College of the Redwoods

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The Optimal Height for Releasing Paratroopers

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Math 55 Differential Equations
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Purpose

When paratroopers are released from a plane during combat they must obviously be dropped high enough so that there is enough time for the parachute to slow the descent and prevent injuries from the impact of landing. At the same time, however, they must be released low enough that they spend a minimal time in the air where they are helplessly exposed to withering enemy fire. The purpose of this project is to determine the optimal height for releasing the paratroopers that will provide for a safe landing, while minimizing the time they are exposed to enemy ground fire.

Background

During the mass aerial deployment of combat paratroopers a continuous stream of soldiers are released simultaneously from each side of the aircraft, jumping out of the plane one after another like lemmings.

- Their parachutes are deployed by means of a $9.1\,\mathrm{m}$ long static line which connects their parachute to the plane and acts as a "rip-cord".
- In order to prevent injuries caused by impact, it is essential that the impact velocity of the paratrooper be between -4.6 and $-5.2\,\mathrm{m/s}$ (About the equivalent of a jump from a $5'$ wall).

Creating the Model

$$\sum F = ma$$

$$ma = -mg - kv$$

This leads to the following system of equations:

$$x'(t) = v$$

$$v'(t) = -g - \frac{k}{m}v$$

Where,

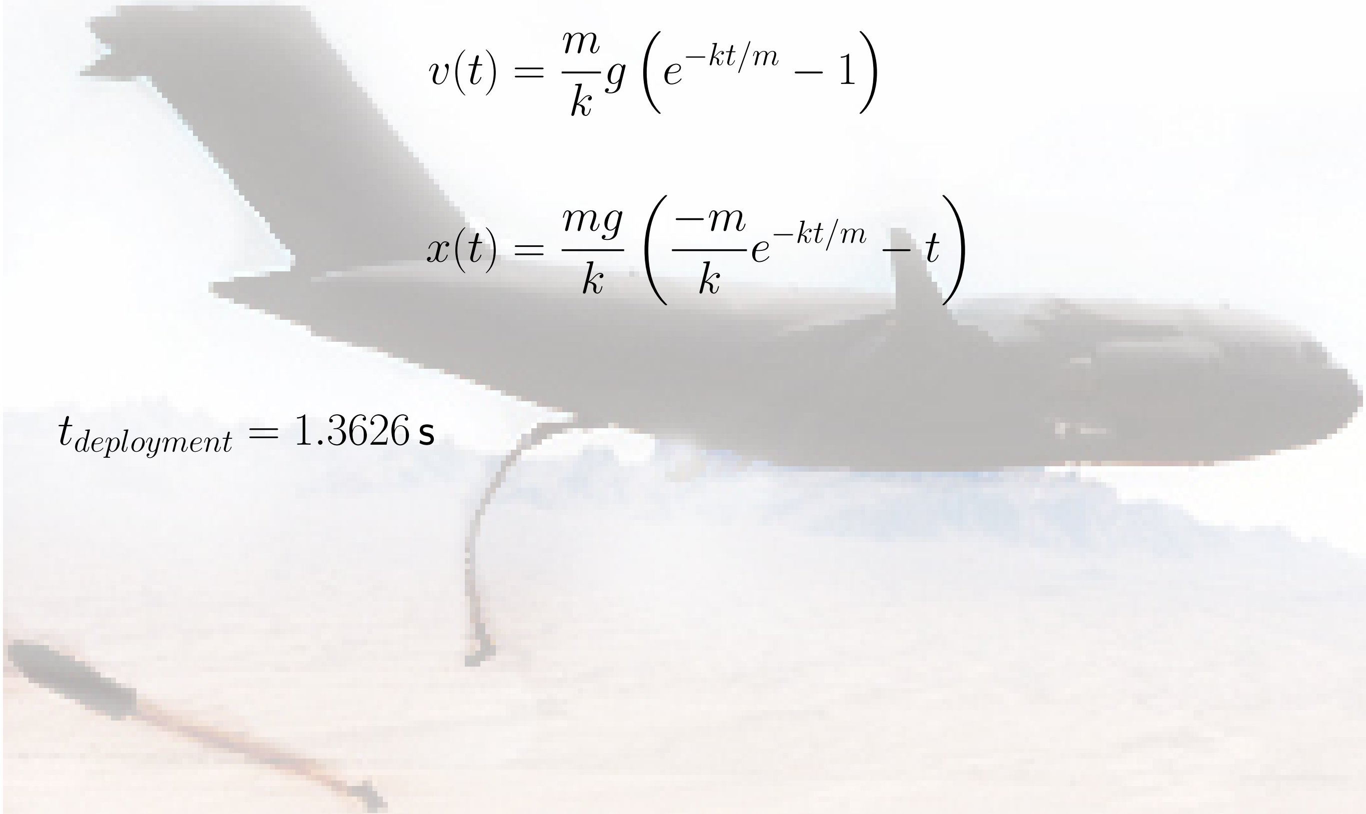
$$k = \begin{cases} k_1 & 0 \leq t < t_d \\ k_2 & t \geq t_d \end{cases}$$

The system of equations also leads to...

$$v(t) = \frac{m}{k} g \left(e^{-kt/m} - 1 \right)$$

$$x(t) = \frac{mg}{k} \left(\frac{-m}{k} e^{-kt/m} - t \right)$$

$t_{deployment} = 1.3626\,\mathsf{s}$



Results of Trial 1

When the solver is used, with the initial conditions

- $v(0) = 0$
- $t_{deployment} = 1.3626\,\mathrm{s}$,

We get the solution $t_i = 3.7417\,\mathrm{s}$

The absolute value of the position at t_i gives us a release height of $24.8\,\mathrm{m}$

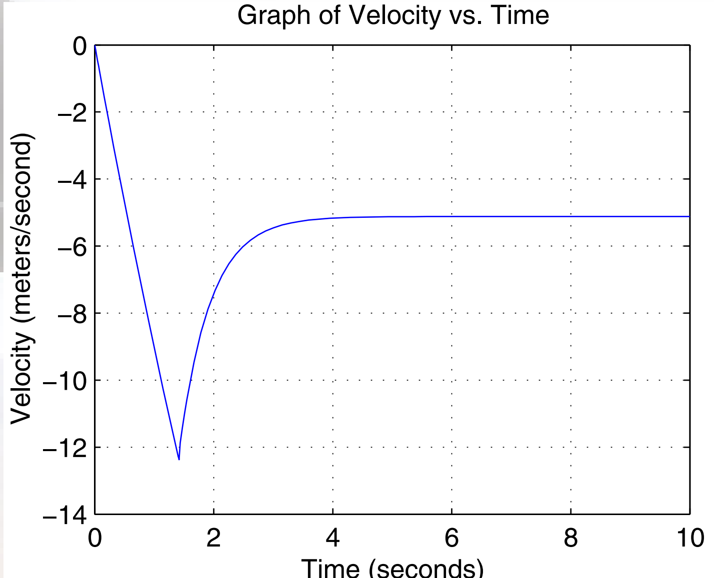


Figure 1: The graph shows the velocity passing through $-5.2\,\mathrm{m/s}$ on the way to a terminal velocity of approximately $5.12\,\mathrm{m/s}$

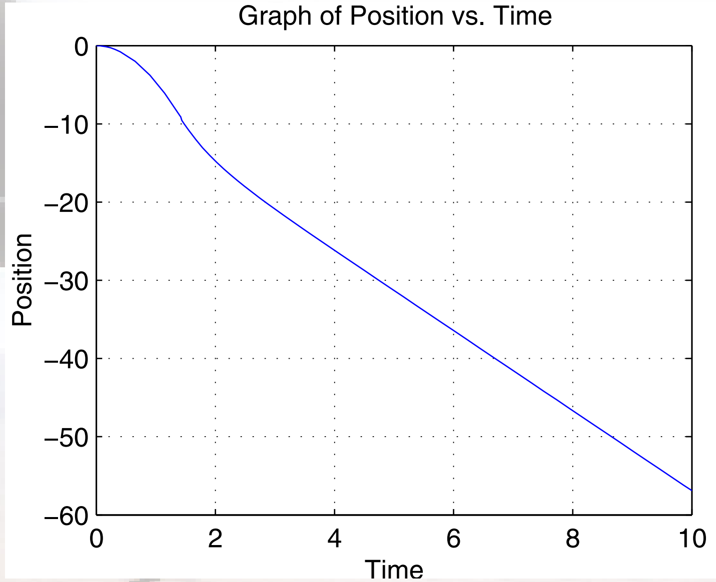


Figure 2: The graph shows the position at t_i is approximately $-24.8\,\mathrm{m}$

Analysis of the Answer

- $24.8\,\mathrm{m}$ seems like a frighteningly height from which to jump out of an aircraft. It is, and it would likely kill you.
- The model does not take into account any motion in the horizontal direction. Although a release height $24.8\,\mathrm{m}$ would give you plenty of time to decelerate in the vertical direction, you would still impact the ground with enough horizontal motion to cause severe injuries, if not death.
- The military has wisely (For once) determined that paratroopers must not only have enough time to decelerate, but also have enough time to determine if their parachute has deployed properly and take corrective action if it has not. This means that they must have time to deploy their reserve parachute.

Results of Trial 2

When the solver is used, with the initial conditions

- $v(0) = 0$
- $t_{deployment2} = 7.3626\,$s,

We get the solution $t_{i2} = 11.3739\,$s

The absolute value of the position at t_{i2} gives us a release height of $224.46\,$m

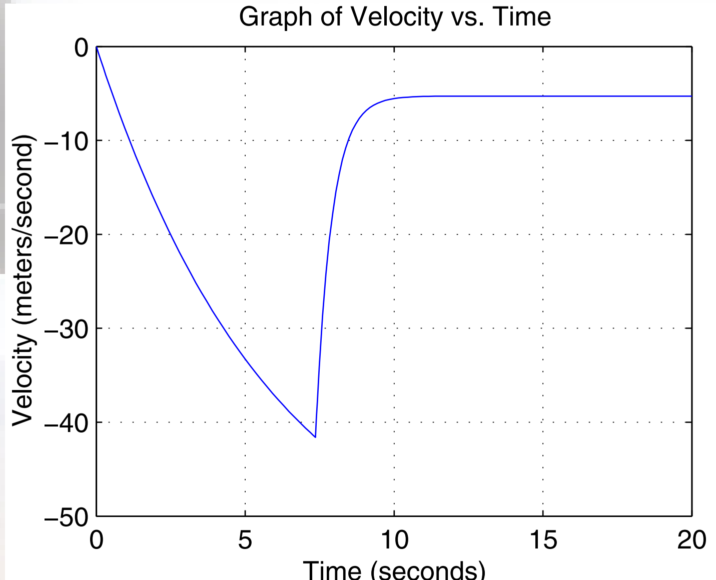


Figure 3: The graph shows the velocity passing through $-5.3\,\mathrm{m/s}$ on the way to a terminal velocity of approximately $-5.28\,\mathrm{m/s}$

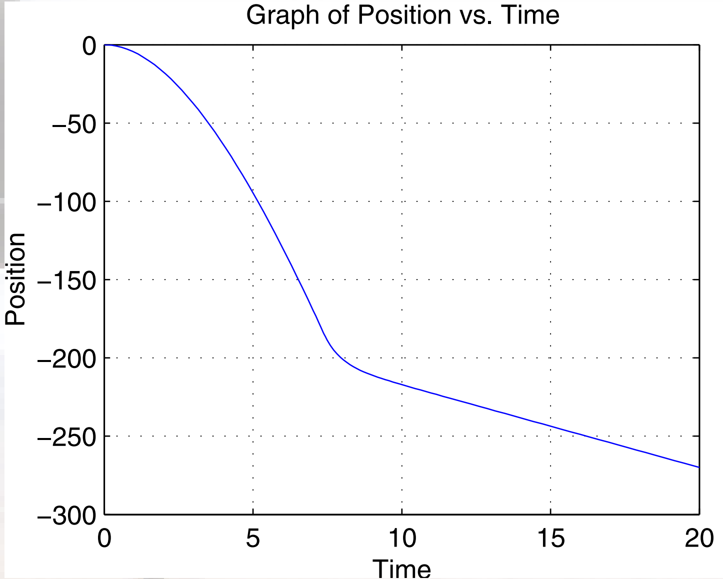


Figure 4: The graph shows the position at t_{i2} is approximately $-224.46\,\mathrm{m}$

Final Answer

We now know that the optimal height for releasing paratroopers is 224.46 m. Just to verify the effects of being dropped from that altitude with a normal parachute deployment, we run the solver again...

The new time of impact is 42.7525 s

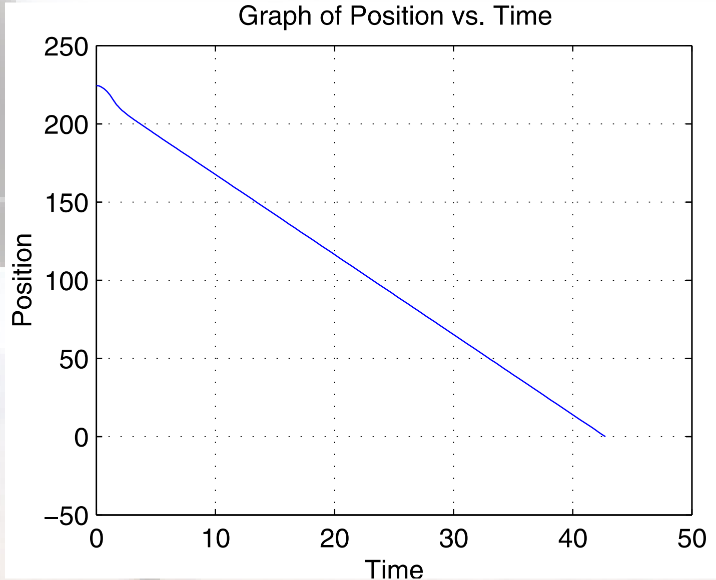


Figure 5: The graph shows the position is equal to 0 at approximately $t = 42.7525\,\mathrm{s}$

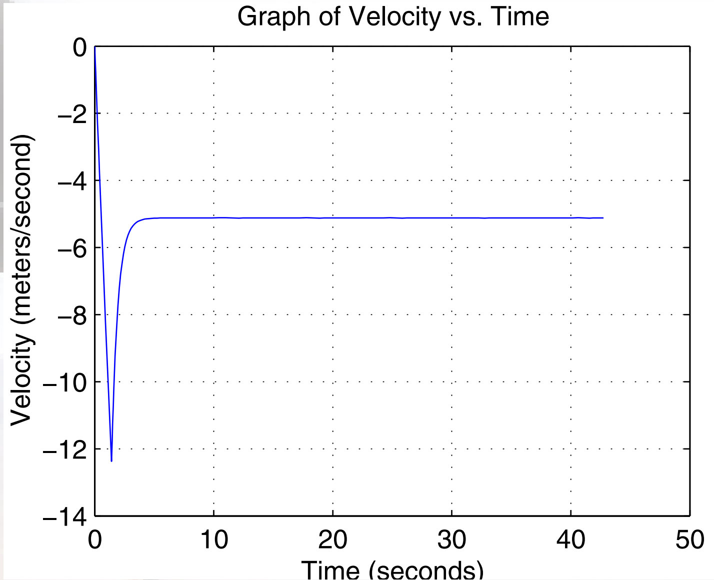


Figure 6: The graph shows the velocity getting arbitrarily close to terminal velocity very early in the jump

References

[1] Arnold, Dave *College of the Redwoods*

[2] Meade, Douglas B.,Allan A. Struthers *Differential Equations in the New Millennium: the Parachute Problem*

[3] Meade, Douglas B. *ODE Models for the Parachute Problem*

[4] Meade, Douglas B. *Maple and the Parachute Problem: Modelling with an Impact*

[5] Stanley, Michael *2/504th Parachute Infantry Regiment*