The SIRS Model for Viral Infection

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Dividing and Describing a Population

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- Susceptible Individuals

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- Recovered Individuals

Population Rates of Change

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- Amount of Decrease, or loss from Infection$= -\beta IS$.
- Thus, the differential equation for the change in the Susceptible Population is

$$\frac{dS}{dt} = -\beta IS + \gamma R,$$

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The rate of change of the Infected Population is given by,

- Amount of Increase, or gain from Susceptible Population$= \beta IS$,
- Amount of Decrease, or loss from Recovery$= -\nu I$.
- Therefore, the rate of change in the Infected Population is given by
$$\frac{dI}{dt} = \beta IS - \nu I.$$

The Recovered Population

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The Recovered Population

- Amount of Increase, or gain from Infected Population= νI,
- Amount of Decrease, or loss from immunity loss= $-\gamma R$.

The Recovered Population

- Amount of Increase, or gain from Infected Population$= \nu I$,
- Amount of Decrease, or loss from immunity loss$= -\gamma R$.
- Consequently, the equation for the rate of change of the Recovered Population is

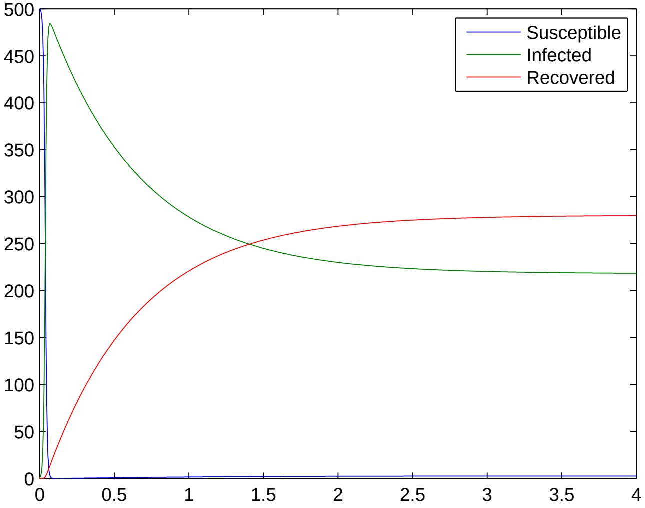
$$\frac{dR}{dt} = \nu I - \gamma R.$$

Summary of Our Equations

$$
\begin{aligned}
\frac{dS}{dt} &= -\beta IS + \gamma R, \\
\frac{dI}{dt} &= \beta IS - \nu I, \\
\frac{dR}{dt} &= \nu I - \gamma R.
\end{aligned}
$$

Figure: A diagram of the flow between the susceptible, infected, and recovered populations.

Three Related Populations



Reducing the SIRS System

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- $\beta IS + \gamma S = \gamma(N - I)$,
- $S = \dfrac{v}{B}$,
- $I = 0$,

The Nullclines of S' and I'

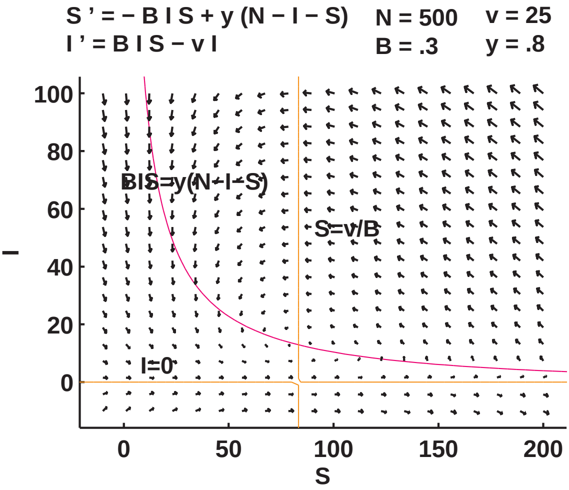

Figure: The Nullclines of I' and S'.

The Jacobian

$$J = \begin{pmatrix} -\beta I - \gamma & -\beta S - \gamma \\ \beta I & \beta S - \nu \end{pmatrix}.$$

Finding the Equilibrium Points

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- $I = \dfrac{\gamma(N - S)}{BS + \gamma}$
- $I = 12.9199$
- $S = \dfrac{v}{B}$
- $S = 83.3333$

A Phase Plane Graph of S and I

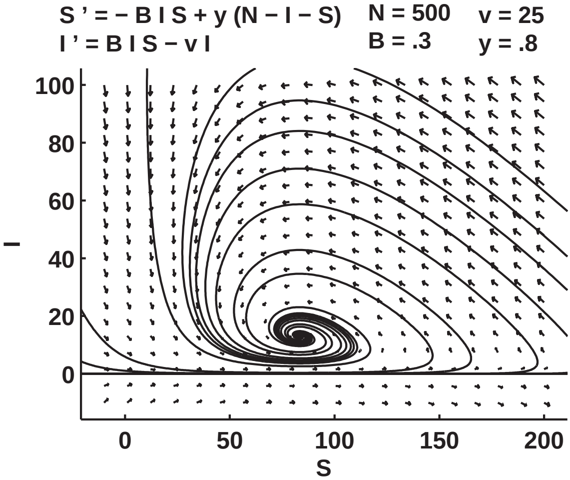

Figure: An Equilibrium point at $(\frac{\nu}{\beta}, \beta IS = \gamma(N - I - S))$.

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Finding the Equilibrium Points

- $I = 0$
- $S = \dfrac{\gamma(N - I)}{BI + \gamma}$
- $S = 500$

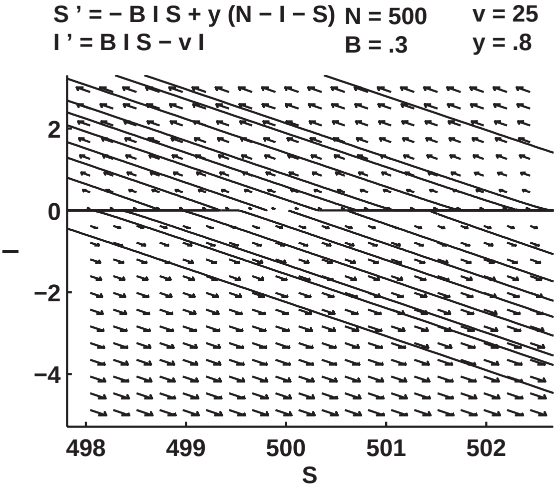

Figure: An Equilibirum Point at $(N, 0)$.

The Threshold Effect

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The Threshold Effect

- $I = \dfrac{\gamma(N - S)}{\beta S - \gamma}$,
- $S = \dfrac{v}{B}$,
- $N > \dfrac{v}{B}$.

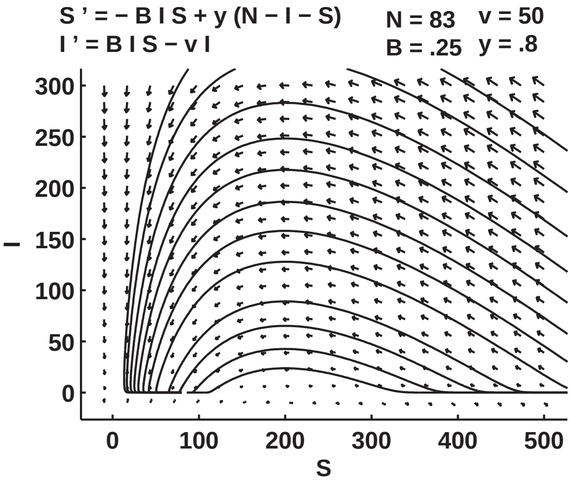

Figure: The plot of I versus S where $N < \frac{\nu}{\beta}$.