A Double Pendulum

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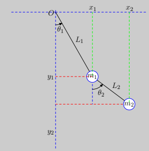
Abstract

In my project I will model the behavior a system consisting of a pendulum hanging from a point with another pendulum attached to the weight at the end of the first pendulum. I will also calculate the energy in the system. In short, I will investigate and attempt to model this system using differential equations.

1. Variables and Parameters

My double pendulum system will consist of two masses connected by weightless bars. The top bar will have length L_1 and mass at the end of this bar will have mass m_1. The bar attached to this mass will have length L_2 and the mass attached the end of this second bar will have mass m_2. I will call the point from which the first pendulum, the pendulum with length L_1 and mass m_1, pivots point O. I will let the angle that the first bar makes with a vertical line drawn down from O be represented by θ_1 and I will let the angle that the second bar makes with a vertical line drawn down from m_1 be represented by θ_2, where counter-clockwise angles are positive. If I set this system in an xy-plane with O being the origin, I can find the position of the masses. I am also going to let the x-position of m_1 to be x_1 and the y-position of m_1 to be y_1. The x and y position of m_2 will be x_2 and y_2. This gives me this list of variables and parameters that correspond to the labels on figure 1:



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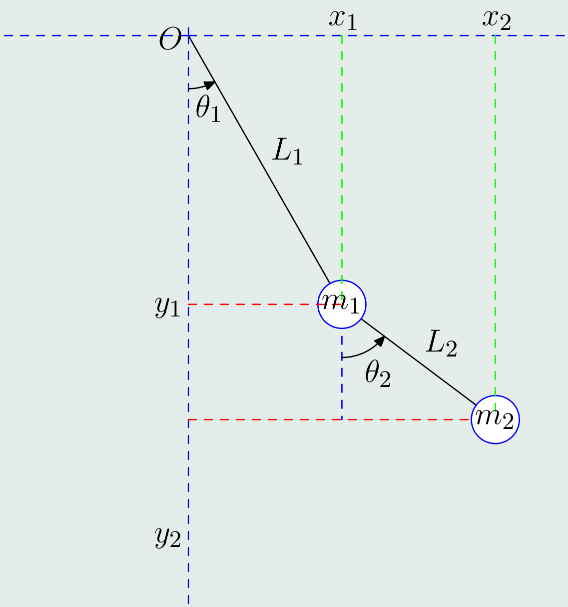
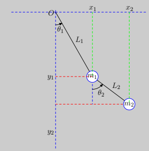


Figure 1: double pendulum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- length of the bar of the first pendulum = L_1
- mass at the end of the first pendulum = m_1
- length of the bar of the second pendulum = L_2
- mass at the end of the second pendulum = m_2
- the point O is the origin and is where the first pendulum pivots from
- angle made by the bar of the first pendulum and the line of rest = θ_1
- angle made by the bar of the second pendulum and the line of rest = θ_2
- the x-position of $m_1 = x_1$
- the y-position of $m_1 = y_1$
- the x-position of $m_2 = x_2$
- the y-position of $m_2 = y_2$

I am also going to let K represent the kinetic energy in the system and P represent the potential or gravitational energy of the system.

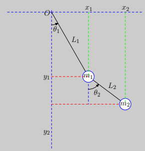
2. Position of Masses

It is very simple to find equations for the x-position and the y-position of the first mass using trigonometry:

$$x_1 = L_1 \sin(\theta_1)$$
$$y_1 = -L_1 \cos(\theta_1)$$

To find the position of the second mass I simply add to the position of the first mass.

$$x_2 = x_1 + L_2 \sin(\theta_2)$$



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From this we get

$$x_2 = L_1 \sin(\theta_1) + L_2 \sin(\theta_2)$$

I do the same thing for the y-position.

$$y_2 = y_1 - L_2 \cos(\theta_2)$$

From this I get

$$y_2 = -L_1 \cos(\theta_1) - L_2 \cos(\theta_2). \tag{1}$$

So I get these formulas for the position of the masses of the pendula:

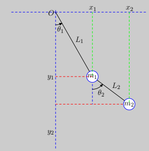
$$\begin{aligned}
x_1 &= L_1 \sin(\theta_1) && (2)\\
y_1 &= -L_1 \cos(\theta_1) && (3)\\
x_2 &= L_1 \sin(\theta_1) + L_2 \sin(\theta_2) && (4)\\
y_2 &= -L_1 \cos(\theta_1) - L_2 \cos(\theta_2) && (5)
\end{aligned}$$

Differentiating I get

$$\begin{aligned}
\dot{x}_1 &= L_1 \cos(\theta_1)\dot{\theta}_1 && (6)\\
\dot{y}_1 &= L_1 \sin(\theta_1)\dot{\theta}_1 && (7)\\
\dot{x}_2 &= L_1 \cos(\theta_1)\dot{\theta}_1 + L_2 \cos(\theta_2)\dot{\theta}_2 && (8)\\
\dot{y}_2 &= L_1 \sin\theta_1\dot{\theta}_1 + L_2 \sin(\theta_2)\dot{\theta}_2. && (9)
\end{aligned}$$

If we square all of the entries in this list I get

$$\begin{aligned}
\dot{x}_1^2 &= L_1^2 \cos^2(\theta_1)\dot{\theta}_1^2 && (10)\\
\dot{y_1}^2 &= L_1^2 \sin^2(\theta_1)\dot{\theta}_1^2 && (11)\\
\dot{x}_2^2 &= L_1^2 \cos^2(\theta_1)\dot{\theta}_1^2 + 2L_1L_2\cos(\theta_1)\cos(\theta_2)\dot{\theta}_1\dot{\theta}_2 + L_2^2\cos^2(\theta_2)\dot{\theta}_2^2 && (12)\\
\dot{y}_2^2 &= L_1^2 \sin^2\theta_1\dot{\theta}_1^2 + 2L_1L_2\sin\theta_1\sin\theta_2\dot{\theta}_1\dot{\theta}_2 + L_2^2\sin^2(\theta_2)\dot{\theta}_2^2. && (13)
\end{aligned}$$



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3. Energy in the System

I must now look at the energy in my double pendulum. This system has two different forms of energy: kinetic energy (the energy of motion) and potential or gravitational energy (the energy available to the system caused by the pull of gravity). The gravitational energy of this system is the gravitational energy in the first pendulum plus the gravitational energy in the second pendulum. Thus I get

$$P = m_1 g y_1 + m_2 g y_2,$$

where P is the potential energy of the system. With substitution from equation (3) and equation (5) I get

$$P = -(m_1 + m_2) g L_1 \cos(\theta_1) - m_2 L_2 g \cos(\theta_2) \tag{14}$$

I must now calculate the kinetic energy of the system. Like with potential energy the total kinetic energy is the sum of the kinetic energies of the two pendula.

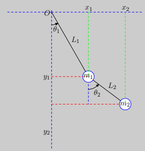
$$K = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2). \tag{15}$$

This brings me to this formula for the kinetic energy of the system:

$$K = \frac{1}{2} m_1 (\dot{x}_1^2 + \dot{y_1}^2) + \frac{1}{2} m_2 (\dot{x}_2^2 + \dot{y}_2^2).$$

Using equations (10), (11), (12) and (13) I get

$$\begin{aligned} K = \frac{1}{2} m_1 (L_1^2 \cos^2(\theta_1)\dot{\theta}_1^2 + L_1^2 \sin^2(\theta_1)\dot{\theta}_1^2) + \\ \frac{1}{2} m_2 [L_1^2 \cos^2(\theta_1)\dot{\theta}_1^2 + 2 L_1 L_2 \cos(\theta_1)\cos(\theta_2)\dot{\theta}_1\dot{\theta}_2 + L_2^2 \cos^2(\theta_2)\dot{\theta}_2^2 + \\ L_1^2 \sin^2\theta_1 \dot{\theta}_1^2 + 2 L_1 L_2 \sin\theta_1 \sin\theta_2 \dot{\theta}_1 \dot{\theta}_2 + L_2^2 \sin^2(\theta_2)\dot{\theta}_2^2]. \end{aligned}$$



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This simplifies to

$$K = \frac{1}{2} m_1 L_1^2 \dot{\theta}_1^2 (\cos^2(\theta_1) + \sin^2(\theta_1)) +$$
$$\frac{1}{2} m_2 [L_1^2 \dot{\theta}_1^2 (\cos^2(\theta_1) + \sin^2(\theta_1)) + L_2^2 \dot{\theta}_2^2 (\cos^2(\theta_2) + \sin^2(\theta_2)) +$$
$$2 L_1 L_2 \dot{\theta}_1 \dot{\theta}_2 (\cos(\theta_1)\cos(\theta_2) + \sin(\theta_1)\sin(\theta_2)].$$

This simplifies to:

$$K = \frac{1}{2} m_1 \dot{\theta}_1^2 L_1^2 + \frac{1}{2} m_2 [\dot{\theta}_1^2 L_1^2 + \dot{\theta}_2^2 L_2^2 + 2\dot{\theta}_1 L_1 \dot{\theta}_2 L_2 \cos(\theta_1 - \theta_2)]. \tag{16}$$

4. Using a Lagrangian

The Lagrangian(L) of a system is the kinetic energy of the system minus the potential energy. This gives me

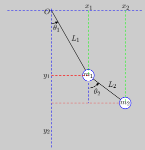
$$L = K - P. \tag{17}$$

Using equations (16) and (14) I get

$$\begin{aligned} L = {} & \frac{1}{2} m_1 (\dot{\theta}_1 L_1)^2 + \frac{1}{2} m_2 [(\dot{\theta}_1 L_1)^2 + (\dot{\theta}_2 L_2)^2 + 2\dot{\theta}_1 L_1 \dot{\theta}_2 L_2 \cos(\theta_1 - \theta_2)] - \\ & [-(m_1 + m_2) g L_1 \cos(\theta_1) - m_2 L_2 g \cos(\theta_2)]. \end{aligned} \tag{18}$$

Simplifying I get

$$\begin{aligned} L = {} & \frac{1}{2}(m_1 + m_2) L_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 L_2^2 \dot{\theta}_2^2 + m_2 L_1 L_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2) + \\ & (m_1 + m_2) g L_1 \cos(\theta_1) + m_2 L_2 g \cos(\theta_2). \end{aligned} \tag{19}$$



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This Euler-Lagrange differential equation must be true:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0 \tag{20}$$

For θ_1 I get

$$\frac{\partial L}{\partial \dot{\theta}_1} = m_1 L_1^2 \dot{\theta}_1 + m_2 L_1^2 \dot{\theta}_1 + m_2 L_1 L_2 \dot{\theta}_2 \cos(\theta_1 - \theta_2) \tag{21}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) = (m_1 + m_2) L_1^2 \ddot{\theta}_1 + m_2 L_1 L_2 \ddot{\theta}_2 \cos(\theta_1 - \theta_2) - m_2 L_1 L_2 \dot{\theta}_2 sin(\theta_1 - \theta_2)(\dot{\theta}_1 - \dot{\theta}_2 \tag{22}$$

$$\frac{\partial L}{\partial \theta_1} = -L_1 g (m_1 + m_2) \sin(\theta_1) - m_2 L_1 L_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2). \tag{23}$$

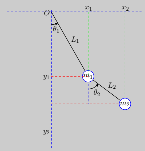
Substituting equations (21), (22), and (23) into the Euler-Lagragnge (equation (20)) I get

$$(m_1 + m_2) L_1^2 \ddot{\theta}_1 + m_2 L_1 L_2 \ddot{\theta}_2 \cos(\theta_1 - \theta_2) + m_2 L_1 L_2 \dot{\theta}_2^2 \sin(\theta_1 - \theta_2) + \\ g L_1 (m_1 + m_2) \sin(\theta_1) = 0$$

Simplifying and solving for $\ddot{\theta}_1$I get

$$\ddot{\theta}_1 = \frac{-m_2 L_2 \ddot{\theta}_2 \cos(\theta_1 - \theta_2) - m_2 L_2 \dot{\theta}_2^2 \sin(\theta_1 - \theta_2) - (m_1 + m_2) g \sin(\theta_1)}{(m_1 + m_2) L_1}. \tag{24}$$

Likewise for θ_2 I get



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$$\frac{\partial L}{\partial \dot\theta_2} = m_2 L_2^2 \dot\theta_2 + m_2 L_1 L_2 \dot\theta_1 \cos(\theta_1 - \theta_2) \tag{25}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta_2}\right) = m_2 L_2^2 \ddot\theta_2 + m_2 L_1 L_2 \ddot\theta_1 \cos(\theta_1 - \theta_2) - m_2 L_1 L_2 \dot\theta_1 \sin(\theta_1 - \theta_2)(\dot\theta_1 - \dot\theta_2) \tag{26}$$

$$\frac{\partial L}{\partial \theta_2} = m_2 L_1 L_2 \dot\theta_1 \dot\theta_2 \sin(\theta_1 - \theta_2) - L_2 m_2 g \sin(\theta_2). \tag{27}$$

Like with θ_1 substituting equations (25), (26), and (27) into the Euler-Lagrange (equation (20) I get

$$m_2 L_2^2 \ddot\theta_2 + m_2 L_1 L_2 \ddot\theta_1 \cos(\theta_1 - \theta_2) - m_2 L_1 L_2 \dot\theta_1 \sin(\theta_1 - \theta_2)(\dot\theta_1 - \dot\theta_2) - \\ m_2 L_1 L_2 \dot\theta_1 \dot\theta_2 \sin(\theta_1 - \theta_2) + L_2 m_2 g \sin(\theta_2) = 0.$$

Dividing through by L_2 and m_2 I get

$$L_2 \ddot\theta_2 + L_1 \ddot\theta_1 \cos(\theta_1 - \theta_2) - L_1 \dot\theta_1^2 \sin(\theta_1 - \theta_2) + g \sin(\theta_2) = 0.$$

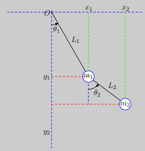
Solving for $\ddot\theta_2$ I get

$$\ddot\theta_2 = \frac{-L_1 \ddot\theta_1 \cos(\theta_1 - \theta_2) + L_1 \dot\theta_1^2 \sin(\theta_1 - \theta_2) - g \sin(\theta_2)}{L_2}. \tag{28}$$

This gives me an equation for $\ddot\theta_1$ that depends on $\ddot\theta_2$ and an equation for $\ddot\theta_2$ that depends on $\ddot\theta_1$. I can use these two equations to make two more equations that have either $\ddot\theta_1$ or $\ddot\theta_2$ in them but not both.

Substituting equation (28) into equation (24) I get

$$\ddot\theta_1 (m_1 + m_2) L_1 = -m_2 L_2 \left[\frac{-L_1 \ddot\theta_1 \cos(\theta_1 - \theta_2) + L_1 \dot\theta_1^2 \sin(\theta_1 - \theta_2) - g \sin(\theta_2)}{L_2}\right] \cos(\theta_1 - \theta_2) - \\ m_2 L_2 \dot\theta_2^2 \sin(\theta_1 - \theta_2) - (m_1 + m_2) g \sin(\theta_1).$$



If I move all the terms that contain $\ddot{\theta}_1$ in them to the left hand side I get

$$\ddot{\theta}_1\left[L_1(m_1+m_2)-m_2L_1\cos^2(\theta_1-\theta_2)\right]=-m_2L_1\dot{\theta}_1^2\sin(\theta_1-\theta_2)\cos(\theta_1-\theta_2)+\\ gm_2\sin(\theta_2)\cos(\theta_1-\theta_2)-m_2L_2\dot{\theta}_2^2\sin(\theta_1-\theta_2)-(m_1+m_2)g\sin(\theta_1).$$

After solving for $\ddot{\theta}_1$, I get

$$\ddot{\theta}_1=\frac{\begin{array}{r}-m_2L_1\dot{\theta}_1^2\sin(\theta_1-\theta_2)\cos(\theta_1-\theta_2)+gm_2\sin(\theta_2)\cos(\theta_1-\theta_2)\\ -m_2L_2\dot{\theta}_2^2\sin(\theta_1-\theta_2)-(m_1+m_2)g\sin(\theta_1)\end{array}}{L_1(m_1+m_2)-m_2L_1\cos^2(\theta_1-\theta_2)}. \tag{29}$$

Likewise I can substitute equation (24) into equation (28) to get

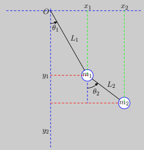
$$\ddot{\theta}_2L_2=L_1\left[\frac{m_2L_2\ddot{\theta}_2\cos(\theta_1-\theta_2)+m_2L_2\dot{\theta}_2^2\sin(\theta_1-\theta_2)+(m_1+m_2)g\sin(\theta_1)}{(m_1+m_2)L_1}\right]\cos(\theta_1-\theta_2)+\\ L_1\dot{\theta}_1^2\sin(\theta_1-\theta_2)-g\sin(\theta_2).$$

Bringing all terms that have $\ddot{\theta}_2$ in them to the left hand side and factoring I get

$$\ddot{\theta}_2\left[\frac{L_2(m_1+m_2)-m_2L_2\cos^2(\theta_1-\theta_2)}{m_1+m_2}\right]=\frac{m_2L_2\dot{\theta}_2^2\sin(\theta_1-\theta_2)\cos(\theta_1-\theta_2)}{m_1+m_2}+\\ g\sin(\theta_1)\cos(\theta_1-\theta_2)+L_1\dot{\theta}_1^2\sin(\theta_1-\theta_2)-g\sin(\theta_2).$$

Then solving for $\ddot{\theta}_2$ I get

$$\ddot{\theta}_2=\frac{\begin{array}{r}m_2L_2\dot{\theta}_2^2\sin(\theta_1-\theta_2)\cos(\theta_1-\theta_2)+g\sin(\theta_1)\cos(\theta_1-\theta_2)(m_1+m_2)\\ +L_1\dot{\theta}_1^2\sin(\theta_1-\theta_2)(m_1+m_2)-g\sin(\theta_2)(m_1+m_2)\end{array}}{L_2(m_1+m_2)-m_2L_2\cos^2(\theta_1-\theta_2)}. \tag{30}$$



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I now have two second order differential equations that I will be able make into four first order differential equation. Setting up different variables for θ_1, θ_2,$\dot{\theta}_1$,$\dot{\theta}_2$ I get this system:

$$z_1 = \theta_1 \tag{31}$$
$$z_2 = \theta_2 \tag{32}$$
$$z_3 = \dot{\theta}_1 \tag{33}$$
$$z_4 = \dot{\theta}_2 \tag{34}$$

Differentiating this list I get

$$\dot{z}_1 = \dot{\theta}_1 \tag{35}$$
$$\dot{z}_2 = \dot{\theta}_2 \tag{36}$$
$$\dot{z}_3 = \ddot{\theta}_1 \tag{37}$$
$$\dot{z}_4 = \ddot{\theta}_2. \tag{38}$$

Using equations (35) and (33) I get

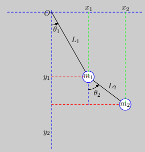
$$\dot{z}_1 = z_3. \tag{39}$$

Using equations (36) and (34) I get

$$\dot{z}_2 = z_4. \tag{40}$$

Using equations (37) and (29) I get

$$\dot{z}_3 = \frac{\begin{array}{r} -m_2 L_1 z_4^2 \sin(z_1 - z_2)\cos(z_1 - z_2) + g m_2 sin(z_2)\cos(z_1 - z_2) \\ -m_2 L_2 z_4^2 \sin(z_1 - z_2) - (m_1 + m_2) g \sin(z_1) \end{array}}{L_1(m_1 + m_2) - m_2 L_1 \cos^2(z_1 - z_2)}. \tag{41}$$



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Using equations (38) and (30) I get

$$\dot{z}_4 = \frac{\begin{array}{r} m_2 L_2 z_4^2 \sin(z_1 - z_2)\cos(z_1 - z_2) + g\sin(z_1)\cos(z_1 - z_2)(m_1 + m_2) \\ +L_1 z_4^2 \sin(z_1 - z_2)(m_1 + m_2) - g\sin(z_2)(m_1 + m_2) \end{array}}{L_2(m_1 + m_2) - m_2 L_2 \cos^2(z_1 - z_2)}. \tag{42}$$

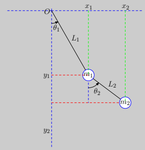
5. Numerical Results Using Matlab

Equations (39), (40), (41) and (42) give me a system of four first order differential equations in four variables. All though these are too complicated for me to solve analytically, I can use the ode45 solver in Matlab to get some numerical results. First I make my function file by letting $\dot{z} = zprime$ be a column vector with four elements and letting z be a column vector with four elements. For the sake of convenience I pass the parameters $m1$, $m2$, $L1$, $L2$ and g into my function.This is the Matlab code I used:

```
function zprime=Pend(t,z,m1,m2,L1,L2,g)
zprime=zeros(4,1);
zprime(1)=z(3);
zprime(2)=z(4);
zprime(3)=(-m2*L1*z(4)^2*sin(z(1)-z(2))*cos(z(1)-z(2))...
    +g*m2*sin(z(2))*cos(z(1)-z(2))-m2*L2*z(4)^2*sin(z(1)-z(2))...
    -(m1+m2)*g*sin(z(1)))/(L1*(m1+m2)-m2*L1*cos(z(1)-z(2))^2);
zprime(4)=(m2*L2*z(4)^2*sin(z(1)-z(2))*cos(z(1)-z(2))...
    +g*sin(z(1))*cos(z(1)-z(2))*(m1+m2)+L1*z(4)^2*sin(z(1)-z(2))*(m1+m2)...
    -g*sin(z(2))*(m1+m2))/(L2*(m1+m2)-m2*L2*cos(z(1)-z(2))^2);
```

Next I set up a script file that defines the parameters $m1$, $m2$, $L1$, $L2$ and g and the initial values of θ_1 ($t1$), θ_2 ($t2$), $\dot{\theta}_1$ ($t1prime$) and $\dot{\theta}_2$ ($t2prime$), calls ode45, saves a column vector of time values (t) and a column vector of z values that has four columns z_1, z_2, z_3 and z_4, which are, according to equations (31)-(34), θ_1, θ_2, $\dot{\theta}_1$ and $\dot{\theta}_2$ respectively. I start with this code:

```
t1=pi/2;
t2=0;
```



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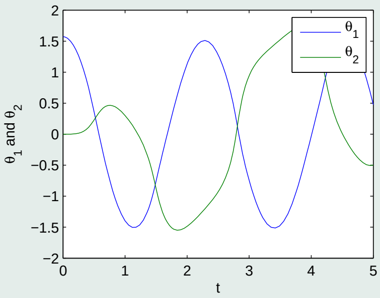


Figure 2: the position of m_1 and m_2 over time

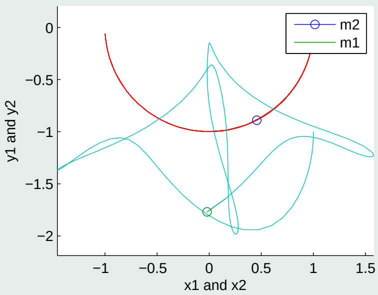
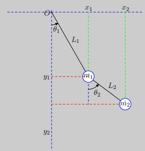


Figure 3: the position of m_1 and m_2 over time

```
t1prime=0;
t2prime=0;
m1=2;
m2=1;
L1=1;
L2=1;
g=9.81;
[t,z]=ode45(@Pend,[0,5],[t1;t2;t1prime;t2prime],[],m1,m2,L1,L2,g);
```

I would like to be able to plot the position of the two masses as time goes forward so that I can check to see if my numerical solution is reasonable so I use equations (2), (3), (4) and (5), replacing θ_1 with the first column of z and θ_2 with the second column of z. So I add this Matlab code:

```
x1=L1*sin(z(:,1));
y1=-L1*cos(z(:,1));
x2=L1*sin(z(:,1))+L2*sin(z(:,2));
y2=-L1*cos(z(:,1))-L2*cos(z(:,2));
```



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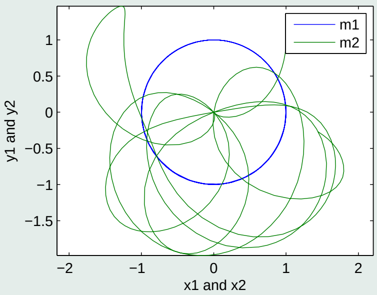


Figure 4: the position of m_1 and m_2 over time

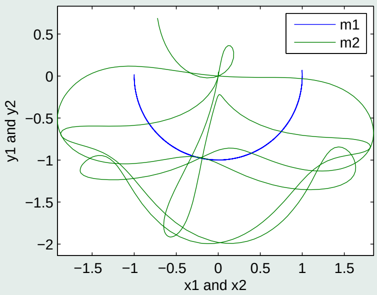


Figure 5: the position of m_1 and m_2 over time

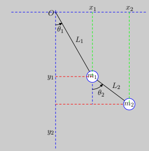
In order to see the numerical results I will plot θ_1 versus time, θ_2 versus time and, in order to see that actual motion of the bobs on the two pendulums I will plot $x1$ verses $y1$ and $x2$ verses $y2$ on the same axes. So I will add this code to my script file:

This Matlab code yields figure 2 and 3.

6. Conclusion

6.1. Types of Behavior

There are three different types of behavior exhibited by this system: chaotic, semi-cyclical and cyclical. Here are some examples. Figure 4 and 5 are examples of chaotic motion of the double pendulum system. Figures 6 and 7 are examples of quasi-cyclical behavior of the system. Figures 8 and 9 are examples of cyclical behavior in the system.



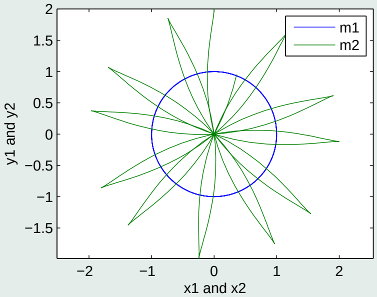


Figure 6: the position of m_1 and m_2 over time

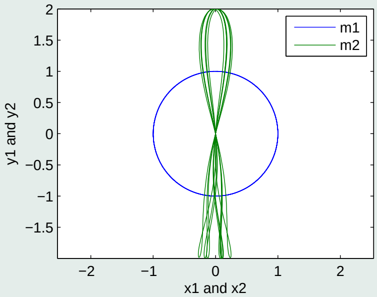


Figure 7: the position of m_1 and m_2 over time

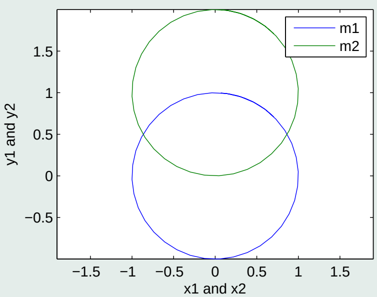


Figure 8: the position of m_1 and m_2 over time

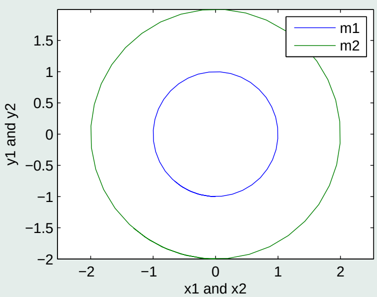
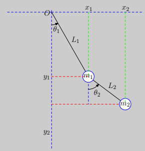


Figure 9: the position of m_1 and m_2 over time



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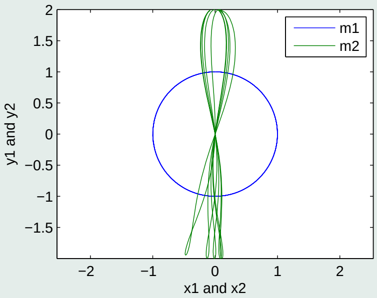


Figure 10: the position of m_1 and m_2 over time with $L_1 = 1$ and $L_2 = 1$

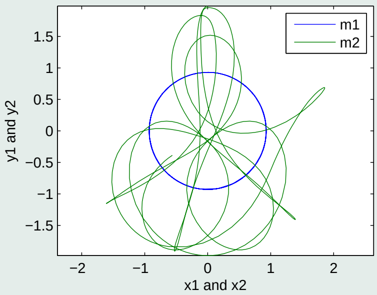
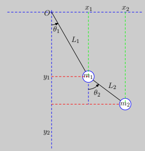


Figure 11: the position of m_1 and m_2 over time with $L_1 = .93$ and $L_2 = 1.05$

6.2. Chaotic behavior of the sysetm

Another characteristic of this system is that small changes in the initial conditions of the system can produce drastic changes in the behavior of the second mass. With $L_1 = 1$, $L_2 = 1$, the initial angles θ_1 and θ_2 set at just a tiny bit under π I get the behavior shown in figure 10 but if I increase the length of L_2 by $\frac{5}{100}$ and decrease L_1 by $\frac{7}{100}$ I get the behavior observed in figure 11. One of these examples exhibits quasi-cyclical behavior while the other shows completely chaotic behavior.



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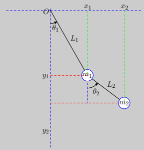
7. Matlab GUI

I also made a Matlab GUI that animates my double pendulum model. The user can vary the length of the pendulum bars, the mass of the pendulum bobs, the initial θ_1, θ_2, $\dot{\theta}_1$ and $\dot{\theta}_2$ values, the time span and the number of points used by ode45 to generate the numeric values for the animation. A large number of points will cause the animation to proceed slower and a small number of points will cause the animation to speed up. This GUI can be accessed at:

`http://online.redwoods.cc.ca.us/instruct/darnold/DEProj/sp08/jaltic/DoublePendulumAnimation.zip`



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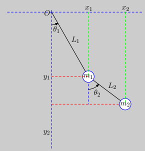
References

[1] Atam P. Arya *Introduction To Classical Mechanics*, Upper Saddle River, NJ: Prentice Hall, 1998.



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