# A Toric Variety from Machine Learning

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## 1 Introduction

## 2 McCulloch-Pitts Process

Given a directed graph $G = (V, E)$ with vertex weights $\beta_i > 0$ and edge weights $\alpha_{ij} > 0$, a *McCulloch-Pitts process, MPP* is an activity-based process with binary states $x \in \{0,1\}^{|V|}$ and transitions $xy$ where state $y$ is one-bit away from state $x$. If $y$ and $x$ differs in the $i$-th bit, we define the transition rate

$$F_{xy} = [\beta_i^{\sigma_i} \alpha_i^{x\sigma_i}]^{1/\tau} \text{ shouldn't it be } \alpha_{ij}?$$

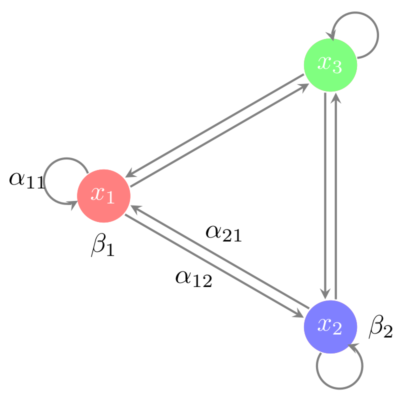


Figure 1: McCulloch-Pitts process with three neurons.

## 3 The Toric Variety

We consider the space of weights $\mathbb{C}^{12} = \{(\beta_i, \alpha_{jk}) \mid 1 \leq i, j, k \leq 3\}$ and the space of transition rates $\mathbb{C}^{24} = \{(F_{xy}) \mid x, y \text{ binary states differing at one bit}\}$. We have a map $f \colon \mathbb{C}^{12} \to \mathbb{C}^{24}$ defined how?. We define the toric variety $X$ as the Zariski closure of the image of $f$.

**Calculation 1.** The variety $X$ has dimension 12 and degree 216. This can be directly calculated after finding the ideal of $X$ via elimination of variables. better: use PolyMake to take advantage of the toric structure