Van Meergeren Art Forgeries

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Abstract

An analysis of art forgery using differential equations.

1 Introduction

In 1945, after the liberation of Belgium from Nazi power, numerous investigations were conducted to track down and imprison Nazi collaborators. One such collaborator was discovered through the records a banker had kept while serving as an intermediary in a sale to Goering of the painting "Woman Taken in Adultery" by the 17th century dutch painter Jan Vermeer. Upon confrontation, he confessed that he was acting on behalf of a third rate Dutch painter H.A. Van Meergeren. On May 29, 1945 Van Meergeren was arrested on the charge of collaborating with the enemy. Less than two months later, on July 12, 1945, Van Meergeren shocked all those involved by stating that he had never sold "Woman Taken in Adultery". He went on to tell that five other presumed Vermeers and two de Houghs (another 17th century dutch painter) were actually his own works. because treason was punishable by life imprisionment and forgery was only punishable to a maximum of four years, many people thought he was merely lying to save being charged with the act of treason. To prove he could have done these works and then treated them so as to make them appear 300 years old, he began painting the Vermeer "Jesus Amongst the Doctors" from his prison cell. The work was nearly completed when Van Meergeren learned the charge of treason had been replaced with the charge of forgery. He immediately stopped paintig so that the secret to aging the paintings would not be revelaed. To prove that the paintings were actually forgeries, a panel of distinguished chemists, physicists, and art historians began a thorough investigation.

Vermeer had been very careful when making these paintings and had actually gotten old paintings that were not worth much, and scraped the paint off to get authentic canvases. He then proceeded to obtain pigments that Vermeer would have used. Van Meergeren also knew that aged paint grew extremely hard and would not dissolve as would fairly new paints. For this reason, he cleverly mixed the chemical phenoformaldehyde into the paint. When the finished painting was heated in the oven, the paint became hard and cracked as would an authentic. However, Van Meergeren had been careless with several of his forgeries and had used the modern pigment cobalt blue. This, along with the traces of the phenoformaldehyde, which had not been discovered until the turn of the 19th century, certainly proved that the origins of the paintings were not quite kosher. Van Meergeren was convicted of art forgery and sentenced to one year imprisonment on October 12, 1947. He died of a heart attack on December 30, 1947.

Even after all the evidence had been gathered, many people still refused to believe that "Disciples at Emmaus" was forged by Van Meergeren. Famed art critic of the time A. Bredius publicly certified "Disciples at Emmaus" an authentic and it was soon after sold to the Rembrandt Society for $170,000. Bredius' argument was that Van Meergeren could not have produced such a work, as "Disciples at Emmaus" was in far more detail and much more technically perfect than any of the other alleged forgeries. He and the Rembrandt Society demanded that the painting be treated as an authentic unless the scientific community could produce concrete proof otherwise. This was attempted in 1967 by scientists at Carnegie Mellon University, and the following paper describes their work.

1.1 Problem Statement

Provide scientific proof that the painting "Disciples at Emmaus" is either an authentic painting from the 17th century or a forgery completed in the early to mid 20th century.

1.2 Solution

It has been shown that the radio activity of a substance is directly proportional to the number of atoms present. If we let $N(t)$ stand for the number of atoms present at time t, then $\frac{dN}{dt}$, the rate at which atoms disintegrate per unit time, is proportional to N. Letting λ stand for the positive decay constant of the substance, a simple model can be constructed.

$$\frac{dN}{dt} = -\lambda N \tag{1}$$

And, solving for N, a general solution can be obtained.

$$\begin{aligned}
\frac{dN}{N} &= -\lambda dt \\
\int \frac{dN}{N} &= \int -\lambda dt \\
\ln N &= -\lambda t + k \\
e^{\ln N} &= e^{-\lambda t + k} \\
N &= e^k e^{-\lambda t} \\
N &= k e^{-\lambda t} \qquad (2)
\end{aligned}$$

Assuming that at time t_0, $N(t_0) = N_0$:

$$
\begin{aligned}
N_0 &= ke^{-\lambda t_0} \\
k &= N_0 e^{\lambda t_0} \qquad (3)
\end{aligned}
$$

And substituting k back into the equation, and solving the IVP, we get:

$$
\begin{aligned}
N &= N_0 e^{\lambda t_0} e^{-\lambda t} \\
N &= N_0 \frac{e^{-\lambda t}}{e^{-\lambda t_0}} \\
N &= N_0 e^{-\lambda(t-t_0)} \qquad (4)
\end{aligned}
$$

Using equation (4) and some algebraic manipulation:

$$
\begin{aligned}
N &= N_0 e^{-\lambda(t-t_0)} \\
\frac{N}{N_0} &= e^{-\lambda(t-t_0)} \\
\ln e^{-\lambda(t-t_0)} &= \ln \frac{N}{N_0} \\
-\lambda(t-t_0) &= \ln \frac{N}{N_0} \\
(t-t_0) &= \frac{\ln \frac{N}{N_0}}{-\lambda} \qquad (5)
\end{aligned}
$$

The decay constant λ can be computed in most cases. For instance, the half-life of lead-210 is known to be 22 years. If we were to let 22 years pass, then the ratio of $N : N_0 = 1 : 2$, and $(t - t_0) = 22$ Making an appropriate substitution into equation (5):

$$
\begin{aligned}
22 &= \frac{\ln \frac{1}{2}}{-\lambda} \\
22 &= \frac{\ln 2^{-1}}{-\lambda} \\
22 &= \frac{\ln 2}{\lambda} \\
\lambda &= \frac{\ln 2}{22} \qquad (6)
\end{aligned}
$$

Small amounts of lead-210 and an even smaller amount of radium-226 is found in all paintings. Since both elements are found in lead oxide commonly known as white lead, which is a pigment artists have used for more that 2000 years. All rocks are known to have a small amount of uranium. The uranium decays to one element which in turn decays to another and so forth and so on. The cycle eventually results in producing lead, a substance which is not radioactive. Uranium, whose half-life is over four billion years, constantly feeds the elements so that as one element decays, the predecessor decays and immediately replaces it. Thus keeping everything in an equilibrium. White lead is made from lead metal which in turn was extracted from lead ore in a process called smelting. In this process, the lead-210 goes along with the metal but 90-95% of the radium-226 and its descendants are removed with other waste products. Thus, most of the lead-210 is cut off and begins to decay with its half-life of 22 years.

Let $y(t)$ be the amount of lead-210 per gram of white lead at time t and let y_0 stand for the amount of lead per gram present at the time of manufacture, t_0. Then let $r(t)$ stand for the number of disintegrations per minute per gram of white lead, at time t. Using λ as the decay constant for lead-210, then

$$\frac{dy}{dt} = -\lambda y + r(t), \quad y(t_0) = y_0 \tag{6}$$

Since there is only a period of three hundred years being examined, and the half-life of radium-226 is 1600 years, $r(t)$ can be considered a constant r. Next, multiplying both sides of the equation by the integrating factor $\mu(t)$ (still yet to be found), we get:

$$\begin{aligned}
\frac{dy}{dt} &= -\lambda y + r \\
\mu(t)\frac{dy}{dt} &= \mu(t)(-\lambda y + r) \\
\mu(t)\frac{dy}{dt} &= -\lambda y\mu(t) + r\mu(t) \\
\mu(t)\frac{dy}{dt} + \lambda y\mu(t) &= r\mu(t) \\
\frac{d}{dt}(y\mu(t)) &= r\mu(t) \\
\int \frac{d}{dt}(y\mu(t)) &= \int r\mu(t) \qquad (8)
\end{aligned}$$

And now finding the integrating factor $\mu(t) = e^{-(-\lambda)\int dt} = e^{\lambda t}$ and substituting for $\mu(t)$ in equation (8):

$$ye^{\lambda t} = \frac{r}{\lambda}e^{\lambda t} + k \tag{9}$$

Examining this at $y(t_0) = y_0$:

$$\begin{aligned}
y_0 e^{\lambda t_0} &= \frac{r}{\lambda}e^{\lambda t_0} + k \\
k &= (y_0 - \frac{r}{\lambda})e^{\lambda t_0} \qquad (10)
\end{aligned}$$

And substituting k back into equation (9):

$$
\begin{aligned}
ye^{\lambda t} &= \frac{r}{\lambda}e^{\lambda t} + (y_0 - \frac{r}{\lambda})e^{\lambda t_0} \\
ye^{\lambda t} &= y_0 e^{\lambda t_0} + \frac{r}{\lambda}(e^{\lambda t} - e^{\lambda t_0}) \\
y &= y_0 \frac{e^{\lambda t_0}}{e^{\lambda t}} + \frac{r}{\lambda}\frac{(e^{\lambda t} - e^{\lambda t_0})}{e^{\lambda t}} \\
y &= y_0 e^{\lambda(t_0 - t)} + \frac{r}{\lambda}(1 - e^{\lambda(t_0 - t)}) \\
y &= \frac{r}{\lambda}(1 - e^{-\lambda(t - t_0)}) + y_0 e^{-\lambda(t - t_0)} \qquad (11)
\end{aligned}
$$

The quantities y and r are values that can be measured and substituted back into the equation. Therefore, if y_0, the amount of lead-210 present at the time of manufacture, were known, $(t - t_0)$ could be computed thus giving the age of the painting. Yet, y_0 cannot be measured directly. It is necessary now to make the assumption that the amount of lead-210 was in radioactive equilibrium with the larger amount of radium-226. This alone is enough to distinguish between a painting dating back 300 years and one painted only twenty years ago.

Proposition 1 *Let us assume that the painting is either a modern forgery or a 300 year old authentic. Then we must set* $(t - t_0) = 300$ *years in equation (11). If the painting is a forgery, then the term* λy_0 *will be absurdly large.*

After some simple algebra we see that:

$$
\begin{aligned}
y &= \frac{r}{\lambda}(1 - e^{-300\lambda}) + y_0 e^{-300\lambda} \\
\lambda y_0 e^{-300\lambda} &= \lambda y - r(1 - e^{-300\lambda}) \\
\lambda y_0 &= \lambda y e^{300\lambda} - r(e^{300\lambda} - 1) \qquad (12)
\end{aligned}
$$

Now, we must determine what exactly is an absurdly large value for λy_0.

Exercise 1.1 *The half-life of uranium-238 is* 4.51×10^9 *years. Let* $N(t)$ *denote the number of uranium-238 atoms per gram of white lead at time* t*. Since the uranium-238 is in radio-active equilibrium with the lead-210, we know from equation (1) that* $\frac{dN_U}{dt} = -\lambda N_u$*. Use the arbitrary approximation* $\frac{dN_u}{dt} = -100\frac{dissentegrations\ per\ minute}{grams\ of\ white\ lead}$*. Compute the number of uranium-238 atoms per gram of ordinary lead at time* t_0*.*

Solution The half life of uranium-238 is an extremely large value. Over the examined period of time, the decay would be so slight that the amount of uranium-238 can be considered a constant under these circumstances. The value of N can be then examined using simple mathematics.

$$
\frac{dN_U}{dt} = -\lambda N_U
$$

$$\begin{aligned}\frac{dN_U}{dt} &= -100 \\ -\lambda N_U &= -100 \\ N_U &= \frac{100}{\lambda} \qquad (13)\end{aligned}$$

And now, we can use an argument identical to that used for equation (6) to find the positive decay constant for uranium-238. However, we must remember that we are trying to find the disintegrations per minute, whereas the half-life is in years. Therefore, A conversion from years to minutes must first be preformed before we continue.

$$\frac{4.51 \times 10^9 \text{ years}}{1 \text{ half-life}} * \frac{365 \text{ days}}{1 \text{ year}} * \frac{24 \text{ hours}}{1 \text{ day}} * \frac{60 \text{ minutes}}{1 \text{ hour}} = 2.3705 \times 10^{15} \frac{\text{minutes}}{\text{half-life}}$$

Now it is possible to use a similar argument to that used in obtaining equation (6) to solve for λ.

$$\begin{aligned}\lambda &= \frac{\ln 2}{(t - t_0)} \\ \lambda &= \frac{\ln 2}{2.3705 \times 10^{15}} \\ \lambda &= 2.9241 \times 10^{-16}\end{aligned}$$

Now, it is possible to plug this in for λ in equation (13) to find the number of uranium-238 atoms.

$$\begin{aligned}N_U &= \frac{100}{\lambda} \\ N_U &= \frac{100}{2.9241 \times 10^{-16}} \\ N_U &= 3.4199 \times 10^{17} \qquad (14)\end{aligned}$$

Exercise 1.2 *Using the number of atoms of uranium-238 per gram of white lead just obtained in the previous exercise, and the facts that a mole of uranium-238 weighs 238 grams and a mole contains* 6.02×10^{23} *atoms, compute the concentration of uranium in the ore.*

Solution This can also be done using elementary mathematics. Since we now know that the number of uranium-238 atoms per gram of white lead is $N_U = 3.4199 \times 10^{17}$, then a conversion can be made.

$$\begin{aligned}&\frac{3.4199 \times 10^{17} \text{ atoms}}{\text{gram of white lead}} * \frac{1 \text{ mole}}{6.02 \times 10^{23} \text{ atoms}} * \frac{238 \text{ grams of uranium}}{1 \text{ mole}} \\ =\ & 238 * \frac{3.4199 \times 10^{17}}{6.02 \times 10^{23}} \frac{\text{grams of uranium}}{\text{gram of white lead}}\end{aligned}$$

This simplifies to

$$\begin{aligned}
\frac{\text{grams of uranium}}{\text{gram of white lead}} &= 238 * \frac{3.4199 \times 10^{17}}{6.02 \times 10^{23}} \\
&= 1.3521 \times 10^{-4} \\
&= 0.00013521 \\
&= 0.013521\% \\
&\approx 0.014\%
\end{aligned}$$

This is a fairly high concentration of uranium since the average amount of uranium in rocks on the earth's crust is about 2.7 parts per million. On the other hand, there are some very rare ores in Western Hampshire whose uranium content is 2-3 percent. Therefore, it is deemed to be safe to say that if the disintegration of lead-210 exceeds 30,000 dpm/(g of white lead), this would be an absurd amount.

Next it is necessary to evaluate

- the present disintegration rate, λy, of the lead-210,
- the disintegration rate, r, of the radium-226, and
- $e^{300\lambda}$

Exercise 1.3 *Lead-210 is a difficult element to measure. After the period of several years, show that polonium-210, a much easier element to measure, has the same half life as lead-210.*

Solution Using equation (11) we know that if $y(t)$ is the amount of lead-210 at time t, then the following equation applies.

$$y = \frac{r}{\lambda}(1 - e^{-\lambda(t-t_0)}) + y_0 e^{-\lambda(t-t_0)}$$

It is also known from equation (4) that if $N(t)$ is the amount of any substance at time (t), then the next equation applies.

$$N = N_0 e^{-\lambda(t-t_0)}$$

So, in this case, we let N stand for the amount of polonium-210. Now, by inspection, we can see that the term $e^{-\lambda(t-t_0)}$ tends to one as we let $t \to \infty$. Therefore, the term $1 - e^{-\lambda(t-t_0)}$ tends to zero as $t \to \infty$. This leaves us with equation (15).

$$y = y_0 e^{-\lambda(t-t_0)} \tag{15}$$

Again, by inspection, one can now see that equation (11) is the same as equation (4) as time increases. Therefore, polonium-210 has the same half-life of lead-210 after several years, and, since we are interested in a time period of between 22 and 300 years, a substitution of the two can be made. The radium-226 and polonium-210 were measured in the paintings and table (1) below was constructed.

Description	^{210}Po Disintegration (λy)	^{226}Ra Disintegration (r)
"Disciples at Emmaus"	8.5	0.8
"Washing of Feet"	12.6	0.26
"Woman Reading Music"	10.3	0.3
"Woman Playing Mandolin"	8.2	0.17
"Lace Make"	1.5	1.4
"Laughing Girl"	5.2	6.0

$e^{300\lambda}$ can be found easily by substituting equation (6) for λ and simplifying.

$$\begin{aligned} e^{300\lambda} &= e^{\frac{300}{22}\ln 2} \\ &= e^{\ln 2^{\frac{150}{22}}} \\ &= 2^{\frac{150}{11}} \end{aligned}$$

With all these calculations performed, it is now possible to determine the authenticity of the paintings in question. Using equation (12), we obtain:

"Disciples at Emmaus"

$$\begin{aligned} \lambda y_0 &= 8.5 * 2^{\frac{150}{11}} - 0.8(2^{\frac{150}{11}} - 1) \\ \lambda y_0 &= 98050.0 \end{aligned}$$

This is unacceptably large. Therefore, the painting is a modern forgery.

"Washing of Feet"

$$\begin{aligned} \lambda y_0 &= 12.6 * 2^{\frac{150}{11}} - 0.26(2^{\frac{150}{11}} - 1) \\ \lambda y_0 &= 1.5713 \times 10^5 \end{aligned}$$

This is unacceptably large. Therefore, the painting is a modern forgery.

"Woman Reading Music"

$$\begin{aligned} \lambda y_0 &= 10.3 * 2^{\frac{150}{11}} - 0.3(2^{\frac{150}{11}} - 1) \\ \lambda y_0 &= 1.2734 \times 10^5 \end{aligned}$$

This is unacceptably large. Therefore, the painting is a modern forgery.

"Woman Playing Mandolin"

$$\begin{aligned} \lambda y_0 &= 8.2 * 2^{\frac{150}{11}} - 0.17(2^{\frac{150}{11}} - 1) \\ \lambda y_0 &= 1.0225 \times 10^5 \end{aligned}$$

This is unacceptably large. Therefore, the painting is a modern forgery.

"Lace Maker"

$$
\begin{aligned}
\lambda y_0 &= 1.5 * 2^{\frac{150}{11}} - 1.4(2^{\frac{150}{11}} - 1) \\
\lambda y_0 &= 1274.8
\end{aligned}
$$

This is a reasonable value for λy_0. Therefore, since the polonium-210 and the radium-238 is almost in radioactive equilibrium, the painting cannot be a recent forgery.

"Laughing Girl"

$$
\begin{aligned}
\lambda y_0 &= 5.2 * 2^{\frac{150}{11}} - 6.0(2^{\frac{150}{11}} - 1) \\
\lambda y_0 &= -10181
\end{aligned}
$$

This is a reasonable value for λy_0. Therefore, since the polonium-210 and the radium-238 is almost in radioactive equilibrium, the painting cannot be a recent forgery.