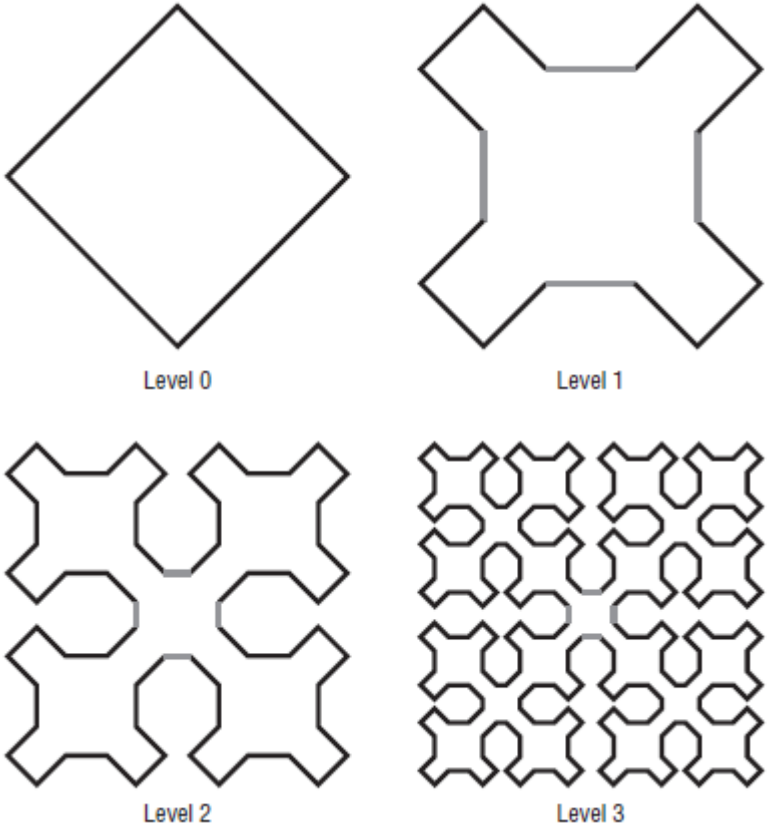
Recursion, Part 1



Agenda

- Basic Algorithms
- Graphical Algorithms
- Summary
- Exercises

Basic Algorithms

- Factorial
- Fibonacci numbers
- Rod Cutting
- Tower of Hanoi

Factorial

- 0! = 1
- N! = N × (N - 1)!

```
Integer: Factorial(Integer: n)
    If (n == 0) Then Return 1
    Return n * Factorial(n - 1)
End Factorial
```

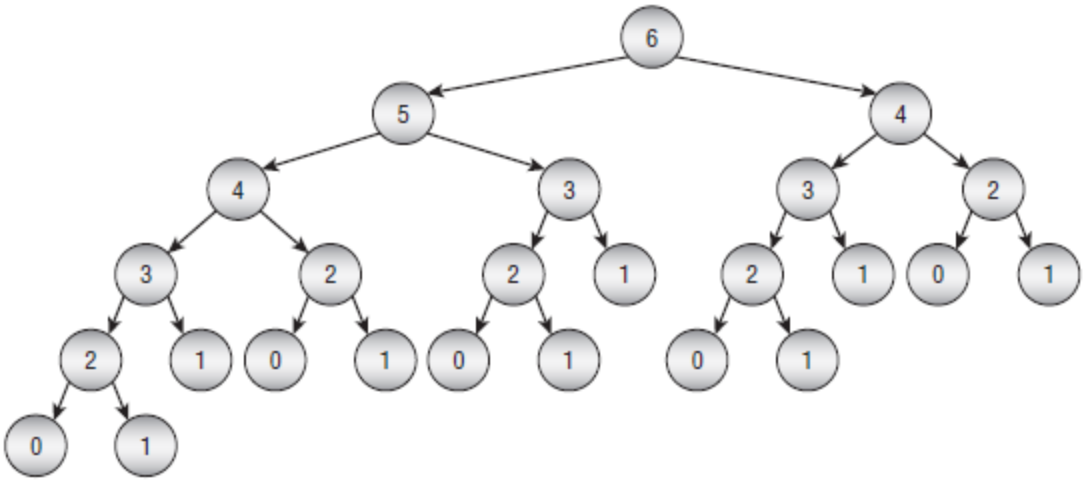
Fibonacci Numbers

- Fibonacci(0) = 0
- Fibonacci(1) = 1
- Fibonacci(n) = Fibonacci(n – 1) + Fibonacci(n – 2) for n > 1

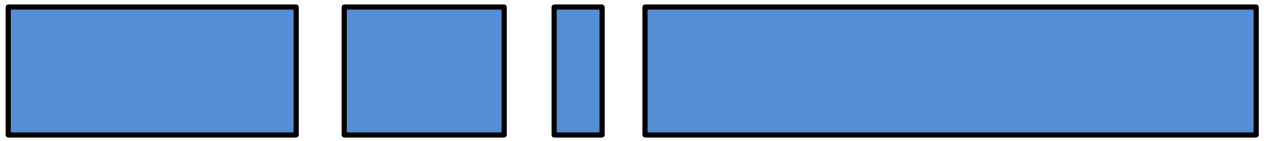
```
Integer: Fibonacci(Integer: n)
    If (n <= 1) Then Return n
    Return Fibonacci(n - 1) + Fibonacci(n - 2);
End Fibonacci
```

Fibonacci Performance

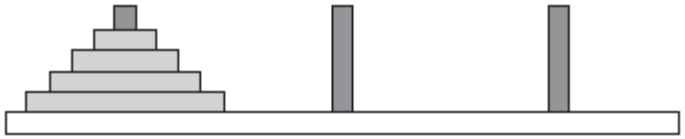
- Lots of duplicated values



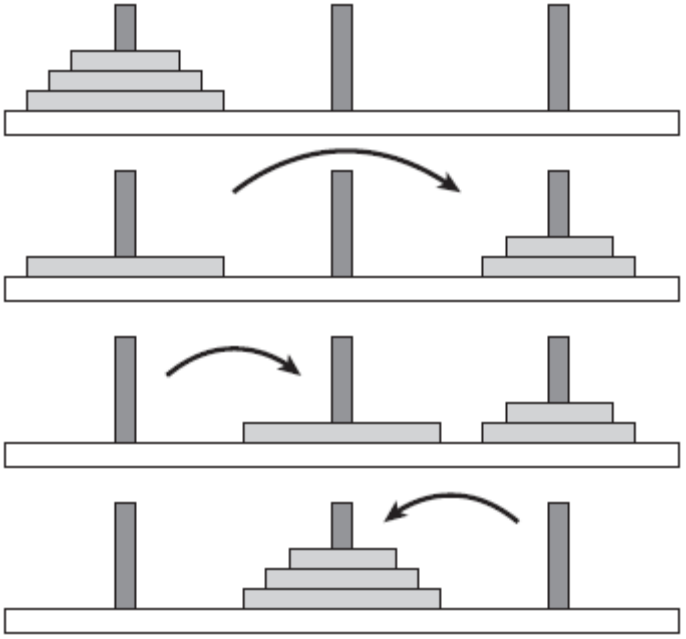
Rod Cutting



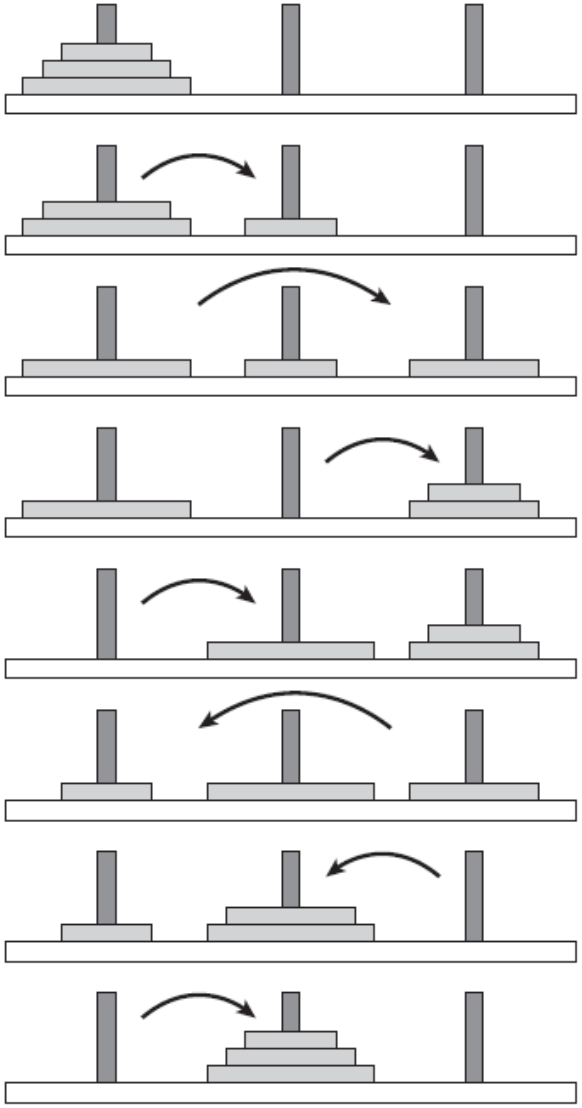
Tower of Hanoi



Tower of Hanoi Recursion



Tower of Hanoi Full Example



Graphical Algorithms

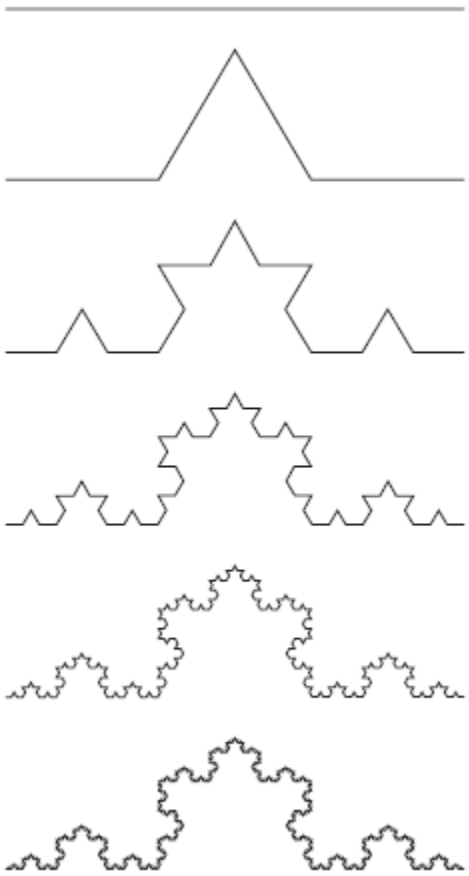
- Koch Curves
- Hilbert Curve
- Sierpiński Curve
- Gaskets
- The Skyline Problem

Koch Curves

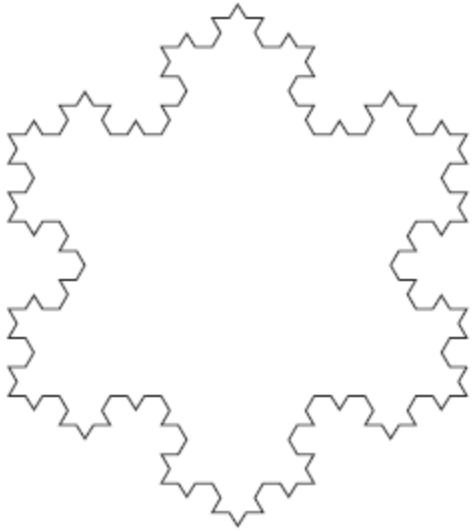
Initiator Generator

- Each part of the larger curve is a copy of the smaller curve suitably scaled and rotated

Koch Levels 0 - 5

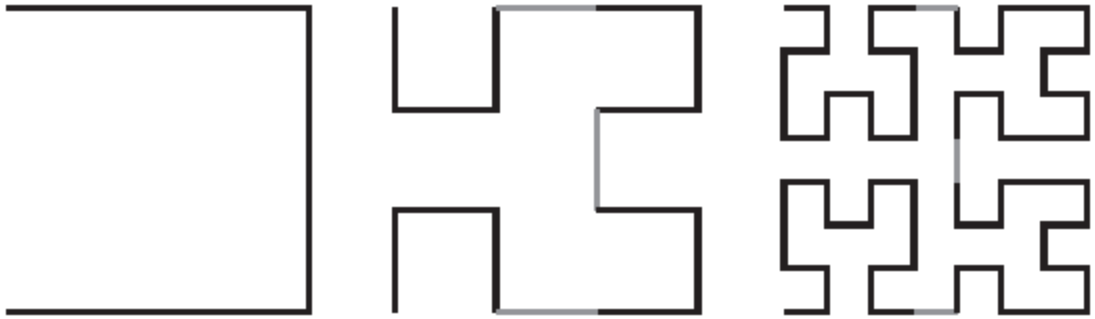


Koch Snowflake



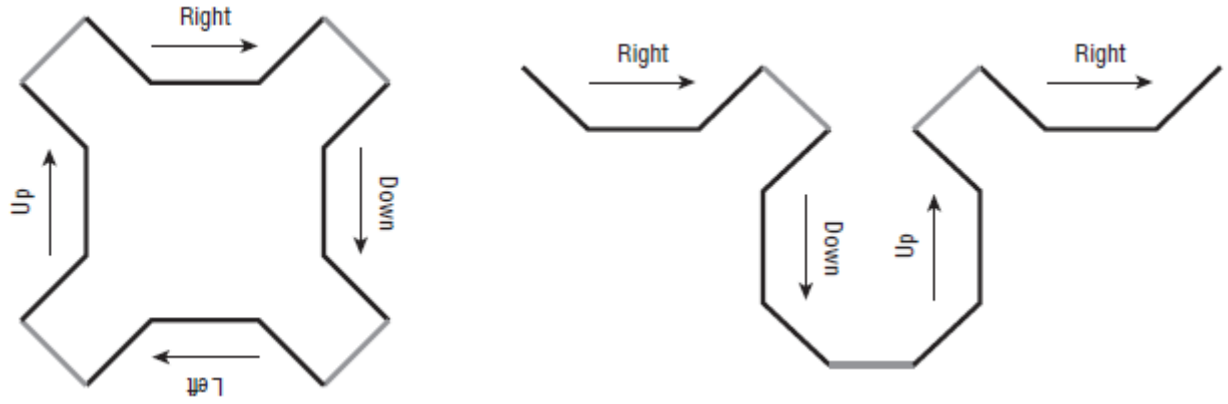
Hilbert Curve

- Each part of the larger curve is a copy of the smaller curve suitably scaled and rotated

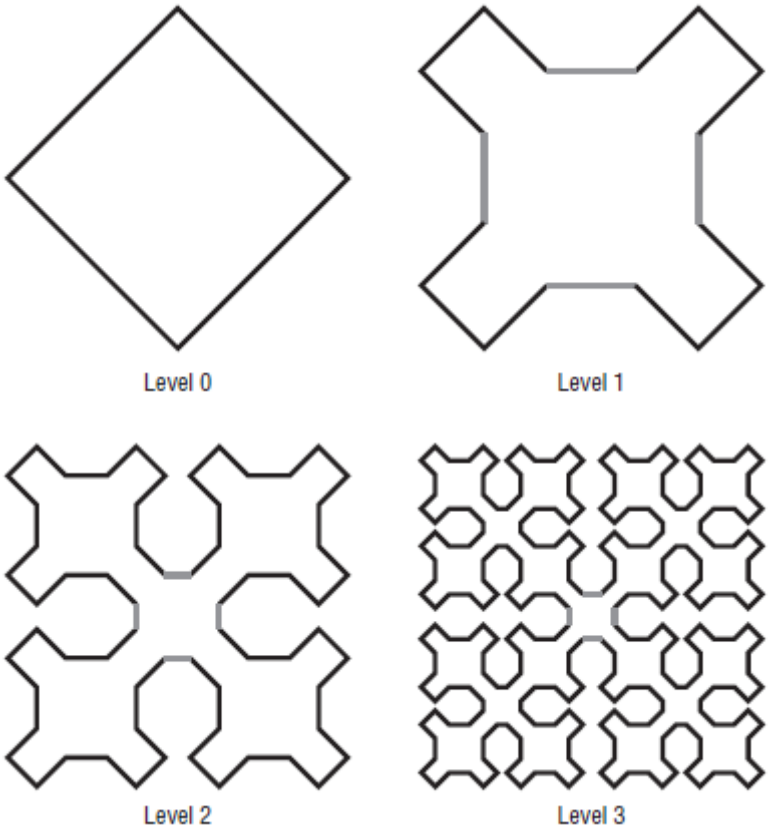


Sierpiński Curve

- Each part of the larger curve is a copy of the smaller curve suitably scaled and rotated



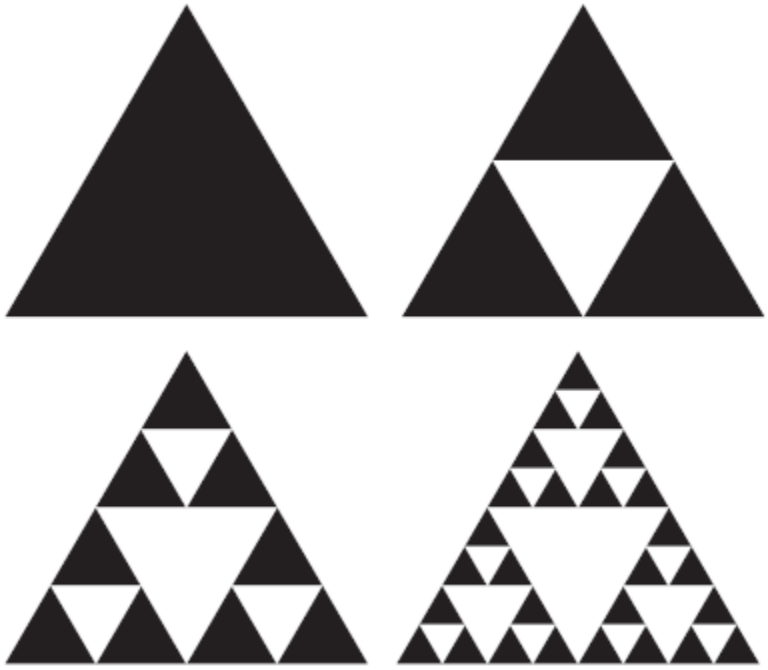
Sierpiński Curve Levels 0 - 3



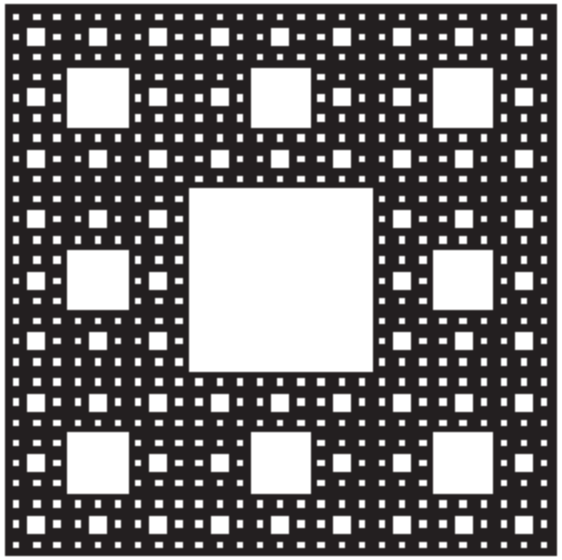
Gaskets

- Divide an area into shapes
- Recursively color some of the shapes

Sierpiński Gasket
(or sieve or triangle)

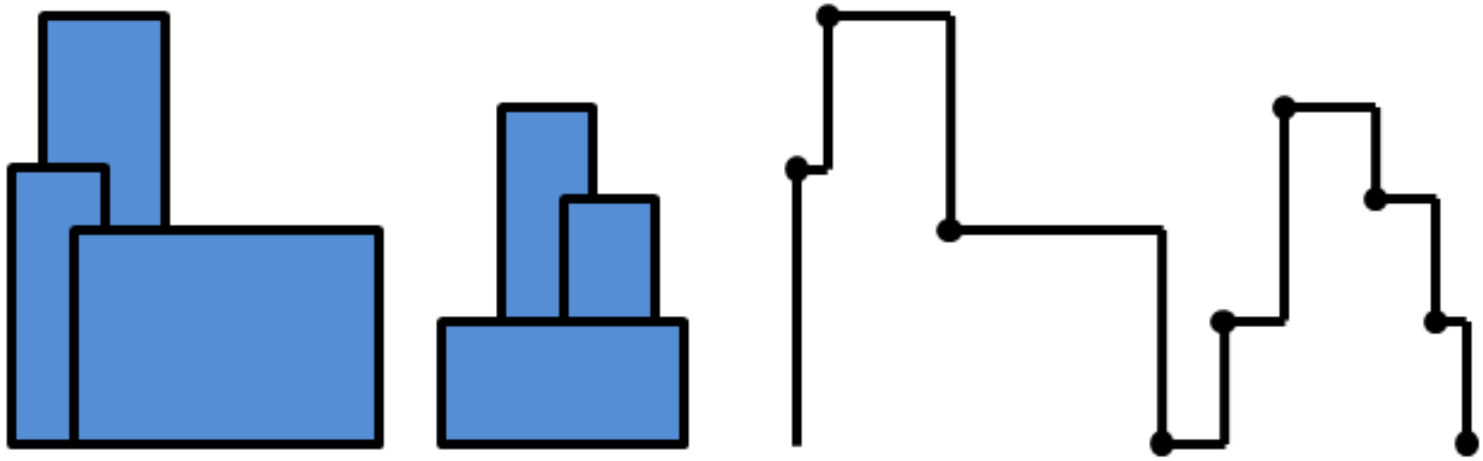


Sierpiński Carpet



The Skyline Problem

- List
- Divide and Conquer



Summary

- Basic Algorithms
  - Factorial
  - Fibonacci numbers
  - Rod Cutting
  - Tower of Hanoi
- Graphical Algorithms
  - Koch Curves
  - Hilbert Curve
  - Sierpiński Curve
  - Gaskets
  - The Skyline Problem

Exercises

- Chapter 9 Exercises 1 – 3, 5, 7, 8, 10, 11.
- Do one of Chapter 9 Exercises 4, 6.
- Do one of Chapter 9 Exercises 7, 9.
- Read *Essential Algorithms, 2e* Chapter 9 pages 252 – 284. (The rest of Chapter 9.)