Test 2 Entanglement

Mr. Phiphat Chomchit 630631028

1. Suppose you have 5 qubits in the state $|\Psi\rangle = \frac{1}{\sqrt{2}}|01010\rangle + \frac{1}{\sqrt{2}}|10101\rangle$. Is state $|\Psi\rangle$ entangled? Why?

Ans:

- Measurement results of entangled qubits are correlated.

$$|B_{00000}\rangle = \frac{1}{\sqrt{2}}|01010\rangle + \frac{1}{\sqrt{2}}|10101\rangle \implies \begin{cases} P(0) = \left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2} \\ P(1) = \left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2} \end{cases}$$

if we measure first qubit and get outcome 0. Calculate the post-measurement state of the qubit pair by removing the term which is no-consistent with the measurement result.

$$|B_{00000}\rangle = \frac{1}{\sqrt{2}}|01010\rangle + \cancel{\frac{1}{\sqrt{2}}|10101\rangle} \implies \frac{\frac{1}{\sqrt{2}}|01010\rangle}{\sqrt{\left|\frac{1}{\sqrt{2}}\right|^2}} = |01010\rangle$$

- Partial and simultaneous measurements give the same outcome.

$$P(01010) = \left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2} \tag{1}$$

$$P(10101) = \left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2} \tag{2}$$

- The state of the entangled qubits cannot be written as the product of the signle-qubit states.

  Proof by contradiction.

  Let $|\Psi\rangle$ can be written as the product of the single-qubit states and

$$|S_i\rangle = a_i|0\rangle + b_i|1\rangle$$

  for $i = 1, 2, 3, 4, 5$

  So, the product of single-qubit states is

$$|S\rangle = |S_1\rangle|S_2\rangle|S_3\rangle|S_4\rangle|S_5\rangle$$

From "$|\Psi\rangle$ can be written as the product of the single-qubit states". So

$$|\Psi\rangle = |S\rangle \tag{3}$$

$$\frac{1}{\sqrt{2}}|01010\rangle + \frac{1}{\sqrt{2}}|10101\rangle = |S\rangle \tag{4}$$

This implies $a_1b_2a_3b_4a_5 = b_1a_2b_3a_4b_5 = \frac{1}{\sqrt{2}}$, otherwise 0.
Consider $a_1a_2a_3a_4a_5 = 0$ That is $a_i = 0$ for some $i \in \{1, 2, 3, 4, 5\}$
case: $a_i = 0$ when i is odd number. Then $a_1b_2a_3b_4a_5 = 0$.
case: $a_i = 0$ when i is even number. Then $b_1a_2b_3a_4b_5 = 0$.
Contradiction, therefore $|\Psi\rangle$ cannot be written as the product of the single-qubit states

2. Write a program to generate $|\Psi\rangle$, run it 1024 times, and come up with an example how to use it in quantum communication.

```
OPENQASM 2.0;
include "qelib1.inc";

qreg q[5];
creg c[5];

h q[0];
x q[1];
x q[3];
cx q[0],q[1];
cx q[0],q[2];
cx q[0],q[3];
cx q[0],q[4];
measure q[0] -> c[0];
measure q[1] -> c[1];
measure q[2] -> c[2];
measure q[3] -> c[3];
measure q[4] -> c[4];
```

Listing 1: IBM Q code

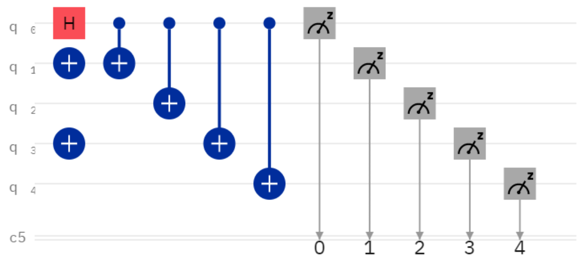

Figure 1: quantum circuit

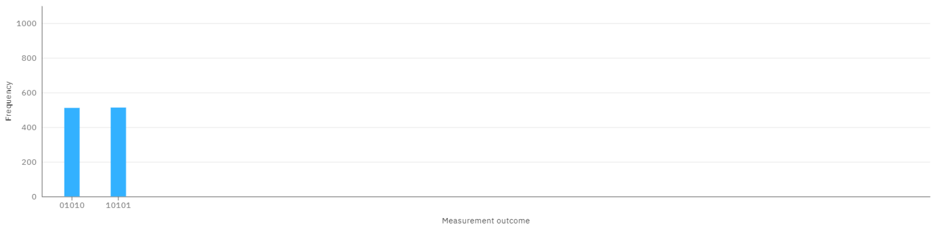

Figure 2: result