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After realizing that Manhattan distance and Euclidean distance both not admissible,   
**I decided to use the *diagonal distance* heuristic function**.

In general: diagonal distance – also known as Chebyshev distance and is calculated by the maximum of differences between X's and Y's of nodes n1 and n2 respectively.

In our search problem: **n2 is always the goal node!**

Let us consider the diagonal distance heuristic function h(n) for a given node n in our search problem – a 2D matrix of chars of size N.   
Each node in our search problem holds a tag, i.e. starting node has tag 'S' and goal node has tag 'G'.

To prove admissibility of this heuristic function, we need to show that it never overestimates the actual cost of reaching the goal node.   
That is, for any node n and its goal state g, h(n) must be less than or equal to the actual cost of reaching g from n, denoted by h\*(n).

By definition of admissibility:

**Proof of Admissibility:**

Consider a node n and its corresponding goal state g.   
Let and be the absolute differences in the x and y coordinates between n and g, respectively.   
Let h(n) represent the Chebyshev distance heuristic value of node n.

We aim to prove that h(n) is less than or equal to the actual cost of reaching g from n, denoted as h\*(n).

To do so, let's examine two cases:

Case 1: Horizontal or vertical movement from n to g:

In this scenario, the actual cost of reaching g from n will be the sum of the absolute differences in the x and y coordinates:

Since the Chebyshev distance heuristic takes the maximum of the absolute differences, we have:

Now, since and are both non-negative, it follows that:

Therefore, the Chebyshev distance heuristic is admissible for horizontal or vertical movement.

Case 2: Diagonal movement from n to g:

In this case, the actual cost of reaching g from n will be the maximum of and ) because diagonal movement allows covering the larger of the two differences in a single step.

Without loss of generality, let's assume is greater than or equal to Thus,

Now, for the Chebyshev distance heuristic, we have:

Since is greater than or equal to it follows that:

But is the actual cost of reaching g from n, so we have:

Therefore, the Chebyshev distance heuristic is also admissible for diagonal movement.

In both cases, we have demonstrated that the Chebyshev distance heuristic never overestimates the actual cost of reaching the goal state. Hence, it is proven to be admissible.

Note: The admissibility of the heuristic function is not affected by the cost from a neighbor of g to g, as long as the heuristic itself never overestimates the actual cost. The cost from a neighbor of g to g does not impact the admissibility proof for the Chebyshev distance heuristic.

**Proof of consistency:**

Let's show local consistency and by that we get global consistency.

To prove local consistency, we need to demonstrate that for any two adjacent nodes (neighbors) n and m, the heuristic value of n is less than or equal to the heuristic value of m plus the cost of moving from n to m, in formal way:

Let's consider two adjacent nodes n and m, which are horizontally, vertically, or diagonally connected in our matrix. Let c(n, m) denote the cost of moving from n to m, which is 1 for horizontal, vertical, and diagonal movements.

The heuristic value for nodes n, m are h(n), h(m) respectively (I already defined what h is 🡪[definition here](#Chebyshev_definition)).

Now, let's analyze the possible cases:

Case 1: Horizontal or vertical movement from n to m:

In this case, the cost of moving from n to m is 1.

Without loss of generality, let's assume   
Therefore, .

Since n and m are adjacent horizontally or vertically, the difference in their x-coordinates or y-coordinates will be at most 1.   
Thus, we have

Therefore, we can conclude that:

Case 2: Diagonal movement from n to m:

In this case, the cost of moving from n to m is also 1.

Without loss of generality, let's assume   
Therefore, .

Since n and m are adjacent horizontally or vertically, the difference in their x-coordinates or y-coordinates will be at most 1.   
Thus, we have

Therefore, we can conclude that:

In both cases, we have shown that , satisfying the local consistency property.

As for global consistency:

To prove global consistency, we need to demonstrate that the consistency property holds for all nodes in our search problem space, not just adjacent nodes.

The local consistency property ensures that the consistency property holds for any two adjacent nodes. By applying this property recursively for all pairs of adjacent nodes in a path from node start ('S') to the goal ('G'), we can establish global consistency.

Since the Chebyshev distance heuristic satisfies the local consistency property for adjacent nodes, this property can be extended to all nodes along a path. Therefore, the Chebyshev distance heuristic is globally consistent.