The Nephroid

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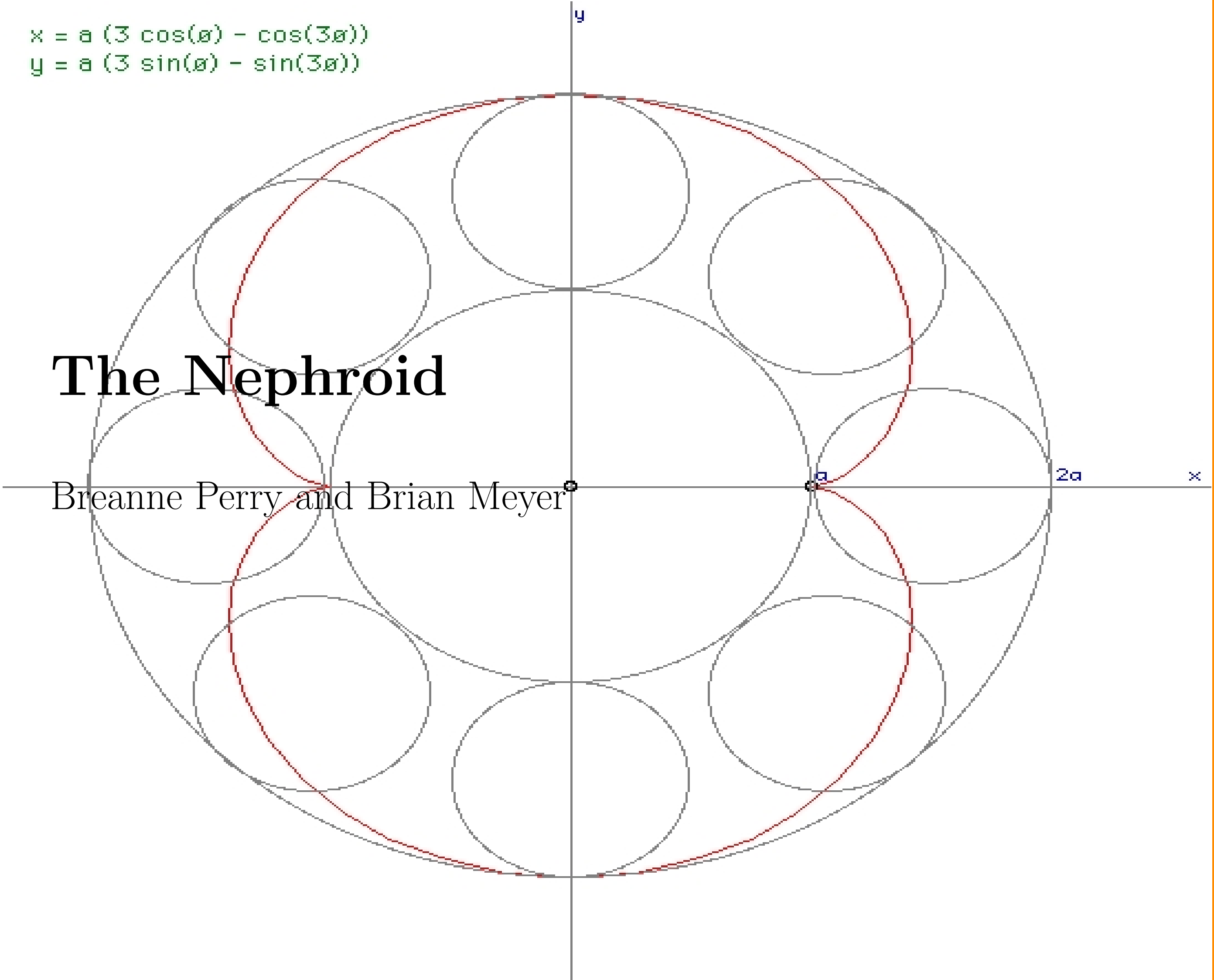


Introduction:Definitions

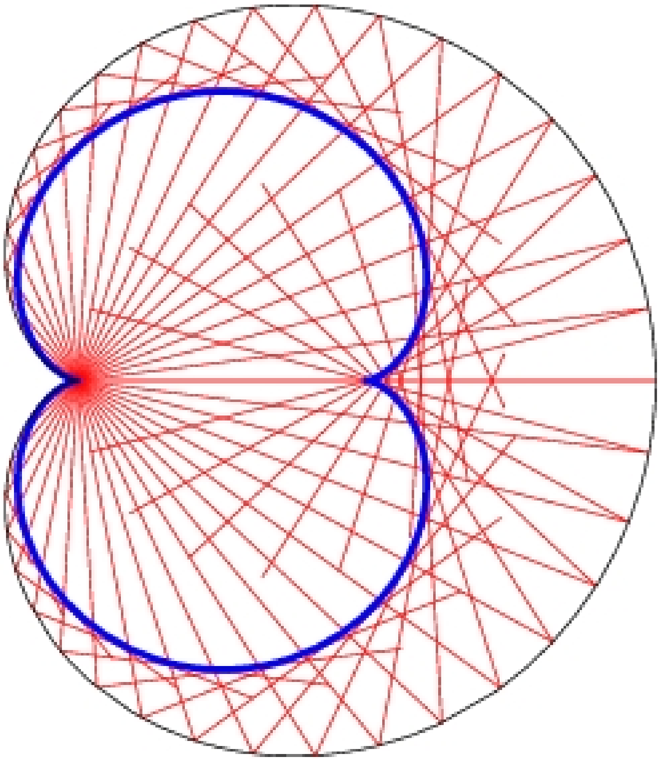
Cycloid:The locus of a point on the outside of a circle with a set radius (r) rolling along a straight line.

Epicycloid:An epicycloid is a cycloid rolling along the outside of a circle instead of along a straight line. The description of an epicycloid depends on the radius of the circle being traversed (b) and the radius of the rolling circle (a).

Nephroid:A bicuspid epicycloid. Nephroid Literally means "kidney shaped". The nephroid arises when the radius of the center circle is twice that of the rolling circle (b=a/2).

Catacaustic:The curve formed by reflecting light off of another curve.

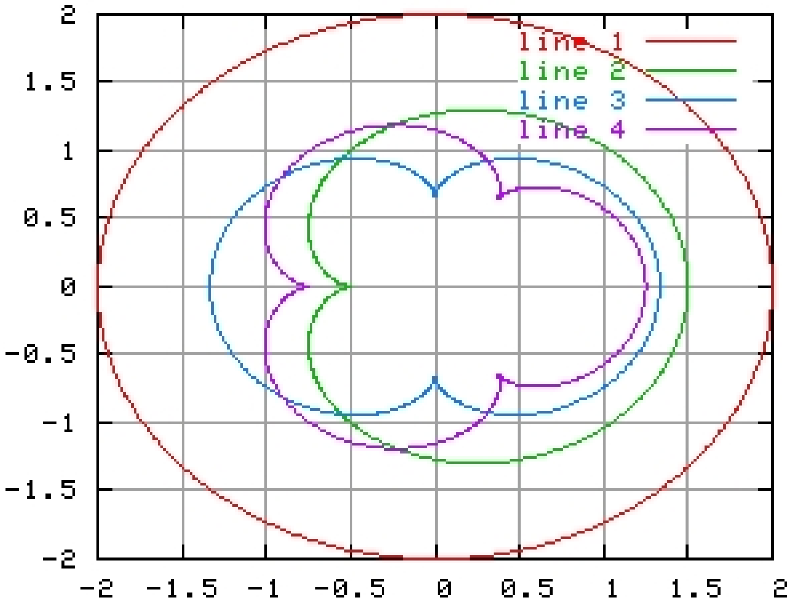
Catacaustic of the Cardiod



History of the Nephroid

The Nephroid was first studied by Christiaan Huygens and Tshirnhausen in 1678, they found it to be the catacaustic of a circle when the light source is at infinity. George Airy proved this to be true through the wave theory of light in 1838. In 1878 Richard Proctor coined the term nephroid in his book *The Geometry of cycloids*. The term Nephroid replaced the existing two-cusped epicycloid.

Some members of the epicycloid family



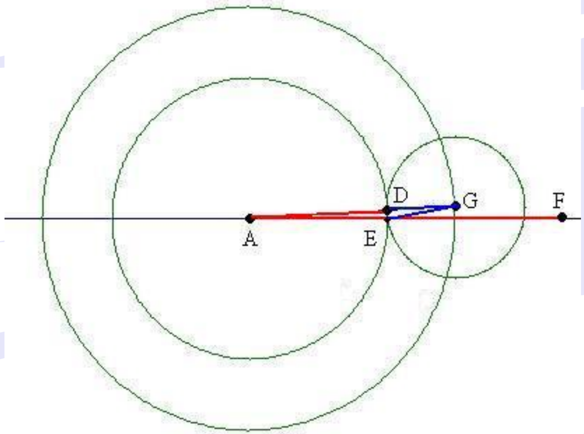
line 2 b=a Cardioid

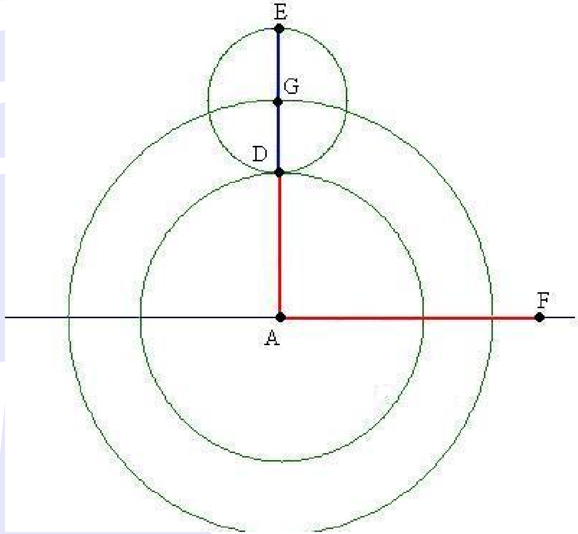
line 3 b=a/2 Nephroid

line 4 b=a/3 three cusped epicycloid

Deriving the formula for the Nephroid

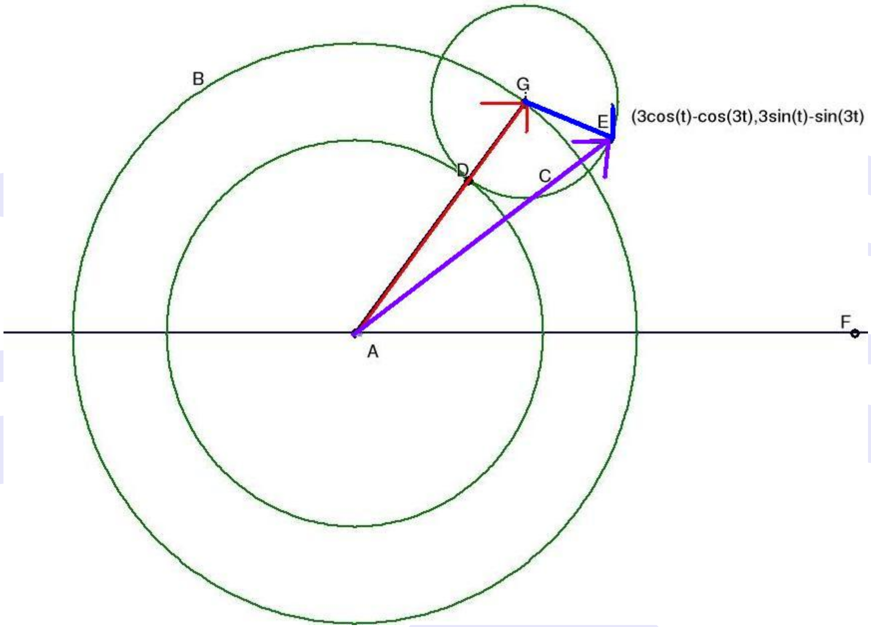
First to determine the relationship between the change in angle from the large circle to the small circle, we analyze the the relationship between point D and E as they traverse the outside of their respected circles. As point D travels $\pi/2$ point E travels π, therefore $\angle DGE$ (blue) is twice as large as $\angle FAD$ (red).



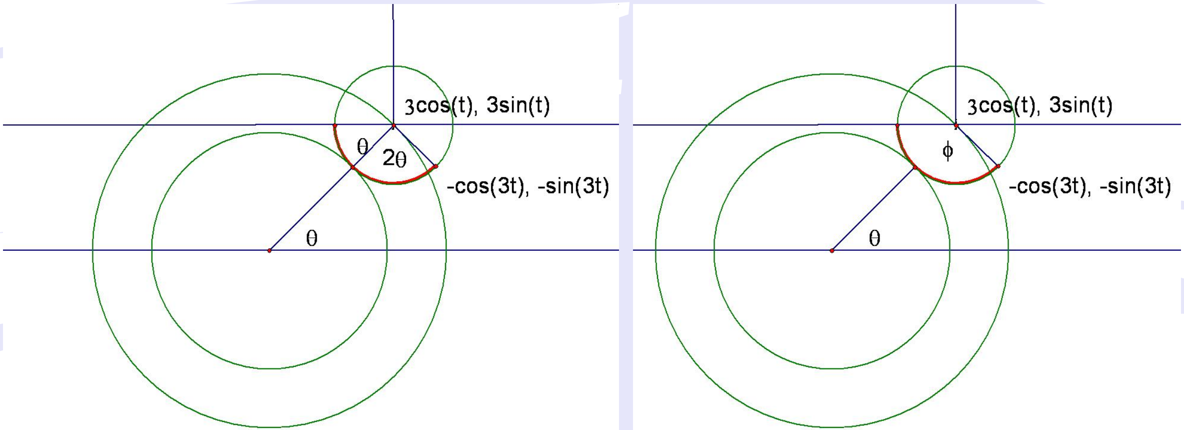


From this relationship we can start the parametrization of the nephroid. We can do this with a vector approach. First we will get to the center of circle C (red vector) then get a vector to point towards the point of interest (blue vector). Then by adding these two vectors (purple vector) we will arrive at the parametric equation for the Nephroid.

$$\vec{AG} + \vec{GE} = \vec{AE}$$



The center of circle C will always travel along a circle with a radius 3a. Therefore, to get to this point we can use $(3a\cos\theta, 3a\sin\theta)$ Then by defining a new coordinate system we are able to locate the point of interest on the outside of the small circle. The outside of the small circle is defined by $(r\cos\theta, r\sin\theta)$. But we know that $r = \frac{1}{2}a$, which in this case equals one. We can define the angle past the second quadrant as ϕ. Since the point is past the second quadrant, we add π, $(\cos(\pi+\phi), \sin(\pi+\phi))$=$(-\sin(\phi), -\cos(\phi))$ and we know, by geometry, that $\phi = 3\theta$. From this we get $(-\sin(3\theta), -\cos(3\theta))$



$$\vec{AE} = \vec{AG} + \vec{GE} \tag{1}$$

$$\vec{AG} = \langle 3\cos(\theta), 3\sin(\theta) \rangle \tag{2}$$

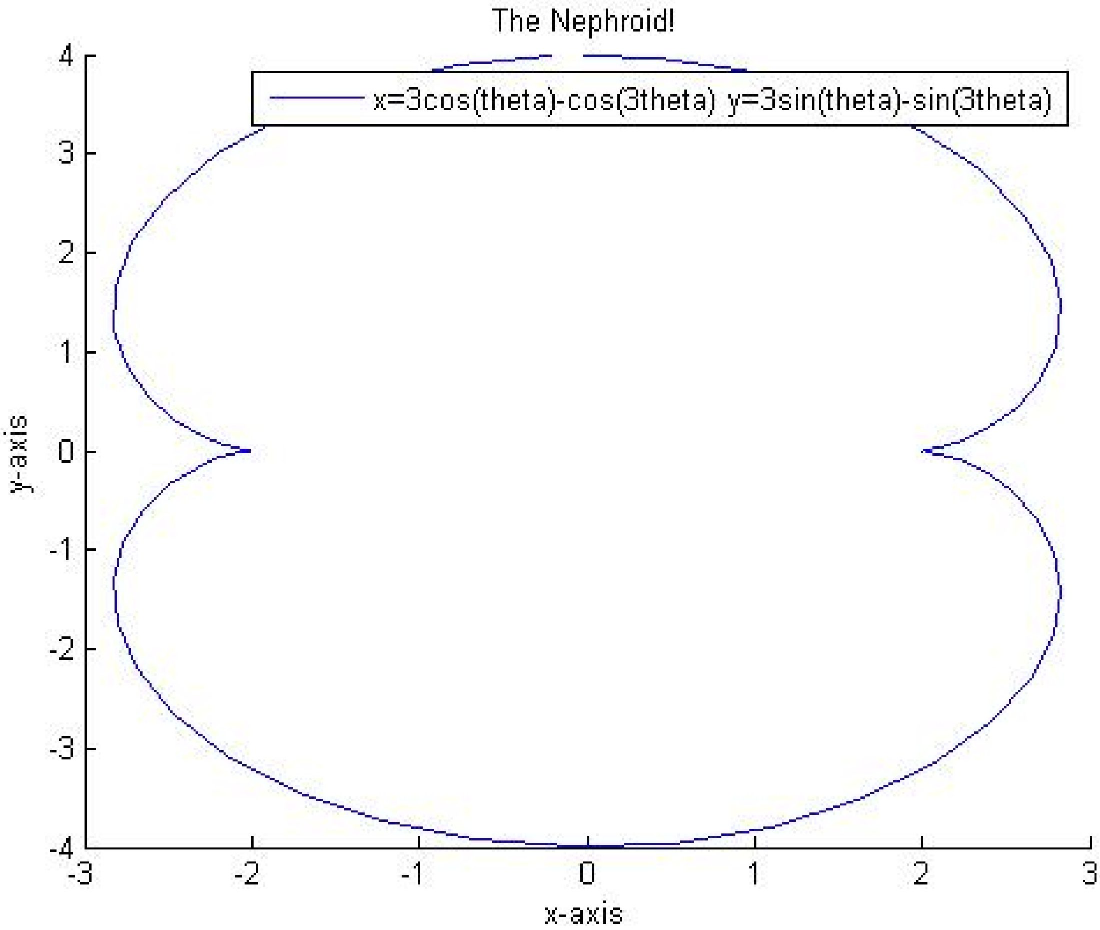
$$\vec{GE} = \langle -\sin(3\theta), -\cos(3\theta) \rangle \tag{3}$$

$$\vec{AE} = \langle 3\cos(\theta) - \cos(3\theta), 3\sin(\theta) - \sin(3\theta) \rangle \tag{4}$$

Therefore, the parametric equations are:

$$x = (3\cos(\theta) - \cos(3\theta)) \tag{5}$$

$$y = (3\sin(\theta) - \sin(3\theta)) \tag{6}$$


The Nephroid!
x=3cos(theta)-cos(3theta) y=3sin(theta)-sin(3theta)
4
3
2
1
0
-1
-2
-3
-4
y-axis
-3
-2
-1
0
1
2
3
x-axis

Creating The Nephroid on Geometers Sketch Pad

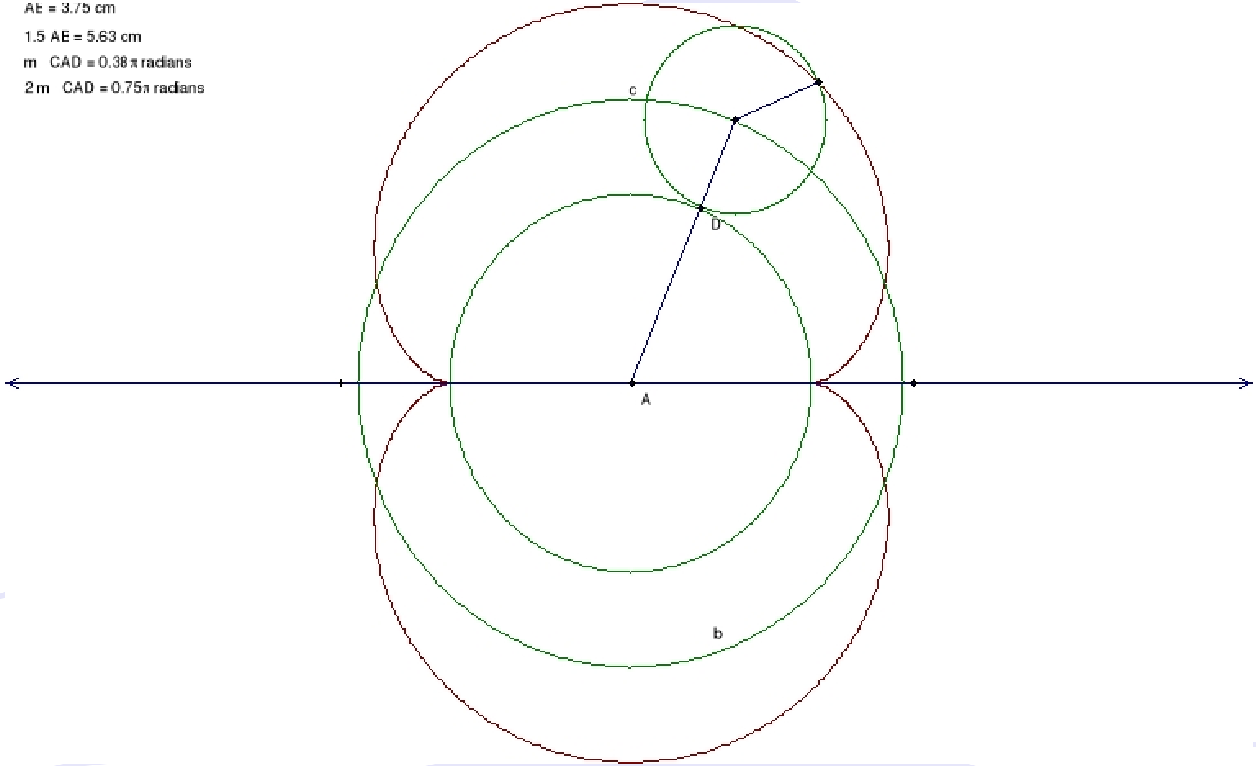
Some previous experience with sketch pad will be required before beginning. First make sure you are in radian mode then create a circle (A) and measure the radius. Next create another circle (B) with a radius $\frac{3}{2}$ times the first circle. Once completed place a line segment from the center of both circles to the outside edge of circle (B). Start a circle (C) from the end of the segment and attach it to the inside circle along the segment. Next create a horizontal line through the center of the first two circles for an x-axis. Measure the angle (θ) between the line and the segment, and multiply this answer by two. Double click the center of circle (c), marking it the center of rotation. Then select the point (d) and rotate it by 2θ. Now create a line segment from the center of circle (c) to the rotated point. Select the center of circle (c) and the rotated point, then create the locus and you have a nephroid.

AE = 3.75 cm

1.5 AE = 5.63 cm

m CAD = 0.38π radians

2 m CAD = 0.75π radians



Works Cited

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