

Romeo and Juliet

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Differential Equations

Spring 1997

Abstract

In our differential equation project we are studied love/hate as a linear case. Although love by its nature is nonlinear, there are many variations that can occur within the limitations of our model.

1 Introduction

“Every love affair has its ups and downs over time,...so it can be modeled by differential equations.” When one thinks of love, the classic tale of Romeo and Juliet comes to mind. In our study of the love/hate model we used Romeo and Juliet as a generic case for different situations.

We began with the general model:

$$\begin{aligned}\frac{dx}{dt} &= ax + by \\ \frac{dy}{dt} &= cx + dy \text{ where } a, b, c, \text{ and } d \text{ are constants}\end{aligned}$$

$$\begin{aligned}x(t) &= \text{Romeo's love/hate for Juliet at time } t \\ y(t) &= \text{Juliet's love/hate for Romeo at time } t\end{aligned}$$

where we picked time t to be measured in years

$$\begin{aligned}x \text{ and/or } y &> 0 \text{ love exists} \\ x \text{ and/or } y &< 0 \text{ love does not exist} \\ x \text{ and/or } y &= 0 \text{ indifference}\end{aligned}$$

Note that the model can also be written in matrix form:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

1.1 Love Affair # 1

Using the system of equations in the following form:

$$\begin{aligned}\frac{dx}{dt} &= ax + by \\ \frac{dy}{dt} &= -by - ay\end{aligned}$$

we are tested the theory that asks,
“Do opposites attract?” In this case we used the matrix:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad (1)$$

Which provided the following graphs:

Graph 1a

This system has a center at $(0,0)$.

Graph 1b

Romeo's feelings show that he really digs Juliet but as soon as his affections are not met, he gets discouraged with Juliet and cools down. In the true spirit of a woman, Juliet starts to like him as soon as his attention fades. Poor Romeo and Juliet, they are caught in a never ending cycle of love and hate. Over the time span of ten years they are only able to achieve simultaneous love one quarter of the time (Graph 1b).

1.2 Love Affair # 2

Identically Cautious

$$\begin{aligned}\frac{dx}{dt} &= ax + by \\ \frac{dy}{dt} &= bx + ay\end{aligned}$$

In this case a represents caution and b represents responsiveness. While Romeo and Juliet respond to each other's advances, they also try to keep from throwing themselves at each other.

We picked the following matrix:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad (2)$$

Graph 2a

This system has a saddle equilibrium point at $(0, 0)$. (Graph 2a)

Graph 2b

The relationship between Romeo and Juliet is explosive. Caution is down and they immediately give in to their extreme mutual love. (Graph 2b)

Graph 2c

Romeo and Juliet have an immediate dislike for one another (Graph 2c) .

Depending on the way Romeo and Juliet initially feel about each other, they either have a love fest or a war. But in either case their feelings are eventually mutual as all trajectories approach the line $y = x$.

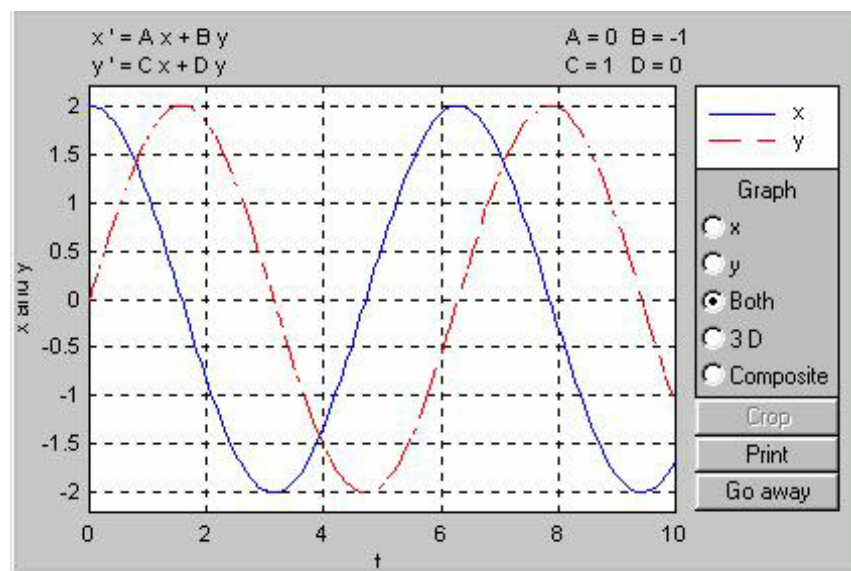
1.3 Love Affair # 3

In this case we used the matrix as follows:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad (3)$$

Graph 3a

The system has a center that occurs at $(0, 0)$ (Graph 3a).



Graph 3b

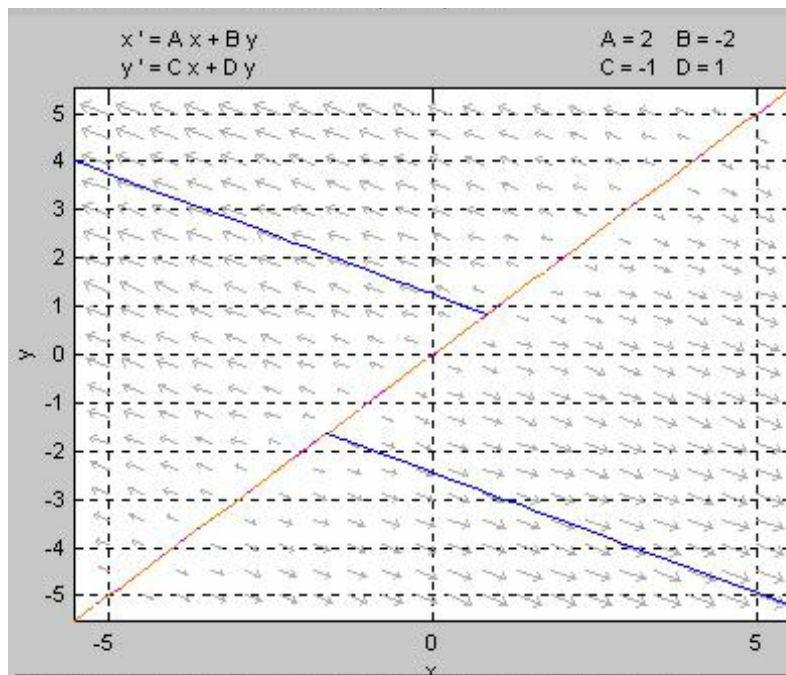
The same never ending cycle of love and hate that we found in love affair #1 exists here as well. The only difference we found is that at time $t=0$, Juliet

loves Romeo and Romeo dislikes Juliet. Again their feelings oscillate, neither sure if they truly love the other. This is a result of the numbers we picked to put into the matrix. Otherwise, the scenario of love one quarter of the time is the same (Graph 3b)

1.4 Love Affair # 4

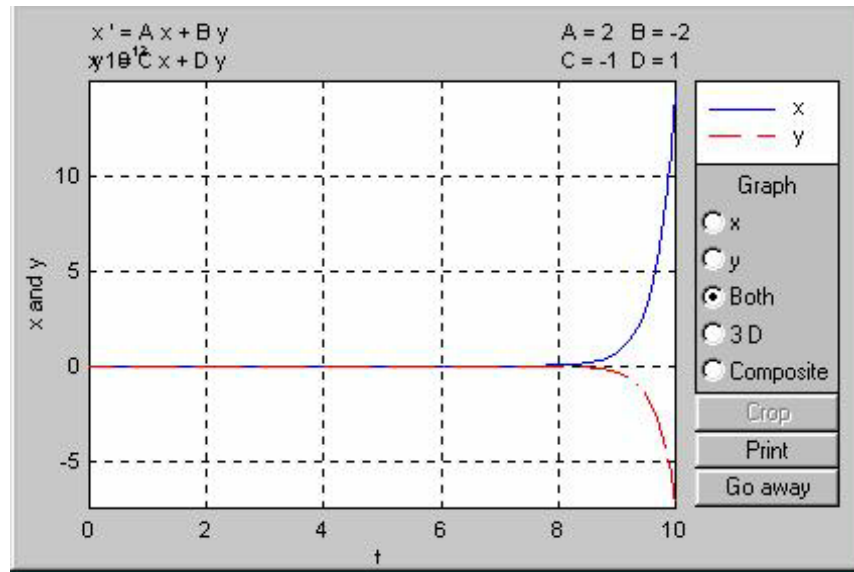
In this case we used the matrix as follows:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad (4)$$



Graph 4a

The system includes straight lines with a nullcline at $y = 0$ (Graph 4a).



Graph 4b

During the first seven years of their knowledge of one another, Romeo and Juliet feel only indifference for each other. At the same time that Romeo realizes his love for Juliet, Juliet realizes that she is repulsed by him. Their feelings shoot off in opposite directions and the possibility of a relationship is nonexistent. Poor lonely Romeo is left to pick up the pieces of his broken heart while Juliet goes on with her everyday life (Graph 4b).

1.5 Love Affair # 5

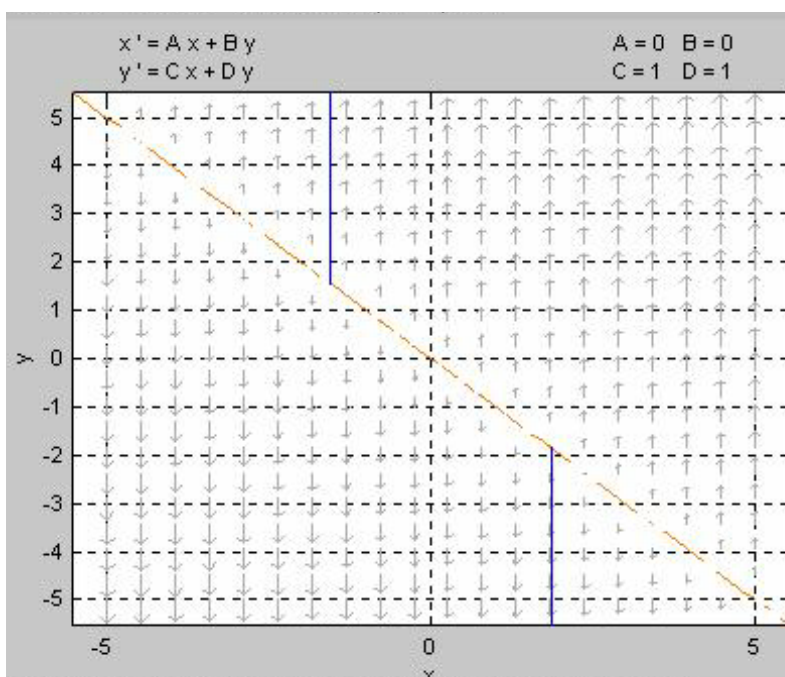
Using the system of equations in the following form:

$$\begin{aligned}\frac{dx}{dt} &= 0 \\ \frac{dy}{dt} &= ax + by\end{aligned}$$

We picked the matrix:

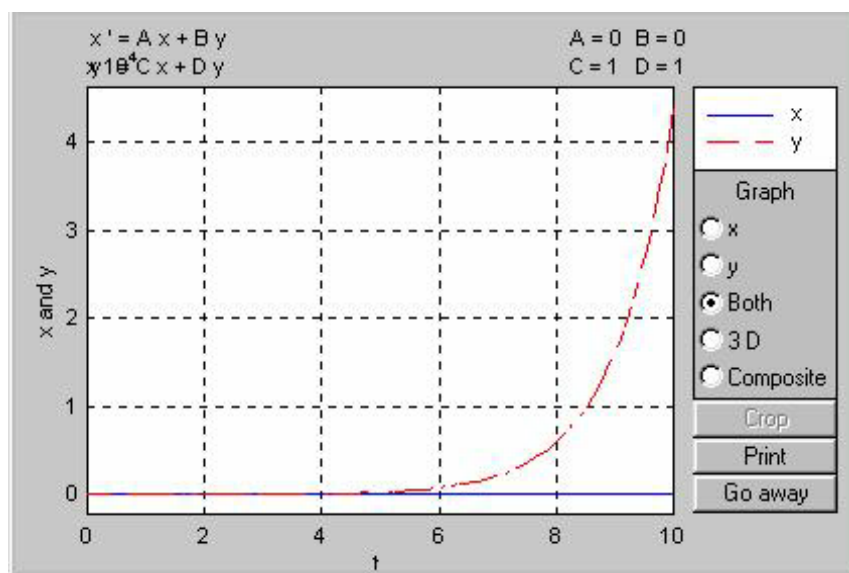
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad (5)$$

Romeo's feelings about Juliet can never be changed. He has no interest what so ever for fair Juliet. In this model we are looking at Juliet's reactions to Romeo's indifference.



Graph 5a

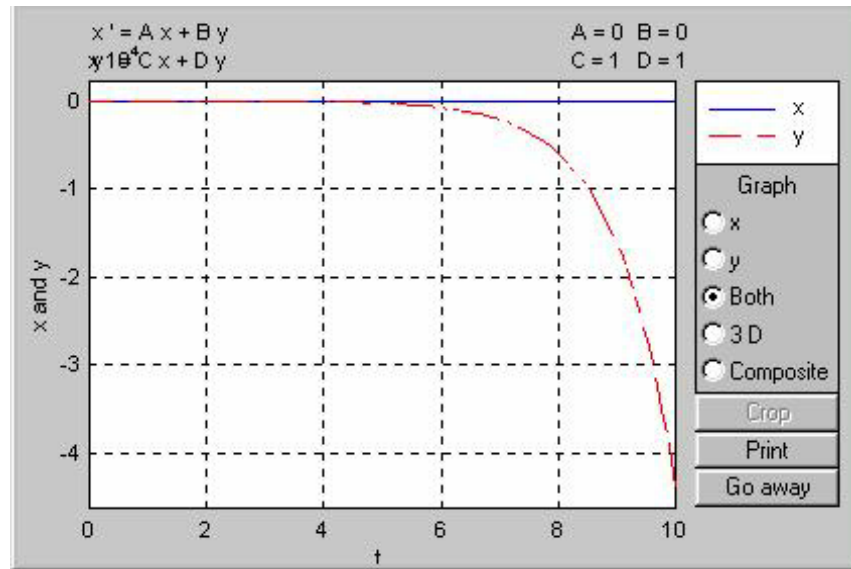
This system has straight line solutions (Graph 5a).



Graph 5b

Romeo's indifference drives Juliet insane with passion. She is so obsessed with Romeo that she devotes every minute of her life to following him (Graph

5b).



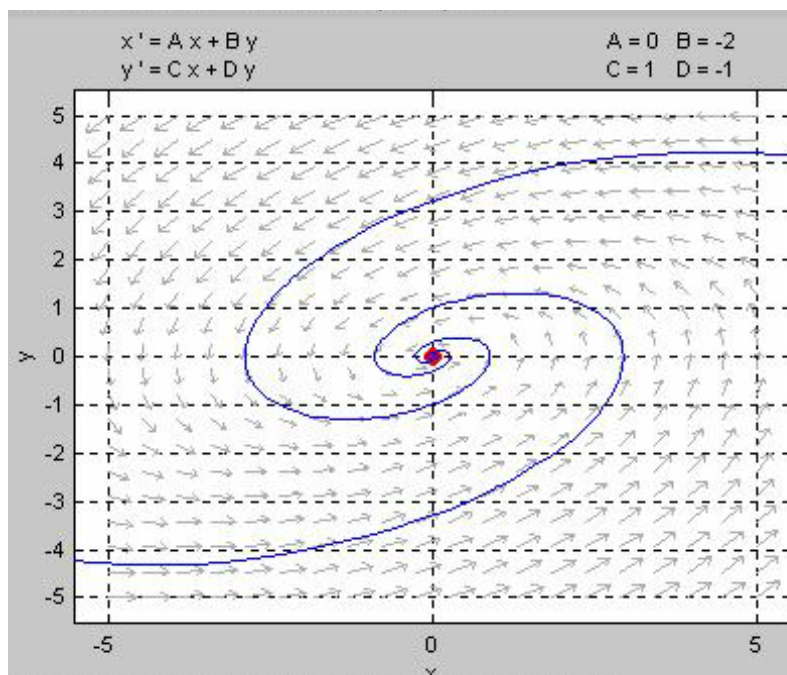
Graph 5c

Juliet needs some excitement in her life. She is completely turned off by Romeo and she ends up hating him (Graph 5c).

1.6 Love Affair # 6

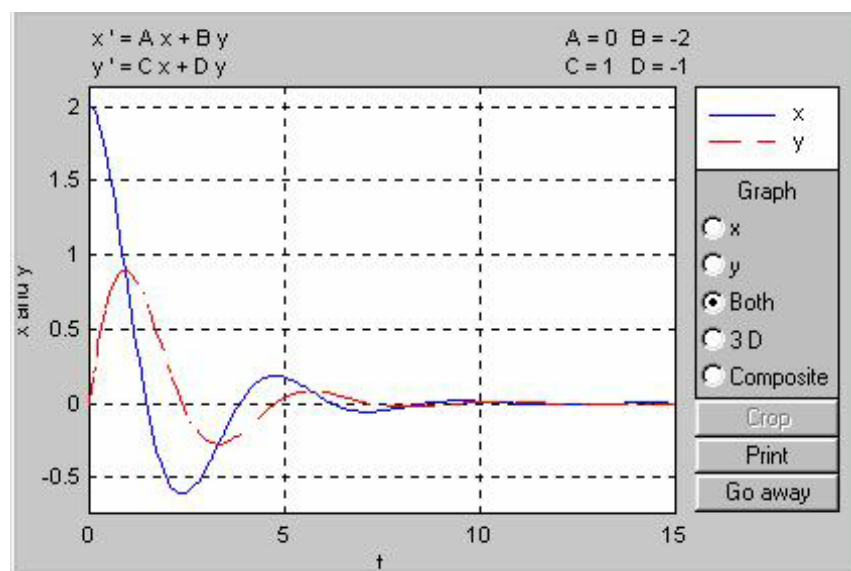
In this case we used the matrix as follows:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad (6)$$



Graph 6a

This system has a spiral sink at the equilibrium point $(0,0)$ (Graph 6a).



Graph 6b

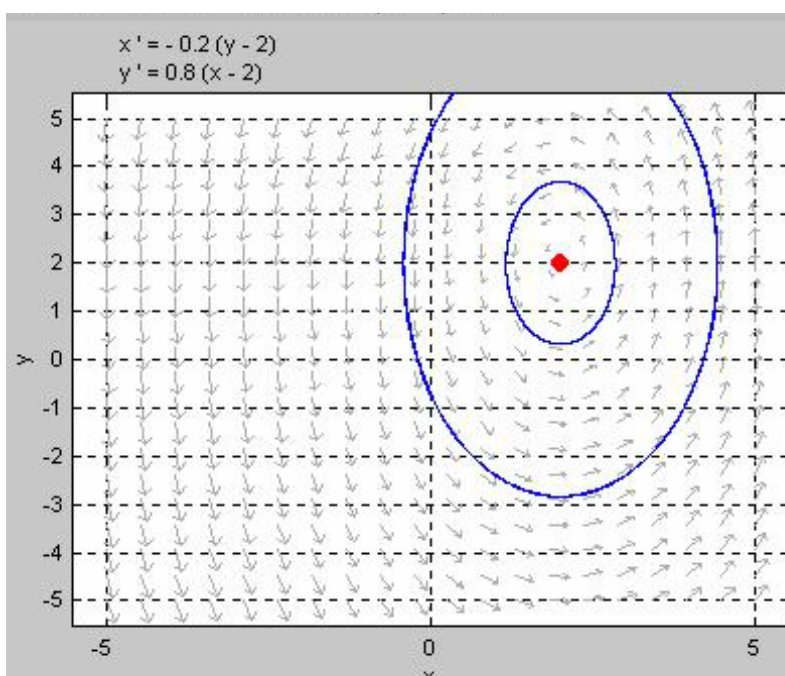
Juliet's love for Romeo is out of control. We tried to help her by damping her reactions with negative values of y . Unfortunately the damping was contagious

and eventually their feelings for each other fizzled out (Graph 6b).

1.7 Love Affair # 7

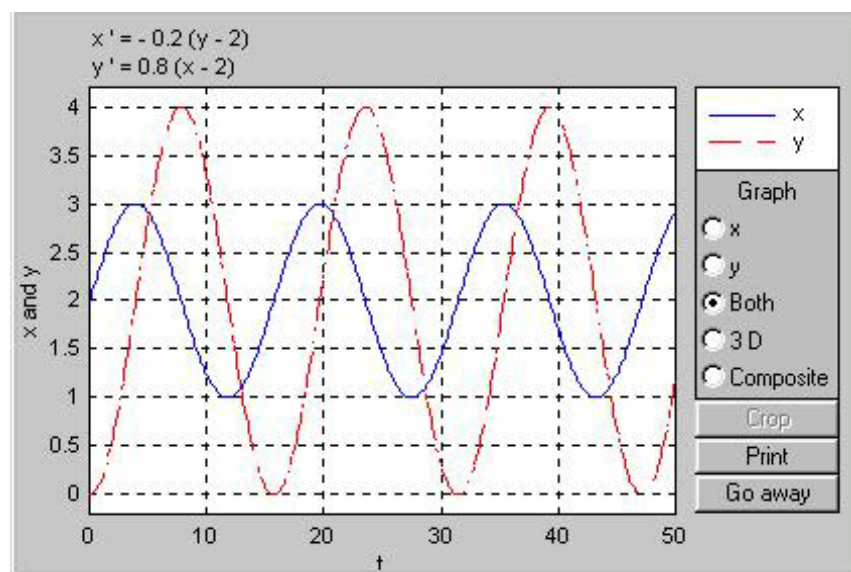
In this case we used the matrix as follows:

$$\begin{aligned}\frac{dx}{dt} &= -0.2(y - 2) \\ \frac{dy}{dt} &= 0.8(x - 2)\end{aligned}$$



Graph 7a

This system has a center at the equilibrium point $(2, 2)$ (Graph 7a).



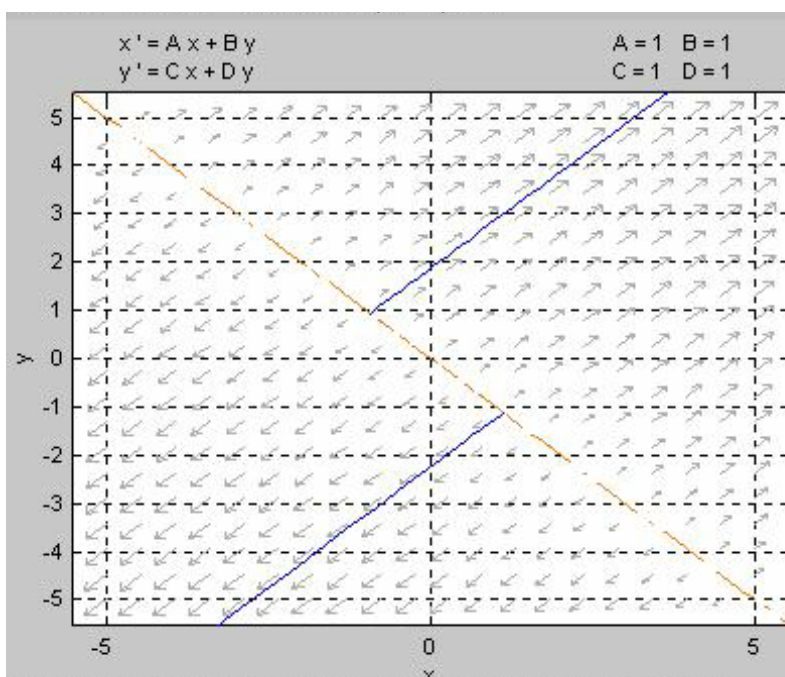
Graph 7b

Juliet responds to Romeo's love but when she becomes too affectionate his love decreases (Graph 7b).

1.8 Love Affair # 8

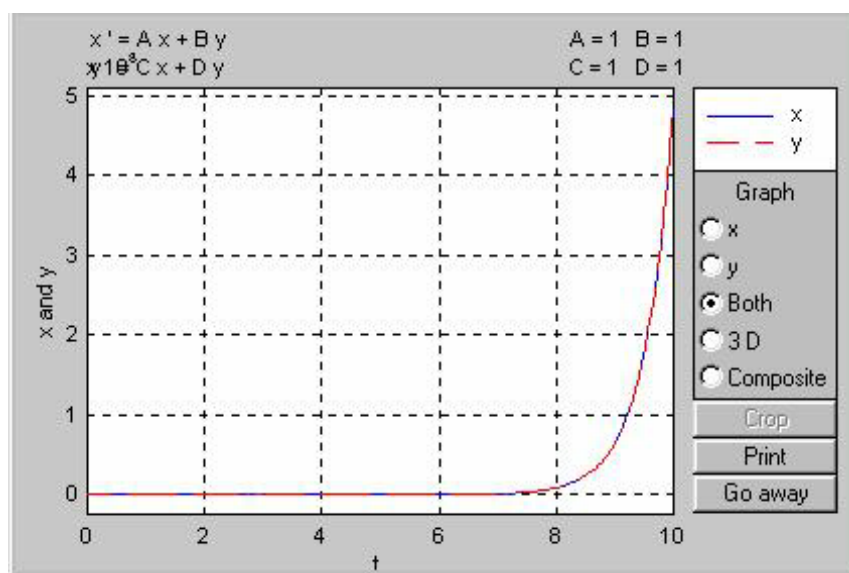
In this case we used the matrix as follows:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad (8)$$



Graph 8a

The system has straight lines and a nullcline at $y = -x$ (Graph 8a).



Graph 8b

Alas! The classic notion of true love forever is supported by this mathematical model. The feelings of love that are held by both Romeo and Juliet

are returned wholeheartedly by the other (Graph 8b).

1.9 Conclusion

Love is a strange and complex thing, not unlike people. When people fall in love any number of situations may occur. As complex as a situation may get, it can still be modeled by a differential equation. While we only dealt with linear equations, we were hardly limited in finding variations of love/hate relationships.