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Homework: Search

1. 3 Jugs
"You have three jugs, measuring 12 gallons, 8 gallons, and 3 gallons, and a water faucet. You can fill the jugs up or empty them out onto the ground, or pour from one jug (A) to another (B) until (1) the target jug (B) is full, or (2) the source jug is empty (A). You need to measure out exactly one gallon."

- States: Water level in 3 jugs. Let jugs be J_{12}, J_8, J_3. Variables are W_{12}, W_8, W_3 water level of each jugs s.t.
$0 \le W_{12} \le 12, 0 \le W_8 \le 8, 0 \le W_3 \le 3$.
State is $W = \{W_12, W_8, W_3\}$

- Actions:
    - $Fill(J_x)$: fill jux x from the faucet.
    Precondition : J_x is not full.
    - $Empty(J_x)$ empty jux x.
    Precondition : J_x is not empty.
    - $Pour(J_x, J_y)$: pour water from jug x to jug y.
    Precondition : J_x is not empty and J_y is not full.

- Transition Model:
    - $Fill(J_x) : W_x = x$.
    - $Empty(J_x) : W_x = 0$.
    - $Pour(J_x, J_y) :$
    $if(y - W_y > W_x)\{$
    $W_y = W_x + W_y$
    $W_x = 0$
    $\}else\{$
    $W_y = y$
    $W_x = W_x - (y - W_y)\}$

- Goal Test:
    - $SUM(W) == 1$
    - $W_{12} == 1 \quad OR \quad W_8 == 1 \quad OR \quad W_3 == 1$

- Step cost:
    - 1 per action.

- Path cost:
    - Sum of step costs in the path.
- Answers (Iterative Deepening Depth First Search):
  Maximum Depth = 7
    - Answers 1:
      $\{0,0,0\} \xrightarrow{Full(J_{12})} \{0,0,12\} \xrightarrow{Pour(J_{12},J_8)} \{0,8,4\} \xrightarrow{Pour(J_{12},J_3)}$
      $\{3,8,1\} \xrightarrow{Empty(J_8)} \{3,0,1\} \xrightarrow{Empty(J_3)} \{0,0,1\}$

    - Answers 2:
      $\{0,0,0\} \xrightarrow{Full(J_{12})} \{0,0,12\} \xrightarrow{Pour(J_{12},J_3)} \{3,0,9\} \xrightarrow{Pour(J_{12},J_8)}$
      $\{3,8,1\} \xrightarrow{Empty(J_8)} \{3,0,1\} \xrightarrow{Empty(J_3)} \{0,0,1\}$

    - Answers 3:
      $\{0,0,0\} \xrightarrow{Full(J_{12})} \{0,0,12\} \xrightarrow{Pour(J_{12},J_8)} \{0,8,4\} \xrightarrow{Empty(J_8)} \{0,0,4\} \xrightarrow{Pour(J_{12},J_3)}$
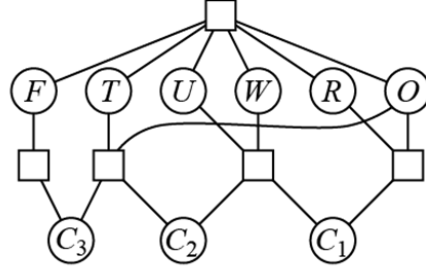 $\{3,0,1\} \xrightarrow{Empty(J_3)} \{0,0,1\}$

    - Answers 4:
      $\{0,0,0\} \xrightarrow{Full(J_{12})} \{0,0,12\} \xrightarrow{Pour(J_{12},J_3)} \{3,0,9\} \xrightarrow{Empty(J_3)} \{0,0,9\} \xrightarrow{Pour(J_{12},J_8)}$
      $\{0,8,1\} \xrightarrow{Empty(J_8)} \{0,0,1\}$

2. Cryptarithmetic

$$
\begin{array}{r}
T\ W\ O \\
+\ T\ W\ O \\
\hline
F\ O\ U\ R
\end{array}
$$

(a)

(b)

Problem is shown on (a). Each letter is a digit. No two letters can have the same digit. Hypergraph (b) shows constraints between variables, where $C1, C2$ and $C3$ are carries, and can have value between 0 and 1. Find the correct digit for each letter.

- Variables : $F, T, U, W, R, O, C_1, C_2, C_3$
- Domains:
    - $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ for F, T, U, W, R, and O
    - $0, 1$ for carry variables C_1, C_2 and C_3
- Constranints: $Alldiff(F, T, U, W, R, O)$
    - $O + O = R + 10C_1$
    - $C_1 + W + W = U + 10C_2$
    - $C_2 + T + T = O + 10C_3$
    - $C_3 = F, T \neq 0, F \neq 0$
- Answer:
    - ■ Find singleton domain :
      From $C_3 = F$ and $F \neq 0$ and $C_3 \in \{0, 1\} \Rightarrow F = 1$
    - ■ Minimum Remaining Values :
      From $C_1, C_2 \in \{0, 1\}$, We assigned $C_1 = C_2 = 1$, so
      $$
      O + O = R + 10 \Rightarrow O \geq 5
      $$
      $$
      W + W = U + 9 \Rightarrow W \geq 5, U \geq 3
      $$
      $$
      T + T = O + 9 \Rightarrow T \geq 5
      $$
    - ■ Minimum Remaining Values :
      From $O, W, T \in \{5, 6, 7, 8, 9\}$ and $U \in \{3, 4, 5, 6, 7, 8, 9\}$ and $R \in \{0, 1, \cdots, 8, 9\}$. We assigned $O = 5$, so $T = 7$ and $R = 0$

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- Minimum Remaining Values :
  From $W \in \{6, 8, 9\}$ and $U \in \{3, 4, 6, 8, 9\}$. We assigned $W = 6$ so $U = 3$

- Check Answer: Our answer is

$$F = 1$$
$$T = 7$$
$$U = 3$$
$$W = 6$$
$$R = 0$$
$$O = 5$$
$$TWO = 765$$
$$FOUR = 1530$$
$$TWO + TWO = 765 + 765 = 1530 = FOUR$$