Rowing Model for a Four

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Abstract

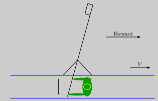
This paper is an application of the derived sweep-rowing stroke model, as applied to Humboldt State's Women's Varsity Open Four.

1. Introduction

The mathematical model for rowing was designed by Maurice Brearley, Neville de Mestre, and Donald Watson. Their Model was designed using the Australian Men's Olympic eight team. In this paper the model will be applied to a Women's Intercollegiate four team. The derivation of the equation will be revisited and then an analysis comparing the times calculated and the times rowed by the team throughout the 2001 season.

2. Notation

The first part of deriving the model for the rowing stroke is defining the variables in order to better understand the derivation.



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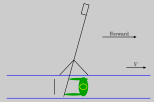
$$
\begin{aligned}
m &= \text{mass of the boat (including the Coxswain)}\\
M &= \text{combined weight of rowers}\\
t &= \text{time from start of the power stroke}\\
v &= \text{velocity of boat at time } t\\
f &= \frac{dv}{dt} = \text{acceleration of boat at time } t\\
T_1 &= \text{duration of power stroke}\\
T_2 &= \text{duration of the recovery}\\
t' &= t - T_1 = \text{time from the start of the recovery}\\
D &= \text{drag of the water on the shell}
\end{aligned}
$$

Also note that the equation for the full stroke is broken into two different equations. The first equation called the power stroke goes form $0 \leq t \leq T_1$, the next part of the stroke is called the recovery, which starts at the end of the power stroke and goes to the beginning of the next stroke seen as $T_1 \leq t \leq T_1 + T_2$. However, because there are two different equations and the recovery needs to also start at 0 its time interval can be adjusted to $0 \leq t' \leq T_2$, where $t' = t - T_1$.

3. The Power Stroke

According to Newton's second law, the forward motion of the boat is equal to the sum of the forces acting on the boat. Because we know the boat is moving forward we know that the positive forward forces are greater than the negative forces.

To assess the forces we need to look at the mechanics of the stroke. The shell is moved through the water using a giant lever or oar. During the stroke the blade of the oar is essentially stationary which allows the boat to be pushed forward through the water at



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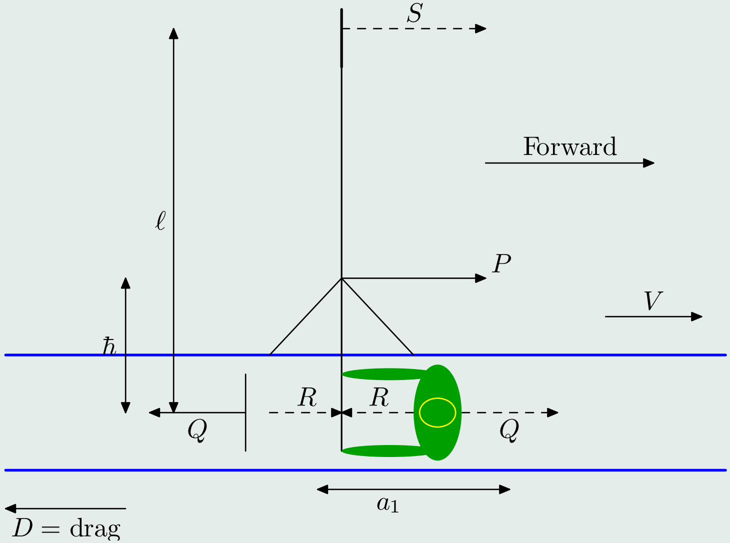


Figure 1: The Power Stroke.

the pivot located at the end of the rigger, as shown in Figure 1. This leads to the force equation

$$\begin{aligned} R\ell &= P(\ell - \hbar) \\ R\ell &= P\ell - P\hbar \\ P\hbar &= P\ell - R\ell \\ \frac{P\hbar}{\ell} &= P - R. \end{aligned}$$

In Figure 1, P is the forward force generated by the stroke at the oar lock of the rigger, and R is the force exerted on the oars by the rowers. We know that R_0 and R_f are both zero because the stroke begins and ends at rest. The maximum force R is applied in the middle of the power stroke, this tells us that the equation for R appears sinusoidal. Therefore, we can infer using the relationship

$$P = \frac{R\ell}{(\ell + \hbar)}$$

that P is also sinusoidal function and can be written

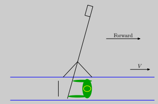
$$P - R = \frac{\hbar}{\ell} P = P_m \sin(n_1 t). \tag{1}$$

R is the force that the rowers exert on the oars during the stroke, both the initial and final force is 0 with a maximum smoothly reached in the middle. The smooth motion is similar to half of a cycle of simple harmonic motion. Thus the equation can be written

$$x_1 = -a_1 \cos(n_1 t),$$

where

$$\begin{aligned} a_1 &= \text{the average change of the rower's position} \\ n_1 &= \frac{\pi}{T_1}. \end{aligned}$$



This equation shows where the rowers are in relationship to their initial position in the boat. Thus x_1'' is the rower's acceleration relative to the boat.

$$x_1'' = n_1^2 a_1 \cos(n_1 t) \tag{2}$$

Now looking at the forces acting on the rowers we get

$$Q - R = M(x_1'' + f) = M(n_1^2 a_1 \cos(n_1 t) + \frac{dv}{dt}) \tag{3}$$

The equation for the motion of the boat is:

$$P - Q - D = M\frac{dv}{dt}. \tag{4}$$

By adding the equations (3) and (4)

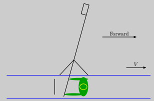
$$\begin{aligned}
P - Q - D + (Q - R) &= M(n_1^2 a_1 \cos(n_1 t) + \frac{dv}{dt}) + m\frac{dv}{dt} \\
P - D - R &= M n_1^2 a_1 \cos(n_1 t) + (M + m)\frac{dv}{dt} \\
(M + m)\frac{dv}{dt} &= P - R - D - M n_1^2 a_1 \cos(n_1 t) \\
(M + m)\frac{dv}{dt} &= \frac{\hbar}{\ell} P - M n_1^2 a_1 \cos(n_1 t) - D
\end{aligned}$$

now substituting $P_m \sin(n_1 t)$ for $\hbar/\ell P$ we get

$$(M + m)\frac{dv}{dt} = P_m \sin(n_1 t) - M n_1^2 a_1 \cos(n_1 t) - a - bv - cv^2$$

from which, with some simplification

$$\frac{dv}{dt} = K_1 \sin(n_1 t) + K_2 \cos(n_1 t) + A + Bv + Cv^2 \tag{5}$$



with the parameters

$$K_1 = P_m/(m+M), \qquad K_2 = -(Mn_1^2 a_1)/(m+M)$$

and

$$A = -a/(m+M), \qquad B = -b/(m+M), \qquad \text{and} \qquad C = -c/(m+M).$$

4. The Recovery

In the recovery the rowers pull themselves back toward the stern in preparation for the next stroke. With their feet strapped into the boat the rowers use their legs to pull themselves back. The rowers motion is the opposite of when they pushed themselves forward in the power stroke, thus again an example of simple harmonic motion, and the equation can be written as:

$$x_2 = a_1 \cos(n_2 t')$$

where

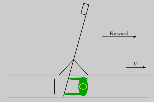
$$\begin{aligned} a_1 &= \text{the average distance traveled by each rower} \\ n_2 &= \frac{\pi}{T_2}. \end{aligned}$$

Thus, the acceleration of the rowers relative to the boat is

$$x_2'' = -n_2^2 a_1 \cos(n_2 t').$$

The motion of the boat relative to the water is $x_2'' + f$. The forward motion of the rowers can be written

$$-F = M(x_2'' + f) = M\left(-n_2^2 a_1 \cos(n_2 t') + \frac{dv}{dt}\right).$$



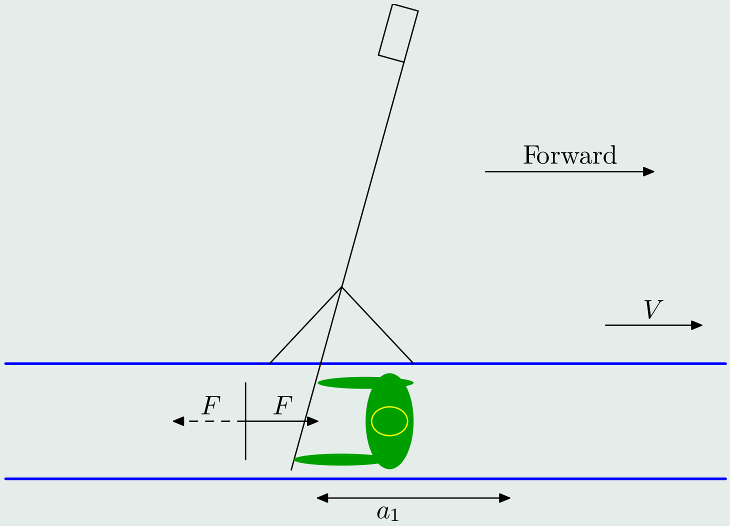


Figure 2: The recovery stroke.

The motion of the boat can be written

$$F - D = m\frac{dv}{dt}.$$

Adding the two equations,

$$\begin{aligned}
F - F - D &= M\left(-n_2^2 a_1 \cos(n_2 t') + \frac{dv}{dt}\right) + m\frac{dv}{dt} \\
-D &= -Mn_2^2 a_1 \cos(n_2 t') + (M+m)\frac{dv}{dt} \\
(M+m)\frac{dv}{dt} &= Mn_2^2 a_1 \cos(n_2 t') - D,
\end{aligned}$$

and manipulations similar to those done in capturing equation (5), we get

$$\frac{dv}{dt} = K_3 \cos(n_2 t') + A + Bv + Cv^2, \tag{6}$$

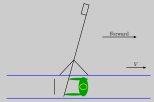
where

$$K_3 = Mn_2^2 a_1/(M+m),$$

and A, B, C are the same as before.

5. Analysis of the Model

In order to get a clear picture of the rowing stroke from the two functions, they must be correlated. This is done by entering the final velocity of one equation into the next. To start we look at the first equation with an initial velocity, 0, V_{00}, symbolizing the start of a



race. The first power stroke is run for T_1, in this case .7 seconds, the total time necessary to complete one power stroke. At the end of T_1 we are left with a velocity of V_{01}. The final velocity for the first power stroke is also the initial velocity of the recovery. Entering V_{01} into the Recovery, as the initial condition and running it from 0 to T_2, .9 seconds for this example, we get a new final velocity, V_{10}. Now entering V_{10} into the power stroke we start the next stroke, which will return V_{11}, that is entered into the recovery, returning V_{20}.

Entering this system into a computer we are able to graph a continuous function of the boats velocity vs. real time shown in Figure 3. One way to determine the time necessary to finish the race for 2000m is to set up the integral

$$2000 = \int_0^t (\text{Function})\, ds$$

but this isn't the way the problem was solved using a computer. Using a numerical solver each stroke was integrated for each of its component parts calculating the distance traveled for each stroke. Next each stroke is added up until 2000m is reached then the function is graphed real time. The final time is then converted into minutes, leaving the calculated time of the race. The numerical example used by Brearley (et al) involving the Olympic rowers had a calculated time of 5 min 43 sec.

6. Testing the Model on Humboldt State

Now is the time of truth. After researching times and rowers for Humboldt's Varsity Open Four, times for their T_1, T_2 splits were found to be .787 and 1.11 seconds respectively. This calculates for a split time of 31.7 strokes per minute. The total force pulled by all the rowers R is 840 N, length of the oar, ℓ, is 3.76m and 1.14m equals $\hbar$. The total mass M of all the rowers is 288 kg and m the weight of the Coxswain and shell is 102 kg.

The difference in the drag of the water on the shell should be negligible between the eight and the four thus neglecting it and using the same polynomial for drag $D = -a - bv - cv^2$

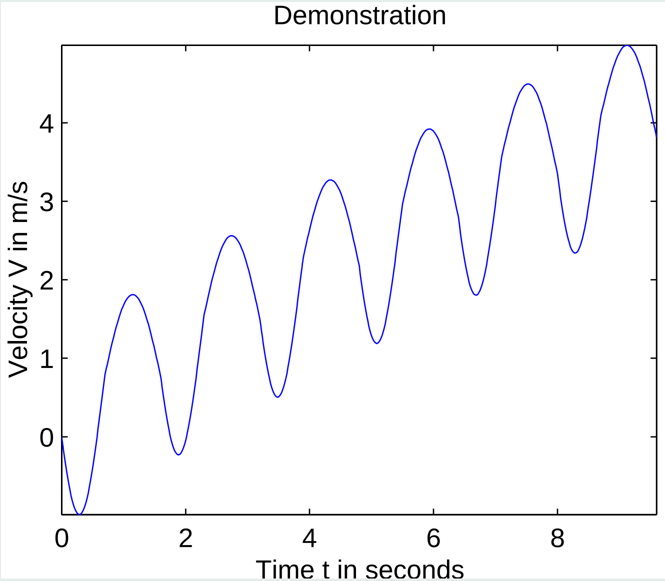
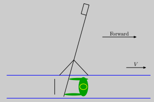


Figure 3: Continuous Function.



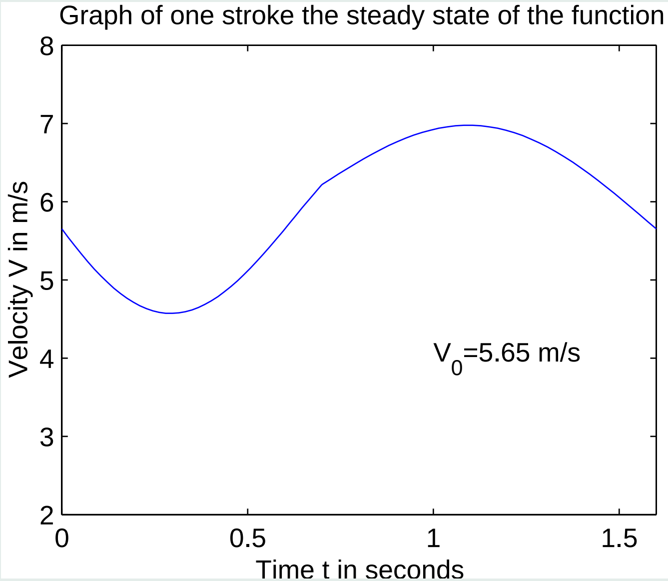
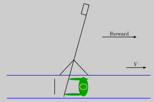


Figure 4: Complete Stroke



shouldn't affect the model too much where $a = 24393, b = -11.22, c = 13.05$. Using the parameters measured for the Varsity Four the values for the equations are as follows.

$$
\begin{aligned}
K_1 &= 2.1429 & A &= -.0636 \\
K_2 &= -5.0314 & B &= .0286 \\
K_3 &= 3.0437 & C &= -0333 \\
n_1 &= \pi/T_1 & n_2 &= \pi/T_2
\end{aligned}
$$

Recall the following equations:

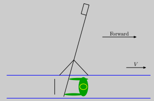
$$
\begin{aligned}
\frac{dv}{dt} &= K_1 \sin(n_1 t) + K_2 \cos(n_1 t) + A + Bv + Cv^2 \\
\frac{dv}{dt} &= K_3 \cos(n_2 t') + A + Bv + Cv^2
\end{aligned}
$$

Using Matlab to solve our equations with the parameters we get the graphs in Figures 5 and 6.

Using the data from the graph including the time needed to complete the total 2000 m for the Women's Four is 9 min 36 seconds, a slightly slower time compared to previous times rowed throughout the year. The last time of the year was 8:35.76.

7. Conclusion

While the times didn't come out exactly as planned they weren't to far off the mark considering that some of the times rowed during the year were a little slower. There is also the factor that the drag was not adjusted for the smaller four shell. This would slow the boat down disproportionate to its size. If the adjustment was made the time would be faster and slightly more accurate.



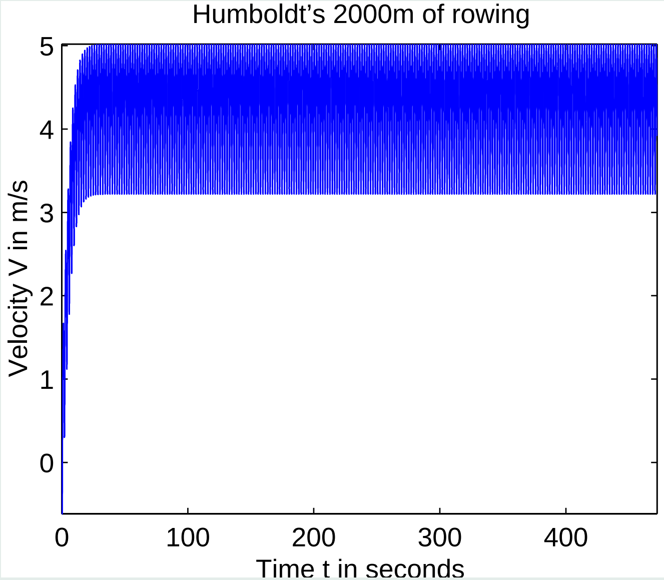


Figure 5: $T_{final} = 574.8$sec.

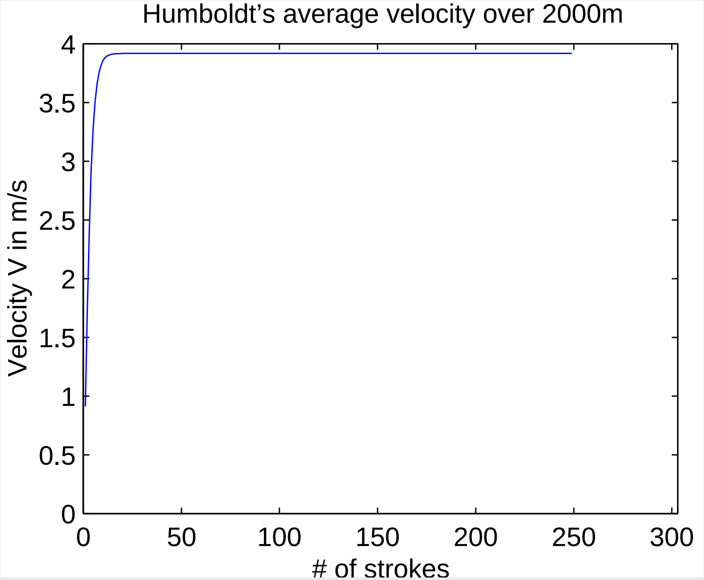


Figure 6: Number of strokes is 248.

Therefore after the consideration are accounted for, we can see that despite the increase in time, the model is a very good model. Looking at the equation of one full stroke in Figure 4 we can see that when the oar enters the water the velocity is sharply reduced and does not come come back up until the more than half-way through the power stroke. The velocity continues to rise through the recovery until the drag of the water slows the boat down.

This representation is extremely close to the behavior of the real situation. This makes the sweep-rowing model very good model especially with its versatility.

References

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