Chapter 5

Higher-Order Linear Differential Equations

Application 5.1
Plotting Second-Order Solution Families

This application deals with the computer-plotting of solution families like those illustrated in Figs. 5.1.6 and Fig. 5.1.7 in the text. Show first that the general solution of the differential equation

$$y'' + 3y' + 2y = 0 \tag{1}$$

is

$$y(x) = c_1 e^{-x} + c_2 e^{-2x}. \tag{2}$$

Then show that the particular solution of Eq. (1) satisfying the initial conditions $y(0) = a,\ y'(0) = b$ is

$$y(x) = (2a+b)e^{-x} - (a+b)e^{-2x} \tag{3}$$

- For Fig. 5.1.6, substitution of $a = 1$ in (3) gives

$$y(x) = (b+2)e^{-x} - (b+1)e^{-2x}. \tag{4}$$

for the solution curve through the point $(0,1)$ with initial slope $y'(0) = b$.

- For Fig. 5.1.7, substitution of $b = 1$ in (3) gives

$$y(x) = (2a+1)e^{-x} - (a+1)e^{-2x}. \tag{5}$$

for the solution curve through the point $(0,a)$ with initial slope $y'(0) = 1$.

In the technology-specific sections following the problems below, we illustrate the use of computer systems like *Maple*, *Mathematica*, and MATLAB to plot simultaneously a family of solution curves like those defined by (4) or (5). Start by reproducing Figs. 5.1.6 and 5.1.7 in the text. Then, for each of the following differential equations, construct both a family of different solution curves satisfying $y(0) = 1$ and a family of different solution curves satisfying the initial condition $y'(0) = 1$.

1. $y'' - y = 0$
2. $y'' - 3y' + 2 = 0$
3. $2y'' + 3y' + y = 0$
4. $y'' + y = 0$ (with general solution $y(x) = c_1 \cos x + c_2 \sin x$)
5. $y'' + 2y' + 2y = 0$ (with general solution $y(x) = e^{-x}(c_1 \cos x + c_2 \sin x)$)

Using *Maple*

Using Eq. (4), the particular solution with $y(0) = 1,\ y'(0) = b$ is defined by

```
partSoln := (b+2)*exp(-x)-(b+1)*exp(-2*x);
```

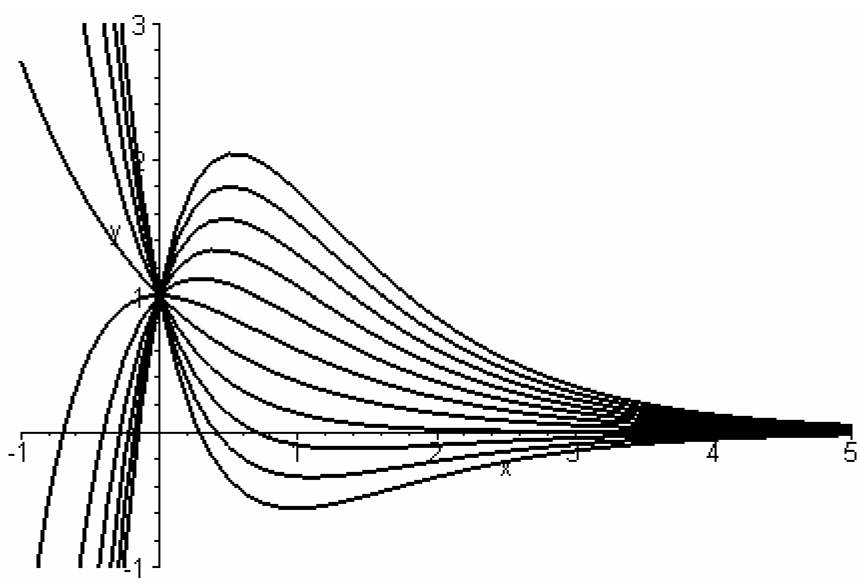
$$partSoln := (b+2)e^{(-x)} - (b+1)e^{-(2x)}$$

The set of such particular solutions with initial slopes $b = -5, -4, -3, \ldots, 4, 5$ is then defined by

```
curves := {seq(partSoln, b =-5..5)}:
```

We plot these 11 curves simultaneously on the x-interval $(-1, 5)$ with the single command

```
plot(curves, x =-1..5, y =-1..3);
```



Using *Mathematica*

Using Eq. (5), the particular solution with $y(0) = a,\ y'(0) = 1$ is defined by

```
partSoln = (2a + 1) Exp[-x] - (a + 1) Exp[-2x]
```

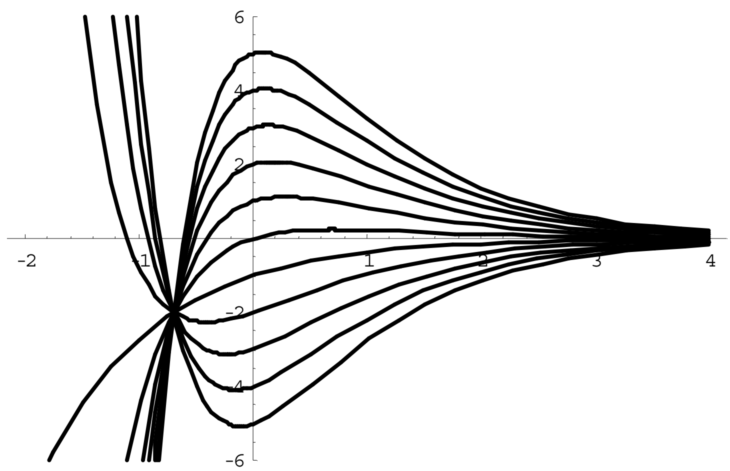
$$(2a+1)e^{-x} - (a+1)e^{-2x}$$

The set of such particular solutions with initial slopes $a = -5, -4, -3, \ldots, 4,\ 5$ is then defined by

```
curves = Table[ partSoln, {a,-5,5} ];
```

We plot these 11 curves simultaneously on the x-interval (–1, 5) with the single command

```
Plot[Evaluate[curves], {x,-2,4}, PlotRange->{-6,6}]
```



Using MATLAB

Using Eq. (5), the particular solution with $y(0) = y'(0) = a$ is defined by

$$y(x) = 3a\,e^{-x} - 2a\,e^{-2x}.$$

We can plot the 11 solution curves with $a = -5, -4, -3, \ldots, 4,\ 5$ on the interval

```
x = -1 : 0.02 : 5;  % x-vector from x=-1 to x=5
```

with the single **for** loop

```
for a = -5 : 5        % for a = -1 to 1 do
    y = 3*a*exp(-x) - 2*a*exp(-2*x);
    plot(x,y,'k')
    axis([-1 5 -7 7]), hold on
    end
```