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The Ways of Digital Sampling

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1. ABSTRACT

In this paper, we introduce important concepts and theorems in signal processing including Nyquist Criterion, aliasing, Fourier transforms, correlation and convolution theorems, noise to signal ratio and mixers. We used the lab equipment to collect some electronic signals and converted them into digital signals. Using these captured data, we tried to demonstrate the application of each theory covered in section 3. We used voltage time series and power spectrum of 1000 kHz frequency with two different sampling rates of 3000 kHz and 1500 kHz to test the Nyquist Criterion and aliasing. Since 3000 kHz $>$ 2000 kHz, our signals (for 3000 kHz sampling rate) were not aliased while our data with 1500 kHz sampling rate were aliased (1500 kHz $<$ 2000 kHz). We also calculated the power spectrum for each sampling rate by applying the Fourier transforms using programming. For our 3000 kHz sampling rate, we showed the spectral leakage and how it can affect the other frequencies present in this spectrum. We failed to show that the autocorrelation function when applied to the voltage, equals the inverse Fourier transform of the power spectrum for our 3000 kHz sampling rate. Meanwhile, we were successful to show how integration time affects the signal-to-noise ratio for different average of N blocks (using our noise data).

Furthermore, using heterodyning process, we worked with SSB and DSB mixers to show the differences in their operations. For our DSB mixer, we chose 700 kHz LO and 665 kHz RF resulting in 1365 kHz output frequency. We successfully predicted the spike in its power spectrum at around 1.36 MHz. For our SSB mixer, we chose 150 MHz LO and 142 MHz RF resulting in 7.5, 8 MHz (approximation) output frequency. We predicted its spikes at -1.5 MHz and 1.5 MHz on its power spectrum, but failed to discover the root of its unexpected spikes. Although we hoped to achieve the theoretical results for each of our experiments, we also expected some failures and inaccuracies. We hope to improve our future experiments with better data capturing and analysis.

2. INTRODUCTION

Fourier series and especially Fourier transforms are important in radio signal processing. It is important to analyze signals in radio astronomy since lots of the data is collected through radio telescopes which also contain noise. To extract useful information from such signals, certain concepts and skills are essential. In this paper, some of the basic and important concepts are introduced and investigated to better understand their use. For instance, power spectrum indicates the power levels of the frequency components present in a signal. There are various ways to obtain the power spectrum (given some voltage data). Also, experimenting with power spectrum requires the use of Fourier transforms, convolution and correlation theorems. Aliasing may impact the power spectrum which is another important concept to consider. In addition, we can affect some frequencies and observe the behavior of the power spectrum. Mixers allow us to do such thing by multiplying the two input signals. We can use different types of mixers such as a double-sideband mixer or a single-sideband mixer. In this paper, all the necessary theorems such as Fourier transforms, Nyquist Criterion, convolution, correlation theorem and other necessary information (mixers, signal-to-noise ratio and such) are discussed in section 3. Our first experiment (section 4) involves the analysis of Nyquist Criterion, aliasing, autocorrelation function, and spectral leakage for our collected data. Our second experiment (section 5) involves signal-to-noise ratio data analysis. In our third experiment (section 6), we use SSB and DSB mixers to capture new data for more data analysis. In section 7, we summarize our experiments' results.

3. BACKGROUND

When converting some electronic signals into digital, it is important to Nyquist sample them. Nyquist sampling allows for the reconstruction of a signal. In a sampled data, the minimum sampling rate which accurately reproduces the spectral frequency, is called the Nyquist Criterion. Also, Nyquist rate is defined as $f_s > 2f_{max}$ where f_s is the sample rate frequency and f_{max} is the maximum frequency. Nyquist rate should not be confused with Nyquist frequency as Nyquist frequency is half of the sample rate. If the Nyquist Criterion is violated then aliasing occurs, such that the different signals cannot be differentiated (not sampled at Nyquist rate).

When we experiment with such signals and conduct data analysis, it is important to be familiar with Fourier Transforms and Discrete Fourier Transforms (DFTs). In order to better understand the universe, Fourier Transforms allow us to transform functions of time (or space) into frequency components (in other words, Fourier Transforms decompose functions of time or space into frequency components). Fourier Transforms allow us to work with sine (or cosine) waves which makes it easier mathematically and use complex exponential functions (including real valued functions). The equations for forward Fourier transforms (F) and inverse Fourier transforms (F^{-1}) are given below ($w = 2\pi\nu$ called the angular frequency coordinate, as the Fourier complement of time):

$$F(\hat{f}(w)) = \frac{1}{2\pi}\int \hat{f}(w)e^{iwt}dw = f(t) \tag{1}$$

$$F^{-1}(f(t)) = \int f(t)e^{-iwt}dt = \hat{f}(w) \tag{2}$$

where $e^{iwt} = cos(wt) + isin(wt)$ and the integral bounds are generally from $-\infty \to \infty$. Introducing some finite bounds to this integral might lead to spectral leakage where power from one frequency contaminates other frequencies. Fourier transform is used for a continuously sampled signals, while a discrete Fourier transform (DFT) is used for discretely sampled signals. In DFTs, the Fourier integral is simply replaced by a summation and the Nyquist criterion is applied. Since we are able to decompose functions of time into functions of frequencies, it is important to understand the meaning of positive and negative frequencies. Frequency is defined as the number of oscillations per second and is often thought of some absolute value concept. In fact, negative and positive frequencies behave the same way and the only difference is as shown: $sin(-wt) = -sin(wt)$ and $cos(-wt) = cos(wt)$ where a negative frequency trails the cosine leading the sine by $\frac{\pi}{2}$. Simply put, a negative frequency can be the phase relationship between the cosine and sine components. Also, positive and negative frequencies have independent information for complex valued functions, while they contain the same information for the real valued functions.

Furthermore, we can define the Fourier transform of the convolution theorem for two functions, as the product of their Fourier transforms. Therefore, the convolution of two functions could also be described as the inverse Fourier transform of the statement above. The convolution theorem for two functions f and g is as follows:

$$F^{-1}(F(f) \cdot F(g)) = \int \hat{f}(w)\hat{g}(w)e^{iwt}dw \tag{3}$$

Also, the correlation theorem is pretty similar to convolution. The product of Fourier transform of one function (f or g) by the complex conjugate of the Fourier transform of the other function, is the Fourier transform of their cross-correlation. The correlation theorem (without further complicating it) is shown below:

$$\int \hat{f}(w)\hat{g}^{*}(w)e^{iwt}dw \tag{4}$$

In radio astronomy, much information about a signal's spectral contents can be obtained through its power spectrum, regardless of the phase. Power spectrum measures the magnitude of the Fourier coefficients of a signal and cross-correlation becomes useful. The power spectrum is equal to the Fourier transform of a signal's auto-correlation function which is defined as the cross-correlation of a function with itself.

Moreove, we can also shift the spectrum to study the heterodyning mixing. Heterodyning is a signal processing technique which allows the mixing of two frequencies to obtain a new signal frequency. The equipment which creates such new frequencies is called a mixer. A real mixer, unlike an ideal mixer, is not ideal (as the name suggests). An ideal mixer multiplies the two input signals. Meanwhile, in a real mixer, the nonlinear diodes perform an estimated multiplication and produce undesired harmonics which are called the intermodulation products. Mixers multiply the input signal by a sine-wave local oscillator (LO). In double-sideband mixer (DSB), a radio frequency (RF) signal goes into one mixer port while the LO goes into the second mixer port which outputs the intermediate frequency (IF) through the third port. Meanwhile, a single-sideband mixer (SSB) consists of two identical DSB mixers and LO goes into both of them and is 90^o shifted (in the right hand mixer). The left hand output is mixed with a cosine while the right hand output extracts the sine component. The left hand output can be neglected. They create the real and imaginary components of a complex function that can provide information through power spectra. But firstly, let's start applying the first introduced theorems into some experiments.

4. EXPERIMENT ONE

Applying the Nyquist theorem to our sampled data files, we have investigated two cases, one is expected to be aliased, while the other is not. We then explored the differences between the two and how the application of Nyquist theorem could help us predict aliasing. We expect the plots of larger sample rates to look more sinuous shaped while the smaller sample rates should trail off. Furthermore, we have worked with Fourier transforms and calculated the power spectrum of our voltage data. Ideally, we should not get any harmonics in the spectrum. We also expect our inverse Fourier transform of the power to be equal to autocorrelation when directly applied to the voltage data. We expect to see complete overlap between their plots. Also, we expect to observe nonzero powers at frequencies other than $\nu_0 = 1000$ kHz due to spectral leakage for our 3000 kHz sampling rate.

4.1. *Methods / Experimental Setup*

Using the generator function in the lab, we captured ten data files of sine waves. The ν_s was fixed at 500 kHz (five data files) and 1000 kHz (another five data files), while the ν inputs were selected to range from 1000 kHz to 3000 kHz. We concluded that this approach could better illustrate the aliasing since it overrides the default filter of our SDR modules. Since our captured data allows us to investigate voltage time series at different frequencies, we are able to calculate the power spectrum P_ν. Power is proportional to voltage squared. However, it is important to consider how the calculation can be affected for complex quantities. When squaring voltage in case of complex quantities, P_ν can be obtained by multiplying the complex conjugate of voltage with itself (as shown by equation below):

$$P_\nu = \tilde{E}(\nu)\tilde{E}^*(\nu) \tag{5}$$

where the $\tilde{E}(\nu)$ is the complex valued function and can be calculated by the forward Fourier transform equation (1), applied to a signal versus time function $E(t)$. Also, the power spectrum can be calculated from the Fourier transform of the autocorrelation function. Assume two arbitrary functions of time ($E(t)$ and $F(t)$). The autocorrelation function is defined as the correlation function (4) applied to these two arbitrary functions when $E(t) = F(t)$. As mentioned in section (3), spectral leakage is produced as a consequence of introducing finite bounds to our Fourier transforms. We can think of these finite bounds as some function called *W(t)* that is unity for $-\frac{T}{2} < t < \frac{T}{2}$, and zero everywhere else. In fact, the finite Fourier transform is equal to the infinite Fourier transform of $E(t)$ and multiplied by $W(t)$. The convolution applied to this leads to the result below:

$$E_{finite}(\nu) = E_{infinite}(\nu) * W(\nu) \tag{6}$$

Where $W(\nu)$ stands for the sinc function which is defined as $\sin x/x$ (the classic result of Fourier transforming our W(t)). Although, 10 data files were collected during our data capture (each contains 2048 samples), we are only going to analyze the data listed below:

- 1000 kHz frequency at 3000 kHz sample rate (no aliasing expected)
- 1000 kHz frequency at 1500 kHz sample rate (expected aliasing)

4.2. *Analysis*

According to Nyquist Criterion (section 3), aliasing happens if $f_{sample} > 2f_{max}$ is violated. From this simple condition, we can predict aliasing for our data. Starting with the first data of 1000 kHz frequency at 3000 kHz sample rate, we predict no aliasing because $3000 > 2(1000) = 2000$ as the Nyquist Criterion is not violated. Meanwhile, for the 1000 kHz frequency at 1500 kHz sample rate, we predict aliasing as the above relation is violated. The plots of voltage time series are helpful in understanding how the value of sample rates can cause aliasing. We plotted three different sample rates of 1500 kHz, 2500 kHz and 3000 kHz in Figure (1) for better demonstration of this phenomenon.

Since we are working with time-ordered voltage data, we can also express the power spectrum for each case. We have applied the Fourier transform (Equation 1) to our voltage data, to achieve the power spectrum as shown in figure 2. As mentioned earlier, the Fourier transform of a power spectrum is equal to the autocorrelation function (according to the correlation theorem). Therefore, we expect the plot of inverse Fourier transform of power to overlap with the autocorrelation function (when applied directly to the voltage time series). However, our plot (for 3000 kHz sample rate) as shown in figure 4 is not what we were exactly expecting. Also, to show the spectral leakage for our

3000 kHz sampling rate data, we applied the Fourier transforms by using *numpy.fft* on our voltage spectrum and padded the input with zeros by choosing a larger n (compared to our length of input at 2048). We then calculated the power spectrum and the frequencies. The results are shown in figure 3.

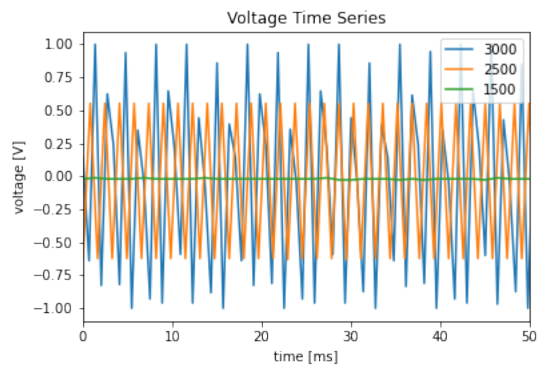
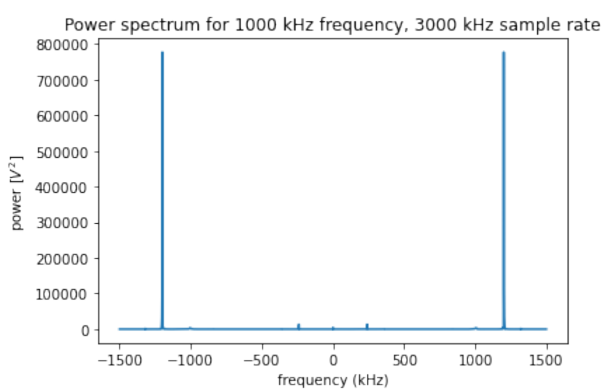


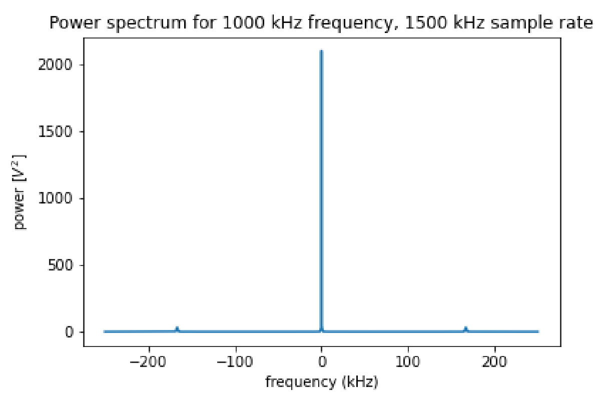
Figure 1: **Voltage Time Series of 1000 kHz frequency**: The sampling rates are 1500 kHz, 2500 kHz and 3000 kHz which are demonstrated on this plot with different colors. These three sampling rates are purposely overplotted to demonstrate the concept of aliasing better. The 3000 kHz sampling rate is not aliased (it doesn't violate Nyquist Criterion) while the 1500 kHz sampling rate is aliased (Nyquist Criterion violation). As the sampling rate increases, the signal appears to behave more like a sine wave (as long as it's following the Nyquist Criterion). This explains the shape of 1500 kHz sampling rate. It has trailed off from a sine wave behavior since it is aliased. The 2500 kHz sampling is plotted as demonstration of increasing the sample rate.

4.3. *Results*

As we expected, our data for 3000 kHz sample rate was not aliased while the 1500 kHz sample rate contained some aliasing. As mentioned above, theoretically, we can check whether aliasing is to be expected or not. By plotting the different sample rates in figure (1), we can conclude that the 3000 kHz sample rate is a sine wave (or at least close), while the 1500 kHz has trailed off from being a sine wave. In fact, larger sample rates indicate no aliasing (as long as they don't violate the Nyquist

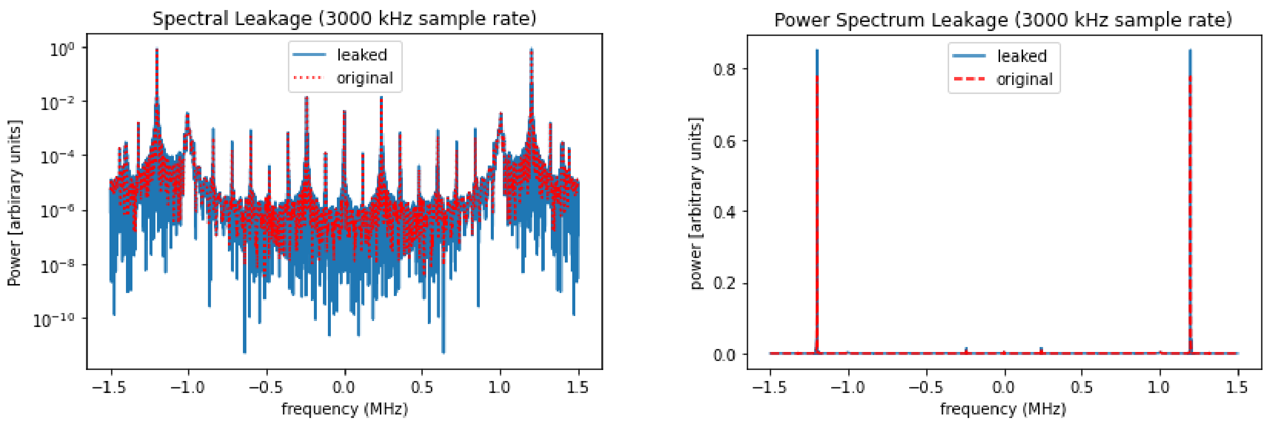


(a)



(b)

Figure 2: **(a) The Power Spectrum of 3000 kHz sample rate**: The application of Fourier transforms on our voltage data allows us to calculate the power spectrum. As demonstrated, the power spectrum for 3000 kHz sample rate contains two spikes (as expected). However, the much smaller spikes are harmonics and we believe it might be caused by the equipment. We are not sure about the real cause of these harmonics. The presence of negative frequency can be thought of the phase relationship between the cosine and sine components where it trails the cosine leading the sine by $\frac{\pi}{2}$. This is the case of no aliasing. **(b) The Power Spectrum of 1500 kHz sample rate**: As shown, the power spectrum for 1500 kHz sample rate contains only one spike. This is an aliased case since $2f_{max}$ exceeds the sample rate. We think the power spectrum only contains one spike since the value of sample rate is close to 1000 kHz frequency, and the $2f_{max}$ is not relatively far from 1500 kHz frequency. The harmonics and negative frequencies appear again (as discussed in figure (a)).



(a) (b)

Figure 3: **(a) Spectral Leakage for 3000 kHz sampling rate**: Both the original spectrum and the leaked spectrum are overplotted on each other to demonstrate how power of one frequency can contaminate other frequencies. It is shown how leakage can affect the shape of our spectrum from its expected (original) form. By using a logarithmic vertical axis, the nonzero powers (other than the $\nu_0 = 1000$) are observed because of spectral leakage. **(b) Power Spectrum Leakage for 3000 kHz sampling rate**: Both the original power spectrum and the leaked power spectrum are overplotted and it is clear that they mimic each other very closely. However, the leaked power spectrum seems to be a bit larger than the original at certain frequencies. This might be evidence how leakage can affect the other frequencies. There are small harmonics present as discussed before. We are not sure the real cause of these harmonics, but guess they might be due to some equipment fault.

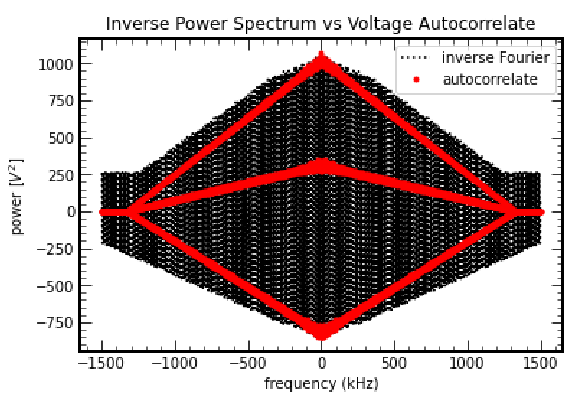


Figure 4: **Inverse Fourier Transform and Autocorrelation Function**: Based on theory, the autocorrelation function (ACF) applied to the voltage time series should equal to the inverse Fourier transform of its power spectrum. If plotted, these two should overlap perfectly. Although, these two do overlap in this plot, it is not a complete overlap and fails our prediction. We are not sure about the cause. Even after following the other group members' successful approach, it fails and we believe it could be due to some coding issue/syntax error.

Criterion, since they look more similar to a sine wave). On the other hand, lower sample rate values (as long as they violate Nyquist Criterion), are less likely to be behaving like a sine wave and we can observe that for the 1500 kHz in our figure (1). Calculating the Fourier transforms of our voltage data by using NumPy then taking the absolute value and squaring it, we plotted the power spectrum for each case. The power spectrum of our 3000 kHz sample rate is shown in figure (2) and two spikes are shown as we have expected. The ideal expectation for our plot was to see only two spikes, but there seems to be some small ones. These small spikes might be harmonics and due to the sampling rate, we expect to see them regardless. Also, the concept of negative frequencies are observed in our power spectrum which we discussed in section (3). The power spectrum for our aliased data (1500 kHz sample rate) is shown in figure (2). The harmonics are observed again. We think in case of aliased data, the harmonics should shower down, but we still need more research to confirm this theory. This spectrum only has one spike. We think if the sampling rate is equal or nearly equal to the observed frequency, then the reconstruction of the aliased wave would result in a horizontal line. The Fourier transform of such horizontal line and achieving the power spectrum of it would result in a single vertical line. The Nyquist is violated and our sampling rate is close enough to our observed frequency. We think this is the cause for the single spike in power spectrum.

On our analysis for the autocorrelation and the inverse Fourier transform of the power, we gained interesting results as shown in figure 4. We were expecting a complete overlap of the autocorrelation and the inverse Fourier transform, but we failed to achieve that. Although, they do overlap, it is not a complete overlap. We are not certain of the cause. By the convolution theorem (3), we could identify the spectral leakage on our 3000 kHz sampling rate. We observed nonzero power at frequencies other than $\nu_0 = 1000$ kHz which shows the evidence of spectral leakage. We were successful in showing how leakage can affect our original spectrum and its power as shown in figure 3.

5. EXPERIMENT TWO

We captured another set of data to experiment to observe how integration time affects the SNR (signal to noise ratio). Analyzing the data which contains noise by a set number of blocks is helpful to such investigation. We believe the power spectrum for a single block should be different from the average power spectrum over all the blocks. We expect the average power spectrum of higher amounts blocks to result in narrower distribution when plotted.

5.1. *Methods / Experimental Setup*

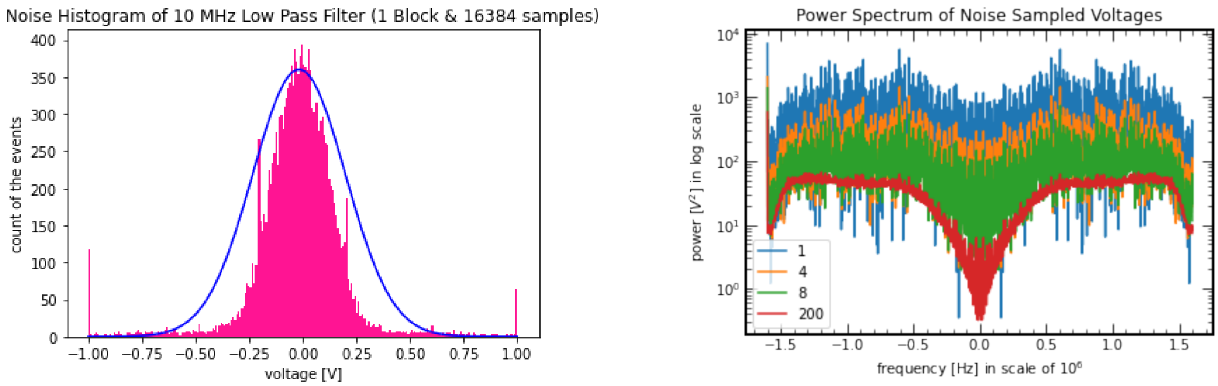
In signal processing, noise is referred to unwanted or unknown fluctuations within a known statistical distribution in signals. The signal to noise ratio (SNR) is the measurement of a signal's intensity (strength) relative to the noise. Using a noise generator in lab, we collected new sets of data that contain noise. We also used a filter to the output to avoid aliasing so we can conduct our experiment more efficiently. We used Fourier transforms (1) to calculate the power spectrum of these sampled voltages. We used 10 MHz low pass filter with 16384 samples. We arbitrary chose 200 blocks as our maximum number of blocks for this analysis.

5.2. *Analysis*

We calculated the mean of our voltage data to be -0.0174 with a standard deviation of 0.216 and variance of 0.0469. We plotted a histogram of our sampled voltages over 250 bins and then overplotted the theoretically-expected Gaussian (figure 5). As mentioned earlier, by applying Fourier transforms, we calculated the power spectrum of a single block and compared it with the average power spectrum of 200 blocks and 8 blocks as shown in figure (5). The power on this plot is demonstrated in log scale and the frequency is in scale of 10^6 Hz.

5.3. *Results*

Comparing the theoretically-expected Gaussian plot on our noise histogram (figure 5), it is evident that our standard deviation value is smaller than our theoretically expected. Meaning that our data is clustered around the mean. Also, harmonics show up in this plot and we think further investigation is needed to discover the cause. Comparing the power spectrum of a single block with the power spectrum of the average of N blocks (which we chose arbitrarily 4, 8 and 200), we noticed that they overlap (figure 5). Separately analyzing and plotting each case (when $N = 200$ or 8), we discovered they are not shifting on y-axis. Instead, the width of these plots changes as N changes. In fact, the



(a)

(b)

Figure 5: **(a) Noise Histogram**: The theoretically-expected Gaussian is shown as the blue figure and is overplotted on the noise histogram of a 10 MHz low pass filter (which contains of only 1 block and 16384 samples). As shown, the theoretically-expected Gaussian appears to be wider than our observation. Our observation's standard deviation appears to be smaller than theory. The harmonics are present again (as discussed in figure 2). **(b) Noise to Signal Ratio (SNR) Power Spectrum**: The power spectrum for the average of N blocks is plotted where $N = (1, 4, 8, 200)$ and 200 is the maximum limit. As N increases, the width of the SNR plot becomes narrower. This demonstrates how integration time affects the SNR. Due to the y-axis scaling in a log base, the SNR figures might appear to be shifting on y-axis. There is no shifting on y-axis, and only factor changing, is the width of SNR (as expected). The x-axis is in scale of 10^6Hz.

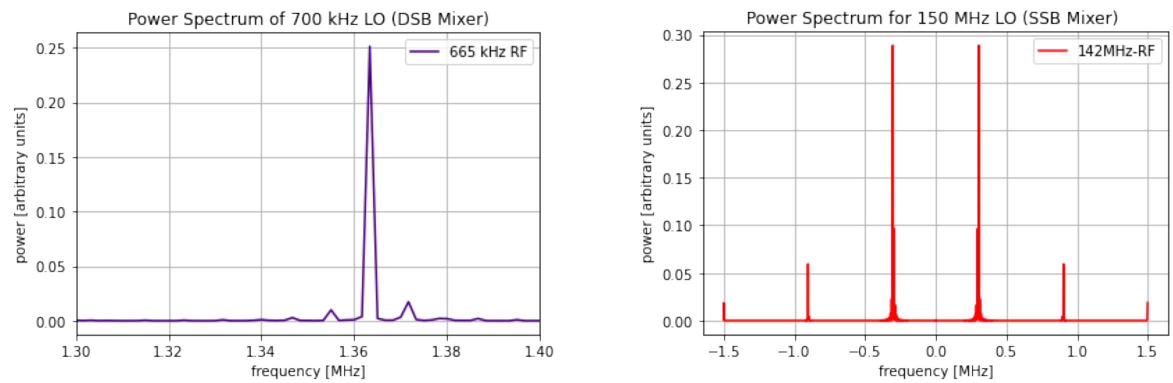
higher value of N (200 for example) results in a narrower distribution while lower values of N behave the opposite as we had expected.

6. EXPERIMENT THREE

In this experiment, we applied the heterodyne mixing to create frequency conversion and worked with DSB and SSB mixers. We were successful in calculating the power spectrum for each mixer. In DSB case, we expect the power spectrum to show a spike at 1.36 MHz since the sum of our LO and RF equals 1365 kHz. We also expect no aliasing (Nyquist Criterion). In SSB case, we expect the power spectrum to spike at approximately 1.5 MHz because the difference between our LO and RF does not get filtered out. We expect aliasing since $f_{sample} = 2f_{max}$ (approximately and the violation of Nyquist Criterion).

6.1. *Methods / Experimental Setup*

When digitally sampling the mixer output, it is important to identify the sum and the difference between our LO and RF frequencies. It can help us predict where to expect spikes on our power spectrum. The DSB mixer only requires one mixing circuit and both the LO and RF are real-valued sine waves, but it cannot distinguish between positive and negative divergence around the LO frequency. As a result, it stacks the two signals on top of each other such that the output frequency can be the sum of two different signals. Meanwhile, the SSB mixer filters out the sum of the frequencies while the difference is not filtered. This allows us to maintain the original components of the original signal that were multiplied by cosine and sine components of the LO. This allows us to distinguish between the components of the two signals whether they appear as positive or negative frequencies (negative frequencies discussed in section 3). As discussed in section 3, we set up our DSB mixer with 700 kHz LO frequency and 665 kHz RF frequency. Since the first block of data might have been faulty, we used the third block for our power spectrum analysis. Setting up the SSB mixer (following 3), we set our LO frequency to 150MHz and the RF at 142 MHz. As mentioned, the first block of data might have been faulty, so we chose the second block for the power spectrum analysis.



(a) (b)

Figure 6: **(a) DSB Mixer Power Spectrum**: The output frequency of DSB mixer is the sum of LO (700 kHz) and RF (665 kHz) which equals to 1365 KHz. Since the maximum frequency is at 3 MHz, the spike is observed around 1.36 MHz (as predicted). There is no aliasing since the Nyquist Criterion is not violated. The frequency axis (x-axis) is zoomed in to better demonstrate the location of the expected spike. The smaller spikes are believed to be harmonics. **(b) SSB Mixer Power Spectrum**: The SSB mixer filtered out the sum of LO (150 MHz) and RF (142 MHz) frequencies while their difference is left unfiltered. The estimate of their difference was calculated to be around 7.5 or 8 MHz. Maximum frequency is set at 3 MHz and we observe aliasing. As expected, there are spikes at 1.5 MHz and -1.5 MHz. The concept of negative frequencies in SSB mixers is shown here (discussed in section 3). Unexpected spikes are observed on this power spectrum and we believe they might be the harmonics. We don't know why these unexpected spikes are larger in magnitude (compared to our expected spikes). We also don't know why they appear in range of (-1,1) MHz.

6.2. *Analysis*

The power spectrum for the DSB mixer is shown in figure (6). We applied the Fourier transforms on the third block of our data to get the power spectrum. For better observation on the occurance of expected spike, we zoomed in on the frequency axis (x-axis) where the spike happens at around 1.36 MHz frequency. This is the case for a 665 kHz RF, 700 kHz LO and 3 MHz maximum frequency. Since the DSB mixer outputs the sum of LO and RF frequencies, we added them together and obtained 1365 kHz frequency. Then we checked whether our signal would be aliased (following the Nyquist Criterion, $(2(1.365) = 2.73$ MHz)

We calculated the power spectrum for our SSB mixer's output frequency (applying Fourier transforms to our second block of voltage data) as shown in figure (6). We got an estimate of 7.5 MHz for our LO and RF frequency difference as the SSB's output signal. Since our maximum frequency is at 3MHz then aliasing should be expected for frequencies higher than 1.5 MHz ($\frac{3}{2}$ MHz). We should expect spikes at 1.5 MHz frequencies for its power spectrum.

6.3. *Results*

For our DSB mixer's output frequency, (as predicted) we observed no aliasing since 2.73 MHz $<$ 3 MHz such that Nyquist Criterion is not violated. We successfully predicted spike at around 1.35 MHz as shown in figure (6). We see some smaller spikes as well and we believe them to be harmonics. Meanwhile, we got some interesting and unexpected results from our SSB mixer's output frequency. We were successful in predicting aliasing since our approximated output frequency 7.5 MHZ $>$ 3 MHZ. We also successfully predicted spikes at around 1.5 MHz as shown in figure (6) and we see negative frequencies in SSB appear (discusses in section 3). However, our expected spikes seem to be small while the unexpected ones are much larger. We think the unexpected spikes could be harmonics, but we are not sure why they appear much larger than our expected spikes. It might have been due to faulty data capture or faulty equipment. Unfortunately, we were not successful in finding this issue's cause. We believe further investigation might be needed.

7. CONCLUSION

We collected various electronic signals and converted them into digital signals for our analysis. Our analysis focused on some important theorems in signal processing. We introduced the Nyquist theorem and demonstrated how the violation of Nyquist Criterion can lead to aliasing. As we expected, we were able to see aliased signals trail off from a sine-wave behavior. We were successful in applying the Fourier transforms to our voltage data and obtained the power spectrum for both cases of aliasing and non aliasing. Unfortunately, our plots contained some harmonics throughout our analysis and we guess it might have been due to our equipment. However, we are not sure, and believe further research is required to discover the true cause. Also, working with data that contained some noise, allowed us to successfully demonstrate how integration time affects the signal-to-noise ratio. By applying Fourier transforms, we successfully achieved the change in width of our SNR and proved to be dependent on the average of N blocks where N was set to be some arbitrary as long as it didn't exceed our maximum limit of 200 blocks. The higher the value of N, the narrower our SNR appeared. Unfortunately, we failed to demonstrate the complete overlap of the inverse Fourier transform of power spectrum with the autocorrelate function (applied to our voltage). In theory, the Fourier transform of power spectrum should equal to the autocorrelate function, but even after following the same approach as some of our team members, this paper fails to achieve the goal. We are not sure why this is happening and after much investigation, we could not find a solution. However, we were successful in showing the spectral leakage for our 3000 kHz sampling rate data and discussed how leakage can affect the other frequencies.

Lastly, we built a double sideband mixer through heterodyning process. We discussed the differences between ideal and real mixers. Also, we constructed a single sideband mixer and tried to show the differences between SSB and DSB theoretically and practically.

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