4 Programming In Matlab

In this chapter, we study the fundamentals of programming in Matlab. We begin with a study of logical arrays and the operators and method that make them so effective in Matlab programming.

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4.1 Logical Arrays

We begin this section by demonstraing how Matlab determines the truth or falsehood of a statement. Enter the following array at the Matlab prompt.

```
>> x=[true false]
x =
     1     0
```

Note that that **true** evaluates to 1, while false evaluates to zero. Moreover, the array stored in the variable **x** in an entirely new type of datatype, called a *logical array*.

```
>> whos
  Name      Size                    Bytes  Class

  x         1x2                         2  logical array
```

Note that each entry in the logical vector **x** takes one byte of storage. Note that this logical type is a completely new datatype. Indeed, it is instructive to compare the vector **x** with the following vector **y**

```
>> y=[1 0]
y =
     1     0
```

On the surface, it appears that the variables **x** and **y** contain exactly the same data, that is until one checks the datatypes with Matlab's **whos** command.

```
>> whos
  Name      Size                    Bytes  Class

  x         1x2                         2  logical array
  y         1x2                        16  double array
```

[1] Copyrighted material. See: http://msenux.redwoods.edu/Math4Textbook/

Note that vector **y** is of class double, using 8 bytes to store each entry.

Alternatively, we can check the class with Matlab's **class** command. On the one hand, the command

```
>> class(x)
ans =
logical
```

informs us that vector **x** has class **logical**. On the other hand, the command

```
>> class(y)
ans =
double
```

informs us that the vector **y** has class **double**.

Like some of the other conversion operators we've seen (like **uint8**), Matlab provides an operator that will convert numeric arrays to logical arrays. Any nonzero real number is converted to a logical 1 (true) and zeros are converted to logical 0 (false).

To see this in action, first enter the following matrix A.

```
>> A=[0 1 3;4 0 0; 5 7 -11]
A =
     0     1     3
     4     0     0
     5     7   -11
```

Note that A has class **double**.

```
>> class(A)
ans =
double
```

Now, convert A to a logical array.

```
>> A=logical(A)
Warning: Values other than 0 or 1 converted to logical 1.
A =
     0     1     1
     1     0     0
     1     1     1
```

Note that all nonzero entries in the original matrix A are now logical 1, while all zeros in the original matrix A are now logical zero. You can test the new class of matrix A with the following command.

```
>> class(A)
ans =
logical
```

Relational Operators

Matlab provides a complete list of *relational operators* (see **Table 4.1**).

Operator	Description
$<$	Less than
$\leq$	Less than or equal to
$>$	Greater than
$\geq$	Greater than or equal to
$==$	Equal to
$\sim=$	Not equal to

Table 4.1. Matlab's relational operators.

The Matlab relational operators compare corresponding elements of arrays with equal dimensions. Relational operators always operate element-by-element. That is, they are "array smart."

For example, consider the vectors **v** and **w**.

```
>> v=[1 3 6]
v =
     1     3     6
>> w=[1 2 4]
w =
     1     2     4
```

The output of the command **v>w** will return a logical vector of the same size as the vectors **v** and **w**. Locations where the specified relation is true receive a 1 and locations where the specified relation is false receive a 0.

```
>> v>w
ans =
     0     1     1
```

Because the first entry of **v** is not larger than the first entry of **w**, the first entry of the answer receives a zero (false). Because the second and third entries of the vector **v** are larger than the second and third entries of the vector **w**, the second and third entries of the answer receive a 1 (true). This is what we mean when we say "Relational operators always operate element-by-element."

Like the other array operators we've seen that act in an element-by-elemnt manner, the arrays being compared must have the same size.

```
>> u=[10 10]
u =
    10    10
>> u>v
??? Error using ==> gt
Matrix dimensions must agree.
```

Comparing with a scalar. One exception to the "same size rule" is when an array is compared to a scalar. In this case, Matlab tests the scalar against every element of the other operand.

```
>> u=rand(1,6)
u =
    0.9501    0.2311    0.6068    0.4860    0.8913    0.7621
>> u>0.5
ans =
     1     0     1     0     1     1
```

Note that the first, third, fifth, and sixth entries of the vector **u** are all greater than 0.5, so the answer receives a 1 (true) in these positions, with the remaining positions receiving a 0 (false).

How Logical Arrays are Used

Matlab uses logical arrays for a type of indexing. A *logical index* designates the elements of a matrix A based on their position in the indexing array, B. In this masking type of operation, every true element in the indexing array is treated as a positional index into the array being accessed

For example, create a matrix A of uniform random numbers.

```
>> A=rand(5)
A =
    0.4565    0.7919    0.9355    0.3529    0.1987
    0.0185    0.9218    0.9169    0.8132    0.6038
    0.8214    0.7382    0.4103    0.0099    0.2722
    0.4447    0.1763    0.8936    0.1389    0.1988
    0.6154    0.4057    0.0579    0.2028    0.0153
```

Next, find the positions of the random numbers that are less than 0.5, and store the result in the matrix B.

```
>> B=A<0.5
B =
     1     0     0     1     1
     1     0     0     0     0
     0     0     1     1     1
     1     1     0     1     1
     0     1     1     1     1
```

Take a moment to check that in each position in B that contains a logical 1 (true), the corresponding position in matrix A contains a uniform random number that is less than 0.5.

There are a number of things that we can do with this information. We could list all the random numbers in A that are less than 0.5 by using matrix B as a logical index into the matrix A.[2]

```
>> A(B)
ans =
    0.4565
    0.0185
    0.4447
    0.1763
    0.4057
    0.4103
    0.0579
    0.3529
    0.0099
    0.1389
    0.2028
    0.1987
    0.2722
    0.1988
    0.0153
```

Or we could set each of these positions to zero.

```
>> A(B)=0
A =
         0    0.7919    0.9355         0         0
         0    0.9218    0.9169    0.8132    0.6038
    0.8214    0.7382         0         0         0
         0         0    0.8936         0         0
    0.6154         0         0         0         0
```

[2] Matlab has roots that began their growth in Fortran, where the entries in arrays were loaded by column. The Matlab command **A(:)** will list all the entries in A, first going down the first column, then the second, etc. With the command **A(B)**, Matlab first lists all the "true" positions in the first column, then the second, etc.

A number of Matlab's functions are designed to execute a test of some sort, then return a logical array. The output of these commands is usually used as a mask or index. For example, create a *magic square* with the following command.

```
>> A=magic(5)
A =
    17    24     1     8    15
    23     5     7    14    16
     4     6    13    20    22
    10    12    19    21     3
    11    18    25     2     9
```

We'll find all the primes in matrix A with Matlab's **ispime** command.

```
>> B=isprime(A)
B =
     1     0     0     0     0
     1     1     1     0     0
     0     0     1     0     0
     0     0     1     0     1
     1     0     0     1     0
```

Each position in matrix B that contains a logical 1 (true) indicates a prime in the corresponding position of matrix A. We can get a listing of the primes in matrix A.

```
 >> A(B)
ans =
    17
    23
    11
     5
     7
    13
    19
     2
     3
```

Or we can replace all of the primes in Matrix A with -1 with this command.

```
>> A(B)=-1
A =
    -1    24     1     8    15
    -1    -1    -1    14    16
     4     6    -1    20    22
    10    12    -1    21    -1
    -1    18    25    -1     9
```

A Practical Example. Although the examples provided thus far provide explanation on how logical arrays are used as indices, they might not seem to have much practical use. Let's look at some examples where logical indexing is helpful in a much more practical way.

▶ **Example 1.** *Plot the graph of* $y = \sqrt{x^2 - 1}$ *over the interval* $[-5, 5]$.

Seems like a simple request.

```
>> x=linspace(-5,5,100);
>> y=sqrt(x.^2-1);
>> plot(x,y)
```

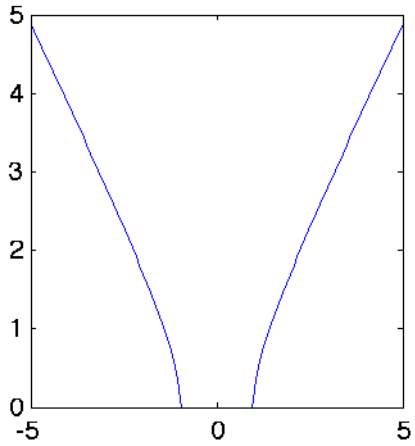


Figure 4.1. Plotting of the graph of $y = \sqrt{x^2 - 1}$.

Seems to have worked (see **Figure 4.1**(a)), but we do receive a warning at the command line when we run this script.

```
Warning: Imaginary parts of complex X and/or Y arguments ignored
```

Complex numbers have seemingly crept into our calculations. They certainly don't exist in the vector **x**, because the command **x=linspace(-5,5,200)** generates **real** numbers between -5 and 5. It must be the vector y that contains the complex numbers, but how to find them?

Real and imaginary parts of complex numbers. A complex number has the form $a + bi$, where $i = \sqrt{-1}$. The *real part* of the complex number $a + bi$ is the number a. The *imaginary part* of the complex number $a + bi$ is the number b. Matlab know all about the real and imaginary parts of complex numbers.

First, check that the variable i contains the complex number i. We do this because i is a popular variable to use in scripts and **for** loops and may have been set to something other than the complex number i.

```
>> i
ans =
        0 + 1.0000i
```

If you don't get this result, type **clear i** and try again.

Enter $z = 3 + 4i$.

```
>> z=3+4i
z =
   3.0000 + 4.0000i
```

The real part of $z = 3 + 4i$ is 3.

```
>> real(z)
ans =
     3
```

The imaginary part of $z = 3 + 4i$ is 4.

```
>> imag(z)
ans =
     4
```

Now, a number r is **real** if and only if the real part of r is identical to the number r. For example, set $r = 4$.

```
>> r=4
r =
     4
```

Now test if its real part is identical to itself.

```
>> real(r)==r
ans =
     1
```

Note that the logical 1 indicates a "true" response. The real part of r equals r. Thus, r is a real number.

On the other hand, set $z = 2 - 3i$.

```
>> z=2-3i
z =
   2.0000 - 3.0000i
```

Test if the real part of z equals z.

```
>> real(z)==z
ans =
     0
```

Note that the logical 0 indicates a "false" response. The real part of z doesn't equal z. Thus, z is not a real number, z is complex.

Let's use this idea to find the positions in the vector **y** that hold complex numbers.

```
>> k=real(y)~=y
k =
  Columns 1 through 10
     0     0     0     0     0     0     0     0     0     0
  Columns 11 through 20
     0     0     0     0     0     0     0     0     0     0
  Columns 21 through 30
     0     0     0     0     0     0     0     0     0     0
  Columns 31 through 40
     0     0     0     0     0     0     0     0     0     0
  Columns 41 through 50
     1     1     1     1     1     1     1     1     1     1
  Columns 51 through 60
     1     1     1     1     1     1     1     1     1     1
  Columns 61 through 70
     0     0     0     0     0     0     0     0     0     0
  Columns 71 through 80
     0     0     0     0     0     0     0     0     0     0
  Columns 81 through 90
     0     0     0     0     0     0     0     0     0     0
  Columns 91 through 100
     0     0     0     0     0     0     0     0     0     0
```

The vector **k** is a logical vector, having logical 1 (true) in each position that y has a complex number. We can use this as an index into the vector **x** to see what values of x are causing complex numbers to appear in the vector **y**.

```
>> x(k)
ans =
  Columns 1 through 6
   -0.9596   -0.8586   -0.7576   -0.6566   -0.5556   -0.4545
  Columns 7 through 12
   -0.3535   -0.2525   -0.1515   -0.0505    0.0505    0.1515
  Columns 13 through 18
    0.2525    0.3535    0.4545    0.5556    0.6566    0.7576
  Columns 19 through 20
    0.8586    0.9596
```

It would appear that values of x between -1 and 1 are creating complex values in the vector **y**. This makes sense because the domain of the function $y = \sqrt{x^2 - 1}$

requires that $x^2 - 1 \geq 0$. Solving for x, $|x| \geq 1$, or equivalently, $x \leq -1$ or $x \geq 1$. For values of x between -1 and 1, the expression $x^2 - 1$ is negative, so the square root produces complex numbers. We can see the complex numbers in **y** with logical indexing.

```
>> y(k)
ans =
  Columns 1 through 3
        0 + 0.2814i        0 + 0.5127i        0 + 0.6527i
  Columns 4 through 6
        0 + 0.7543i        0 + 0.8315i        0 + 0.8907i
  Columns 7 through 9
        0 + 0.9354i        0 + 0.9676i        0 + 0.9885i
  Columns 10 through 12
        0 + 0.9987i        0 + 0.9987i        0 + 0.9885i
  Columns 13 through 15
        0 + 0.9676i        0 + 0.9354i        0 + 0.8907i
  Columns 16 through 18
        0 + 0.8315i        0 + 0.7543i        0 + 0.6527i
  Columns 19 through 20
        0 + 0.5127i        0 + 0.2814i
```

Note that the real part of each of these complex numbers is zero.

So, how does Matlab's **plot** command handle complex numbers. The answer is found by typing **help plot**. The relevant part of the helpfile follows.

```
PLOT(Y) plots the columns of Y versus their index.
    If Y is complex, PLOT(Y) is equivalent to
    PLOT(real(Y),imag(Y)). In all other uses of PLOT,
    the imaginary part is ignored.
```

In the case at hand, we used **plot(x,y)**, so this paragraph of the help file for the **plot** command would indicate that Matlab simply ignores the imaginary parts of the vector y when the entries are complex. For the output of **y(k)** above, when Matlab ignores the imaginary part, that is equivalent to saying that Matlab considers all of these complex numbers to equal their real part, or zero. This can be seen by adding the command **axis([-5,5,-5,5])** to produce the image in **Figure 4.2**.

```
>> axis([-5,5,-5,5])
```

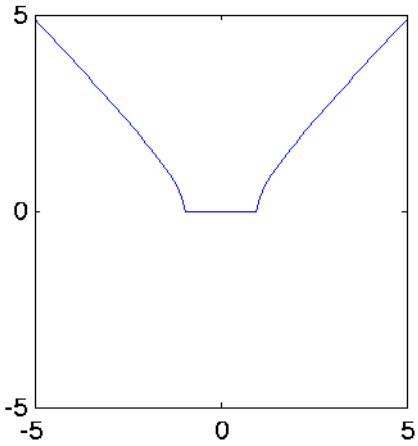


Figure 4.2. The **axis** command reveals that the **plot** command believes that all y-values equal zero for values of x between -1 and 1.

Note the horizontal line connecting the right- and left-hand halves of the hyperbola in **Figure 4.2**. This line simply does not belong in this image, as x-values between -1 and 1 are not in the domain of of $y = \sqrt{x^2 - 1}$. However, Matalb simply ignores the imaginary part of the complex numbers in the vector **y**, considers them to be zero, and plots this "false" horizontal line.

Let's get rid of this "false" horizontal line by setting all the offending complex number entries in the vector **y** to **NaN** (Matlab's "Not a Number.")

```
>> y(k)=NaN;
```

If you remove the suppressing semicolon at the end of this command, you will be able to see that all the complex numbers in the vector **y** have been replaced by **NaN**.

The good news is the fact that Matlab's **plot** command ignores these **NaN**'s and does not plot them. The following command will produce the image in **Figure 4.3**.

```
>> plot(x,y)
>> axis([-5,5,-5,5])
```

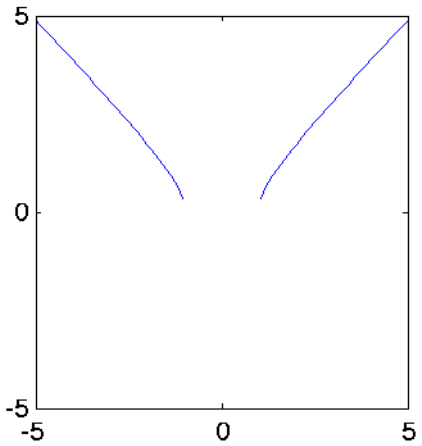


Figure 4.3. Removing the complex numbers from the plot.

Preplanning. All of these contortions with complex numbers can be avoided with proper preplanning. As we've said, the domain of $y = \sqrt{x^2 - 1}$ is $\{x : x \leq -1 \text{ or } x \geq 1\}$.

We'll start by defining an anonymous function, making it "array smart."

```
>> f=@(x) sqrt(x.^2-1)
f =
    @(x) sqrt(x.^2-1)
```

Test the anonymous function at $x = 0$, where $f(0) = \sqrt{0^2 - 1} = \sqrt{-1} = i$.

```
>> f(0)
ans =
        0 + 1.0000i
```

Note that $f(0) = i$ is a correct response.

We'll first plot the left-hand branch of the hyperbola on $\{x : x \leq -1\}$.

```
>> x=linspace(-5,-1);
>> y=f(x);
>> plot(x,y)
```

Because each of the entries in the vector **x** lie in the domain of the function $y = \sqrt{x^2 - 1}$, no complex numbers are produced when evaluating the function on the entries in **x**.

Next, we'll plot the right-hand branch of the hyperbola on $\{x : x \geq 1\}$. Again, no complex numbers are produced because we are using x-values that lie in the domain of the function.

```
>> x=linspace(1,5);
>> y=f(x);
>> line(x,y)
```

Remember that the **line** command is used to append to an existing plot. We could also have used a **hold on**, followed by a **plot(x,y)**, but the **line** command does not require that we "hold" the plot.

Finally, adjust the window boundaries with the **axis** command.

```
>> axis([-5,5,-5,5])
```

The result is an image indentical to that in **Figure 4.3**.

Complex numbers are not so easily avoided in other situations. Let's look at an example that uses Matlab's **mesh** command.

▶ **Example 2.** *Sketch the surface* $z = \sqrt{x^2 - y^2}$ *on the rectangular domain* $D = \{(x, y) : -2 \leq x, y \leq 2\}$.

Let's again use an anonymous function, making it "array smart."

```
>> f=@(x,y) sqrt(x.^2-y.^2)
f =
    @(x,y) sqrt(x.^2-y.^2)
```

Test the anonymous function. Note that $f(5,3) = \sqrt{5^2 - 3^2} = 4$.

```
>> f(5,3)
ans =
     4
```

That looks correct.

Now, the following commands produce the image in **Figure 4.4**(a). At first glance, it appears that all is well.

```
>> [x,y]=meshgrid(linspace(-2,2,50));
>> z=f(x,y);
>> mesh(x,y,z)
```

However, this time Matlab ignores the imaginary part of the complex numbers in **z** without even providing a warning. This is evident in the graph of $z = \sqrt{x^2 - y^2}$ where you see that a large number of points in the mesh have z-value equal to zero. These points in the xy-plane are where Matlab is ignoring the imaginary part of the complex entries in z and plotting only the real part (which is zero).

We can check to see if the matrix z contains any complex numbers.

```
>> isreal(z)
ans =
     0
```

The false response indicates the presence of complex numbers in matrix z. We can replace all the complex entries in z with these commands.

```
>> k=real(z)~=z;
>> z(k)=NaN;
```

You may want to remove the suppressing semicolons and view the output of the last two commands.

Matlab's **mesh** command refuses to plot the **Nan**'s and produces the image in **Figure 4.4**(b).

```
>> mesh(x,yz)
```

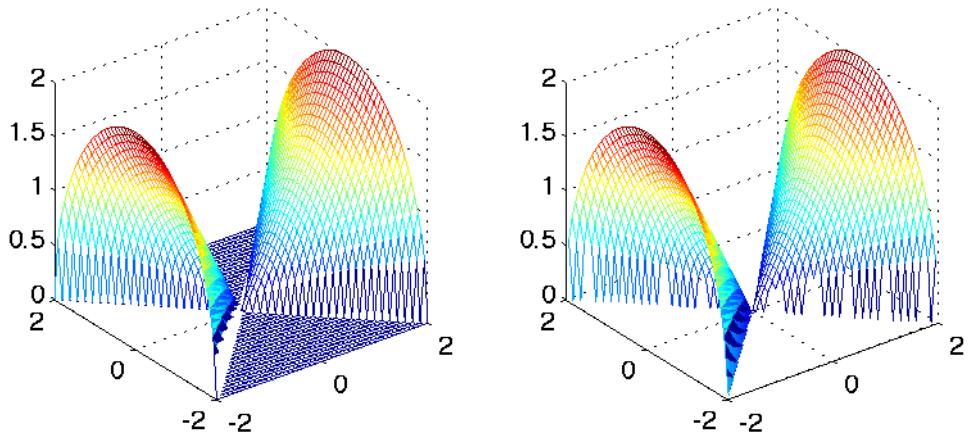


Figure 4.4. Removing the complex entries removes the false zero plane.

Logical Operators

The Matlab symbols **&**, **|**, and **~** are the logical operators **AND**, **OR**, and **NOT**, respectively. Truth tables for each logical operator are gathered in **Table 4.2**. Remember that logical 1 is true and logical 0 is false.

- The statment "A and B" is true if and only if both statements A and B are true. This is reflected in the truth table for **&** (**AND**) in **Table 4.2**(a).
- The statement "A or B" is true if and only if one or the other of statements A and B are true. This is reflected in te truth table for **|** (**OR**) in **Table 4.2**(b).
- The statement "not A" simply reverses the truth or falsehood of statement A. This is reflected in the truth table for **~** (**NOT**) in **Table 4.2**(c).

Each of the logical operators **&**, **|**, and **~** acts *element-wise*. Moreover, the output is of type logical. For example, create two logical vectors **u** and **v**.

```
>> u=logical([1 1 1 0 0]);
>> v=logical([1 0 1 0 1]);
```

A	B	$A \,\&\, B$
1	1	1
1	0	0
0	1	0
0	0	0

(a) **AND**

A	B	$A \mid B$
1	1	1
1	0	1
0	1	1
0	0	0

(b) **OR**

A	$\sim A$
1	0
0	1

(c) **NOT**

Table 4.2. Truth tables for Matlab's logical operators.

The first and third entries of vectors **u** and **v** are logical 1's (true), hence the first and third entries of **u&v** will be logical 1's (true). According to **Table 4.2**(a), all other entries of **u&v** will be logical 0's (false).

```
>> u, v, u&v
u =
     1     1     1     0     0
v =
     1     0     1     0     1
ans =
     1     0     1     0     0
```

On the other hand, **Table 4.2**(b) indicates that **u|v** (**OR**) will be false if and only if both entries are false. In the case of the vectors **u** and **v**, the fourth entry of each vector is logical 0 (false), so the fourth entry of **u|v** will be logical 0 (false). All other entries of **u|v** will be logical 1's (true).

```
>> u, v, u|v
u =
     1     1     1     0     0
v =
     1     0     1     0     1
ans =
     1     1     1     0     1
```

Finally, **~u** simply reverses the truth or falsehood of each entry of the vector **u**.

```
>> u, ~u
u =
     1     1     1     0     0
ans =
     0     0     0     1     1
```

If the vectors and/or matrices involved are not logical arrays, the logical operators first convert the operands to logical arrays. First, create a "magic" matrix A.

```
>> A=magic(3)
A =
     8     1     6
     3     5     7
     4     9     2
```

Create a *diagonal matrix* D with the vector $[5, 6, 7]$ on its main diagonal.

```
>> D=diag([5,6,7])
D =
     5     0     0
     0     6     0
     0     0     7
```

We could first convert these to logical matrices and then "and" them.

```
>> LA=logical(A), LD=logical(D), LA&LD
Warning: Values other than 0 or 1 converted to logical 1.
LA =
     1     1     1
     1     1     1
     1     1     1
Warning: Values other than 0 or 1 converted to logical 1.
LD =
     1     0     0
     0     1     0
     0     0     1
ans =
     1     0     0
     0     1     0
     0     0     1
```

However, this is not necessary, because the operator **&** will convert each of its operands to a logical type before performing the **AND** operation.

```
>> A, D, A&D
A =
     8     1     6
     3     5     7
     4     9     2
D =
     5     0     0
     0     6     0
     0     0     7
ans =
     1     0     0
     0     1     0
     0     0     1
```

Finally, because the logical operators act element-wise, the arrays used as operands must have the same dimensions.

```
>> u=[2,3], v=[4 5 6], u|v
u =
     2     3
v =
     4     5     6
??? Error using ==> or
Inputs must have the same size.
```

Let's look at some powerful applications of these operators.

▶ **Example 3.** *Find all entries of matrix* **magic(5)** *that are* **not** *prime.*

Start by entering the matrix and using Matlab's **isprime** function to find the prime entries.

```
>> M=magic(5); B=isprime(M);
```

We saw in an earlier example that **M(B)** will list all of the primes in the matrix M. So, let's use the logical operator ~ to list all the nonprimes. For convenience and purposes of comparison, we list the matrix M.

```
>> M
M =
    17    24     1     8    15
    23     5     7    14    16
     4     6    13    20    22
    10    12    19    21     3
    11    18    25     2     9
```

Now we list all the nonprimes in the matrix M.

```
>> M(~B)
ans =
     4
    10
    24
     6
    12
    18
     1
    25
     8
    14
    20
    21
    15
    16
    22
     9
```

Even more succinctly, try the command **M(~isprime(M))** to achieve the same result.

Let's look at another example.

▶ **Example 4.** *Sketch the surface* $z = x^2 + y^2$ *on the rectangular domain* $d = \{(x, y) : -1 \leq x, y \leq 1\}$, *but only for those pairs* (x, y) *that satisfy* $y < x$ *and* $y > -x$.

Let's again use an anonymous function, making it "array smart."

```
>> f=@(x,y) x.^2+y.^2
f =
    @(x,y) x.^2+y.^2
```

Test the function. Note that $f(3, 4) = 3^2 + 4^2 = 25$.

```
>> f(3,4)
ans =
    25
```

That checks.

Next, create the rectangular grid and evaluate z at each point (x, y) in the grid.

```
[x,y]=meshgrid(linspace(-1,1,50));
z=f(x,y);
```

Now, find those values of the grid that satisfy $y < x$ and $y > -x$.

```
k=(y<x) & (y>-x);
```

We want to eliminate from the mesh all points that do **not** satisfy this requirement.

```
z(~k)=NaN;
```

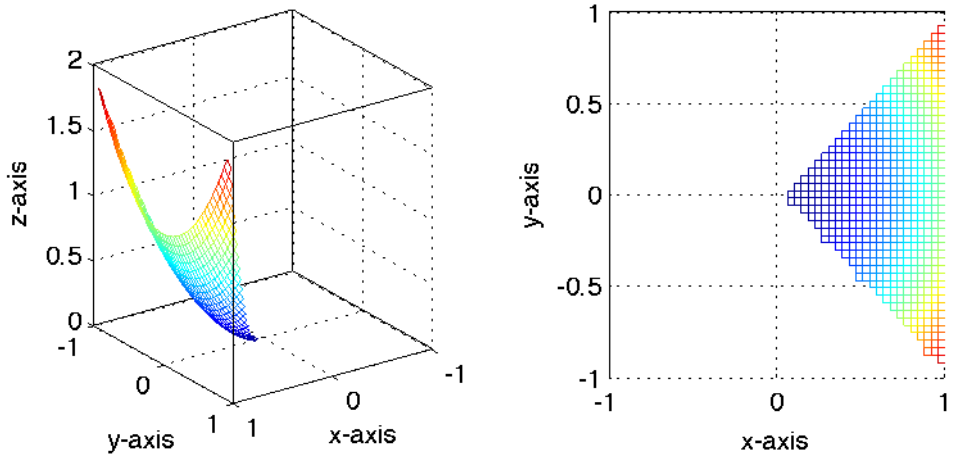
We can now draw the resulting mesh, label the axes, and provide an orientation.

```
mesh(x,y,z)
xlabel('x-axis')
ylabel('y-axis')
zlabel('z-axis')
box on
view(145,20)
```

The result is shown in **Figure 4.5**(a).

An interesting view is provided by the following command, the result of which is shown in **Figure 4.5**(b). In this view, we've declined to rotate the xy-axes, but elevated our eye 90°, so that we are gazing directly down at the xy-plane. This view clearly shows that we have sketched the surface of a region in the rectangular domain restricted to (x, y) pairs satisfying $y < x$ and $y > -x$.

```
view(0,90)
```



(a) (b)

Figure 4.5. The surface $z = x^2 + y^2$ restricted to points where $y < x$ and $y > -x$.

4.1 Exercises

In **Exercises 1-6**, first enter the following vectors.

```
>> v=[1,3,5,7], w=[5,2,5,4]
v =
     1     3     5     7
w =
     5     2     5     4
```

In each exercise, first predict the output of the given command, then validate your response with the appropriate Matlab command. *Note: The idea here is not to simply enter the command. Rather, spend some time thinking, then predict the output before you enter the command to verify your conclusion.*

1. **v>w**

2. **v>=w**

3. **v<=w**

4. **v<w**

5. **v==w**

6. **v~=w**

Store the following "magic" matrix in the variable A.

```
>> A=magic(5)
A =
  17   24    1    8   15
  23    5    7   14   16
   4    6   13   20   22
  10   12   19   21    3
  11   18   25    2    9
```

In **Exercises 7-12**, first predict the output of the given command, then validate your response with the appropriate Matlab command. *Note: The idea here is not to simply enter the command. Rather, spend some time thinking, then predict the output before you enter the command to verify your conclusion.*

7.

```
>> A>14
```

8.

```
>> A<=12
```

9.

```
>> (A>3) & (A<=20)
```

10.

[3] Copyrighted material. See: http://msenux.redwoods.edu/IntAlgText/

```
>> (A<5) | (A>=21)
```

11.

```
>> ~(A<=6)
```

12.

```
>> (A>6) & ~(A>8)
```

---

13. Set **P=pascal(5)**. Write a Matlab command that will list all entries of matrix P that are less than 20 but not equal to 1.

14. Set **P=pascal(5)**. Write a Matlab command that will list all entries of matrix P that are not equal to 1.

15. Set **P=pascal(5)**. Write a Matlab command that will list all entries of matrix P that are less than or equal to 4 or greater than 20.

16. Set **P=pascal(5)**. Write a Matlab command that will list all entries of matrix P that are greater than 3 but not greater than 35.

When a and b are positive integers, the Matlab command **mod(a,b)** returns the remainder when a is divided by b. Use this command to produce the requested entries of the given matrix in **Exercises 17-20**.

17. Set **A=magic(4)**. Write a Matlab command that will return all entries of matrix A that are divisible by 2. *Hint: An integer k is divisible by 2 if the remainder is zero when k is divided by 2.*

18. Set **A=magic(4)**. Write a Matlab command that will return all entries of matrix A that are odd integers. *Hint: A integer k is odd if its remainder is 1 when k is divided by 2.*

19. Set **A=magic(4)**. Write a Matlab command that will return all entries of matrix A that are divisible by 3.

20. Set **A=magic(4)**. Write a Matlab command that will return all entries of matrix A that are divisible by 5.

---

21. Execute the following command to list the primes less than or equal to 100.

```
>> primes(100)
```

Secondly, create a vector **u** holding the integers from 1 to 100, inclusive. Use Matlab's **isprime** command and logical indexing to pick out and list all the primes in vector **u**. Compare the results.

22. Create a vector **u** holding the integers from 100 to 1000, inclusive. Use Matlab's **isprime** command and logical indexing to pick out all the primes in vector **u**. Use Matlab's **max** command on the result to find the largest

prime between 100 and 1000.

In **Exercises 23-26**, perform each of the following tasks for the given function.

i. Write an "array smart" anonymous function f for the given function. Test your anonymous function before proceeding.
ii. Set **x=linspace(-10,10,200)** and evaluate the function with **y=f(x)**.
iii. Use the **plot(x,y)** command to plot the function.
iv. Use **axis([-10,10,-10,10])** to set the window boundaries.
v. Use logical indexing to set all of the complex entries in the vector **y** to **NaN**. Open a second figure window with the command **figure**. Replot the result and reset the window boundaries as above, if necessary. Add axis labels and a title and turn the grid on.

23. $f(x) = 2 + \sqrt{x+5}$

24. $f(x) = 3 - \sqrt{x-3}$

25. $f(x) = \sqrt{9-x^2}$

26. $f(x) = \sqrt{x^2-25}$

In **Exercises 27-30**, use "advanced planning" to plot the given function on a subset of the domain $[-10, 10]$ to avoid complex entries when evaluating the given function. In each case, set the window boundaries with the command **axis([-10,10,-10,10])**, turn on the grid, and add axes labels and a title.

27. The function in **Exercise 23**.

28. The function in **Exercise 24**.

29. The function in **Exercise 25**.

30. The function in **Exercise 26**.

In **Exercises 31-34**, perform each of the following tasks for the given function.

i. Write an "array smart" anonymous function f for the given function. Test your anonymous function before proceeding.
ii. Set:

```
x=linspace(-3,3,40);
y=x;
[x,y]=meshgrid(x,y);
```

Evaluate the function with **z=f(x,y)**.
iii. Use **mesh(x,y,z)** to plot the surface defined by the function.
iv. Use logical indexing to set all of the complex entries in the vector **z** to **NaN**. Open a second figure window with the command **figure**. Replot the surface. Add axis labels and a title.

31. $f(x, y) = \sqrt{1+x}$

32. $f(x, y) = \sqrt{1-y}$

33. $f(x, y) = \sqrt{9-x^2-y^2}$

34. $f(x, y) = \sqrt{x^2+y^2-1}$

In **Exercises 35-40**, perform each of the following tasks for the given func-

tion.

i. Write an "array smart" anonymous function f for the given function. Test your anonymous function before proceeding.
ii. Set:

```
x=linspace(-3,3,40);
y=x;
[x,y]=meshgrid(x,y);
```

Evaluate the function with **z=f(x,y)**.
iii. Use logical indexing to replace all entries in **z** with **NaN** that do not satisfy the given constraint.
iv. Use **mesh(x,y,z)** to plot the surface defined by the function and constraint. Add axis labels and a title.
v. Open a second figure window with the command **figure**. Replot the surface and orient the view with **view(0,90)**. Add axis labels and a title. Does the pictured region satisfy the given constraint?

35. $f(x,y) = 12 - x - y$ where

$$x > -1.$$

36. $f(x,y) = 10 - 2x + y$ where

$$y \leq 3.$$

37. $f(x,y) = 14 + x - 2y$ where

$$x + y < 0.$$

38. $f(x,y) = 14 + x - 2y$ where

$$x - y \geq 0.$$

39. $f(x,y) = 11 - 2x + 2y$ where

$$x + y < 0 \quad \text{and} \quad x \geq -2.$$

40. $f(x,y) = 10 + 2x - 3y$ where

$$x - 2y \leq 0 \quad \text{or} \quad y \leq 0.$$

4.1 Answers

1.

```
>> v>w
ans =
     0     1     0     1
```

3.

```
>> v<=w
ans =
     1     0     1     0
```

5.

```
>> v==w
ans =
     0     0     1     0
```

7.

```
>> A>14
ans =
   1     1     0     0     1
   1     0     0     0     1
   0     0     0     1     1
   0     0     1     1     0
   0     1     1     0     0
```

9.

```
>> (A>3) & (A<=20)
ans =
   1     0     0     1     1
   0     1     1     1     1
   1     1     1     1     0
   1     1     1     0     0
   1     1     0     0     1
```

11.

```
>> ~(A<=6)
ans =
   1     1     0     1     1
   1     0     1     1     1
   0     0     1     1     1
   1     1     1     1     0
   1     1     1     0     1
```

13.

```
>> P((P<20) & ~(P==1))
ans =
     2
     3
     4
     5
     3
     6
    10
    15
     4
    10
     5
    15
```

15.

```
>> P((P<=4) | (P>20))
ans =
     1
     1
     1
     1
     1
     1
     2
     3
     4
     1
     3
     1
     4
    35
     1
    35
    70
```

17.

```
>> A(mod(A,2)==0)
ans =
    16
     4
     2
    14
    10
     6
     8
    12
```

19.

```
>> A(mod(A,3)==0)
ans =
     9
     3
     6
    15
    12
```

21.

```
>> u=1:100;
>> k=isprime(u);
>> u(k)
ans =
  Columns 1 through 4
     2     3     5     7
  Columns 5 through 8
    11    13    17    19
  Columns 9 through 12
    23    29    31    37
  Columns 13 through 16
    41    43    47    53
  Columns 17 through 20
    59    61    67    71
  Columns 21 through 24
    73    79    83    89
  Column 25
    97
```

23. Define the anonymous function.

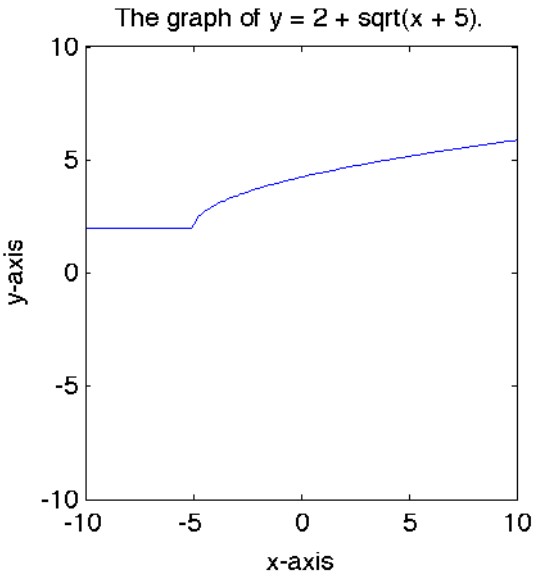
```
f=@(x) 2+sqrt(x+5);
```

Evaluate the function on $[-10, 10]$ and plot the result.

```
x=linspace(-10,10,200);
y=f(x);
plot(x,y)
```

Adjust the window boundaries.

```
axis([-10,10,-10,10])
```

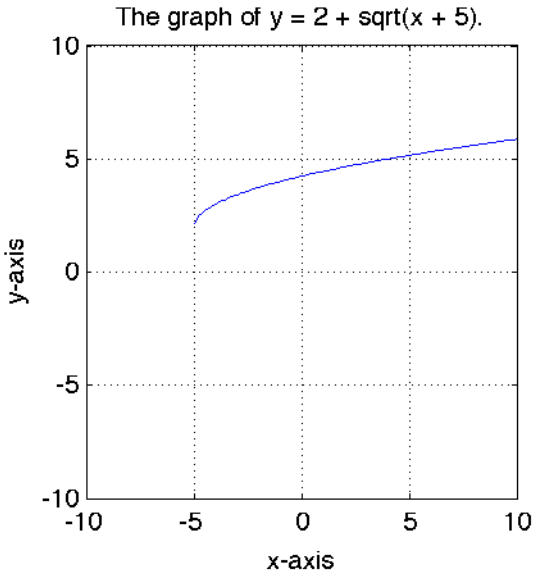


Eliminate complex numbers.

```
k=real(y)~=y;
y(k)=NaN;
```

Open a new figure window and replot. Turn on the grid.

```
figure
plot(x,y)
axis([-10,10,-10,10])
grid on
```



25. Define the anonymous function.

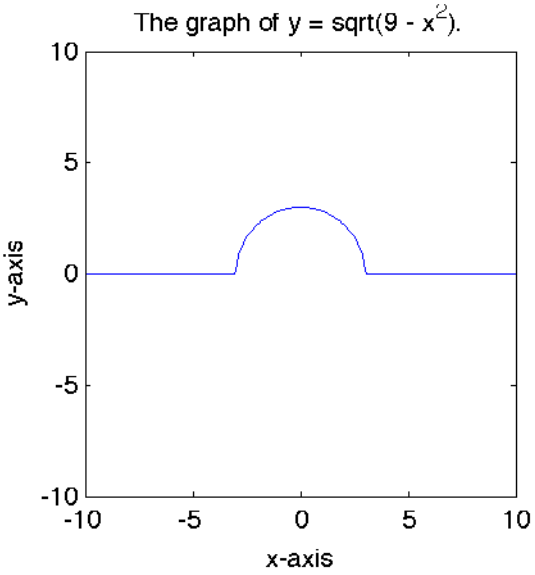
```
f=@(x) sqrt(9-x.^2);
```

Evaluate the function on $[-10, 10]$ and plot the result.

```
x=linspace(-10,10,200);
y=f(x);
plot(x,y)
```

Adjust the window boundaries.

```
axis([-10,10,-10,10])
```

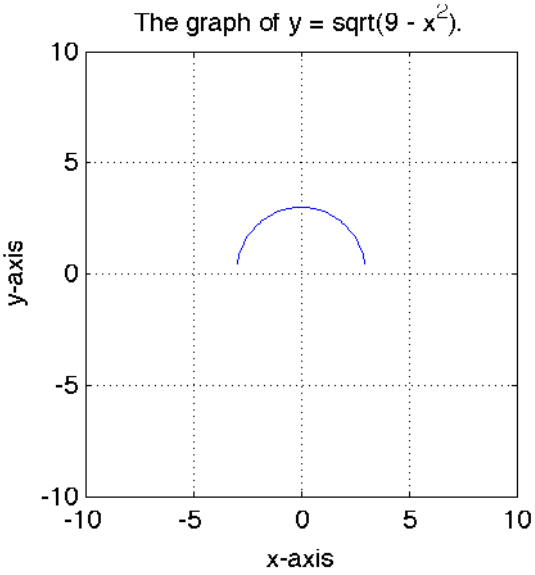


Eliminate complex numbers.

```
k=real(y)~=y;
y(k)=NaN;
```

Open a new figure window and replot. Turn on the grid.

```
figure
plot(x,y)
axis([-10,10,-10,10])
grid on
```



27. Define the anonymous function.

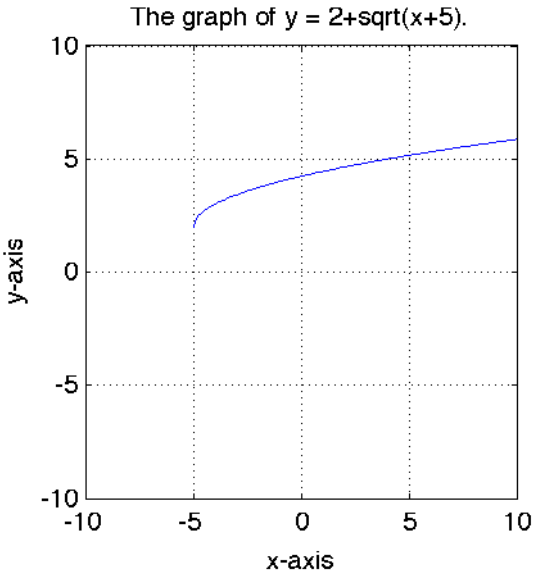
```
f=@(x) 2+sqrt(x+5);
```

The domain of $y = 2 + \sqrt{x+5}$ is the set of all real numbers greater than or equal to -5. Evaluate the function on $[-5, 10]$ and plot the result.

```
x=linspace(-5,10,200);
y=f(x);
plot(x,y)
```

Adjust the window boundaries and add a grid.

```
axis([-10,10,-10,10])
grid on
```



29. Define the anonymous function.

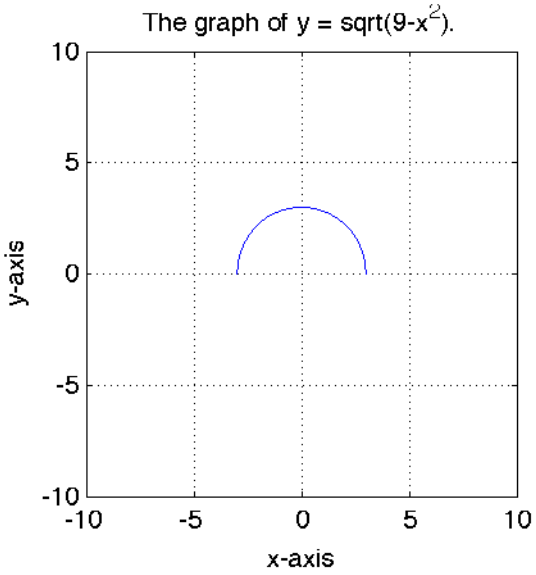
```
f=@(x) sqrt(9-x.^2);
```

The domain of $y = \sqrt{9 - x^2}$ is the set of all real numbers greater than or equal to -3 and less than or equal to 3. Evaluate the function on $[-3, 3]$ and plot the result.

```
x=linspace(-5,10,200);
y=f(x);
plot(x,y)
```

Adjust the window boundaries and add a grid.

```
axis([-10,10,-10,10])
grid on
```



31. Define the anonymous function.

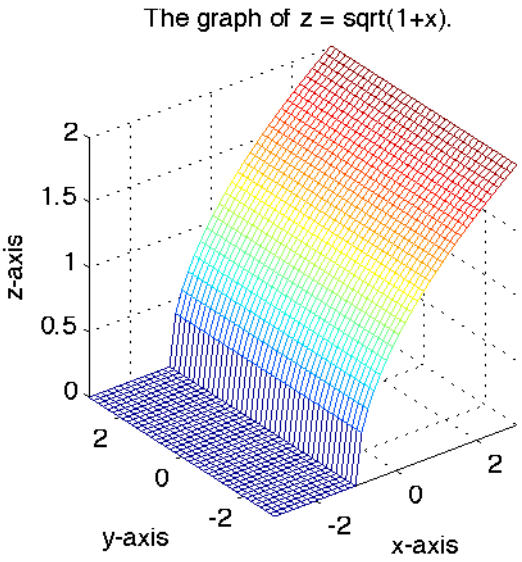
```
f=@(x,y) sqrt(1+x);
```

Set up the grid.

```
x=linspace(-3,3,40);
y=x;
[x,y]=meshgrid(x,y);
```

Evaluate the function at each (x, y) pair in the grid and plot the resulting surface.

```
z=f(x,y);
mesh(x,y,z)
```

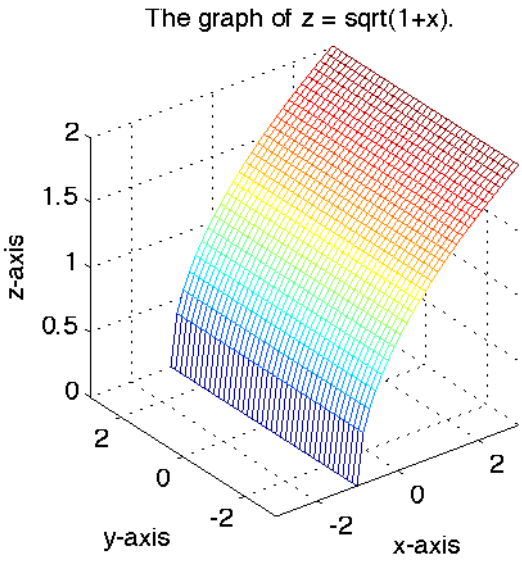


Eliminate complex numbers from the matrix z.

```
k=real(z)~=z;
z(k)=NaN;
```

Open a new figure window and replot the surface.

```
figure
mesh(x,y,z)
```



33. Define the anonymous function.

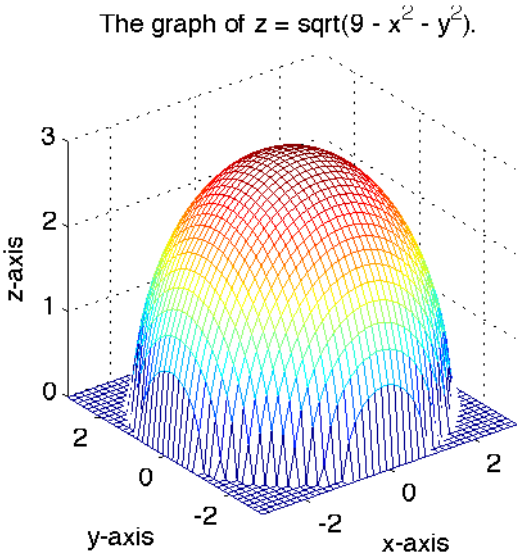
```
f=@(x,y) sqrt(9-x.^2-y.^2);
```

Set up the grid.

```
x=linspace(-3,3,40);
y=x;
[x,y]=meshgrid(x,y);
```

Evaluate the function at each (x, y) pair in the grid and plot the resulting surface.

```
z=f(x,y);
mesh(x,y,z)
```

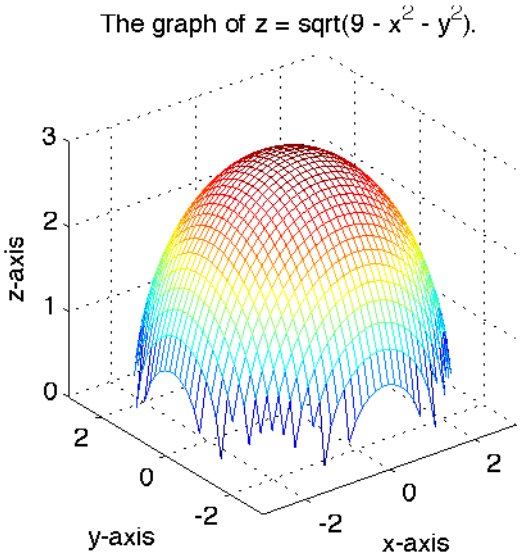


Eliminate complex numbers from the matrix z.

```
k=real(z)~=z;
z(k)=NaN;
```

Open a new figure window and replot the surface.

```
figure
mesh(x,y,z)
```



35. Define the anonymous function.

```
f=@(x,y) 12-x-y;
```

Set up the grid.

```
x=linspace(-3,3,40);
y=x;
[x,y]=meshgrid(x,y);
```

Evaluate the function at each (x, y) pair in the grid.

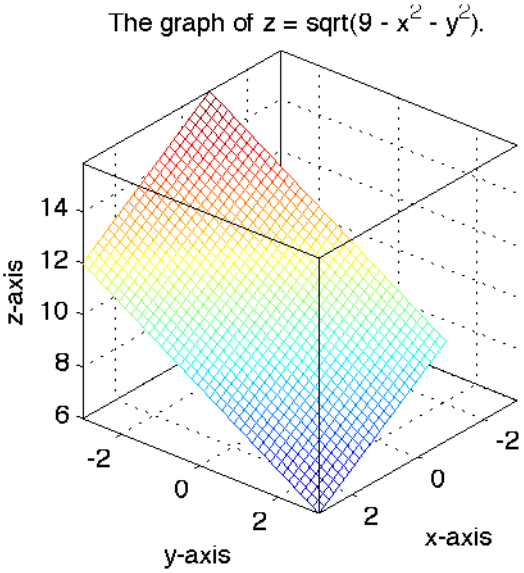
```
z=f(x,y);
```

Determine where $x > -1$ and set all entries in z equal to **NaN** where this contraint is not satisfied.

```
k=x>-1;
z(~k)=NaN;
```

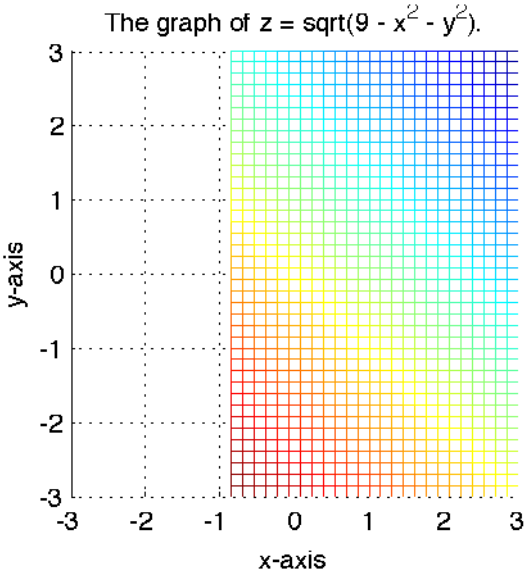
Draw the surface, adjust the orientation, and turn the box on for depth.

```
mesh(x,y,z)
view(130,30)
box on
```



Open a new figure window, replot, and adjust the view so that the eye stares down the z-axis directly at the xy-plane.

```
figure
mesh(x,y,z)
view(0,90)
```



37. Define the anonymous function.

```
f=@(x,y) 14+x-2*y;
```

Set up the grid.

```
x=linspace(-3,3,40);
y=x;
[x,y]=meshgrid(x,y);
```

Evaluate the function at each (x, y) pair in the grid.

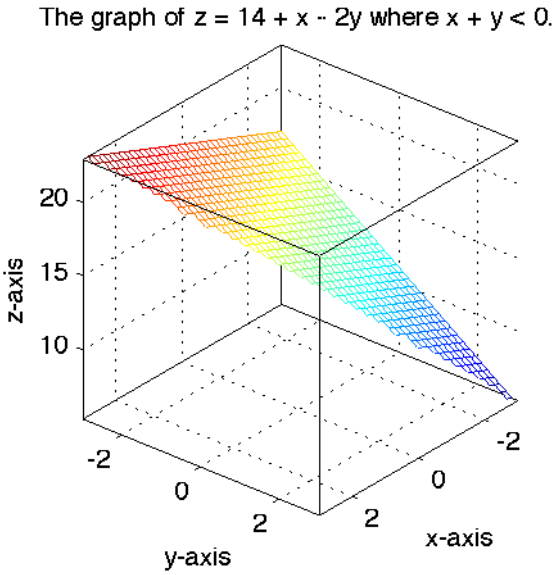
```
z=f(x,y);
```

Determine where $x > -1$ and set all entries in z equal to **NaN** where this contraint is not satisfied.

```
k=x+y<0;
z(~k)=NaN;
```

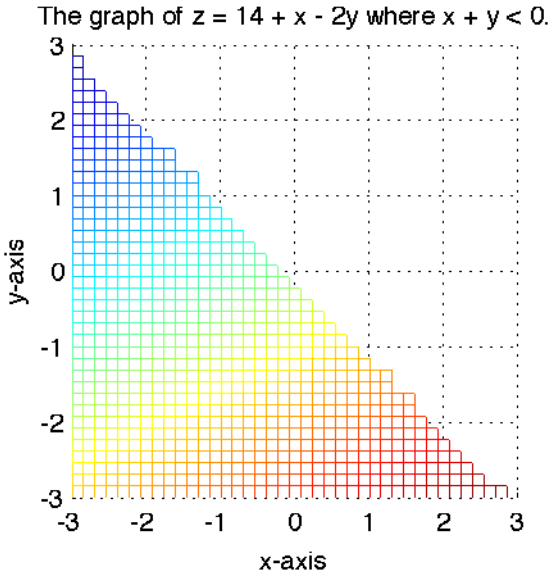
Draw the surface, adjust the orientation, and turn the box on for depth.

```
mesh(x,y,z)
view(130,30)
box on
```



Open a new figure window, replot, and adjust the view so that the eye stares down the z-axis directly at the xy-plane.

```
figure
mesh(x,y,z)
view(0,90)
```



39. Define the anonymous function.

```
f=@(x,y) 11-2*x+2*y;
```

Set up the grid.

```
x=linspace(-3,3,40);
y=x;
[x,y]=meshgrid(x,y);
```

Evaluate the function at each (x, y) pair in the grid.

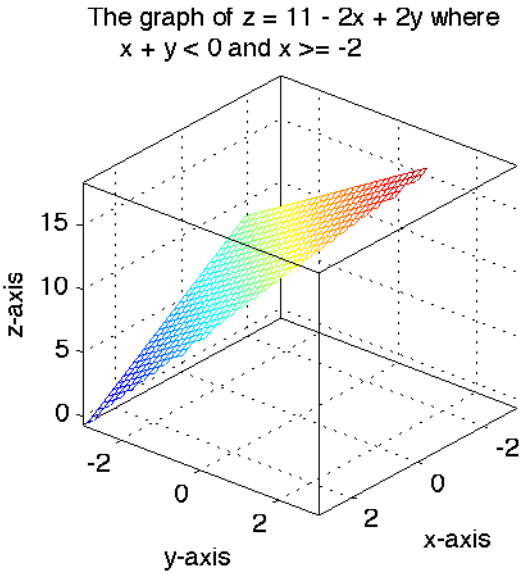
```
z=f(x,y);
```

Determine where $x > -1$ and set all entries in z equal to **NaN** where this contraint is not satisfied.

```
z=f(x,y);
k=(x+y<0) & (x>=-2);
z(~k)=NaN;
```

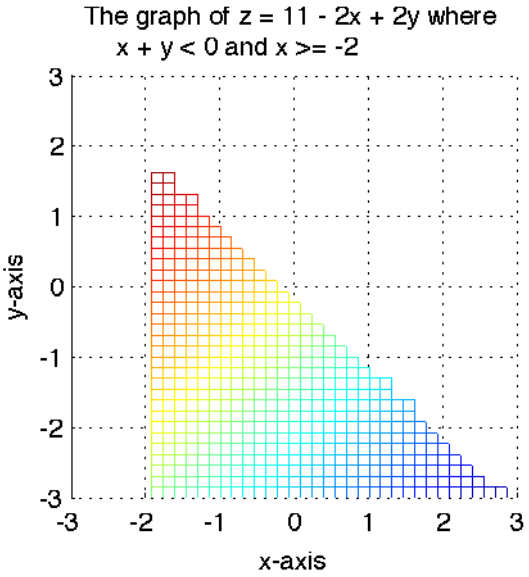
Draw the surface, adjust the orientation, and turn the box on for depth.

```
mesh(x,y,z)
view(130,30)
box on
```



Open a new figure window, replot, and adjust the view so that the eye stares down the z-axis directly at the xy-plane.

```
figure
mesh(x,y,z)
view(0,90)
```



4.2 Control Structures in Matlab

In this section we will discuss the control structures offered by the Matlab programming language that allow us to add more levels of complexity to the simple programs we have written thus far. Without further ado and fanfare, let's begin.

If

If evaluates a logical expression and executes a block of statements based on whether the logical expressions evaluates to true (logical 1) or false (logical 0). The basic structure is as follows.

```
if logical_expression
 statements
end
```

If the **logical_expression** evaluates as true (logical 1), then the block of statements that follow **if logical_expression** are executed, otherwise the statements are skipped and program control is transferred to the first statement that follows **end**. Let's look at an example of this control structure's use.

Matlab's **rem(a,b)** returns the remainder when a is divided by b. Thus, if a is an even integer, then **rem(a,2)** will equal zero. What follows is a short program to test if an integer is even. Open the Matlab editor, enter the following script, then save the file as **evenodd.m**.

```
n = input('Enter an integer: ');
if (rem(n,2)==0)
    fprintf('The integer %d is even.\n', n)
end
```

Return to the command window and run the script by entering **evenodd** at the Matlab prompt. Matlab responds by asking you to enter an integer. As a response, enter the integer 12 and press `Enter`.

```
>> evenodd
Enter an integer: 12
The integer 12 is even.
```

Run the program again. When prompted, enter the nonnegative integer 17 and press `Enter`.

```
>> evenodd
Enter an integer: 17
```

There is no response in this case, because our program provides no alternative if the input is odd.

Besides the new conditional control structure, we have two new commands that warrant attention.

1. Matlab's **input** command, when used in the form **n = input('Enter an integer: ')**, will display the string as a prompt and wait for the user to enter a number and hit `Enter` on the keyboard, whereupon it stores the number input by the user in the variable *n*.
2. The command **fprintf** is used to print formatted data to a file. The default syntax is **fprintf(FID,FORMAT,A,...)**.
   i. The argument **FID** is a file identifier. If no file identifier is present, then **fprintf** prints to the screen.
   ii. The argument **FORMAT** is a format string, which may contain contain *conversion specifications*[5] from the C programming language. In the **evenodd.m** script, **%d** is conversion specification which will format the first argument following the format string as a signed decimal number. Note the **\n** at the end of the format string. This is a *newline* character, which creates a new line after printing the format string to the screen.
   iii. The format string can be followed by zero or more arguments, which will be substituted in sequence for the C-language conversion specifications in the format string.

Else

We can provide an alternative if the **logical_expression** evaluates as false. The basic structure is as follows.

[5] For more information on conversion specifications, type **doc fprintf** at the command prompt.

```
if logical_expression
  statements
else
  statments
end
```

In this form, if the **logical_expression** evaluates as true (logical 1), the block of statements between **if** and **else** are executed. If the **logical_expression** evaluates as false (logical 0), then the block of statements between **else** and **end** are executed.

We can provide an alternative to our **evenodd.m** script. Add the following lines to the script and resave as **evenodd.m**.

```
n = input('Enter an integer: ');
if (rem(n,2)==0)
    fprintf('The integer %d is even.\n', n)
else
    fprintf('The integer %d is odd.\n', n)
end
```

Run the program, enter 17, and note that we now have a different response.

```
>> evenodd
Enter an integer: 17
The integer 17 is odd.
```

Because the logical expression **rem(n,2)==0** evaluates as false when $n = 17$, the **fprintf** command that lies between **else** and **end** is executed, informing us that the input is an odd integer.

Elseif

Sometimes we need to add more than one alternative. For this, we have the following structure.

```
if logical_expression1
  statements
elseif logical_expression2
  statements
else
  statements
end
```

Program flow is as follows.

1. If **logical_expression1** evaluates as true, then the block of statements between **if** and **elseif** are executed.
2. If **logical_expression1** evaluates as false, then program control is passed to **elseif**. At that point, if **logical_expression2** evaluates as true, then the block of statements between **elseif** and **else** are executed. If the logical expression **logical_expression2** evaluates as false, then the block of statments between **else** and **end** are executed.

You can have more than one **elseif** in this control structure, depending on need. As an example of use, consider a program that asks a user to make a choice from a menu, then reacts accordingly.

First, ask for input, then set up a menu of choices with the following commands.

```
a=input('Enter a number a: ');
b=input('Enter a number b: ');
fprintf('\n')
fprintf('1) Add a and b.\n')
fprintf('2) Subtract b from a.\n')
fprintf('3) Multiply a and b.\n')
fprintf('4) Divide a by b.\n')
fprintf('\n')
n=input('Enter your choice: ');
```

Now, execute the appropriate choice based on the user's menu selection.

```
if n==1
 fprintf('The sum of %0.2f and %0.2f is %0.2f.\n',a,b,a+b)
elseif n==2
 fprintf('The difference of %0.2f and %.2f is %.2f.\n',a,b,a-b)
elseif n==3
 fprintf('The product of %.2f and %.2f is %.2f.\n',a,b,a*b)
elseif n==4
 fprintf('The quotient of %.2f and %.2f is %.2f.\n',a,b,a/b)
else
 fprintf('Not a valid choice.\n')
end
```

Save this script as **mymenu.m**, then execute the command **mymenu** at the command window prompt. You will be prompted to enter two numbers a and b. As shown, we entered 15.637 for a and 28.4 for b. The program presents a menu of choices and prompts us for a choice.

```
Enter a number a: 15.637
Enter a number b: 28.4

1). Add a and b.
2). Subtract b from a.
3). Multiply a and b.
4). Divide a by b.

Enter your choice:
```

We enter 1 as our choice and the program responds by adding the values of a and b.

```
Enter your choice: 1

The sum of 15.64 and 28.40 is 44.04.
```

Some comments are in order.

1. Note that after several **elseif** statments, we still list an **else** command to catch invalid entries for n. The choice for n should match a menu designation, either 1, 2, 3, or 4, but any other choice for n causes the statement following **else** to

be executed. This is a good programming practice, having a sort of "otherwise block ' as a catchall for unexpected responses by the user.

2. We use the conversion specification **%0.2f** in this example, which outputs a number in fixed point format with two decimal places. Note that this results in rounding in the output of numbers with more than two decimal places.

Switch, Case, and Otherwise

As the number of **elseif** entries in a conditional control structure increases, the code becomes harder to understand. Matlab provides a much simpler control construct designed for this situation.

```
switch expression
  case value1
    statements
  case value2
    statements
  ...
  otherwise
    statements
end
```

The **expression** can be a scalar or string. **Switch** works by comparing the input **expression** to each **case** value. It executes the statements following the first match it finds, then transfers control to the line following the **end**. If **switch** finds no match, it executes the statment block following the **otherwise** statement.

Let's create a second script having the same menu building code.

```
a=input('Enter a number a: ');
b=input('Enter a number b: ');
fprintf('\n')
fprintf('1) Add a and b.\n')
fprintf('2) Subtract b from a.\n')
fprintf('3) Multiply a and b.\n')
fprintf('4) Divide a by b.\n')
fprintf('\n')
n=input('Enter your choice: ');
fprintf('\n')
```

However, instead of using an **if ...elseif...else...end** control structure to select a block of statments to execute based on the user's choice of menu item, we will implement a **switch** structure instead.

```
switch n
 case 1
  fprintf('The sum of %.2f and %.2f is %.2f.\n',a,b,a+b)
 case 2
  fprintf('The difference of %.2f and %.2f is %.2f.\n',a,b,a-b)
 case 3
  fprintf('The product of %.2f and %.2f is %.2f.\n',a,b,a*b)
 case 4
  fprintf('The quotient of %.2f and %.2f is %.2f.\n',a,b,a/b)
 otherwise
  fprintf('Not a valid choice.\n')
end
```

Save the script as **mymenuswitch.m**, change to the command window, and enter and execute the command **mymenuswitch** at the command prompt. Note that the behavior of the program **mymenuswitch** is identical to that of **mymenu**.

```
Enter a number a: 15.637
Enter a number b: 28.4

1) Add a and b.
2) Subtract b from a.
3) Multiply a and b.
4) Divide a by b.

Enter your choice: 1

The sum of 15.64 and 28.40 is 44.04.
```

If the user enters an invalid choice, note how control is passed to the statment following **otherwise**.

```
Enter a number a: 15.637
Enter a number b: 28.4

1) Add a and b.
2) Subtract b from a.
3) Multiply a and b.
4) Divide a by b.

Enter your choice: 5

Not a valid choice.
```

Loops

A **for** loop is a control structure that is designed to execute a block of statements a predetermined number of times. Here is its general use syntax.

```
for index=start:increment:finish
  statements
end
```

As an example, we display the squares of every other integer from 5 to 13, inclusive.

```
for k=5:2:13
    fprintf('The square of %d is %d.\n', k, k^2)
end
```

The output of this simple loop follows.

```
The square of 5 is 25.
The square of 7 is 49.
The square of 9 is 81.
The square of 11 is 121.
The square of 13 is 169.
```

Another looping construct is Matlab's **while** structure who's basic syntax follows.

```
while expression
  statements
end
```

The **while** loop executes its block of statements as long as the logical controlling **expresssion** evaluates as true (logical 1). For example, we can duplicate the output of the previous **for** loop with the following **while** loop.

```
k=5;
while k<=13
    fprintf('The square of %d is %d.\n', k, k^2)
    k=k+2;
end
```

When using **while**, it is not difficult to fall into a trap of programming a loop that iterates indefinitely. In the example above, the loop will iterate as long a **k<=13**, so it is imperative that we increment the value of k in the interior of the loop, as we do with the command **k=k+2**. This command adds 2 to the current value of k, then replaces k with this incremented value. Thus, the first time through the loop, $k = 5$, the second time through the loop, $k = 7$, etc. When k is no longer less than or equal to 13, the loop terminates.

The simple use of **for** and **while** loops to generate the squares of every other integer from 5 to 13 can be accomplished just as easily with array operations.

```
k=5:2:13;
A=[k; k.^2];
fprintf('The square of %d is %d.\n', A)
```

The use of **fprintf** in this example warrants some explanation. First, the command **A=[k; k.^2]** generates a matrix with two rows, the first row holding the values of k, the second the squares of k.

```
A =
     5     7     9    11    13
    25    49    81   121   169
```

Again, with roots deeply planted in Fortran, Matlab enters the entries of matrix A in columnwise fashion. The following command will print all entries in the matrix A.

```
>> A(:)
ans =
     5
    25
     7
    49
     9
    81
    11
   121
    13
   169
```

It is important to note the order in which the entries were extracted from the matrix A, that is, the entries from the first column, followed by the entries from the second column, etc, until all of the entries of matrix A are on display. This is precisely the order in which the **fprintf** command picks off the entries of matrix A in our example above. The first time through the loop, **fprintf** replaces the two occurrences of **%d** in its format string with 5 and 25. The second time through the loop, the two occurrences of **%d** are replaced with 7 and 49, and so on, until the loop terminates.

For additional help on **fprintf**, type **doc fprintf** at the Matlab command prompt.

Although a good introductory example of using loops, producing the squares of integers is not a very interesting task. Let's look at a more interesting example of the use of **for** and **while** loops.

▶ **Example 1.** *For a number of years, a number of leading mathematicians believed that the infinite series*

$$\sum_{k=1}^{\infty} \frac{1}{k^2} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots \tag{4.1}$$

converged to a finite sum, but none of them could determine what that sum was. Until 1795, that is, when Leonhard Euler produced the suprising result that the sum of the series equaled $\pi^2/6$. *Use a* **for** *loop to sum the first twenty terms of the series and compare this* partial sum *with* $\pi^2/6$ *(compute the relative*

error). Secondly, write a **while** *loop that will determine how many terms must be summed to produce an approximation of* $pi^2/6$ *that is correct to 4 significant digits.*

We wish to sum the first 20 terms, so we begin by setting $n = 20$. We will store the running sum in the variable s, which we initialize to zero. A **for** loop is used to track the running sum. On each pass of the **for** loop, the sum is updated with the command **s=s+1/k^2**, which takes the previous value of the running sum stored in s, adds the value of the next term in the series, and stores the result back in s.

```
n=20;
s=0;
for k=1:n
    s=s+1/k^2;
end
```

The true sum of the series in $\pi^2/6$. Hence, we compute the relative error with the following statement.

```
rel=abs(s-pi^2/6)/abs(pi^2/6);
```

We can then use **fprintf** to report the results.

```
fprintf('The sum of the first %d terms is %f.\n', n, s)
fprintf('The relative error is %e.\n', rel)
```

You should obtain the following results.

```
The sum of the first 20 terms is 1.596163.
The relative error is 2.964911e-02.
```

Note that the relative error is less than 5×10^{-2}, so our approximation of $\pi^2/6$ (using the first 20 terms of the series) contains only 2 significant digits.

The next question asks how many terms must be summed to produce and approximation of $\pi^2/6$ correct to 4 significant digits. Thus, the relative error in the approximation must be less than 5×10^{-4}.

We start by setting **tol=5e-4**. The intent is to loop until the relative error is less than this tolerance, which signifies that we have 4 signicant digits in our approximation of $\pi^2/6$ using the infinite series. The variable s will again contain the running sum and we again initialize the running sum to zero. The variable k contains the term number, so we initialize k to 1. We next initialize **rel_error** to 1 so that we enter the **while** loop at least once.

```
tol=5e-4;
s=0;
k=1;
rel_error=1;
```

Each pass through the loop, we do three things:

1. We update the running sum by adding the next term in the series to the previous running sum. This is accomplished with the statement **s=s+1/k^2**.
2. We increment the term number by 1, using the statement **k=k+1** for this purpose.
3. We calculate the current relative error.

We continue to loop while the relative error is greater than or equal to the tolerance.

```
while rel_error>=tol
    s=s+1/k^2;
    k=k+1;
    rel_error=abs(s-pi^2/6)/abs(pi^2/6);
end
```

When the loop is completed, we use **fprintf** to output results.

```
fprintf('The actual value of pi^2/6 is %f.\n', pi^2/6)
fprintf('The sum of the first %d terms is %f.\n', k-1, s)
fprintf('The relative error is %f.\n', rel_error)
```

The results follow.

```
The actual value of pi^2/6 is 1.644934.
The sum of the first 1216 terms is 1.644112.
The relative error is 0.000500.
```

Note that we have agreement in approximately 4 significant digits.

This example demonstrates when we should use a **for** loop and when a **while** loop is more appropriate. We offer the following advice.

What type of loop should I use? When you know in advance the precise number of times the loop should interate, use a **for** loop. On the other hand, if you have no predetermined knowledge of how many times you will need the loop to execute, use a **while** loop.

For $k = A$

Matlab also supports a type of **for** loop whose block of statements are executed for each column in a matrix A. The syntax is as follows.

```
for k = A
  statements
end
```

In this construct, the columns of matrix A are stored one at a time in the variable k while the following statments, up to the **end**, are executed. For example, enter the matrix A.

```
>> A=[1 2;3 4]
A =
     1     2
     3     4
```

Now, enter and exercute the following loop.

```
>> for k=A, k, end
k =
     1
     3
k =
     2
     4
```

Note that k is assigned a *column* of matrix A at each iteration of the loop.

This column assignment does not contradict the use we have already investigated, i.e., **for k=start:increment:finish**. In this case, the Matlab construct **start:increment:finish** creates a *row vector* and k is assigned a new column of the row vector at each iteration. Of course, this means that k is assigned a new element of the row vector at each iteration.

As an example of use, let's plot the *family* of functions defined by the equation

$$y = 2Cte^{-t^2}, \tag{4.2}$$

where C is one of a number of specific constants. The following code produces the family of curves shown in **Figure 4.6**. At each iteration of the loop, C is assigned the next value in the row vector **[-5,-3,-1,0,1,3,5]**.

```
t=linspace(-4,4,200);
for C=[-5,-3,-1,0,1,3,5]
    y=-2*C*t.*exp(-t.^2);
    line(t,y)
end
```

Break and Continue

There are two sitautions that frequently occur when writing loops.

1. If a certain state is achieved, the programmer wishes to terminate the loop and pass control to the code that follows the end of the loop.
2. The programmer doesn't want to exit the loop, but does want to pass control to the next iteration of the loop, thereby skipping any remaining code that remains in the loop.

As an example of the first situation, consider the following snippet of code. Matlab's **break** command will cause the loop to terminate if a prime is found in the loop index k.

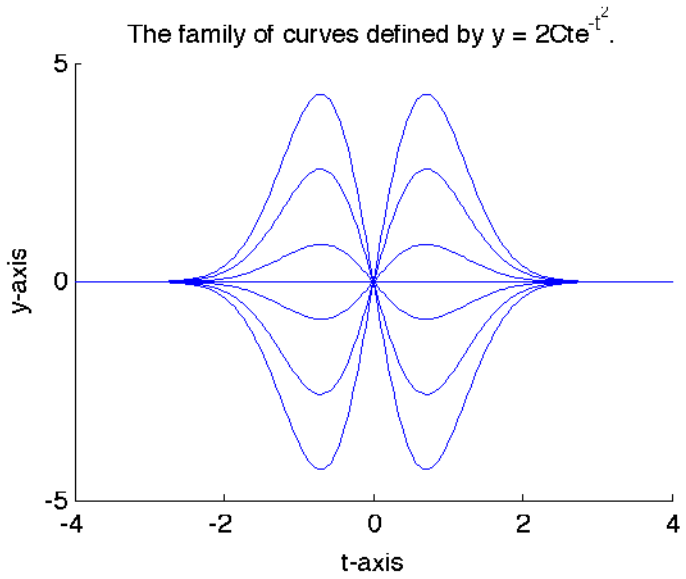


Figure 4.6. A family of curves for $C = -5, -3, -1, 0, 1, 3$, and 5.

```
for k=[4,8,12,15,17,19,21,24]
    if isprime(k)
        fprintf('Prime found, %d, exiting loop.\n',k)
        break
    end
    fprintf('Current value of index k is: %d\n',k)
end
```

Note that the output lists each value of k until a prime is found. The **fprinft** command issues a warning message and the **break** command terminates the loop.

```
Current value of index k is: 4
Current value of index k is: 8
Current value of index k is: 12
Current value of index k is: 15
Prime found, 17, exiting loop.
```

In this next code snippet, we want to print the multiples of three that are less than or equal to 20. At each iteration of the **while** loop, Matlab checks to see if k is divisible by 3. If not, it increments the counter k, then the **continue** statment

that follows skips the remaining statements in the loop, and returns control to the next iteration of the loop.

```
N=20;
k=1;
while k<=N
    if mod(k,3)~=0
        k=k+1;
        continue
    end
    fprintf('Multiple of 3: %d\n',k)
    k=k+1;
end
```

If a multiple of 3 is found, **fprintf** is used to output the result in a nicely formatted statement.

```
Multiple of 3: 3
Multiple of 3: 6
Multiple of 3: 9
Multiple of 3: 12
Multiple of 3: 15
Multiple of 3: 18
```

Any and All

The command **any** returns **true** if any element of a vector is a nonzero number or is logical 1 (true).

```
>> v=[0 0 1 0 1]; any(v)
ans =
     1
```

On the other hand, **any** returns **false** if non of the elements of a vector are nonzero numbers or logical 1's.

```
>> v=[0 0 0 0]; any(v)
ans =
     0
```

The command **all** returns **true** if all of the elements of a vector are nonzero numbers or are logical 1's (true).

```
>> v=[1 1 1 1]; all(v)
ans =
     1
```

On the other hand, **all** returns **false** if the vector contains any zeros or logical 0's.

```
>> v=[1 0 1 1 1]; all(v)
ans =
     0
```

The **any** and **all** command can be quite useful in programs. Suppose for example, that you want to pick out all the numbers from 1 to 2000 that are multiples of 8, 12, and 15. Here's a code snippet that will perform this task. Each time through the loop, we use the **mod** function to determine the remainder when the current value of k is divided by the numbers in **divisors**. If "all" remainders are equal to zero, then k is divisible by each of the numbers in **divisors** and we append k to a list of multiples of 8, 12, and 15.

```
N=2000;
divisors=[8,12,15];
multiples=[];
for k=1:N
    if all(mod(k,divisors)==0)
        multiples=[multiples,k];
    end
end
fmt='%5d %5d %5d %5d %5d\n';
fprintf(fmt,multiples)
```

The resulting output follows.

```
 120   240   360   480   600
 720   840   960  1080  1200
1320  1440  1560  1680  1800
1920
```

Vectorizing the Code. In this case we can avoid loops altogether by using logical indexing.

```
k=1:2000;
b=(mod(k,8)==0) & (mod(k,12)==0) & (mod(k,15)==0);
fmt='%5d %5d %5d %5d %5d\n';
fprintf(fmt,k(b))
```

The reader should check that this code snippet produces the same result, i.e., all multiples of 8, 12, and 15 from 1 to 2000.

Nested Loops

It's possible to nest one loop inside another. You can nest a second **for** loop inside a primary **for** loop, or you can nest a **for** loop inside a **while** loop. Other combinations are also possible. Let's look at some situations where this is useful.

▶ **Example 2.** *A* Hilbert Matrix H *is an* $n \times n$ *matrix defined by*

$$H(i,j) = \frac{1}{i+j-1}, \tag{4.3}$$

where $1 \le i, j \le n$. *Set up nested* **for** *loops to construct a Hilbert Matrix of order* $n = 5$.

We could create this Hilbert Matrix using hand calculations. For example, the entry in row 3 column 2 is

$$H(3,2) = \frac{1}{3+2-1} = \frac{1}{4}.$$

These calculations are not difficult, but for large order matrices, they would be tedious. Let's let Matlab do the work for us.

```
n=5;
H=zeros(n);
```

Now, doubly nested **for** loops achieve the desired result.

```
for i=1:n
    for j=1:n
        H(i,j)=1/(i+j-1);
    end
end
```

Set rational display format, display the matrix H, then set the format back to default.

```
format rat
H
format
```

The result is shown in the following *Hilbert Matrix* of order 5.

```
H =
    1          1/2        1/3        1/4        1/5
    1/2        1/3        1/4        1/5        1/6
    1/3        1/4        1/5        1/6        1/7
    1/4        1/5        1/6        1/7        1/8
    1/5        1/6        1/7        1/8        1/9
```

Note that the entry is row 3 column 2 is 1/4, as predicted by our hand calculations above. Similar hand calculations can be used to verify other entries in this result.

How does it work? Because $n = 5$, the primary loop becomes **for i=1:5**. Similarly, the inner loop becomes **for j=1:5**. Now, here is the way the nested structure proceeds. First, set $i = 1$, then execute the statement **H(i,j)=1/(i+j-1)** for $j = 1, 2, 3, 4$, and 5. With this first pass, we set the entries $H(1,1)$, $H(1,2)$, $H(1,3)$, $H(1,4)$, and $H(1,5)$. The inner loop terminates and control returns to the primary loop, where the program next sets $i = 2$ and again executes the statement **H(i,j)=1/(i+j-1)** for $j = 1$, 2, 3, 4, and 5. With this second pass, we set the entries $H(2,1)$, $H(2,2)$, $H(2,3)$, $H(2,4)$, and $H(2,5)$. A third and fourth

pass of the primary loop occur next, with $i = 3$ and 4, each time iterating the inner loop for $j = 1$, 2, 3, 4, and 5. Finally, on the last pass through the primary, the program sets $i = 5$, then executes the statement **H(i,j)=1/(i+j-1)** for $j = 1$, 2, 3, 4, and 5. On this last pass, the program sets the entries $H(5, 1)$, $H(5, 2)$, $H(5, 3)$, $H(5, 4)$, and $H(5, 5)$ and terminates.

Fourier Series — An Application of a For Loop

Matlab's **mod(a,b)** works equally well with real numbers, providing the remainder when a is divided by b. For example, if you divide 5.7 by 2, the quotient is 2 and the remainder is 1.7. The command **mod(5.7,2)** should return the remainder, namely 1.7.

```
>> mod(5.7,2)
ans =
    1.7000
```

The following commands produce what engineers call a *sawtooth curve*, shown in **Figure 4.7**(a).

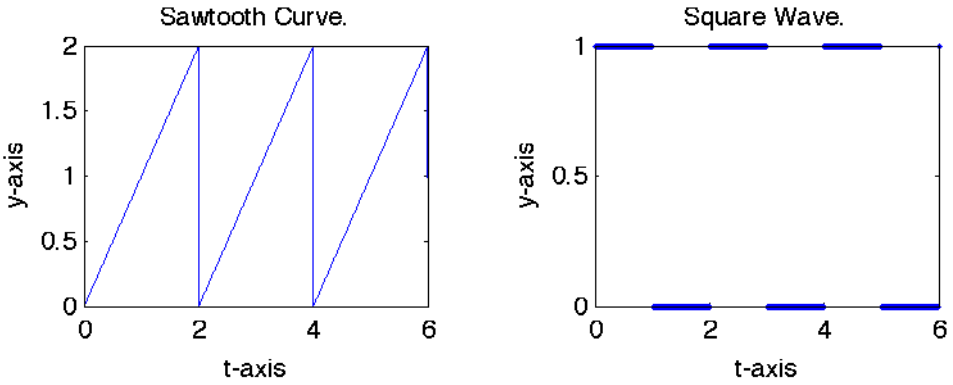
```
t=linspace(0,6,500);
y=mod(t,2);
plot(t,y)
```

In **Figure 4.7**(a), we plot the remainders when the time is divided by 2. Hence, we get the a periodic function of period 2, where the remainders grow from 0 to just less than 2, then repeat every 2 units.

With the following adjustment, we produce what engineers call a *square wave* (shown in **Figure 4.7**(b)). Recall that **y<1** returns true (logical 1) when y is less than 1, and false (logical 0) when y is not less than 1.

```
y=(y<1);
plot(t,y,'*')
```

This type of curve can emulate a switch that is "on" for one second (y-value 1), then off for the next second (y-value 0), and then periodically repeats this "on-off" cycle over its domain.



(a) (b)

Figure 4.7. Generating a square wave with **mod**.

Let's make one last adjustment, increasing the amplitude by 2, then shifting the graph downward 1 unit. This produces the *square wave* shown in **Figure 4.8**.

```
y=2*y-1;
plot(t,y,'*')
```

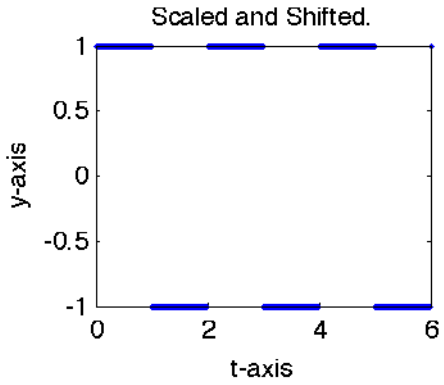


Figure 4.8. A square wave with period 2.

Note that this square waves alternates between the values 1 and -1. The curve equals 1 on the first half of its period, then -1 on its second half. This pattern then repeats with period 2 over the remainder of its domain.

Fourier Series. Using advanced mathematics, it can be shown that the following *Fourier Series* "converges" to the square wave pictured in **Figure 4.8**

$$\sum_{n=0}^{\infty} \frac{4}{(2n+1)\pi} \sin(2n+1)\pi t \tag{4.4}$$

It is certainly remarkable, if not somewhat implausible, that one can sum a series of sinusoidal function and get the resulting square wave picture in **Figure 4.8**.

We will generate and plot the first five terms of the series (**4.4**), saving each term in a row of matrix A for later use. First, we initialize the time and the number of terms of the series that we will use, then we allocate appropriate space for matrix A. Note how we wisely save the number of points and the number of terms in variables, so that if we later decide to change these values, we won't have to scan every line of our program making changes.

```
N=500;
numterms=5;
t=linspace(0,6,N);
A=zeros(numterms,N);
```

Next, we use a **for** loop to compute each of the first 5 terms (**numterms**) of series (**4.4**), storing each term in a row of matrix A, then plotting the sinusoid in three space, where we use the angular frequency as the third dimension.

```
for n=0:numterms-1
    y=4/((2*n+1)*pi)*sin((2*n+1)*pi*t);
    A(n+1,:)=y;
    line((2*n+1)*pi*ones(size(t)),t,y)
end
```

We orient the 3D-view, add a box for depth, turn on the grid, and annotate each axis to produce the image shown in **Figure 4.9**(a).

```
view(20,20)
box on
grid on
xlabel('Angular Frequency')
ylabel('t-axis')
zlabel('y-axis')
```

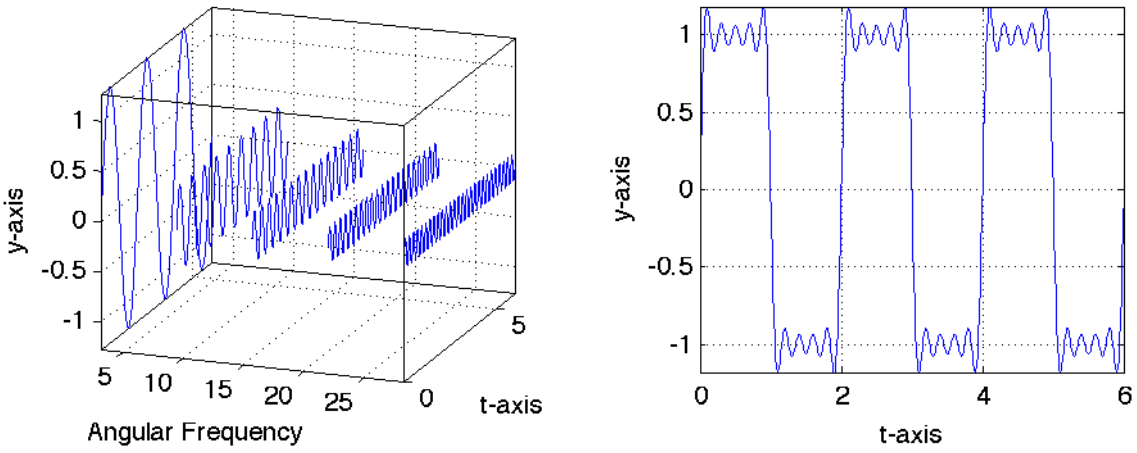
As one would expect, after examining the terms of series (**4.4**), each each consecutive term of the series is a sinusoid with increasing angular frequency and decreasing amplitude. This is evident with the sequence of terms shown in **Figure 4.9**(a).

However, a truly remarkable result occurs when we add the sinusoids in **Figure 4.9**(a). This is easy to do because we saved each individual term in a row of matrix A. Matlab's **sum** command will sum the columns of matrix A, effectively summing the first five terms of series (**4.4**) at each instant of time t.

```
figure
plot(t,sum(A))
```

We "tighten" the axes, add a grid, then annotate each axis. This produces the image in **Figure 4.9**(b). Note the striking similarity to the square wave in **Figure 4.8**.

```
axis tight
grid on
xlabel('t-axis')
ylabel('y-axis')
```

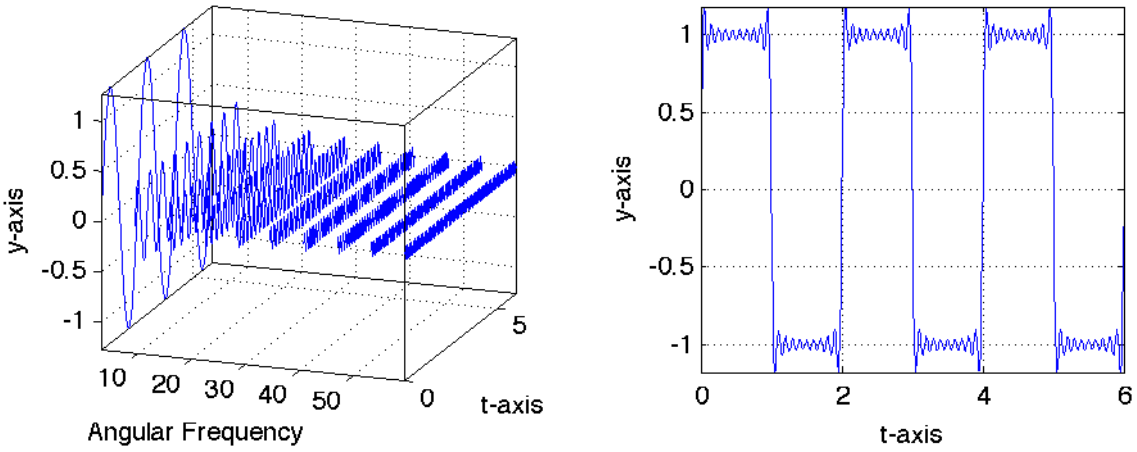


(a) (b)

Figure 4.9. Approximating a square wave with a Fourier series (5 terms).

Because we smartly stored the number of terms in the variable **numterms**, to see the effect of adding the first 10 terms of the fourier series (**4.4**) is a simple matter of changing **numterms=5** to **numterms=10** and running the program again. The ouptut is shown in **Figures 4.10**(a) and (b). In **Figure 4.10**(b), note the even closer resemblance to the square wave in **Figure 4.8**, except at the ends, where the "ringing" exhibited there is known as *Gibb's Phenomenon*. If you

one day get a chance to take a good upper-division differential equations course, you will study Fourier series in more depth.



(a) (b)

Figure 4.10. Adding additional terms of the Fourier series (4.4).

4.2 Exercises

1. Write a **for** loop that will output the cubes of the first 10 positive integers. Use **fprintf** to output the results, which should include the integer and its cube. Write a second program that uses a **while** loop to produce an indentical result.

2. Without appealing to the Matlab command **factorial**, write a **for** loop to output the factorial of the numbers 10 through 15. Use **fprintf** to format the output. Write a second program that uses a **while** loop to produce an indentical result. *Hint: Consider the* **prod** *command.*

3. Write a single program that will count the number of divisors of each of the following integers: 20, 36, 84, and 96. Use **fprintf** to output each result in a form similar to "The number of divisors of 12 is 6."

4. Set **A=magic(5)**. Write a program that uses nested **for** loops to find the sum of the squares of all entries of matrix A. Use **fprintf** to format the output. Write a second program that uses array operations and Matlab's **sum** function to obtain the same result.

5. Write a program that uses nested **for** loops to produce *Pythagorean Triples*, positive integers a, b and c that satisfy $a^2+b^2 = c^2$. Find all such triples such that $1 \le a, b, c \le 20$ and use **fprintf** to produce nicely formatted results.

6. Another result proved by Leonhard Euler shows that

$$\frac{\pi^4}{90} = 1 + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \cdots.$$

Write a program that uses a **for** loop to sum the first 20 terms of this series. Compute the relative error when this sum is used as an approximation of $\pi^4/90$. Write a second program that uses a **while** loop to determine the number of terms required so that the sum approximates $\pi^4/90$ to four significant digits. In both programs, use **fprintf** to format your output.

7. Some attribute the following series to Leibniz.

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots$$

Write a program that uses a **for** loop to sum the first 20 terms of this series. Compute the relative error when this sum is used as an approximation of $\pi/4$. Write a second program that uses a **while** loop to determine the number of terms required so that the sum approximates $\pi/4$ to four significant digits. In both programs, use **fprintf** to format your output.

8. *Goldbach's Conjecture* is one of the most famous unproved conjectures in all of mathematics, which is remarkable in light of its simplistic statement:

[6] Copyrighted material. See: http://msenux.redwoods.edu/IntAlgText/

> *All even integers greater than 2 can be expressed as a sum of two prime integers.*

Write a program that expresses 1202 as the sum of two primes in ten different ways. Use **fprintf** to format your output.

9. Here is a simple idea for generating a list of prime integers. Create a vector **primes** with a single entry, the prime integer 2. Write a program to test each integer less than 100 to see if it is a prime using the following procedure.

i. If an integer is divisible by any of the integers in **primes**, skip it and go to the next integer.
ii. If an integer is **not** divisible by any of the integers in **primes**, append the integer to the vector **primes** and go to the next integer.

Use **fprintf** to output the vector **primes** in five columns, right justified.

10. There is a famous stoy about Sir Thomas Hardy and Srinivasa Ramanujan, which Hardy relates in his famous work "A Mathematician's Apology," a copy of which resides in the CR library along with the work "The Man Who Knew Infinity: A Life of the Genius Ramanujan."

> *I remember once going to see him when he was ill at Putney. I had ridden in taxi cab number 1729 and remarked that the number seemed to me rather a dull one, and that I hoped it was not an unfavorable omen. "No," he replied, "it is a very interesting number; it is the smallest number expressible as the sum of two cubes in two different ways."*

Write a program with nested loops to find integers a and b (in two ways) so that $a^3 + b^3 = 1729$. Use **fprintf** to format the output of your program.

11. Write a program to perform each of the following tasks.

i. Use Matlab to draw a circle of radius 1 centered at the origin and inscribed in a square having vertices $(1, 1)$, $(-1, 1)$, $(-1, -1)$, and $(1, -1)$. The ratio of the area of the circle to the area of the square is $\pi : 4$ or $\pi/4$. Hence, if we were to throw darts at the square in a rondom fashion, the ratio of darts inside the circle to the number of darts thrown should be approximately equal to $\pi/4$.
ii. Write a **for** loop that will plot 1000 randomly generated points inside the square. Use Matlab's **rand** command for this task. Each time random point lands within the unit circle, increment a counter **hits**. When the **for** loop terminates, use **fprintf** to output the ratio darts that land inside the circle to the number of darts thrown. Calculate the relative error in approximating $\pi/4$ with this ratio.

12. Write a program to perform each of the following tasks.

i. Prompt the user to enter a 3×4 matrix. Store the result in the matrix A.

ii. Use **if..elseif..else** to provide a menu with three choices: (1) Switch two rows of the matrix; (2) Multiply a row of the matrix by a scalar; and (3) Subtract a scalar multiple of a row from another row of the matrix.
iii. Perform the task in the requested menu item, then return the resulting matrix to the user.
   a. In the case of (1), your program should prompt the user for the rows to switch.
   b. In the case of (2), your program should prompt the user for a scalar and a row that will be multiplied by the scalar.
   c. In the case of (3), your program should prompt the user for a scalar and two row numbers, the first of which is to be multiplied by the scalar and subtracted from the second.

13. The solutions of the quadratic equation $ax^2+bx+c=0$ are given by the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}.$$

The discriminant $D = b^2-4ac$ is used to predict the number of roots. There are three cases.

1. If $D > 0$, there are two real solutions.
2. If $D = 0$, there is one real solution.
3. If $D < 0$, there are no real solutions.

Write a program the performs each of the following tasks.

i. The program prompts the user to input a, b, and c.
ii. The programs computes the discriminant D which it uses with the conditional **if..elseif..else** to determine and output the number of solutions.
iii. The program should also output the solutions, if any.

Use well crafted format strings with **fprintf** to output all results.

14. Plot the modified *sawtooth* curve.

```
N=500;
t=linspace(0,6*pi,N);
y=mod(t+pi,2*pi)-pi;
line(t,y)
```

It can be shown that the Fourier series

$$\sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n} \sin nx$$

"converges" to the sawtooth curve. Perform each of the following tasks.

i. Sketch each of the individual terms in 3-space, as shown in the narrative.
ii. Sketch the sawtooth, hold the graph, then sum the first five terms of the series and superimpose the plot of the result.

15. Logical arrays are helpful when it comes to drawing piecewise functions. For example, consider the piecewise definition

$$f(x) = \begin{cases} 0, & \text{if } -\pi \le x < 0 \\ \pi - x, & \text{if } 0 \le x \le \pi. \end{cases}$$

Enter the following to plot the piecewise function f.

```
N=500;
x=linspace(-pi,pi,N);
y=0*(x<0)+(pi-x).*(x>=0);
plot(x,y,'.')
```

A Fourier series representation of the function f is given by

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos nx + b_n \sin nx \right],$$

where $a_0 = \pi/2$, and

$$a_n = \frac{1 - \cos n\pi}{n^2 \pi} \quad \text{and} \quad b_n = \frac{1}{n}.$$

Hold the plot of f, then superimpose the Fourier series sum for $n = 1$ to $n = 5$. *Hint: This is a nice place for anonymous functions, for example, set:*

```
b = @(n) 1/n;
```

4.2 Answers

1. This **for** loop will print the cubes of the first 10 positive integers.

```
for k=1:10
    fprintf('The cube of %d is %d.\n',k,k^3)
end
```

This **while** loop will print the cubes of the first 10 positive integers.

```
k=1;
while k<=10
    fprintf('The cube of %d is %d.\n',k,k^3)
    k=k+1;
end
```

3. The following program will print the number of divisors of 20, 36, 84, and 96.

```
for k=[20, 36, 84, 96]
    count=0;
    divisor=1;
    while (divisor<=k)
        if mod(k,divisor)==0
            count=count+1;
        end
        divisor=divisor+1;
    end
    fprintf('The number of divisors of %d is %d.\n',k,count)
end
```

5. This program will print Pythagorean Triples so that $1 \leq a, b, c \leq 20$.

```
N=20;
for a=1:N
  for b=1:N
     for c=1:N
        if (c^2==a^2+b^2)
            fprintf('Pythagorean Triple: %d, %d, %d\n', a, b, c)
        end
     end
  end
end
```

7. The following loop will sum the first 20 terms.

```
N=20;
s=0;
for k=1:N;
    s=s+(-1)^(k+1)/(2*k-1);
end
```

Compute the relative error in approximating $\pi/4$ with the sum of the first 20 terms.

```
rel=abs(s-pi/4)/abs(pi/4);
```

Output the results.

```
fprintf('The actual value of pi/4 is %.6f.\n',pi/4)
fprintf('The sum of the first %d terms is %f.\n',N,s)
fprintf('The relative error is %.2e.\n',rel)
```

9. The **mod(m,primes)==0** comparison will produce a logical vector which contains a 1 in each position that indicates that the current number m is divisible by the number in the **prime** vector in the corresponding position. If "any" of these are 1's, then the number m is divisible by at least one of the primes in the current list **primes**. In that case, we continue to the next value of m. Otherwise, we append m to the list of primes.

```
primes=2;
for m=3:100
    if any(mod(m,primes)==0)
        continue
    else
        primes=[primes,m];
    end
end
```

We create a format of 5 fields, each of width 5, then **fprintf** the vector **primes** using this format.

```
fmt='%5d %5d %5d %5d %5d\n';
```

The resulting list of primes is nicely formatted in 5 columns.

```
    2     3     5     7    11
   13    17    19    23    29
   31    37    41    43    47
   53    59    61    67    71
   73    79    83    89    97
```

11. We open a figure window and set a lighter background color of gray.

```
fig=figure;
set(fig,'Color',[0.95,0.95,0.95])
```

We draw a circle of radius one, increase its line width, change its color to dark red, set the axis equal, then turn the axes off.

```
t=linspace(0,2*pi,200);
x=cos(t);
y=sin(t);
line(x,y,'LineWidth',2,'Color',[0.625,0,0])
axis equal
axis off
```

We draw the square in counterclockwise order of vertices, starting at the point $(1, 1)$. In that order, we set the x- and y-values of the vertices in the vectors **x** and **y**. We then draw the square with a thickened line width and color it dark blue.

```
x=[1,-1,-1,1,1]; y=[1,1,-1,-1,1];
line(x,y,'LineWidth',2,'Color',[0,0,0.625])
```

Next, we set the number of darts thrown at 1000. Then we plot one dart at the origin, change its linestyle to 'none' (points will not be connected with line segements), choose a marker style, and color it dark blue.

```
N=1000;
dart=line(0,0,...
    'LineStyle','none',...
    'Marker','.',...
    'Color',[0,0,0.625]);
```

In the loop that follows, we use the handle to this initial dart to add x- and y-data (additional darts). First, we set the number of **hits** (darts that land inside the circle) to zero (we don't count the initial dart at $(0, 0)$).

```
hits=0;
for k=1:N
 x=2*rand-1;
 y=2*rand-1;
 set(dart,...
   'XData',[get(dart,'XData'),x],...
   'YData',[get(dart,'YData'),y])
    %drawnow
    if (x^2+y^2<1)
        hits=hits+1;
    end
end
```

In the body of the loop, we choose uniform random numbers between -1 and 1 for both x and y. We then add these new x- and y-values to the **XData** and **YData** of the existing dart by using its handle **dart**. We do this for x by

first "getting" the **XData** for the dart with **get(dart,'XData')**, then we append the current value of x with **[get(dart,'XData'),x]**. The **set** command is used to set the updated list of x-values. A similar task updates the dart's y-values. At each iteration of the loop, we also increment the **hits** counter if the dart lies within the border of the circle, that is, if $x^2 + y^2 < 1$. You can uncomment the **drawnow** command to animate the dart throwing at the expense of waiting for 1000 darts to be thrown to the screen.

Finally, we output the requested data to the command window.

```
fprintf('%d of %d darts fell inside the circle.\n',hits,N)
fprintf('The ratio of hits to throws is %f\n\n',hits/N)

fprintf('The ratio of the area of the circle to the area\n')
fprintf('the area of the square is pi/4, or approximately %f.\n\n',pi/4)

rel_err=abs(hits/N-pi/4)/abs(pi/4);
fprintf('The relative error in approximating pi/4 wiht the ratio\n')
fprintf('of hits to total darts thrown is %.2e.\n',rel_err)
```

15. We begin by using logical relations to craft the piecewise function.

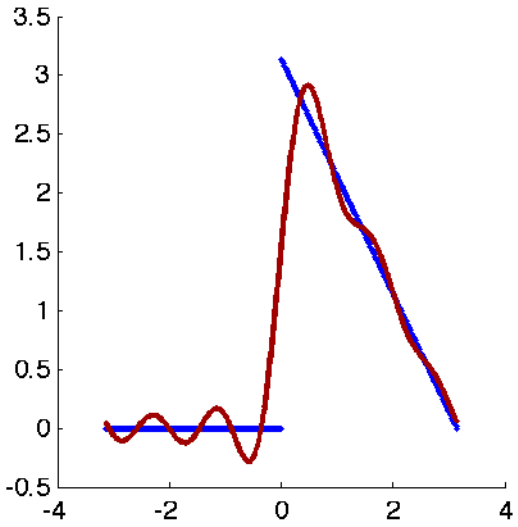
```
N=500;
x=linspace(-pi,pi,N);
y=0*(x<0)+(pi-x).*(x>=0);
line(x,y,...
    'LineStyle','none',...
    'Marker','.')
```

Next, we compose two anonymous functions for the coefficients a_n and b_n.

```
a=@(n) (1-cos(n*pi))/(n^2*pi);
b=@(n) 1/n;
```

Note that $a_0 = \pi/2$, so $a_0/2 = \pi/4$, which must be added to the sum of the first five terms of the series. Thus, we'll start our running sum at $s = \pi/4$. The **for** loop then sums the first five terms. We use another **line** command to plot the result.

```
s=pi/4;
N=5;
for k=1:N
    s=s+(a(k)*cos(k*x)+b(k)*sin(k*x));
end
line(x,s,'LineWidth',2,'Color',[0.625,0,0])
```



4.3 Functions in Matlab

You've encountered function notation in your mathematics courses, perhaps in algebra, trigonometry, and/or calculus. For example, consider the function defined by

$$f(x) = 2x^2 - 3. \tag{4.5}$$

This function expects a single input x, then outputs a single number $2x^2 - 3$. Sometimes an alternative notation is used to reinforce this concept that the input x is mapped to the output $2x^2 - 3$, namely $f : x \to 2x^2 - 3$. For example,

$$f : 3 \longrightarrow 2(3)^2 - 3.$$

That is, $f : 3 \to 15$. Using the notation in (4.5), we proceed in a similar manner.

$$f(3) = 2(3^2) - 3.$$

That is, $f(3) = 15$. Note that there is one input, 3, and one output, 15. The goal of this section is to learn how Matlab emulates this input-output behavior of functions.

Anonymous Functions

As we've seen in previous work, we can use *anonymous functions* in Matlab to emulate the behavior of mathematical functions like $f(x) = 2x^2 - 3$. The general syntax of an anoymous function is

```
fhandle = @(arglist) expr
```

where **expr** is the body of the function, the part that churns your input to produce the output. It can consist of any **single** valid Matlab expression. To the left of **expr** is **(arglist)**, which should be a comma separated list of all input variables to be passed to the function. To the left of the argument list is Matlab's **@** symbol, which creates a *function handle* and stores it in the variable **fhandle**.

You execute the function associated with the function handle with the following syntax.

```
fhandle(arg1, arg2, ..., argN)
```

[7] Copyrighted material. See: http://msenux.redwoods.edu/Math4Textbook/

That is, you specify the variable holding the function handle, followed by a comma separated list of arguments in parentheses.

▶ **Example 1.** *Create a function to emulate the behavior of the mathematical function* $f(x) = 2x^2 - 3$.

We use the syntax **fhandle = @(arglist) expr** to create and store a function handle in the variable f.

```
>> f = @(x) 2*x^2 - 3
f =
    @(x) 2*x^2 - 3
```

We now execute the function by specifying the function handle associated with it, followed by a single argument (the input) in parentheses.

```
>> f(3)
ans =
    15
```

However, note what happens when we pass an array as input to this function.

```
>> x=0:5
x =
     0     1     2     3     4     5
>> f(x)
??? Error using ==> mpower
Matrix must be square.

Error in ==> @(x) 2*x^2 - 3
```

We've failed to make our function "array smart." You cannot square a vector, as we are reminded in the error message "Matrix must be square." We need to use array operators to make our anonymous function "array smart."

```
>> f = @(x) 2*x.^2 -3
f =
    @(x) 2*x.^2 -3
```

Note that this time we used the "dot raised to" operator **.^** which will operate elementwise and square each entry of the input. Note that using an array as input is now successful, and the function is evaluated at each entry of the input vector **x=0:5**.

```
>> f(x)
ans =
    -3    -1     5    15    29    47
```

Even though we've made our function array smart, it will still operate on individual scalar entries.

```
>> f(5)
ans =
    47
```

Making Functions Array Smart. Many of Matlab's routines require that a function passed as an argument be "array smart." You need to be aware of this if you pass a function to one of these routines.

Let's look at another example.

▶ **Example 2.** *Create a function to emulate the mathematical behavior of the function defined by* $f(x, y) = 9 - x^2 - y^2$.

Note that there are two inputs to the function f. As an example, we evaluate the function at the point $(x, y) = (2, 3)$.

$$f(2, 3) = 9 - 2^2 - 3^2.$$

Thus, $f(2, 3) = -4$.

We define an anonymous Matlab function to emulate the behavior of f. Note that the the **@** symbol is followed by a comma delimited list of two variables x and y, contained in parentheses.

```
>> f= @(x,y) 9 - x^2 - y^2
f =
    @(x,y) 9 - x^2 - y^2
```

We evaluate the function f at $(x, y) = (2, 3)$ as follows.

```
>> f(2,3)
ans =
    -4
```

Note that this agrees with our hand calculation above.

It's possible to use constants in your anonymous function that are previously defined in your current workspace.

▶ **Example 3.** *Write a function to evaluate* $f(x, y, z) = Ax + By + Cz$ *at the point* $(-1, 2, -3)$*, where* $A = 2$*,* $B = 3$*, and* $C = -4$ *are previously defined in the current workspace.*

Set the constants $A = 2$, $B = 3$, and $C = -4$.

```
>> A=2; B=3; C=-4;
```

Now, define the anonymous function as follows.

```
>> f = @(x,y,z) A*x + B*y + C*z
f =
    @(x,y,z) A*x + B*y + C*z
```

Now, evaluating the function at $(x, y, z) = (-1, 2, -3)$ by hand,

$$f(-1, 2, -3) = A(-1) + B(2) + C(-3) = (2)(-1) + (3)(2) + (-4)(-3) = 16.$$

Checking in Matlab.

```
>> f(-1,2,-3)
ans =
    16
```

It's important to note that changing the values of A, B, and C after the function definition will have no effect on the anonymous function.

```
>> A=1; B=1; C=1;
>> f(-1,2,-3)
ans =
    16
```

If you want these new constants to be used in the function, you must reevaluate the anonymous function, as the anonymous function retains the values of any constants as they existed in the workspace at the moment of the anonymous function's conception.

```
>> f = @(x,y,z) A*x + B*y + C*z
f =
    @(x,y,z) A*x + B*y + C*z
>> f(-1,2,-3)
ans =
    -2
```

We'll leave it to our readers to check the accuracy of this last result.

Finally, it is possible to construct an anonymous function with no input.

```
>> t = @() datestr(now)
t =
    @() datestr(now)
```

You must call such a function with an empty argument list.

```
>> t()
ans =
04-Mar-2007 00:34:07
```

Function M-Files

Note that the anonymous function syntax **fhandle = @(args) expr** only allows for a single Matlab expression in its definition body. Moreover, only one output variable is allowed. In this section we will look at *Function M-files*, which are similar to scripts, but allow for both multiple input and output and as many lines as needed in the body of the function.

Like script files, you write function m-files in the Matlab editor. The first line of the function m-file must contain the following syntax.

```
function [out1, out2, ...] = funname(in1, in2, ...)
```

The keyword **function** must be the first word in the function m-file. This alerts the Matlab interpreter that the file contains a function, not a script. Next comes a comma delimited list of output arguments (surrounding brackets are required if more than one output variable is used), then an equal sign, followed by the function name. The function name **funname** must be string composed of letters, digits, and underscores. The first character of the name must be a letter. A valid length for the function name varies from machine to machine, but can be determined by the command **namelengthmax**.

```
>> namelengthmax
ans =
    63
```

If you name your function **funname**, as above, you must save the function m-file as **funname.m**. That is, you must attach **.m** to the function name when saving the function m-file.

After the function name comes a comma delimited list of input variables, surrounded by required parentheses. Input arguments are passed by value and all variables in the body of the function are "local" to the function (we'll have more to say about local and global variables later). In the body of the function m-file,

you must assign a value to each output variable before the function definition is complete.

Before getting too ambitious, let's start with some simple examples of function M-files that emulate the behavior of the anonymous functions in **Examples 1** through **3**.

▶ **Example 4.** *Write a function m-file to emulate the behavior of the function* $f(x) = 2x^2 - 3$.

Open the Matlab editor and enter the following lines.

```
function y=f(x)
y=2*x^2-3;
```

Note how we've followed the syntax rules outlined above.

i. The first word on the first line is the keyword **function**.
ii. We are using y as the output variable. Because there is only a single output variable, the surrounding brackets are not required.
iii. The function name is **f**.
iv. Finally, surrounded in required parentheses is a single input variable x.

Save the file as **f.m**. Note how we've attached **.m** to the function name **f**.

In **Example 1**, we found that $f(3) = 15$. Let's test the current function. Return to the command window and enter the following at the Matlab prompt.

```
>> f(3)
ans =
    15
```

As in **Example 1**, the current function is not "array smart," so attempting to evaluate the function on an array will fail.

```
>> x=0:5
x =
     0     1     2     3     4     5
>> f(x)
??? Error using ==> mpower
Matrix must be square.

Error in ==> f at 2
y=2*x^2-3;
```

Return to the editor and make the following change to the functon m-file.

```
function y=f(x)
y=2*x.^2-3;
```

Note that we have replaced the regular exponentiation symbol $\hat{}$ with the array exponentiation symbol .$\hat{}$ ("dot raised to"). This makes our function "array smart" and we can now evaluate the function at each entry of the vector **x**.

```
>> f(x)
ans =
    -3    -1     5    15    29    47
```

In the next example, we use a function m-file to emulate the behavior in **Example 2**.

▶ **Example 5.** *Write a function m-file for* $f(x, y) = 9 - x^2 - y^2$. *Evaluate the function at* $(x, y) = (2, 3)$.

Open the Matlab editor and enter the following lines.

```
function z=f(x,y)
z=9-x^2-y^2;
```

Note that this time we have two input variables, x and y. The function name is **f**, so we again save the file as **f.m**. Evaluate the function at $(x, y) = (2, 3)$ at the Matlab prompt in the command window.

```
>> f(2,3)
ans =
    -4
```

This agrees with the result in **Example 2**.

Let's stretch this example a bit further. Move to the command window and set the following variables at the Matlab command prompt.

```
>> dogs=3; cats=3;
```

Now, evaluate the function at these variables and store the result in the variable **pets**.

```
>> pets=f(dogs,cats)
pets =
    -4
```

Note that this produces an identical result, highlighting two important points.

i. The names of the variables in command workspace do not have to match the names of the input arguments in the function definition. In this example, the *values* of the variables **dogs** and **cats** are passed to the arguments **x** and **y** in the function m-file.
ii. The value of the output variable in the command workspace does not need to match the value of the output variable in the function defintion. In this example, the value of the output variable **z** is returned and stored in the command workspace variable **pets**.

In **Example 3**, there were three constants involved, A, B, and C. We can pass these additional constants to the function if we add them to the list of input arguments in the function defintion.

▶ **Example 6.** *Evaluate the function* $f(x, y, z) = Ax + By + Cz$ *at* $(x, y, z) = (-1, 2, -3)$, *where* $A = 2$, $B = 3$, *and* $C = -4$.

Enter the following lines in the Matlab editor.

```
function w=f(x,y,z,A,B,C)
w=A*x+B*y+C*z;
```

Save the file as **f.m**.

Set $A = 2$, $B = 3$, and $C = -4$ in the command workspace at the Matlab prompt.

```
>> A=2; B=3; C=-4;
```

We evaluate the function by passing the appropriate values to the function arguments x, y, and z. In addition, we pass the values of A, B, and C in the command workspace to the functions arguments A, B, and C.

```
>> f(-1,2,-3,A,B,C)
ans =
    16
```

Note that this matches the result in **Example 3**.

Again, it is not required that the names of the command workspace variables match those of the input and output arguments in the function definition. We could just as easily use the current function defintion to compute the cost of a purchase at a pet store.

```
>> guppies=10; tadpoles=12; goldfish=4;
>> per_guppy=0.50; per_tadpole=0.35; per_goldfish=0.85;
>>cost=f(guppies,tadpoles,goldfish,per_guppy,
per_tadpole,per_goldfish)
cost =
   12.6000
```

Global Variables. Alternatively, we could declare the constants A, B, and C as global variables at the command prompt in Matlab's workspace. You should declare the variables as global before you assign values to them. As we already have values assigned to variables A, B, and C in the command window workspace, we will clear them, declare them to be global, then assign values.

```
>> clear A B C
>> global A B C
>> A=2; B=3; C=-4;
```

You must also declare these constants as global in the function m-file.

```
function w=f(x,y,z)
global A B C
w=A*x+B*y+C*z;
```

Resave the function m-file as **f.m**. Note that we've made two changes.

i. We've removed A, B, and C as input arguments. The first line of the function file now reads **function w=f(x,y,z)**.
ii. We've declared the variables A, B, and C as global. Now, any changes to the global variables in the command workspace will be recognized by the variables in the function workspace, and vice-versa, any change to the global variables in the function's workspace will be recognized by the command workspace. In effect, the two workspaces share the values assigned to the global variables.

Evaluate the function at $(x, y, z) = (-1, 2, -3)$.

```
>> f(-1,2,-3)
ans =
    16
```

More Than One Output

To this point, anonymous functions can seemingly do everything that we've demonstrated via function m-files in **Examples 4-6**. However, two key facts about anonymous functions explain why we should continue to explore function m-files in more depth.

1. Anonymous functions are allowed only one output.
2. The body of an anonymous function is allowed exactly one valid Matlab expression.

Let's look at a function m-file that sends two outputs back to the calling program or Matlab command workspace.

▶ **Example 7.** *Write a function m-file that will compute the area and circumference of a circle of radius r.*

Open the editor and enter the following lines.

```
function [area,circumference]=area_and_circumference(r)
area=pi*r^2;
circumference=2*pi*r;
```

Save the file as **area_and_circumference.m**. Set the radius of a particular circle at the Matlab prompt.

```
>> radius=12;
```

Call the function **area_and_circumference** with input argument **radius** and store the resulting area and circumference in the workspace variables A and C.

```
>> [A,C]=area_and_circumference(radius)
A =
  452.3893
C =
   75.3982
```

Functions return output variables in the order in which they are listed in the function definition line within the m-file. Thus,

```
[area,circumference]=area_and_circumference(r)
```

declares that **area** will be the first output returned and **circumference** will be the second.

It is not required that the calling statement (whether from the command line, a script, or another function) contain the same number of output variables as defined by the function m-file. If you specify fewer output variables in the calling statement than are defined in the function m-file, the output variables in the

calling statement will be filled in the order defined in the function m-file. For example, the following command returns and stores the area.

```
>> myarea=area_and_circumference(radius)
myarea =
  452.3893
```

The order of the output variables in the function definition control the order in which results are returned to the calling statement, not the names of the output variables in the calling statement.

```
>> mycircum=area_and_circumference(radius)
mycircum =
  452.3893
```

This could be a crucial error and illustrates why function m-files should contain documentation explaining their use.

Documenting Help in Function M-Files. The first contiguous block of comment files in a function m-file are displayed to the command window when a user enters **help funname**, where **funname** is the name of the function. You terminate this opening comment block with a blank line or an executable statement. After that, any later comments do not appear when you type **help funname**.

The first line of the comment block is special and is used by Matlab's **lookfor** and should therefore contain the function name and a brief description of the function. Ensuing lines should contain syntax of use and sometimes it is useful to provide examples of use. Type **help svd** for an example of a well crafted help file and then type **lookfor svd** to see how the first line of the help file is special.

With these thoughts in mind, let's craft some documentation for our function **area_and_circumference**. Add the following lines to the m-file and save.

```
function [area,circumference]=area_and_circumference(r)
%AREA_AND_CIRCUMFERENCE finds the area and circumference
%of a circle.
%   [A,C]=AREA_AND_CIRCUMFERENCE(R) returns the area A and
%   circumference C of a circle of radius R.
area=pi*r^2;
circumference=2*pi*r;
```

Type the following to view the help.

```
>> help area_and_circumference
 AREA_AND_CIRCUMFERENCE finds the area and circumference
 of a circle.
    [A,C]=AREA_AND_CIRCUMFERENCE(R) returns the area A and
    circumference C of a circle of radius R.
```

"Look For" functions that have the the word "area" in the first line of their help blocks.

```
>> lookfor area
AREA_AND_CIRCUMFERENCE finds the area and circumference
POLYAREA Area of polygon.
...
```

Note that our **area_and_circumference** function is listed along with a number of other function m-files.

No Output

It is not required that a function m-file produce output variables. There are times when we will want to write functions that take input arguments, but then are self contained. The remainder of work needed to be processed is accomplished within the function m-file itself.

▶ **Example 8.** *The equation* $ax^2 + bx + c = 0$, *called a* quadratic equation, *has solutions provided by the* quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{4.6}$$

Write a function m-file that takes as input the coefficients a, b, *and* c *of the quadratic equation, then, based on the result of the discriminant* $D = b^2 - 4ac$, *produces the solutions of the quadratic equation.*

We begin with the function line and a short help block.

```
function quadratic(a,b,c)
%QUADRATIC finds solutions of ax^2+bx+c=0.
%   QUADRATIC(A,B,C) uses the value of the discriminant
%   D=B^2-4AC to determine if the quadratic equation
%   has two real solutions, 1 real solution, or no real
%   solutions, and prints the solutions to the screen.
```

Note the first line: **function quadratic(a,b,c)**. The function **quadratic** requires three input arguments A, B, and C (the coefficients of the quadratic equation $ax^2 + bx + c = 0$), but provides no output.

First, calculate the value of the discriminant.

```
D = b^2 - 4*a*c;  %the disciminant
```

If the disciminant is positive, there are two real solutions, given by the quadratic formula (4.6). If the discriminant is zero, then the quadratic formula provides a single solution, namely $x = -b/(2a)$. If the discriminant is negative, then there are no real solutions (we cannot take the square root of a negative number). This calls for an **if..elseif..else** conditional.

```
if D>0
    x1=(-b-sqrt(D))/(2*a);
    x2=(-b+sqrt(D))/(2*a);
    fprintf('The discriminant is %d.\n',D)
    fprintf('Hence, there are two solutions.\n')
    fprintf('The first solution is: %f\n',x1)
    fprintf('The second solution is: %f\n',x2)
elseif D==0
    x=-b/(2*a);
    fprintf('The discriminant is %d.\n',D)
    fprintf('Hence, there is exactly one solution.\n')
    fprintf('The solution is: %f\n',x)
else
    fprintf('The discriminant is %d.\n',D)
    fprintf('Hence, there are no real solutions.\n')
end
```

Save the function as **quadratic.m**.

As a first test, consider quadratic equation $x^2 - 2x - 3 = 0$. Note that the discriminant is $D = (-2)^2 - 4(1)(-3) = 16$ and predicts two real roots. Indeed, the quadratic factors as $(x + 1)(x - 3) = 0$. Thus, the equation has solutions $x = -1$ or $x = 3$.

```
>> quadratic(1,-2,-3)
The discriminant is 16.
Hence, there are two solutions.
The first solution is: -1.000000
The second solution is: 3.000000
```

So far, so good. As a second test, consider the quadratic $x^2 - 4x + 4 = 0$. The discriminant is $D = (-4)^2 - 1(1)(4) = 0$ and predicts exactly one real root. Indeed, the quadratic factors as $(x-2)^2 = 0$. Thus, the equation has a single real root $x = 2$.

```
>> quadratic(1,-4,4)
The discriminant is 0.
Hence, there is exactly one solution.
The solution is: 2.000000
```

Finally, consider the quadratic equation $x^2 - 2x + 2 = 0$. The discriminant is $D = (-2)^2 - 4(1)(2) = -4$, so the quadratic equation has no real solutions.

```
>> quadratic(1,-2,2)
The discriminant is -4.
Hence, there are no real solutions.
```

4.3 Exercises

In **Exercises 1-6**, write an anonymous function to emulate the given function. In each case, evaluate your anonymous function at the given value(s) of the independent variable.

1. $f(x) = x^2 - 3x - 4, \quad f(1).$

2. $f(x) = 5 - x - x^2 - 2x^4, \quad f(1).$

3. $f(t) = e^{-0.25t}(2\cos t - \sin t), \quad f(1).$

4. $g(t) = e^{0.10t}(\cos 2t - 3\sin 2t), \quad g(1).$

5. $h(u, v) = \cos u \cos v, \quad h(1, 2).$

6. $f(u, v) = \dfrac{\sin(u^2 + v^2)}{u^2 + v^2}, \quad f(1, 2).$

In **Exercises 7-12**, write a function M-file to emulate the function in the given exercise. In each case, evaluate the given function at the given value of the independent variable.

7. The function and evaluation given in **Exercise 1**.

8. The function and evaluation given in **Exercise 2**.

9. The function and evaluation given in **Exercise 3**.

10. The function and evaluation given in **Exercise 4**.

11. The function and evaluation given in **Exercise 5**.

12. The function and evaluation given in **Exercise 6**.

13. Consider the anonymous function

```
f=@(x) x.*exp(-C*x.^2);
```

Write a **for** loop that will plot this fmaily of functions on the interval $[-1, 1]$ for C in $\{-1, -0.8, -0.6, \ldots 1\}$.

14. Consider the anonymous function

```
f=@(x) C*exp(t-t.^2/2);
```

Write a **for** loop that will plot this family of functions on the interval $[-2, 4]$ for C in $\{-1, -0.8, -0.6, \ldots 1\}$.

15. Consider the function

$$f(x) = xe^{-Cx^2}.$$

Write a function M-file that will emulate this function and allow the passing of the parameter c. That is, your first line of your function should look as follows:

```
function y=f(x,c)
```

In cell mode (or a script M-file), write

a **for** loop that will plot this family of functions on the interval $[-1, 1]$ for C in $\{-1, -0.8, -0.6, \ldots 1\}$.

16. Consider the function

$$f(x) = ce^{x-x^2/2}.$$

Write a function M-file that will emulate this function and allow the passing of the parameter c. That is, your first line of your function should look as follows:

```
function y=f(x,c)
```

In cell mode (or a script M-file), write a **for** loop that will plot this family of functions on the interval $[-2, 4]$ for C in $\{-1, -0.8, -0.6, \ldots 1\}$.

17. The sequence

$$1, 1, 2, 3, \ldots, a_{n-2}, a_{n-1}, a_n, \ldots$$

where $a_n = a_{n-2} + a_{n-1}$, is called a *Fibonacci* sequence. Write a function that will return the nth term of the Fibonacci sequence. The first line of your function should read as follows.

```
function term=fib(n)
```

Call this function from cell mode (or a script M-file) and format the output.

18. Adjust the function M-file created in **Exercise 17** so that it returns a vector containing the first n terms of the Fibonacci sequence. Call this function from cell mode (or a script M-file) and format the output.

19. Euclid proved that there were an infinite number of primes. He also believed that there was an infinite number ot *twin primes*, primes that differ by 2 (like 5 and 7 or 11 and 13), but no one has been able to prove this conjecture for the past 2300 years. Write a function that will produce all twin primes between two inputs, integers a and b.

20. In mathematics, a *sexy prime* is a pair of primes $(p, p+6)$ that differ by 6. Write a function that will produce all sexy primes between two inputs, integers a and b.

21. The factorial of nonnegative integer n is denoted by the mathematic notation $n!$ and is defined as follows.

$$n! = \begin{cases} 1, & \text{if } n = 0 \\ n(n-1)(n-2)\cdots 1, & \text{if } n \neq 0 \end{cases}$$

Write a function M-file **myfactorial** that returns the factorial when a nonnegative integer is received as input. Use cell mode (or a script M-file) to test your function at $n = 0$ and $n = 5$. *Hint: Consider using Matlab's* **prod** *function.*

22. The *binomial coefficient* is defined by

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

Write a function M-file called **binom** that takes two inputs, n and k, then returns the binomial coefficient. Use the **myfactorial** function written in **Exercise 21**. Use cell mode to test your function for $n = 5$ and $k = 2$.

23. The first few rows of *Pascal's Triangle* look like the following.

```
1
1     1
1     2     1
1     3     3     1
1     4     6     4     1
```

The nth row holds the binomial coefficients

$$\binom{n}{k}, \quad k = 0, 1, \ldots, n.$$

Write a function to produce Pascal's Triangle for an arbitrary number **r** of rows. Your first line should read as follows.

```
function pascalTriangle(r)
```

This function should use nested **for** loops to produce the rows of Pascal's Triangle. The function should also use the **fprintf** command to print each row to the screen. Use cell mode to test your function by entering the following line in your cell enabled editor.

```
pascalTriangle(8)
```

Your function should employ the **binom** function written in **Exercise 20**. *Hint: The format to use for the* **fprintf** *command is the tricky part of this exercise. Before entering the inner loop to compute the coefficients for a particular row, set* **format=''** *(the empty string), then build the string in the inner loop with* **format=[format,'%6d']**. *Once the inner loop completes, you'll need to add a line return character before executing* **fprintf(format,row)**.

24. Rewrite the **quadratic** function m-file of **Example 8** by replacing the **if..elseif..else** conditional with Matlab's **switch..case..otherwise** conditional.

25. Rewrite the **quadratic** function m-file of **Example 8** to produce two complex solutions of the quadratic equation $ax^2+bx+c=0$ in the case where the discriminant $D = b^2 - 4ac < 0$.

26. Write a function called **mysort** that will take a column vector of numbers as input, sort them in ascending order, then return the sorted list as a column vector. Test the speed of your routine with the following commands.

```
in=rand(10000,1);
>> tic, mysort(in); toc
```

Compare the speed of your sorting routine with Matlab's.

```
>> tic, sort(in); toc
```

Note: This is a classic problem in computer science, a ritual of passage, like crossing the equator for the first time. There are very sophisticated algorithms for attacking this problem, but try to see what you can do with it on your own instead of searching for a sort routine on the internet.

27. Write a function called **mymax** that will take a column vector of numbers as input and find the largest entry. The routine should spit out two numbers, the largest entry m and its position k in the vector. Test the speed of your routine with the following commands.

```
in=rand(1000000,1);
>> tic, [m,k]=mymax(in); toc
```

Compare the speed of your maximum routine with Matlab's.

```
>> tic, [m,k]=max(in); toc
```

28. Write a function called **mymin** that will take a column vector of numbers as input and find the smallest entry. The routine should spit out two numbers, the smallest entry m and its position k in the vector. Test the speed of your routine with the following commands.

```
in=rand(10000,1);
>> tic, [m,k]=mymin(in); toc
```

Compare the speed of your maximum routine with Matlab's.

```
>> tic, [m,k]=min(in); toc
```

29. According to *Wikipedia*, the solutions to the *cubic equation*

$$ax^3 + bx^2 + cx + d = 0$$

are found as follows. First, compute

$$q = \frac{3ac - b^2}{9a^2}$$

and

$$r = \frac{9abc - 27a^2d - 2b^3}{54a^3}.$$

Next, compute

$$s = \sqrt[3]{r + \sqrt{q^3 + r^2}}$$

and

$$t = \sqrt[3]{r - \sqrt{q^3 + r^2}}.$$

The solutions are, first,

$$x_1 = s + t - \frac{b}{3a},$$

then

$$x_2 = -\frac{s+t}{2} - \frac{b}{3a} + \frac{\sqrt{3}}{2}(s-t)i,$$

and

$$x_3 = -\frac{s+t}{2} - \frac{b}{3a} - \frac{\sqrt{3}}{2}(s-t)i,$$

where $i = \sqrt{-1}$. Write a function to compute the roots of the cubic equation. The first line should read as follows.

```
function [x1,x2,x3]=cardano(a,b,c,d)
```

Set up a test to verify that your implementation of the solution of the cubic equation is working correctly.

4.3 Answers

1.

```
>> f=@(x) x^2-3*x-4;
>> f(1)
ans =
    -6
```

3.

```
>> f=@(t) exp(-0.25*t)*(2*cos(t)-sin(t));
>> f(1)
ans =
    0.1862
```

5.

```
>> h=@(u,v) cos(u)*cos(v);
>> h(1,2)
ans =
   -0.2248
```

7. The function M-file:

```
function y=f(x)
y=x^2-3*x-4;
```

Calling the function from the command line.

```
>> f(1)
ans =
    -6
```

9. The function M-file:

```
function y=f(t)
y=exp(-0.25*t)*(2*cos(t)-sin(t));
```

Calling the function from the command line.

```
>> f(1)
ans =
    0.1862
```

11. The function M-file:

```
function z=h(u,v)
z=cos(u)*cos(v);
```

Calling the function from the command line.

```
>> h(1,2)
ans =
   -0.2248
```

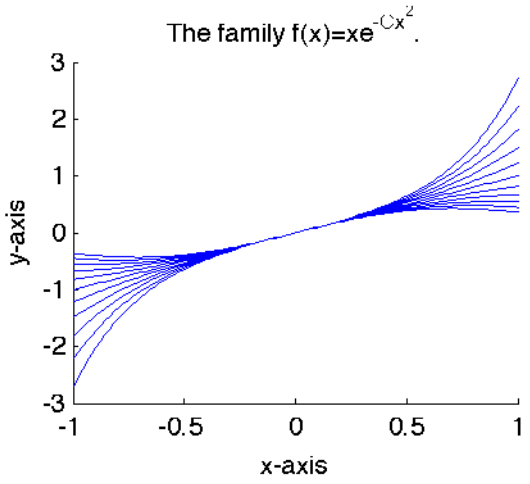
13. Set the domain.

```
x=linspace(-1,1);
```

Write the **for** loop.

```
for C=-1:0.2:1
    f=@(x) x.*exp(-C*x.^2);
    y=f(x);
    line(x,y)
end
```

Annotating the plot produces the following image.



15. Write the function M-file.

```
function y=f(x,c)
y=x.*exp(-c*x.^2);
```

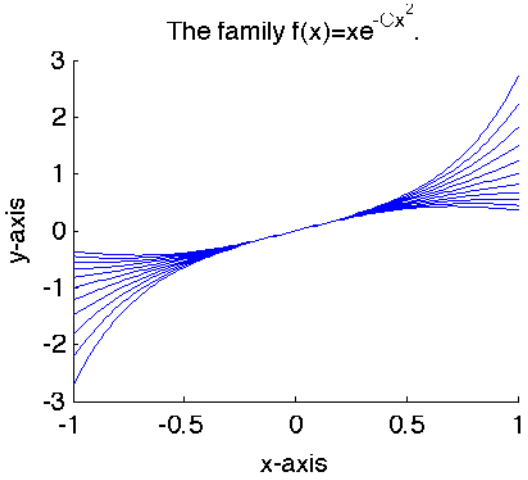
Set the domain.

```
x=linspace(-1,1);
```

Write the **for** loop.

```
for C=-1:0.2:1
    y=f(x,C);
    line(x,y)
end
```

Annotating the plot produces the following image.



17. Save the following as **fib.m**.

```
function c=fib(n)
if n==1
    term=1;
    return
elseif n==2
    term=1;
    return
else
    a=1;
    b=1;
    for k=3:n
        c=a+b;
        a=b;
        b=c;
    end
end
```

At the command line or in the editor with cell mode enabled, enter and execute the following as a test.

```
n=7;
nterm=fib(n);
fprintf('The n = %d term of the sequence is: %d\n', n,nterm)
```

The output is correct.

```
The n = 7 term of the sequence is: 13
```

Readers should check the behavior for other values of n.

19. Save these lines as **twinprimes.m**. The || is Matlab's "short-circuit or." If the first the first part of the "or" expression evaluates as true, then the interpreter doesn't bother with the second part of the "or" expression, as it is already true. Note that we've also included an error message, which when executed, displays to the command window and stops program execution.

```
function twins=twinprimes(a,b)
if round(a)~=a || round(b)~=b
    error('The inputs a and b must be integers.')
end
candidates=a:b;
twins=[];
candidates=candidates(isprime(candidates));
len=length(candidates);
for k=1:(len-1)
    if candidates(k+1)-candidates(k)==2
        twins=[twins;candidates(k:k+1)];
    end
end
```

Test the function.

```
>> twins=twinprimes(1,100)
twins =
     3     5
     5     7
    11    13
    17    19
    29    31
    41    43
    59    61
    71    73
```

Looks good. We leave to the reader to check that we have all of the twin primes between 1 and 100.

21. Save the following lines as **myfactorial.m**. Again we're error checking. If n is not an integer of if it is not the case that n is nonnegative, we send an error message to the command window and halt program execution. The && is Matlab's "short-circuit and." If the first part of the and statement evaluates as false, the interpreter doesn't bother with the second part of the "and" statement, saving some time.

```
function fact=myfactorial(n)
if round(n)~=n or ~(n>=0)
    error('n must be a nonnegative integer')
end
if n==0
    fact=1;
else
    fact=prod(1:n);
end
```

Checking.

```
>> fact=myfactorial(0)
fact =
     1
>> fact=myfactorial(5)
fact =
   120
```

Looks correct.

23. Enter the following lines and save a **pascalTriangle.m**.

```
function pascalTriangle(rows)
for n=0:rows
    row=[];
    format='';
    for k=0:n
        row=[row,binom(n,k)];
        format=[format,'%6d'];
    end
    format=[format,'\n'];
    fprintf(format,row)
end
```

This function will only work if you have written a successful binomial coefficient function **binom.m** (see **Exercise 22**), either with a persnalized **myfactorial.m** (see **Exercise 21**) or Matlab's built in **factorial.m**. Execute the following at the command prompt.

```
>> pascalTriangle(5)
     1
     1     1
     1     2     1
     1     3     3     1
     1     4     6     4     1
     1     5    10    10     5     1
```

27. Enter the following code in the editor and save as **mymax.m**.

```
function [m,k]=mymax(v)
m=v(1);
k=1;
for i=1:length(v)
    if v(i)>m
        m=v(i);
        k=i;
    end
end
```

Test it on something where you can easily spot the maximum, as is the case in the 5th position of the following vector.

```
>> v=[2 4 7 8 11 8 4 2 5]
v =
     2     4     7     8    11     8     4     2     5
>> [m,i]=mymax(v)
m =
    11
i =
     5
```

Now for a comparison. First, **mymax**.

```
>> tic, [m,i]=mymax(v); toc
Elapsed time is 0.107614 seconds.
```

Now using Matlab's **max** routine.

```
>> tic, [m,i]=max(v); toc
Elapsed time is 0.040215 seconds.
```

4.4 Variable Scope in Matlab

We now know the general structure and use of a function. In this section we'll look closely at the *scope of a variable*, which is loosely defined as the range of functions that have access to the variable in order to set, acquire, or modify its value.

The Base Workspace

Matlab stores variables in parts of memory called workspaces. The *base workspace* is the part of memory that is used when you are entering commands at the Matlab prompt in the command window. You can clear all variables from your base workspace by entering the command **clear**[10] at the Matlab prompt in the command window.

```
>> clear
```

The command **whos** will list all variables in the current workspace. When entered at the prompt in the Matlab command window, the usual behavior is to list all variables in the *base workspace*.

```
>> whos
```

Because we've cleared all variables from the base workspace, there is no output from the whos command.

Create a new variable in the base workspace by entering the following command at the Matlab prompt in the command window.

```
>> M=magic(3);
```

Now when we enter the **whos** command, this new variable is listed as present in the base workspace.

---

[9] Copyrighted material. See: http://msenux.redwoods.edu/Math4Textbook/

[10] The **clear** command is much more extensive than described in this narrative. For example, **clear functions** will clear all compiled M-functions in memory and the command **clear global** will clear all global variables. For a complete description of the **clear** command and its capabilities, type **help clear** at the Matlab command prompt.

```
>> whos
  Name      Size                    Bytes  Class

  M         3x3                        72  double array
```

Scripts and the Base Workspace

When you execute a script file from the Matlab prompt in the command window, the script uses the base workspace. It can access any variables that are present in the base workspace, and any variables created by the script remain in the base workspace when the script finishes. As an example, open the Matlab editor and enter the following lines.

```
x=1:5
s=sum(x)
p=prod(x)
```

Save the script as **sumprod.m** and return to the Matlab command window. Execute the script by typing its name at the command prompt. The script finds the sum and product of the integers from 1 to 5.

```
>> sumprod
x =
     1     2     3     4     5
s =
    15
p =
   120
```

Examine the current state of the base workspace by typing the **whos** command at the command prompt.

```
>> whos
  Name      Size                    Bytes  Class

  M         3x3                        72  double array
  p         1x1                         8  double array
  s         1x1                         8  double array
  x         1x5                        40  double array
```

Note that the variables x, s, and p, created in the script, remain present in the base workspace when the script completes execution.

If a variable exists in the base workspace, then a script executed in the base workspace has access to that variable. As an example, enter the following lines in the Matlab editor and save the script as **usebase.m**.

```
fprintf('Changing the base variable p.\n')
p=1000;
fprintf('The base variable p is now %d.\n',p)
```

Execute the script.

```
>> usebase
Changing the base variable p.
The base variable p is now 1000.
```

Now, at the Matlab prompt, examine the contents of the variable p in the base workspace.

```
>> p
p =
        1000
```

Note that the script had both access to and the ability to change the base workspace variable p.

Function Workspaces

Functions do not use the base workspace. Each function has its own workspace where it stores its variables. This workspace is kept separate from the base workspace and all other workspaces to protect the integrity of the data used by the function.

As an example, let's start by clearing the base workspace.

```
>> clear
>> whos
```

Note that the base workspace is now empty.

Next, open the Matlab editor and enter the following lines. Save the function as **localvar.m**. Note that the function expects no input, nor does it return any output to the caller.

```
function localvar
p=12;
fprintf('In the function localvar, the value of p is: %d\n',p)
```

Execute the function by typing its name at the command prompt.

```
>> localvar
In the function localvar, the value of p is: 12
```

Now, examine the base workspace with the **whos** command.

```
>> whos
```

No variable p! There is no variable p in the base workspace because the variable p was created in the *function workspace*. Variables that are created in function workspace exist only until the function completes execution. Once the function completes execution, the variables in the function workspace are gone.

The function workspace is completely separate from the base workspace. As an example, set a value for p in the base workspace by entering the following command at the command prompt.

```
>> p=24
p =
    24
```

Examine the base workspace.

```
>> whos
  Name      Size                    Bytes  Class

  p         1x1                         8  double array
```

Now, execute **localvar** again.

```
>> localvar
In the function localvar, the value of p is: 12
```

At the command prompt, examine the contents of the variable p in the base workspace.

```
>> p
p =
    24
```

Still 24! This is clear evidence that the base workspace and the function workspace are separate creatures. While the function is executing, it uses the value of p in its *function workspace* and prints a message that p is 12. However, the value of p in the function workspace is completely separate from the value of p in the base workspace. Once the function completes execution, the value of p in the function workspace no longer exists. Furthermore, note that when the function sets **p=12** in its function workspace, this has absolutely no effect on the value of p in the base workspace.

Passing by Value. Most of today's modern computer languages allow the user at least two distinct ways to pass a variable as an argument to a function or subroutine: *by reference* or *by value*. Passing by reference means that the existing memory address for the variable is passed to the function or subroutine. Because it now has direct access to the memory location of the variable, the function or

subroutine is free to make changes to the value of the variable in that particular memory location.

On the other hand, Matlab passes variables **by value** to function arguments. This means that the function that receives this value has no idea where the variable is actually located in memory and is unable to make changes to the variable in the caller's workspace. In MatlabSpeak, the variable that is passed to the function is *local* to the function, it exists in the function's workspace. If the function makes an attempt to set or modify this variable, it does so to the variable in its workspace, not in the caller's workspace.

For eample, make the following adjustments to the function **localvar** and save as **localvar.m**.

```
function localvar(p)
fprintf('Function localvar received this value of p: %d\n',p)
p=12;
fprintf('Function localvar changed the value of p to: %d\n',p)
```

Examine the base workspace variable p. If needed, reset this variable with the command **p=24**.

```
>> p
p =
    24
```

Execute the function **localvar** by passing it the variable p from the base workspace.

```
>> localvar(p)
Function localvar received this value of p: 24
Function localvar changed the value of p to: 12
```

Now, examine the variable p in the base workspace.

```
>> p
p =
    24
```

Still 24! Again, the base workspace and function workspace are separate animals! The **value** of the base workspace variable p is passed to the function workspace of **localvar** with the command **localvar(p)**. The value of the variable p is changed in the function workspace, but because only the **value** of the workspace variable was passed to **localvar**, the value of the base workspace variable p remains unchanged.

Separate Workspaces. The idea of separate workspaces is Matlab's strategy to protect users from unintentionally changing the value of a variable in a workspace.

Forcing a Change of Variable in the Caller's Workspace. It's possible to have a function change the value of a variable in the caller's workspace, but you have to go to some lengths to do so. For example, adjust the function **localvar** so that it returns the value of p as output.

```
function x=localvar(p)
fprintf('Function localvar received this value of p: %d\n',p)
p=12;
fprintf('Function localvar changed the value of p to: %d\n',p)
x=p;
```

Note that the value of p is changed in the workspace of function **localvar** and returned to the caller via the output variable x. Execute the function at the command prompt as follows.

```
>> p=localvar(p);
Function localvar received this value of p: 24
Function localvar changed the value of p to: 12
```

Now, examine the contents of the variable p in the base workspace.

```
>> p
p =
    12
```

We've finally succeeded in changing the value of p in the base workspace, but note that we had to work hard to do so. This time we changed the value of p in the

function **localvar**, then returned it in the output variable. Because we assigned the output of the function **localvar** to the base workspace variable p with the command **p=localvar(p)**, the value of p in the base workspace changed. Note that the command **localvar(p)** would have no effect on the workspace variable p. It's the fact that we assign the output that causes the change to the base workspace variable p.

Global Variables

One way to allow a function access to variables in a caller's workspace is to declare them as *global variables*. You must do this in the caller's workspace and in the function that wants access to these variables. As an example, open the editor, enter the following lines, and save the function M-file as **localvar.m**.

```
function localvar
global P
fprintf('In localvar, the value of P is: %d\n', P)
```

Note that this adaption of **localvar** receives no input nor does it output anything. However, it does declare P as a global variable. Consequently, if the caller does the same in its workspace, the function **localvar** should be able to access the global variable P.

In the command window, we'll assign a value to P. Global variables should first be declared before they are assigned a value, after which we assign P a value of 50.

```
>> global P
>> P=50
P =
    50
```

Execute the function **localvar** by entering the command **localvar** at the Matlab prompt in the command window.

```
>> localvar
In localvar, the value of P is: 50
```

This is a clear indication that the variable P is shared by the base workspace and the workspace of the function **localvar**.

Global Variables.

- Global variables names are usually longer and more descriptive than local variables. Although not required, most Matlab programmers will use all uppercase letters in naming global variables, such as **WIDTH** or **HEIGHT**.
- There is a certain amount of risk involved when using global variables and it is recommended that you use them sparingly. For example, declaring a variable global in a function might override the use of a global variable of the same name in another function. This error can be very hard to track down. Another difficulty arises when the programmer wishes to change the name of a global variable and has to track down and change its implementation in a number of functions.

In a later section, we will discuss the use of *Nested Functions* in Matlab, which allows the nested functions to see any variables declared in the function in which they find themselves nested. In many cases, the use of nested functions is far superior to employing numerous global variables.

Persistent Variables

We probably will have little use for *persistent* variables, but we'll mention them for the sake of completeness. Sometimes you might want a local variable defined in a function to retain its value between repeated calls to the function.

As an example, open the editor, enter the following lines, then save the function M-file as **addTerm.m**.

```
function addTerm(term)
persistent SUM_T
if isempty(SUM_T)
    SUM_T=0;
end
SUM_T=SUM_T+term
```

Some comments are in order.

1. You can declare and use persistent variables in a function M-file only.
2. Only the function in which the persistent variable is declared is allowed access to the variable.
3. When the function exits, Matlab does not clear the persistent variable from memory, so it retains its value from one function call to the next.

When the persistent variable is first declared, it is set to the empty matrix. On the first call of the function, we need a different reaction than on ensuing function calls. If the variable **SUM_T** is empty, we initialize it to zero. On the next call of the function, the variable **SUM_T** is no longer empty, so it is updated by incrementing **SUM_T** by **term**.

To get a sense of how this function will perform, we intentionally left off a suppressing semicolon in the command **SUM_T=SUM_T+term** so that we can track progress of the persistent variable between repeated function calls. Enter the following command at the Matlab prompt in the command window.

```
>> for k=1:5,addTerm(1/k),end
SUM_T =
     1
SUM_T =
    1.5000
SUM_T =
    1.8333
SUM_T =
    2.0833
SUM_T =
    2.2833
```

Although the above should clearly demonstrate that **SUM_T** retains its value between successive function calls, another run of the loop should cement the idea of a persistent variable firmly in our minds.

```
>> for k=1:5,addTerm(1/k),end
SUM_T =
    3.2833
SUM_T =
    3.7833
SUM_T =
    4.1167
SUM_T =
    4.3667
SUM_T =
    4.5667
```

Readers should carefully check (with calculator in hand) that these values are correct.

To clear a persistent variable from memory, you can use either clear the function from memory **clear addTerm** or you can use the command **clear all**.

4.4 Exercises

1. Clear the workspace by entering the command **clear all** at the Matlab prompt in the command window. Type **whos** to examine the base workspace and insure that it is empty. Open the editor, enter the following lines, then save the file as **VariableScope.m**. Execute the cell in cell-enabled mode.

```
%% Exercise #1
clc
P=1000;
r=0.06;
t=10;
I=P*r*t;
fprintf('Simple interest gained is %.2f.\n',I)
```

Examine the base workspace by entering **whos** at the command window prompt. Explain the output.

2. Clear the workspace by entering the command **clear all** at the Matlab prompt in the command window. Type **whos** to examine the base workspace and insure that it is empty. At the command prompt, enter and execute the following command.

```
>> P=1000; r=0.06; t=10;
```

Examine the workspace:

```
>> whos
  Name      Size                    Bytes  Class

  P         1x1                         8  double array
  r         1x1                         8  double array
  t         1x1                         8  double array
```

Add the following cell to the file **VariableScope.m**.

```
%% Exercise #2
clc
I=P*r*t;
fprintf('Simple interest gained is %.2f.\n',I)
```

State the output of the script. Use **whos** to examine the base workspace variables. Anything new? Explain.

3. Clear the workspace by entering the command **clear all** at the Matlab prompt in the command window. Type **whos** to examine the base workspace and insure that it is empty. At the command prompt, enter and execute the following command.

```
>> P=1000; r=0.06; t=10;
```

Examine the workspace:

```
>> whos
  Name      Size                    Bytes  Class

  P         1x1                         8  double array
  r         1x1                         8  double array
  t         1x1                         8  double array
```

Open the editor, enter the following lines, and save the function M-file as **simpleInterest1.m**.

```
function I=simpleInterest1
I=P*r*t;
fprintf('Simple interest gained is %.2f.\n',I)
```

Add the following cell to **VariableScope.m**.

```
%% Exercise #3
clc
I=simpleInterest1;
fprintf('Simple interest gained is %.2f.\n',I)
```

Execute the cell and then state what happens and why.

4. Let's try an approach different to the attempt in **Exercise 3**. Open the editor, enter the following lines, and save the file as **simpleInterest2.m**.

```
function I=simpleInterest2
global P R T
I=P*R*T;
```

Add the following cell to **VariableScope.m**.

```
%% Exercise #4
clc
clear all
global P R T
P=1000; R=0.06; T=10;
I=simpleInterest2;
fprintf('Simple interest gained is %.2f.\n',I)
```

Execute the cell and state the resulting output. Explain the difference between the result in **Exercise 3** and the current exercise.

5. Enter the following lines in the editor and save the file as **clearFunction**.

```
function clearFunction
clear all
```

Add the following cell to **VariableScope.m**.

```
%% Exercise #5
clc
clear all
P=1000; R=0.06; T=10;
clearFunction;
whos
```

Execute the cell and explain the resulting output. Why do the variables P, R, and T still remain in the base workspace? Didn't we clear them with **clearFunction**?

6. Enter the following lines in the editor and save the file as **simpleInterest3.m**.

```
function I=simpleInterest3
P=5000;
R=0.10;
T=8;
I=P*R*T
```

Add the following cell to **VariableScope.m**.

```
%% Exercise #6
clc
clear all
P=1000; R=0.06; T=10;
dbstop simpleInterest3 5
simpleInterest3
whos
```

The command **dbstop simpleInterest3 5** puts a *breakpoint* at line 5 of the function **simpleInterest3**. When the script executes the function on the next line, the Matlab Debugger halts execution at line 5 of the function **simpleInterest3** and displays the debug prompt and the current line.

```
5   I=P*R*T
K>>
```

To see where you are, enter the following command at the debug prompt.

```
K>> dbstack
> In simpleInterest3 at 5
```

This tells us we are in **simpleInterest3** at line 5. Note that the variables P, R, and T are declared in lines previous to line 5. Examine their contents in the workspace of function **simpleInterest3**.

```
K>> P, R, T
P =
        5000
R =
    0.1000
T =
     8
```

Notice that we are in the function workspace, not the base workspace. Now end the debugging session and allow the function to complete its execution with the following command.

```
K>> dbcont
```

When execution of **simpleInterst3** completes, we are returned to the caller, in this case the base workspace. Examine the contents of P, R, and T in the base workspace.

```
>> P, R, T
P =
        1000
R =
    0.0600
T =
    10
```

This is clear evidence that the base workspace and the function workspace are separate entities.

4.4 Answers

1. Script output:

```
Simple interest gained is 600.00.
```

Worspace variables:

```
>> whos
  Name      Size                    Bytes  Class

  I         1x1                         8  double array
  P         1x1                         8  double array
  r         1x1                         8  double array
  t         1x1                         8  double array
```

3. We obtain an error:

```
??? Undefined function or variable 'P'.

Error in ==> simpleInterest1 at 2
I=P*r*t;
```

This happens because the variable P is in the base workspace and the function **simpleInterest1** executes in its own workspace where there exists no instance of the variable P. In short, functions cannot see the variables in the base workspace unless you pass them to the function as arguments or make them global in the base workspace and function.

5. No. The **clear all** in **clearFunction** clears the function workspace, not the base workspace where the cell enable script is operating.

4.5 Subfunctions in Matlab

Up to this point, our programs have been fairly simple, usually taking on a single task, such as solving a quadratic equation. As programs become more complex and perform multiple tasks, modern programmers break programs into a series of self contained modules. In this manner, various parts of the program can be written separately, tested, and once stable, can be integrated into the program as a whole.

In older programming languages, these modules were known as subroutines. In modern, object oriented languages such as C++ and Java, the modules are objects, created from classes, that contain their own variables and methods for manipulating data and performing tasks. These objects can then be reused and assembled to create sophisticated programs.

To attack a complex programming task in Matlab, we must first break the task up into manageable units. Each of these units or modules are then coded as **functions** and tested until we are certain they perform the simple task for which they are designed. Once satisfied that the individual functions are performing as designed, we must then integrate them into a complete and functioning program.

In this section we will discuss two ways that we can accomplish this modular design.

1. We can write each of the modules as functions in separate files. We then write a master script file which calls each of the function M-files as needed.
2. We can write a primary function in place of the script file. We can then incorporate the modules as *subfunctions* directly in this primary function file. That is, we write one self contained file.

Actually there is also a third way to proceed. We can write a primary file in place of a script file. However, by signalling the end of this primary function with the Matlab keyword **end**, we signal Matlab that out subfunctions (each of which must also use the keyword **end**) are *nested functions*. A primary function written in this manner emulates some of the behavior of an object in an object oriented language, with its own variables and methods (subfunctions). We'll see in a later section that the rules for variable scope will change significantly when using nested functions.

We will investigate each of these options in some detail. In the next section, we investigate the first of these options, calling a function M-file from within the body of a script M-file.

[12] Copyrighted material. See: http://msenux.redwoods.edu/Math4Textbook/

Calling Functions From Script Files

We are going to tackle a programmng project that will perform rational arithmetic for the user of the program. Options will include reducing, adding, subtracting, multiplying, and dividing fractions input by the user of our program.

Contemplating the design of such a program, here are some possible smaller tasks that need implmentation.

1. Present the user with a menu of choices: reduction, addition, subtraction, multiplication, and division.
2. Routines will need to be written for each possible user choice on the menu.
3. The program will need to query the user for a menu choice, then react in the appropriate manner.

Note that this design is preliminary. In the process of writing a complex program, difficulties with the original design will surely arise. When that occurs, we will need to make modifications of the original design, which in turn will create the need for design of new modules not considered in the first design.

However, one clear place that we can start is on the design of a menu of choices for the user of our program. Remember, the menu should query what action the user wants to perform, reduction, addition, subtraction, or division. With this thought in mind, we write the following lines and save the script as **rationalArithmetic.m**.

```
clc
fprintf('Rational Arithmetic Menu:\n\n')
fprintf('1). Reduce a fraction to lowest terms.\n')
fprintf('2). Add two fractions.\n')
fprintf('3). Subtract two fractions.\n')
fprintf('4). Multiply two fractions.\n')
fprintf('5). Divide two fractions.\n')
fprintf('6). Exit program.\n\n')
n=input('Enter number of your choice: ');
```

Go to the Matlab prompt in the command window and execute the script by typing the following.

```
>> rationalArithmetic
```

The script clears the command window, puts up a menu of choices, then queries the user for a response. The script accepts the user response from the keyboard and saves it in the variable n. Because we haven't programmed it to do anything else yet, the script takes no further action and simply exits.

```
Rational Arithmetic Menu:

1). Reduce a fraction to lowest terms.
2). Add two fractions.
3). Subtract two fractions.
4). Multiply two fractions.
5). Divide two fractions.
6). Exit program.

Enter number of your choice: 1
```

Now, let's think modularly. Putting up a menu is a task that will be repeated over and over as the user employs our program to perform rational arithmetic. So, instead of placing the code in the script, let's put it into a function. Enter the following lines in the editor and save the file as **rationalMenu**.

```
function n=rationalMenu
clc
fprintf('Rational Arithmetic Menu:\n\n')
fprintf('1). Reduce a fraction to lowest terms.\n')
fprintf('2). Add two fractions.\n')
fprintf('3). Subtract two fractions.\n')
fprintf('4). Multiply two fractions.\n')
fprintf('5). Divide two fractions.\n')
fprintf('6). Exit program.\n\n')
n=input('Enter number of your choice: ');
```

Test the function **rationalMenu** to make sure that it functions properly. Enter the following command at the Matlab prompt in the command window.

```
n=rationalMenu
```

The response feels the same. A menu is presented and the user responds by entering a number to indicate their menu choice.

```
Rational Arithmetic Menu:

1). Reduce a fraction to lowest terms.
2). Add two fractions.
3). Subtract two fractions.
4). Multiply two fractions.
5). Divide two fractions.
6). Exit program.

Enter number of your choice: 3
n =
     3
```

Note that Matlab stores the user response in the variable n in the command window workspace (base workspace).

At this point, users can replace all current lines in the Matlab script file **rationalArithmetic** with a single line, namely **n=rationalMenu**. Running the script file **rationalArithmetic** will call and execute the function **rationalMenu** with the expected result.

However, we'd like to add a bit more functionality to the script. Specifically, we'd like to add a loop that continually replays the menu until the user makes the choice: 6) Exit program, whereupon, the script should exit. Open the script M-file **rationalArithmetic** in the editor and replace all existing lines with the lines that follow. Resave the script as **rationalArithmetic**.

```
finished=false;
while (~finished)
    n=rationalMenu;
    switch n
        case 1
        case 2
        case 3
        case 4
        case 5
        case 6
            finished=true;
    end
end
```

Run the script at the Matlab prompt in the command window.

```
>> rationalArithmetic
```

In response to the prompt, try entering choices 1 through 5. Note that in each case, the menu refreshes and you are prompted for another choice. Selecting choice #6 exits the script and returns you to the Matlab command prompt. When the user makes the choice to exit the program, the variable **finished** is assgined the value **true** and the **while** loop terminates.

Hopefully, readers can now ascertain where we're heading with this structure. We code specific individual tasks as functions and save them as function M-files in the same directory containing the script **rationalArithmetic**. We then can access these functions by calling them from the script. Once the program is fully implemented, we'll have a number of function M-files collected in a directory that are driven from a script residing in the same directory.

We will pursue this design and structure no further (functions and a master script file), as we will now consider how we can implement a similar design with subfunctions.

Subfunctions

To use subfunctions to accomplish our programming task, we make a simple but subtle change to our structure. We take the script file and change it into a function by adjusting the first line so that it contains the keyword **function**.

```
function rationalArithmetic
finished=false;
while (~finished)
    n=rationalMenu;
    switch n
        case 1
        case 2
        case 3
        case 4
        case 5
        case 6
            finished=true;
    end
end
```

Execute this function by typing the following at the Matlab command prompt.

```
rationalArithmetic
```

You should note no change in the behavior of the program.

So, where's the advantage? The advantage lies in the fact that instead of creating *external* function M-files that are called from with a script, we can actually include the code for the **rationalMenu** function as a *subfunction* inside the primary function. Open the editor, enter the following lines, and save the file as **rationalArithmetic.m**.

```
function rationalArithmetic
finished=false;
while (~finished)
    n=rationalMenu;
    switch n
        case 1
        case 2
        case 3
        case 4
        case 5
        case 6
            finished=true;
    end
end

function n=rationalMenu
clc
fprintf('Rational Arithmetic Menu:\n\n')
fprintf('1). Reduce a fraction to lowest terms.\n')
fprintf('2). Add two fractions.\n')
fprintf('3). Subtract two fractions.\n')
fprintf('4). Multiply two fractions.\n')
fprintf('5). Divide two fractions.\n')
fprintf('6). Exit program.\n\n')
n=input('Enter number of your choice: ');
```

Readers should note no difference in performance when they enter the command **ratinalArithmetic** at the Matlab prompt. The savings here is the fact that when

we replace the driving script file with a function M-file, this primary function can contain as many subfunctions as needed to complete the programming task. In this way, the program is *self contained*. One file contains everything needed by the program to perform the task for which it was designed.

Adding Functionality to our Program

In this section, we will write subfunctions that will reduce a fraction to lowest terms. This corresponds to the first choice on the menu of choices produced by the subfunction **rationalMenu**.

How to proceed? We reduce a fraction to lowest terms in two steps. First, we find a greatest common divisor for both numerator and denominator, then divide both numerator and denominator by this greatest common denominator.

For example, to reduce the rational number 336/60 to lowest terms, we proceed as follows.

1. Find the greatest common divisior of 336 and 60, which is $\gcd(336, 60) = 12$.
2. Divide numerator and denominator by the greatest common divisor.

$$\frac{336}{60} = \frac{336 \div 12}{60 \div 12} = \frac{28}{5}$$

Two tasks call for two subfunctions, one to find the gcd and a second to do the actual reduction.

The Euclidean Algorithm. Early in our mathematical education we learn that if you take an nonegative integer a and divide it by a positive integer b, there is an integer quotient q and remainder r such that

$$a = bq + r, \qquad 0 \leq r < b.$$

We can use this *Euclidean Algorithm* to find the greatest common divisor of 336 and 60. We proceed as follows.

1. When 376 is divided by 60, the quotient is 5 and the remainder is 36. That is,

$$336 = 5 \cdot 60 + 36.$$

2. When 60 is divided by 36, the quotient is 1 and the remainder is 24. That is,

$$60 = 1 \cdot 36 + 24.$$

3. When 36 is divided by 24, the quotient is 1 and the remainder is 12. That is,

$$36 = 1 \cdot 24 + 12.$$

4. When 24 is divided by 12, the quotient is 2 and the remainder is 0. That is,

$$24 = 2 \cdot 12 + 0.$$

The last nonzero remainder is the greatest common divisor. That is,

$$\gcd(336, 60) = 12.$$

Let's write a Matlab function that will implement the Euclidean Algorithm. Open the Matlab editor and enter the following lines. Save the file as **rationalGCD.m**.

```
function d=rationalGCD(a,b)
while true
    a=mod(a,b);
    if a==0
        d=b;
        return
    end
    b=mod(b,a);
    if b==0
        d=a;
        return
    end
end
```

We must test this function. Enter the following at the Matlab prompt.

```
>> d=rationalGCD(336,60)
d =
    12
```

Note that this is the correct response, matching our hand calculations above.

Our code warrants some comments.

1. Recall that the command **a=mod(a,b)** stores in a the remainder when a is divided by b. If the remainder a is zero, as we saw above, then b is the GCD, which we assign to the output variable d, then issue a **return**. The **return** command exits the function and returns control to the caller, in this case the command window.
2. Because a now contains the remainder from the first division, **b=mod(b,a)** stores in b the remainder when b is divided by a. This is equivalent to the

second step of our hand calculation above. If the remainder b is zero, then a is the GCD, which we assign to the output variable d, then issue a **return**.

3. Iterate until done.

Now that we have a function that will find the greatest common divisor, let's craft a function that will reduce a fraction to lowest terms.

Reducing to Lowest Terms. Our function will take as input the numerator and denominator of the rational number. It will then call **rationalGCD** to find the greatest common divisor of the numerator and denominator. It will then divide both numerator and denominator by the greatest common divisor and output the results. Open the Matlab editor, enter the following lines, then save the file as **rationalReduce.m**.

```
function [num,den]=rationalReduce(num,den)
d=rationalGCD(num,den);
num=num/d;
den=den/d;
```

Let's test this function on the fraction 336/60, which we know reduces to 28/5.

```
>> [num,den]=rationalReduce(336,60)
num =
    28
den =
     5
```

This is the correct response. The calling script of function will need to take responsibility for formatting the output.

Adding Subfunctions to Primary Function. We've tested **rationalGCD** and **rationalReduce** and found them to produce the correct results. We will now add each of these functions as subfunctions to our existing primary function **rationalArithmetic**. Open the file **rationalArithmetic** and add the following lines at the very bottom of the file **rationalArithmetic.m**.

```
function [num,den]=rationalReduce(num,den)
d=rationalGCD(num,den);
num=num/d;
den=den/d;
```

Next, add the following lines at the very bottom of the file **rationalArithmetic.m**.

```
function d=rationalGCD(a,b)
while true
    a=mod(a,b);
    if a==0
        d=b;
        return
    end
    b=mod(b,a);
    if b==0
        d=a;
        return
    end
end
```

Actually, the order of the subfunctions is irrelevant, but collecting them at the bottom of the primary function M-file **rationalArithmetic.m** is a good practice, making them easy to find and separating them from the main body of code. However, the order in which they appear at the bottom of the primary function file **rationalArithmetic.m** makes no difference to execution.

Activating Menu Items. We now need to write some code so that when the user selects the first menu item (Reduce a fraction to lowest terms), two things will happen:

1. The user will be asked to input the numerator and denominator of the fraction she wants reduced.
2. The program will output an equivalent fraction that is reduced to lowest terms.

With these thoughts in mind, add these lines to **rationalArithmetic.m** just after **case 1**.

```
clc
fprintf('Reduce a rational number to lowest terms.\n\n')
num=input('Enter the numerator: ');
den=input('Enter the denominator: ');
fprintf('You''ve entered the fraction %d/%d.\n\n',num,den)
[num,den]=rationalReduce(num,den);
fprintf('When reduced to lowest terms: %d/%d\n\n',num,den)
fprintf('Press any key to continue.\n')
pause
```

Resave the file as **rationalArithmetic.m**. Test the result by entering the command **rationalArithmetic** at the Matlab prompt. The following menu appears.

```
Rational Arithmetic Menu:

1). Reduce a fraction to lowest terms.
2). Add two fractions.
3). Subtract two fractions.
4). Multiply two fractions.
5). Divide two fractions.
6). Exit program.

Enter number of your choice:
```

Enter 1 as your choice and hit the `Enter` key. Enter the following responses to the prompts.

```
You have chosen to reduce a rational number to lowest terms.

Enter the numerator: 336
Enter the denominator: 60
You've entered the fraction 336/60.

When reduced to lowest terms: 28/5

Press any key to continue.
```

Pressing any key returns the user to the main menu where she can make another choice. Selecting 6). Exit the program will exit the program.

A few comments are in order.

1. The **fprintf** commands are self explanatory.
2. The heart of this code is the call to the **rationalReduce** subfunction, which takes the user's input for the numerator and denominator, then returns the reduced form of the numerator and denominator. The ensuing **fprintf** commands format the result.
3. The **pause** command is new, but simply explained. It halts program execution until the user strikes a key.

It should now be a simple matter to complete menu items #4 and #5. You need only multiply two rational numbers as follows,

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d},$$

then reduce the result. For the division, you need only invert and multiply,

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c},$$

then reduce the result.

To accomplish these tasks in code requires two subfunctions, **rationalMultiply** and **rationalDivide**. You will also have to write code that requests input, performs the requested operation, then formats the output after **case 4** and **case 5**.

Addition and subtraction are a bit trickier.

Finding a Least Common Denominator. To add and subtract fractions, the program will need to find a least common denominator (LCD). For example, to add 5/12 and 3/8, we make equivalent fractions with a commond denominator, then add.

$$\frac{5}{12} + \frac{3}{8} = \frac{10}{24} + \frac{9}{24} = \frac{19}{24}$$

Additionally, you must insure that the answer is reduced to lowest terms.

So, how do we write a subfunction that will produce a least common denominator? The following fact from number theory reveals the result.

$$\text{lcd}(a, b) = \frac{ab}{\text{gcd}(a, b)}$$

Thus,

$$\text{lcd}(8, 12) = \frac{8 \cdot 12}{\text{gcd}(8, 12)} = \frac{96}{4} = 24.$$

Thus, the code for the least common denominator is clear. Open the editor and enter the following lines. Save the file as **rationalLCD.m**.

```
function m=rationalLCD(a,b)
d=rationalGCD(a,b);
m=(a*b)/d;
```

Test the result at the command line.

```
>> m=rationalLCD(8,12)
m =
    24
```

This is the correct response. Now copy the following lines and enter them at the bottom of the primary function **rationalArithmetic.m** as follows.

```
function m=rationalLCD(a,b)
d=ratGCD(a,b);
m=(a*b)/d;
```

Completing rationalArithmetic.m

We'll leave it to our readers to complete the remaining tasks in the primary function **rationalArithmetic**. Here is what remains to be done.

1. You'll need to write subfunctions **rationalAdd** and **rationalSubtract**. Each should take two fractions as input, find an LCD, make equivalent fractions, add (or subtract), reduce, then send the result back to the caller.
2. Write code that requests input, performs the expected operation, then formats the output after **case 2** and **case 3**.

Try to proceed as we have in the foregoing narrative. First write the standalone function **rationalAdd**. You might consider using the following first line.

```
function [num,den]=rationalAdd(num1,den1,num2,den2)
```

Note that the input calls for two fractions: **num1** and **den1** are the numerator and denominator of the first fraction input by the user, **num2** and **den2** are

the numerator and denominator of the second fraction input by the user. You will want to make calls to **rationalLCD** and **rationalReduce** in the standalone function **rationalAdd**. Once you have the function **rationalAdd** tested and working, paste it as a subfunction at the bottom of the file **rationalArithmetic**.

This is an imporant idea. Write the code for the smaller task. Test it thoroughly. When you gain confidence in the result, then drop it in the larger program and coordinate it with the existing code.

4.5 Exercises

1. Complete the program **rationalArithmetic**. Implement four subfunctions, **rationalAdd**, **rationalSubtract**, **rationalMultiply** and **rationalDivide**, then add code to the corresponding **case** structure that will prompt the user for input, call the appropriate subfunction, then format the output. Test your function thoroughly.

2. Although Matlab handles the arithmetic of complex numbers quite nicely, in this exercise you will write your own function to perform complex arithmetic. First, a bit of review. Recall that $i = \sqrt{-1}$ and $i^2 = -1$. With these facts, coupled with the knowledge that the usual laws of arithmetic apply equally well to the complex numbers (commutative, associative, distributive, etc.), it is not difficulty to show each of the following results.

i.) $(a+bi)+(c+di) = (a+c)+(b+d)i$

ii.) $(a+bi)-(c+di) = (a+c)-(b+d)i$

iii.) $(a+bi)(c+di) = (ac-bd)-(ad+bc)i$

iv.) $\dfrac{a+bi}{c+di} = \dfrac{(ac+bd)+(bc-ad)i}{c^2+d^2}$

It is instructive to provide a proof of each of these results. Here is your task. Name your primary function **complexArithmetic**, then perform each of the following tasks.

a.) Write a subfunction **complexMenu** that will offer the user a choice of four operations, addition, subtraction, multiplication, division, and exiting the program. This routine should ask the user for a numerical response and return the choice to the calling function in the output variable n.

b.) The primary function should have a **case** structure that responds to the user's choice in n. Each item in the case structure should prompt the user to enter two complex numbers as row vectors, i.e., as **[a,b]** and **[c,d]**, representing $a+bi$ and $c+di$, respectively. This input should be sent as row vectors to the appropriate subfunction, then the returned row vector should be formatted and printed to the screen with **fprintf** in a manner you deem fitting.

c.) Write four subfunctions, **complexAdd**, **complexSubtract**, **complexMultiply**, and **complexDivide**. Each subfunction should accept two row vectors as

input and return a row vector as output. For example, the first line of the subfunction **complexDivide** should read as follows.

```
function w=complexDivide(u,v)
```

Note that the function receives the row vectors $[a, b]$ and $[c, d]$ in the arguments **u** and **v**, respectively. The output of **complexDivide** should be returned as a **row vector** in the output argument **w**. Before including your subfunction in the primary function, test it thoroughly. One suggestion would be to turn on rational display with **format rat**, then enter **(2+3i)/(2+i)** at the Matlab prompt and note the result. Now, feed $[2, 3]$ and $[2, 1]$ to your standalone function **complexDivide** and compare the result. Once the subfunction is written and tested, add it as a subfunction to the primary function **complexArithmetic**.

Thoroughly test the primary function **complexArithmetic** for accuracy, insuring that each of the five menu choices function properly and provide accurate output.

3. On November 13, 1843, Sir William Rowan Hamilton presented his first paper on the quaternions to the Royal Irish Academy. It was entitled *On a new Species of Imaginary Quantities connected with a theory of Quaternions* and was published in Volume 2 of the *Proceedings of the Royal Irish Academy*. It is interesting to read about the discovery in Sir William's own words in a letter to his son.

> Every morning in the early part of the above-cited month, on my coming down to breakfast, your (then) little brother William Edwin, and yourself, used to ask me, "Well, Papa, can you *multiply* triplets"? Whereto I was always obliged to reply, with a sad shake of the head: "No, I can only *add* and *subtract* them."
>
> But on the 16th day of the same month — which happened to be a Monday, and a Council day of the Royal Irish Academy — I was walking in to attend and preside, and your mother was walking with me, along the Royal Canal, to which she had perhaps driven; and although she talked with me now and then, yet an *under-current* of thought was going on in my mind, which gave at last a *result*, whereof it is not too much to say that I felt at once the importance. An *electric* circuit seemed to *close*; and a spark flashed forth, the herald (as I *foresaw*, *immediately*) of many long years to come of definitely directed thought and work, by *myself* if spared, and at all events on the part of *others*, if I should even be allowed to live long enough distinctly to communicate the discovery. Nor could I resist the impulse — unphilosophical as it may have been — to cut with a knife on a stone

> of Brougham Bridge, as we passed it, the fundamental formula with the symbols, i, j, k; namely,
>
> $$i^2 = j^2 = k^2 = ijk = -1, \tag{4.7}$$
>
> which contains the *Solution of the Problem*, but of course, as an inscription, has long since mouldered away. A more durable notice remains, however, on the Council Books of the Academy for that day (October 16th, 1843), which records the fact, that I then asked for and obtained leave to read a Paper on Quaternions, at the *First General Meeting* of the session: which reading took place accordingly, on Monday the 13th of the November following.

This letter and other correspondence by Sir William, along with a collection of his work and papers, can be found at the following URL.

http://www.maths.tcd.ie/pub/HistMath/People/Hamilton/

The quaternions grew out of Hamilton's desire to extend the geometry associated with complex numbers. A complex number has the form $a + bi$, where a and b are arbitrary real numbers. Hamilton first attempted to extend this to another dimension, working with numbers having the triplet form $a+bi+cj$, but soon discoved that another dimension was needed. Thus, the general form of a quaternion is $a + bi + cj + dk$. The discovery of the relationship (**4.7**) initiated Hamilton's journey into the world of quaternions.

To this day, the quaternions remain influential. Students in abstract algebra meet the definition when they begin their work on the theory of groups. The geometry of quaternions enables rotations and reflections in four dimensional space. The role of the quaternions in quantum mechanics and physics is nicely documented at the following URL.

http://world.std.com/~sweetser/quaternions/qindex/qindex.html

Sir William had no difficulty determining how to add two quaternions. In short,

$$(a+bi+cj+dk)+(e+fi+gj+hk) = (a+e)+(b+f)i+(c+g)j+(d+h)k. \tag{4.8}$$

The definition of subtraction established by Sir William is not unexpected.

$$(a+bi+cj+dk)-(e+fi+gj+hk) = (a-e)+(b-f)i+(c-g)j+(d-h)k \tag{4.9}$$

Scalar multiplication was easily established by Sir William.

$$\alpha(a + bi + cj + dk) = (\alpha a) + (\alpha b)i + (\alpha c)j + (\alpha d)k \tag{4.10}$$

We now reach the point where Sir William was confounded, multiplication of two quaternions. His startling breakthrough, captured during the walk with his wife along the Royal Canal, allowed him to make the further developments.

$$ij = k = -ji, \qquad jk = i = -kj, \qquad ki = j = -ik \tag{4.11}$$

Students of vector calculus will recognize the familiar relationships amongst i, j, and k, if they think of i, j, and k as unit vectors along the x, y, and z axes in the usual 3-space orientation used in vector calculus.

Of course, the relationships in (**4.11**) demonstrate that multiplication of the quaternions is *not commutative*. Changing the order of the factors changes the product. In general, if u and v are quaternions, it is not the case that uv equals vu.

Sir William was able to demonstrate that multiplication was an associative operation ($(uv)w = u(vw)$) and that multiplication is distributive with respect to addition ($u(v + w) = uv + uw$). With associativity and the distributive law in hand, a straightforward (albeit messy) calculation shows that the product of the quaternions $u = u_1 + u_2i + u_3j + u_4k$ and $v = v_1 + v_2i + v_3j + v_4k$ is

$$\begin{aligned} uv = {} & (u_1v_1 - u_2v_2 - u_3v_3 - u_4v_4) + (u_1v_2 + u_2v_1 + u_3v_4 - u_4v_3)i \\ & + (u_1v_3 + u_3v_1 + u_4v_2 - u_2v_4)j + (u_1v_4 + u_4v_1 + u_2v_3 - u_3v_2)k. \end{aligned} \tag{4.12}$$

When you studied the complex numbers, before making the definition of division, you paused to develop the *complex conjugate*. In a similar manner, if $u = u_1 + u_2i + u_3j + u_4k$, then the conjugate of this quaternion is the quaternion

$$\overline{u} = \overline{u_1 + u_2i + u_3j + u_4k} = u_1 - u_2i - u_3j - u_4k. \tag{4.13}$$

Note that the conjugate of a quaternion if very similar to the conjugate of a complex number. That is, if z is the complex number $z = a + bi$, then the conjugate of z is $\overline{z} = a - bi$. Geometrically, the conjugate of the complex number z is the reflection of z across the real axis, as shown in **Figure 4.11**.

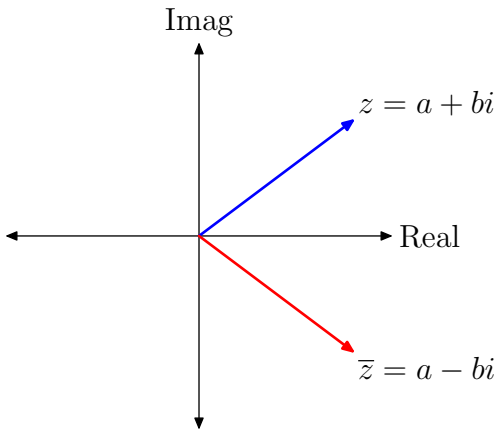


Figure 4.11. The conjugate is a reflection across the real axis.

Moreover, recall that multiplying a complex number by its conjugate produces the square of length of the complex number. That is,

$$z\overline{z} = (a + bi)(a - bi) = a^2 - b^2 i^2 = a^2 + b^2.$$

Note that in **Figure 4.12**, the Pythagorean Theorem gives the length of the complex number $z = a + bi$ as $\sqrt{a^2 + b^2}$. Hence, $z\overline{z} = a^2 + b^2$ give the square of the length.

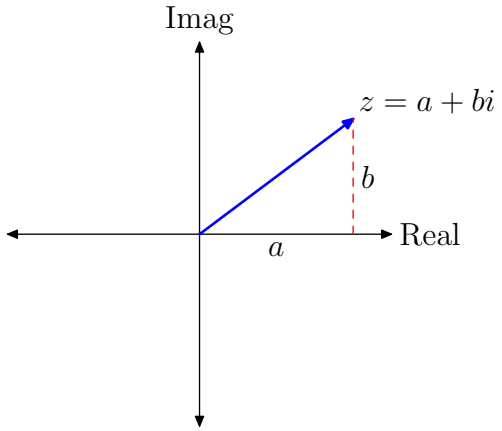


Figure 4.12. The magnitude of the complex number is $\sqrt{a^2 + b^2}$.

In similar fashion, if v is the quaternion $v = v_1 + v_2 i + v_3 j + v_3 k$, then one can use **equation (4.12)** to show that

$$v\overline{v} = (v_1^2 + v_2^2 + v_3^2 + v_4^2) + 0i + 0j + 0k = v_1^2 + v_2^2 + v_3^2 + v_4^2.$$

Thus, in a sense, $v\overline{v}$ gives the square of the length of the quaternion. That is, $v\overline{v}$ is a scalar, not a quaternion.

We are now in a position where we can divide one quaternion by another. If the quaternion is nonzero, then it has nonzero length and an easy calculation reveals that

$$v \cdot \frac{\overline{v}}{v\overline{v}} = 1.$$

Thus,

$$v^{-1} = \frac{\overline{v}}{v\overline{v}}. \tag{4.14}$$

When writing your program, take note that $v\overline{v}$ is a real number. So, if you use **quaternionMultiply** to compute $v\overline{v}$, you will need to divide $\overline{v}$ by the *first component* of the result returned by **quaternionMultiply**.

Division is now easily defined.

$$\frac{u}{v} = uv^{-1}. \tag{4.15}$$

We begin our programming task. Save a primary function as **quaternionArithmetic**, then perform each of the following tasks.

a.) The primary function should store the quaternions

$$u = 1 + 2i + 3j + 4k \qquad \text{and} \qquad v = 5 + 6i + 7j + 8k$$

as row vectors **u=[1,2,3,4]** and **v=[5,6,7,8]**. It should then use **fprintf** to format and output four results to the screen, $u + v$, $u - v$, uv, and u/v.

b.) Write and implement each of the following subfunctions:

i.) **function w=quaternionAdd(u,v)**

ii.) **function w=quaternionSubtract(u,v)**

iii.) **function v=quaternionConjugate(u)**

iv.) **fucntion v=quaternionInverse(u)**

v.) **function w=quaternionDivide(u,v)**

Note that the input and output to each of these routines are quaternions, which should be stored a row vectors of length four. Before adding each subfunction to your primary function, test each function thoroughly for accuracy.

Some hints are in order:

- **quaternionAdd** and **quaternionSubtract** should use **equations (4.8)** and **4.9**, respectively.
- **quaternionMultiply** should use **equation (4.12)**.
- **quaternionConjugate** should use **equation (4.13)**.
- **quaternionInverse** should use **equation (4.14)**, which in turn requires a call of the subfunction **quaternionConjugate** to complete the calculation.
- **quaternionDivide** should use **equation (4.15)**, which in turn will require calls to **quaternionInverse** and **quaternionMultiply** to complete the calculation.

Because there is no user input involved, you should be able to open the editor in cell mode and simply call **quaternionArithmetic** and publish the result to HTML. Include your function **quaternionArithmetic** as a comment. Your grade on this assignment will reflect the accuracy of your answers, so you might want to compare and contrast your answers with your classmates. One way to do this is through the Discussion Board on Blackboard.

4.6 Nested Functions in Matlab

Normally, we include subfunctions in a primary function as follows.

```
function output=primary(input)
statements---including subfunction calls

% first subfunction
function out=one(in)
statements

% second subfunction
function out=two(in)
statments
```

To write a *nested function*, write one or more functions within the body of another function in an M-file. You must terminate any nested function (as well as the primary function) with an **end** statement.

```
function output=primary(input)
statements---including subfunctionc calls

% first subfunction
    function out=one(in)
        statements
    end %nested function one

% second subfunction
    function out=two(in)
        statments
    end %nested function two

end %function primary
```

M-file functions do not normally require a terminating **end** statement. However, if an M-file contains one or more nested functions, you must terminate *all* functions (including any subfunctions) in the M-file with an **end** statement, whether or not they contain nested functions.

Variable Scope in Nested Functions

So, what's the big deal? Why should we bother to study *nested functions*? What do they offer that we don't already have with the existing functions and subfunctions we've already studied?

The answer lies in *variable scope*. As a short example, open the editor and enter the following lines. Save the function M-file as **varScope1.m**.

```
function varScope1
x=3;
nestfun1

function nestfun1
fprintf('In nestfun1, x equals: %d\n',x)
```

Note that we have a primary function named **varScope1** which contains one subfunction named **nestfun1**. The primary assigns the variable x the scalar 3, then calls the subfunction with the command **nestfun1**. In turn, the subfunction **nestfun1** attempts to print the value of the variable x **fprintf** with a nice format string.

Remember that the primary function **varScope1** and the subfunction **nestfun1** have separate workspaces. The variable x exists in the primary workspace, but the subfunction workspace is independent of the primary workspace and thus has no knowledge of the variable x. Thus, when we run the function, we should not be surprised by the error message.

```
>> varScope1
??? Undefined function or variable 'x'.

Error in ==> varScope1>nestfun1 at 6
fprintf('In nestfun1, x equals: %d\n',x)

Error in ==> varScope1 at 3
nestfun1
```

This is precisely the error message we expected to see. The variable x exists in the primary function workspace, but the subfunction workspace has no knowledge of the variable x. When the subfunction attempts to print the contents of the variable x, an error occurs.

We've seen a couple of ways we can fix this M-file so that the value of x is printed by the subfunction. One way would be to declare the variable x global in both the primary function and the subfunction.

```
function varScope1
global x
x=3;
nestfun1

function nestfun1
global x
fprintf('In nestfun1, x equals: %d\n',x)
```

Running this M-file produces the desired effect. The subfunction now has knowledge of the global variable x and its **fprintf** statement is successful.

```
>> varScope1
In nestfun1, x equals: 3
```

We've also seen that we can pass the value of the variable x to an input argument of the subfunction.

```
function varScope1
x=3;
nestfun1(x)

function nestfun1(x)
fprintf('In nestfun1, x equals: %d\n',x)
```

As readers can see, this also works.

```
>> varScope1
In nestfun1, x equals: 3
```

Nested Functions Redefine the Rules of Variable Scope. If we take our same example, and nest the subfunction, a whole new set of rules are put into place for variable scope, the first of which follows.

The First Rule for Variable Scope in Nested Functions. A variable that has a value assigned to it by the primary function can be read or overwritten by a function nested at any level within the primary.

Enter the following lines exactly as they appear below into the editor and save the function M-file as **varScope2.m**.

```
function varScope2
x=3;
nestfun1

function nestfun1
fprintf('In nestfun1, x equals: %d\n',x)
end  %end nestfun1

end  %end varScope2
```

If you are using the Matlab editor, select all of the above lines in the editor with the key sequence Ctrl+a (Cmd-a on the Mac), then "smart indent" the highlighted selection with Ctrl+i (Cmd-i on the Mac). The resulting indentation is shown in what follows.

```
function varScope2
x=3;
nestfun1

    function nestfun1
        fprintf('In nestfun1, x equals: %d\n',x)
    end  %end nestfun1

end  %end varScope2
```

We've used comments to indicate where the primary and nested functions "end." Note that the combination of the **end** statements, the "smart" indentation in the editor, and our helpful comments all serve to highlight the fact that the function **nestfun1** is "nested" within the primary function **varScope2**. According to the *First Rule for Variable Scope in Nested Functions*, the nested function **nestfun1** should have access to the variable x that is assigned the value of 3 in the primary function **varScope2**.

```
>> varScope2
In nestfun1, x equals: 3
```

Very interesting! No global variables, no passing of values as arguments, but because **nestfun1** is "nested" within the primary function, the nested function **nestfun1** has access to all variables assigned in the primary function.

Graphical User Interfaces

In this section we introduce our readers to Matlab's GUI's *Graphical User Interfaces*). Matlab has a full complement of **uicontrols** (*User Interface Controls*), typical of what you commonly see in most of today's modern sofware: edit boxes, popupmenus, push- and toggle-buttons, sliders, radio buttons, and checkboxes, a few of which we will introduce in this section.

We're going to write our first GUI, keeping it low-key as we introduce the concepts. Our gui design can be described with the following series of tasks:

1. Plot a function.
2. Provide radio buttons to change the color of the graph of the function, offering choices of red, green, or blue.
3. Provide a popup menu with choices of different line styles for the plot: solid, dotted, etc.
4. Provide an edit box that allows the user to change the linewidth.

Let's begin. Open the editor and enter the following lines, then save the function M-file as **plotGUI.m**.

```
function plotGUI

close all
clc

% plot domain
xmin=0;
xmax=4*pi;

% some colors
figure_color=[0.8,0.9,0.8];
panel_color=[1,0.9,0.8];
buttongroup_color=[0.9,0.9,0.8];
```

This function takes no input and export no output. However, it is possible to write GUI functions that accept input and export data back to the caller.

Next we close all open figure windows and clear the command window. This is helpful during the development of the GUI, but once we have a tested and working program, we'll remove these two commands from the GUI.

We set limits on the domain of our plot, which we will also use when we initialize an axes on our figure window. It's helpful to put these here and not have to change them in several places in our program.

Finally, we initialize some colors we will use for our GUI components. Note that each color takes the form of a row vector with three entries, each of which must be a number between 0 and 1. The first entry is the percentage of red, the second the percentage of green, and the third entry is the percentage of blue. Most modern computers have color wheels where you can view a color and the percentages of red, green, and blue (in the RGB system) to make up the color.

Next, we'll add a figure window to the GUI. Add the following lines to the function M-file **plotGUI** and save.

```
hFigure=figure(...
    'Units','Pixels',...
    'Position', [100 100 700 500],...
    'Color',figure_color,...
    'MenuBar','none',...
    'ToolBar','none',...
    'NumberTitle','off',...
    'Name','Plot GUI');
```

Run the program. Some things to note:

1. We set **Units** to **Pixels**. Thereafter, all positioning commands are made in Pixels. Think of a pixel as one dot on your computer screen. Typical resolution on modern displays are of the order of 1024 by 768 pixels, but can be higher (or lower) depending on personal display settings on your computer.
2. Next we set the **Position** of the figure window at **[100 100 700 500]**. The first two measurements (100 and 100) are the location of the lower left-hand corner of the figure window, relative to the lower left-hand corner of your computer screen. The next two measurements (700 and 500) are the width and height of the figure window. All of these measurements are in pixels
3. Finally, we set the color, then turn off the menubar and toolbar as well as the usual number title that is displayed in a typical figure window (Figure 1,

Figure 2, etc.). Then we give our figure window a meaningful name for the project, namely **Plot GUI**.

4. Note that we assign a *handle* to the figure window named **hFigure**. Every Matlab object can be associated with a handle for further reference.

We will now add an axes to the figure window. Note that the axes object will be a "child" of the figure window (equivalently, the figure window is a "parent" of the axes object). Add the following lines to the file **plotGUI** and save.

```
hAxes=axes(...
    'Parent',hFigure,...
    'Units','Pixels',...
    'Position',[50 50 400 400],...
    'Xlim',[xmin,xmax],...
    'XGrid','on',...
    'YGrid','on');
```

Run the program. Things to note:

1. Note that the "Parent" of the axes object is set to **hFigure**, which is the numerical handle assigned to the figure window designated above. This makes the axes object a "child" of the figure window whose numerical handle is **hFigure**.
2. The unit of measurement for the axes object is also set to "Pixels."
3. The position of the axes object is set to **[50 50 400 400]**. The first two numbers (50 and 50) set the lower left-hand corner of the axes object, relative to the lower left-hand corner of its parent, in this case the figure window designated by the handle **hFigure**.
4. The limits on the x-axis are set to **[xmin,xmax]** and the grid in both the x- and y-directions is turned 'on' (equivalent to using the **grid on** command).
5. Finally, note that we've assigned a handle **hAxes** to our axes object for further reference.

Now that we have a figure window and axes object, let's create some data and place a plot of the data on the axes object. Enter the following lines in **plotGUI** and save.

```
% plot data
x=linspace(xmin,xmax);
y=sin(2*x);
hLine=line(x,y);
```

If you've been running the program, note that we've left some room on the right for our controls. First, let's create a **uipanel** (user interface panel) in this free area of the figure window to contain the various controls of the GUI which we will create later. Enter the following lines in **plotGUI** and save.

```
hPanel=uipanel(...
  'Parent',hFigure,...
  'Units','Pixels',...
  'Position',[475,50,200,400],...
  'BackgroundColor',panel_color);
```

Running the program at this stage should produce the result shown in **Figure 4.13**.

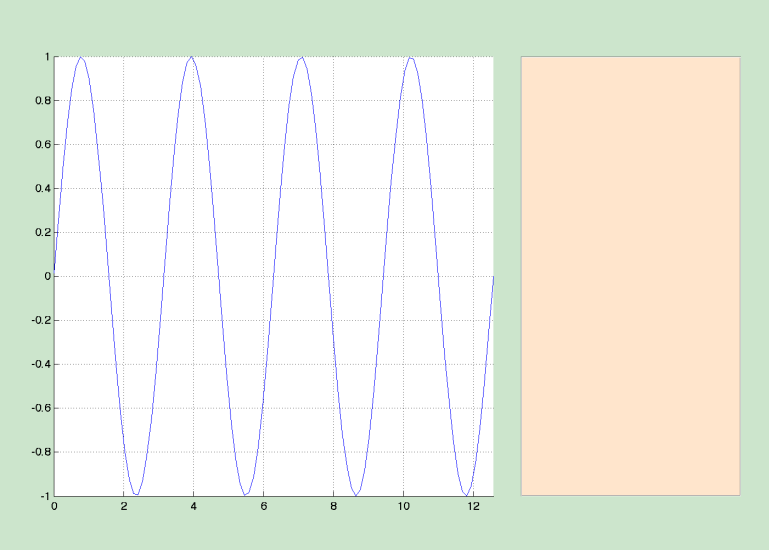


Figure 4.13. Figure acting as parent to child axes and panel objects.

Note that the **uipanel** is a child object of the figure window having handle **hFigure**. Hence, when we set the position with the vector **[475,50,200,400]**, the first two entries (475 and 50) designate the lower left-hand corner of the **uipanel** as measured from the lower left-hand corner of the 'Parent' figure window. The next two entries of the position vector, 200 and 400, designate the width and the height of the **uipanel**.

Button Groups and Radio Buttons

. We're now going to place three radio buttons in the **uipanel**, one for each of the colors red, green, and blue. If the user selects the 'red' radio button, then

we'll color the graph red. Similar strategies will apply to the 'green' and 'blue' radio buttons.

We could set the radio buttong directly in the **uipanel**. However, if we first place our three radio buttons in what is known as a **uibuttongroup**, only one of them can be selected at a time. So, enter the following lines in **plotGUI** and save.

```
hButtonGroup=uibuttongroup(...
    'Parent',hPanel,...
    'Units','Pixels',...
    'Position',[10,275,180,100],...
    'BackgroundColor',buttongroup_color}
```

Because the **uibuttongroup** is a child of the **uipanel** with handle **hPanel**, when we position the **uibuttongroup** with the vector **[10,275,180,100]**, the first two entries (10 and 275) are the lower left-hand corner of the **uibuttongroup**, as measured from the lower left-hand corner of the **uipanel** with handle **hPanel**. The second two entries, 180 and 100, designate with width and height of the **uibuttongroup**.

Next, we add a radio button to the **uibuttongroup**. We use a **uicontrol** for this purpose, with **Style** set to **Radio**. It is important to note that the parent of this **uicontrol** is the **uibuttongroup** having handle **hButtonGroup**. Thus, when we set the position to **[10,10,160,20]**, the fist two entries (10 and 10) are the lower left-hand corner of the radion **uicontrol**, as measured from the lower left hand corner of its parent (**uibuttongroup** with handle **hButtonGroup**).

```
r1=uicontrol(...
    'Style','Radio',...
    'Parent',hButtonGroup,...
    'Units','Pixels',...
    'Position', [10,10,160,20],...
    'String','Blue',...
    'BackgroundColor',buttongroup_color);
```

The **BackgroundColor** should be self explanatory, and when we set **String** to **Blue**, this prints the message 'Blue' to the default position (to the immediate right of the radio button.

We add two more radio buttons (one for red and one for green) in a similar manner. First, green.

```
r2=uicontrol(...
    'Style','Radio',...
    'Parent',hButtonGroup,...
    'Units','Pixels',...
    'Position', [10,37,160,20],...
    'String','Green',...
    'BackgroundColor',buttongroup_color);
```

Then, red.

```
r3=uicontrol(...
    'Style','Radio',...
    'Parent',hButtonGroup,...
    'Units','Pixels',...
    'Position', [10,65,160,20],...
    'String','Red',...
    'BackgroundColor',buttongroup_color);
```

Note that each radio button has a unique handle (**r1**, **r2**, and **r3**) for later reference.

Adding the button group and radio buttons produce the GUI shown in **Figure 4.14**.

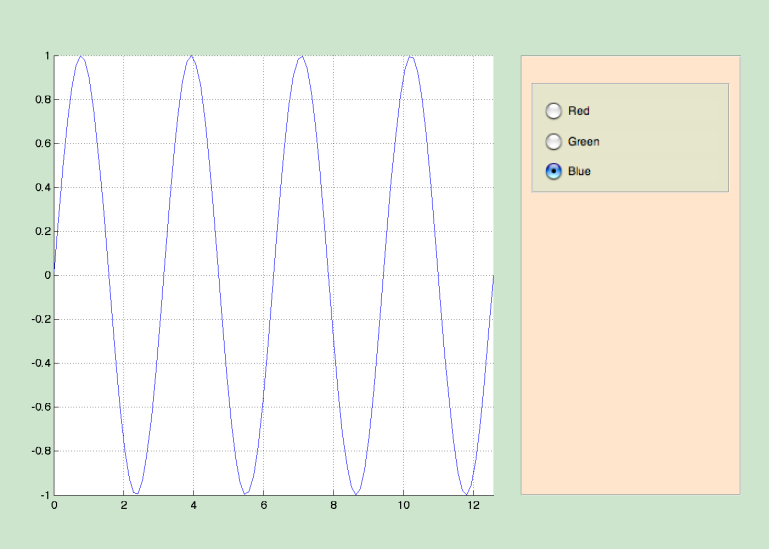


Figure 4.14. Adding a button group and radio buttons.

At this point, you can try clicking each of the radio buttons, red, blue, or green. Although they are not programmed to do anything at this point, note that only one of them can be selected at a time. This behavior (one at a time selection) is due to the fact that we grouped them inside a **uibuttongroup**.

Adding a Function Callback to the UIButtonGroup

Note that the handle **hButtonGroup** can be used to get or set property-value pairs for the **uibuttongroup**. You can obtain information on all possible property-value pairs for a **uibuttongroup** by typing **doc uibuttongroup** at the Matlab prompt. We are particularly interested in the **SelectionChangeFcn** property, to which we must assign a handle to a function. This sort of function is referred to as a *callback function* and will be executed whenever a new radio button is selected with the mouse. (Hence the property name, **SelectionChangeFcn**). It doesn't matter what name you use for the callback function, but writers of Matlab GUI's tend to use a descriptive name that includes the substring 'callback.' Thus, we'll name the callback **colorSelection_callback**.

Add one more property-value pair to the **uibuttongroup** as follows.

```
hButtonGroup=uibuttongroup(...
    'Parent',hPanel,...
    'Units','Pixels',...
    'Position',[10,275,180,100],...
    'BackgroundColor',buttongroup_color,...
    'SelectionChangeFcn',@colorSelection_callback);
```

Note that this is not new code. We simply appended the last property-value pair to the existing code that initialized the **uibuttongroup**.

Now, whenever you click a radio button with your mouse, the callback function named **colorSelection_callback** will be executed. You can give it a try at this point, but the callback function does not yet exist in our GUI code, so you will receive an error message attesting to that fact.

What we need to do next is write the function callback. So, go to the end of the file **plotGUI.m**, just prior to the closing **end** of the primary function **plotGUI**, and add the following lines. We'll fill in the body of this function callback in a moment, but just add these lines at present.

```
    function colorSelection_callback(hObject,eventdata)
    end % end colorSelection_callback
```

Note that this callback function is *nested* inside the primary function **plotGUI**. Note the **end** command, which you must use to terminate any nested function.

Some comments are in order.

- The name of the callback must match the name of the function handle assigned to the property **SelectionChangeFcn** in the **uibuttongroup**.
- Two input arguments are required. You may use any names you wish.
  i. The first argument, which we have named **hObject**, receives the value of the handle of the calling object. In this case, **hObject** receives the handle of the **uibuttongroup**, namely **hButtonGroup**.
  ii. The second argument, which we have named **eventdata**, is passed a structure containing data describing the state of the calling object when triggered by an event (in this case the clicking of a radio button by the mouse). In this case, the structure **eventdata** contains two fields, one named **OldValue**, which contains the handle of the previously selected radio button, and a second named **NewValue**, which contains the handle of the newly selected radio button. Note that the possible handles contained in these fields are three: **r1**, **r2**, or **r3**. You can access the value of these fields by typing the structure name, a period, then the field desired. For example, to get the handle to the newly selected radion button, type **eventdata.NewValue**. To get the handle of the previously selected radio button, type **eventdata.OldValue**.

We'll now code the body of the callback function. First, we get the value of the currently selected radio button and query the handle for the value of 'String,' which could be either 'Red', 'Green', or 'Blue' (see the initialization strings for each radio button described and coded earlier). We set up a switch structure, which will color the plot red, green, or blue, depending on whether **lineColor** equals 'Red', 'Green', or 'Blue'.

```
function colorSelection_callback(hObject,eventdata)
    lineColor=get(eventdata.NewValue, 'String');
    switch lineColor
        case 'Red'
            set(hLine,'Color','r')
        case 'Green'
            set(hLine,'Color','g')
        case 'Blue'
            set(hLine,'Color','b')
    end
end % end colorSelection_callback
```

The thing to note is the fact that the nested callback has access to all variables defined in the outer primary function. In particular, **hLine** is a handle to the sinusoidal plot and it was defined in the primary function. Hence, the nested function **colorSelect_callback** has access to this variable by default. It is not necessary to pass the variable by value or to declare it global. Note that in each **case** of the **switch** structure, the code uses the handle **hLine** to set the 'Color' property of the plot to the appropriate color.

At this point, you should be able to run the function without error. The GUI will initialize, after which you can click the radio buttons and watch the plot change to the color associated with the selected radio button.

Popup Menus

In this section we will add a popup menu containing choices of line styles for our plot. However, before adding the popup **uicontrol**, we will first add some explanatory text by adding a **uicontrol** containing *static text*. Static text is not dynamic[15], as the name implies, and is used specifically to annotate your GUI with help messages and instructions.

So, immediately following the code for your last radio button (the code with handle **r3**), enter the following lines.

```
hLineStyleText=uicontrol(...
    'Style','text',...
    'Parent',hPanel,...
    'Position', [10 240 180 20],...
    'String', 'LineStyle Choices',...
    'HorizontalAlignment','Left',...
    'BackgroundColor', panel_color);
```

Again, comments are in order.

- The 'Style' of the **uicontrol** is 'text'. This declares the control as static text. The text that is actually printed on your GUI is the value of the 'String' property, in this case "LineStyle Choices." Underneath this static text, we will soon place a popup menu with choices for line styles.
- Not that this **uicontrol** is a child of the **uipanel** with handle **hPanel**. Consequently, any positioning is measured from the lower left-hand corner of the **uipanel**.

[15] Although static text cannot be modified by the user, it can be modified by the program code.

Next, we deal with the popup menu. Following the code for our static text, enter these lines to intialize a popup menu of line style choices.

```
hLineStylePopup=uicontrol(...
    'Style','popup',...
    'Parent',hPanel,...
    'Units','Pixels',...
    'Position', [10 225 180 15],...
    'String',{'Solid' 'Dotted' 'DashDot' 'Dashed' 'None'},...
    'BackgroundColor', buttongroup_color,...
    'Callback', @LineStylePopup_callback);
```

Here are a few words of explanation. First, some simple comments.

- The 'Style' of the **uicontrol** is 'popup'. This declares the control to be a popup menu.
- Again, the 'Parent' of this **uicontrol** is the **uipanel** with handle **hPanel**. Thus, the position is again measured from the lower left=hand corner of the **uipanel**.

The 'String' property of the **uicontrol** is a *cell* containing the individual strings that we wish to appear on the popup menu. A cell is a Matlab data structure that allows the user to collect different data types in a set.

One of the simplest ways to build a cell in Matlab is shown in the code that follows. You should enter this code at the Matlab prompt, not in the code of our GUI that is under construction.

```
>> C={'dashed', 1:5, magic(3)}
C =
    'dashed'    [1x5 double]    [3x3 double]
```

Note that we have collected three different data types in a cell, delimiting the entries with curly braces, and assigned the result to the variable C. The first entry is a string, the second a row vector, and the third a 3×3 matrix.

You can access the contents of the individual elements of a cell by using curly braces with the usual indexing. For example, to obtain the contents of the first entry in C, enter the following code (again, at the Matlab prompt, not in the code for the GUI).

```
>> C{1}
ans =
dashed
```

In similar fashion, we can access the contents of the second entry in C with the following command.

```
>> C{2}
ans =
     1     2     3     4     5
```

We will leave it to our readers to access the magic matrix that is the third entry in C.

Our use of cells at this juncture will be at a very elementary level. For a full discussion of cell structure in Matlab, we refer our readers to the Matlab documentation.

To continue with our discussion of the popup **uicontrol**, after setting the background color, note that we set the 'Callback' property of the **uicontrol** to a function handle **LineStylePopup_callback**. At this point, you can run the GUI and note the added presence of our two new **uicontrols** in **Figure 4.15**, the static text and the popup menu. However, selecting an item on the popup menu with the mouse will lead to an error, because the callback function **LineStylePopup_callback** has not yet been defined.

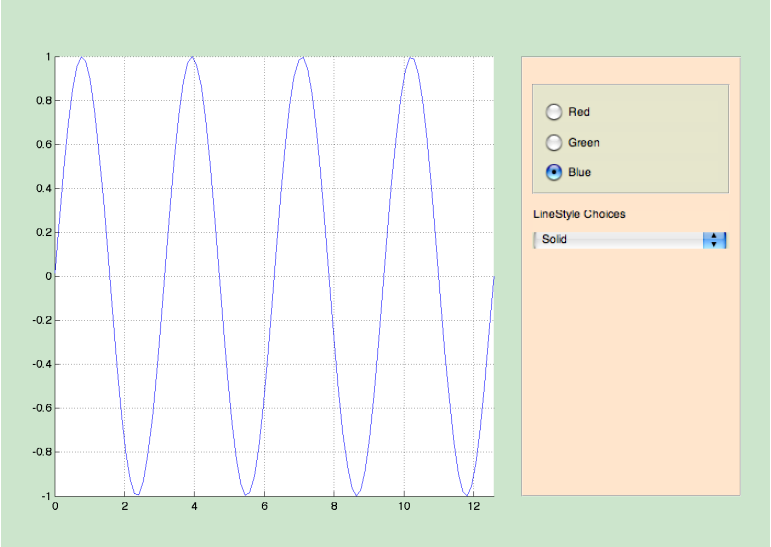


Figure 4.15. Adding a static text and popup menu.

So, let's defined the callback for the popup menu of line styles. Just below the callback function **colorSelection_callback**, add the following lines.

```
function LineStylePopup_callback(hObject,eventdata)
    lineStyleChoices=get(hObject,'String');
    lineStyleChoice=get(hObject,'Value');
    lineStyle=lineStyleChoices{lineStyleChoice};
    switch lineStyle
        case 'Solid'
            set(hLine,'LineStyle','-')
        case 'Dotted'
            set(hLine,'LineStyle',':')
        case 'DashDot'
            set(hLine,'LineStyle','-.')
        case 'Dashed'
            set(hLine,'LineStyle','--')
        case 'None'
            set(hLine,'LineStyle','none')
    end
end % end LineStylePopup_callback
```

We will make several comments regarding this callback.

- The first line contains the function keyword and the name of the callback, in this case **LineStylePopup_callback**. The descriptive name of this callback is a reminder of it is designed to provide. Note the oblibatory arguments, **hObject** and **eventdata**. The argument **hObject** will receive the handle of the calling object (in this case, the handle of the popup menu **uicontrol**). In this example, the argument **eventdata** is not used, but it is still required. As more evolutions of Matlab are updated, plans are in the works to make more use of the argument **eventdata** (as we did with our **uibuttongroup**, but for the meantime, it's not used in this callback, but it is still required.
- Because **hObject** is passed the handle to the popup **uicontrol**, we can use it to access two properties of that control, the 'String' and 'Value' properties.
  i. The 'String' property contains a cell of strings, each of which describe a line style.
  ii. The 'Value' property contains a number indicating which entry on the popup menu was selected by the user.

The cell of strings is stored in **lineStyleChoices** and the number indicating the choice is stored in **lineStyleChoice**. We obtain the string describing the choice with cell indexing. Note the use of the curly braces.

- A switch structure is then used to set the line style based on the string describing the choice that is stored in **lineStyle**. Because the callback is a nested function, it can access the handle **hLine** of the plot, then use it to set the line style according to the choice in **lineStyle**.

At this point, you can run the GUI again. Select the 'Red' radio button and the 'Dashed' linestyle from the popup menu. The result is shown in **Figure 4.16**.

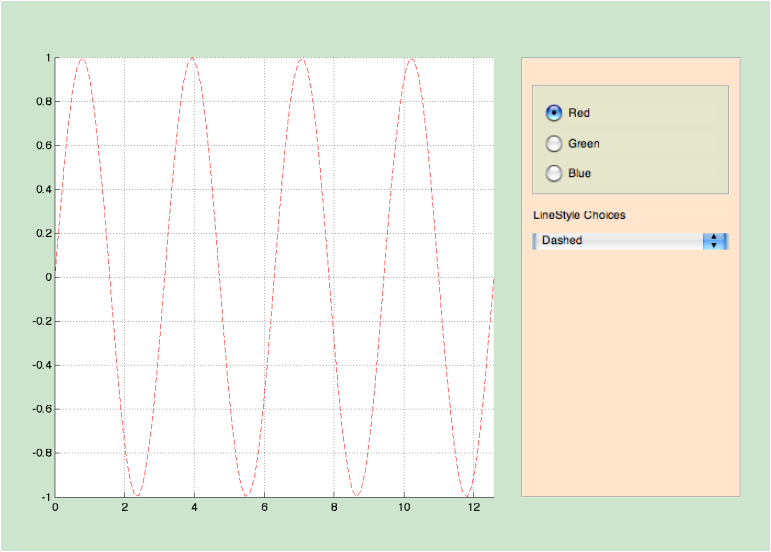


Figure 4.16. Dashed and red.

Edit Boxes

We will add one last **uicontrol** to our GUI, an *edit box*. An edit box provides an area where a user can enter text. Unlike static text, however, an edit box is dynamic. The user can change what is entered there, and on the other hand, your program can also update the contents of an edit box.

However, before we create an edit box on our GUI, we'll first place some static text above the planned location for the edit box. The text will provide the user with information relevant for the edit box. Enter the following lines just below the last **uicontrol**, the popup **uicontrol** for line styles. As static text controls were explained above, and there is nothing new about this current entry other than the 'String' value, we offer the code without comment.

```
hLineWidthText=uicontrol(...
    'Style','text',...
    'Parent',hPanel,...
    'Position', [10 180 180 20],...
    'String', 'Enter Desired LineWidth: (1-10)',...
    'HorizontalAlignment','Left',...
    'BackgroundColor', panel_color);
```

Next, to initialize the edit box for our GUI, enter the following lines just below the last **uicontrol**, the static text control with handle **hLineWidthText**.

```
hLineWidthEditbox=uicontrol(...
    'Style','edit',...
    'Parent',hPanel,...
    'Units','Pixels',...
    'Position', [10 150 180 25],...
    'String','1',...
    'BackgroundColor', buttongroup_color,...
    'Callback', @LineWidthEditbox_callback);
```

Some comments are in order.

- Note that the 'Style' is 'edit', so this **uicontrol** is a dynamic edit box, one where both user and program can modify the text inside.
- The parent is the **uipanel**, so the position of the edit box will be measured from the lower left-hand corner of the panel with handle **hPanel**.
- We intialize 'String' to '1'. This will be the intial value that appears in the edit box. It represents a line width of 1 point (1 pt).
- We assign the function handle **@LineWidthEditbox_callback** to the 'Callback' property.

At this point, if you attempt to modify the text in the dit box, you will receive an error as the callback function **LineWidthEditbox_callback** is not yet written. Let's take care of that immediately and write the callback.

As usual, the obligatory arguments **hObject** and **eventdata** are used.

```
function LineWidthEditbox_callback(hObject,eventdata)
    lineWidth=str2double(get(hObject,'String'));
    if (lineWidth>=1) && (lineWidth<=10)
        set(hLine,'LineWidth',lineWidth);
    else
        set(hObject,'String','Invalid Input---Try again');
    end
end % end LineStyleEditbox_callback
```

Some comments are in order.

- Because **hObject** contains the handle of the calling object, **hLineWidthEditbox** in this case, we can use this handle to 'get' the value of the 'String' property, i.e., the string entered in the edit box. Because this value is a string, we use **str2double** to change it to a number (double precision, the Matlab default), storing the result in **lineWidth**. This is necessary because the 'LineWidth' property of our plot expects a number as its value.
- We use an **if..else..end** construct to determine if the number is greater than or equal to 1 *and* less than or equal to 10. If this test is true, then we use the handle to our plot to set the 'LineWidth' property to the value of **lineWidth**. If not, then we alert the user that the input is invalid and ask them to try again, i.e., enter a linewidth in the edit box between 1 and 10.

Run the program. Choose, red, dashed, and enter 2 for the linewidth to produce the image in **Figure 4.17**.

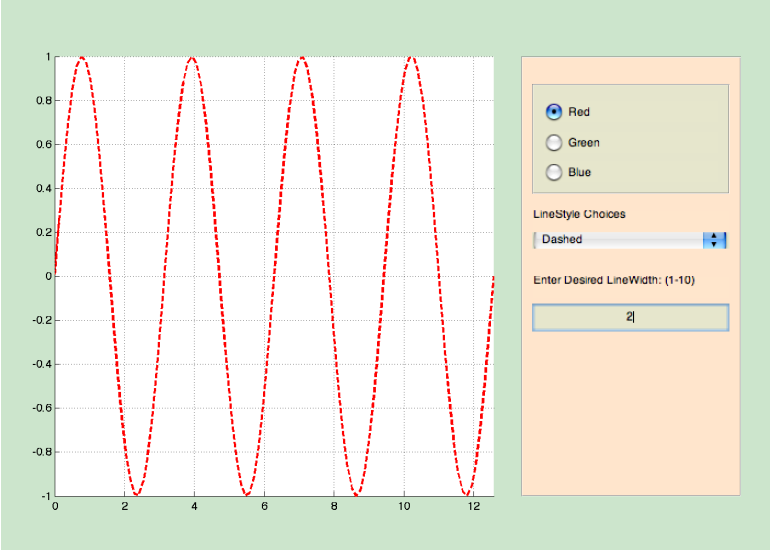


Figure 4.17. Adding an edit box for linewidth.

Appendix

Here we share a complete listing of **plotGUI** for convenience.

```
function plotGUI

close all
clc

% plot domain
xmin=0;
xmax=4*pi;

% some colors
figure_color=[0.8,0.9,0.8];
panel_color=[1,0.9,0.8];
buttongroup_color=[0.9,0.9,0.8];

hFigure=figure(...
    'Units','Pixels',...
    'Position', [100 100 700 500],...
    'Color',figure_color,...
    'MenuBar','none',...
    'ToolBar','none',...
    'NumberTitle','off',...
    'Name','Plot GUI');

hAxes=axes(...
    'Parent',hFigure,...
    'Units','Pixels',...
    'Position',[50 50 400 400],...
    'Xlim',[xmin,xmax],...
    'XGrid','on',...
    'YGrid','on');

hPanel=uipanel(...
    'Parent',hFigure,...
    'Units','Pixels',...
    'Position',[475,50,200,400],...
    'BackgroundColor',panel_color);
```

```
hButtonGroup=uibuttongroup(...
    'Parent',hPanel,...
    'Units','Pixels',...
    'Position',[10,275,180,100],...
    'BackgroundColor',buttongroup_color,...
    'SelectionChangeFcn',@colorSelection_callback);

r1=uicontrol(...
    'Style','Radio',...
    'Parent',hButtonGroup,...
    'Units','Pixels',...
    'Position', [10,10,160,20],...
    'String','Blue',...
    'BackgroundColor',buttongroup_color);

r2=uicontrol(...
    'Style','Radio',...
    'Parent',hButtonGroup,...
    'Units','Pixels',...
    'Position', [10,37,160,20],...
    'String','Green',...
    'BackgroundColor',buttongroup_color);

r3=uicontrol(...
    'Style','Radio',...
    'Parent',hButtonGroup,...
    'Units','Pixels',...
    'Position', [10,65,160,20],...
    'String','Red',...
    'BackgroundColor',buttongroup_color);

hLineStyleText=uicontrol(...
    'Style','text',...
    'Parent',hPanel,...
    'Position', [10 240 180 20],...
    'String', 'LineStyle Choices',...
    'HorizontalAlignment','Left',...
    'BackgroundColor', panel_color);
```

```
hLineStylePopup=uicontrol(...
    'Style','popup',...
    'Parent',hPanel,...
    'Units','Pixels',...
    'Position', [10 225 180 15],...
    'String',{'Solid' 'Dotted' 'DashDot' 'Dashed' 'None'},...
    'BackgroundColor', buttongroup_color,...
    'Callback', @LineStylePopup_callback);

hLineWidthText=uicontrol(...
    'Style','text',...
    'Parent',hPanel,...
    'Position', [10 180 180 20],...
    'String', 'Enter Desired LineWidth: (1-10)',...
    'HorizontalAlignment','Left',...
    'BackgroundColor', panel_color);

hLineWidthEditbox=uicontrol(...
    'Style','edit',...
    'Parent',hPanel,...
    'Units','Pixels',...
    'Position', [10 150 180 25],...
    'String','1',...
    'BackgroundColor', buttongroup_color,...
    'Callback', @LineWidthEditbox_callback);

% plot data
x=linspace(xmin,xmax);
y=sin(2*x);
hLine=line(x,y);

    function colorSelection_callback(hObject,eventdata)
        lineColor=get(eventdata.NewValue, 'String');
        switch lineColor
            case 'Red'
                set(hLine,'Color','r')
            case 'Green'
                set(hLine,'Color','g')
            case 'Blue'
                set(hLine,'Color','b')
        end
    end % end colorSelection_callback
```

```
    function LineStylePopup_callback(hObject,eventdata)
        lineStyleChoices=get(hObject,'String');
        lineStyleChoice=get(hObject,'Value');
        lineStyle=lineStyleChoices{lineStyleChoice};
        switch lineStyle
            case 'Solid'
                set(hLine,'LineStyle','-')
            case 'Dotted'
                set(hLine,'LineStyle',':')
            case 'DashDot'
                set(hLine,'LineStyle','-.')
            case 'Dashed'
                set(hLine,'LineStyle','--')
            case 'None'
                set(hLine,'LineStyle','none')
        end
    end % end LineStylePopup_callback

    function LineWidthEditbox_callback(hObject,eventdata)
        lineWidth=str2double(get(hObject,'String'));
        if (lineWidth>=1) && (lineWidth<=10)
            set(hLine,'LineWidth',lineWidth);
        else
            set(hObject,'String','Invalid Input---Try again');
        end
    end % end LineStyleEditbox_callback

end  % end plotGUI
```

4.6 Exercises

---

1. Write a Matlab GUI that contains one axes object. Set each of the following properties of the axes object to the indicated value.

i. Set both 'XLim' and 'YLim' properties to $[-10, 10]$.
ii. Set both 'XTick' and 'YTick' properties to **-10:2:10**.
iii. Set both 'XGrid' and 'YGrid' properties to 'on'.
iv. Set the 'ButtonDownFcn' property of the axes object to the callback function handle **@changeBackgroundColor_Callback**.

Use a cell to contain of list of strings representing colors.

```
color_list={'r','g','b','m','c','w'};
```

Write the callback function **changeBackgroundColor_Callback** that will change the 'Color' property of the axes object each time the mouse is clicked on the axes object. The colors should cycle through the list of colors in the order presented in **color_list**, returning to the first color of the list when the last color is used.

2. Write a Matlab GUI that contains one axes object. Set each of the following properties of the axes object to the indicated value.

i. Set both 'XLim' and 'YLim' properties to $[-10, 10]$.
ii. Set both 'XTick' and 'YTick' properties to **-10:2:10**.
iii. Set both 'XGrid' and 'YGrid' properties to 'on'.
iv. Set the 'ButtonDownFcn' property of the axes object to the callback function handle **@plotCurrentPoint_Callback**.

Write the callback function **plotCurrentPoint_Callback** so that it uses the **line** command to plot a point at the position of the mouse click. Set the 'LineStyle' to 'none', 'Marker' to 'o', 'MarkerSize' to 12, 'MarkerFaceColor' to 'r', and 'MarkerEdgeColor', to 'k'. *Hint: There's a bit of trickiness here as 'CurrentPoint' returns a* 2×3 *array. Try the following experiment. At the command prompt, type:*

```
>> figure
>> axes
>> grid
>> x=get(gca,'CurrentPoint')
```

The response might be something like the following:

```
x =
    0.3744    0.5658    1.0000
    0.3744    0.5658    0.0000
```

Note that **x(1,1)** *and* **x(1,2)** *contains the coordinates you need to use in your GUI.*

3. Write a Matlab GUI that contains one axes object. Set each of the following properties of the axes object to the indicated value.

i. Set both 'XLim' and 'YLim' properties to $[-10, 10]$.
ii. Set both 'XTick' and 'YTick' properties to **-10:2:10**.
iii. Set both 'XGrid' and 'YGrid' properties to 'on'.
iv. Set the 'ButtonDownFcn' property of the axes object to the callback function handle **@cursorCoordinates_Callback**.

Use Matlab's **text** command to print the coordinates of the origin on the screen.

```
hText=text(0,0,'(0,0)')
set(hText,'HorizontalAlignment','center')
```

Write the callback function **cursorCoordinates_Callback** that gets the 'CurrentPoint' of a mouse click in the axes object. Set the 'Position' property of the text object to the coordinates provided by 'CurrentPoint'. Concatenate the following string.

```
textStr=['(',num2str(x(1,1)),',',num2str(x(1,2)),')'];
```

Assign **textStr** as the value of the 'String' property of the text object.

4. Write a Matlab GUI that contains one axes object. Set each of the following properties of the axes object to the indicated value.

i. Set both 'XLim' and 'YLim' properties to $[-10, 10]$.
ii. Set both 'XTick' and 'YTick' properties to **-10:2:10**.

iii. Set both 'XGrid' and 'YGrid' properties to 'on'.
iv. Set the 'ButtonDownFcn' property of the axes object to the callback function handle **@cursorUpdateText_Callback**.

Create a panel as a child of the figure object with Matlab's **uipanel** command. Inside the panel, create a static text **uicontrol** as a child of the panel. Set the 'String' property of this control to '(0,0)' to start with. Write the callback function **cursorUpdateText_Callback** that gets the 'CurrentPoint' of a mouse click in the axes object and stores it in the variable **x**. Concatenate the following string.

```
textStr=['(',num2str(x(1,1)),',',num2str(x(1,2)),')'];
```

Assign **textStr** as the value of the 'String' property of the static text **uicontrol**. Thus, clicking the mouse in the axes should update the text object with the coordinates of the mouse cursor in the axes object.

5. Write a Matlab GUI that contains one axes object. Set each of the following properties of the axes object to the indicated value.

i. Set both 'XLim' and 'YLim' properties to $[-10, 10]$.
ii. Set both 'XTick' and 'YTick' properties to **-10:2:10**.
iii. Set both 'XGrid' and 'YGrid' properties to 'on'.
iv. Set the 'ButtonDownFcn' property of the axes object to the callback function handle **@updateLine_Callback**.

Initialize a line object with the command **hLine=line(0,0)**. Write a callback function **updateLine_Callback** that gets the 'CurrentPoint' of the mouse click in the axes object. Append the x- and y-coordinates of this new point to the value of the 'XData' and 'YData' properties of the line object having handle **hLine**. For example, to update the x-data, you will need these lines in the body of the callback.

```
newPoint=get(hObject,'CurrentPoint');
x=get(hLine,'XData');
set(hLine,'XData',[x,newPoint(1,1)])
```

Similar code is required to update the 'YData'. Thus, each time you click your mouse in the axes, a new line segmented is added, connecting the last point plotted to the current point clicked with the mouse.

6. Write a Matlab GUI that opens a figure with handle **hFig**. Set the property **WindowButtonMotionFcn** of the figure object to the function callback handle **@mouseCursor_Callback**. Create a **uipanel** object as a child of the figure object. Initialize a static text **uicontrol** that is a child of the **uipanel**. Initialize the 'String' property of this static text control to the empty string.

Write the callback with the form:

```
function mouseCursor_Callback(hObject,eventdata)
```

Get the current point of the mouse position in the figure window with:

```
x=get(hObject,'CurrentPoint');
```

Concatenate a text string as follows:

```
coordsStr=['(',num2str(x(1,1)),',',num2str(x(1,2)),')'];
```

Finally, set the 'String' property of the text **uicontrol** to the value **coordStr**. Moving the mouse in the figure window should show the pixel coordinates of the mouse in the static text control.

7. Write a Matlab GUI that opens a figure window with handle **hFig**. Set the **WindowButtonMotionFcn** property of the figure window to the callback **@mouseCursor_Calback**. Create an axes object with handle **hAx**. Set each of the following properties of the axes object to the indicated value.

i. Set both 'XLim' and 'YLim' properties to $[-10, 10]$.
ii. Set both 'XTick' and 'YTick' properties to **-10:2:10**.
iii. Set both 'XGrid' and 'YGrid' properties to 'on'.

Use the axes 'Position' property to locate and size the axes object within the figure window. Create a **uipanel** object as a child of the figure object. Initialize a static text **uicontrol** that is a child of the **uipanel**. Initialize the 'String' property of this static text control to the empty string. Write the callback with the form:

```
function mouseCursor_Callback(hObject,eventdata)
```

In the callback, get the current point of the mouse position in the figure window with:

```
newPoint=get(hObject,'CurrentPoint');
```

Next, get the position of the axes in the figure window with:

```
pos=get(hAx,'Position');
```

Test if the cursor is "in" the axes object with:

```
inAxes=(newPoint(1,1)>=pos(1)) && ...
            (newPoint(1,1)<pos(1)+pos(3)) && ...
            (newPoint(1,2)>pos(2)) && ...
            (newPoint(1,2)<pos(2)+pos(4));
```

Now, if the previous logical **inAxes** is **true**, get the 'CurrentPoint' in the axes (which differs from the 'CurrentPoint' in the figure window — it's given in axes coordinates), then use this value to set the 'String' property of the static text control. That is:

```
if inAxes
       x=get(hAx,'CurrentPoint');
       coordsStr=['(',num2str(x(1,1)),',',num2str(x(1,2)),')'];
       set(hText,'String',coordsStr);
end
```

Moving the mouse in the figure window should show mouse cursor coordinates in the static text control only when the mouse passes over the axes object. The coordinates will also be in axes coordinates, not figure pixel coordinates.

8. Adjust the GUI in **Exercise 7** so that the cursor will draw a path as it is dragged over the axes object. This is best accomplished by declaring and intializing a line object in the primary function.

```
hLine=line;
set(hLine,'XData',[],'YData',[]);
```

Now, if the mouse cursor is **inAxes**, then get 'CurrentPoint' from the axes object and use it to update the 'XData' and 'YData' properties of the line object, much as you did in **Exercise 5**.

9. The Matlab command **eval** is used to execute a string as a Matlab command. For example, try the following:

```
>> str='x=-3:3; y=x.^2';
>> eval(str)
y =
     9     4     1     0     1     4     9
```

Note that Matlab's **eval** command simply executes the string as a command. You can also store the output of the **eval** command as follows:

```
>> x=-3:3;
>> y=eval('x.^2')
y =
     9     4     1     0     1     4     9
```

This gives us a way that we can interact with the user who wants to plot an equation. We set up an edit box, the user types in the equation, then we use the **eval** command to evaluate the equation on a domain, then plot the result.

Write a Matlab GUI that contains one axes object. Set each of the following properties of the axes object to the indicated value.

i. Set both 'XLim' and 'YLim' properties to $[-10, 10]$.
ii. Set both 'XTick' and 'YTick' properties to **-10:2:10**.
iii. Set both 'XGrid' and 'YGrid' properties to 'on'.

Set up a **uicontrol** edit box in the figure window. If you wish, you can make things look nicer by first adding a panel, then placing your edit box inside the panel. Make the edit box large enough so that the user has room to enter his equation. You might set up a static text **uicontrol** in front of the edit box with its 'String' property set to 'y = ' so that the user knows that he should only enter the right-hand side of the equation $y = f(x)$.

Set the 'Callback' property of the edit box to the handle **@plotFunction_Callback**. In writing the callback **plotFunction_Callback**, you will have to vectorize the equation string before evaluating it with **eval**, setting all the *, /, and ^ to their "dot" equivalents. Matlab provides the **vectorize** command for this purpose. For example, try the following at the command prompt.

```
>> str='x^2*sin(x)/exp(x+1)';
>> vectorize(str)
ans =
x.^2.*sin(x)./exp(x+1)
```

In the callback, you will want retrieve the string from the edit box, vectorize it, then evaluate the string on the domain $[-10, 10]$ and store the result. You can then plot the resulting data.

```
x=linspace(-10,10);
eqnStr=get(hEditEquation,'String');
eqnStr=vectorize(eqnStr);
y=eval(eqnStr);
plot(x,y)
```

A more sophisticated approach would be to create an empty line object in the primary function just prior to the callback.

```
hLine=line;
set(hLine,'XData',[],'YData',[]);
```

Then, instead of using the **plot** command, you can update the data in the line object.

```
set(hLine,'XData',x,'YData',y);
```

10. Following the hints given in **Exercise 9**, write a Matlab GUI that emulates the behavior of a graphing calculator. First, set up an axes object with the properties set as follows.

i. Set both 'XLim' and 'YLim' properties to $[-10, 10]$.
ii. Set both 'XTick' and 'YTick' properties to **-10:2:10**.
iii. Set both 'XGrid' and 'YGrid' properties to 'on'.

Create three panels. One panel will hold an edit box where the user enters his equation, a second will hold edit boxes for the window parameters xmin, xmax, xscl, ymin, ymax, and yscl. You might also want to create an edit box in this second panel that contains the number of points to be plotted. Should a graph have a "jagged" appearance, increasing the number in this edit box would smoothen the graph. A third panel should contain a **uicontrol** that is a pushbutton. This pushbutton should have its 'String' property set to 'Graph' and 'Callback' property set to the handle **@plotFunction_Callback**.

The design should allow the user to enter his equation, window parameters, and number of points. When the user clicks the pushbutton with his mouse, the callback function **plotFunction_Callback** will grab the window parameter data and adjust 'Xlim', 'YLim', 'XTick', and 'YTick' of the axes object accordingly. Then the user's equation should be drawn over the domain [Xmin,Xmax].

This problem could potentially be expanded into a semester ending project, adding 'Zoom' menu or a 'Calc' menu for finding zeros and extrema. This project is very open-ended with lots of potential for further implementation of ideas and utilities.

4.6 Answers

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2. linePlot.m

4. cursorText.m

8. mouseDraw.m