Portfolio Optimization Under a Minimax Rule

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This paper provides a new portfolio selection rule. The objective is to minimize the maximum individual risk and we use an l_∞ function as the risk measure. We provide an explicit analytical solution for the model and are thus able to plot the entire efficient frontier. Our selection rule is very conservative. One of the features of the solution is that it does not explicitly involve the covariance of the asset returns.
(*Portfolio Selection; Risk Averse Measures; Bicriteria Piecewise Linear Program; Efficient Frontier; Kuhn-Tucker Conditions*)

1. Introduction

The portfolio selection problem is of both theoretical and practical interest. Markowitz (1952) laid the foundations for this line of research with his mean-variance (M-V) model. While Markowitz used the portfolio variance as a risk measure, other risk definitions have been proposed. Konno (1990) and Konno and Yamazaki (1991) used the mean absolute deviation as their risk measure. The mean absolute deviation corresponds to an l_1 function, whereas the variance corresponds to an l_2 function. In this paper we propose a more conservative portfolio selection rule whereby the investor minimizes the maximum risk of the individual assets. This risk measure corresponds to an l_∞ function.

The classic M-V model can be solved analytically for the efficient frontier (see Merton 1972) when short selling is permitted. It has been found that the composition of the optimal portfolio can be very sensitive to estimation errors in the expected returns of the underlying assets; see Chopra and Ziemba (1998), Hensel and Turner (1998), Chopra et al. (1992) and Best and Grauer (1991a, 1991b, 1992). In the case of large-scale optimization problems the relationships between the inputs and the optimal portfolio tends to be obscured (Best and Grauer 1991a). Our model provides a clear connection between the expected returns of the assets and their importance in the optimal portfolio. Under our decision rule there are two steps in the solution. First we rank the individual assets in terms of their expected returns and risks. Second, we compute the optimal properties based on the information contained in the rankings. The ranking rule consists of inequalities among the expected returns. This enables us to see more clearly how the composition of the portfolio varies. There are two important differences between our model and conventional models, such as the M-V model. In our model we do not allow for short selling. We impose this restriction to obtain a simple analytical solution. It is a weakness of the model. In our model correlations among the

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