Test 2 Superdense Coding

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1. This homework needs your understanding of basis change/rotation in linear algebra. The application is to use a qubit to encode classical bits. Initially, Alice and Bob share an EPR pair and there is a quantum channel from Alice to Bob. Alice wants to send classical bits to Bob. Superdense coding is a protocol where Alice sends one qubit to Bob, but the amount of transmitted information is two classical bits. This is done as follows.

   Alice has two classical bits, b_1 and b_2, and Alice shares an EPR pair with Bob. Recall that the EPR pair is

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

   We assume that the first qubit belongs to Alice and the second Bob. Depending on which pair of classical bits Alice wants to send, the operations are as follows.

b_1b_2	Alice's Operation	State after Alice's Operation
00	Nothing	$\frac{1}{\sqrt{2}}\|00\rangle + \frac{1}{\sqrt{2}}\|11\rangle$
01	X gate	$\frac{1}{\sqrt{2}}\|10\rangle + \frac{1}{\sqrt{2}}\|01\rangle$
10	Z gate	$\frac{1}{\sqrt{2}}\|00\rangle - \frac{1}{\sqrt{2}}\|11\rangle$
11	XZ gate	$\frac{1}{\sqrt{2}}\|10\rangle - \frac{1}{\sqrt{2}}\|01\rangle$

   Now Alice passes her qubit through the quantum channel to Bob. Now Bob has access to both qubits and applies CNOT gate to the first (control) and second (target) qubits, then applies H gate to the first qubit, and lastly measures both qubits to obtain the classical bits Alice wants to send.

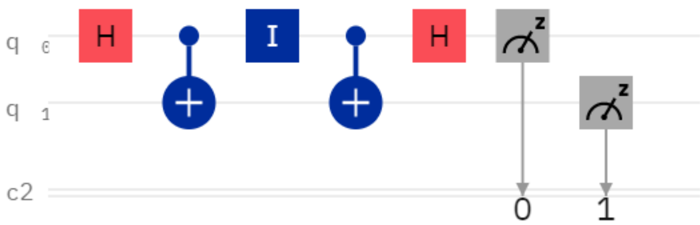
   Your job is to implement this superdense coding quantum circuit and demonstrates it works for each case of 2 classical bits.

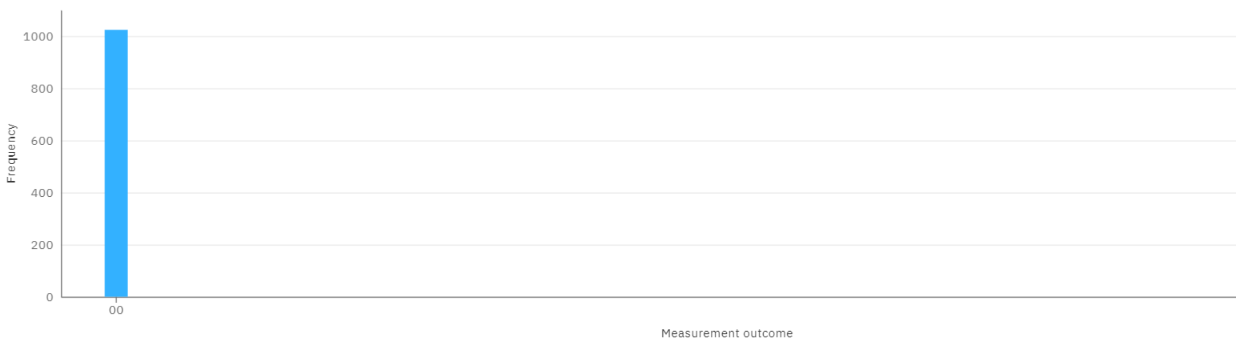
Ans.

1. Nothing

```
OPENQASM 2.0;
include "qelib1.inc";
//initial qubits
qreg q[2];
creg c[2];

//entangle state
h q[0];
cx q[0],q[1];

//encoding
id q[0];

//decoding
cx q[0],q[1];
h q[0];
measure q[0] -> c[0];
measure q[1] -> c[1];
```

Listing 1: Superdense Coding (Nothing)

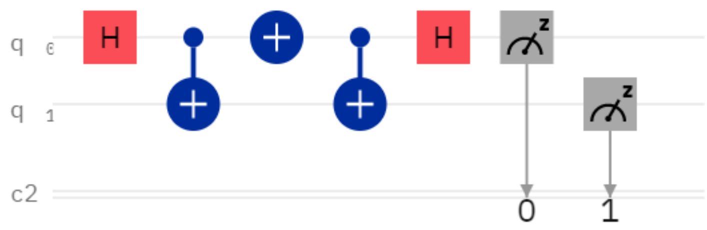


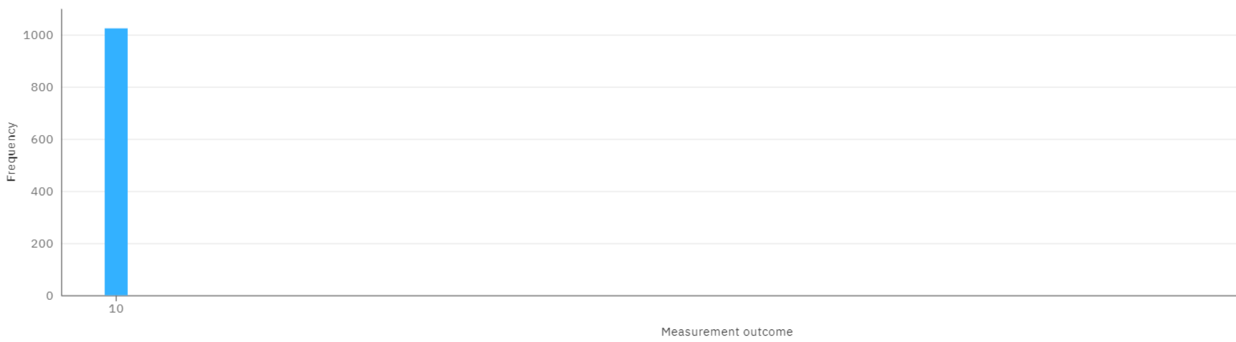


2. X gate

```
OPENQASM 2.0;
include "qelib1.inc";
//initial qubits
qreg q[2];
creg c[2];

//entangle state
h q[0];
cx q[0],q[1];

//encoding
x q[0];

//decoding
cx q[0],q[1];
h q[0];
measure q[0] -> c[0];
measure q[1] -> c[1];

```

Listing 2: Superdense Coding (X-gate)

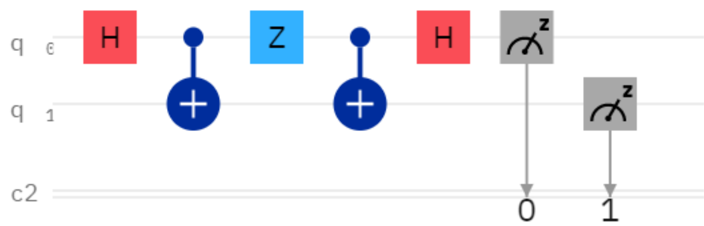


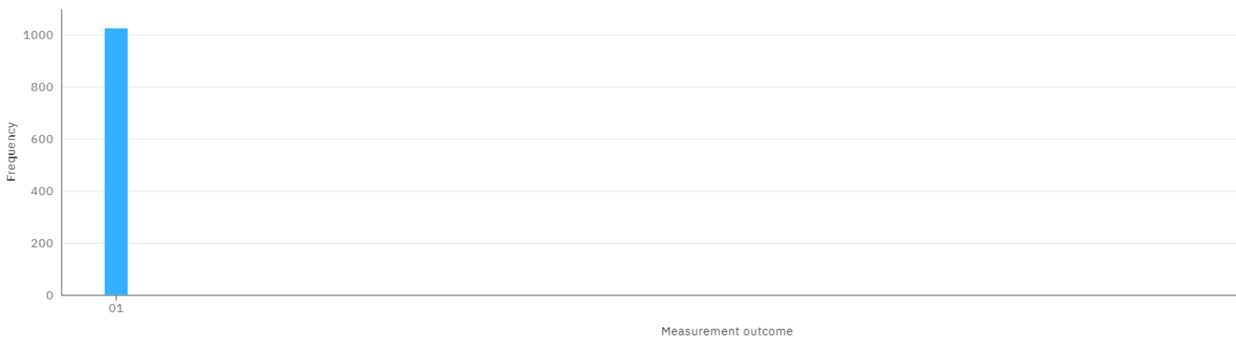


3. Z gate

```
OPENQASM 2.0;
include "qelib1.inc";
//initial qubits
qreg q[2];
creg c[2];

//entangle state
h q[0];
cx q[0],q[1];

//encoding
z q[0];

//decoding
cx q[0],q[1];
h q[0];
measure q[0] -> c[0];
measure q[1] -> c[1];

```

Listing 3: Superdense Coding (Z-gate)





4. XZ gate

```
OPENQASM 2.0;
include "qelib1.inc";
//initial qubits
qreg q[2];
creg c[2];

//entangle state
h q[0];
cx q[0],q[1];

//encoding
x q[0];
z q[0];

//decoding
cx q[0],q[1];
h q[0];
measure q[0] -> c[0];
measure q[1] -> c[1];
```

Listing 4: Superdense Coding (XZ-gate)