Chapter 10

Laplace Transform Methods

Application 10.1
Computer Algebra Transforms and Inverse Transforms

If $f(t) = t\cos(3t)$ then the definition of the Laplace transform gives the improper integral

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty t\,e^{-st}\cos 3t\,dt \qquad (1)$$

whose evaluation would appear to require a fairly tedious integration by parts. Consequently, a computer algebra system is useful for the quick calculation of Laplace transforms.

In the paragraphs that follow we illustrate the use of appropriate *Maple*, *Mathematica*, and MATLAB commands to find Laplace transforms and inverse transforms. You can use these computer algebra commands to check the answers to Problems 11–32 in Section 10.1 of the text, as well as a few interesting problems of your own selection.

Using *Maple*

First we load *Maple*'s **int**egral **trans**forms package `inttrans` with the command

```
with(inttrans):
```

and define (as an expression) the function $f(t)$ whose transform appears in (1):

```
f := t*cos(3*t):
```

Then the Laplace transform of f is given by

```
F := laplace(f, t, s);
```

$$F := \frac{\cos\left(2\arctan\left(3\frac{1}{s}\right)\right)}{s^2+9}$$

```
F := simplify(expand( F ));
```

$$F := \frac{s^2 - 9}{\left(s^2 + 9\right)^2}$$

Thus we obtain the Laplace transform $F(s) = (s^2 - 9)/(s^2 + 9)^2$.

As illustrated above, it's a good idea to routinely "expand and simplify" the result when calculating a Laplace transform. We can inverse Laplace transform to recover the original function $f(t)$ with the *Maple* command

```
invlaplace(F, s, t);
```

$$t\cos(3t)$$

Thus we are (as desired) back where we started.

Remark Note carefully the order of s and t in the commands above — first t, then s when transforming; first s, then t when inverse transforming.

Using *Mathematica*

First we load the Laplace transforms package **`Calculus:LaplaceTransform`** with the command

```
Needs["Calculus`LaplaceTransform`"]
```

and define (as an expression) the function $f(t)$ whose transform appears in (1):

```
f = t Cos[3 t];
```

Then the Laplace transform of f is given by

```
F = LaplaceTransform[f, t, s]
```

$$\frac{2s^2}{(9+s^2)^2} - \frac{1}{9+s^2}$$

```
F = F // Expand // Simplify
```

$$\frac{-9+s^2}{(9+s^2)^2}$$

Thus we obtain the Laplace transform $F(s) = (s^2 - 9)/(s^2 + 9)^2$.

As illustrated above, it's a good idea to routinely "expand and simplify" the result when calculating a Laplace transform. We can inverse Laplace transform to recover the original function $f(t)$ with the *Mathematica* command

```
InverseLaplaceTransform[F, s, t] // Expand // Simplify
```

$t\cos(3t)$

Thus we are (as desired) back where we started.

Remark Note carefully the order of s and t in the commands above — first t, then s when transforming; first s, then t when inverse transforming.

Using MATLAB

First we define (as an expression) the function $f(t)$ whose transform appears in (1):

```
syms t s
f = t*cos(3*t);
```

Then the Laplace transform of f is given by

```
F = laplace(f)
F =
1/(s^2+9)*cos(2*atan(3/s))

F = simplify(expand(F))
F =
(s^2-9)/(s^2+9)^2
```

Thus we obtain the Laplace transform $F(s) = (s^2-9)/(s^2+9)^2$.

As illustrated above, it's a good idea to routinely "expand and simplify" the result when calculating a Laplace transform. We can inverse Laplace transform to recover the original function $f(t)$ with the MATLAB command

```
ilaplace(F)
ans =
t*cos(3*t)
```

Thus we are (as desired) back where we started.

Remark Note that the Laplace transform of a function of t is automatically a function of the variable s, while the inverse transform of a function of s is automatically a function of the variable t. So if we adhere to the usual notations, then variables need not be specified in calculating transforms and inverse transforms with MATLAB.