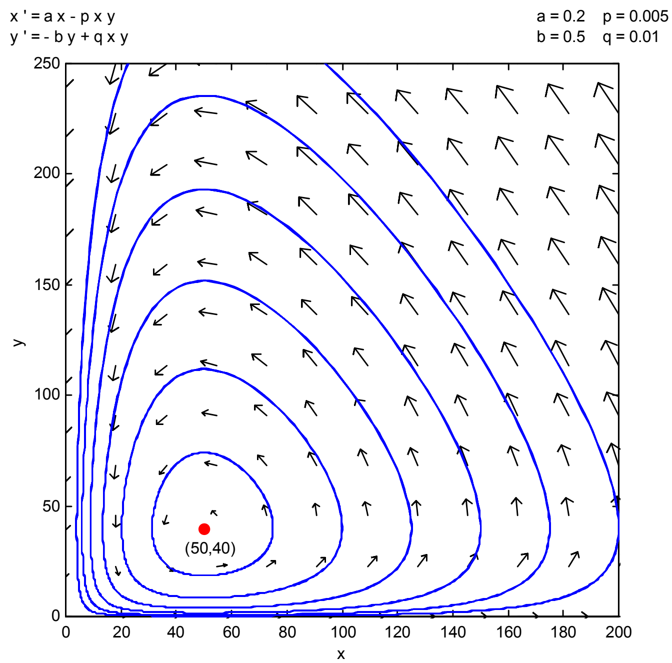
Application 9.3

Predator-Prey and Your Own Game Preserve

The closed trajectories in the figure below represent periodic solutions of a typical predator-prey system, but provide no information as to the actual periods of the population oscillations these solutions describe.

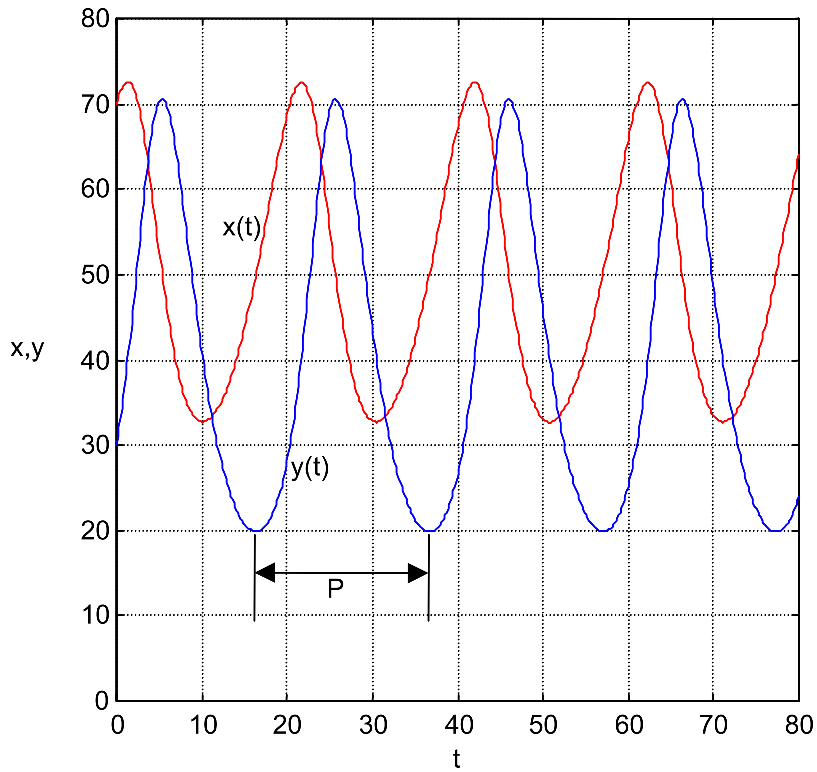


The period P of a particular solution $(x(t), y(t))$ can be gleaned from the graphs of x and y as functions of t. The figure on the next page shows these graphs for the particular solution satisfying the initial conditions $x(0) = 70,\ y(0) = 30$. The labeled period P indicates how the period with which the x- and y-populations oscillate can be measured — at least approximately — on such a figure.

Investigation 1

You own a large forest hunting preserve that you originally stocked with F_0 foxes and R_0 rabbits on January 1, 1999. The following differential equations model the numbers $R(t)$ of rabbits and $F(t)$ of foxes t months later.

$$\begin{aligned} \frac{dR}{dt} &= 0.01\,p\,R - 0.0001\,a\,RF \\ \frac{dF}{dt} &= -0.01\,q\,F - 0.0001\,b\,RF \end{aligned} \qquad (1)$$



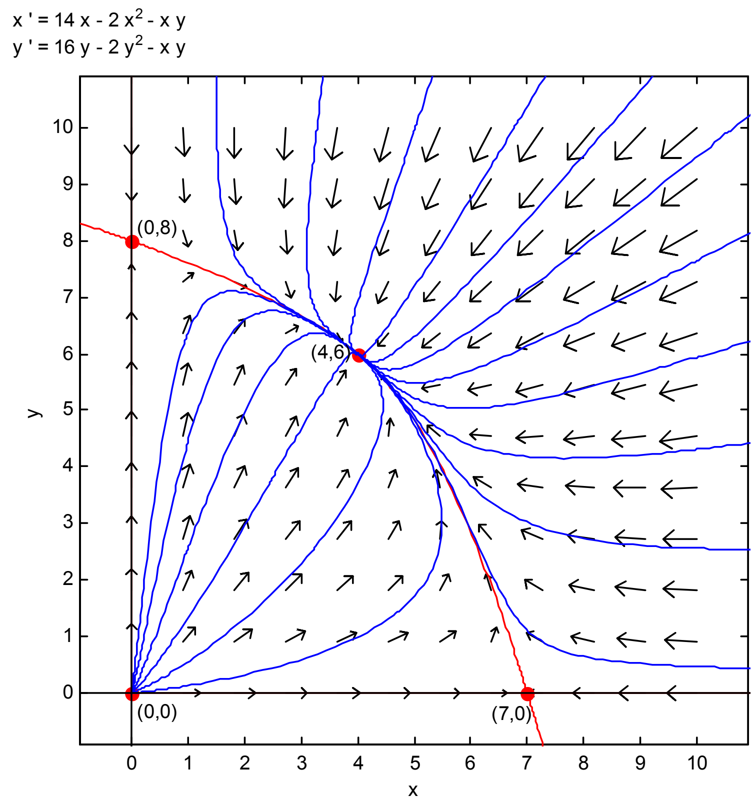
where p and q are the two largest digits (with $p < q$), and a and b are the two smallest nonzero digits (with $a < b$) in your student ID number.

The numbers of foxes and rabbits will oscillate periodically, out of phase with each other (like the functions $x(t)$ and $y(t)$ in the figure above). Pick your initial numbers F_0 of foxes and R_0 of rabbits -- perhaps several hundred of each -- so that the resulting solution curve in the RF-plane is a fairly eccentric closed curve. (The eccentricity may be increased if you start with a largish number of rabbits and a smallish number of foxes, as any hunting preserve owner would naturally do -- since foxes eat rabbits.)

Your task then is to determine

- The period of oscillation of the rabbit and fox populations;
- The maximum and minimum numbers of rabbits, and the calendar dates on which they first occur; and
- The maximum and minimum numbers of foxes, and the calendar dates on which they first occur.

With computer software that can plot both RF-trajectories and tR- and tF-solution curves like those above, you can "zoom in" graphically on the points whose coordinates provide the requested information.



Investigation 2

For a more general ecological system to investigate, let a, b, c, d be the four smallest nonzero digits (in any order) and m, n the two largest digits in your student ID number. Then consider the system

$$\frac{dx}{dt} = x(P - ax \pm by), \qquad \frac{dy}{dt} = y(Q \pm cx - dy) \tag{2}$$

Where $P = ma - (\pm nb)$ and $Q = nd - (\pm mc)$, with the same choice of plus/minus signs in dx/dt and P and (independently) in dy/dt and Q — so that (m, n) is a critical point of the system. Then use the methods of the Section 6.1 application to plot a phase plane portrait for this system in the first quadrant of the *xy*-plane. In particular, determine the long-term behavior (as $t \to \infty$) of the two populations, in terms of their initial populations $x(0) = x_0$ and $y(0) = y_0$. For instance, the figure above shows a phase plane portrait for the system

$$\frac{dx}{dt} = x(14 - 2x - y), \qquad \frac{dy}{dt} = y(16 - x - 2y).$$

We see a nodal source at (0, 0), a nodal sink at (4, 6), and saddle points at (7, 0) and (0, 8). It follows that, if x_0 and y_0 are both positive, then $x(t) \to 4$ and $y(t) \to 6$ as $t \to \infty$.

In the sections that follow we use the simple predator-prey system

$$\frac{dx}{dt} = x - xy, \qquad \frac{dy}{dt} - y + xy. \tag{3}$$

to illustrate the *Maple*, *Mathematica*, and MATLAB techniques needed for these investigations.

Using *Maple*

To plot a solution curve for the system in (3) we need only load the **DEtools** package and use the **DEplot** function. For instance, if we first define the differential equations

```
deq1 :=  diff(x(t),t) =  x - x*y:
deq2 :=  diff(y(t),t) = -y + x*y:
```

then the commands

```
with(DEtools):
DEplot([deq1,deq2], [x,y], t=0..25,
       {[x(0)=1, y(0)=3]}, stepsize=0.1,
       linecolor=blue, arrows=none);
```

plot the xy-solution curve with initial conditions $x(0)=1,\ y(0)=3$ on the interval $0 \le t \le 25$ with step size $h = 0.1$. Next, the command

```
DEplot([deq1,deq2], [x,y], t=0..25,
       {[x(0)=1, y(0)=3]}, stepsize=0.1,
       scene = [t,x], linecolor=blue, arrows=none);
```

plots the corresponding tx-solution curve, on which the approximate period of oscillation of the prey population can be measured.

Using *Mathematica*

To plot a solution curve for the system in (3) we need only define the differential equations

```
deq1 = x'[t] ==  x[t] - x[t]*y[t];
deq2 = y'[t] == -y[t] + x[t]*y[t];
```

and then use **NDSolve** to integrate numerically. For instance, the command

```
soln = NDSolve[ {deq1,deq2, x[0]==1, y[0]==3},
                {x[t],y[t]}, {t,0,25} ]
```

yields an approximate solution on the interval $0 \le t \le 25$ satisfying the initial conditions $x(0)=1,\; y(0)=3$. Then the command

```
ParametricPlot[
     Evaluate[{x[t],y[t]} /. soln], {t,0,25}]
```

plots the corresponding *xy*-solution curve, and the command

```
Plot[Evaluate[ x[t] /. soln ], {t,0,25}]
```

plots the corresponding *tx*-solution curve, on which the approximate period of oscillation of the prey population can be measured.

Using MATLAB

To plot a solution curve for the system in (3) we need only define the system by means of the m-file

```
function  yp = predprey(t,y)
% predprey.m
yp = y;
x = y(1);
y = y(2);
yp(1) = x - x.*y;
yp(2) = -y + x.*y;
```

and then use **ode23** to integrate numerically. For instance, the command

```
[t,x] = ode23('predprey', [0:0.1:25], [1;3]);
```

yields an approximate solution on the interval $0 \le t \le 25$ satisfying the initial conditions $x(0)=1,\; y(0)=3$. Then the command

```
plot(x(:,1), x(:,2))
```

plots the corresponding *xy*-solution curve, and the command

```
plot(t, x(:,1))
```

plots the corresponding *tx*-solution curve, on which the approximate period of oscillation of the prey population can be measured.