing V and E and $\mathbf{v}$ is a matrix containing the eigenvectors.

$$\begin{bmatrix} a-\lambda & -c\\ -f & d-\lambda\end{bmatrix}\begin{bmatrix} v_1\\ v_2\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

From this, we see that $v_1=1$ and $v_2=(a-\lambda)/c$ and our eigenvectors are

$$\begin{bmatrix}1\\(a-\lambda_1)/c\end{bmatrix}, \text{and} \begin{bmatrix}1\\(a-\lambda_2)/c\end{bmatrix} \tag{14}$$

Therefore, the solution becomes

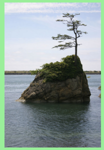
$$\begin{bmatrix}V\\E\end{bmatrix}=\begin{bmatrix}c_1e^{\lambda_1 t}\\c_2e^{\lambda_2 t}\end{bmatrix}\begin{bmatrix}1 & 1\\(a-\lambda_1)/c & (a-\lambda_2)/c\end{bmatrix}$$

which yields

$$\begin{aligned}V_h(t) &= c_1e^{\lambda_1 t}+c_2e^{\lambda_2 t}\\ E_h(t) &= c_1\frac{a-\lambda_1}{c}e^{\lambda_1 t}+c_2\frac{a-\lambda_2}{c}e^{\lambda_2 t}\end{aligned} \tag{15}$$

These are the solutions of the associated homogeneous equation (8) and are only a part of the general solution. The other half of the general solution is called the particular solution and is the solution to the nonhomogeneous differential system that is the original linear model. The particular solution is

$$\begin{aligned}V_p(t) &= e^{\lambda_1 t}\int\left[e^{-\lambda_1 t}e^{\lambda_2 t}\int e^{-\lambda_2 t}\left(\frac{dV_\tau}{dt}-dV_\tau-cE_\tau\right)dt\right]dt.\\ E_p(t) &= e^{\lambda_1 t}\int\left[e^{-\lambda_1 t}e^{\lambda_2 t}\int e^{-\lambda_2 t}\left(\frac{dE_\tau}{dt}-aE_\tau-fV_\tau\right)dt\right]dt.\end{aligned} \tag{16}$$



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where the terms V_τ and E_τ have been defined previously in equation (5). Hence, the general solution to the vegetation model is

$$\begin{aligned} V &= V_h + V_p \\ V &= c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} + e^{\lambda_1 t} \int \left[e^{-\lambda_1 t} e^{\lambda_2 t} \int e^{-\lambda_2 t} \left(\frac{dV_\tau}{dt} - dV_\tau - cE_\tau \right) dt \right] dt. \end{aligned} \tag{17}$$

and the general solution to the erosion model becomes

$$\begin{aligned} E &= E_h + E_p \\ E &= c_1 \frac{a-\lambda_1}{c} e^{\lambda_1 t} + c_2 \frac{a-\lambda_2}{c} e^{\lambda_2 t} + e^{\lambda_1 t} \int \left[e^{-\lambda_1 t} e^{\lambda_2 t} \int e^{-\lambda_2 t} \left(\frac{dE_\tau}{dt} - aE_\tau - fV_\tau \right) dt \right] dt. \end{aligned} \tag{18}$$

3. Qualitative Analysis

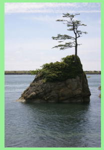
3.1. The Vegetation-Erosion Chart

Though it may be helpful to predict certain long-term behavior, sometimes it is important to visualize a current condition and its possible influence on the sustainability of an environment. As equation (5) allows, we can redefine the original differential system as

$$\begin{aligned} \frac{dV}{dt} &= aV - cE + V_\tau \\ \frac{dE}{dt} &= dE - fV + E_\tau \end{aligned} \tag{19}$$

For the qualitative analysis to be applicable in a situation, one must consider the effects of a precise and current level of vegetation development and erosion. This being said, the rate at which the vegetation and erosion are changing with respect to time is zero. Therefore, $dV/dt = 0$ and $dE/dt = 0$. Equation (**??**) becomes

$$\begin{aligned} cE &= aV + V_\tau \\ -dE &= -fV + E_\tau \end{aligned} \tag{20}$$



Solving these equations both for E we obtain two linear approximations of long term behaviors of erosion in a watershed.

$$\begin{aligned} E_v &= \frac{a}{c}V + \frac{V_\tau}{c} \\ E_e &= \frac{f}{d}V + \frac{E_\tau}{d} \end{aligned} \tag{21}$$

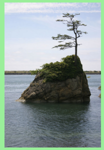
Where E_v is the equation derived from the original differential equation containing dV/dt and E_e is the equation derived from the original differential equation containing dE/dt. These equations represent the nullclines of the system; however, it is difficult to actually visualize the affects of the human and ecological stresses unless we can compare these behaviors with the behavior of the system when it is lacking these stresses. For this reason, we need also to obtain equations in which E_τ and V_τ both equal zero. As you may recall, E_τ and V_τ have previously been defined as the combination of many ecological and human stresses on an environment. The lack of these terms in the model can predict the behavior of the watershed in a natural setting. This second set of linear equations is

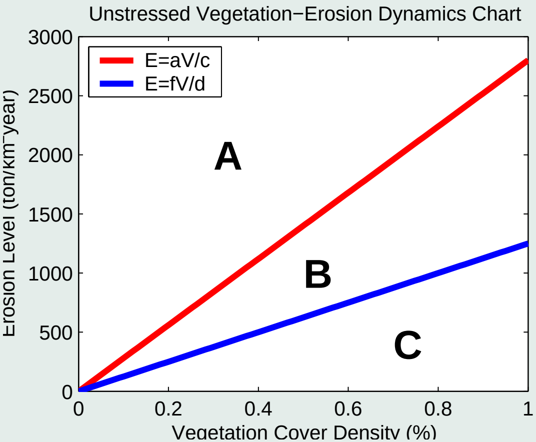
$$\begin{aligned} E_v &= \frac{a}{c}V \\ E_e &= \frac{f}{d}V \end{aligned}$$

When we graph these two sets of equations we can produce the Vegetation-Erosion Charts. Notice that in these figures (1(a)) and (1(b)), V is always represented with a percentage out of 100 because it is the percentage of ground being covered by trees and shrubs. The two vegetation erosion charts give us a more clear example of the differences in a watershed with and without stresses from the environment and human influence.

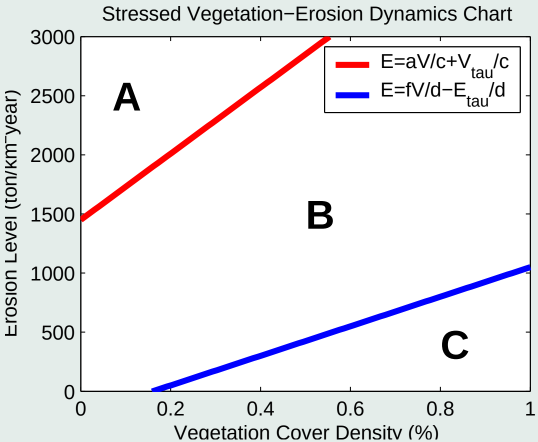
The bold-faced lines pictured in figure 1(a) and 1(b) divide the figures into three distinct sections labeled A, B,and C. These sections indicate important characteristics about the enclosed zone. For example, section A indicates a detrimental state for the watershed. In this section, vegetation is decreasing ($dV/dt < 0$) and erosion is increasing ($dE/dt > 0$). A environment that finds itself located in this high risk section is in dire need of erosion control and reforestation efforts. Without attention, it is likely that the watershed will be overwhelmed with erosion and be destroyed.

The section labeled B, is a borderline or medium risk section. Some principles of B are beneficial to the watershed while others are hurtful. Ecosystems with VCD and erosion ratings which place them within zone B often still need help to increase vegetation because although we have a decrease in erosion ($dE/dt < 0$), we also have decreasing vegetation ($dV/dt < 0$).





(a)



(b)

Figure 1: Vegetation-Erosion Charts

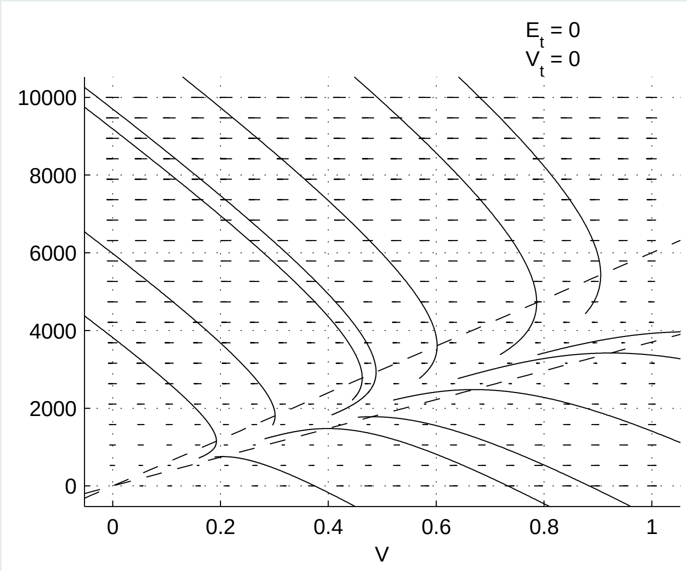
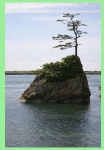


Figure 2: Plot of Several Solutions in the Phase Plane

The most beneficial section for an environment would be the section labeled C, which is considering extremely low risk for a watershed. In this area, Vegetation is increasing and erosion is decreasing meaning that $dV/dt > 0$ and $dE/dt < 0$. In this region, reforestation and anti-erosion efforts have put the watershed on the path to complete recovery.

The vegetation-erosion chart is a representation of the system plotted in the phase plane. In the phase plane, we see vegetation plotted against erosion rather than against time. The lines that divide the figure into the labeled sections are the nullclines. These play an important role in the understanding of the system.

As we plot solutions within the phase plane that exists with no stresses from the environment or humans, we notice that there is a source equilibrium point at the origin. All solutions plotted in Figure **??** are shown moving forward in time from an initial point located in section B between the two nullclines. There exists a straight line solution such that all initial conditions located above that solution will eventually decay to zero vegetation and be destroyed, while all those beginning below the



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straight line solution will eventually recover to 100 percent vegetation.

When we compare Figure 1(a) to its similar counterpart Figure 1(b), we see the change in the likelihood of a system falling into sections A, B, C. Notice that in Figure 1(a) Section A incorporates nearly half of the area of the chart, however, with human involvement (including negative and positive involvement), this area is reduced to a noticeably smaller region. In some studies, a watershed's ability to survive or be destroyed is completely determined by its location on the vegetation-erosion chart.

4. Case Studies

4.1. Heishui

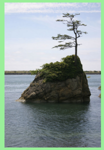
The Heishui River Basin is a river located near the Yangtze River in China. Because of its exceptionally high erosion rate and very low VCD, the Heishui River Basin is the perfect case in which to apply the model. In order to demonstrate the applicability of the developed model, certain measures were taken to restore the vegetation cover density and relieve the stressful erosion levels. Trees in the area were reforested at a rate of four percent annually starting in 1978 and anti-erosion dams were constructed that reduced the crippling $7,243$ km/ton^2 per year of original erosion by 650 km/ton^2 per year. With an initial VCD of only 7.6%, it would take a great deal of human interference and to restore the river basin to an acceptable state.

Because, in our case, the reforestation and anti-erosion efforts greatly outweigh the importance of other ecological stresses such as precipitation and air pollution, V_τ and E_τ are constants.

$$\begin{aligned} V_\tau(t) &= V_{\tau_0} = .04 \\ E_\tau(t) &= E_{\tau_0} = -650 \end{aligned} \tag{22}$$

Substituting these two values into our particular solution found in equation (16) we can solve for the general solution of this system. The fact that these values are constants makes the integration much simpler because $dV_0/dt = 0$ and the remainder of the grouped integrand becomes a constant and can be moved in front of the integrals. Integrating the remaining $e^{\lambda_1 t}$ and $e^{\lambda_2 t}$ terms we can produce a surprisingly very simple particular solution.

$$\begin{aligned} V_p(t) &= e^{\lambda_1 t} \int \left[e^{-\lambda_1 t} e^{\lambda_2 t} \int e^{-\lambda_2 t} \left(\frac{dV_\tau}{dt} - dV_\tau - cE_\tau \right) dt \right] dt. \\ &= e^{\lambda_1 t} \int \left[e^{-\lambda_1 t} e^{\lambda_2 t} \int e^{-\lambda_2 t} \left(-dV_\tau - cE_\tau \right) dt \right] dt. \end{aligned}$$



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$$
\begin{aligned}
&= (-dV_\tau - cE_\tau)\, e^{\lambda_1 t} \int \left[e^{-\lambda_1 t} e^{\lambda_2 t} \int e^{-\lambda_2 t}\, dt \right] dt. \\
&= (-dV_\tau - cE_\tau)\, e^{\lambda_1 t} \int \left[e^{-\lambda_1 t} e^{\lambda_2 t} \left(\frac{-1}{\lambda_2} e^{-\lambda_2 t} \right) \right] dt. \\
&= \left(\frac{dV_\tau + cE_\tau}{\lambda_2} \right) e^{\lambda_1 t} \int e^{-\lambda_1 t}\, dt. \\
&= \left(\frac{dV_\tau + cE_\tau}{\lambda_2} \right) e^{\lambda_1 t} \left(\frac{-1}{\lambda_1} e^{-\lambda_1 t} \right) \\
&= \frac{-dV_\tau - cE_\tau}{\lambda_1 \lambda_2} \qquad (23)
\end{aligned}
$$

After manually simplifying $\lambda_1 \lambda_2$ we produce a denominator equal to $ad - cf$ Therefore, the particular solution becomes

$$
V_p(t) = \frac{-dV_\tau - cE_\tau}{ad - cf} \qquad (24)
$$

A similar method is used for determining the particular solution to the Erosion dynamic.

$$
E_p(t) = \frac{-fV_\tau + aE_\tau}{ad - cf} \qquad (25)
$$

The combination of these two particular solutions and the homogeneous solutions in equation (15) give us the following general solutions for the vegetation and erosion levels in the Heishui River Basin:

$$
\begin{aligned}
V(t) &= c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} - \frac{dV_\tau + cE_\tau}{ad - cf} \\
E(t) &= c_1 \frac{a - \lambda_1}{c} e^{\lambda_1 t} + c_2 \frac{a - \lambda_2}{c} e^{\lambda_2 t} - \frac{fV_\tau + aE_\tau}{ad - cf}
\end{aligned} \qquad (26)
$$

In order to solve complete the solution we must substitute an initial condition into each equation. These initial conditions are given as

$$
\begin{aligned}
V(1978) &= .076 \\
E(1978) &= 7,243
\end{aligned}
$$

After the substitution, the coefficients can be placed into an augmented matrix and reduced in order to solve for the needed c_1 and c_2 values. However, one cannot complete the reduction without full

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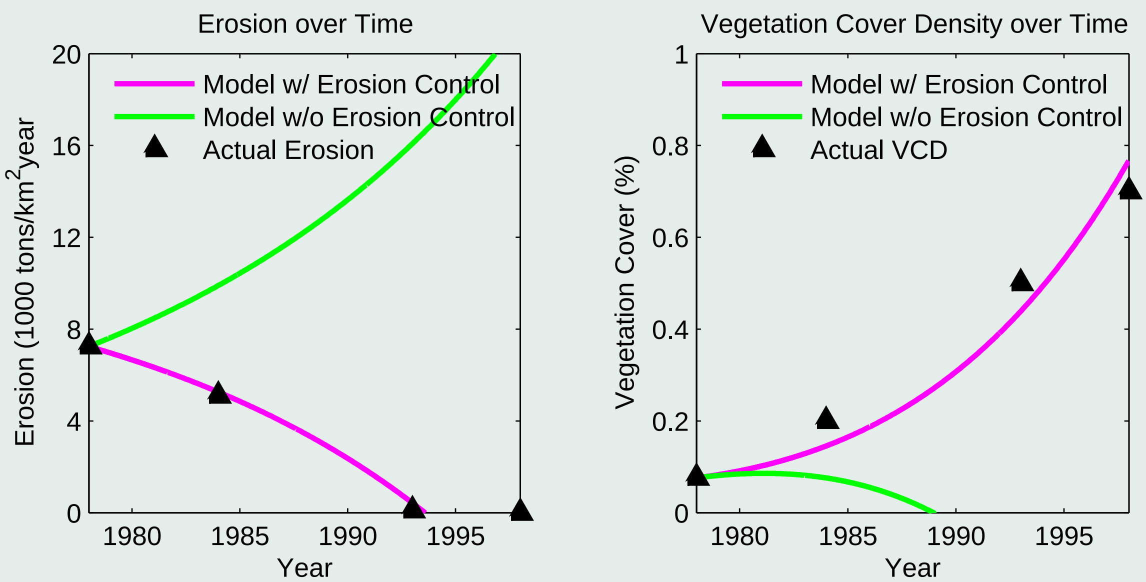


(a) (b)

Figure 3: Graphs of Vegetation and Erosion in the Heishui River Basin

knowledge of the values of the parameters a, d, c, and f. In this case, certain climatic, morphological, and soil conditions were measured over long periods to ensure proper labeling of the parameters

$$a = 0.03 year^{-1}$$
$$c = 0.000005 km^2/ton$$
$$d = 0.054 year^{-1}$$
$$f = 200 ton/km^2 year^2$$

The knowledge of the value of these parameters is the final piece in the completion of the Heishui model. We now can use equation (26) to simulate the long term behavior of the river basin. Figure 3 shows graphs of the vegetation and erosion plotted over time. As you can see, the actual data points measured

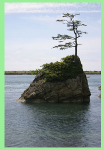
(plotted with solid black triangles) fit the estimated model behavior (plotted in magenta), proving that the methodology is accurate and applicable. The rate at which the VCD is increasing with respect to time increases exponentially over time, solidifying the understanding that greater vegetation promotes further plant-life. If the current efforts for reforestation and anti-erosion continue, it is shown that the watershed has the ability to reach 100 percent vegetation cover density by the year 2002. Also pictured, the erosion levels almost linearly reduce over time and reach zero tons/km^2 by 1994. Figure 3 also presents the long term behavior of the system with no erosion control. One might find it reasonable to think that mere reforestation efforts would have beneficial influences on a troubled watershed; however, as the model predicts, when $V_{\tau_0} = .04$ and $E_{\tau_0} = 0$, meaning that erosion control dams are not built, the vegetation is quickly stunted by the huge erosion levels and is completely obliterated by the year 1992. Simultaneously, erosion levels would quickly escalate to over $20,000$ ton/km^2 by 2002 and the watershed would be destroyed. Therefore, the implementation of reforestation alone is simply not enough to restore an ecosystem to full function. The visualization of the vegetation's relationship with the erosion in river basin can be accurately portrayed through the vegetation-erosion charts.

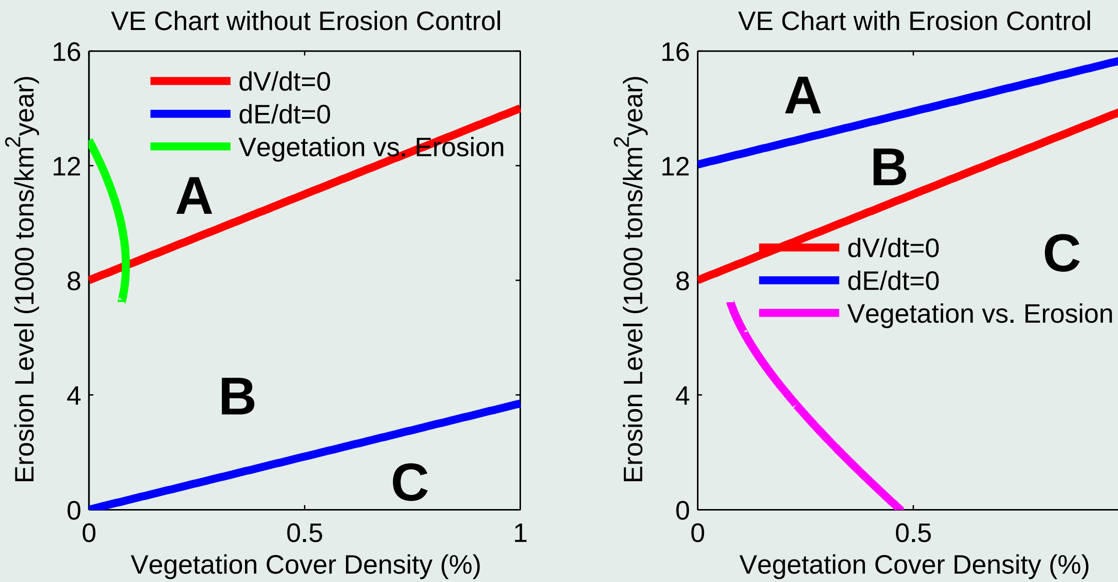
4.2. Qualitative Analysis of Heishui River Basin

Notice that in figure 4 the Vegetation-Erosion Chart pictured that does not includes erosion mitigation efforts has a much ratio of high and medium risk zones to low risk zones. It is not hard to see that a watershed may have a difficult time making a complete recovery if erosion efforts were non-existent. In Figure 4(b) however, a troubled watershed has a great chance of becoming located within zone C and can therefore be expected to recover. We can plot the solution starting at specific initial values inside the phase plane of the Vegetation-Erosion Charts.

As we can see from Figure 4(a), when anti-erosion efforts are ceased, the solution, moving forward in time from the point $(.04, 7243)$, reaches zero vegetation and excessive erosion, just as our original plot of erosion versus time indicates. Also Figure 4(b) shows that with erosion control, the vegetation eventually reaches one hundred percent and the erosion will soon drop to 0 ton/km^2 annually.

With the study of effective erosion prevention, we learn that, although reforestation efforts are an important aspect in the rebuilding of a weakened ecosystem, it was only in the combination of reforestation and anti-erosion efforts that the Heishui River Basin could be salvaged.





(a) (b)

Figure 4: Vegetation-Erosion Charts of Heishui River Basin

5. Conclusion

Vegetation and erosion are a pair of interactive and dynamic actions that are imperative to the survival of a watershed. Having been studied for only about twenty years, the vegetation-erosion dynamic is a fairly new science and is quickly making its mark in the scientific and civil engineering worlds. With ecosystem survival being an important and delicate issue today, it is imperative to understand and predict the long-term behaviors of high risk watersheds and to develop the technology to prevent these damaged systems from being destroyed. Through the use of differential equations and the vegetation-erosion dynamic developed in these pages, we hope to refine our skills in watershed management.

Because vegetation is affected by so many ecological and human factors, the quantification of these factors is the only way to predict the evolution of a system under certain conditions. The use of vegetation-erosion charts can describe a system as increasing in vegetation and decreasing in erosion, increasing in erosion and decreasing in vegetation, or decreasing in both erosion and vegetation. Human efforts, a junction of reforestation and erosion mitigation efforts, can move a system from one condition to another depending on the severity of the problem and the distance that the system lies from the nullclines in the VE Charts.

By applying the vegetation-erosion equations to case studies completed in the Heishui River Basin in China, we can prove the applicability of the model and the absolute importance of erosion control in aiding recovery. This fact ensures that the model could be used to predict any watershed's long term behavior and the necessary actions to be taken in order to guarantee recovery.

References

[1] Wang, Z.Y., G.H. Huang, G.Q. Wang, and J. Gao. "Modeling of Vegetation-Erosion Dynamics in Watershed Systems." *Journal of Environmental Engineering*. Vol 130, No. 7, July 1, 2004.