Mr. Phiphat Chomchit 630631028

**Homework: Search**

1. 3 Jugs
   "You have three jugs, measuring 12 gallons, 8 gallons, and 3 gallons, and a water faucet. You can fill the jugs up or empty them out onto the ground, or pour from one jug ($A$) to another ($B$) until (1) the target jug ($B$) is full, or (2) the source jug is empty ($A$). You need to measure out exactly one gallon."

   - States: Water level in 3 jugs. Let jugs be $J_{12}, J_8, J_3$. Variables are $W_{12}, W_8, W_3$ water level of each jugs s.t.
     $0 \le W_{12} \le 12, 0 \le W_8 \le 8, 0 \le W_3 \le 3$.
     State is $W = \{W_12, W_8, W_3\}$

   - Actions:
     - $Fill(J_x)$: fill jux x from the faucet.
       **Precondition :** $J_x$ is not full.
     - $Empty(J_x)$ empty jux x.
       **Precondition :** $J_x$ is not empty.
     - $Pour(J_x, J_y)$: pour water from jug x to jug y.
       **Precondition :** $J_x$ is not empty and $J_y$ is not full.

   - Transition Model:
     - $Fill(J_x) : W_x = x$.
     - $Empty(J_x) : W_x = 0$.
     - $Pour(J_x, J_y) :$
       $if(y - W_y > W_x)\{$
       $W_y = W_x + W_y$
       $W_x = 0$
       $\}else\{$
       $W_y = y$
       $W_x = W_x - (y - W_y)\}$

   - Goal Test:
     - $SUM(W) == 1$
     - $W_{12} == 1 \quad OR \quad W_8 == 1 \quad OR \quad W_3 == 1$

   - Step cost:
     - 1 per action.

- Path cost:
  - Sum of step costs in the path.
- Answers (Iterative Deepening Depth First Search):
  - Answers 1:
    $$\{0,0,0\} \xrightarrow{Full(J_{12})} \{0,0,12\} \xrightarrow{Pour(J_{12},J_8)} \{0,8,4\} \xrightarrow{Pour(J_{12},J_3)}$$
    $$\{3,8,1\} \xrightarrow{Empty(J_8)} \{3,0,1\} \xrightarrow{Empty(J_3)} \{0,0,1\}$$

  - Answers 2:
    $$\{0,0,0\} \xrightarrow{Full(J_{12})} \{0,0,12\} \xrightarrow{Pour(J_{12},J_3)} \{3,0,9\} \xrightarrow{Pour(J_{12},J_8)}$$
    $$\{3,8,1\} \xrightarrow{Empty(J_8)} \{3,0,1\} \xrightarrow{Empty(J_3)} \{0,0,1\}$$

  - Answers 3:
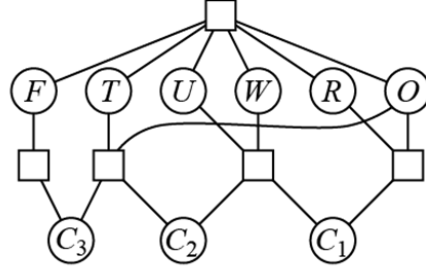    $$\{0,0,0\} \xrightarrow{Full(J_{12})} \{0,0,12\} \xrightarrow{Pour(J_{12},J_8)} \{0,8,4\} \xrightarrow{Empty(J_8)} \{0,0,4\} \xrightarrow{Pour(J_{12},J_3)}$$
    $$\{3,0,1\} \xrightarrow{Empty(J_3)} \{0,0,1\}$$

  - Answers 4:
    $$\{0,0,0\} \xrightarrow{Full(J_{12})} \{0,0,12\} \xrightarrow{Pour(J_{12},J_3)} \{3,0,9\} \xrightarrow{Empty(J_3)} \{0,0,9\} \xrightarrow{Pour(J_{12},J_8)}$$
    $$\{0,8,1\} \xrightarrow{Empty(J_8)} \{0,0,1\}$$

2. Cryptarithmetic



$$
\begin{array}{r}
T\ W\ O \\
+\ T\ W\ O \\
\hline
F\ O\ U\ R
\end{array}
$$

(a)                                                          (b)

Problem is shown on $(a)$. Each letter is a digit. No two letters can have the same digit. Hypergraph $(b)$ shows constraints between variables, where $C1, C2$ and $C3$ are carries, and can have value between 0 and 1. Find the correct digit for each letter.

- Variables : $F, T, U, W, R, O, C_1, C_2, C_3$
- Domains:
    - $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ for $F, T, U, W, R$, and $O$
    - $0, 1$ for carry variables $C_1, C_2$ and $C_3$
- Constranints: $Alldiff(F, T, U, W, R, O)$
    - $O + O = R + 10C_1$
    - $C_1 + W + W = U + 10C_2$
    - $C_2 + T + T = O + 10C_3$
    - $C_3 = F, T \neq 0, F \neq 0$
- Answer:
    - ■ Find singleton domain :
      From $C_3 = F$ and $F \neq 0$ and $C_3 \in \{0, 1\} \Rightarrow F = 1$
    - ■ Minimum Remaining Values :
      From $C_1, C_2 \in \{0, 1\}$, We assigned $C_1 = C_2 = 1$, so

$$
O + O = R + 10 \Rightarrow O \geq 5
$$
$$
W + W = U + 9 \Rightarrow W \geq 5, U \geq 3
$$
$$
T + T = O + 9 \Rightarrow T \geq 5
$$

    - ■ Minimum Remaining Values :
      From $O, W, T \in \{5, 6, 7, 8, 9\}$ and $U \in \{3, 4, 5, 6, 7, 8, 9\}$ and $R \in \{0, 1, \cdots, 8, 9\}$. We assigned $O = 5$, so $T = 7$ and $R = 0$

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- Minimum Remaining Values :
  From $W \in \{6, 8, 9\}$ and $U \in \{3, 4, 6, 8, 9\}$. We assigned $W = 6$ so $U = 3$
- Check Answer: Our answer is

$$F = 1$$
$$T = 7$$
$$U = 3$$
$$W = 6$$
$$R = 0$$
$$O = 5$$
$$TWO = 765$$
$$FOUR = 1530$$
$$TWO + TWO = 765 + 765 = 1530 = FOUR$$