TurbOPark

A Turbulence Optimized Park model

Prepared Nicolai Gayle Nygaard, January 2022
Checked Jesper Grønnegaard Pedersen, January 2022
Accepted Sidse Damgaard Hansen, January 2022
Approved Peter Krastins, January 2022

Disclaimer: This note provides a brief description of the TurbOPark wake model. A full description of the model and its validation will be provided elsewhere. At the time of writing the model has only been validated for offshore use, and in combination with a blockage and flow model.

1 Model input and output variables

Symbol	Variable name in code	Description
Model inputs		
u_0	`u0`	(vector of length `n_u0`) Free wind speeds to calculate waked wind speeds and turbine power for
ϕ	`direction`	(scalar) Direction of free wind
η_i	`u_corr`	(vector of length `n_WTG`) Correction factor relating the freestream wind speed at the reference height and location to the freestream wind speed at the hub height and location of each turbine
X	`x_utm`	(vector of length `n_WTG`) x-coordinates of the turbines in UTM coordinates
Y	`y_utm`	(vector of length `n_WTG`) y-coordinates of the turbines in UTM coordinates
$z_{H,i}$	`hub_height`	(vector of length `n_WTG`) Hub height of turbines
$P_i(u)$, $C_{T,i}(u)$, D_i	`power_curve`	Array of structs, one for each unique turbine type in the calculation. Each struct contains the turbine power curve $P_i(u)$, thrust curve $C_{T,i}(u)$, and rotor diameter D_i
-	`power_curve_index`	(vector of integers of length `n_WTG`) Index for each turbine pointing to the relevant entry in the `power_curve` array
I_0	`ti0`	(vector of length `n_u0` or scalar) Ambient turbulence intensity as a decimal number
A	`A`	(scalar - optional) Wake expansion calibration parameter

σ_{max}^{rel}	`sigma_max_rel`	(scalar - optional) Factor defining an effective wake width
Model outputs		
P	`power`	(matrix of size `[n_WTG, n_u0]`) Power output for each turbine accounting for wake effects
u_w	`u`	(matrix of size `[n_WTG, n_u0]`) Wind speed at each turbine position accounting for wake effects

2 Ordering the turbines

For a given wind direction ϕ the Cartesian turbine coordinates (X, Y) are rotated to a new set of coordinates (x, y), where the x-axis is aligned with the wind direction, such that the wind vector is pointing along the positive x-axis:

$$[x, y] = \begin{bmatrix} -\sin\phi & -\cos\phi \\ \cos\phi & -\sin\phi \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} \tag{1}$$

The turbines are then sorted from most upwind (lowest x value) to most downwind (largest x value). The indexing used below assumes this ordering, hence $x_i \geq x_j$ for $i > j$. By definition turbine 1 at (x_1, y_1) is unwaked since no turbines are upstream of it.

3 Freestream wind speed scaling

The freestream wind speed at the reference location and height, $\boldsymbol{x}_0 = (x_0, y_0, z_0)$, is u_0. The reference position could for example be specifying the location and height of wind measurements obtained before the wind farm was constructed. The unwaked wind speed at the location and hub height of turbine i is

$$u_{0i} = u_{0i}(\boldsymbol{x}_i, \boldsymbol{x}_0, \boldsymbol{\theta}) = \eta_i(\boldsymbol{x}_i, \boldsymbol{x}_0, \boldsymbol{\theta}) u_0 = \eta_i u_0 \tag{2}$$

Here η_i is a wind speed scaling factor from the reference position, where u_0 is known, to the position of turbine i. It depends on the location and hub height of the turbine, $\boldsymbol{x}_i = (x_i, y_i, z_{H,i})$, relative to $\boldsymbol{x}_0$. The wind speed scaling contains $\boldsymbol{\theta}$, a vector of parameters which can include wind direction, shear coefficient, and other variables that can influence the wind speed variation across the 3D volume enclosing the turbines and the reference position. In the provided examples the wind speed correction factor includes a horizontal gradient and a shear-correction adjusting the freestream wind speed from the reference height to the turbine hub heights. The same wind speed correction factor can be used to couple the wake model to a blockage model as described in [1].

4 Wind speed deficit

The wind speed deficit is defined as

$$\frac{\Delta u_i(\boldsymbol{x})}{u_0(\boldsymbol{x})} = \frac{u_0(\boldsymbol{x}) - u_{w,i}(\boldsymbol{x})}{u_0(\boldsymbol{x})} \tag{3}$$

The wind speed in the wake of turbine i is $u_{w,i}(\boldsymbol{x})$. Note that in this formulation the background flow velocity $u_0(\boldsymbol{x}) = \eta(\boldsymbol{x})u_0$ is explicitly dependent on the position $\boldsymbol{x} = (x, y, z)$.

TurbOPark uses the Gaussian wind speed deficit model by Bastankhah and Porté-Agel [2]. Introducing the downstream distance $\hat{x}_i(x) = x - x_i$ this is written as

$$\frac{\Delta u_i(\boldsymbol{x})}{u_0(\boldsymbol{x})} = C_i(\hat{x}_i) \exp\left(-\frac{r^2}{2\sigma_{w,i}^2(\hat{x}_i)}\right) \tag{4}$$

Here $\sigma_{w,i}(\hat{x}_i)$ is the characteristic wake width, which describes how fast the wind speed deficit decreases from its peak value $C_i(\hat{x}_i)$ towards zero as one moves radially from the wake centreline. The radial distance from the wake centreline is given by

$$r = \sqrt{(y - y_i)^2 + \left(z - z_{H,i}\right)^2} \tag{5}$$

Note that the deficit is only defined for $\hat{x}_i > 0$. The peak wind speed deficit is derived using momentum conservation [2]:

$$C_i(\hat{x}_i) = 1 - \sqrt{1 - \frac{C_{T,i}(u_{in,i})}{8(\sigma_{w,i}(\hat{x}_i)/D_i)^2}} \tag{6}$$

Here $u_{in,i}$ is the inflow wind speed at turbine i, which has a rotor diameter of D_i. If the turbine is not waked, then $u_{in,i} = u_{0i}$, otherwise the inflow wind speed is calculated from the wind speed in the wakes of the upstream turbines as shown in section 5. Due to the expansion of the wake $\sigma_{w,i}(\hat{x}_i)$ increases monotonically with increasing $\hat{x}_i$. The peak velocity deficit $C_i(\hat{x}_i)$ therefore decays with increasing downstream distance and vanishes asymptotically. With the definitions above this means that $u_{w,i}(\boldsymbol{x}) \to u_0(\boldsymbol{x})$ as $\hat{x}_i \to \infty$.

The wake expansion and hence the functional form of $\sigma_{w,i}(x)$ is the unique feature of TurbOPark. In TurbOPark the wake expansion is driven by the combination of the ambient, atmospheric turbulence, characterised by the turbulence intensity I_0, and the turbulence generated in the wake itself. The latter contribution to the turbulence intensity is modelled by the expression proposed by Frandsen [3] and depends on the turbine thrust coefficient and downstream distance. By integrating the wake expansion rate along the downstream path, we arrive at an analytical expression for the characteristic wake width:

$$\frac{\sigma_{w,i}(\hat{x}_i)}{D_i} = \epsilon_i + \frac{AI_0}{\beta}\left(\sqrt{(\alpha + \beta\hat{x}_i/D_i)^2 + 1} - \sqrt{1 + \alpha^2} - \ln\left[\frac{\left(\sqrt{(\alpha + \beta\hat{x}_i/D_i)^2 + 1} + 1\right)\alpha}{\left(\sqrt{1 + \alpha^2} + 1\right)(\alpha + \beta\hat{x}_i/D_i)}\right]\right) \tag{7}$$

Following [2] the initial characteristic wake width (at $\hat{x}_i = 0$) is

$$\sigma_{w,i}(0)/D_i = \epsilon_i = 0.25\left(\frac{1 + \sqrt{1 - C_{T,i}(u_{in,i})}}{2\sqrt{1 - C_{T,i}(u_{in,i})}}\right)^{1/2} \tag{8}$$

And we have defined the constants $\alpha = c_1 I_0$ and $\beta = c_2 I_0/\sqrt{C_{T,i}(u_{in,i})}$. The parameters $c_1 = 1.5$ and $c_2 = 0.8$ are taken from the Frandsen turbulence model [3], and based on validation on data from 19 offshore wind farms the recommended value for the wake expansion calibration parameter is

$$A = 0.04$$

The given model parameters apply for the specific model setup described in this note. Modification of the wake superposition method (see Section 5) or how the ground effect is handled (Section 6) will require a re-calibration of the model.

5 The wake at a downstream turbine

The wind speed deficit at the position of turbine j downstream from turbine i ($x_j > x_i$) is

$$\frac{\Delta u_{ij}(r)}{u_{0j}} = \frac{u_{0j} - u_{w,i}^{j}}{u_{0j}} = C_i(x_{ij}) \exp\left(-\frac{r^2}{2\sigma_{w,i}^2(x_{ij})}\right) \tag{9}$$

The streamwise separation between the two turbines is $x_{ij} = x_j - x_i$, and the cross-stream position is still generic. The impact of the wake from turbine i depends on the extent of its overlap with the rotor of turbine j. To account for this, we use the average of the deficit over the downstream rotor disk:

$$\delta_{ij} \equiv \frac{1}{A_j} \int_{A_j} \frac{\Delta u_{ij}}{u_{0j}} dA = \frac{C_i(x_{ij})}{A_j} \int_{A_j} \exp\left(-\frac{r^2}{2\sigma_{w,i}^2(x_{ij})}\right) dA \tag{10}$$

The integral is over all the (y, z) points inside the rotor disk of turbine j, which has an area of $A_j = \pi D_j^2/4$. We evaluate the integral numerically and have implemented it as a look-up table based on discretized values of the lateral separation between two turbines and the rotor radius of the downstream turbine, both normalized by the characteristic wake width, see Appendix A for more details.

The distance between the rotor centres of the two turbines is

$$r_{ij} = \sqrt{(y_i - y_j)^2 + (z_{H,i} - z_{H,j})^2} \tag{11}$$

If this is large compared with the characteristic wake width $\sigma_{w,i}(x_{ij})$, the rotor-averaged deficit is insignificant. For computational efficiency we do not calculate the wake impact from turbine i on turbine j if $r_{ij} > \sigma_{max}^{rel} \sigma_{w,i}(x_{ij})/2 + D_j/2$.

The rotor-averaged wind speed deficit defines $\bar{u}_{w,i}^{j}$ which is the rotor-averaged wind speed in the wake from turbine i at the position of turbine j:

$$\delta_{ij} = \int_{A_j} \frac{u_{0j} - u_{w,i}^{j}}{u_{0j}} dA = 1 - \frac{\bar{u}_{w,i}^{j}}{u_{0j}} \tag{12}$$

Multiple overlapping wakes are superposed using the method applied by Katić et al. [4] to give the combined wind speed deficit δ_j at turbine j

$$\delta_j = \frac{u_{0j} - \bar{u}_w^j}{u_{0j}} = \sqrt{\sum_{i=1}^{j-1} \delta_{ij}^2} \tag{13}$$

The effective rotor-averaged waked wind speed $\bar{u}_w^j$, taking overlapping wakes from all upstream turbines into account, then defines the inflow wind speed at turbine j

$$u_{in,j} = \bar{u}_w^j = u_{0j}(1 - \delta_j) = \eta_j u_0(1 - \delta_j) \tag{14}$$

This wind speed is used to calculate the waked power of the turbine using the power curve $P_j(u_{in,j})$ and it is used with the thrust curve to calculate the wind speed deficit in the wake originating at this turbine through $C_{T,j}(u_{in,j})$.

6 Ground effect

To indirectly include the effect of the ground plane we use the method of image turbines [5]. At every turbine position an image turbine is added to the array mirroring the turbine through the ground plane. The hub height of the image turbine is the negative of the turbine hub height. These image turbines generate their own wakes, which are merged with all the other wakes in the wind farm using the superposition method.

7 References

[1] N. G. Nygaard, S. T. Steen, L. Poulsen and J. G. Pedersen, "Modelling cluster wakes and wind farm blockage," *Journal of Physics: Conference Series,* vol. 1618, p. 062072, 2020.

[2] M. Bastankhah and F. Porté-Agel, "A new analytical model for wind-turbine wakes," *Renewable Energy,* vol. 70, p. 116, 2014.

[3] S. T. Frandsen, "Risø-R-1188(EN) Turbulence and turbulence generated structural loading in wind turbine clusters," Risø, Roskilde, Denmark, 2007.

[4] I. Katić, J. Højstrup and N. O. Jensen, "A simple model for cluster efficiency," in *Proceedings of the European Wind Energy Association conference and exhibition*, Rome, Italy, 1986.

[5] P. B. S. Lissaman and E. R. Bate, Jr. , "AV FR 7058 Energy effectiveness of wind energy collection systems," AeroVironment Inc., Pasadena, California, 1977.

Appendix A

To calculate the integral in equation (10) we first change from cartesian to polar coordinates. Using a polar coordinate system with the first axis aligned with the line between the centre of the wake and the centre of the downstream turbine (see Figure 1), the integral can be written as

$$\frac{1}{A_j}\int_{A_j} \exp\left(-\frac{r^2}{2\sigma_{w,i}^2(x_{ij})}\right) dA = \frac{1}{A_j}\int_0^{2\pi}\int_0^{R_j} \exp\left(-\frac{\rho^2 + r_{ij}^2 - 2\rho r_{ij}\cos\theta}{2\sigma_{w,i}^2(x_{ij})}\right)\rho\, d\rho d\theta$$

Here we have defined $R_j \equiv D_j/2$.

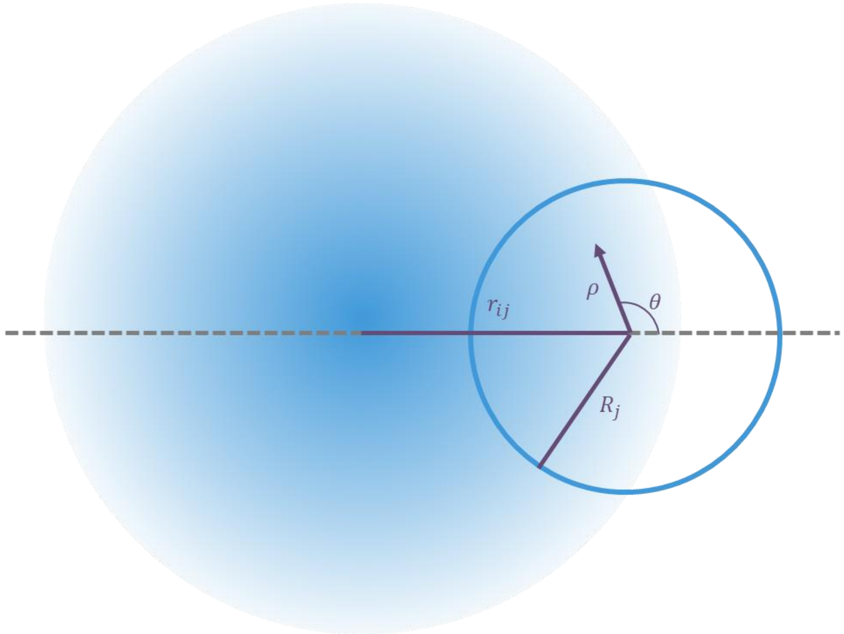

Figure 1: Illustration of how the polar coordinate system is chosen. The fading circle illustrates the gaussian wake from turbine i, and the smaller circle the downstream rotor area of turbine j. Note that the centre of the downstream rotor may be above or below the wake centre in the cartesian coordinate system.

Finally, all lengths are normalized with the characteristic wake width $\sigma_{w,i}(x_{ij})$, giving $\bar{R}_j = \frac{R_j}{\sigma_{w,i}(x_{ij})}$, $\bar{r}_{ij} = \frac{r_{ij}}{\sigma_{w,i}(x_{ij})}$, and $\bar{\rho} = \frac{\rho}{\sigma_{w,i}(x_{ij})}$. This yields the following integral

$$\frac{1}{\pi\bar{R}_j{}^2}\int_0^{2\pi}\int_0^{\bar{R}_j} \exp\left(-\frac{\bar{\rho}^2 + \bar{r}_{ij}^2 - 2\bar{\rho}\bar{r}_{ij}cos\theta}{2}\right)\bar{\rho}\, d\bar{\rho} d\theta$$

This integral is then solved numerically to create a look-up-table covering all relevant combinations of $\bar{R}_j$ and $\bar{r}_{ij}$.