$X \tau X \tau topologia$
$, X \in \tau$
$\tau \forall A_1, A_2 \in \tau A_1 \cap A_2 \in \tau$
$\tau \forall \mathcal{B} \subseteq \tau \bigcup_{A \in \mathcal{B}} A \in \tau$
$\tau aperti \tau$
$X \neq \tau (X, \tau) spazio\ topologico$
$Y \subseteq X chiuso Y$
X
$F_\tau X$
$, X \in F_\tau$
F_τ
F_τ
$X \neq$
$\tau_B = \{, X\}$
$\tau_D = (X)$
$\tau \tau' X \tau pi\ fine \tau' \tau \supset \tau' \tau' \tau \tau \tau'$
$X \neq [d] X \times X$
$\forall x, y \in X : d(x, y) \geq 0 (d(x, y) = 0 \iff x = y)$
$d(x, y) = d(y, x)$
$d(x, z) \leq d(x, y) + d(y, z)$
$d distanza (X, d) spazio\ metrico$
$^n d_E$
$X d_D = \{ 0\ x = y$
$1 x \neq y$
$(X, d) palla\ aperta\ centrata\ in\ x\ di\ raggio\ r$

$(X, d) \tau_d$

$\tau_d X topologia\ indotta$
$\in \tau_d X \in \tau_d X$
$A_1, \ldots, A_n X A_k x A_k x \in A_k B^k(x) \tau_d A_k \tau_d$
$x \in A_j x A_j \exists [r_i] x \subseteq A_i \subseteq A_j$
$(X, \tau) metrizzabile d \tau = \tau_d d$
$(^n, d_E)(^n, \tau_E)$
$(X, d_D) d_D \tau_D [1/2] x = \{x\}$
$(X, d)(X, \tau_d)$
$A \in \tau_d$
$y \in [r] x r' [r'] x \subseteq [r] x r' < r - d(x, y)$

$A A x x A A$
$(X, d)(X, d') topologicamente\ equivalenti$
$\mathrm{B}^E_r(x) \mathrm{B}_r(x) d(x, y) \sqrt{x - y x - y}$

$d_E \mathrm{B}_r(0)$

$r' r''$

$x \in (X, \tau) intorno x \tau U_x A x \in A \subseteq U_x$
$(X, \tau) base\ di\ aperti \mathcal{B} \tau \mathcal{B}$
$base\ di\ intorni x x$

$A^1 U_x A'_{x \in A} U_x A' A A A = A'$
$(X, \tau) \chi \subseteq X \tau_\chi [A \cap \chi] A \in \tau \chi$
$\cap \chi = \chi X \cap \chi = \chi$
$a_1, \ldots, a_n \in \tau_\chi \forall k \exists A_k \in \tau : a_k = A_k \cap \chi$

$A_k (X, \tau)$
$a_{k k \in I} \subseteq \tau_\chi A_k$

$(\chi, \tau_\chi) sottospazio (X, \tau)$
$A \iff A$
A
$A A$
$chiusura Y$

$Y Y$

$Y Y = Y \cap C$
$parte\ interna Y Y$

$Y Y$

$x_1, x_2, x_3, \ldots, x_k x_j R^N$
$XX[R]TXT_2{}^0$
X
$R^n g_0 X \hookrightarrow R^n X g_x = g_0|_{T_x X} x T_x X$
$R^N \mathcal{E}[XY] f\, liscia XY \mathcal{E} X \subseteq R^N Y = R^N U_{r,K,\epsilon} r \in N K \subseteq X \epsilon > 0$

k
$K_i X U_{r,K_i,\epsilon} U_{r,\epsilon} K[f] XY \mathcal{E}$
$U_{r,K,\epsilon,(U,\phi),(U',\phi')} = [g] X R^N g(U) \subseteq U' \phi' \circ g \circ \phi^{-1} \in U_{r,K,\epsilon}(\phi' \circ g \circ \phi^{-1}) \phi \circ g \circ \phi^{-1} R^N R^N K_i U_{r,K_i,(U_i,\psi_i)(U'_i,\psi'_j)} U_{r,\epsilon,(U}$
$XY \mathcal{E}(X,Y) = \{XY lisce\} X$
$(X,Y) = \{XY immersioni\}$
$\forall x \in X\ [Df_x] T_x X T_{f(x)} Y$
$xxf(x)$
$(X,Y) = \{XY embedding\}$
Xf
$(X,Y) = \{XY diffeomorfismi\}$
$\mathcal{E}(X,Y)$

gfg
$g_n f x_n y_n g_n(x_n) = g_n(y_n) X x_n x_0 y_n y_0 x_0 \neq y_0 f f(x_0) = f(y_0) f x_0 = y_0 x_0$
$x_0 x_0 x_n y_n{}^n g_n(y_n) - g_n(x_n) = 0 \exists z_n t.c._{z_n} g_n(y_n - x_n) = 0 z_n x_0 z_n \in [x_n, y_n]$
$v_n \frac{y_n - x_n}{\|y_n - x_n\|} \in S^{n-1} S^n v_n v_0 \in S^{n-1}{}_{x_0} f(v_0) = 0 f$
gfg
$gg(X) \subseteq Y X g(X) Y Y \subseteq^n$
$XY g(x) = Y g$
$\mathcal{B}\mathcal{B}'^n M^{\mathcal{B}}_{\mathcal{B}'} \det M^{\mathcal{B}}_{\mathcal{B}'} > 0$
n
$\det M^{\mathcal{B}}_{\mathcal{B}} = \det = 1$
$\det M^{\mathcal{B}}_{\mathcal{B}''} = \det(M^{\mathcal{B}}_{\mathcal{B}'} M^{\mathcal{B}'}_{\mathcal{B}''}) = \det M^{\mathcal{B}}_{\mathcal{B}'} \cdot \det M^{\mathcal{B}'}_{\mathcal{B}''}$
$(\det M^{\mathcal{B}}_{\mathcal{B}'}) = (\det M^{\mathcal{B}'}_{\mathcal{B}})$
$orientazione^n$
$XA = \{U_j, \phi_j\} X\, atlante\, orientato \{[\det \lambda_{ji}] U_i \cap U_j \backslash \{0\}\}^+$
$compatibili$
$orientazione\, orientabile\, non\ orientabile$

$(W, V_0, V_1) triade W W = V_0 V_1$
$W V_0 V_1$
$V_0 V_1 W V_0 = V_1 =$
$X_0 X_1 n X_0\, cobordante X_1 X_0 X_1$

${}_0 X_1 \exists W, V_0, V_1, f_0, f_1 : \{\ W = V_0 V_1$
$[f_0] X_0 V_0 diffeomorfismo$
$[f_1] X_1 V_1 diffeomorfismo$

2
$V 2 \dim(V) = n U H 101$
$10H$
$(v, v) = 0, \forall v \in V$
V
$V = U \perp U \perp \ldots \perp U V n U \dim(V) = n$
$V = H \perp H \perp \ldots \perp H V n H \dim(V) = 2n$
$V = (H \perp H \perp \ldots \perp H) \perp (U \perp U \perp \ldots \perp U)$
$U \perp H \simeq 3U U \perp HU$
$U \perp H = [100001010] W [1001] u, v, z U \perp H W = (u+v, u+z) W u + v u + z V = W \perp U U$
V
$X V = \eta_1(X)[f] S_1 S_2 (\eta_1(S_1), \beta_1)(\eta_1(S_2), \beta_2)$
$X(\eta(S), \beta)$
$\eta(S^2)$
$\eta(S^2) \dim \eta(S^2) = 0 [f] S^1 S^2 \eta(S^2) S^2 f S^2 / f(S^1) f [f] S^1 S^2 [p] S^{1n} p [p \circ f] S^{1n} P \in f(S^1 0)^2 [r_t] x t x P$

P

$f : S^1 \to S^2$

$\eta(T_2) = H$

a

b

$\eta(S^1 \times S^1 = T_2) \eta(T_2) = 2 \times 2 H = 01$
$10 \eta \beta H \eta(S^1$

a

b b

a

$(x, y) \sim (x', y') \Leftrightarrow \exists (n, m) \in^2 (x', y') - (x, y) = (n, m)$
$^{222} \Leftarrow S^1 \times S^1 \subseteq C \times C (x, y) \Leftarrow (e^{2\pi i x}, e^{2\pi i y})(p, q) \in^2 pq$