Velocity and Aerodynamic Drag

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Abstract

This paper will discuss the *Velocity Dependence of Aerodynamic Drag* and some of the basics that determine the dependence. First we will show the velocity dependence for a relatively small Reynolds Number, second for a large Reynolds Number $> 250,000$. Finally we will discuss the implications and examples of both approximations.

1 Brief History

Velocity Dependence of Aerodynamic Drag is simply what dependence the drag of an object will be on the velocity of this object. Traditionally mathematicians and engineers approximated it to be $F_d = bv$ for all ranges of velocity. This is simply saying that as the velocity of object increases the drag force will be proportional to its velocity. That is its linearly proportional to its velocity. As time has passed it has been found that the force due to drag is not linearly proportional to the velocity. It has been found that it is proportional to $F_d = bv^n$ where n is a factor of the Reynolds Number of that object. There are many factors that affect drag, but Reynolds Number is one of the most significant. *The Velocity Dependence of Aerodynamic Drag* also can't be mathematically derived accurately that is. The proof of the velocity dependence is dependent of wind–tunnel experiments to verify the dependence of v and the value of b.

1.1 Discussion of Small and Large *R*eynolds Number

The general factors of drag, aerodynamic friction, are density and viscosity of the fluid, air being considered a fluid. The definition of Reynolds Number is

$$R = \frac{\rho d v}{\mu}, \tag{1}$$

where,

$$\begin{aligned} \rho &= \text{density of the fluid,} \\ v &= \text{velocity of the body in the fluid,} \\ \mu &= \text{is the viscosity of the fluid,} \\ d &= \text{is a characteristic length.} \end{aligned}$$

Indy Car Statistics									
Speed (Mph)	25	50	90	133	162.5	178	193	215	220
Drag Force(LBS)	30	40	125	250	375	500	625	750	875

Table 1: Drag Force Data.

The *Reynolds Number* for a body that is large in size and slow in velocity could produce a equivalent *Reynolds Number* of a very small object that travels with a high velocity. This seems like it isn't possible logically until we evalute the definition of /emphReynolds Number and see that it is.

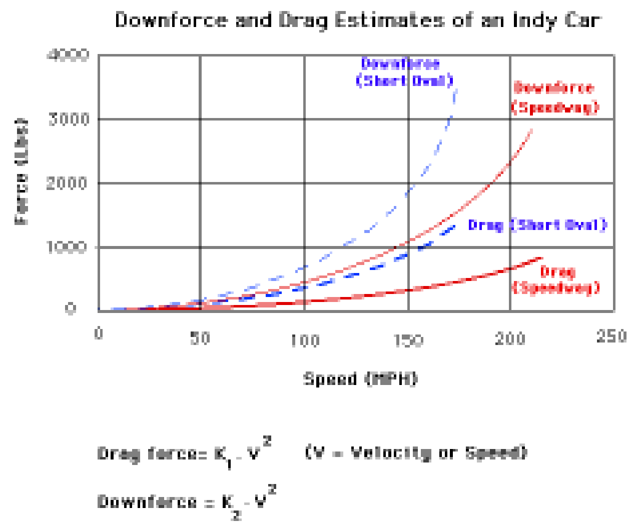

Figure 1: Drag Estimates of an Indy Car (Aerodynamics in Car Racing)

Figure 1 was taken from a website that researches Indy Car Aerodynamics. They concluded that the drag force due to aerodynamic drag is proportional to the square of the velocity, as shoen in Figure 1. When I saw their data (Table 1), I graphed it fit and fit a equation to it which is $F_d = 0.015v^{2.0061}$. These data points and equation can be seen by the Figure 2. When we look at fit the equation we see that it is $F_d = bv^{2.0061}$ which is close to a squared velocity term.

Now that we have verified our model for a *Reynolds Number* that is relatively large (See Table 2), we can move on to the implications of this finding.

1.2 Discussion of Models

Now let's look at some models that will show how drastic of an error it would be to use a linear model rather than a squared model. First let's look at a sky diver that is falling from a plane without an open parachute (See Figure 3). We find from the force diagram that the Differential Equation is

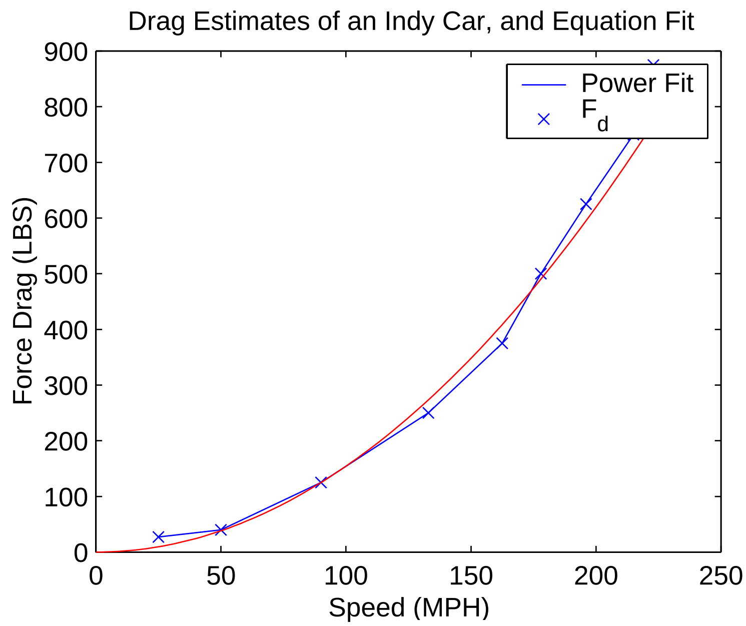

Figure 2: Drag Estimates of an Indy Car, and Equation Fit

Typical *Reynolds Numbers*		
Object	Characteristic Length	Typical *Reynolds Number*
Submarine	Length	3000,000,000
Small Aircraft	Chord	5,000,000
Parachutes	Diameter	2,500,000
Sky Diver	Diameter	1,000,000
Baseball	Diameter	250,000
Model Airplanes	Chord	50,000
Butterfly	Chord	7,000
Dust Particle	Diameter	1

Table 2: Reynold's Numbers of Aerodynamic bjects.

$$v' = g - \frac{k}{m}v \tag{2}$$

where $g = 9.8\,\mathrm{m/s^2}$, and $m = 70\,\mathrm{kg}$ and $k = 0.25$. This is the first equation and now I would like to introduce the second, which is a squared dependency model.

$$v' = g - \frac{k}{m}v^2 \tag{3}$$

We will be using the same constants for these models. The Model 2 is an separable ODE but the second is much more complicated, so I will use Dfield5 to analyse both systems. The first system can been seen in Figure 4 which leads us to conclude that the terminal velocity of the sky diver would be approximately 2750 m/s which with out an in depth discussion of terminal velocity can be said to be grossly un–correct; But if you look at Figure 5, you will see that the terminal velocity is about 53 m/s, which is reasonable. Terminal velocity is calculated by Equation 4.

$$v_{(ter\min al)} = \sqrt{\frac{mg}{k}} = \sqrt{\frac{(70)(9.8)}{\frac{1}{4}}} \approx 50\,\mathrm{m/s} \approx 120\,\mathrm{mph} \tag{4}$$

2 Conclusion

We have now looked at what the correct *Velocity Dependence of Aerodynamic Drag*, which is $n = 2$ from our previous equation $F_d = bv^n$, or simply $F_d = bv^2$. This model is still only an approximation of what the true coefficients are. To find the true coefficients, a wind tunnel would need to be used. Objects that have an extremely large *Reynolds Number* such as an asteroid, will have a velocity dependence of $n = 4$, and such things as snails will definitely have a linear dependence or, $n = 1$. To conclude, I would recommend visiting *Dave*

Figure 3: Force Diagram Sky Diver

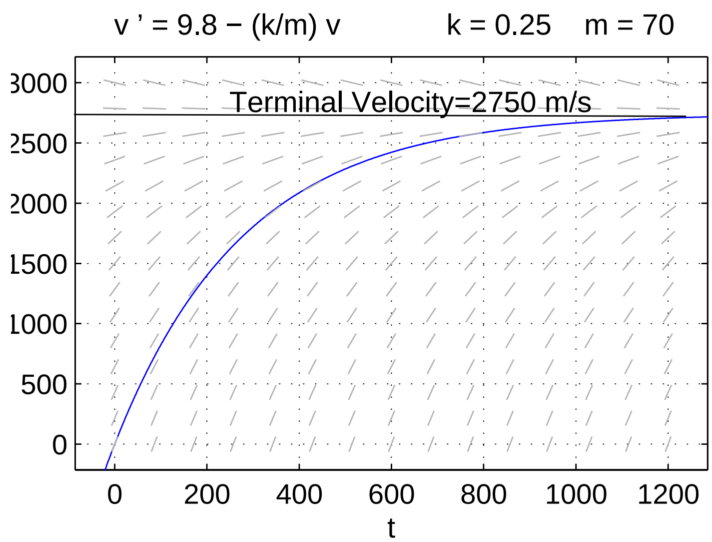

Figure 4: Parachute Model of Linear Dependence

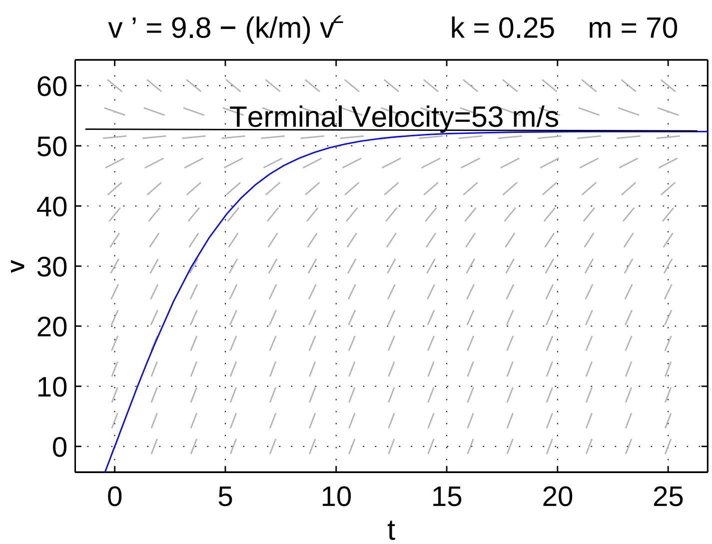

Figure 5: Parachute Model of Squared Dependence

Arnolds Differential Equations Project site to explore other differential equation projects and to look at my presentation that explains the types of Aerodynamic Drag (air friction) and shows the causes of certain non–streamlined objects. It also shows the affect that an object experiences as it breaks the sound barrier.

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